Program proposal for GGI, Florence:

Large-N Gauge Theories

Organizers: A. Armoni (Swansea), T. Cohen (Maryland), G. Grignani (Perugia), H. Neuberger (Rutgers), L. Yaffe (Seattle), A. Zaffaroni (Milano Bicocca).

1 Abstract

We propose a 10–12 week workshop in the spring of 2010 entitled "Large-N Gauge Theories." We would like the workshop to include a one week conference early in the program. We propose to organize a program focused on diverse aspects of large-N gauge theories: phenomenology, formal, and lattice. We believe that it will be very fruitful to bring together researchers interested in various approaches to large-N physics. Past workshops where lattice and gauge theory experts interacted with string and AdS/CFT experts have been very successful, and this is a model we would like to emulate.

2 Scientific Motivations

In 1974 't Hooft proposed to approximate QCD by an $SU(N)$ gauge theory, where $\lambda \equiv g^2 N$ is kept fixed and N is sent to ∞ [1]. 't Hooft showed that in this limit the dynamics of the gauge theory is dominated by planar graphs, and the expansion in Feynman diagrams can be organized as a topological expansion. This idea is known as the "'t Hooft large-N limit." 't Hooft also proposed that in this limit the gauge theory may have a description in terms of string theory, where the tree level is described by planar diagrams and the non-planar corrections are related to string interactions. The 't Hooft large-N limit has led to several significant phenomenological successes (see the review [2]), among them an explanation of Zweig's rule, the behavior of Regge trajectories of mesons, and the Witten-Veneziano formula [3, 4]. 't Hooft's solution to two-dimensional QCD in this same limit is also noteworthy [5].

Part of the motivation for the large-N limit of gauge theories lies in the hope that it might ultimately provide the means for performing reliable computations in the large coupling phase of non-Abelian gauge theories in four dimensions. To support these hopes, one may note the significant simplifications which occur in the large N-limit for matrix-models, allowing one to discriminate the phases of broken and unbroken symmetries [6]. It was shown by Brezin, Itzykson, Parisi and Zuber [6] in 1978 that a class of functional integrals in the large N-limit can be calculated by steepest descent methods. In this limit a particular configuration (or gauge orbit) totally dominates the functional integral and the distributions of eigenvalues of the underlying unitary matrices have a sharply determined $N = \infty$ saddle point. The change in their distribution when varying λ drives possible phase transitions [7]. The large N-limit of matrix models has opened a large field of research which has been important both for gauge theory and strings.

It has long been understood that the large N limit of gauge theories is a type of classical limit [8]. Quantum fluctuations of appropriate (gauge invariant) observables vanish as $N \to \infty$ and the quantum dynamics reduces to classical dynamics on a (complicated, infinite dimensional) $N = \infty$ phase space. These features motivate efforts to reformulate QCD in terms of a new set of string-like degrees of freedom which would usefully describe its behavior in the strong coupling regime.

The most remarkable progress in the understanding of gauge/string duality over the last thirty years emerged indirectly from the study of gravitational effects in superstring theory. In 1998 Maldacena conjectured that large-N $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) has a dual description in terms of type IIB supergravity on $AdS_5 \times S^5$ [9]. This conjecture, known as AdS/CFT duality, relates a non-confining gauge theory to a string theory. Soon after, the duality was generalized to confining backgrounds (such as the Klebanov-Strassler and Maldacena-Nunez solutions), giving the first explicit realizations of 't Hooft's idea in a four-dimensional setup. These well established examples of dualities involve Yang-Mills theories with fields in the adjoint representation only. Quarks in the fundamental representation and flavors can be introduced in the quenched approximation allowing one to study QCD-like theories. Holographic ideas are deeply involved in this duality: string theory, which is a theory of quantum gravity, has a dual description as a quantum field theory living on the boundary of the background spacetime.

In a more recent development, advances in understanding of gauge theory have come from the observation that planar, conformal $\mathcal{N} = 4$ SYM is an integrable system in the one-loop approximation [10] and, very likely, at higher loops as well [11]. This extends and generalizes in a beautiful way the previously observed integrable structures of one-loop QCD, which also happens to be conformal. Integrability is indispensable for resolving the complicated mixing problem in order to find the anomalous dimensions of conformal operators. A key part of this analysis involves use of a Bethe ansatz which reduces the spectral problem to the solution of a system of finitely many non-linear equations.

For the last decade, the duality between $\mathcal{N} = 4$ superconformal Yang-Mills theory and type IIB string theory on $AdS_5 \times S^5$ has been celebrated as the simplest example of an exact duality between gauge theory and string theory. Recently, another very simple example relating a three dimensional conformal theory to AdS_4 has been constructed. Following developments initiated by Bagger, Lambert and Gustavsson [12, 13, 14] in finding the superconformal world-volume theory for multiple M2-branes, Aharony, Bergman, Jafferis and Maldacena constructed a new $\mathcal{N} = 6$ superconformal Chern-Simons theory (ABJM theory) [15] which should be the world-volume theory of multiple M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$. These authors have conjectured a new duality between ABJM theory and type IIA string theory on $AdS_4 \times \mathbb{C}P^3$. This is believed to be another exact duality between gauge theory and string theory and it opens the way for the study of conformal and non-conformal gauge theories in three dimensions using supergravity.

To date, the quantitative understanding of the gauge/string duality has been limited to a few tractable cases. Beyond these examples, only qualitative similarities between string and gauge theories have been understood. One major limitation is that at present we can only do explicit computations in the supergravity regime, which is the low energy limit of string theory. However, at least a few properties derived using supergravity calculations seem to be universal and robust enough to apply to much wider class of strongly interacting gauge theories, including QCD. One of the aims of the proposed workshop is to try to understand how to go beyond the supergravity approximation. This would make it possible to realize the old idea of solving QCD in the large-N limit through a string theory dual.

Even though no gravity dual of QCD is known today, many efforts to use AdS/CFT (or gauge/string) duality to study strongly coupled gauge theories are motivated by the hope that results will provide useful models for understanding QCD. In judging the utility of holographic models, it is important to bear in mind the paucity of alternative approaches. There are no known systematic methods capable of studying non-perturbative dynamical properties of QCD in strongly coupled regimes. While static equilibrium properties can be analyzed using Euclidean lattice simulations, these numerical methods are not directly applicable to dynamical observables such as transport properties of the quark-gluon plasma (QGP). Hence, a useful formulation of QCD in terms of string-like degrees of freedom could shed light on some of its most fundamental properties. Recent progress in this direction has come from the use of AdS/CFT duality.

A major application of gauge/string duality involves the study of strongly coupled non-Abelian plasma. At zero temperature $\mathcal{N} = 4$ super Yang-Mills theory and QCD have very different properties, but at non-zero temperature (above the deconfinement temperature in QCD) they share many important features. For example, both theories undergo a confinement/deconfinement phase transition to a non-Abelian plasma phase with Debye screening, finite spatial correlation lengths, and long distance near-equilibrium dynamics described by neutral fluid hydrodynamics. However, $\mathcal{N} = 4$ super Yang-Mills theory has the advantage that it is much easier to study in the large N-limit than QCD. There are good reasons to be believe that many physically relevant quantities should not be highly sensitive to the precise make-up of the plasma, and hence $\mathcal{N} = 4$ plasma should provide a useful model for a QCD plasma at temperatures where the plasma is strongly coupled. Gauge/string duality can be used to compute not only thermodynamic quantities, but also transport coefficients, response to high energy projectiles, emission spectra, and other dynamic properties. The ratio of the shear viscosity to the entropy density has been found to be a universal property of strongly coupled gauge theories with a gravity dual. The universality of this ratio is related to universal properties of black hole horizons. Remarkably, the value of this ratio computed using the AdS/CFT correspondence can be compared with estimates extracted from modeling of heavy ion experiments performed at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven, NY. From these experiments there is evidence that an effectively equilibrated quark-gluon plasma forms, that it behaves as a strongly coupled liquid, and that the value of the ratio of the shear viscosity to the entropy of this plasma is close to the value suggested by AdS/CFT duality.

Obtaining a better understanding of the confinement/deconfinement phase transition and of both the confining and deconfining phases of gauge theories will be very useful and will have many applications. A quantitative understanding of the confining phase of gauge theories has eluded theoretical physicists for decades, and continues to be a very active field of research. Moreover, the deconfining phase is under intense study both theoretically and experimentally, with ongoing heavy ion experiments at RHIC soon to be complemented by higher energy heavy ion collisions at the LHC accelerator at CERN.

In addition to the above-described developments involving gauge/string duality, there has also been recent noteworthy progress in understanding the relation between the large N limits of different gauge theories. As long as appropriate symmetry realization conditions are satisfied, it is now understood that there is a rich network of exact non-perturbative equivalences relating the large N limits of theories with different gauge groups, matter content, actions, and spacetime volumes. (See, for example, Refs. [16, 17, 18, 19, 20, 21] and references therein.) The existence of these large N equivalences has important phenomenological utility [20, 21], and practical implications for efficient simulations of large N gauge theories [16]. Further studies of connections between large N limits of different theories is clearly worthwhile.

The large N limit has played an important role in elucidating numerous aspects of hadron phenomenology. One of the critically important points is the emergence at large N of a contracted SU($2N_f$) spin-flavor symmetry for the ground-state band of baryons[2]. This symmetry implies the existence of model-independent relations between different observables which hold to leading order at large N; moreover, there are certain "gold-plated" relations for which the $1/N$ corrections vanish. These "gold-plated" relations appear to hold to considerable accuracy in the real world. The extension of this analysis to excited baryons has been a major recent push in hadron phenomenology [22]. Despite these advances in making model-independent predictions based on large N, hadronic physics remains largely within the domain of models. Large N considerations have played a critical role in helping to justify models and characterizing their limitations. Large N analysis [23] was critical in clarifying the spurious predictions [24] of pentaquark states in chiral soliton models. Experimental claims by numerous groups of the discovery of pentaquarks near this predicted value created a sensation; however, higher statistic studies suggest that these claims were erroneous [25, 26].

Below we list in a bit more detail possible topics of the envisioned program.

3 Topics

1. Formal aspects of AdS/CFT duality: quantization of type IIB strings on $AdS_5 \times S^5$. Integrability of the worldsheet theory. Integrability of $\mathcal{N} = 4$ SYM. Understanding of the AdS_4/CFT_3 duality. Quantization of type IIA strings on $AdS_4 \times \mathbb{C}P^3$. Integrability in the AdS_4/CFT_3 correspondence. Bethe Ansatz and spin chains.

2. Wilson loops and large N gauge theory amplitudes at strong coupling via the AdS/CFT duality. Dual superconformal symmetry.

3. Matrix models, phase transitions and large N-behavior.

4. Attempts to describe QCD (or $\mathcal{N} = 1$ super QCD) via gravity/string theory: Klebanov-Strassler, Maldacena-Nunez models with and without extra flavors. Sakai-Sugimoto and AdS/QCD. Flavor physics.

5. Finite temperature and plasma physics via holographic QCD. Phenomenology of the quark-gluon plasma.

6. Large-N phenomenology. Mesons, glueballs, baryons and domain-walls in large-N theories. Connections to hadronic models.

7. New large N expansions, planar equivalence. Large-N dualities. Chiral theories.

8. Lattice simulations of large-N theories. Reduced (Eguchi-Kawai) models. Phase transitions in planar theories.

4 Potential key participants

O. Aharony (Weizmann), L. Alday (Princeton), N. Beisert (AEI-MPI), D. Berenstein (UCSB), B. Bringoltz (Seattle), R. Brower (Boston), S. Catterall (Syracuse), L. del Debbio (Edinburgh), T. DeGrand (Colorado), J. Erlich (Williamsburg) S. Gubser (Princeton), D. Gross (UCSB), C. Herzog (Princeton), D. Kaplan (Seattle), V. Kazakov (Paris), I. Klebanov (Princeton), G. Korchemsky (Orsay), P. Kovtun (Victoria), C. Kristjansen (Copenhagen), J. Kuti (San Diego), R. Lebed (Arizona), J. Maldacena (Princeton), A. Manohar (San Diego), D. Mateos (Barcelona), J. Minahan (Uppsala), C. Nunez (Swansea), R. Pisarski (Brookhaven), M. Rangamani (Durham), G. Semenoff (Vancouver), M. Shifman (Minnesota), D. Son (Seattle), M. Stephanov (Chicago), M. Staudacher (AEI-MPI), A. Starinets (Oxford), S. Sugimoto (Tokyo), G. 't Hooft (Utrecht), M. Teper (Oxford), M. Ünsal (Stanford), G. Veneziano (CdF and CERN), K. Zarembo (Paris-Uppsala).

5 Dates

Our aim is to have a 10–12 weeks program on spring 2010. We wish to organize a 3–5 day workshop near the beginning of the program.

References

[1] G. 't Hooft, "A planar diagram theory for strong interactions," Nucl. Phys. B **72**, 461 (1974).

[2] A. V. Manohar, "Large N QCD," arXiv:hep-ph/9802419.

[3] E. Witten, "Current algebra theorems for the U(1) Goldstone boson," Nucl. Phys. B **156**, 269 (1979).

[4] G. Veneziano, "U(1) without instantons," Nucl. Phys. B **159**, 213 (1979).

[5] G. 't Hooft, "A two-dimensional model for mesons," Nucl. Phys. B **75**, 461 (1974).

[6] E. Brezin, C. Itzykson, G. Parisi and J. B. Zuber, "Planar diagrams," Commun. Math. Phys. **59**, 35 (1978).

[7] D. J. Gross and E. Witten, "Possible third order phase transition in the large N lattice gauge theory," Phys. Rev. D **21**, 446 (1980).

[8] L. G. Yaffe, "Large N limits as classical mechanics," Rev. Mod. Phys. **54**, 407 (1982).

[9] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[10] J. A. Minahan and K. Zarembo, "The Bethe-ansatz for $\mathcal{N} = 4$ super Yang-Mills," JHEP **0303**, 013 (2003) [arXiv:hep-th/0212208].

[11] N. Beisert, B. Eden and M. Staudacher, "Transcendentality and crossing," J. Stat. Mech. **0701**, P021 (2007) [arXiv:hep-th/0610251].

[12] J. Bagger and N. Lambert, "Modeling multiple M2's," Phys. Rev. D **75**, 045020 (2007) [arXiv:hep-th/0611108].

[13] A. Gustavsson, "Algebraic structures on parallel M2-branes," arXiv:0709.1260 [hep-th].

[14] J. Bagger and N. Lambert, "Gauge symmetry and supersymmetry of multiple M2-branes," Phys. Rev. D **77**, 065008 (2008) [arXiv:0711.0955 [hep-th]].

[15] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, "$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," arXiv:0806.1218 [hep-th].

[16] M. Unsal and L. G. Yaffe, "Center-stabilized Yang-Mills theory: confinement and large N volume independence," arXiv:0803.0344 [hep-th].

[17] P. Kovtun, M. Unsal and L. G. Yaffe, "Volume independence in large N_c QCD-like gauge theories," JHEP **0706**, 019 (2007) [arXiv:hep-th/0702021].

[18] P. Kovtun, M. Unsal and L. G. Yaffe, "Necessary and sufficient conditions for non-perturbative equivalences of large N_c orbifold gauge theories," JHEP **0507**, 008 (2005) [arXiv:hep-th/0411177].

[19] A. Armoni, M. Shifman and M. Unsal, "Planar limit of orientifold field theories and emergent center symmetry," Phys. Rev. D **77**, 045012 (2008) [arXiv:0712.0672 [hep-th]].

[20] A. Armoni, M. Shifman and G. Veneziano, "SUSY relics in one-flavor QCD from a new $1/N$ expansion," Phys. Rev. Lett. **91**, 191601 (2003) [arXiv:hep-th/0307097].

[21] A. Armoni, M. Shifman and G. Veneziano, "From super-Yang-Mills theory to QCD: Planar equivalence and its implications," arXiv:hep-th/0403071.

[22] T. D. Cohen and R. F. Lebed, "New relations for excited baryons in large N_c QCD," Phys. Rev. Lett. **91**, 012001 (2003).

[23] T. D. Cohen, "Chiral soliton models, large N_c consistency and the Θ^+ exotic baryon " Phys. Lett. **B581**, 175 (2004).

[24] D. Diakonov, V. Petrov and M. Polyakov, " Exotic Anti-Decuplet of Baryons: Prediction from Chiral Solitons, " Z. Phys. **A359**, 305, (1997).

[25] CLAS collaboration, "Search for the Θ^+ pentaquark in the reaction $\gamma d \to pK^- K^+ n$", Phys. Rev. Lett. **96**, 212001 (2006).

[26] CLAS collaboration, "Search for the Θ^+ pentaquark in the $\gamma d \to n\Lambda K^+$ reaction measured with CLAS " , Phys. Rev. Lett. **97**, 032001 (2006).