SFT-2021
Lectures on Statistical Field Theories
Galileo Galilei Institute
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Program of lectures

Alexander Altland ((University of Cologne)
The Sachdev-Ye-Kitaev model. (6h)

Since its introduction in 2015, the SYK model has become a paradigm bridging between different fields, notably the physics of strong correlations, quantum chaos, and the physics of holographic quantum matter. Defined in terms of a deceptively simple model Hamiltonian, the model displays a plethora of phenomena characteristic for strongly interacting many body systems, including exponential instabilities, non-Fermi liquid behavior, strong spectral correlations, and many body localization. At the same time, it is simple enough to be amenable to highly resolved numerical diagnostics and analytic approaches. It is perhaps the only system where parameter free comparison between analytic theories and numerical data is possible in a massively correlated context. These lectures will introduce the SYK model and discuss its applications in condensed matter physics, quantum chaos, and localization theory.

Prerequisites: Basic concepts of many body theory (second quantization, path integrals, a little bit of diagrammatic perturbation theory). Although not strictly required, familiarity with elementary concepts of quantum chaos (basics of random matrix theory, Anderson localization at the level of, e.g. sections IB and IIIA of the ancient lecture notes, available here, will be helpful in putting the story into a larger context.

Olalla Castro-Alvaredo (City University of London)
Entanglement in Quantum Field Theory: An Introduction to Branch Point Twist Fields and their Applications. (6h)

In this course I will introduce branch point twist fields and explain how they emerge in the context of computing entanglement measures. I will focus mainly on massive 1+1D integrable quantum field theory.

In Lecture 1 I will introduce some common entanglement measures such as the entanglement entropy and explain their main features in 1+1D theories, both at and away from criticality. I will then explain how the same measures and features can be derived from branch point twist field correlators.

In Lecture 2 I will focus on the technicalities of the computation of branch point twist field correlators, focussing on massive 1+1D integrable quantum field theory. I will introduce the form factor programme and its generalization to branch point twist fields. I will explain how the form factor equations resulting from this programme can be solved.

In Lecture 3 I will discuss two or three (time permitting) applications of the ideas discussed in Lectures 1 and 2. All my examples will use form factor technology. I will discuss how form factors can help us obtain massive corrections to the entanglement entropy of a large subsystem, how they can allow us to say something about the time-dependence of the entanglement entropy in an out-of-equilibrium situation, and finally, how the solutions to the form factor equations are not always unique and what this means.

Bibliography: Troughout my course I will refer to some of my own work and you can find all my publications at https://olallacastroalvaredo.weebly.com/publications.html, A lot of what I will discuss comes from paper [28] and the subsequent review article [24].
Beyond these papers my course will assume a certain basic knowledge of integrable quantum field theory and conformal field theory. A paper that covers a lot of the fundamental ideas I usually take for granted is the classic: https://projecteuclid.org/euclid.aspm/1529259764
Finally, since a lot of my course is about form factors and integrability, anyone wanting to explore this further can learn much from the book: https://global.oup.com/academic/product/statistical-field-theory-9780198788102?cc=gb&lang=en&

Filippo Colomo (INFN, Florence)
Limit-shape phenomena in statistical mechanics. (6h)

Standard arguments of statistical mechanics ensure that bulk properties of physical systems should not depend on boundary conditions. A prototypical counterexample is offered by dimer models, where the presence of geometric constraints can induce long range order effects. Consequently, suitable boundary condition may modify thermodynamics, with order parameters and free energy density acquiring spatial dependence, and the possible emergence of spatial phase separation and limit shapes. Such phenomena are by now clearly understood in dimer models, but much remains to be done for an interactive exactly solvable extension thereof, the six-vertex model. In these lectures, we aim at providing an introductory (and very partial) picture of the state of the art in the field.

The course is introductory, with no particular prerequisite. However, a basic knowledges on random matrix model, and on quantum integrability could help.

Bibliography:
- Richard Kenyon, *"Introduction to the dimer model"*, available here .
- Jean-Marie Stéphan, *"Extreme boundary conditions and random tilings"*, arXiv:2003.06339
- K. Palamarchuk and N. Reshetikhin, *"The six-vertex model with fixed boundary conditions"*, *arXiv:1010.5011*

Balázs Pozsgay (Budapest University of Technology and Economics)
Introduction to the Bethe Ansatz. (10h)

In this lecture course we will introduce the Bethe Ansatz, one of the central methods used to solve one dimensional integrable models. We will present the solution of integrable spin chains (such as the famous Heisenberg spin chain and its generalizations), but we will also treat the Lieb-Liniger model (1D delta-function interacting Bose gas) in passing, leaving many details regarding this model to the separate course by Andrea Trombettoni. We will focus on the coordinate Bethe Ansatz solution of the models, and discuss general properties of the Bethe Ansatz wave function, such as completely elastic and factorized scattering of the particles. We will discuss the finite volume spectrum, and also the thermodynamic limit of these models. Short discussion of correlation functions will also be given, together with a brief introduction to the so-called nested Bethe Ansatz, which is used to solve multi-component systems. If time permits, we will also discuss the algebraic formulation of the method.

Andrea Trombettoni (University of Trieste)

Lieb-Lininger model for the one-dimensional Bose gas (6h)

In the lecture course I will discuss properties of one-dimensional (1D) Bose gases as described by the Lieb-Liniger (LL) model and its extensions. After discussing the reduction from the full 3D system to the 1D limit, I will present equilibrium properties of the LL Hamiltonian, applying to this model results and concepts from the course by Balázs Pozsgay. The many-body Bethe Ansatz wave functions, the LL integral equations and the Yang-Yang thermodynamics are discussed together with an analysis of the weak- and strongly-interacting limits. Finally a discussion of phase fluctuations in 1D Bose gases and selected dynamical problems will be presented, and put in connection with experimental setups.