GGI lectures on the theory of fundamental interactions

Firenze 12-18 January 2015

Aspects of Neutrino Physics (F. Feruglio)

Bibliography

- on-line slides

- Lectures

W. Grimus hep-ph/0307149

P. Hernandez hep-ph/1010.4131

L. Maiani hep-ph/1406.5503

- Book

"Fundamentals of Neutrino Physics and Astrophysics" by C. Giunti and C. Kim

- PDG: Section on "Neutrino Masses, Mixing and Oscillations"

- Further readings:

General: A. Strumia and F. Vissani, hep-ph/0606054

Oscillation formula: L. Stodolsky hep-ph/9802387

GUTS: R. Mohapatra hep-ph/9801235

Leptogenesis: P. Di Bari hep-ph/1206.3168

Lepton Flavor Violation: F. Deppisch hep-ph/1206.5212,

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GGI lectures on the theory of fundamental interactions 2015

Firenze, 12-16 January 2015

Aspects of neutrino physics (I)
Neutrino Masses, Mixing and Oscillations: the data

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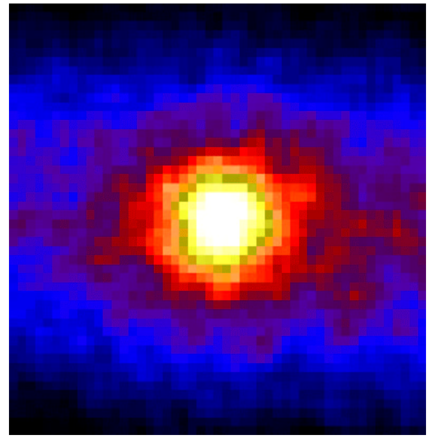
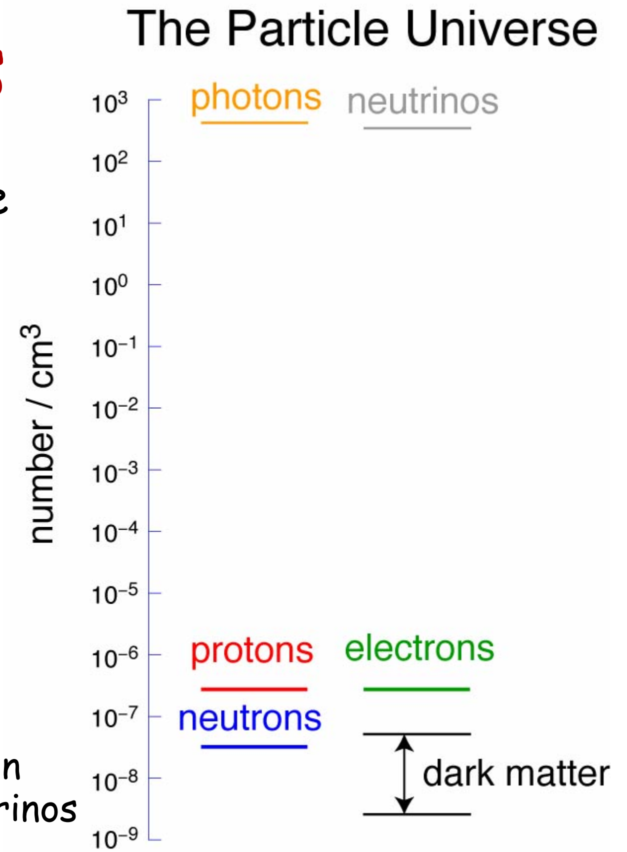
General remarks on neutrinos

the more abundant particles in the universe after the photons: about 300 neutrinos per cm^3

produced by stars: most of the sun energy emitted in neutrinos. As I speak more than 1 000 000 000 000 000 solar neutrinos go through your bodies each second.



this is a picture of the sun reconstructed from neutrinos

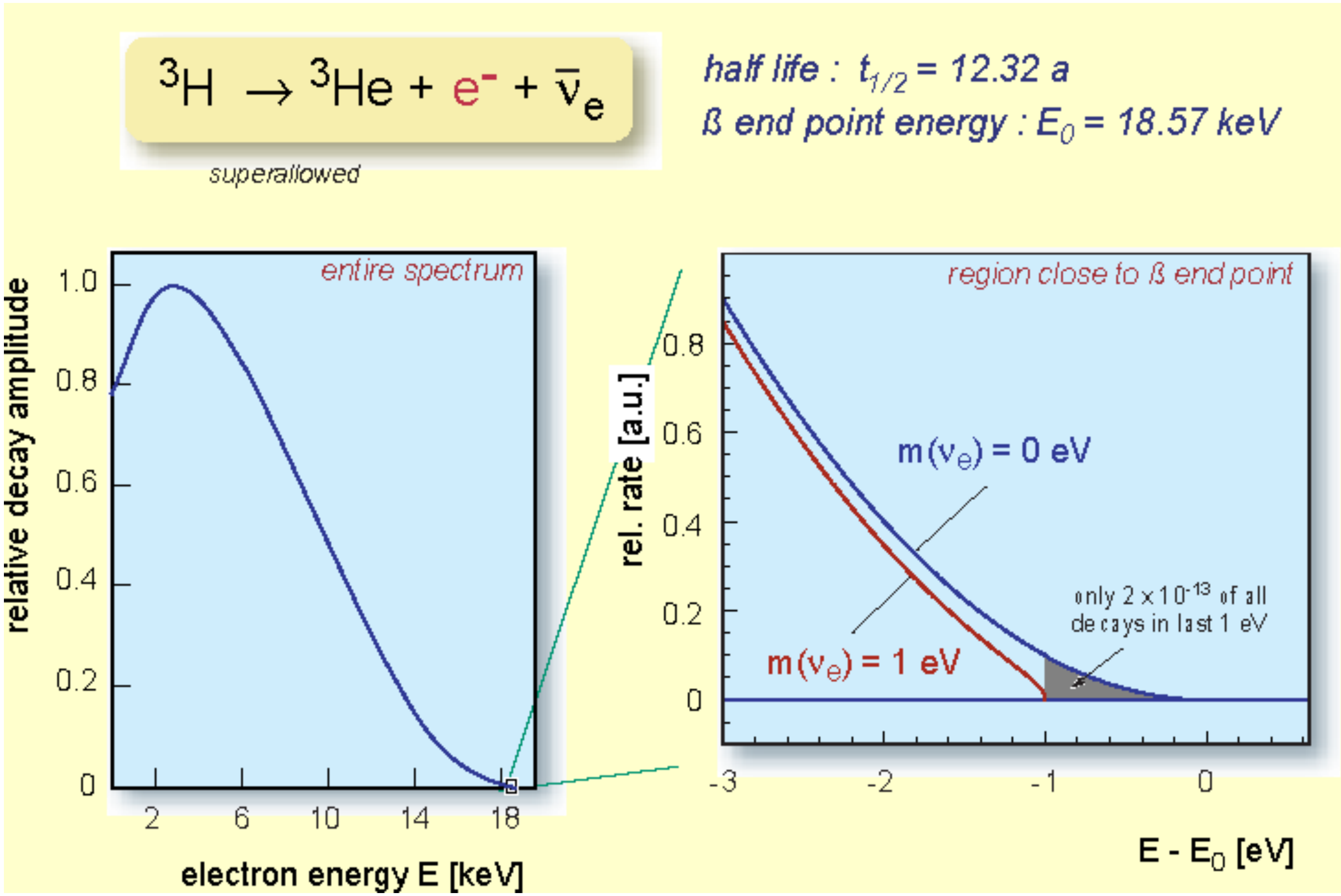


from Murayama talk Aspen 2007

electrically neutral and extremely light:
they can carry information about extremely large length scales
e.g. a probe of supernovae dynamics: neutrino events from a supernova explosion first observed 27 years ago

in particle physics:
they have a tiny mass (1 000 000 times smaller than the electron's mass)
the discovery that they are massive allows us to explore, at least in principle, extremely high energy scales, otherwise inaccessible to present laboratory experiments

Upper limit on neutrino mass (laboratory)



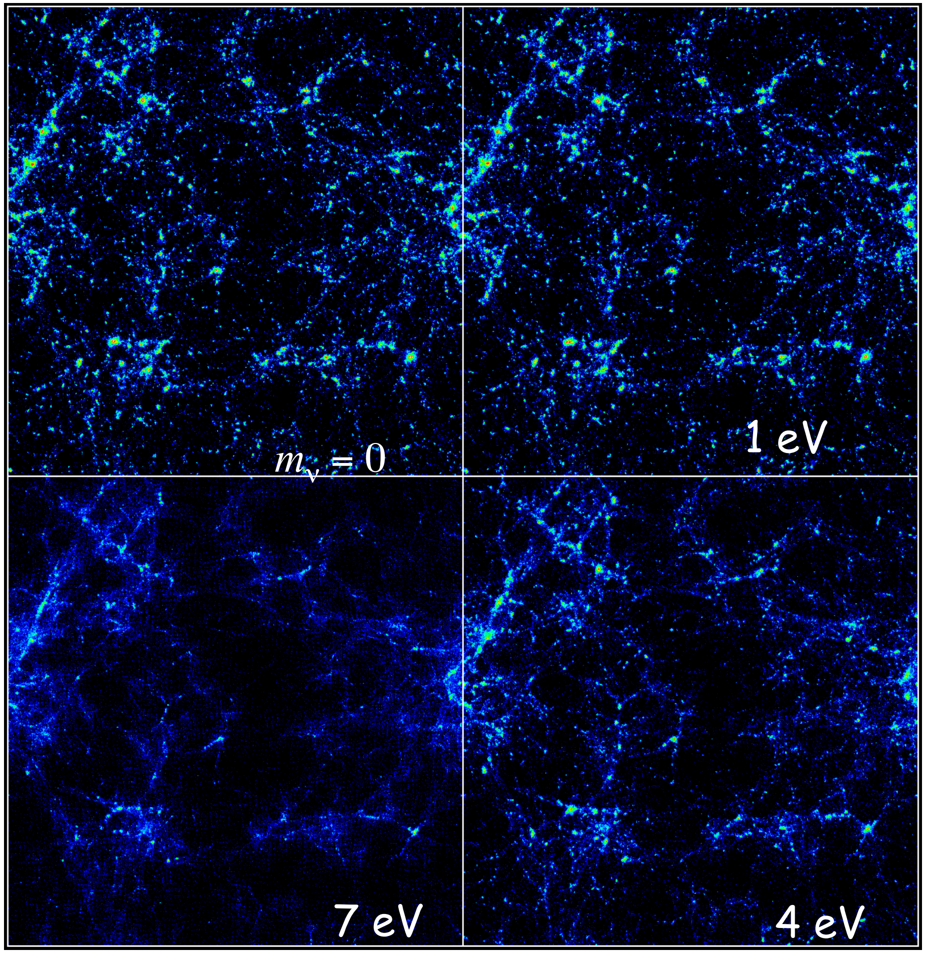
$$m_\nu < 2.2\, eV \quad (95\%\ CL)$$

Upper limit on neutrino mass (cosmology)

massive ν suppress the formation of small scale structures

$$\sum_i m_i < 0.2 \div 1 \quad eV$$

depending on
- assumed cosmological model
- set of data included
- how data are analyzed



$$k_{\rm nr} \approx 0.026 \left(\frac{m_\nu}{1\,{\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}.$$

The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,{\rm eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right)$$

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)} P(\vec{k})$$

Two-flavour neutrino oscillations (ν_e, ν_μ)

here ν_e are produced with average energy E

source

$\longleftarrow$ L $\longrightarrow$

detector

here we measure

$$P_{ee} \equiv P(\nu_e \rightarrow \nu_e)$$

neutrino interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}}_{U} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L \gamma^\mu \nu_l$$

as before, but

$$\gamma/2 = \vartheta$$

$$t \approx L$$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_2^2} \approx \frac{m_2^2}{2E} - \frac{m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E}$$

$$P_{ee} = \left|\langle \nu_e | \psi(L) \rangle\right|^2 = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

no dependence on the phase α more on this later on ....

to see any effect, if Δm^2 is tiny, we need both θ and L large

regimes

$$P_{ee} = |\langle \nu_e | \psi(L) \rangle|^2 = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\frac{\Delta m^2 L}{4E} << 1 \qquad P_{ee} \approx 1$$

$$\frac{\Delta m^2 L}{4E} >> 1 \qquad \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \approx \frac{1}{2} \qquad P_{ee} \approx 1 - \frac{\sin^2 2\vartheta}{2}$$

by averaging over ν_e energy at the source

$$\frac{\Delta m^2 L}{4E} \approx 1 \qquad P_{ee} = P_{ee}(E)$$

useful relation $\frac{\Delta m^2 L}{4E} \approx 1.27 \left(\frac{\Delta m^2}{1\,eV^2}\right)\left(\frac{L}{1\,Km}\right)\left(\frac{E}{1\,GeV}\right)^{-1}$

source	L(km)	E(GeV)	$\Delta m^2(eV^2)$
ν_e, ν_μ (atmosphere)	10^4 (Earth diameter)	1-10	10^{-4} - 10^{-3}
anti- ν_e (reactor)	1	10^{-3}	10^{-3}
anti- ν_e (reactor)	100	10^{-3}	10^{-5}
ν_e (sun)	10^8	10^{-3} - 10^{-2}	10^{-11} - 10^{-10}

neglecting matter effects

Three-flavour neutrino oscillations $(\nu_e, \nu_\mu, \nu_\tau)$

survival probability as before, with more terms

$$P_{ff} = P(\nu_f \to \nu_f) = \left|\langle \nu_f | \psi(L) \rangle\right|^2 = 1 - 4\sum_{k<j} |U_{fk}|^2 |U_{fj}|^2 \sin^2\left(\frac{\Delta m_{jk}^2 L}{4E}\right)$$

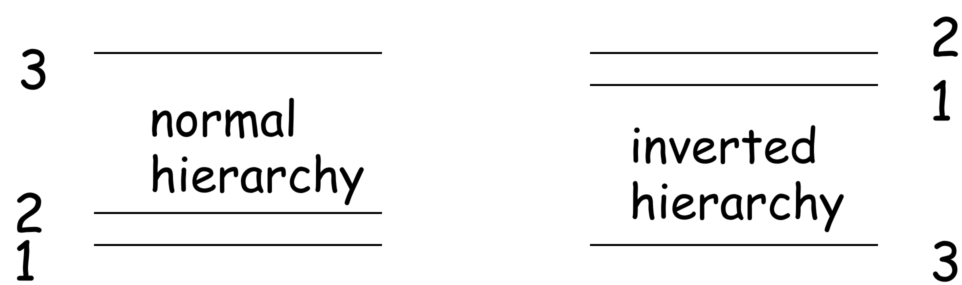
similarly, we can derive the disappearance probabilities $P_{ff'} = P(\nu_f \to \nu_{f'})$

conventions: $[\Delta m_{ij}^2 \equiv m_i^2 - m_j^2]$

$m_1 < m_2$

$\Delta m_{21}^2 < \left|\Delta m_{32}^2\right|, \left|\Delta m_{31}^2\right|$ i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



Mixing matrix $U=U_{PMNS}$ (Pontecorvo,Maki,Nakagawa,Sakata)

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi} \nu_i$$

$$(f = e, \mu, \tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

$$U_{PMNS} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{-i\delta} & c_{13}\, s_{23} \\ -c_{12}\, s_{13}\, c_{23}\, e^{-i\delta} + s_{12}\, s_{23} & -s_{12}\, s_{13}\, c_{23}\, e^{-i\delta} - c_{12}\, s_{23} & c_{13}\, c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

three mixing angles $\vartheta_{12},\ \vartheta_{13},\ \vartheta_{23}$

three phases (in the most general case) δ $\quad \underbrace{\alpha,\ \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \to \nu_{f'})$

oscillations can only test 5 combinations

$\Delta m^2_{21}, \Delta m^2_{32},$ $\vartheta_{12},\ \vartheta_{13},\ \vartheta_{23}$ δ

structure of the mixing matrix

$$\begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{-i\delta} & c_{13}\,s_{23} \\ -c_{12}\,s_{13}\,c_{23}\,e^{-i\delta} + s_{12}\,s_{23} & -s_{12}\,s_{13}\,c_{23}\,e^{-i\delta} - c_{12}\,s_{23} & c_{13}\,c_{23} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Analysis of Oscillations Data

we anticipate that there are two small parameters

$$|\alpha| \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| \approx 0.03$$

$$\Delta m^2_{21} << \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right|$$

$$|U_{e3}|^2 \approx \sin^2\vartheta_{13} \approx 0.02$$

we first consider experiments not sensitive to Δm^2_{21} (L not very large, E not very small) and we set $\Delta m^2_{21} = 0$

EXERCISE
derive $P_{ee}, P_{\mu\mu}, P_{\mu e}$ in the limit $\Delta m^2_{21} = 0$ (vacuum osc., no matter effects)

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \qquad \Delta \equiv \frac{\Delta m^2_{13} L}{4E} \quad [\Delta m^2_{21} = 0]$$

$$P_{ee} = 1 - 4|U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2\Delta$$

$$P_{\mu\mu} = 1 - 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \sin^2\Delta$$

$$P_{\mu e} = P_{e\mu} = 4|U_{\mu 3}|^2 |U_{e3}|^2 \sin^2\Delta$$

similarly, $P_{\tau\tau}, P_{\tau\mu}, P_{\mu\tau}, P_{\tau e}, P_{e\tau}$ only depend on U_{f3} and Δ for $\Delta m^2_{21} = 0$

we are testing the third column

$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & U_{e3} \\ \cdot & \cdot & U_{\mu 3} \\ \cdot & \cdot & U_{\tau 3} \end{pmatrix}$$

we also consider the limit $\vartheta_{13} = 0$
we are left with one frequency and one mixing angle

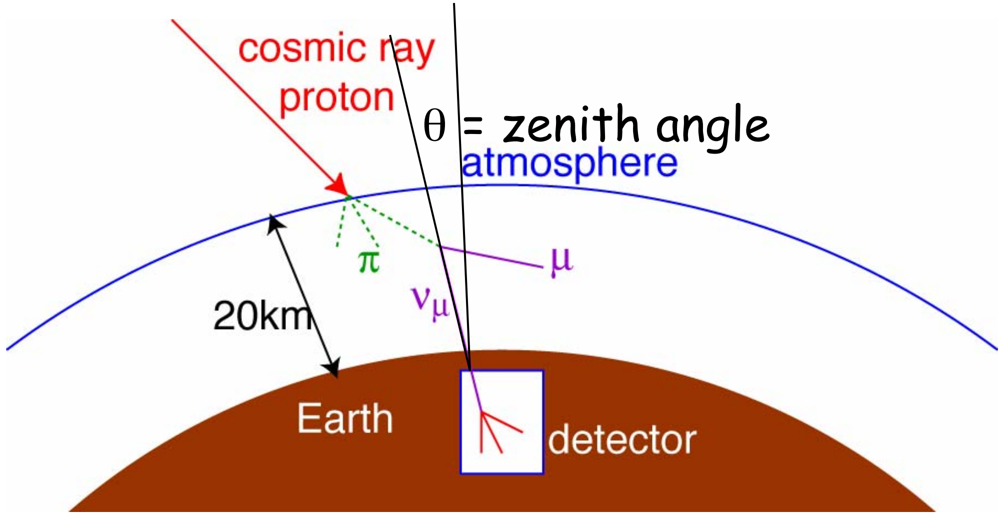
$$|U_{e3}|^2 \approx \sin^2 \vartheta_{13} \approx 0$$

$$P_{ee} = 1$$
$$P_{\mu\mu} = 1 - \sin^2 2\vartheta_{23} \sin^2 \Delta$$
$$P_{\mu e} = P_{e\mu} = 0$$

$$P_{\tau\tau} = P_{\mu\mu}$$
$$P_{\tau\mu} = P_{\mu\tau} = \sin^2 2\vartheta_{23} \sin^2 \Delta$$
$$P_{\tau e} = P_{e\tau} = 0$$

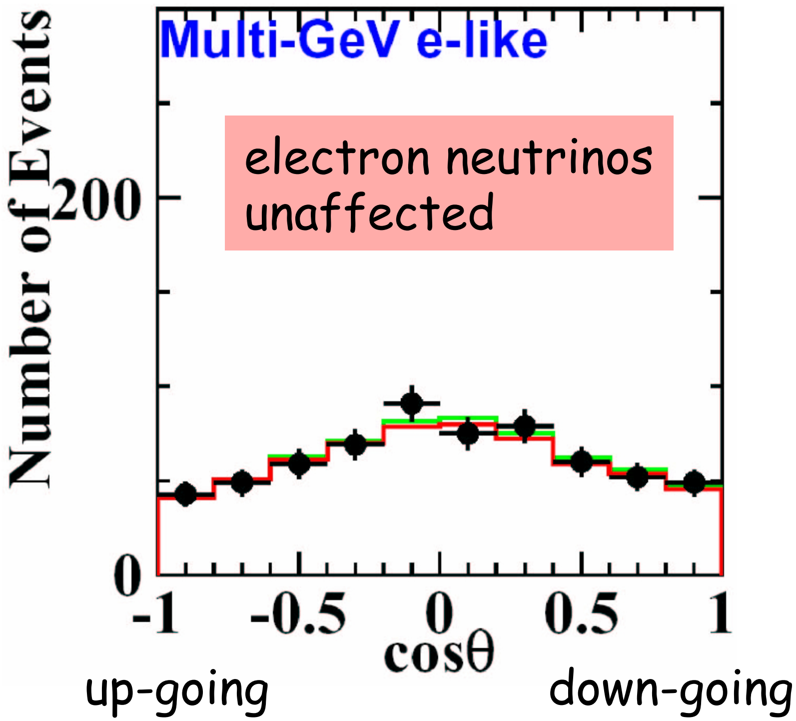
two-flavour oscillations

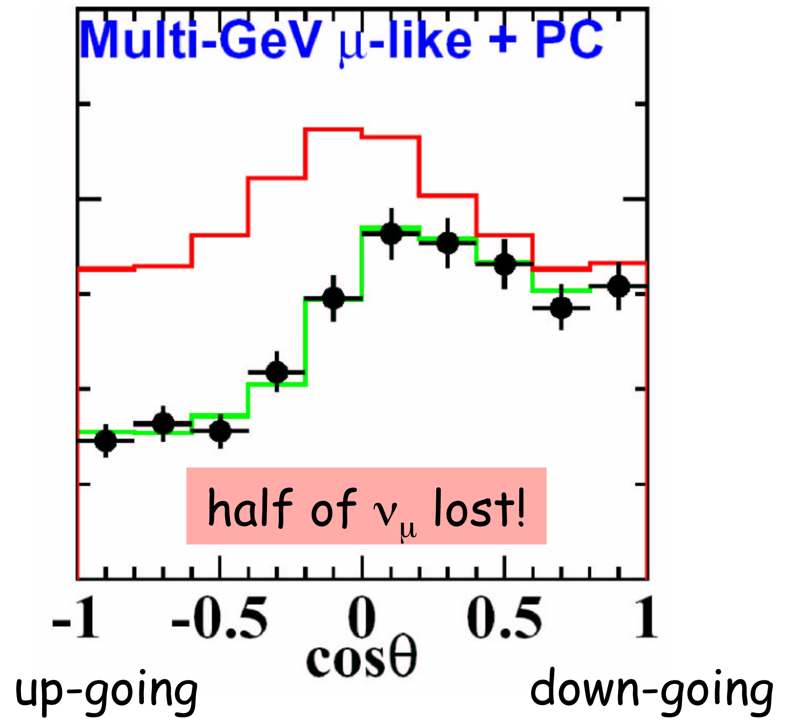
Atmospheric neutrino oscillations



Electron and muon neutrinos (and antineutrinos) produced by the collision of cosmic ray particles on the atmosphere

Experiment:
SuperKamiokande (Japan)





electron neutrinos do not oscillate

by working in the approximation $\Delta m_{21}^2 = 0$

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \approx 1$$

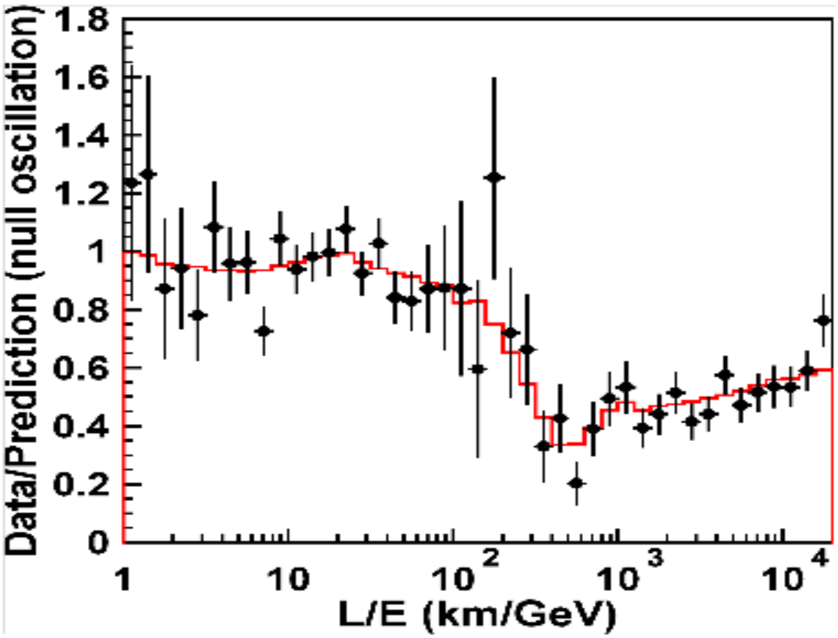
for $U_{e3} = \sin\vartheta_{13} \approx 0$

muon neutrinos oscillate

$$P_{\mu\mu} = 1 - \underbrace{4|U_{\mu3}|^2(1-|U_{\mu3}|^2)}_{\sin^2 2\vartheta_{23}} \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

$$|\Delta m_{32}^2| \approx 2 \cdot 10^{-3} \quad eV^2$$

$$\sin^2\vartheta_{23} \approx \frac{1}{2}$$



$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & -\frac{1}{\sqrt{2}} \\ \cdot & \cdot & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

maximal mixing!
not a replica of the quark mixing pattern

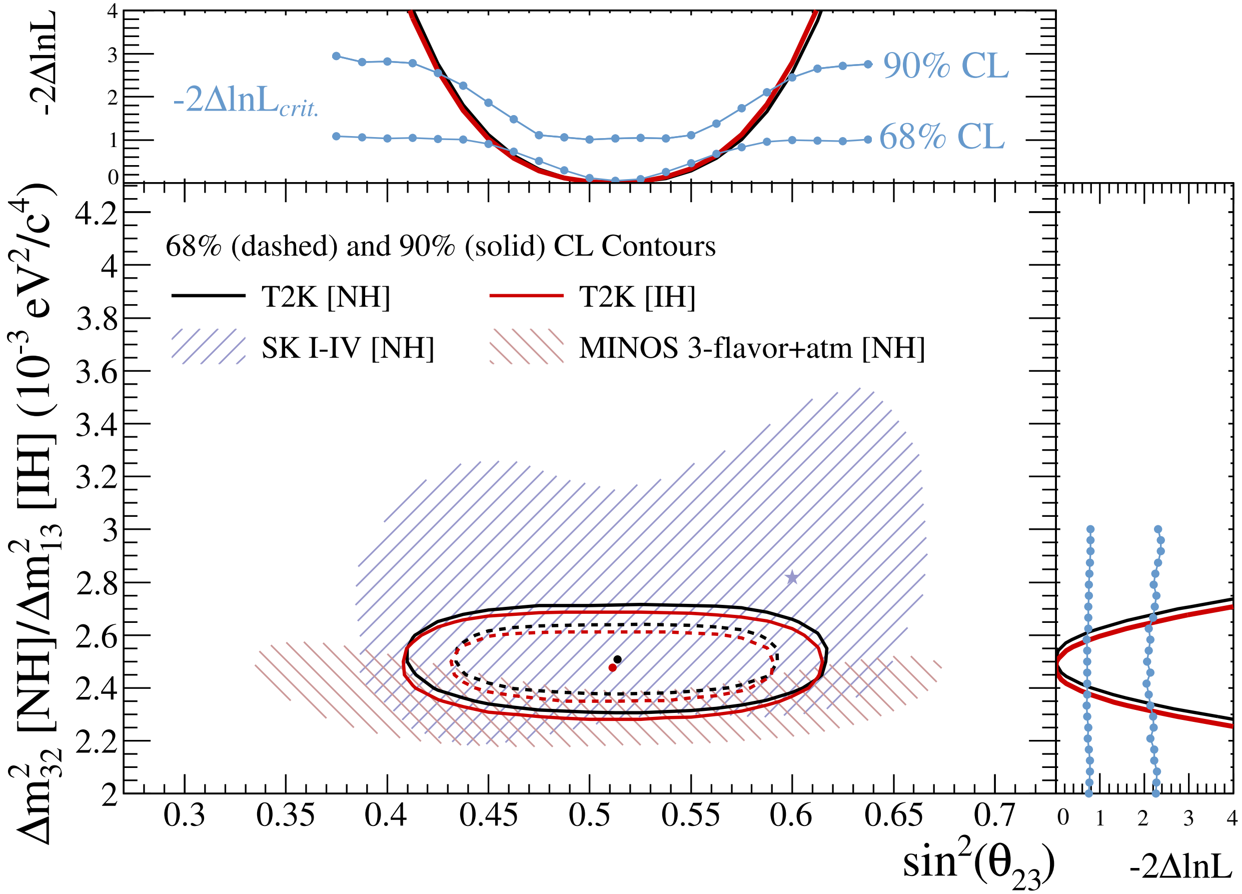
other terrestrial experiments measuring $P_{\mu\mu}$

man made neutrino beams

K2K (Japan, from KEK to Kamioka mine L ≈ 250 Km E ≈ 1.3 GeV)
MINOS (USA, from Fermilab to Soudan mine L ≈ 735 Km E ≈3 GeV)
T2K (Japan, from Tokai,J-Park to Kamioka mine L ≈ 295 Km E ≈ 0.6 GeV)
OPERA (CERN-Italy, from CERN to LNGS L ≈ 732 Km E ≈ 17 GeV)
all sensitive to Δm_{32}^2 close to 10^{-3} eV^2

OPERA energy optimized to maximize τ production, via CC events
by the end of 2014 4 τ events have been seen

recent results from T2K [hep-ex/1403.1532]



$$\sin^2\vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & (NO) \\ 0.511 \pm 0.055 & (IO) \end{cases}$$

$$\Delta m^2_{32} = (2.51 \pm 0.10) \times 10^{-3}\ eV^2 \quad (NO)$$
$$\Delta m^2_{13} = (2.48 \pm 0.10) \times 10^{-3}\ eV^2 \quad (IO)$$

KamLAND

previous experiments were sensitive to Δm^2 close to 10^{-3} eV^2
to explore smaller Δm^2 we need larger L and/or smaller E

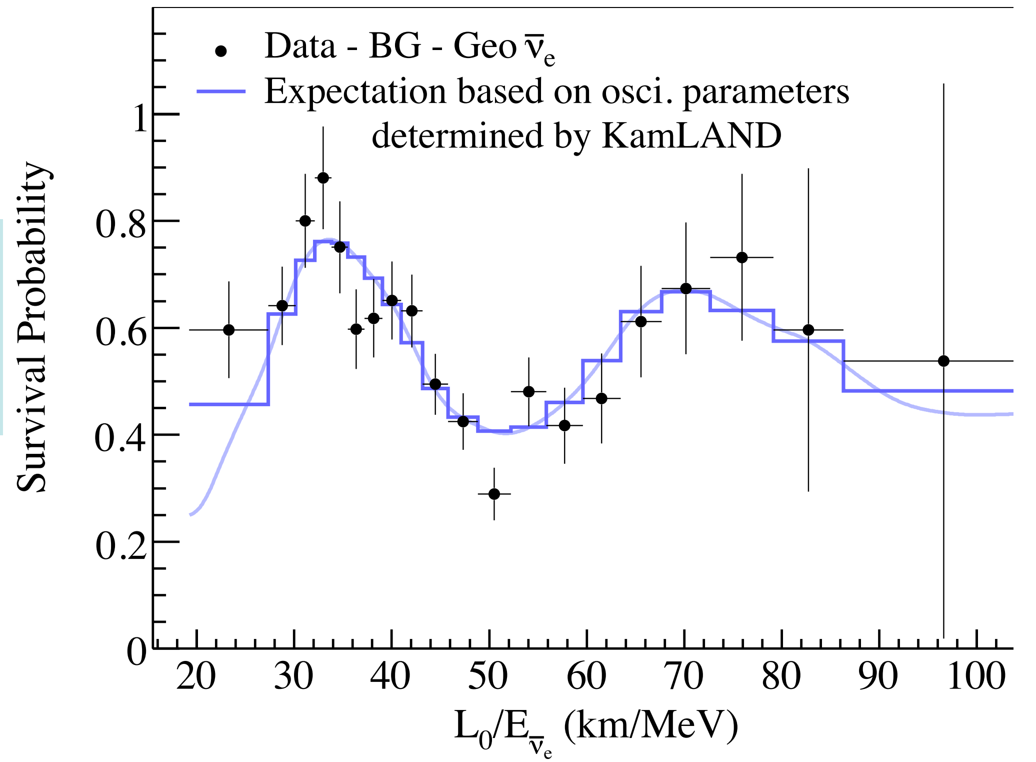
KamLAND experiment exploits the low-energy electron anti-neutrinos (E≈3 MeV) produced by Japanese and Korean reactors at an average distance of L≈180 Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2

by working in the approximation

$U_{e3} = \sin\vartheta_{13} = 0$ we get

[Exercise]

$$P_{ee} = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta_{12}} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$



$$\Delta m_{21}^2 \approx 8 \cdot 10^{-5} \quad eV^2$$

$$\sin^2\vartheta_{12} \approx 1/3$$

EXERCISE

estimate Δm^2_{21} from position of second oscillation dip in previous plot

$$\Delta m^2_{21} = 6\pi \left.\frac{E}{L}\right|_{dip} \approx 6\pi \times \frac{1}{50}\, MeV/Km = 7.5 \times 10^{-5}\, eV^2$$

EXERCISE

work out P_{ee} by keeping U_{e3} non-vanishing

$$P_{ee} \approx |U_{e3}|^4 + (1-|U_{e3}|^2)^2 \left(1-\sin^2 2\vartheta_{12} \sin^2 \Delta_{21}\right)$$

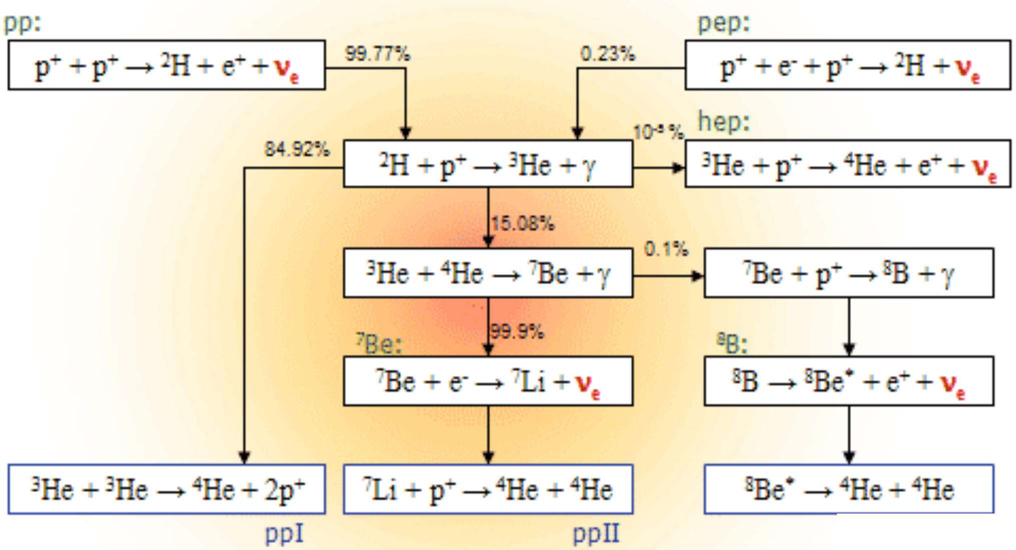
$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

by unitarity

this pattern is called tri-bimaximal completely different from the quark mixing pattern: two angles are large

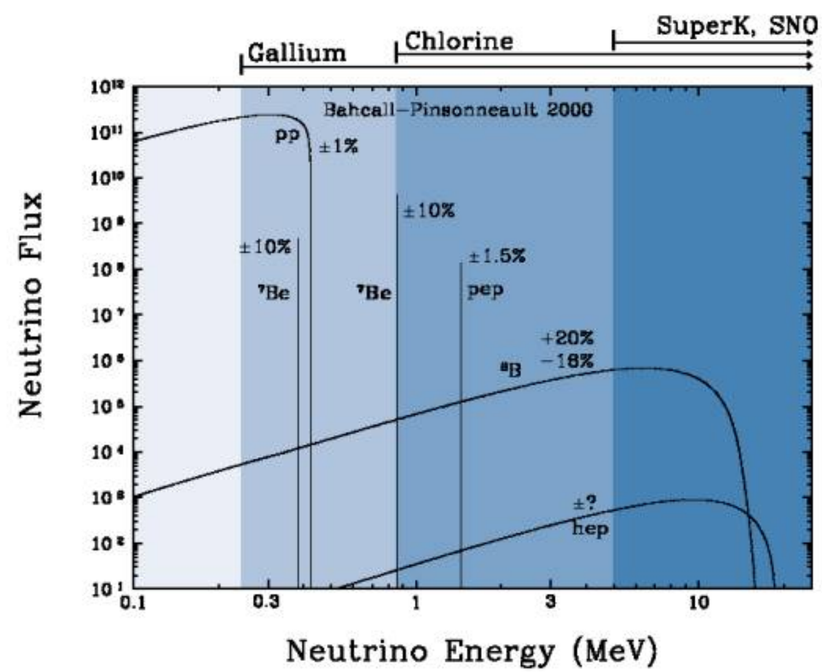
historically Δm_{21}^2 and $\sin^2 \theta_{12}$ were first determined by solving the solar neutrino problem, i.e. the disappearance of about one third of solar electron neutrino flux, for solar neutrinos above few MeV. The desire of detecting solar neutrinos, to confirm the thermodynamics of the sun, was the driving motivation for the whole field for more than 30 years. Electron solar neutrinos oscillate, but the formalism requires the introduction of matter effects, since the electron density in the sun is not negligible. Experiments: SuperKamiokande, SNO, Borexino

Solar Neutrinos



ν_e produced in the core of the sun through several chains/reactions

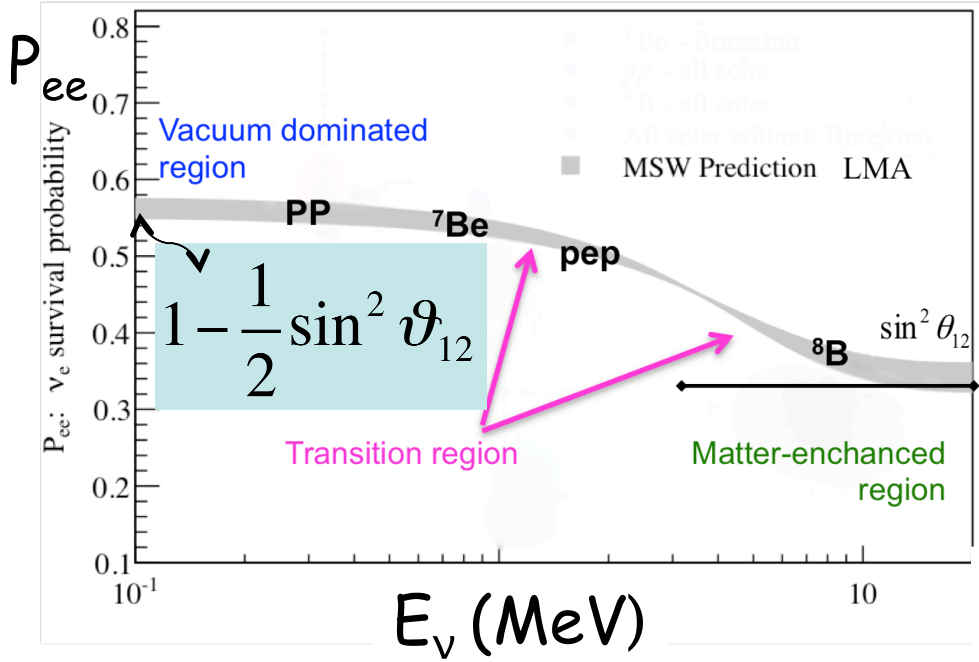
with different energy spectrum



most neutrinos come from pp fusion $E_{max} \approx 0.4$ MeV

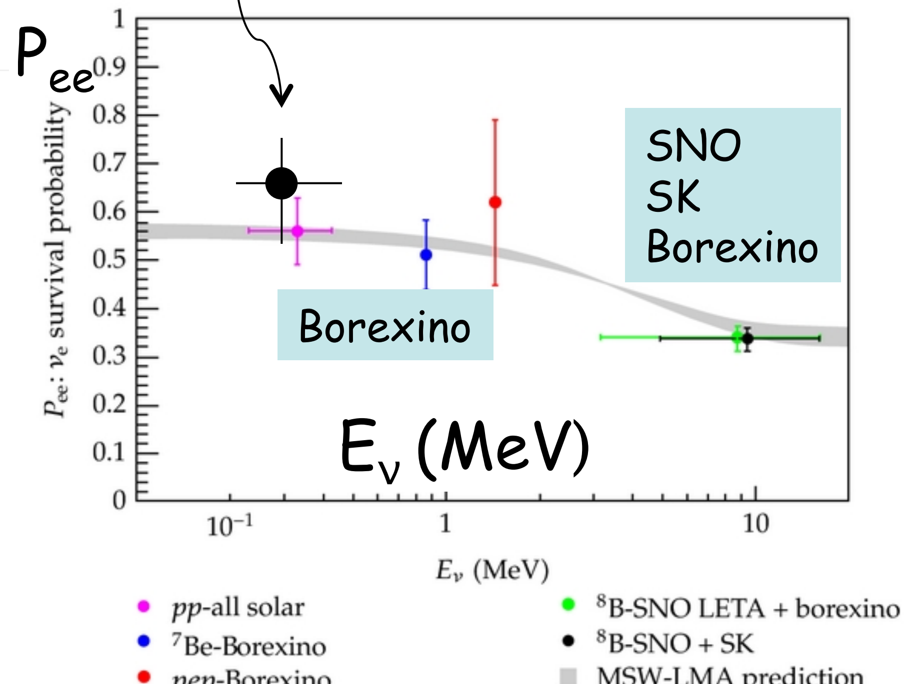
most energetic neutrinos come from ^{8}B decay $E_{max} \approx 15$ MeV

Theory prediction for P_{ee}



$\sin^2\vartheta_{12}$

[Borexino, Nature 512 (2014) 383]



experiments reveal solar neutrinos through different processes and have different energy thresholds

ϑ_{13} from disappearance experiments

These experiments have been realized with reactors. Electron anti-neutrinos are produced by a reactor (E≈3 MeV, L≈1 Km) (by CPT the survival probability in vacuum is the same for neutrinos and antineutrinos and matter effects are negligible).
In this range of (L,E) oscillations driven by Δm^2_{21} are negligible and the survival probability P_{ee} only depends on ($|U_{e3}|$, Δm^2_{31}).

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}}\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$E \approx 3\ MeV$$
$$L \approx 1\ Km$$

Experiment	Near Detectors	Far Detectors
CHOOZ (France)	–	(1) 1050m
Double CHOOZ	> 2014	(1) 1050m
Reno (Korea)	(1) 290m	(1) 1380m
Daya Bay (China)	(4) (360-530)m	(4) (1600-2000)m

before 2012 there was only an upper bound on $|U_{e3}|$ by CHOOZ
today (end 2014) the value of ϑ_{13} is dominated by the Daya Bay result

$$\sin^2 2\vartheta_{13} = 0.085 \pm 0.005 \qquad |\Delta m^2_{13}| = 2.44^{+0.10}_{-0.11} \times 10^{-3}\ eV^2$$

$$|U_{e3}|^2 = \sin^2\vartheta_{13} = 0.0215 \pm 0.0013 \qquad \vartheta_{13} = (8.4 \pm 0.3)^0$$

ϑ_{13} from appearance experiments

These experiments use a muon-neutrino beam from an accelerator and look for conversion of muon-neutrinos into electron-neutrinos. The (L,E) range is such that they are mainly sensitive to Δm^2_{31}

Experiment	E(GeV)	L(Km)
T2K (Japan)	0.6	295
MINOS (USA)	3	735

at the LO (neglecting Δm^2_{21} and matter effects)

$$P_{\mu e} = 4|U_{\mu 3}|^2 |U_{e3}|^2 \sin^2\Delta = \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

however in this case corrections from Δm^2_{21} and matter effects are non-negligible

EXERCISE

by expanding $P_{\mu e}$ to first order in $\alpha = \Delta m^2_{21}/\Delta m^2_{13}$ show that

$$\begin{aligned} P_{\mu e} \quad = \quad & \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\sin^2\Delta_{13} \\ & -8\alpha J_{CP}\Delta_{13}\sin^2\Delta_{13} \\ & -8\alpha J_{CP}\frac{\cos\delta}{\sin\delta}\Delta_{13}\cos\Delta_{13}\sin\Delta_{13} \\ & +O(\alpha^2)+ \textit{matter effects} \end{aligned}$$

$$\Delta_{13} = \frac{\Delta m^2_{31}L}{4E}$$

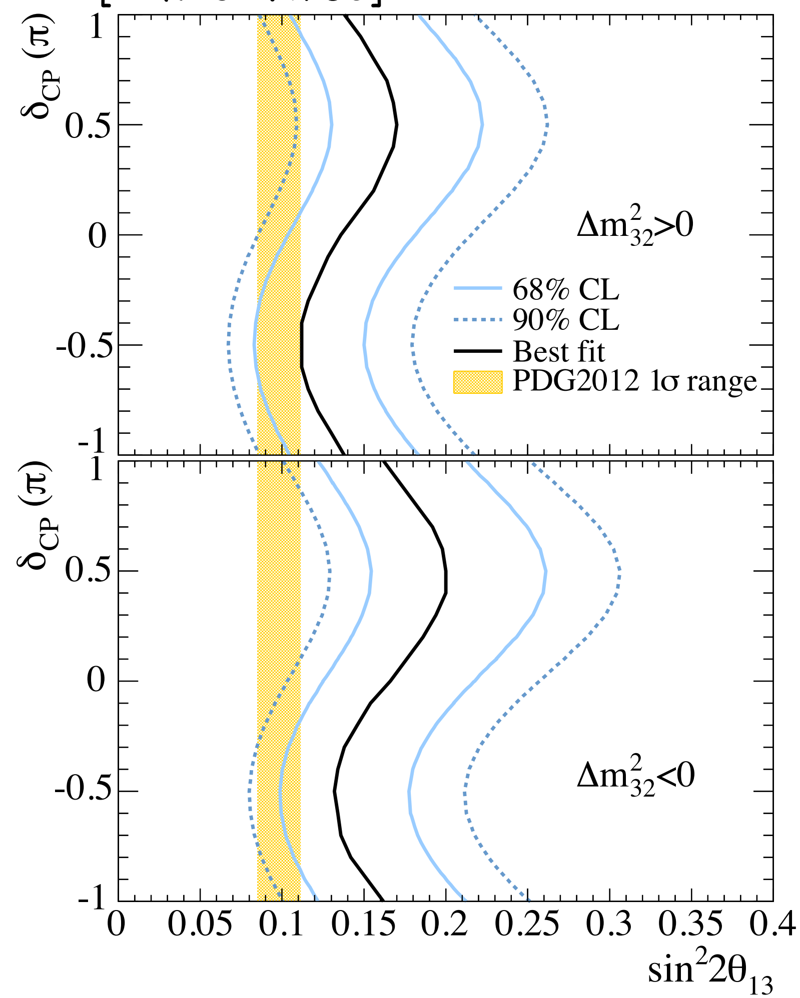
$$\begin{aligned} J_{CP} &= \mathrm{Im}\left(U_{\mu 3}U^*_{e3}U^*_{\mu 2}U_{e2}\right) \\ &= \frac{1}{8}\sin 2\vartheta_{12}\sin 2\vartheta_{23}\sin 2\vartheta_{13}\cos\vartheta_{13}\sin\delta \end{aligned}$$

T2K works near the first oscillation maximum where $|\Delta_{13}|=\pi/2$

$$P_{\mu e} = \sin^2\vartheta_{23}\sin^2 2\vartheta_{13} - 4\pi|\alpha| J_{CP} + O(\alpha^2) + \textit{matter effects}$$

the relative subleading corrections are O(20%) and are sensitive to sinδ

[T2K 1311.4750]



At present (end 2014) agreement with the value of ϑ_{13} determined by reactor disappearance experiments requires

$$\sin\delta \approx -1$$

$$\delta \approx \frac{3}{2}\pi$$

i.e. maximal CP violation in the lepton sector

main detection processes

Neutrinos	Experiment	Process
atmospheric ν	SK K2K, MINOS, T2K, Opera	$\nu N \to l X$
solar ν	SK, Borexino SNO	$\nu_X e \to \nu_X e$ $\nu_X D \to \nu_X pn, \nu_e D \to e pp$
reactor ν	KamLand, Chooz, DoubleChooz, Reno, Daya Bay	$\bar{\nu}_e p \to e^+ n \quad (e^+ D\gamma)$

Summary of data

$m_\nu < 2.2\ eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

$$\Delta m^2_{atm} \equiv \begin{cases} \Delta m^2_{31} = (2.462 \pm 0.033) \times 10^{-3}\ \mathrm{eV}^2 & \text{NO} \\ \Delta m^2_{32} = -(2.453 \pm 0.047) \times 10^{-3}\ \mathrm{eV}^2 & \text{IO} \end{cases}$$

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.55^{+0.18}_{-0.17}) \times 10^{-5}\ \mathrm{eV}^2$$

$$\sin^2 \vartheta_{13} = 0.0223^{+0.0011}_{-0.0010} \qquad \delta_{CP} = (259^{+76}_{-69})$$

$$\sin^2 \vartheta_{23} = [0.451^{+0.026}_{-0.020}] \oplus [0.580^{+0.024}_{-0.039}]$$

$$\sin^2 \vartheta_{12} = 0.311^{+0.013}_{-0.012}$$

[G.-Garcia, Maltoni, Salvado, Schwetz 1209.3023]

violation of individual lepton number implied by neutrino oscillations

Summary of unkowns

absolute neutrino mass scale is unknown [but well-constrained!]

sign $[\Delta m^2_{atm}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

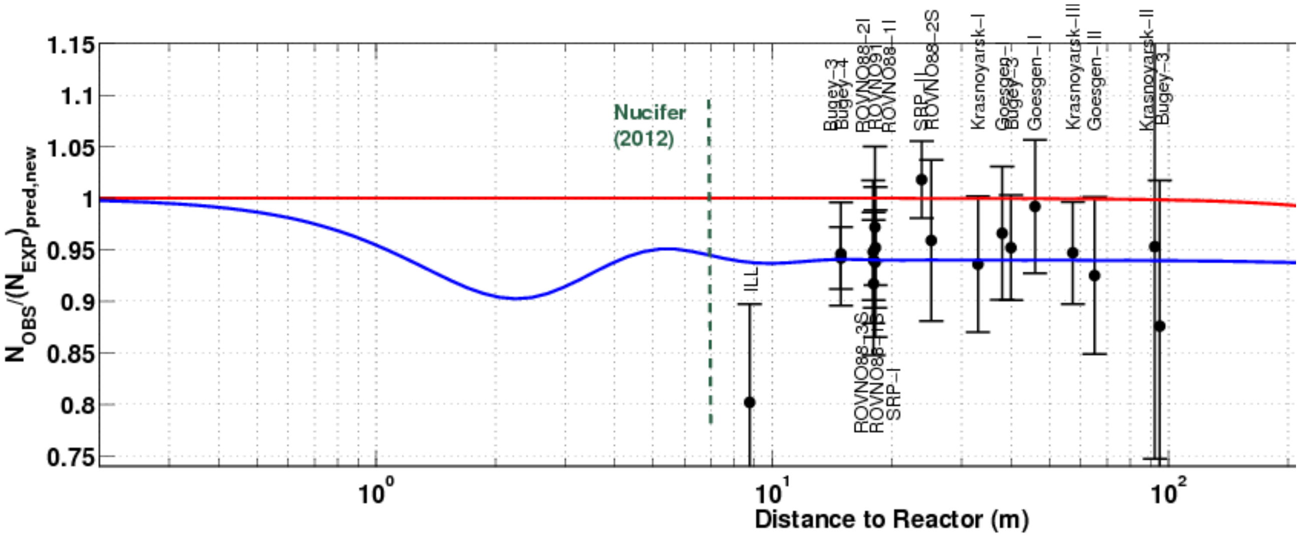
δ, α, β unknown

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established sterile neutrinos ?

1 reactor anomaly (anti-ν_e disappearance)

re-evaluation of reactor anti-ν_e flux: new estimate 3.5% higher than old one



$$(\Phi_{exp} - \Phi_{th}) / \Phi_{th} \approx -6\%$$

[th. uncertainty?]

very SBL L ≲ 100 m

$$\vartheta_{es} \approx 0.2$$

$$\Delta m^2 \approx m_s^2 \geq 1\, eV^2$$

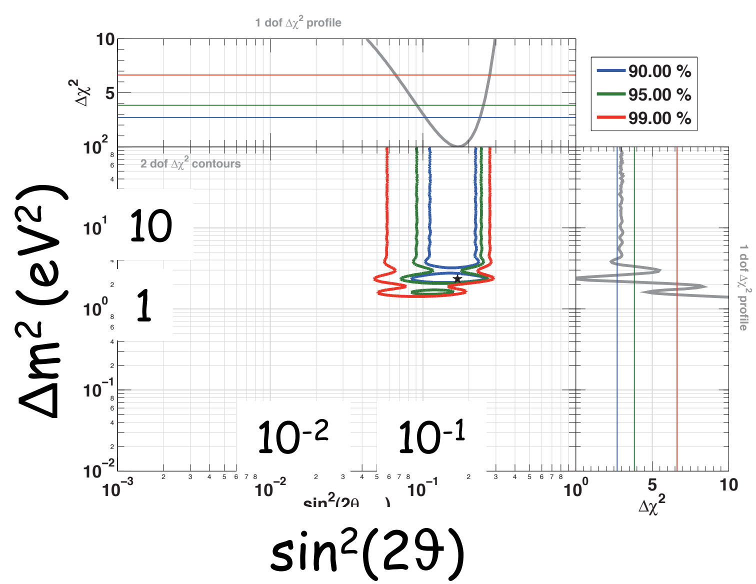
10

supported by the Gallium anomaly

ν_e flux measured from high intensity radioactive sources in Gallex, Sage exp

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

[error on σ or on Ge extraction efficiency]



… but disfavoured by cosmological limits

2 long-standing claim

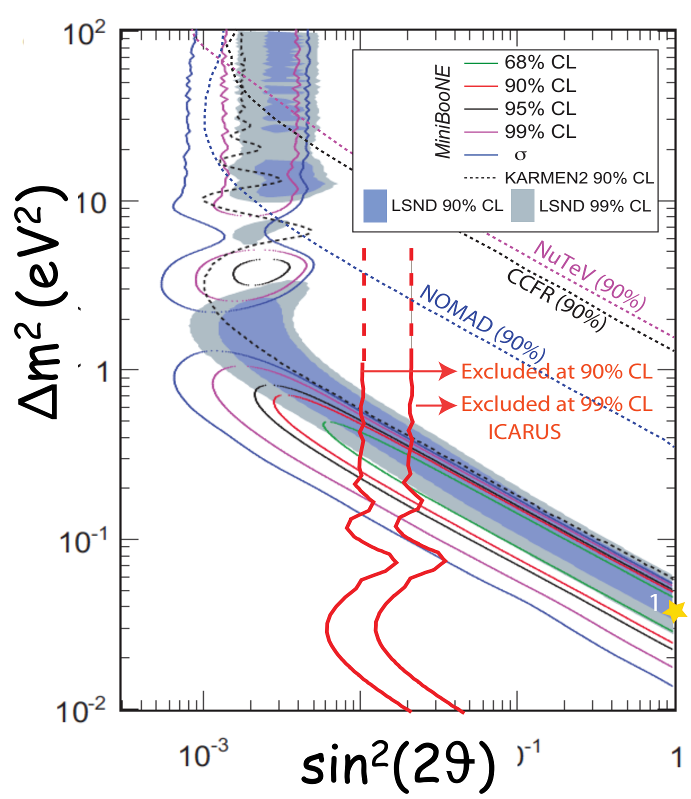
evidence for $\nu_\mu \to \nu_e$ appearance in accelerator experiments

exp		$E(MeV)$	$L(m)$	
$LSND$	$\bar{\nu}_\mu \to \bar{\nu}_e$	$10 \div 50$	30	3.8σ
$MiniBoone$	$\nu_\mu \to \nu_e$ $\bar{\nu}_\mu \to \bar{\nu}_e$	$300 \div 3000$	541	3.8σ [signal from low-energy region]

parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

$$\Delta m^2 \approx 0.5\, eV^2$$



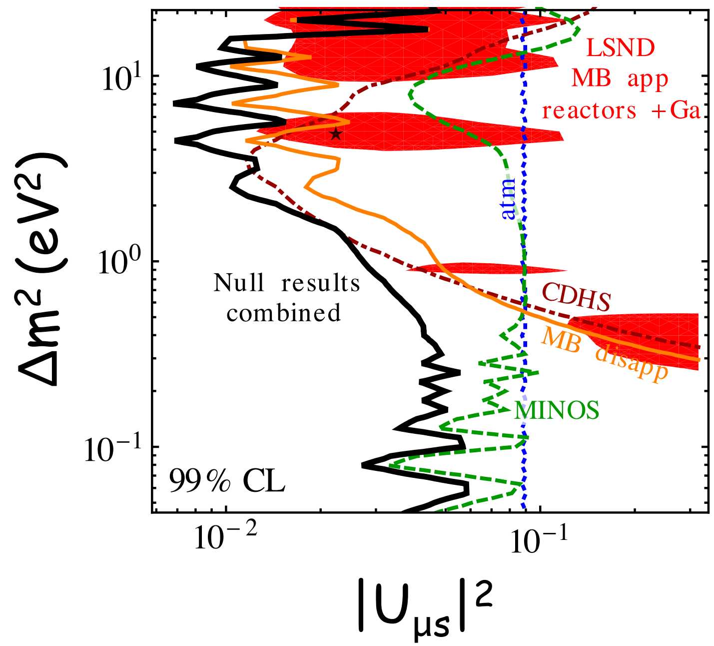
3

interpretation in 3+1 scheme: inconsistent (more than 1s disfavored by cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \;\Rightarrow\; \vartheta_{\mu s} \approx 0.2$$

predicted suppression in ν_μ disappearance experiments: undetected

by ignoring LSND/Miniboone data the reactor anomaly can be accommodated by $m_s \geq 1$ eV and $\vartheta_{es} \approx 0.2$ [not suitable for Warm DM]



GGI lectures on the theory of fundamental interactions 2015

Firenze, 12-16 January 2015

Aspects of neutrino physics (II)
Neutrino Masses, Mixing and Oscillations: Implication for Physics BSM

Ferruccio Feruglio
Universita' di Padova

Lecture 1
Neutrino Masses

Summary of data

$m_\nu < 2.2\ eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

$$\Delta m^2_{atm} \equiv \begin{cases} \Delta m^2_{31} = (2.462 \pm 0.033) \times 10^{-3}\ \mathrm{eV}^2 & \text{NO} \\ \Delta m^2_{32} = -(2.453 \pm 0.047) \times 10^{-3}\ \mathrm{eV}^2 & \text{IO} \end{cases}$$

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.55^{+0.18}_{-0.17}) \times 10^{-5}\ \mathrm{eV}^2$$

$$\sin^2 \vartheta_{13} = 0.0223^{+0.0011}_{-0.0010} \qquad \delta_{CP} = (259^{+76}_{-69})$$

$$\sin^2 \vartheta_{23} = [0.451^{+0.026}_{-0.020}] \oplus [0.580^{+0.024}_{-0.039}]$$

$$\sin^2 \vartheta_{12} = 0.311^{+0.013}_{-0.012}$$

[G.-Garcia, Maltoni, Salvado, Schwetz 1209.3023]

violation of individual lepton number implied by neutrino oscillations

Summary of unkowns

absolute neutrino mass scale is unknown [but well-constrained!]

sign $[\Delta m^2_{atm}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

δ, α, β unknown

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established

Beyond the Standard Model

a non-vanishing neutrino mass is the first evidence of the incompleteness of the Standard Model [SM]

in the SM neutrinos belong to SU(2) doublets with hypercharge Y=-1/2
they have only two helicities (not four, as the other charged fermions)

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2)$$

the requirement of invariance under the gauge group G=SU(3)xSU(2)xU(1) forbids pure fermion mass terms in the lagrangian. Charged fermion masses arise, after electroweak symmetry breaking, through gauge-invariant Yukawa interactions

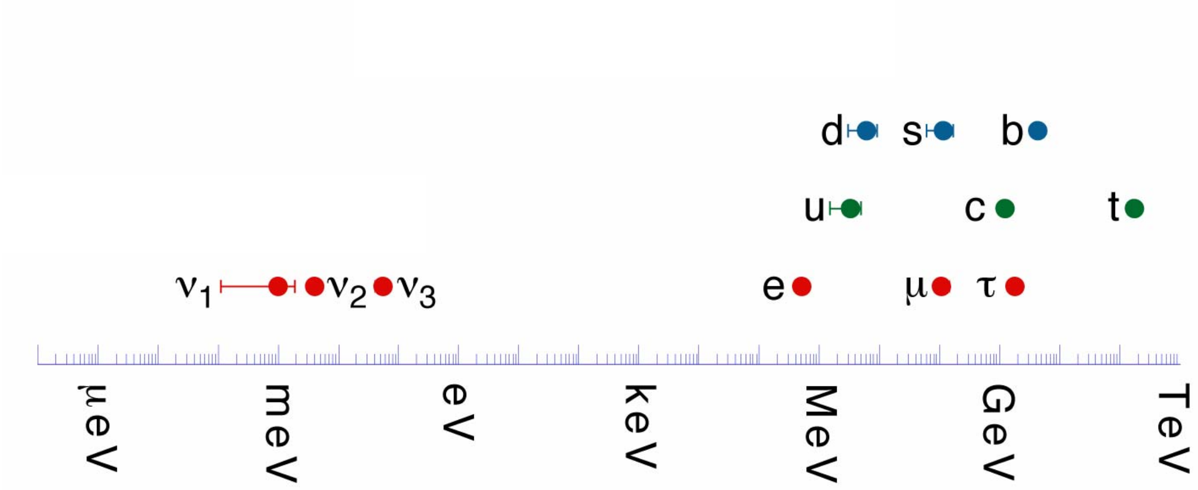
$$\Phi \quad \underbrace{\Psi\Psi'}_{\text{same helicity}}$$

not even this term is allowed for SM neutrinos, by gauge invariance

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

How to modify the SM?

the SM, as a consistent QFT, is completely specified by

0. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

1. particle content — three copies of (q, u^c, d^c, l, e^c)
   one Higgs doublet Φ

2. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(0.+1.+2.) leads to the SM Lagrangian, L_{SM}, possessing an additional, accidental, global symmetry: (B-L) -> EXERCISE

0. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!
   We could extend G, but, to allow for neutrino masses, we need to modify 1. (and/or 2.) anyway…

Exercise 1: anomalies of B and L_i

the anomaly of the baryonic current and the individual leptonic currents are proportional to tr[Q {T^A,T^B}] and tr[Q {Y,Y}] where Q=(B,L_i) and (T^A,Y) are the generators of the electroweak gauge group

compute these traces in the SM with 3 fermion generations

$$\frac{1}{2}\mathrm{tr}[B\{T^A,T^B\}] = 3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{4}(up)+\frac{1}{4}(down)\right]\delta^{AB} = \frac{3}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{T^A,T^B\}] = 1(L_i)\times\left[\frac{1}{4}(nu)+\frac{1}{4}(e)\right]\delta^{AB} = \frac{1}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[B\{Y,Y\}] = 3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{18}(Doubl)-\frac{10}{18}(Singl)\right] = -\frac{3}{2}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{Y,Y\}] = 1(L_i)\times\left[\frac{1}{2}(Doubl)-1(Singl)\right] = -\frac{1}{2}$$

(B+L) is anomalous, (B/3-L_i) [and (B-L)] are anomaly-free

First possibility: modify (1), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

- add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)
- ask for (global) invariance under B-L (no more automatically conserved as in the SM)

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = -d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) - e^c y_e (\Phi^+ l) - \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

a generic problem of this approach

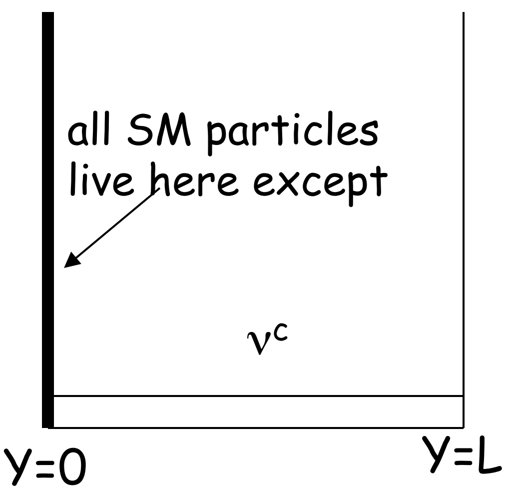
the particle content can be modified in several different ways in order to account for non-vanishing neutrino masses (additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,...). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \le 10^{-12}$$

Quite a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$

$$= \frac{1}{\sqrt{L}} \nu_0^c (\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale) then neutrino Yukawa coupling is suppressed

Second possibility: abandon (2) renormalizability

A disaster?

$$L = L^{SM}_{d\leq 4} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

a new scale Λ enters the theory. The new (gauge invariant!) operators L_5, L_6,... contribute to amplitudes for physical processes with terms of the type

$$\frac{L_5}{\Lambda} \rightarrow \frac{E}{\Lambda} \qquad \frac{L_6}{\Lambda^2} \rightarrow \left(\frac{E}{\Lambda}\right)^2 \qquad \ldots$$

the theory cannot be extrapolated beyond a certain energy scale $E \approx \Lambda$.
[at variance with a renormalizable (asymptotically free) QFT]

If $E \ll \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \approx 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will *look like* a renormalizable theory!

$$\frac{E}{\Lambda} \approx \frac{10^2\, GeV}{10^{15}\, GeV} = 10^{-13}$$

an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top} !

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{\left(\tilde{\Phi}^+ l\right)\left(\tilde{\Phi}^+ l\right)}{\Lambda} =$$

$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a unique operator!
[up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ) with respect to the neutrino mass term of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = -\nu^c y_\nu (\tilde{\Phi}^+ l) - \frac{1}{2}\nu^c M \nu^c + h.c.$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out'' the field ν^c

$$L_{eff}(l) = \frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + \ldots$$

terms suppressed by more powers of M^{-1}

this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.

Exercise 2

derive the see-saw relation by integrating out the fields ν^c through their e.o.m. in the heavy M limit. Compute the 1st order corrections in p/M

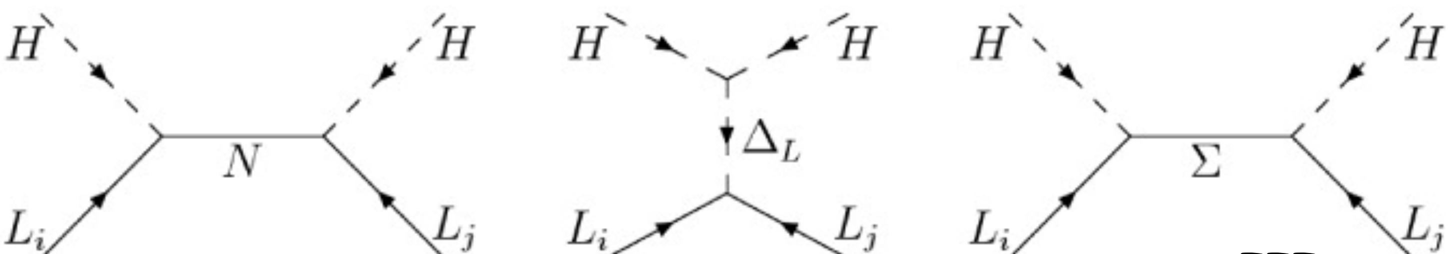
equations of motion of ν^c

$$\begin{pmatrix} \nu^c \\ \bar{\nu}^c \end{pmatrix} = \begin{pmatrix} i\bar{\sigma}^\mu \partial_\mu & -M^+ \\ -M & i\sigma^\mu \partial_\mu \end{pmatrix}^{-1} \begin{pmatrix} y_\nu^* \bar{\omega} \\ y_\nu \omega \end{pmatrix} = \begin{pmatrix} -M^{-1} y_\nu \omega \\ -M^{*-1} y_\nu^* \bar{\omega} \end{pmatrix} + \ldots \qquad \omega \equiv (\tilde{\Phi}^+ l)$$

$$L_{eff} = i\bar{l}\,\bar{\sigma}^\mu \partial_\mu l + \frac{1}{2}\left[\omega(y_\nu^T M^{-1} y_\nu)\omega + h.c.\right] + i\bar{\omega}(y_\nu^+ M^{+-1} M^{-1} y_\nu)\bar{\sigma}^\mu \partial_\mu \omega + O(M^{-3})$$

d-=5 d-=6 renormalizes the KE of ν by v^2/M^2

there are 3 types of see-saw depending on the particle we integrate out
they all give rise to the same d=5 operator



type I $\qquad$ type II $\qquad$ type III

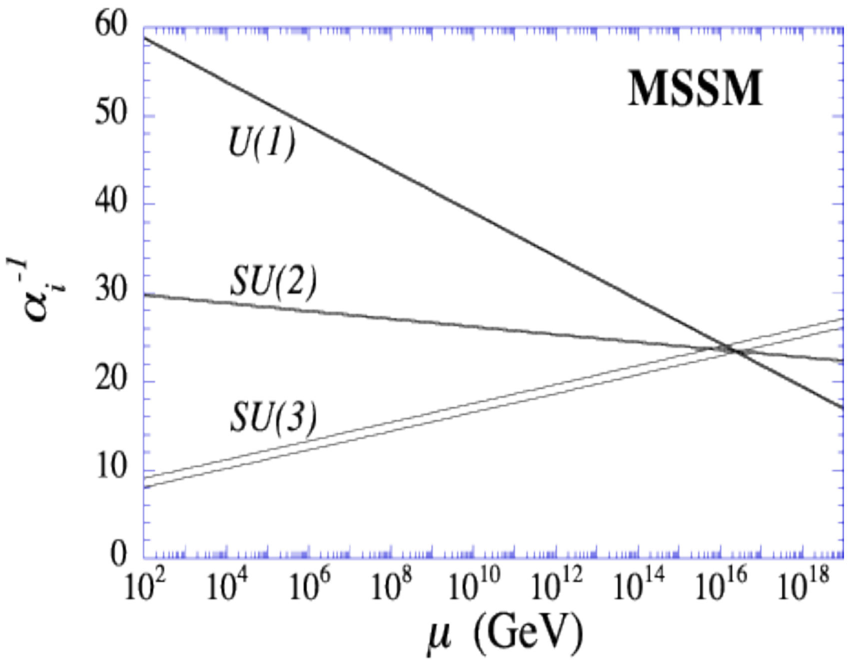
$$y_N^T (M_N)^{-1} y_N \qquad y_\Delta \frac{\mu}{M_\Delta^2} \qquad y_\Sigma^T (M_\Sigma)^{-1} y_\Sigma$$

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs):
the SM gauge group G is embedded into a simple group such as SU(5), SO(10),...



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

Unity of All Elementary-Particle Forces
Phys. Rev. Lett. 32, (1974) 438
Howard Georgi and S. L. Glashow

Georgi, H.; Quinn, H.R. and Weinberg, S.
Hierarchy of interactions in unified gauge theories.
Phys. Rev. Lett. 33 (1974) 451

Exercise 3: gauge coupling unification

0^{th} order approximation

justify this
$$\sqrt{\frac{5}{3}}g_Y = g_2 = g_3 \qquad \sin^2\vartheta_W = \frac{g_Y^2}{g_Y^2+g_2^2} = \frac{3}{8} \approx 0.375$$

include 1-loop running

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(m_Z)} + \frac{b_i}{2\pi}\log\frac{Q}{m_Z}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{MSSM} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{SM} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}$$

knowledge of b.c. M_{GUT} and $\alpha_U = \alpha(M_{GUT})$ would allow to predict $\alpha_i(m_Z)$
in practice, we use as inputs

$$\alpha_{em}^{-1}(m_Z)\Big|_{\overline{MS}} = 127.934 \qquad \sin^2\vartheta(m_Z)\Big|_{\overline{MS}} = 0.231$$

to predict
[MSSM]

$$\alpha_3(m_Z)\Big|_{\overline{MS}} = \frac{7\alpha_{em}(m_Z)}{15\sin^2\vartheta(m_Z)-3} \approx 0.118$$

$$\alpha_U = \frac{28\alpha_{em}(m_Z)}{36\sin^2\vartheta(m_Z)-3} \approx \frac{1}{25}$$

$$\log\left(\frac{M_{GUT}}{m_Z}\right) = \pi\frac{3-8\sin^2\vartheta(m_Z)}{14\alpha_{em}(m_Z)} \quad\Rightarrow\quad M_{GUT} \approx 2\times 10^{16}\,\text{GeV}$$

[corrections from 2-loop RGE,
threshold corrections at M_{SUSY},
threshold corrections at M_{GUT}]

Exercise 4: effective lagrangian for nucleon decay

recognize that, the with the SM particle content, the lowest dimensional operators violating B occur at d=6. Make a list of them

$$\frac{1}{\Lambda_B^2} \times \begin{cases} q q u^{c+} e^{c+} & q q q l \\ q l u^{c+} d^{c+} & u^c u^c d^c e^c \end{cases}$$

color and SU(2) indices contracted

notice that they respect $\Delta B=\Delta L$: nucleon decay into antileptons
e.g. $p \to e^+ \pi^0$, $n \to e^+ \pi^-$ [$n \to e^- \pi^+$ suppressed by further powers of Λ_B]

naïve estimate

$$\tau_p \approx \frac{\Lambda_B^4}{m_p^5}$$

assuming

$$\tau_p(p \to e^+ \pi^0) > 1.4 \times 10^{34} \, ys \qquad \text{[SK]}$$

we get

$$\Lambda_B > 2.6 \times 10^{16} \quad \text{GeV}$$

in GUTs Λ_B is related to the scale M_{GUT} at which the grand unified symmetry is broken down to SM gauge group
the observed proton stability is guaranteed by the largeness of M_{GUT}

In SUSY extensions of the SM the lowest dimensional operators violating B occur at d=5: why?

flavor puzzle made simpler in SU(5) ?

$\bar{5} = (l, d^c) \qquad 10 = (q, u^c, e^c) \qquad 1 = \nu^c$

Higgs

$\Phi_5 = (\Phi_D, \Phi_T)$

$$L_Y = -10\, y_u\, 10\, \Phi_5 - \bar{5}\, y_d\, 10\, \Phi_5^+ - 1\, y_\nu\, \bar{5}\, \Phi_5 - \frac{1}{2} 1 M 1 + h.c.$$

$y_d = y_e^T$

$m_b = m_\tau$ — O.K.

$m_s = m_\mu$, $m_d = m_e$ — wrong, but not by orders of magnitude; can be fixed with additional Higgs: $m_s \approx m_\mu / 3$, $m_d \approx 3\, m_e$

suppose that y_u, y_e, y_ν and M/Λ are anarchical matrices [O(1) matrix elements] and that the observed hierarchy is due to the wave function renormalization of matter multiplets (we will see how later on)

$$10 \to F_{10}\, 10 \qquad \bar{5} \to F_{\bar{5}}\, \bar{5} \qquad 1 \to F_1\, 1$$

$$F_X = \begin{pmatrix} \lambda^{Q_{X_1}} & 0 & 0 \\ 0 & \lambda^{Q_{X_2}} & 0 \\ 0 & 0 & \lambda^{Q_{X_3}} \end{pmatrix}$$

$\lambda \approx 0.22$

$Q_{X_1} \geq Q_{X_2} \geq Q_{X_3}$

F_1 dependence cancels in m_ν

$\Upsilon_u = F_{10}\, y_u\, F_{10} \qquad \Upsilon_d = F_{\bar{5}}\, y_d\, F_{10} \qquad \Upsilon_e = F_{10}\, y_d^T\, F_{\bar{5}} \qquad m_\nu \propto F_{\bar{5}}\, y_\nu^T M^{-1} y_\nu\, F_{\bar{5}}$

large mixing in lepton sector suggests $F_{\bar{5}} \approx \mathrm{diag}(1,1,1)$

hierarchy mostly due to F_{10} $\quad m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$

large l mixing corresponds to a large d^c mixing: unobservable in weak int. of quarks

how can a wave function renormalization (effectively) arise?

several possibilities

here (Exercise 5): bulk fermions in a compact extra dimension S^1/Z_2

$$\mathcal{L} = i\bar{\Psi}_1\Gamma^M\partial_M\Psi_1 + i\bar{\Psi}_2\Gamma^M\partial_M\Psi_2 - m_1\varepsilon(y)\bar{\Psi}_1\Psi_1 + m_2\varepsilon(y)\bar{\Psi}_2\Psi_2 - \left[\delta(y)\frac{y}{\Lambda}\bar{f}_1(h+v)f_2 + h.c.\right]$$

$$\Psi_1 = \begin{pmatrix} E_1 \\ \bar{f}_1 \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_2 \\ \bar{E}_2 \end{pmatrix}$$

solve the e.o.m. for the fermion zero modes with the b.c.

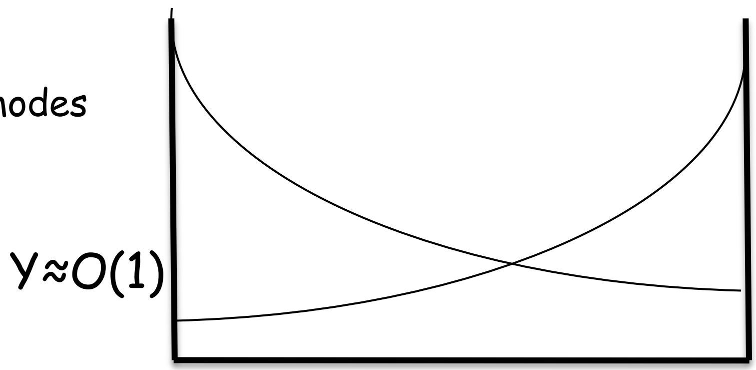
$$-\gamma_5\partial_y\Psi^0_{1,2} \pm m_{1,2}\,\varepsilon(y)\Psi^0_{1,2} = 0$$

$$\Psi_1(-y) = +\gamma_5\Psi_1(y)$$

$$\Psi_2(-y) = -\gamma_5\Psi_2(y)$$

$$f_i^0(y) = \sqrt{\frac{2m_i}{1-e^{-2m_i\pi R}}}e^{-m_i y}$$

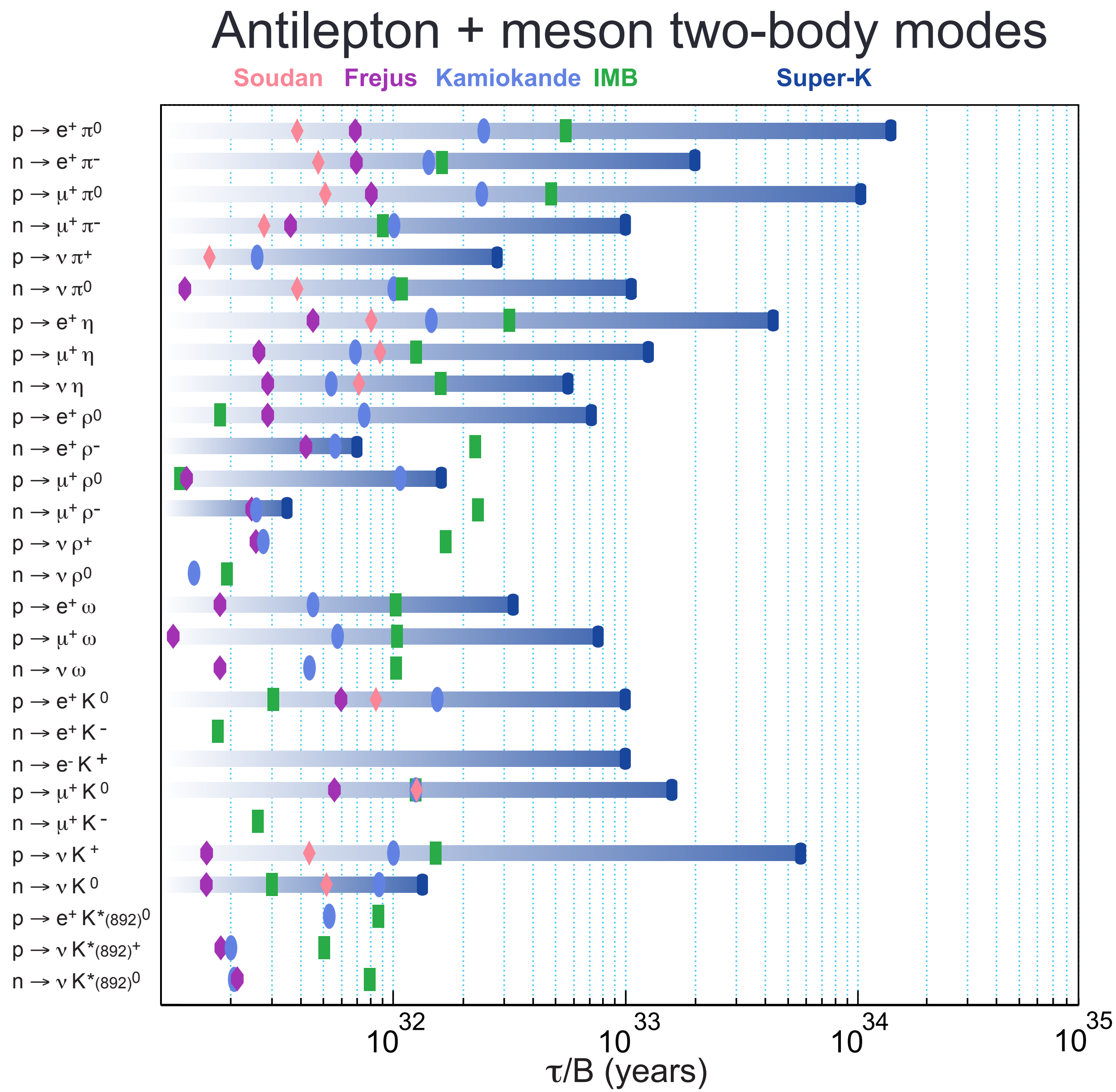
vanishing zero-modes for $(E_1, \bar{E}_2)$



$$\mathcal{L}_Y = -\frac{1}{\Lambda\pi R}\bar{f}_1(F_1 y F_2)(h+v)f_2$$

$$F_i = \sqrt{\frac{x_i}{1-e^{-x_i}}} \approx \begin{cases} e^{-x_i/2} & x_i >> 1 \\ 1 & x_i \approx 0 \\ \sqrt{-x_i} & x_i << -1 \end{cases}$$

Back up slides



Antilepton + meson two-body modes
Soudan
Frejus
Kamiokande
IMB
Super-K
p → e+ π0
n → e+ π-
p → μ+ π0
n → μ+ π-
p → ν π+
n → ν π0
p → e+ η
p → μ+ η
n → ν η
p → e+ ρ0
n → e+ ρ-
p → μ+ ρ0
n → μ+ ρ-
p → ν ρ+
n → ν ρ0
p → e+ ω
p → μ+ ω
n → ν ω
p → e+ K0
n → e+ K-
n → e- K+
p → μ+ K0
n → μ+ K-
p → ν K+
n → ν K0
p → e+ K*(892)0
p → ν K*(892)+
n → ν K*(892)0
10^32
10^33
10^34
10^35
τ/B (years)

Flavor symmetries I (the hierarchy puzzle)

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

call ξ_i the generic small parameter. A modern approach to understand why $\xi_i << 1$ consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When $\xi_i = 0$ the theory becomes invariant under a flavour symmetry F

Example: why $y_e << y_{top}$? Assume $F = U(1)_F$

$F(t)=F(t^c)=F(h)=0$ — $y_{top}(h+v)t^c t$ — allowed

$F(e^c)=p>0$ $F(e)=q>0$ — $y_e(h+v)e^c e$ — breaks $U(1)_F$ by (p+q) units

if $\xi = \langle\varphi\rangle/\Lambda < 1$ breaks U(1) by one negative unit — $y_e \approx O(\xi^{p+q}) << y_{top} \approx O(1)$

provides a qualitative picture of the existing hierarchies in the fermion spectrum

GGI lectures on the theory of fundamental interactions 2015

Firenze, 12-16 January 2015

Aspects of neutrino physics (III)
Neutrino Masses, Mixing and Oscillations: Leptogenesis and Hierarchy problem

Ferruccio Feruglio
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The see-saw (continue)

2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu\right] v^2$$

example

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

$\delta \ll 1$
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} \ll \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\bar{B}})}{s} \approx 6 \times 10^{-10}$$

Sakharov conditions met by the see-saw theory
1. (B-L) violation at high-temperature and (B+L) violation by pure SM interactions
2. C and CP violation by additional phases in see-saw Lagrangian (more on this later)
3. out-of-equilibrium condition

restrictions imposed by leptogenesis on neutrinos

here: thermal leptogenesis
dominated by lightest ν^c
no flavour effects]

active neutrinos should be light

out-of-equilibrium controlled by rate of RH neutrino decays

$$\frac{M_1}{8\pi}(y_\nu y_\nu^+)_{11} < \left.\frac{T^2}{M_{Pl}}\right|_{T\approx M_1}$$

$$\frac{(y_\nu y_\nu^+)_{11} v^2}{M_1} \equiv \tilde{m}_1 < 10^{-3} \text{ eV}$$

Exercise 6: compute this

more accurate estimate $m_i < 0.15$ eV

RH neutrinos should be heavy

[efficiency factor ≤1
washout effects]

$$\eta_B \approx 10^{-2}\, \varepsilon_1\, \eta$$

$$\varepsilon_1 = \frac{\Gamma(\nu_1^c \to l\Phi) - \Gamma(\nu_1^c \to \bar{l}\Phi^*)}{\Gamma(\nu_1^c \to l\Phi) + \Gamma(\nu_1^c \to \bar{l}\Phi^*)} = -\frac{3}{16\pi}\sum_{j=2,3}\frac{M_1}{M_j}\frac{\mathrm{Im}\{[(yy^+)_{1j}]^2\}}{(yy^+)_{11}} \approx 0.1\times\frac{M_1 m_i}{v^2}$$

[Yukawas y in mass eigenstate basis for ν^c_i]

$M_1 > 6\times 10^8$ GeV

more refined bound [Davidson and Ibarra 0202239]

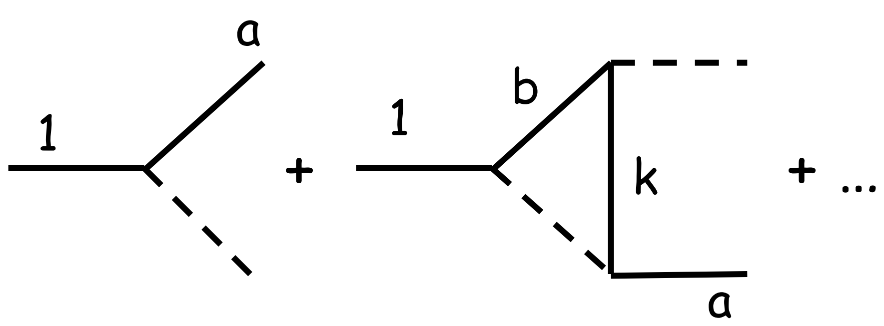
$$\left|\varepsilon_1^{\infty}\right| \leq \varepsilon_1^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}(m_3 - m_1)$$

$$T_R \approx M_1 > (4\times 10^8 \div 2\times 10^9)\ GeV$$

in conflict with the bound on T_R in SUSY models to avoid overproduction of gravitinos

$$T_R^{SUSY} < 10^{7-9}\ GeV$$

Exercise 7: reconstruct the flavour structure of ε_1



$$\mathcal{A}(\nu_1^c \to l_a \Phi) \propto y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}$$

$$\mathcal{A}(\nu_1^c \to \bar{l}_a \Phi^*) \propto y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}$$

$$\varepsilon_1 \propto \frac{\left|y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}\right|^2 - \left|y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}\right|^2}{\left|y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}\right|^2 + \left|y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}\right|^2} \approx \frac{\mathrm{Im}(W)\,\mathrm{Im}\{[(yy^+)_{1k}]^2\}}{(yy^+)_{11}}$$

[sums understood]

$$\mathrm{Im}(W) \approx \frac{M_1}{M_k}$$

Exercise 8: count the number of physical parameters in the type I see-saw model distinguish between moduli and phases

y_e, y_ν and M depend on (18+18+12)=48 parameters, 24 moduli and 24 phases

we are free to choose any basis leaving the kinetic terms canonical (and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad \nu^c \to \Omega_{\nu^c} \nu^c \qquad l \to \Omega_l l \qquad [U(3)^3]$$

these transformations contain 27 parameters (9 angles and 18 phases) and effectively modify y_e, y_ν and M

$$y_e \to \Omega^T_{e^c} y_e \Omega_l \qquad y_\nu \to \Omega^T_{\nu^c} y_\nu \Omega_l \qquad M \to \Omega^T_{\nu^c} M \Omega_{\nu^c}$$

so that we can remove 27 parameters from y_e, y_ν and M

we remain with 21 parameters: 15 moduli and 6 phases
the moduli are 9 physical masses and 6 mixing angles

the same count in the quark sector would give a total of 9 moduli (6 masses amd 3 mixing angles) and 0 phases <- wrong
how the above argument should be modified, in general?

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles and 3 phases, as in lecture 1

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is

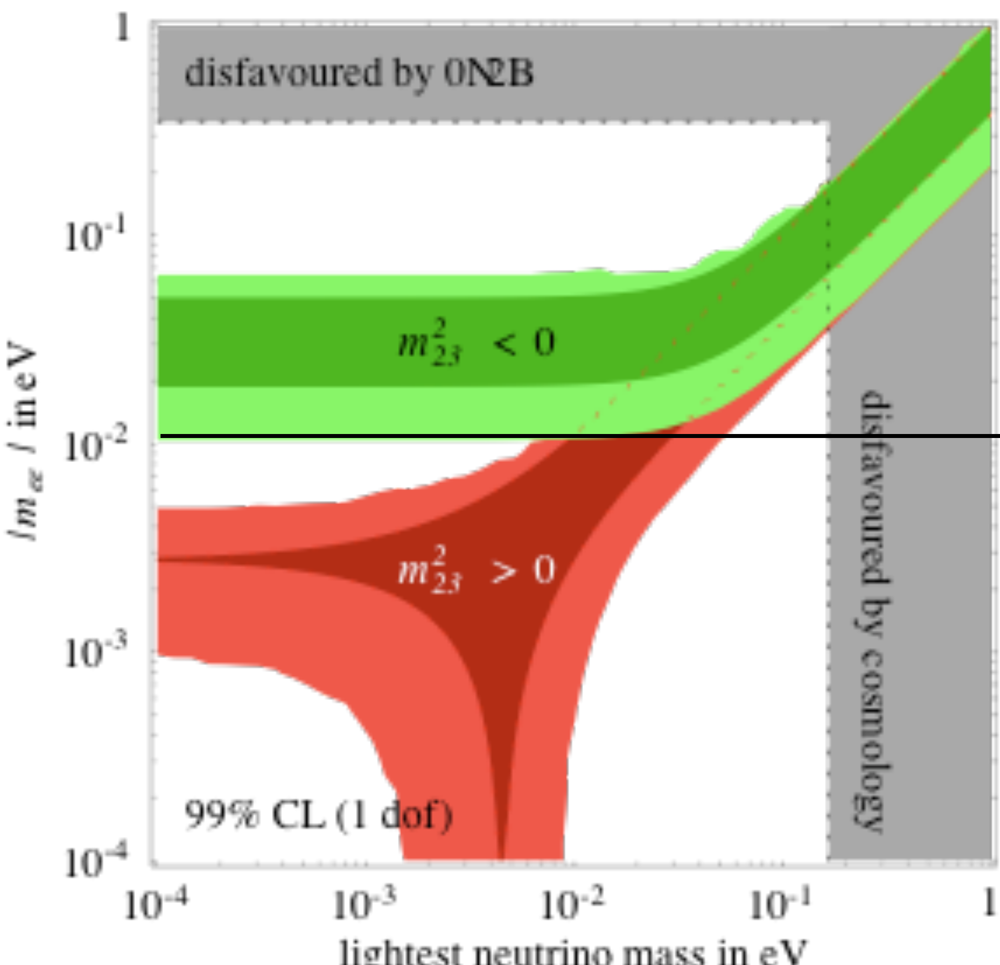
$0\nu\beta\beta$ decay: $(A,Z) \to (A,Z+2)+2e^-$

this would discriminate L_5 from other possibilities, such as Example 1.

The decay in 0νββ rates depend on the combination $|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12} e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13} e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



from the current knowledge of $(\Delta m^2_{ij}, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

future expected sensitivity on $|m_{ee}|$

10 meV

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

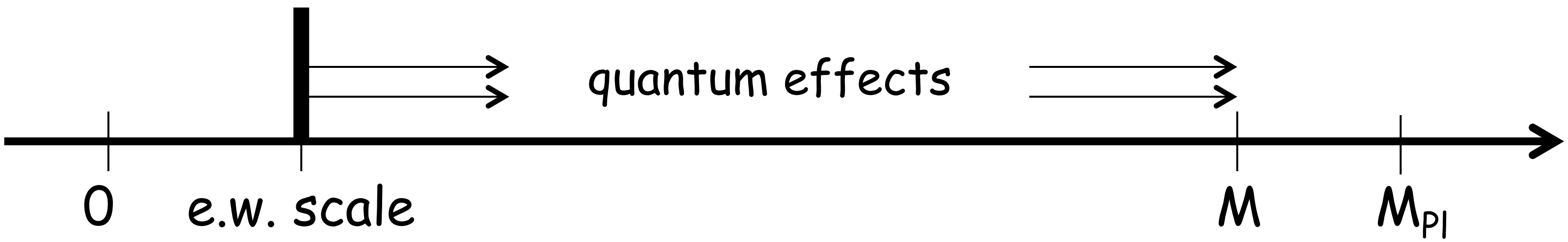
Neutrinos and the Higgs boson

1. neutrinos and the hierarchy problem
2. neutrinos and the stability of the electroweak vacuum

1. Why

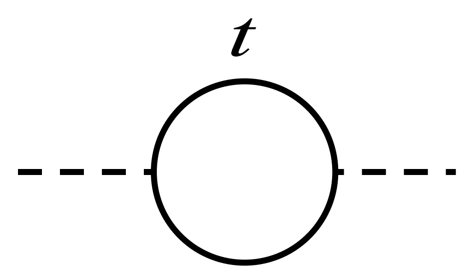
any new particle threshold: $M_{GUT,...}$

e.w. scale << ..., M_{Pl} ? ⟺ sensitivity of m_h to UV physics



often discussed in terms of quadratic divergences

$$\delta m_h^2 \propto \frac{y_t^2}{16\pi^2}\Lambda^2$$



but

-- what represents exactly Λ ? Any evidence from experiment?

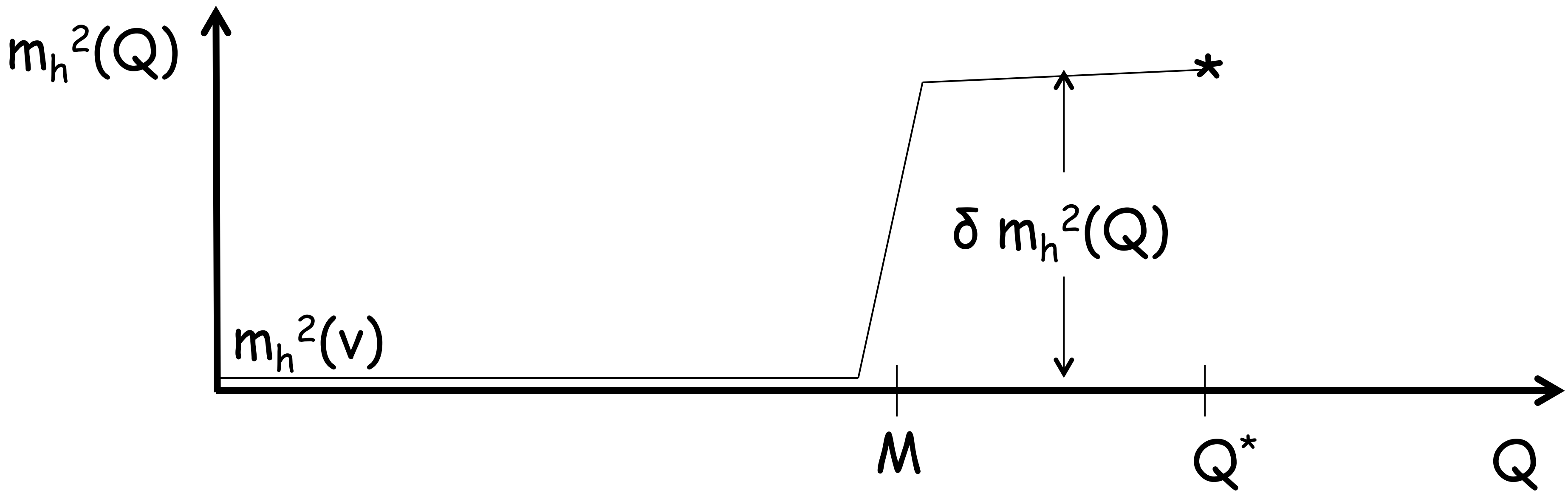
-- can we get rid of Λ in some suitable scheme ?

-- technical aspect obscure physics

hierarchy problem can be formulated entirely in terms of renormalized quantities with no reference to regulators

assumption: coupling y of Higgs particle to an heavy state of mass M

running Higgs mass $$\delta m_h^2(Q) \approx \frac{y^2}{16\pi^2} M^2 \log\frac{Q}{M} \qquad Q > M$$



fine-tune the initial conditions at Q^* such that

$$m_h^2(v) \approx m_h^2(Q^*) - \frac{y^2}{16\pi^2} M^2 \log\frac{Q^*}{M}$$

consider type I see-saw

heavy state ν^c	mass M
Yukawa coupling	y_ν

we will see in a moment

$$\delta m_h^2(Q) \approx -\frac{y_\nu^2}{4\pi^2} M^2 \log\frac{Q}{M} \qquad Q > M$$

by using $m_\nu \approx \frac{y_\nu^2 v^2}{M}$ to eliminate the y^2 dependence

$$\left|\delta m_h^2(Q)\right| \approx \frac{1}{4\pi^2} \frac{m_\nu M^3}{v^2} \log\frac{Q}{M} < v^2$$

$$M < 1.4 \times 10^7 \quad \text{GeV}$$

$$\left[\begin{array}{c} \log\frac{Q}{M} \approx 1 \\ m_\nu \approx 0.05\ \text{eV} \end{array}\right]$$

$$y_\nu \approx \sqrt{\frac{m_\nu M}{v^2}} < 10^{-4}$$

too small for thermal leptogenesis ?

Exercise 9: derive the threshold corrections to $m_\sigma^2(Q)$ in the toy model

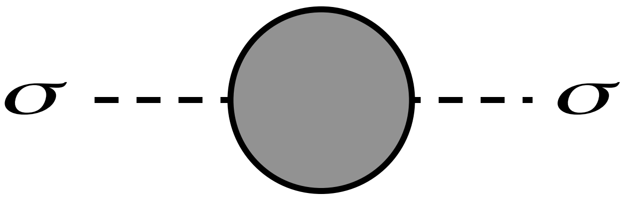
$$L = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + i\bar{\xi}\bar{\sigma}^\mu \partial_\mu \xi - \frac{1}{2}\left[\xi^T \mathcal{M} \xi + h.c.\right]$$

$$\xi = \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$$

$$\mathcal{M}(\sigma) = \begin{pmatrix} 0 & y(\sigma+v) \\ y(\sigma+v) & M \end{pmatrix}$$

assume $m_\sigma^2(0) = 0$

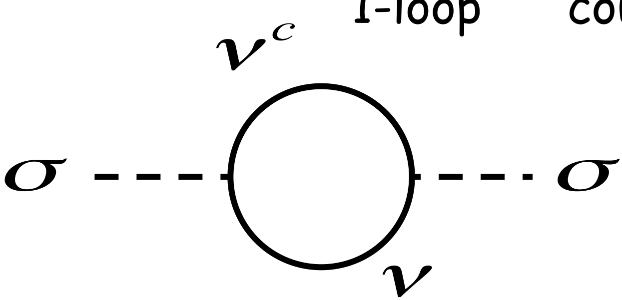
1. start from the 1-loop renormalized self-energy



$= i\left[Q^2 - \Pi_f(Q^2)\right]$

$$\Pi_f(Q^2) = \underbrace{\Pi(Q^2)}_{\text{1-loop}} - \underbrace{\Pi(0) - Q^2\Pi'(0)}_{\text{counterterms}}$$

OS scheme $\begin{cases} \Pi_f(0) = 0 \\ \Pi'_f(0) = 0 \end{cases}$



$= -i\,\Pi(Q^2)$

2. evaluate 1-loop diagram $-i\Pi(Q^2)$ in the limit $0 \approx m_1 << m_2 \approx M$

$$m_{1,2} = \frac{1}{2}\left(M \pm \sqrt{M^2 + 4y^2v^2}\right) \approx \begin{cases} -y^2v^2/M \\ M + y^2v^2/M \end{cases}$$

in dimensional regularization

$$\Pi(Q^2) = \frac{y^2}{2\pi^2}\int_0^1 dx\left[(D - \log\Omega)(2\Omega - Q^2x(1-x)) + \Omega\right]$$

$$D = \frac{2}{\varepsilon} - \gamma + \log 4\pi$$

$$\Omega = -Q^2x(1-x) + M^2x$$

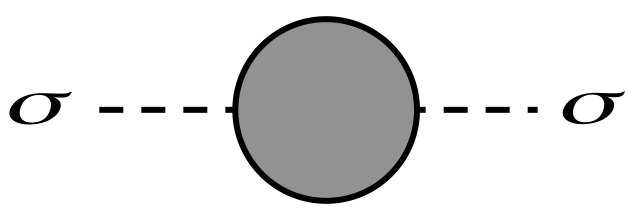
3. compute $\Pi_f(Q^2)$

$$\Pi_f(Q^2) = \frac{y^2}{2\pi^2}\int_0^1 dx\left[-2Q^2x(1-x) - (2M^2x - 3Q^2x(1-x))\log\frac{\Omega}{M^2x}\right]$$

finite

relevant limits $Q^2 << M^2$ $\quad \Pi_f(Q^2) = -\frac{y^2}{12\pi^2}\frac{Q^4}{M^2} + \ldots$

decoupling

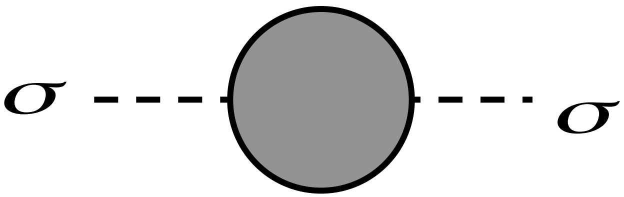


$$= iQ^2\left[1 + \frac{y^2}{12\pi^2}\frac{Q^2}{M^2} + \ldots\right]$$

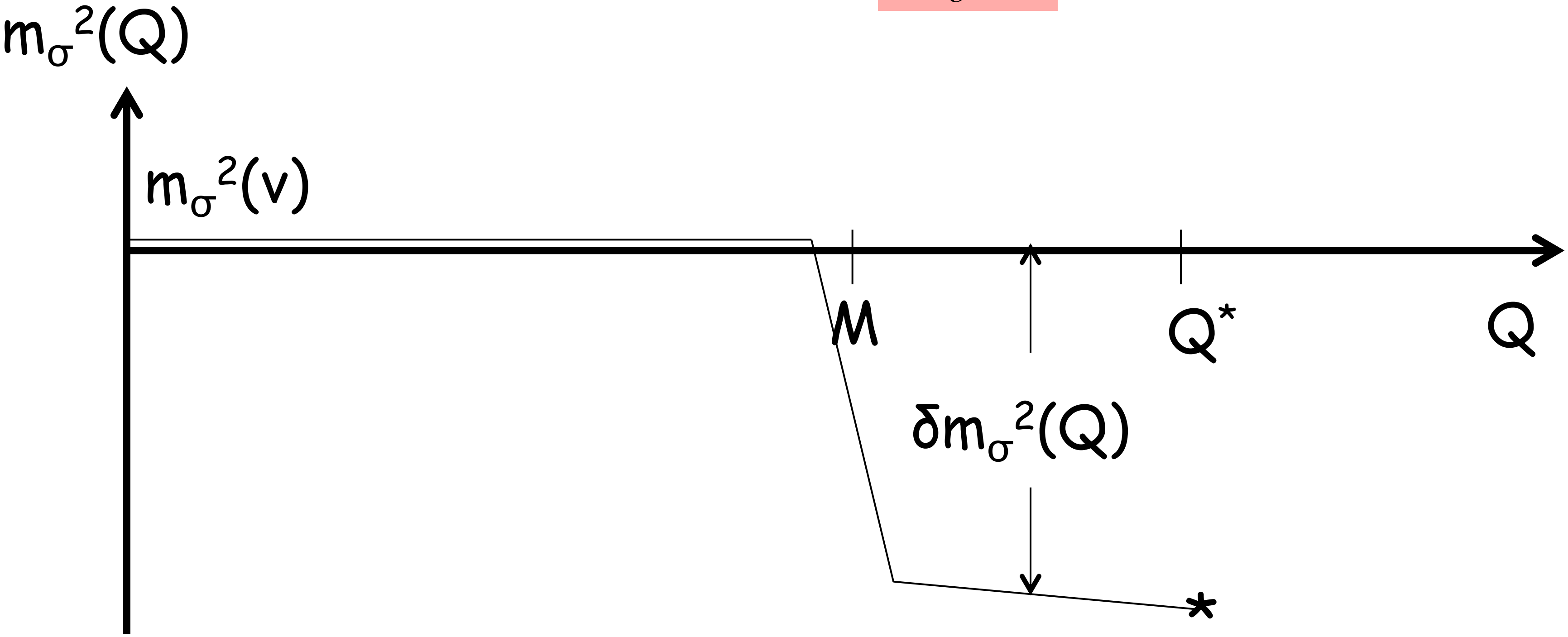
$$m_\sigma^2(Q) = 0$$

$Q^2 >> M^2$

$$\Pi_f(Q^2) = \frac{y^2}{2\pi^2}\left[Q^2\left(-\frac{3}{4}+\frac{1}{4}\log\frac{-Q^2}{M^2}\right) - M^2\log\frac{-Q^2}{M^2}\right]+\dots$$



$$= i\left(Q^2 + \underbrace{\frac{y^2}{2\pi^2}M^2\log\frac{-Q^2}{M^2}}_{-m_\sigma^2(Q)}\right)\left(1+O(y^2)\right)$$



similar conclusions in type II and type III see-saw where threshold corrections are dominated by 2-loop gauge interactions

type III $\delta m_h^2(Q) \approx -\frac{72 g^4}{(4\pi)^4} M^2 \log\frac{Q}{M}$ $\quad Q > M$ $\quad M < 940$ GeV

type II $M < 200$ GeV

ways out

the initial conditions at the scale Q^* are fine-tuned to an accuracy of order (e.w. scale)/M

the threshold correction at the scale M is almost cancelled by an other contribution, as e.g. in supersymmetry with a splitting between neutrinos and sneutrinos of order 4π x (e.w. scale)

the Higgs is not an elementary particle and dissolves above a compositness scale ~ TeV
gap between the e.w. scale and the compositeness scale if the Higgs is a PGB

2. neutrinos and the stability of the electroweak vacuum

for the current values

$$m_h = (125.66 \pm 0.34) \quad \text{GeV}$$

$$m_t = (173.2 \pm 0.9) \quad \text{GeV}$$

$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

the Higgs potential develops an instability at

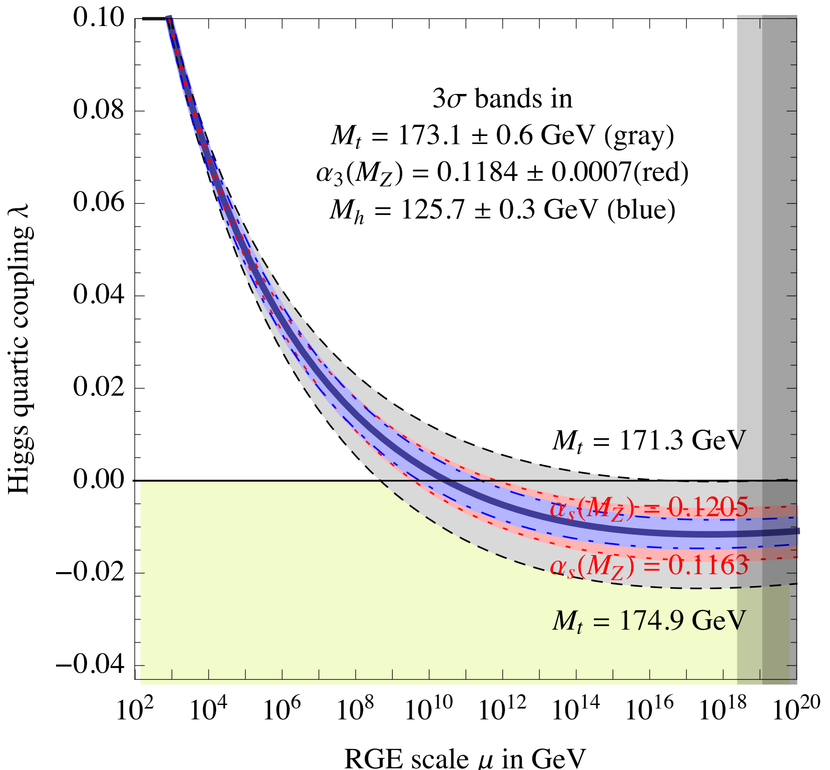
$$10^9\,\text{GeV} < \Lambda < 10^{15}\,\text{GeV}$$

assumption: only SM all the way up to the scale Λ

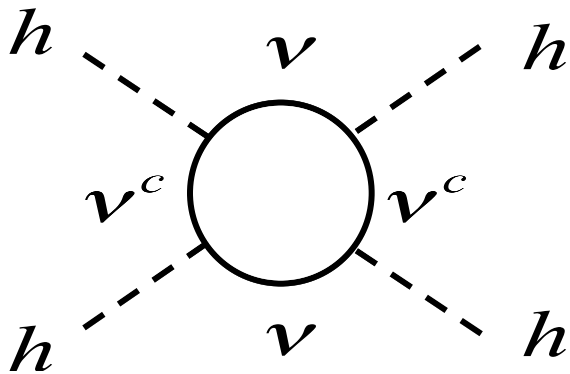
for large values of the field h

$$V(h) \approx \frac{\lambda}{4} h^4$$

$$(4\pi)^2 \frac{d\lambda}{dt} = -6y_t^4 + \frac{3}{8}[2g^4 + (g^2 + g'^2)] + \underbrace{12\lambda y_t^2 - 3\lambda(g^2 + 3g'^2)}_{O(\lambda)} + \underbrace{24\lambda^2}_{O(\lambda^2)} + \ldots$$

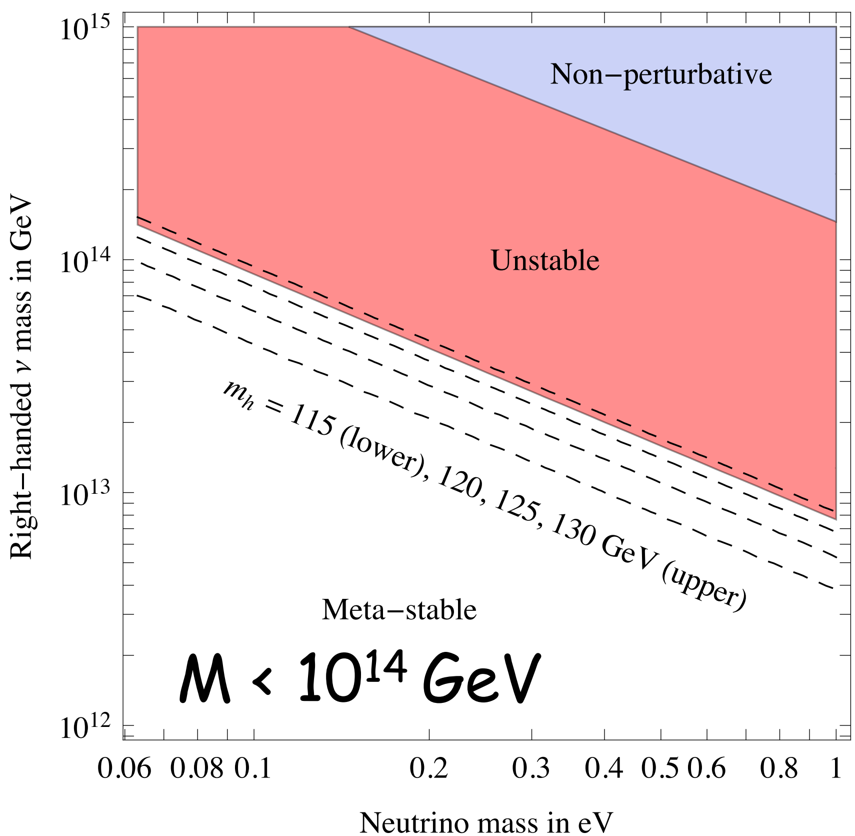


above the scale M a new contribution to β_λ arises from neutrino Yukawa couplings



$$\delta\beta_\lambda = -2\,\mathrm{tr}(y_\nu y_\nu^+ y_\nu y_\nu^+) < 0$$

contributes to instability above M



the larger M,
the larger the contribution

$$y_\nu \approx \sqrt{\frac{m_\nu M}{v^2}}$$

the bound applies only to the portion of SM parameter space that guarantees a stable vacuum in the limit y_ν=0
(m_t on the lower side
α_S on the higher side)

Back up slides

IFUP-TH/2008-9

Type-III see-saw at LHC

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Abstract

Neutrino masses can be generated by fermion triplets with TeV-scale mass, that would manifest at LHC as production of two leptons together with two heavy SM vectors or higgs, giving rise to final states such as $2\ell + 4j$ (that can violate lepton number and/or lepton flavor) or $\ell + 4j + \not{E}_T$. We devise cuts to suppress the SM backgrounds to these signatures. Furthermore, for most of the mass range suggested by neutrino data, triplet decays are detectably displaced from the production point, allowing to infer the neutrino mass parameters. We compare with LHC signals of type-I and type-II see-saw.

GGI lectures on the theory of fundamental interactions 2015

Firenze, 12-16 January 2015

Aspects of neutrino physics (IV)
Neutrino Masses, Mixing and Oscillations:
Neutrinos and Lepton Flavor Violation

Ferruccio Feruglio
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Lecture 3
Neutrinos and LFV

LFV expected at some level

neutrino masses and $U_{PMNS} \neq 1$ ⇒ L_i violated ($i=e,\mu,\tau$)

evidence for lepton flavor conversion

direct	$\nu_e \to \nu_\mu, \nu_\tau$	sol, LBL exp
indirect	$\nu_\mu \to \nu_\tau$	atm

should show up in processes with charged leptons

Process	Relative probability	Present Limit	Experiment	Year	prospects
$\mu \to e\gamma$	1	5.7×10^{-13}	MEG	2012	6×10^{-14}
$\mu^- \mathrm{Ti} \to e^- \mathrm{Ti}$	$Z\alpha/\pi$	4.3×10^{-12}	SINDRUM II	2006	$10^{-15} \div 10^{-16}$
$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$	$Z\alpha/\pi$	7×10^{-13}	SINDRUM II	2006	$10^{-15} \div 10^{-16}$
$\mu \to eee$	α/π	4.3×10^{-12}	SINDRUM	1988	$10^{-15} \div 10^{-16}$
$\tau \to \mu\gamma$	$(m_\tau/m_\mu)^{2\div 4}$	3.3×10^{-8}	B-factories	2011	
$\tau \to e\gamma$	$(m_\tau/m_\mu)^{2\div 4}$	4.5×10^{-8}	B-factories	2011	

Table 1: Relative sensitivities and experimental limits of the main CLFV processes.

here: focus on radiative decays of charged leptons in the SM, minimally extended to accommodate e.g. Dirac neutrinos

$$BR(\mu \to e\gamma) \approx \frac{3\alpha}{32\pi} \left| U^*_{\mu i} U_{ei} \frac{m_i^2}{m_W^2} \right|^2 \approx 10^{-53}$$

Exercise 10: reproduce this

[solution in Cheng and Li]

[unobservable also within type I see-saw] $m_i \approx 0.05\, eV \quad U_{fi} \approx O(1)$

depleted by
-- weak interactions
-- loop factor
-- GIM mechanism (mixing angle large, but neutrino masses tiny)

<-> GIM suppression for quarks:
small mixing angles
large top mass

a good place to look for BSM physics

general parametrization of LFV effects BSM

$$L = L_{SM} + \sum_i c_i^5 \frac{O_i^5}{\Lambda} + \sum_i c_i^6 \frac{O_i^6}{\Lambda^2} + \ldots$$

O^d_i gauge invariant operators dimension d

low-energy effective Lagrangian in the lepton sector

$$L = L_{SM} + i\frac{e}{\Lambda^2} e^c\left(\sigma^{\mu\nu}F_{\mu\nu}\right)\mathcal{Z}(\Phi^+ l) + \frac{1}{\Lambda^2}[\text{4-fermion}] + h.c. + \ldots$$

[relation between the scale Λ and new particle masses M' can be non-trivial in a weakly interacting theory g Λ/4π≈M']

$\mathcal{Z}_{ij}$ a matrix in flavour space

$$L_Y = -e^c y_e(\Phi^+ l) + h.c. + \ldots$$

in the basis where charged leptons are diagonal

$\mathrm{Im}\left[\mathcal{Z}\right]_{ii}$	d_i	electric dipole moments
$\mathrm{Re}\left[\mathcal{Z}\right]_{ii}$	$a_i = \frac{(g-2)_i}{2}$	anomalous magnetic moments
$\left\|\left[\mathcal{Z}\right]_{ij}\right\|^2 \quad (i \neq j)$	$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)}$	radiative decays $\mu \to e\gamma \quad \tau \to \mu\gamma \quad \tau \to e\gamma$
[4-fermion operators]	other LFV transitions	$\mu \to eee \quad \tau \to \mu\mu\mu \quad \tau \to eee \quad \ldots$

$$BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$$

$$\frac{\mathcal{Z}_{\mu e}}{\Lambda^2} < 2 \times 10^{-9}\ \text{TeV}^{-2}$$

→ either the scale of new physics is very large or flavour violation from New Physics is highly non-generic

$$\Lambda > 2 \times 10^4 \left[\sqrt{\mathcal{Z}_{\mu e}}\right] TeV$$

not a specific problem of the lepton sector

here: constraints from flavour physics on d=6 |ΔF|=2 operators

FLAVOUR PROBLEM

Operator	Bounds on Λ in TeV ($c_{ij}=1$) Re	Bounds on Λ in TeV ($c_{ij}=1$) Im	Bounds on c_{ij} ($\Lambda = 1$ TeV) Re	Bounds on c_{ij} ($\Lambda = 1$ TeV) Im	Observables
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\,d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\|q/p\|, \phi_D$
$(\bar{c}_R\,u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\|q/p\|, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\,d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\,s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

TABLE I: Bounds on representative dimension-six $\Delta F = 2$ operators. Bounds on Λ are quoted assuming an effective coupling $1/\Lambda^2$, or, alternatively, the bounds on the respective c_{ij}'s assuming $\Lambda = 1$ TeV. Observables related to CPV are separated from the CP conserving ones with semicolons. In the B_s system we only quote a bound on the modulo of the NP amplitude derived from Δm_{B_s} (see text). For the definition of the CPV observables in the D system see Ref. [15].

[Isidori, Nir, Perez, 2010]

Minimal Flavour Violation (quarks)

[Chivukula. Georgi 1987
D'Ambrosio, Giudice, Isidori, Strumia 2002]

- useful benchmark: a framework where the only source of flavour violation beyond the SM are the Yukawa coupling. Well-defined in the quark sector.

- in the limit $y_u = y_d = 0$, the SM lagrangian is invariant under a $U(3)^3$ flavour symmetry

$$G_q = SU(3)_{u^c} \times SU(3)_{d^c} \times SU(3)_q \times \ldots$$

$$q = (1,1,3) \quad u^c = (\bar{3},1,1) \; d^c = (1,\bar{3},1)$$

if the Yukawa couplings y_u and y_d are promoted to non-dynamical fields (spurions) transforming conveniently, the SM lagrangian remains formally invariant under the flavour group G_q

$$L_{SM} = \ldots - d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) + h.c.$$

$$y_u = (3,1,\bar{3}) \qquad y_d = (1,3,\bar{3})$$

- MFV assumes that new operators coming from New Physics do not involve any additional field/spurions and that they are still invariant under G_q
[additional assumption: no additional sources of CPV other than those in $y_{u,d}$]

Exercise 11: build the leading operator contributing to b -> s γ in MFV

a convenient basis:

$$y_d = \hat{y}_d \qquad y_u = \hat{y}_u V_{CKM}$$

$\hat{y}_{u,d}$ diagonal

leading order MFV invariant

$$i\frac{e}{\Lambda^2} d^c\left(\sigma^{\mu\nu}F_{\mu\nu}\right)Z^d(\Phi^+ q) + h.c.$$

$$Z^d = y_d y_u^+ y_u$$

$$= \frac{2\sqrt{2}}{v^3}\left(\hat{m}_d V_{CKM}^+ \hat{m}_u^2 V_{CKM}\right)$$

$$\hat{m}_u \approx \text{diag}(0,0,m_t)$$

$$b \to s\gamma \quad \Leftrightarrow \quad \left(Z^d\right)^*_{32}, \quad \left(Z^d\right)_{23}$$

$$\left(Z^d\right)^*_{32} = \frac{2\sqrt{2}}{v^3} m_b \left(m_t^2 V_{tb} V_{ts}^*\right)$$

$$\left(Z^d\right)_{23} = \frac{2\sqrt{2}}{v^3} m_s \left(m_t^2 V_{tb} V_{ts}^*\right)$$

MFV is nothing but the GIM mechanism extended to BSM contributions

$\left[b^c\left(\sigma F\right)s\right]^+$ dominates over by (m_t/m_b) $s^c\left(\sigma F\right)b$

$$BR(B \to X_s\gamma) = (3.55 \pm 0.24 \pm 0.09)\times 10^{-4}$$

➡ $\Lambda > 6.1\, TeV$

Exercise 12: build the leading operator with ΔF=2 in MFV

same basis as before:

$$y_d = \hat{y}_d \qquad y_u = \hat{y}_u V_{CKM} \qquad \hat{y}_{u,d} \text{ diagonal}$$

leading MFV invariant

$$\bar{q}_{Li}\gamma^{\mu}(y_u^{+}y_u)_{ij}\, q_{Lj}\, \bar{q}_{Lk}\gamma_{\mu}(y_u^{+}y_u)_{kl}\, q_{Ll}$$

looking at the down quark sector and selecting i=k=d,s and j=l=b we get the MFV operator contributing to ΔB=2

$$O_{MFV}(|\Delta B| = 2) = \frac{c}{\Lambda_{NP}^2}\, y_t^4 (V_{tb}V_{tq}^*)^2\, \bar{q}_L\gamma^{\mu}b_L\, \bar{q}_L\gamma_{\mu}b_L \qquad (q = d,s)$$

where we used $\hat{m}_u \approx \text{diag}(0,0,m_t)$

again same CKM suppression as in the SM. Now the bound on the scale of New Physics reads

$$\Lambda_{NP} > 5.9\ TeV$$

define 2 New Physics parameters

$$\Delta_q \equiv \frac{M_{12}^{q}}{M_{12}^{q,SM}} \qquad (q=d,s)$$

[O_{MFV} modify M_{12} for B_d and B_s in the same way: i.e Δ_d and Δ_s are identical and real in MFV]

bound on the scale of New Physics in MFV

Operator	Bound on Λ	Observables
$H^\dagger\left(\overline{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} Q_L\right)(eF_{\mu\nu})$	6.1 TeV	$B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$
$\frac{1}{2}(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L)^2$	5.9 TeV	ϵ_K, Δm_{B_d}, Δm_{B_s}
$H_D^\dagger\left(\overline{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} T^a Q_L\right)(g_s G^a_{\mu\nu})$	3.4 TeV	$B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right)\left(\overline{E}_R\gamma_\mu E_R\right)$	2.7 TeV	$B \to X_s\ell^+\ell^-$, $B_s \to \mu^+\mu^-$
$i\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right) H_U^\dagger D_\mu H_U$	2.3 TeV	$B \to X_s\ell^+\ell^-$, $B_s \to \mu^+\mu^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right)\left(\overline{L}_L\gamma_\mu L_L\right)$	1.7 TeV	$B \to X_s\ell^+\ell^-$, $B_s \to \mu^+\mu^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right)(eD_\mu F_{\mu\nu})$	1.5 TeV	$B \to X_s\ell^+\ell^-$

TABLE II: Bounds on the scale of new physics (at 95% C.L.) for some representative $\Delta F = 1$ [27] and $\Delta F = 2$ [12] MFV operators (assuming effective coupling $\pm 1/\Lambda^2$), and corresponding observables used to set the bounds.

[Isidori, Nir, Perez, 2010]

Minimal Flavour Violation (leptons)

extension of MFV to leptons is ambiguous:
we can describe neutrino masses in several ways

1 B-L conserved, pure Dirac neutrino masses
just copy the quark sector

$$G_l = SU(3)_{\nu^c} \times SU(3)_{e^c} \times SU(3)_l \times \ldots$$

$$l = (1,1,3) \quad \nu^c = (\bar{3},1,1) \; e^c = (1,\bar{3},1)$$

$$y_\nu = (3,1,\bar{3})$$

$$y_e = (1,3,\bar{3})$$

$$i\,\frac{e}{\Lambda^2} e^c \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{Z}(\Phi^+ l) + h.c.$$

choose as basis:

$$y_e = \hat{y}_e \qquad y_\nu = \hat{y}_\nu U^+_{PMNS}$$

$$\mathcal{Z} = y_e y_\nu^+ y_\nu$$

$$= \frac{2\sqrt{2}}{v^3}\left(\hat{m}_e U_{PMNS} \hat{m}_\nu^2 U^+_{PMNS}\right)$$

dominant contribution to μ -> e γ

$$\left(\mathcal{Z}\right)^*_{21} = \frac{2\sqrt{2}}{v^3} m_\mu \left(U^*_{\mu i} U_{ei} m_i^2\right)$$

$$\approx 10^{-28}$$

μ -> e γ unobservable
even for $\Lambda \approx 1$ TeV

2 B-L violated, neutrino masses from d=5 operator

[Cirigliano, Grinstein, Isidori, Wise 2005]

$$L = \ldots + e^c y_e (\Phi^+ l) + \frac{1}{2\Lambda_L}\left(\tilde{\Phi}^+ l\right) w \left(\tilde{\Phi}^+ l\right) + h.c.$$

an important assumption: $\Lambda_L \neq \Lambda$

$$G_l = SU(3)_{e^c} \times SU(3)_l \times \ldots$$

$$l = (1,3) \qquad e^c = (\bar{3},1)$$

$$y_e = (3,\bar{3})$$

$$w = (1,\bar{6})$$

the only sources of G_l breaking

$$y_e = \sqrt{2}\frac{m_e^{diag}}{v} \qquad w = \frac{2\Lambda_L}{v^2} U^* m_\nu^{diag} U^+$$

spurions expressed in terms of known quantities and Λ_L

$$\mathcal{Z} = y_e w^+ w$$

$$= \frac{4\sqrt{2}}{v^3}\frac{\Lambda_L^2}{v^2}\left(\hat{m}_e U_{PMNS} \hat{m}_\nu^2 U_{PMNS}^+\right)$$

μ -> e γ dominated by

$$\left(\mathcal{Z}\right)_{21}^* = \frac{4\sqrt{2}}{v^3}\frac{\Lambda_L^2}{v^2} m_\mu \left(U_{\mu i}^* U_{ei} m_i^2\right)$$

enhancement factor can be huge $\quad \dfrac{\Lambda_L^2}{v^2}$

experimental bound satisfied by $(\Lambda_L/\Lambda)<10^9$

μ -> e γ observable if $\Lambda_L >> \Lambda$

[qualitatively similar conclusion when MFV extended to the type I see-saw case]

Exercise 13: show that

+ for normal hierarchy
- for inverted hierarchy

$$\mathcal{Z}_{ij} = \frac{4\sqrt{2}}{v^3}\frac{\Lambda_L^2}{v^4}\left[\Delta m^2_{sol} U_{i2} U^*_{j2} \pm \Delta m^2_{atm} U_{i3} U^*_{j3}\right]$$

and estimate

$$\frac{R_{\mu e}}{R_{\tau\mu}} = \frac{BR(\mu \to e\gamma)}{BR(\tau \to \mu\gamma)} \times \frac{BR(\tau \to \mu\nu_\tau\bar{\nu}_\mu)}{BR(\mu \to e\nu_\mu\bar{\nu}_e)}$$

solution

$$\frac{R_{\mu e}}{R_{\tau\mu}} \approx \left|\frac{2}{3} r \pm \sqrt{2}\sin\vartheta_{13} e^{i\delta}\right|^2 \approx (0.035 \div 0.055)$$

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

from present bound on μ -> e γ

$$R_{\tau\mu} < (1.0 \div 1.6) \times 10^{-11}$$

hints:

-- use unitarity relation for U_{PMNS}

-- use approximate values

$$U_{\mu 3} \approx U_{\tau 3} \approx 1/\sqrt{2}$$

$$U_{e2} \approx U_{\mu 2} \approx -U_{\tau 2} \approx 1/\sqrt{3}$$

LFV in the limit of vanishing neutrino masses

MFV extended to the lepton sector reproduces the GIM suppression
in particular LF is conserved when $m_i=0$

GIM suppression can be evaded in several models of fermion masses
e.g. in partial compositness where elementary fermions acquire a mass
through their mixing with a composite sector

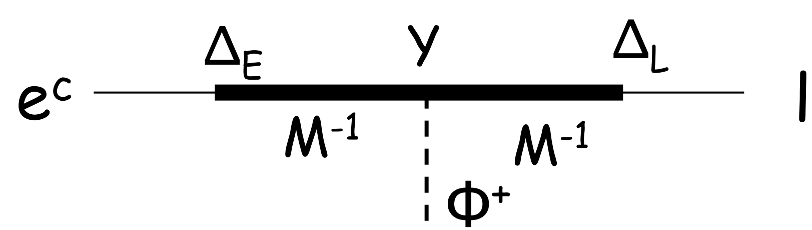
a toy model

$$
\begin{aligned}
L_Y &= -e^c\Delta_E E - L^c\Delta_L l && \Leftrightarrow \text{ elementary-composite mixing}\\
&\quad - E^c M E - L^c M L && \Leftrightarrow \text{ Dirac masses for composite fermions}\\
&\quad - E^c Y(\Phi^+ L) - (L^c\tilde{\Phi}^+)\tilde{Y} E + h.c. && \Leftrightarrow \text{ Yukawa coupling of composite fermions}
\end{aligned}
$$

by integrating out the composite sector [Exercise 14]

$$L_Y = -e^c y_e(\Phi^+ l) + h.c.$$

$$y_e = (\Delta_E M^{-1})Y(M^{-1}\Delta_L) + \ldots$$



higher-orders in (Φ/M)

Exercise 15

compute the corrections to previous LO relations by using the equation of motion for the composite sector. Start with 1 generation and then discuss the 3 generation case.

write L_Y in matrix notation

$$L_Y = -\begin{pmatrix} e^c & E^c & L^c \end{pmatrix} \begin{pmatrix} 0 & \Delta_E & 0 \\ 0 & M & Y\Phi^+ \\ \Delta_L & \tilde{\Phi}^+\tilde{Y} & M \end{pmatrix} \begin{pmatrix} l \\ E \\ L \end{pmatrix} + h.c.$$

write the e.o.m. for the composite fields (E^c,L^c) and (E,L) in the limit of negligible kinetic term and substitute them back into L_Y

$$L_Y = e^c \begin{pmatrix} \Delta_E & 0 \end{pmatrix} \begin{pmatrix} M & Y\Phi^+ \\ \tilde{\Phi}^+\tilde{Y} & M \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \Delta_L \end{pmatrix} l + h.c.$$

expand this expression in powers of the Higgs field

At the LO $y_e = F_{E^c} Y F_L$

$$F_{E^c} = \Delta_E M^{-1} \qquad F_L = M^{-1}\Delta_L$$

an intriguing possibility (anarchic scenario):

-- Yukawa coupling Y in the composite sector are O(1)

-- fermion mass hierarchy entirely due to the amount of mixing F

it arises is many SM extensions

split fermions in an Extra Dimension

$$F_{X_i} = \sqrt{\frac{2\mu_i}{1-e^{-2\mu_i r}}}$$

ED	μ_i	r
Flat $[0,\pi R]$	M_i/Λ	$\Lambda\pi R$
Warped $[R,R']$	$1/2 - M_i R$	$\log R'/R$

M_i = bulk mass of fermion X_i

$Y_{u,d}$ = O(1) Yukawa couplings between bulk fermions and a Higgs localized at one brane

no symmetry:
hierarchy produced by geometry

fermion masses from abelian flavour symmetries $Q(X_i) \geq 0$

$$F_{X_i} = \text{diag}\left(\lambda^{Q(X_1)}, \lambda^{Q(X_2)}, \lambda^{Q(X_3)}\right) \qquad \lambda = \frac{\langle\varphi\rangle}{\Lambda}$$

chiral multiplets X_i of the MSSM coupled to a superconformal sector

[Nelson-Strassler 0006251]

$$F_{X_i} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{\gamma_i}{2}} < 1$$

γ_i anomalous dimension of X_i

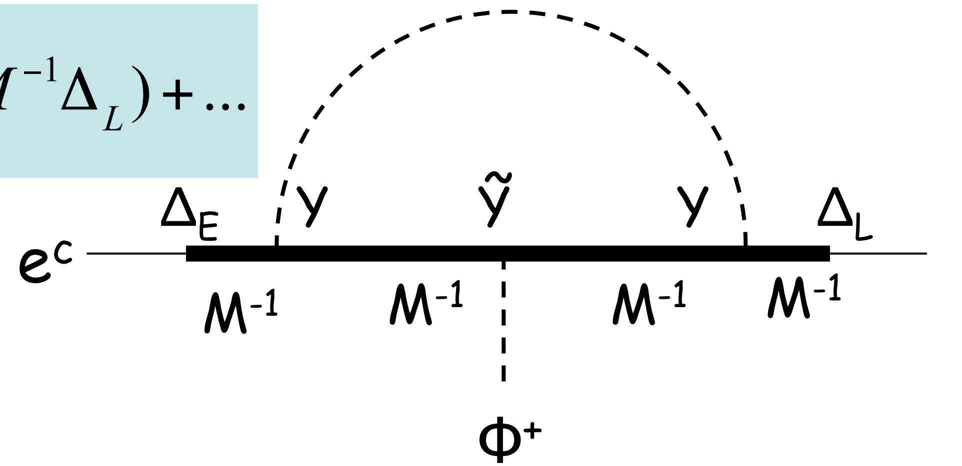
so far neutrino are massless
do we expect LFV in our toy model?

one-loop contribution to lepton dipole operator from Higgs exchange
(assuming M proportional to identity)

$$\frac{Z}{\Lambda^2} \approx \frac{1}{16\pi^2 M^2}(\Delta_E M^{-1}) Y \tilde{Y} Y (M^{-1}\Delta_L) + \dots$$

in general these combinations not diagonal in the same basis

$$y_e = (\Delta_E M^{-1}) Y (M^{-1}\Delta_L) + \dots$$



LFV not suppressed by neutrino masses and unrelated to (B-L) breaking scale

rough estimate

$$\frac{Z_{\mu e}}{\Lambda^2} < 2\times 10^{-9}\ \text{TeV}^{-2} \quad \Rightarrow \quad M > 10\ \text{TeV}$$

$$\Delta_E \approx \Delta_L$$

$$\frac{\Delta_f}{M} \approx \sqrt{\frac{m_f}{v}}$$

$$Y \approx \tilde{Y} \approx O(1)$$

Exercise 16: reproduce flavour pattern of Z from a spurion analysis

$$\frac{Z}{\Lambda^2} \approx \frac{1}{16\pi^2 M^2}(\Delta_E M^{-1}) Y \tilde{Y} Y (M^{-1}\Delta_L) + \ldots$$

-- identify the maximal flavour symmetry G of our toy model

-- identify the transformation properties of the spurions Δ_L, Δ_E, Y, $\tilde{Y}$, that guarantee the invariance of L_y

-- using previous tools, build the relevant dipole operator invariant under G

summary

- LFV expected in charged leptons = CLFV

- CLFV probes physics beyond the νSM [=SM minimally extended to accommodate ν masses]

  observable rates for CLFV require new physics at a scale well below the GUT or the L-violation scales [$\Lambda \ll \Lambda_L$ in our example of MFV]

- GIM suppression in CLFV is a special feature of MFV: it can be violated in models of fermion masses and relation to neutrino masses and mixing angles can be more indirect