Cosmology

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of the Russian Academy of Sciences

Department of Particle Physics and Cosmology
Physics Faculty
Moscow State University

Tentative outline

Lecture 1

1. Introduction: homogeneous Universe
   - 1.1 Metric, redshift
   - 1.2 Friedmann equation and covariant energy conservation
   - 1.3 Radiation, matter and Λ
   - 1.4 Sample solutions
     - 1.4.1 Particle horizon
     - 1.4.2 de Stter space; event horizon
   - 1.5 Stages of real Universe
   - 1.6 Conformal times of various epochs

Lecture 2

2. Cosmological perturbations I
   - 2.1 Helicity decomposition
   - 2.2 Tensor modes
     - 2.2.1 Equations
     - 2.2.2 Regimes of evolution
     - 2.2.3 Solution in subhorizon regime
   - 2.3 Vector modes

Cont'd

Lectures 3,4

3. Cosmological perturbations II

Lecture 3

3.1 Scalar modes
- 3.1.1 Equations
- 3.1.2 Sample solutions
- 3.1.2 Classification: adiabatic mode, entropy (isocurvature) modes

Lecture 4

3.2 Adiabatic mode
- 3.2.1 Initial condition
- 3.2.2 Summary of experimental results
- 3.2.3 Evolution of baryon-photon component
- 3.2.4 Evolution of dark matter
- 3.2.5 After recombination. BAO, structure formation

Cont'd

Lecture 5

4. Elements of theory of CMB temperature anisotropy
   - 4.1 Sachs–Wolfe, Doppler
     and integrated Sachs–Wolfe effects
   - 4.2 Large angular scales
   - 4.3 Intermediate angular scales
   - 4.4 What do we learn from CMB temperature anisotropy

Lecture 6

5. Elements of theory of CMB polarization
   - 5.1 More about photon last scattering
   - 5.2 Reionization
   - 5.2 Polarization from Thomson scattering
   - 5.3 E- and B-modes
   - 5.4 Effect of scalar perturbations
   - 5.4 Effect of tensor perturbations

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Lectures 7,8

6. Inflation
   - 6.1 Main idea
   - 6.2 Inflaton as a driver
   - 6.3 Generation of tensor perturbations
   - 6.4 Generation of scalar perturbations: inflaton mechanism
   - 6.5 Estimating non-Gaussianity
   - 6.6 Curvaton mechanism. Examples of non-Gaussianity
   - 6.7 Example of generation of entropy mode: axion misalignment

Lecture 9

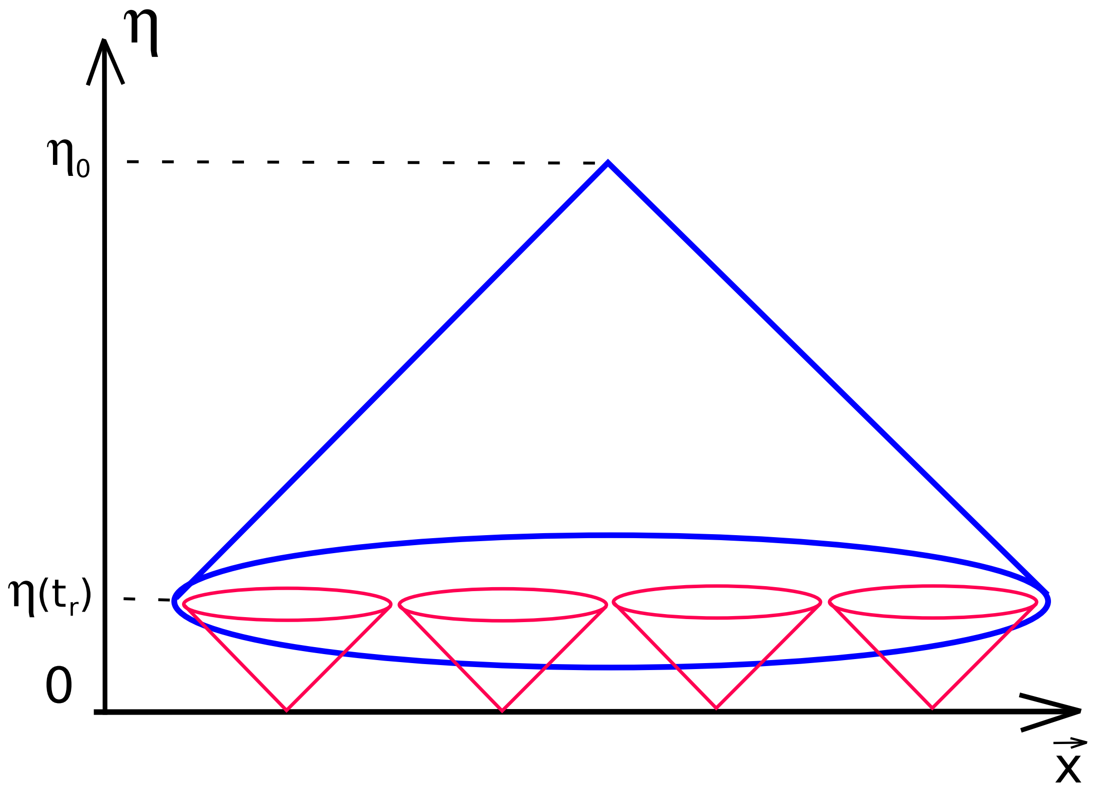
7. Alternatives to inflation
   - 7.1 Bouncing Universe
   - 7.2 Genesis
   - 7.3 Generating scalar perturbations

8. Conclusions

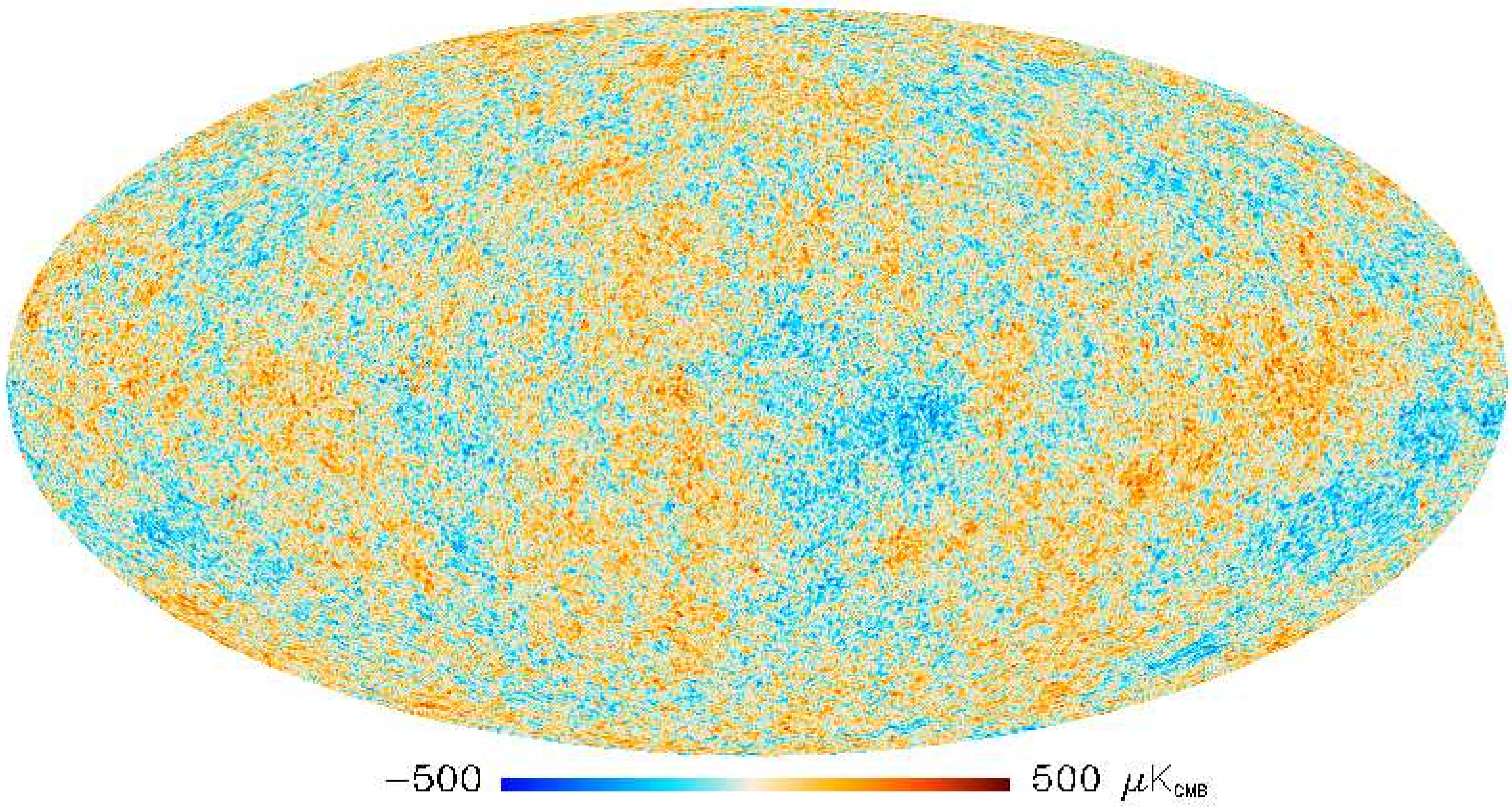
$\hbar = c = k_B = 1$ calculator:

ppc.inr.ac.ru/eng/uc.php

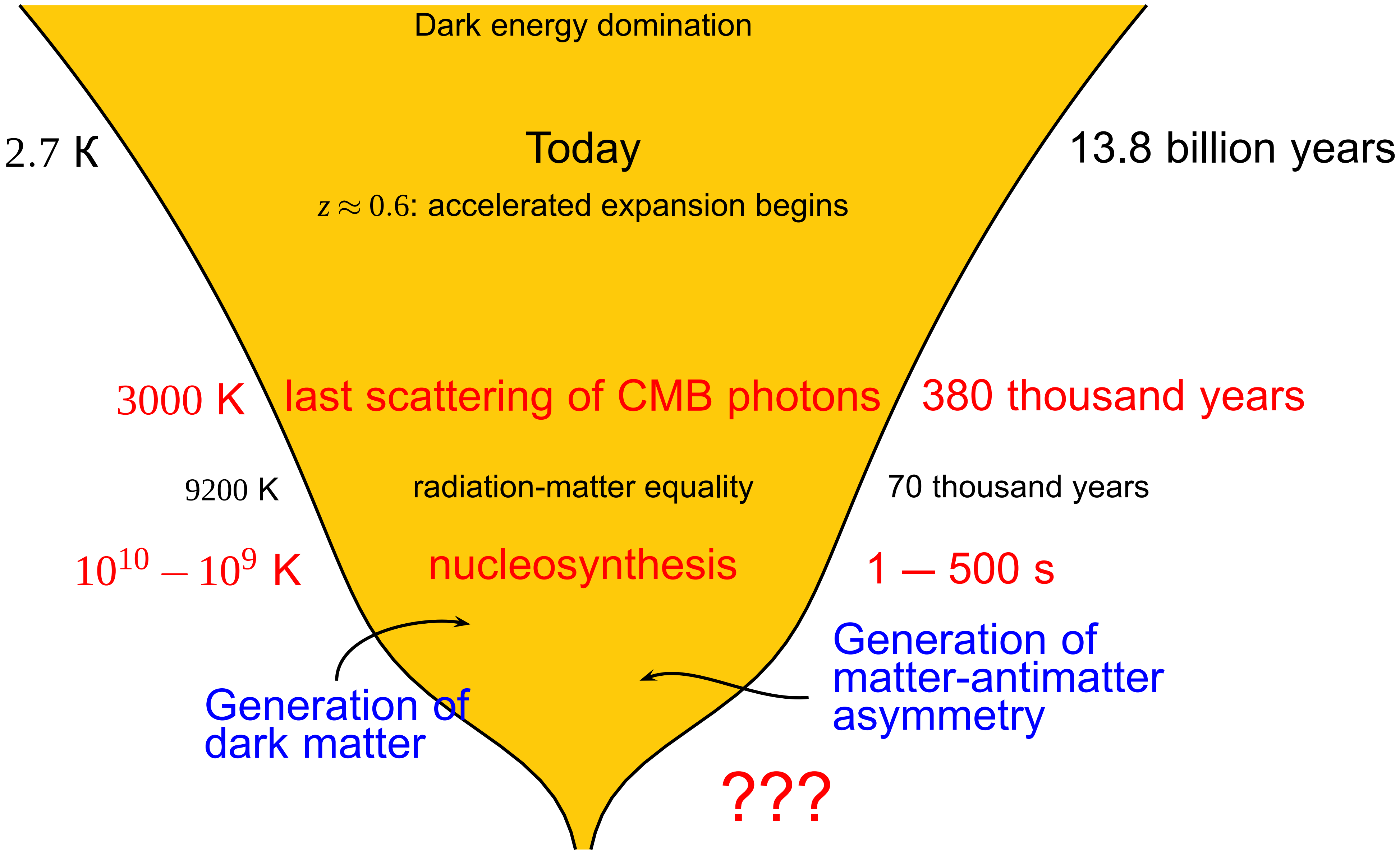
Causal structure of space-time in hot Big Bang theory (i.e., assuming that the Universe started right from the hot epoch)



Angular size of horizon at recombination $\approx 2^o$.



There are perturbations which were superhorizon at the time of recombination, angular scale $\gtrsim 2^o$. Causality: they could not be generated at hot epoch!



Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
3000 K
last scattering of CMB photons
380 thousand years
9200 K
radiation-matter equality
70 thousand years
$10^{10} - 10^{9}$ K
nucleosynthesis
1 — 500 s
Generation of matter-antimatter asymmetry
Generation of dark matter
???

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3.1 Scalar modes

3.1.1 Conformal Newtonian gauge

3.1.2 Equations

3.1.3 Sample solutions: single component fluids

3.1.3.1 Master equation

3.1.3.2 Radiation

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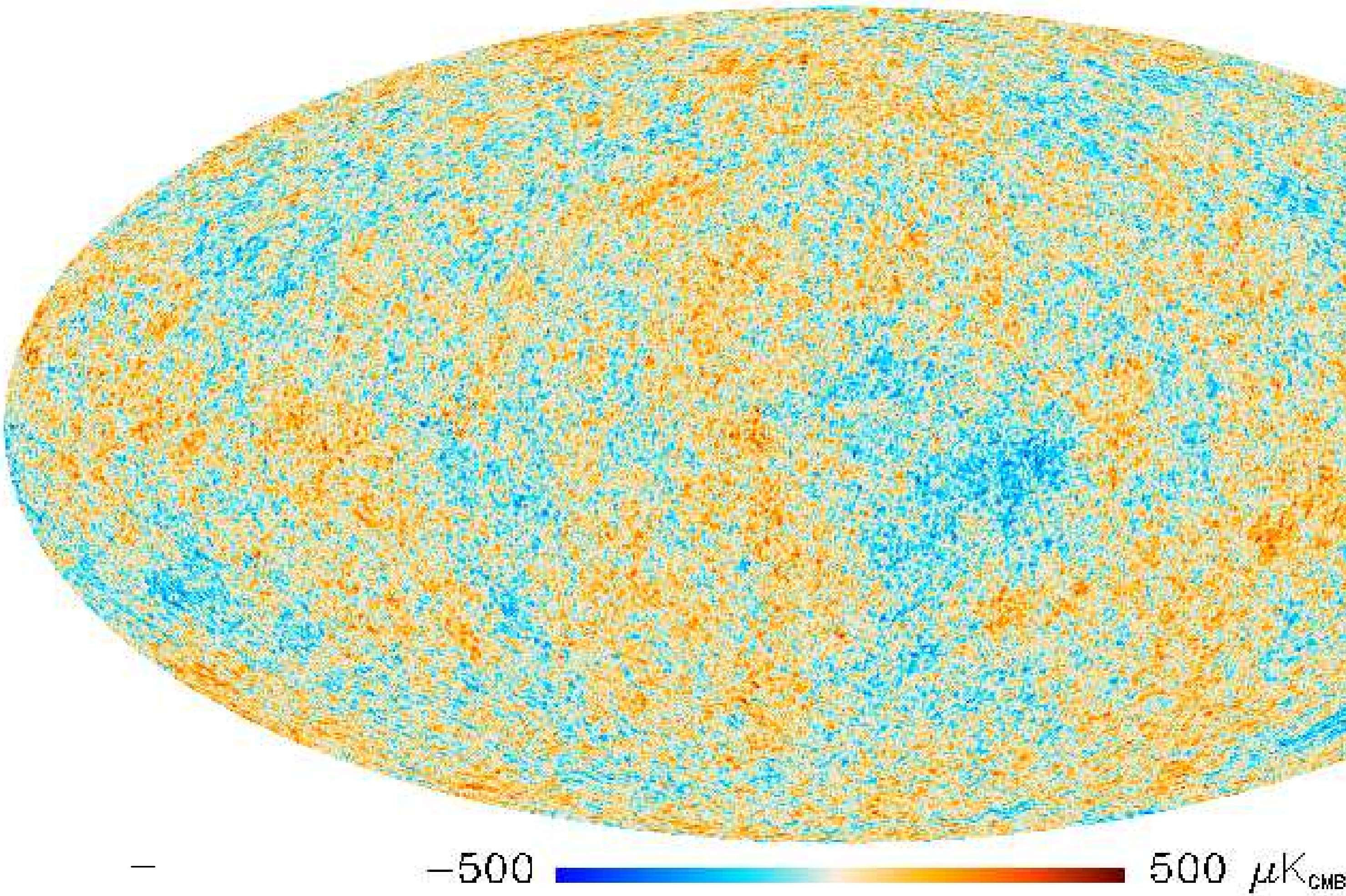
$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck

Einstein:

$$-\Delta\Psi + 3\frac{a'}{a}\Psi' - 3\frac{a'^2}{a^2}\Phi = 4\pi G a^2 \delta\rho_{tot} \, ,$$

$$-\Psi' + \frac{a'}{a}\Phi = -4\pi G[(\rho+p)v]_{tot} \, ,$$

$$\Psi'' - \frac{1}{3}(\Delta\Phi + \Delta\Psi) + \frac{a'}{a}\left(2\Psi' - \Phi'\right) - 2\frac{a''}{a}\Phi + \frac{a'^2}{a^2}\Phi = -4\pi G a^2 [\delta p]_{tot} \, ,$$

$$\Delta\left(\Phi + \Psi\right) = -12\pi G a^2 [(\rho+p)\pi]_{tot} \, .$$

Covariant conservation:

$$\delta\rho' + 3\frac{a'}{a}(\delta\rho + \delta p) + (\rho+p)\left(\Delta v + 3\Psi'\right) = 0 \, ,$$

$$[(\rho+p)v]' + \delta p + (\rho+p)\left(4\frac{a'}{a}v + \pi + \Phi\right) = 0 \, .$$

Ideal fluids

Einstein eqs.:

$$\Delta\Phi - 3\frac{a'}{a}\Phi' - 3\frac{a'^2}{a^2}\Phi = 4\pi G a^2 \cdot \delta\rho_{tot}$$

$$\Phi' + \frac{a'}{a}\Phi = -4\pi G a^2 \cdot [(\rho + p)v]_{tot}$$

$$\Phi'' + 3\frac{a'}{a}\Phi' + \left(2\frac{a''}{a} - \frac{a'^2}{a^2}\right)\Phi = 4\pi G a^2 \cdot \delta p_{tot}$$

Covariant conservation

$$\delta\rho' + 3\frac{a'}{a}(\delta\rho + \delta p) + (\rho + p)(\Delta v - 3\Phi') = 0$$

$$[(\rho + p)v]' + 4\frac{a'}{a}(\rho + p)v + \delta p + (\rho + p)\Phi = 0\ ,$$

Single component fluid

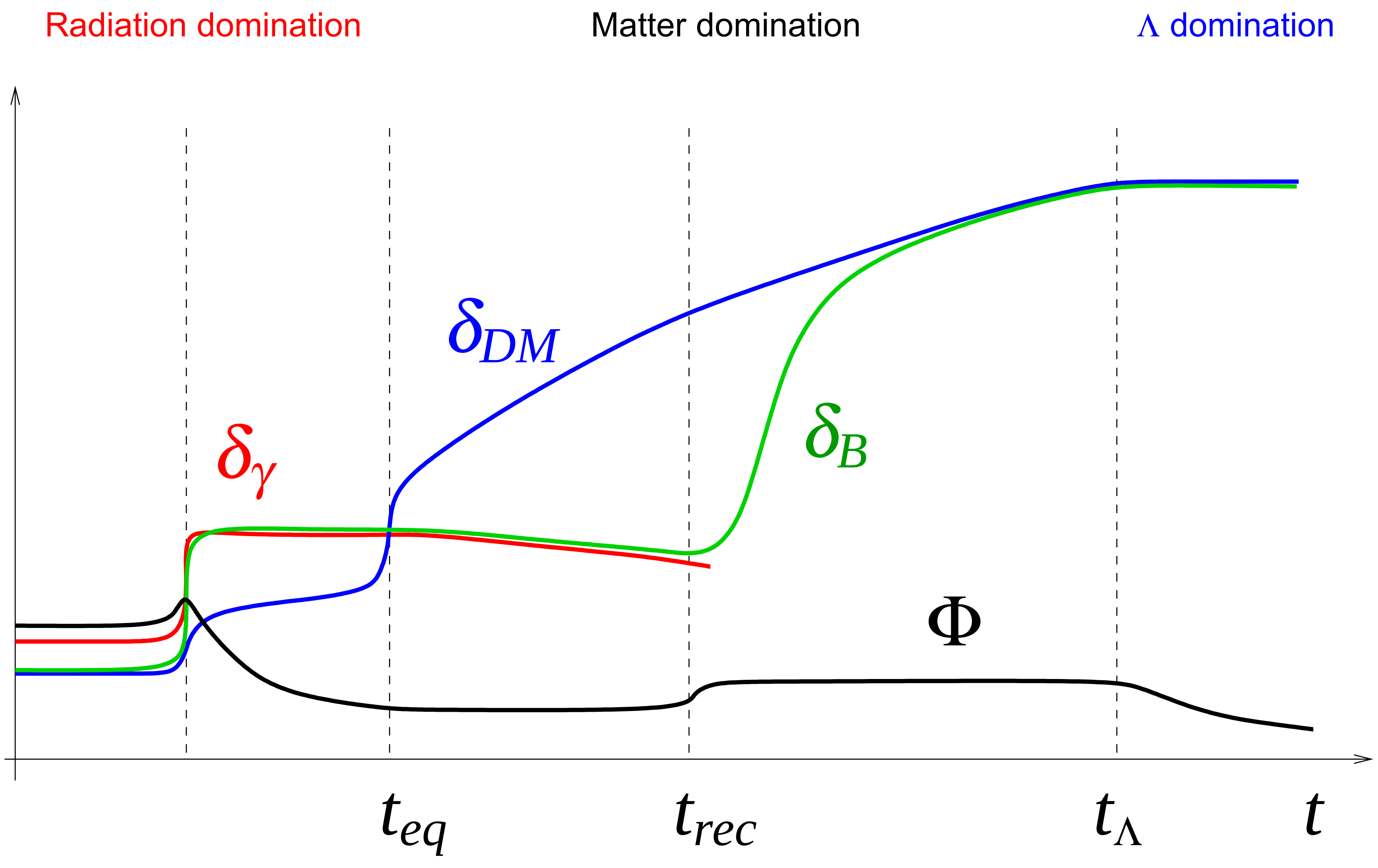
Master equation ($u_s^2 = w$)

$$\Phi'' + 3\,\frac{a'}{a}(1+u_s^2)\Phi' + \left[2\frac{a''}{a} - \frac{a'^2}{a^2}(1-3u_s^2)\right]\Phi + u_s^2\,k^2\,\Phi = 0\ .$$

Perturbation in energy density:

$$4\pi G a^2\,\delta\rho = -\left(k^2\Phi + 3\frac{a'}{a}\Phi' + 3\frac{a'^2}{a^2}\Phi\right)$$

Growth of perturbations (linear regime)



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Einstein:

$$-\Delta\Psi+3\frac{a'}{a}\Psi'-3\frac{a'^2}{a^2}\Phi=4\pi Ga^2\delta\rho_{tot}\ , \tag{1}$$

$$-\Psi'+\frac{a'}{a}\Phi=-4\pi Ga^2[(\rho+p)v]_{tot}\ , \tag{2}$$

$$\Psi''-\frac{1}{3}(\Delta\Phi+\Delta\Psi)+\frac{a'}{a}\left(2\Psi'-\Phi'\right)-2\frac{a''}{a}\Phi+\frac{a'^2}{a^2}\Phi=-4\pi Ga^2[\delta p]_{tot}\ , \tag{3}$$

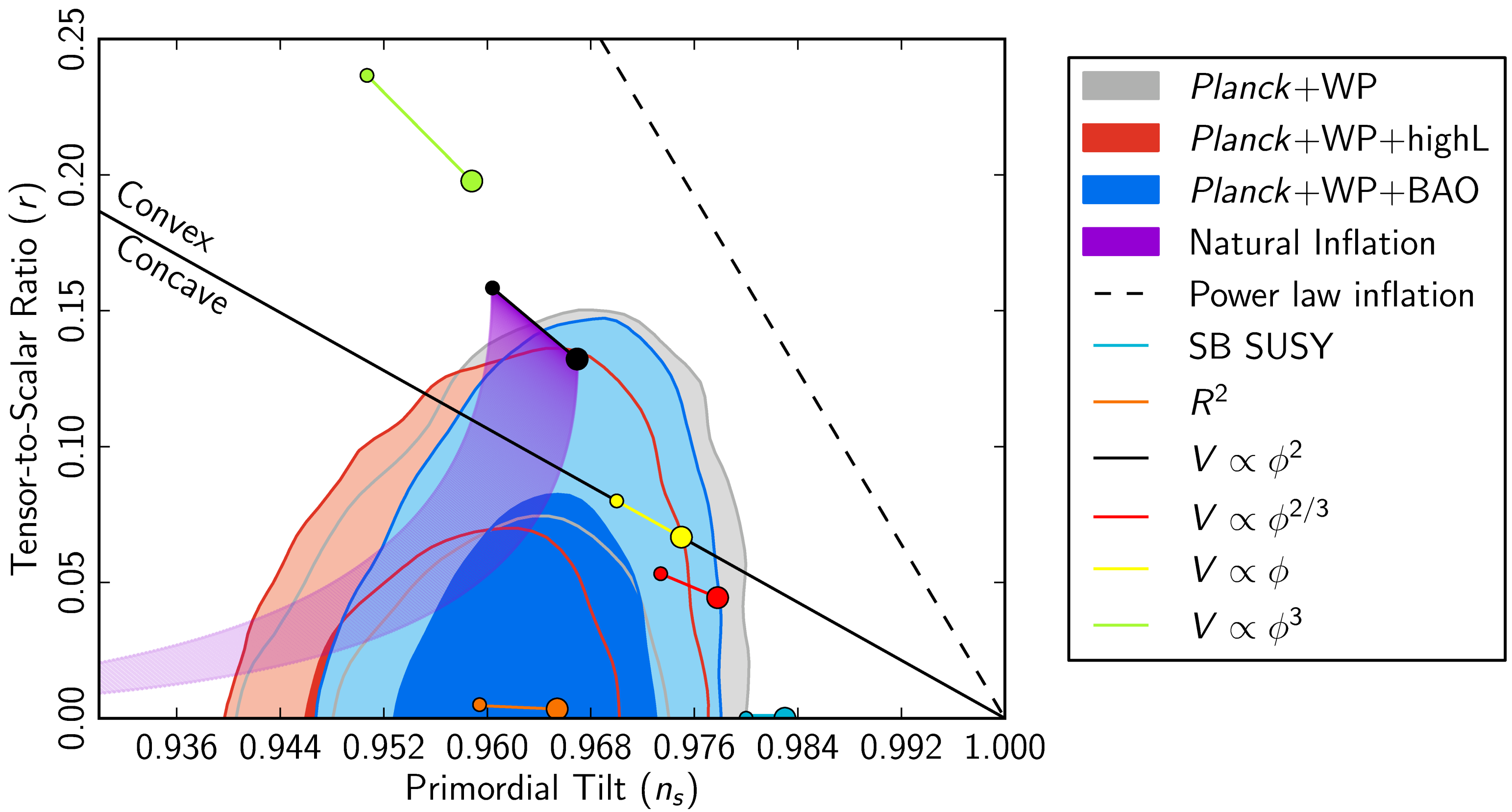
$$\Delta(\Phi+\Psi)=-12\pi Ga^2[(\rho+p)\pi]_{tot}\ . \tag{4}$$

Covariant conservation:

$$\delta\rho'+3\frac{a'}{a}(\delta\rho+\delta p)+(\rho+p)\left(\Delta v+3\Psi'\right)=0\ , \tag{5}$$

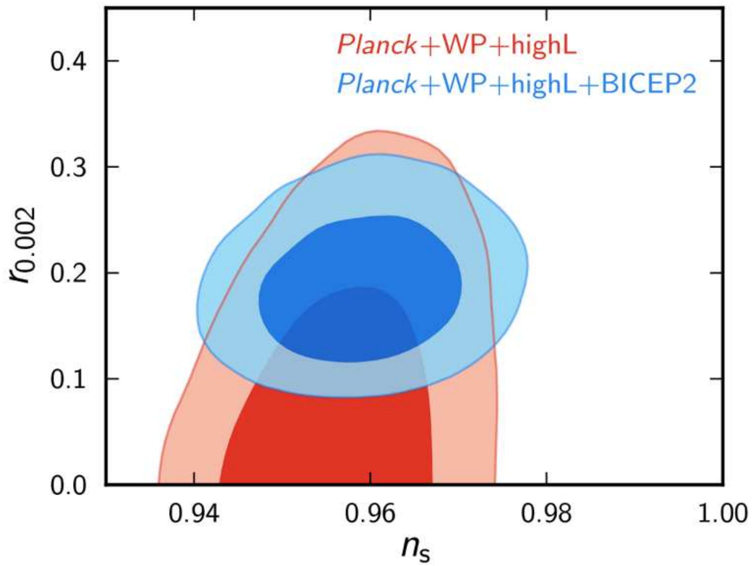
$$[(\rho+p)v]'+\delta p+(\rho+p)\left(4\frac{a'}{a}v+\pi+\Phi\right)=0\ . \tag{6}$$

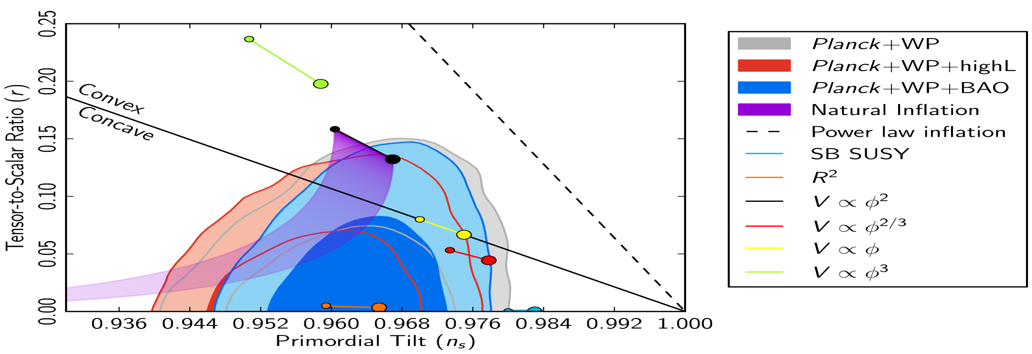
Planck + all: Scalar tilt vs tensor power



BICEP-2 claim $r = 0.2$

- $r = 0.2$ is large: 10% contribution to δT at low multipoles, $l \lesssim 30$.

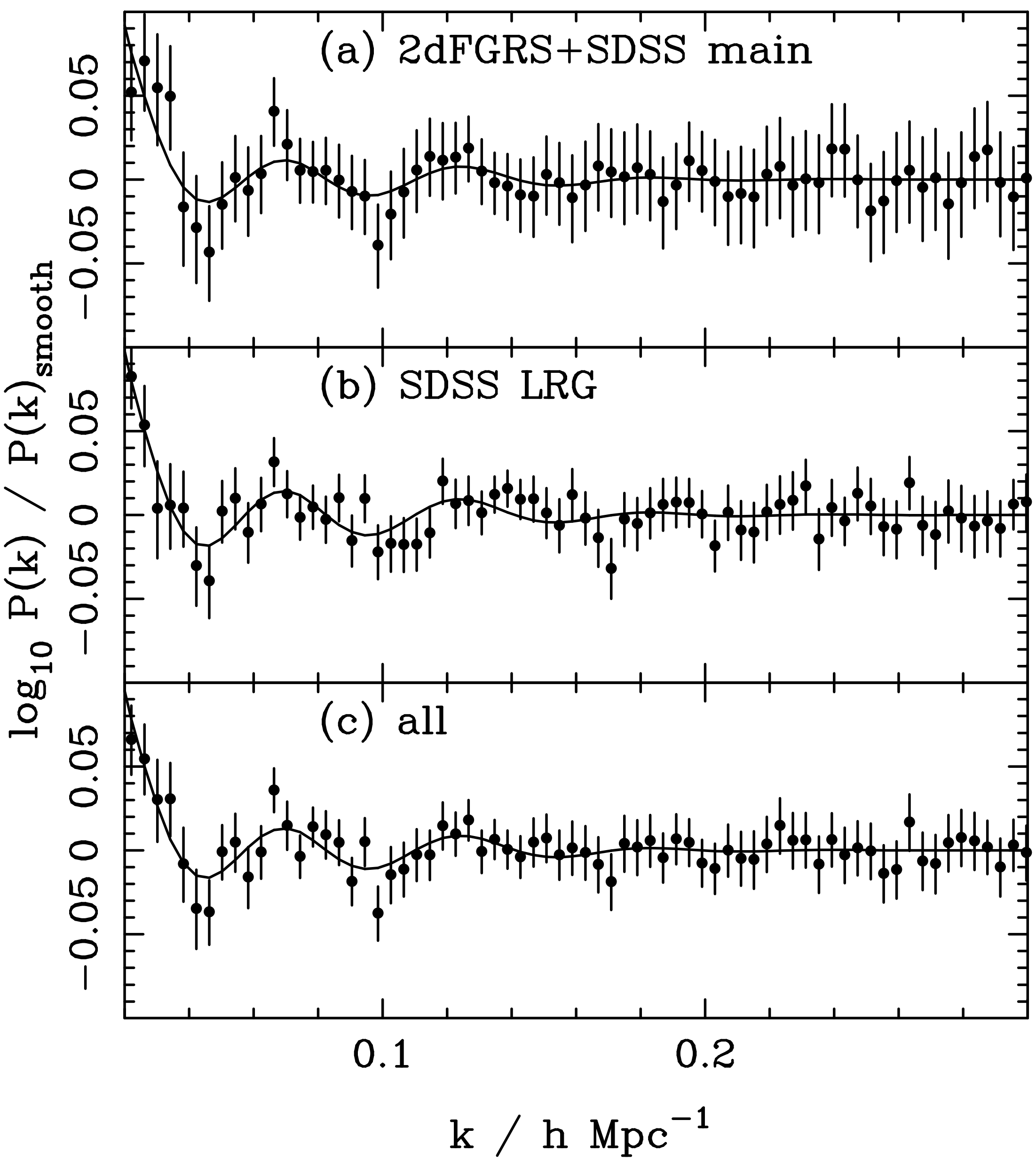




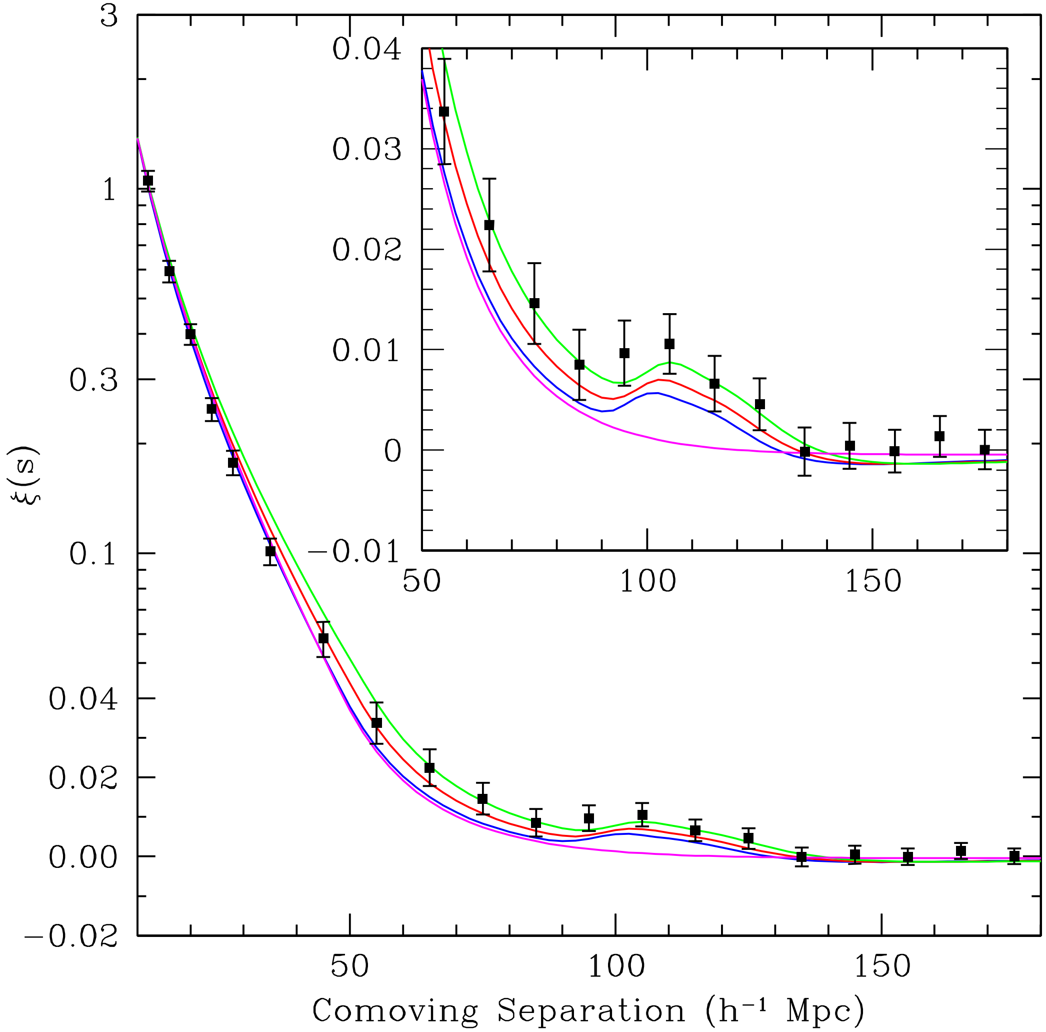
BICEP2 and Planck with $dn_s/d\ \log k = -0.02$ (very large)
Inflation: $dn_s/d\ \log k \approx -0.001$

Planck + all

BAO in power spectrum



BAO in correlation function



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Lecture 9

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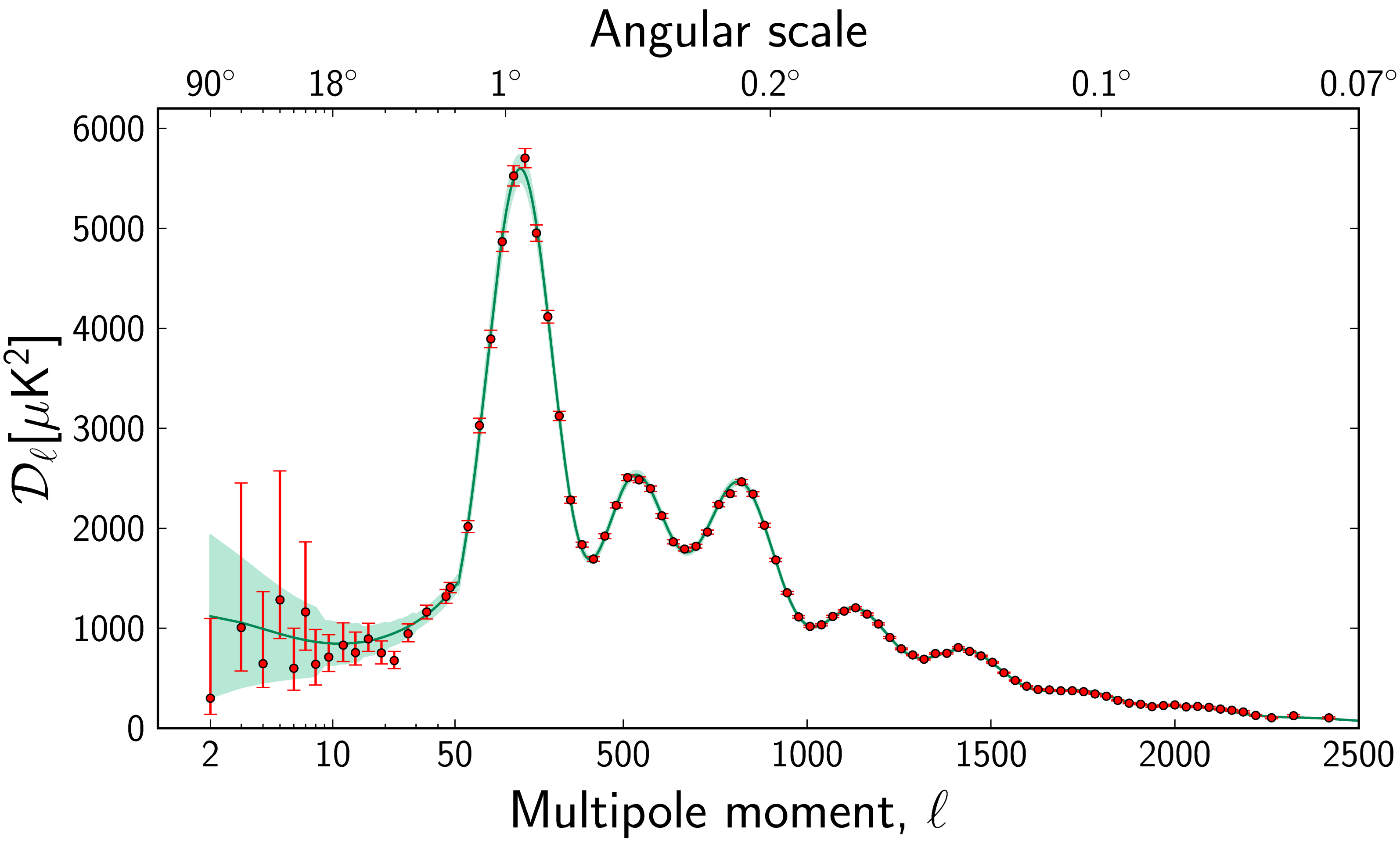
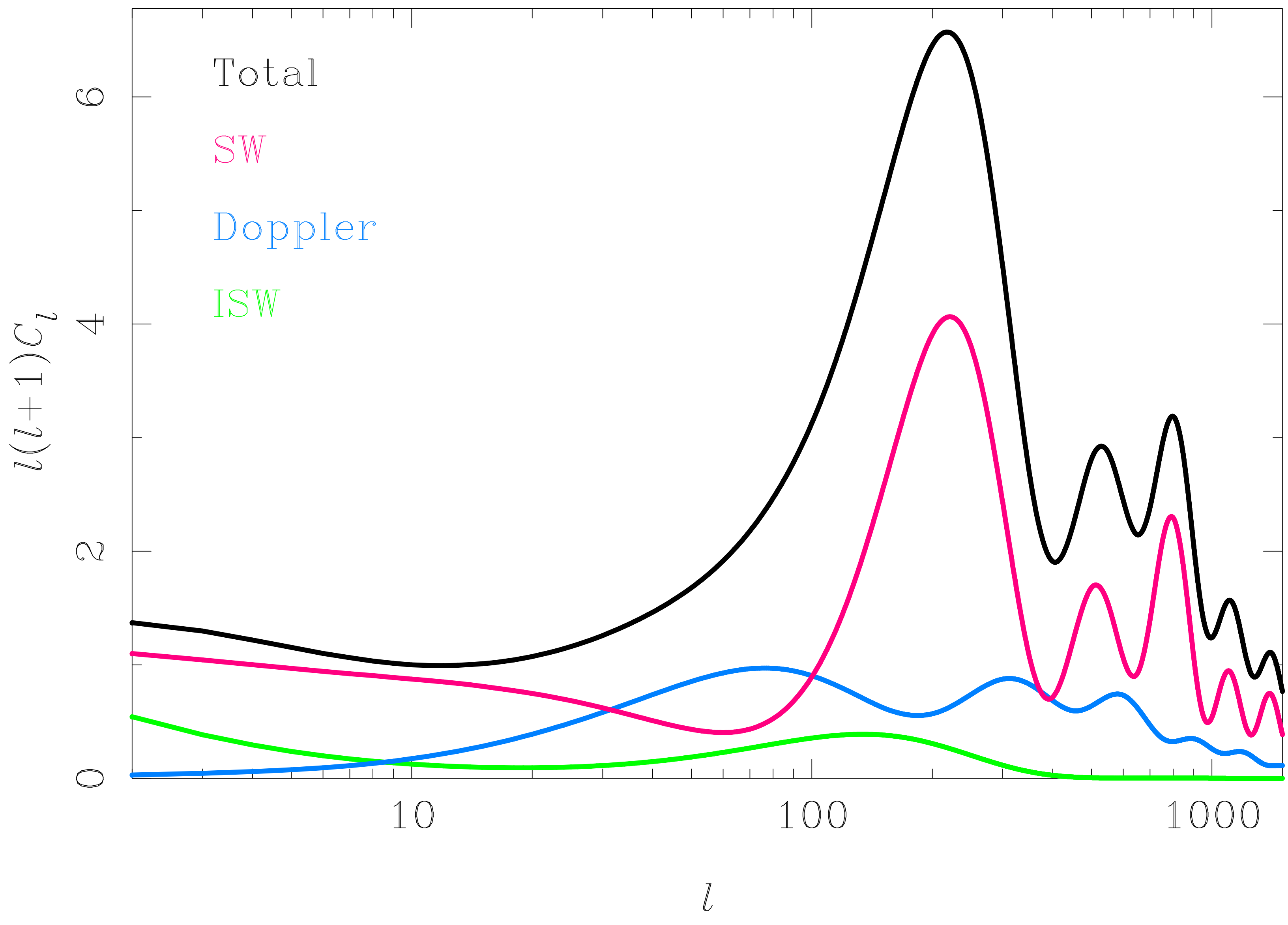
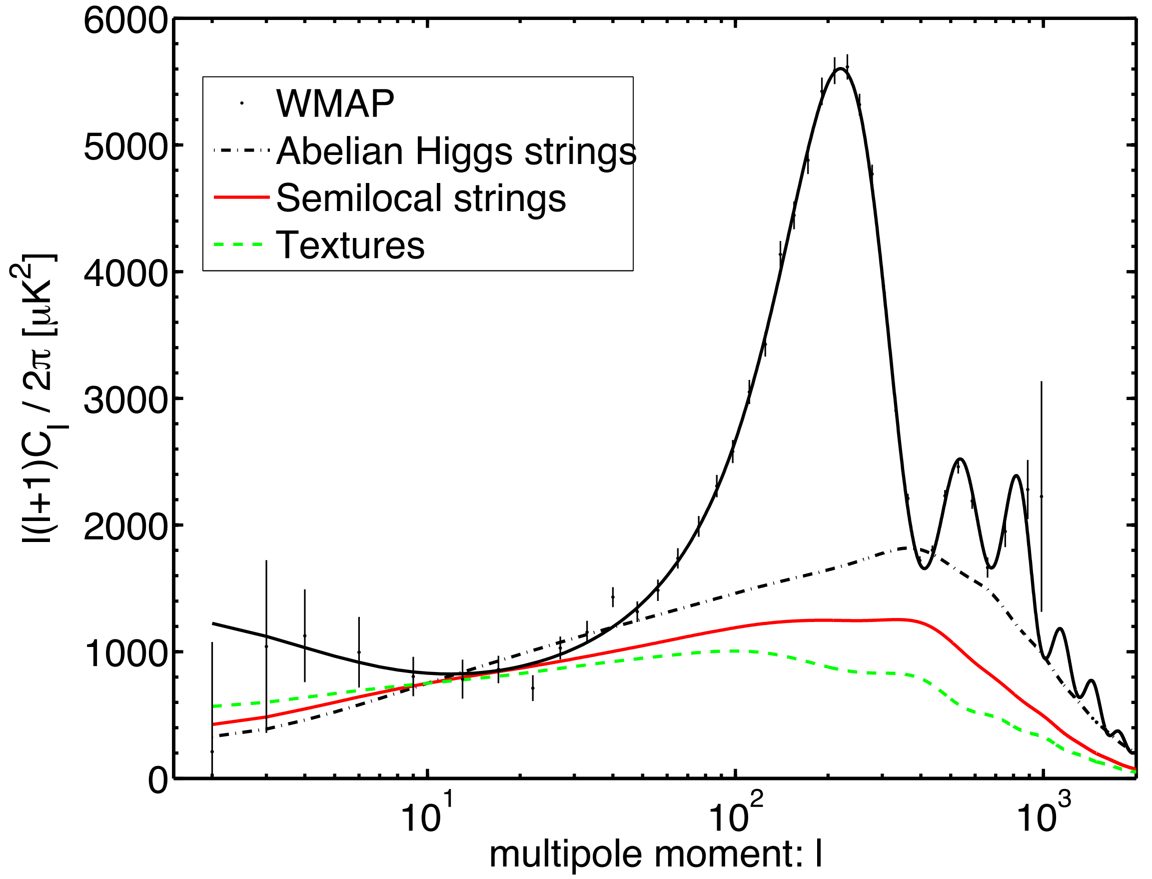
CMB temperature angular spectrum



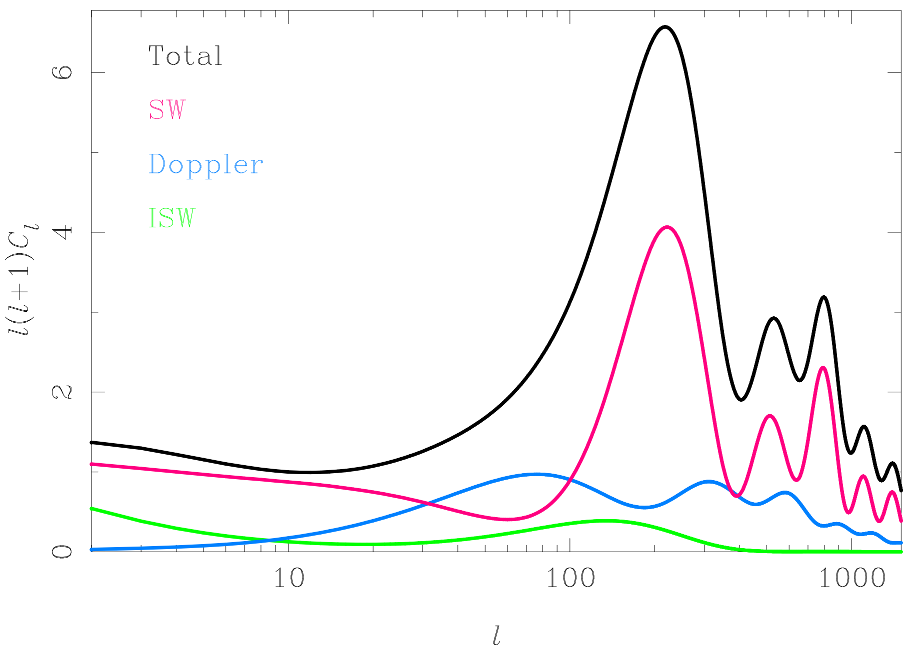
Three contributions



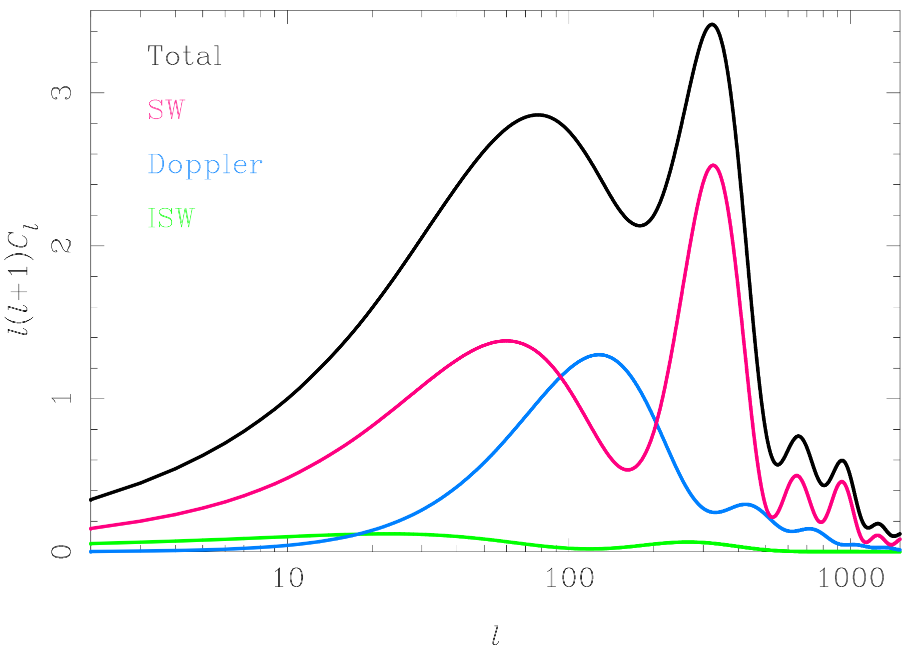
Top. defects are not seeds for perturbations



Effects of adaiabatic and entropy perturbations

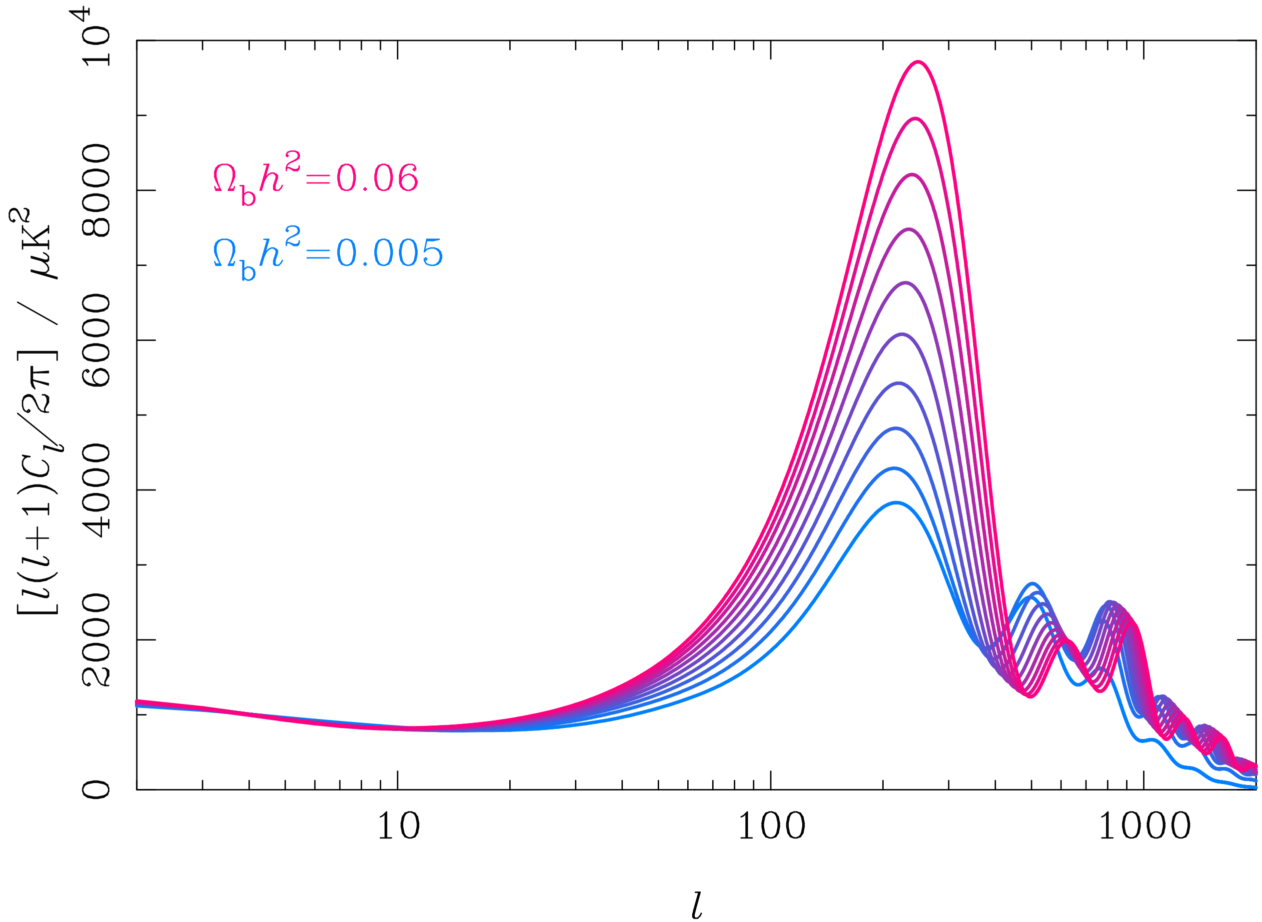


adiabatic perturbations

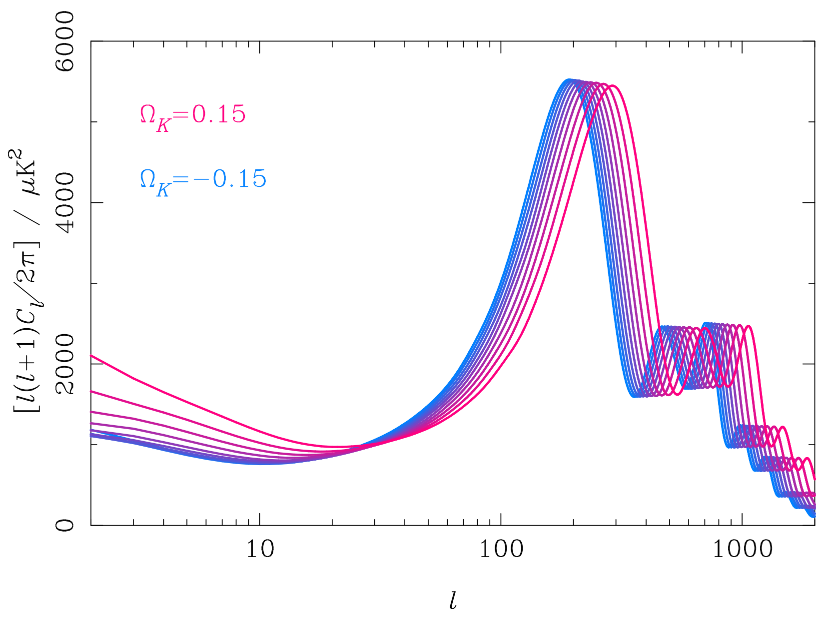


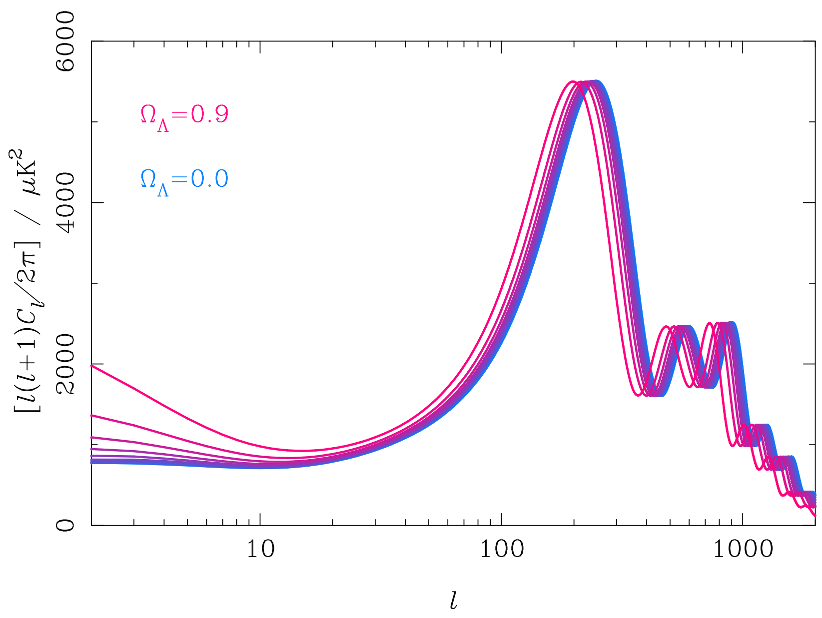
entropy perturbation

Effect of baryons



Effect of curvature (left) and Λ





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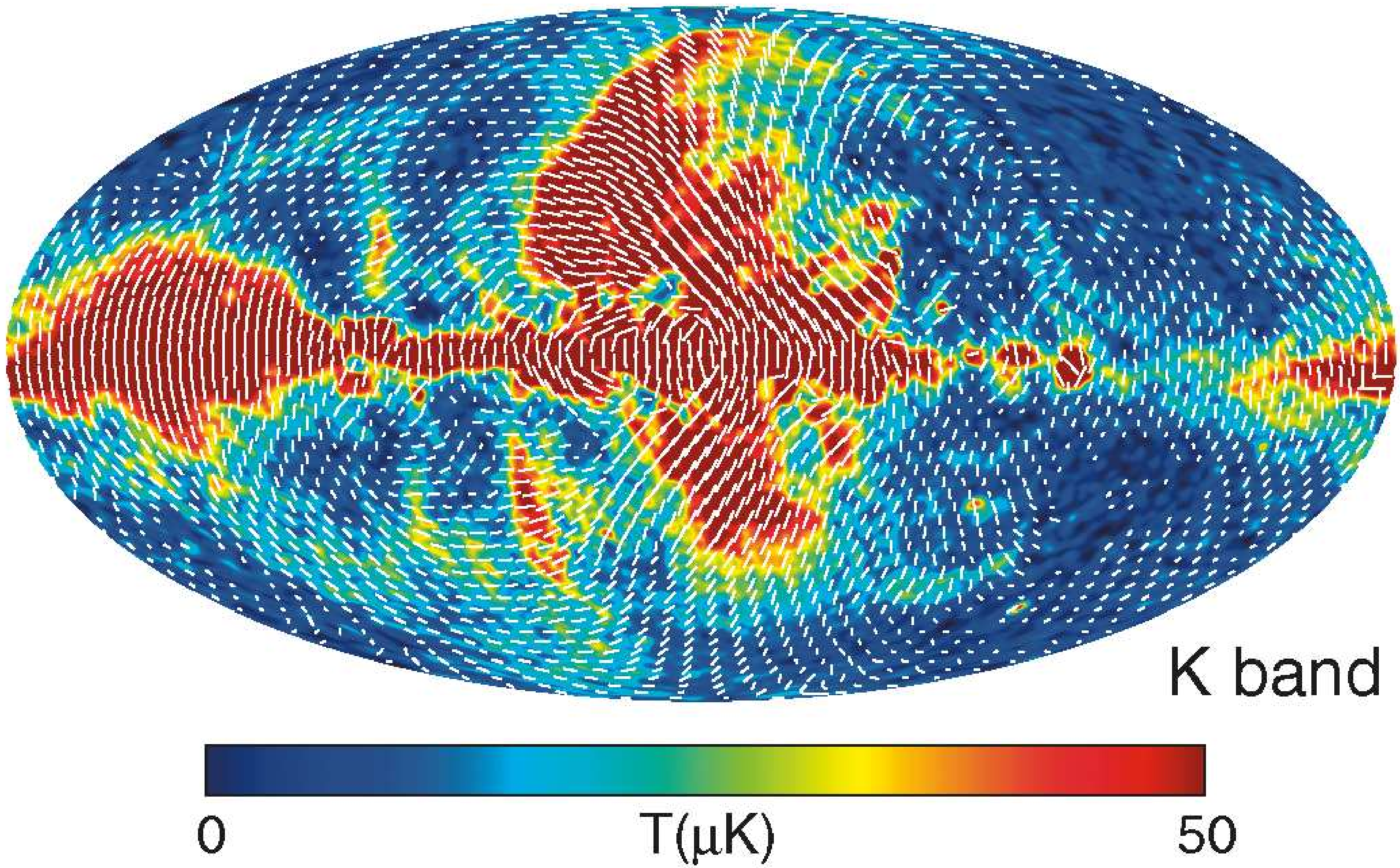
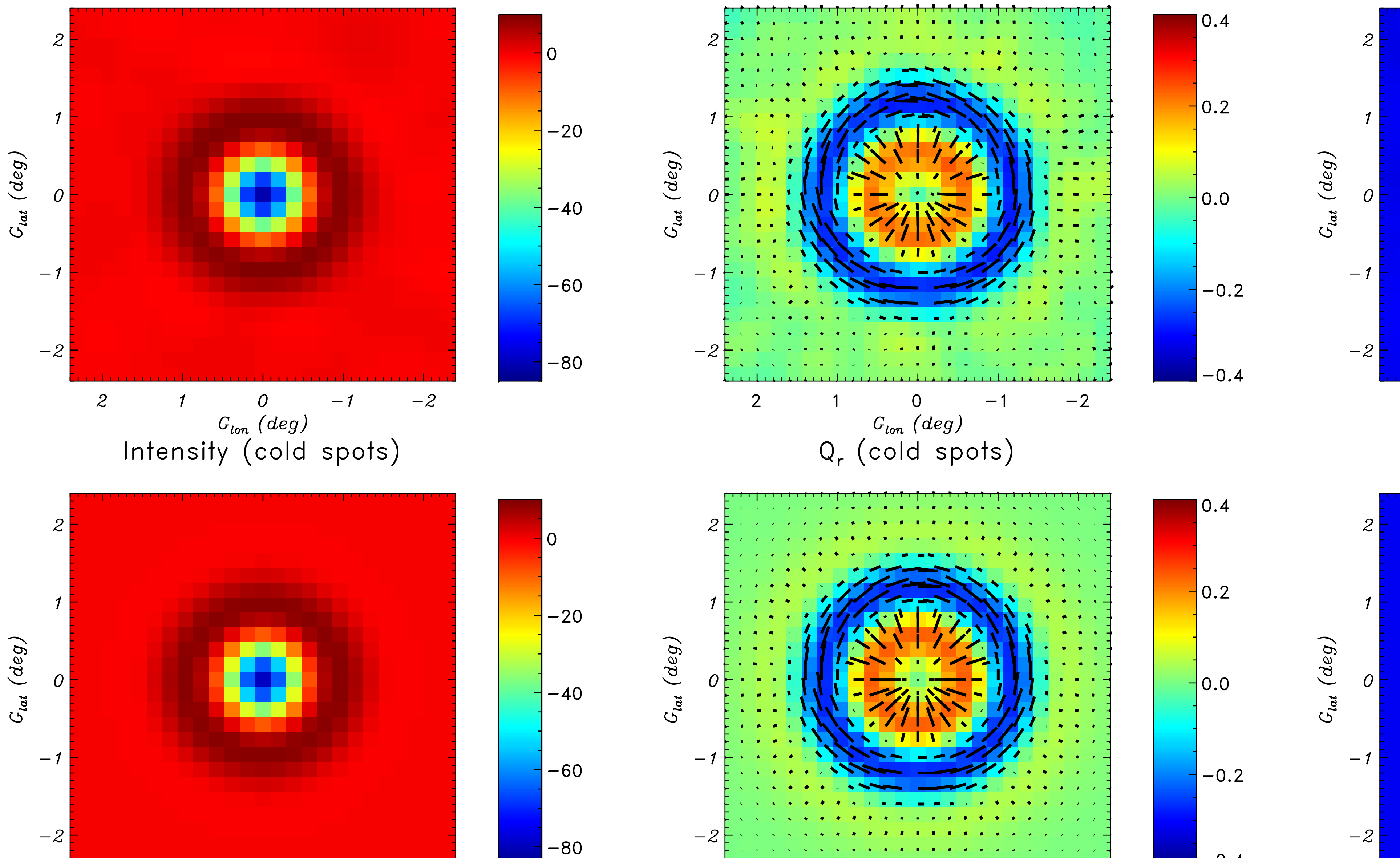
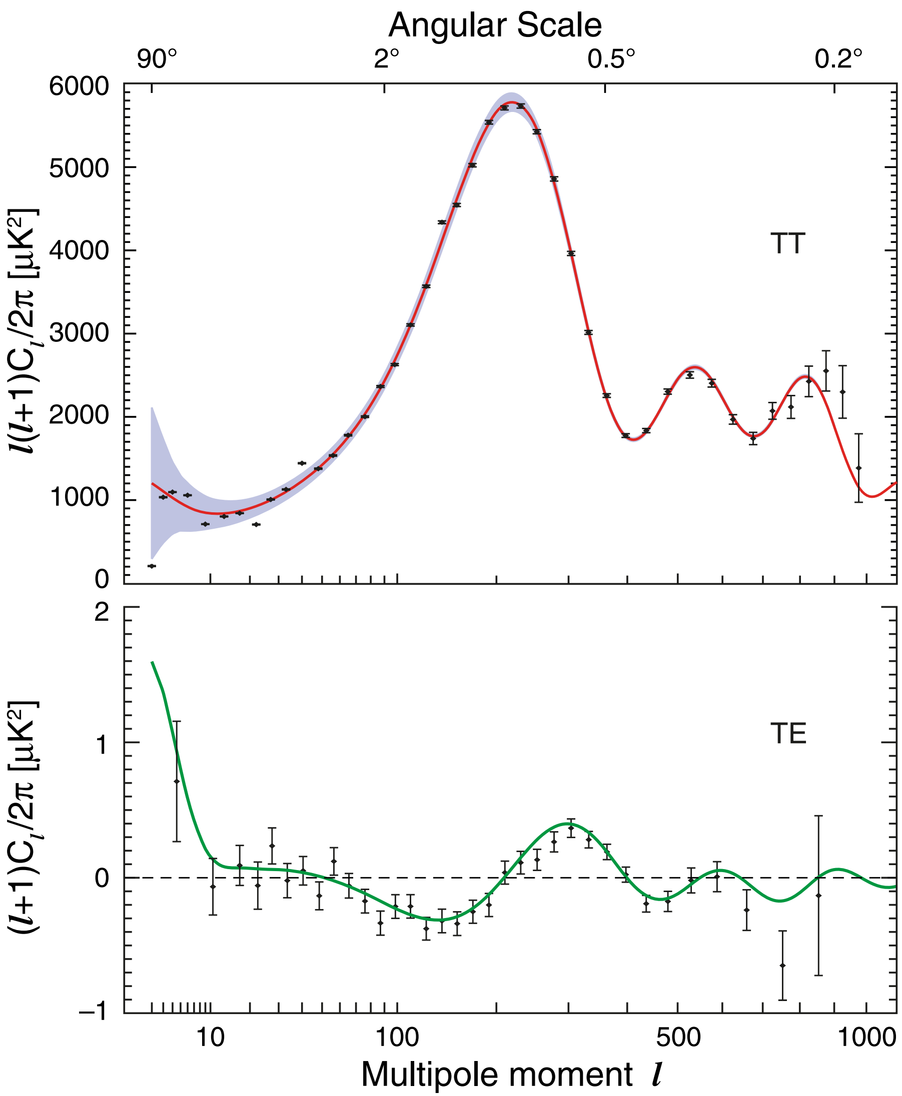
WMAP polarization sky



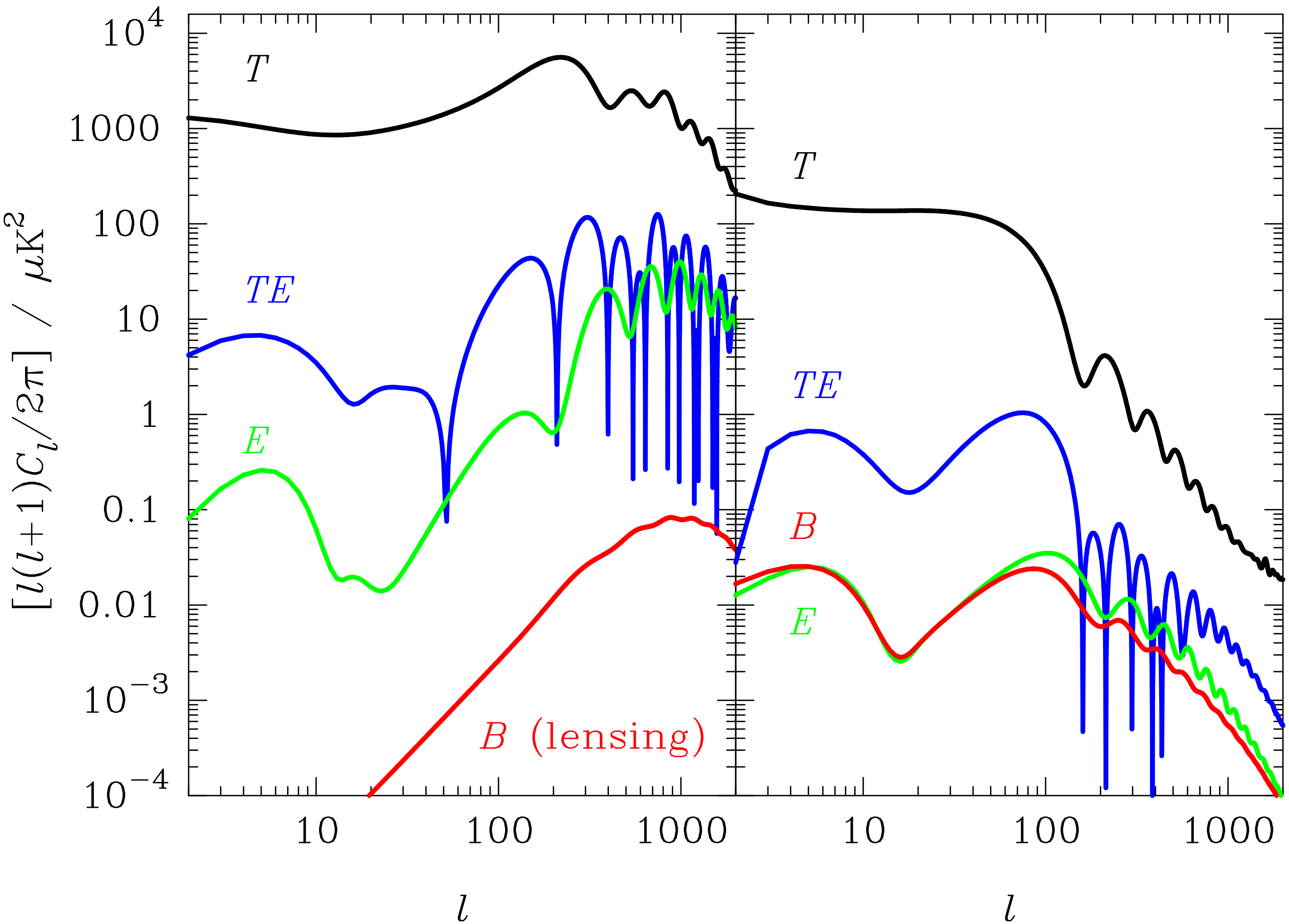
Planck polarization spots



WMAP cross correlation spectrum TE



Effects of scalar (left) and tensor (right) pertubations



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3.2 Adiabatic mode

- 3.2.1 Initial condition: ζ
- 3.2.2 Summary of experimental results
- 3.2.3 Summary: at recombination
- 3.2.4 After recombination: BAO.

Cont’d

Lecture 5

4. Elements of theory of CMB temperature anisotropy

   4.1 Sachs–Wolfe, Doppler
   and integrated Sachs–Wolfe effects

   4.2 Large angular scales

   4.3 Intermediate angular scales

   4.4 What do we learn from CMB temperature anisotropy

Lecture 6

5. Elements of theory of CMB polarization

5.1 Polarization in Thomson scattering

5.2 More about photon last scattering

5.3 Reionization

5.4 Polarization tensor. E- and B-modes

5.4 Effect of scalar perturbations

5.4 Effect of tensor perturbations

Cont'd

Lecture 7

6. Inflation

Lecture 8

Inflation

- 6.5 Estimating non-Gaussianity
- 6.6 Curvaton mechanism. Examples of non-Gaussianity
- 6.7 Example of generation of entropy mode:
  axion misalignment

Lecture 9

7. Alternatives to inflation
   - 7.1 Bouncing Universe
   - 7.2 Genesis
   - 7.3 Generating scalar perturbations

8. Conclusions

Slow roll

Equations

$$3H\dot{\phi} = -V'$$

$$H^2 = \frac{8\pi}{3}\frac{V}{M_{Pl}^2}$$

Parameters

$$\varepsilon = \frac{V'^2 M_{Pl}^2}{16\pi V^2}$$

$$\eta = \frac{V'' M_{Pl}^2}{8\pi V}$$

Cosmology

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Tentative outline

Lecture 2

2. Cosmological perturbations I
   - 2.1 Generalities
     - 2.1.1 Perturbed metric. Gauge invariance.
     - 2.1.2 Perturbed energy-momentum tensor.
   - 2.2 Helicity decomposition
   - 2.3 Tensor modes
     - 2.3.1 Equation
     - 2.3.2 Regimes of evolution
     - 2.3.3 Solutions in superhorizon and subhorizon regimes. Matching.
   - 2.4 Digression: conformal times of recombination and radiation-matter equality
   - 2.5 Vector modes

Cont'd

Lectures 3,4

3. Cosmological perturbations II

Lecture 3

- 3.1 Scalar modes
  - 3.1.1 Conformal Newtonian gauge
  - 3.1.2 Equations
  - 3.1.3 Sample solutions: single component fluids
    - 3.1.3.1 Master equation
    - 3.1.3.2 Radiation
    - 3.1.3.3 Matter
    - 3.1.3.4 Matter at Λ-domination
  - 3.1.2 Classification: adiabatic mode, entropy (isocurvature) modes

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Lecture 7

6. Inflation

    6.1 Main idea
    6.2 Inflaton as a driver
    6.3 Generation of tensor perturbations
    6.4 Generation of scalar perturbations: inflaton mechanism

Lecture 8

Inflation, part 2

6.5 Single field: estimating non-Gaussianity

6.6 Single field: eternal inflation

6.7 Curvaton mechanism. Examples of non-Gaussianity and entropy modes

6.8 Comments on reheating

Lecture 9

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    7.1 Null Energy Condition and its violation

    7.2 Galilean Genesis

    7.3 Contracting Universe: Belinsky–Lifshits–Khalatnikov problem and way out

    7.4 Generating scalar perturbations: conformal models

8. Overall conclusions

Problems for exam

1. Consider single field inflation in slow roll regime, at which the slow roll parameter ε decreases in time. Take any inflaton potential consistent with the CMB and galaxy distribution data. Show that during the period at inflation, which is responsible for generating the adiabatic perturbations, the inflaton field rolls down at least by

$$\Delta\phi \gtrsim 10r \cdot M_{Pl} \ ,$$

where r is the tensor-to-scalar ratio. [This means, in particular, that the discovery of tensor modes with $r \sim 0.2$, as originally claimed by BICEP-2, would imply that the variation of the inflaton over the relevant period of time at inflationary epoch was super-Planckian.]

2. Relatively short gravity waves, created at inflation, after horizon re-entry at radiation domination can be viewed as a collection of gravitons (just like electromagnetic waves emitted by antenna can be viewed as a collection of photons). Assuming that the Hubble parameter H some 60 e-foldings before inflation end is known, calculate the average (over enesemble of universes) number of gravitons $\langle N(k, \Delta k) \rangle$ in the present visible Universe in the interval of momenta from k/a_0 to $(k + \Delta k)/a_0$, and relative variance of this number

$$\frac{\sqrt{(\langle N^2(k, \Delta k) \rangle - \langle N(k, \Delta k) \rangle^2}}{\langle N(k, \Delta k) \rangle} .$$

Dropping the assumption about the value of the Hubble parameter, calculate these quantities for the inflaton potential $V = m^2 \phi^2 / 2$. Give numerical estimates in the latter case for $k/a_0 = 1$ Mpc^{-1}, $\Delta k = k$.

3. Consider Minkowski space and generalized Galileon theory with the Lagrangian

$$L = e^{4\pi} F(Y) + e^{2\pi} K(Y) \Box \pi$$

where

$$Y = e^{-2\pi} (\partial \pi)^2 .$$

Let F and K be chosen in such a way that there is a solution

$$e^{\pi} = \frac{\text{const}}{-t} , \qquad t < 0 .$$

Let this solution be healthy, i.e., there are neither ghosts nor gradient instabilities among preturbations about this solution. Find the power specrum of perturbations $\delta\pi$ about this solution in the regime $k|t| \ll 1$, where k is the spatial momentum of a mode.