Introduction to CFT and Conformal Bootstrap

Andreas Stergiou, King's College London

These lectures aim to provide a self-contained introduction to the modern conformal bootstrap method. The study of conformal field theory (CFT) will first be motivated through a description/reminder of its relevance for critical phenomena. Ideas of the renormalization group (RG) will be reviewed, and the "old" way of studying CFTs as endpoints of RG flows will be explained, with an emphasis on the ε expansion method. The set of ideas necessary to understand the conformal bootstrap method will then be introduced, and both analytic and numerical implementations of the conformal bootstrap method will be discussed.

Day 1: General remarks, motivation for the study of CFTs, phase transitions, critical exponents, renormalization group, ε expansion.

Day 2: Conformal transformations, conformal group/algebra, operators, correlation functions.

Day 3: Radial quantization, unitarity, operator product expansion, conformal blocks.

Day 4: Crossing symmetry, conformal bootstrap basics, analytic bootstrap.

Day 5: Numerical bootstrap, global symmetries, mixed correlators.

Suggested reading:

1. David Tong's lectures on Statistical Field Theory (http://www.damtp.cam.ac.uk/user/tong/sft/sft.pdf),
2. Slava Rychkov's EPFL Lectures on Conformal Field Theory in $D \geq 3$ Dimensions (arXiv:1601.05000),
3. David Simmons-Duffin's TASI Lectures on the Conformal Bootstrap (arXiv:1602.07982),
4. Shai Chester's Weizmann Lectures on the Numerical Conformal Bootstrap (arXiv:1907.05147).

Review article:

David Poland, Slava Rychkov, Alessandro Vichi, "The Conformal Bootstrap: Theory, Numerical Techniques, and Applications", Rev. Mod. Phys. 91 015002 (2019), arXiv:1805.04405 [hep-th].

Gauge theories in $2 + 1$ dimensions and dualities

Chong Wang, Perimeter Institute

(Dated: September 23, 2022)

We will (roughly) cover the following topics:

1. **Overview of critical gauge theories in condensed matter physics**: deconfined quantum critical point (DQCP), Dirac spin liquids, fractional quantum Hall transitions
2. **Dualities in** $(1 + 1)d$: Ising model and bosonization revisited
3. **A duality web in** $(2 + 1)d$: boson-vortex duality, boson-fermion duality ($3d$ Chern-Simons bosonization), fermion-fermion duality. Evidences for the dualities: symmetry and anomaly, phase diagram, coupled-wire construction
4. **The half-filled Landau level**: Halperin-Lee-Read theory of composite fermi liquid, particle-hole symmetry and parity anomaly in the half-filled Landau level, Son's Dirac composite fermion theory
5. **Deconfined criticality**: dualities, symmetries and anomalies in DQCP and beyond

Main reference: https://arxiv.org/abs/1810.05174 (Most of the lectures will be based on this reference, I am also attaching a slightly updated version)

Lattice models, numerical simulations and applications

Anders W. Sandvik, Boston University

In this series of lectures I will discuss quantum lattice models, some of the numerical methods used to study them, as well as analysis techniques allowing us to connect results to the quantum field theory formalism. The focus will be on quantum spin-1/2 models, for which very efficient quantum Monte Carlo (QMC) algorithms are available and results can be obtained for large system sizes at low temperatures (or in their ground states). I will also briefly discuss QMC methods for fermion models as well as generic systematic variational methods based on matrix-product and tensor-product states (the former related to the density-matrix RG method). I will discuss how to connect simulation results to field theories through RG scaling dimensions, which can be extracted using finite-size scaling methods (also some times referred to as "phenomenological RG"). After discussing models and motivations from experimental and theoretical perspectives, I will give an overview of numerical methods (focusing on principles rather than technical details) and their capabilities. I will explain the principles of finite-size scaling both in equilibrium and in systems undergoing classical or quantum annealing processes—the latter an emergent methodology implemented in programmable qubit devices that can be used to emulate quantum spin models. To illustrate timely applications of the methods, I will discuss QMC studies of deconfined quantum criticality and related phenomena as well as, time permitting, quantum annealing of spin glass models.

The 10 lectures (45 min each) will be organized as 4 different sections as follows:

I) Introduction and overview of the lecture series (1 lecture)
- experimental motivations (materials and qubit devices)
- lattice models and different types of states (phases)
- classical and quantum phase transitions

II) Simulation methods (4 lectures)
- exact diagonalization
- path integrals and stochastic series expansion QMC methods
- auxiliary field QMC methods for fermions
- spectral functions from QMC with analytic continuation
- matrix- and tensor-product states

III) Criticality and scaling behavior (2 lectures)
- critical exponents and RG scaling dimensions
- finite-size scaling (phenomenological RG)
- classical and quantum annealing, dynamic finite-size scaling

IV) Illustrative examples (3 lectures)
- J-Q models and deconfined quantum criticality
- 2D Disordered quantum spin systems, critical (random-singlet) phase
- quantum annealing of spin glasses

Recommended reading

Lectures notes on quantum spin models, exact diagonalization and QMC:
https://arxiv.org/abs/1101.3281

A very brief introduction to matrix-product states can be found here:
https://arxiv.org/abs/1002.1657

State-of-the art calculations with tensor-product states are discussed here:
https://journals.aps.org/prx/abstract/10.1103/PhysRevX.9.031041
This goes much beyond what will be covered in the lectures but should be useful for those who want to go further.

Auxiliary-field QMC for fermions is discussed here (manual for the ALF library, which I will not discuss but should be useful for those who want to go further with this approach):
https://scipost.org/SciPostPhys.3.2.013/pdf

An extensive article on analytic continuation of QMC data:
https://arxiv.org/abs/2202.09870
Only a brief intro to such methods will be given in the lectures.

For a concise introduction to static and dynamic finite-size scaling, see the supplementary information (pages 16-24) of
https://arxiv.org/abs/2207.13800
The main topic of this article is quantum annealing of a spin glass on a D-wave device (superconducting qubits), which I will discuss briefly as an example of emergent platforms for emulation of quantum-many-body systems.

Some recent works on J-Q models and their applications to deconfined quantum criticality and related phenomena (some of the section IV lectures will draw on these works):
https://arxiv.org/abs/2003.14305
https://arxiv.org/abs/1804.07115
https://arxiv.org/abs/1804.06108