SFT-2025
Lectures on Statistical Field Theories
Galileo Galilei Institute
Florence, 3-14 February 2025

Program of lectures

John Chalker (University of Oxford)
Random quantum circuits (6h)

I will discuss random quantum circuits as models for chaotic many-body quantum dynamics. These models offer a setting in which it is possible to study dynamics using reasonably simple analytical calculations. Two main ways of characterising dynamics are - in the Heisenberg picture - via operator spreading, and - in the Schrödinger picture - via entanglement spreading. Random quantum circuits allow us to examine both of these in spatially extended systems with local interactions. In addition, dynamics can be characterised by the spectral properties of the evolution operator, and I will show how random Floquet circuits give access to spectral properties of the evolution operator in many-body chaotic systems.

A short review article is the area is:

- Fisher, Khemani, Nahum and Vijay, *"Random quantum circuits"*, Annual Reviews of Condensed Matter Physics 14, 335 (2023).

It covers some but not all of the lecture material, and also includes some topics that will not be discussed in the lectures.

Fabian Essler (University of Oxford)
Field theory approaches to low-dimensional many-particle quantum systems (10h)

I will give an introduction to bosonisation and renormalisation group approaches to one dimensional spin systems. There are many excellent books and review articles on the subject, e.g.

- A.O. Gogolin, A.A. Nersesyan and A.M. Tsvelik *"Bosonization and Strongly Correlated Systems"*, Cambridge University Press 1998;
- T. Giamarchi, *"Quantum Physics in one Dimension"*, Clarendon Press Oxford, 2004;
- D. Senechal, *"An Introduction to Bosonization"*, arXiv:cond-mat/9908262 .

There are no prerequisites beyond second quantisation and introductory quantum field theory.

Werner Krauth (ENS, Paris & University of Oxford)
The second Markov-chain revolution (6h+3h)

The Markov-chain Monte Carlo method is an outstanding tool in science that, essentially, consists in evaluating very high-dimensional integrals using a stochastic approach. A first "revolution", in the 1950s, introduced the concept of sampling, and built on its analogy with equilibrium statistical mechanics. It lead, for example, to the famous Metropolis and heat-bath algorithms. Recent decades have witnessed a second revolution, where the concepts of mixing, stopping, coupling, and lifting, etc., have lead to a better understanding, and to much faster, and sometimes even "perfect" sampling

methods. They often build on the notion of non-reversibility and map onto non-reversible physical dynamics.
The course will introduce to Markov chains and Monte Carlo methods from a modern vantage point. While stressing the mathematical foundations, it will be entirely based on several dozen of concrete miniature programs that treat single- and many-particle systems, spin models, card-shuffling problems, among others.

There are no prerequisites, but it would be useful if at least some students came with a laptop, maybe even with Python installed on it (but that is not necessary).

Silvia Pappalardi (University of Cologne)
Chaotic dynamics, the Eigenstate Thermalization Hypothesis and beyond (6h)

These lectures aim to introduce the current understanding and recent advancements in chaotic dynamics: "a nickname for the investigation of quantum systems which do not permit exact solutions." We will begin by defining thermalization in many-body quantum systems and explore how Random Matrix Theory reveals universality in chaotic many-body dynamics, culminating in the Eigenstate Thermalization Hypothesis (ETH). The lectures will cover the primary achievements of the ETH standard formulation as well as its limitations. Building on this foundation, we will present the full version of ETH, which incorporates correlations among matrix elements essential for describing dynamical correlations across different times. The course concludes with an exploration of recent developments, highlighting how Free Probability theory—an extension of probability for non-commuting random variables—provides a framework for understanding this ansatz.

Selected bibliography:

- (The first and still the best): M. Srednicki, *The approach to thermal equilibrium in quantized chaotic systems*, J. Phys. A: Math. Gen. 32 1163 (1999).
- (The review): L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, *From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics*, Advances in Physics, 65, (2016).
- (Physics-friendly introduction to Free Probability): R. Speicher, *Lecture Notes on "Free Probability Theory"*, ArXiv:1908.08125.
- (Full ETH): L. Foini and J. Kurchan, *Eigenstate thermalization hypothesis and out-of-time order correlators*, Phys. Rev. E 99, 042139 (2019).

Hubert Saleur (CEA, Saclay & USC, Los Angeles)
Conformal loop ensembles (10h)

Conformal loop ensembles are statistical ensembles of non-intersecting loops in two dimensions, which turn out to be conformal invariant for a wide range of their parameters. Their properties have been studied for many years by physicists and mathematicians alike. My goal in these lectures is to give a (superficial) review of the topic, and mostly use it as an opportunity to introduce and discuss many concepts of modern mathematical physics. These include sigma models and topological angles, diagram algebras, extended symmetries, vertex models and the Yang-Baxter equation, conformal invariance, operator product expansions, and, hopefully, the bootstrap. While the lectures will be self-contained, they will often only touch upon important topics for which further reading will be suggested. A reasonable knowledge of (classical and quantum) statistical mechanics, path integrals and basic quantum field theory will be useful but probably not indispensable.