II. Problems of (questions for) the Standard Model

R. Barbieri

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Problems of (questions for) the SM

0. Which rationale for matter quantum numbers?

$$|Q_p + Q_e| < 10^{-21} e$$

1. Phenomena unaccounted for

neutrino masses
Dark matter
matter-antimatter asymmetry
inflation?

2. Why $\theta \lesssim 10^{-10}$? $\theta G_{\mu\nu} \tilde{G}^{\mu\nu}$

Axions

3. $\mathcal{O}_i : d(\mathcal{O}_i) \leq 4$ only?

neutrino masses
Gravity
Are the protons forever?

4. Lack of calculability (a euphemism)

the hierarchy problem
the flavour paradox

Why $|Q_p + Q_e| < 10^{-21} e$?

(recall Einstein's lesson from $m_{in} = m_{grav}$)

$$\Psi = Q(3,2)_{1/6} \;\; u(\bar{3},1)_{-2/3} \;\; d(\bar{3},1)_{1/3} \;\; L(1,2)_{-1/2} \;\; e(1,1)_1$$

Ψ = next-to-simplest rep of $\mathcal{G}$:

?

chiral, anomaly-free, vector-like under $SU(3) \times U(1)_{em}$

However:

1. A simpler rep: $\Xi = (3,2)_0 \;\; (\bar{3},1)_{1/2} \;\; (\bar{3},1)_{-1/2}$

2. What if ν_R are added?

?

$$\tilde{\Psi} = Q(3,2)_y \;\; u(\bar{3},1)_{-y-1/2} \;\; d(\bar{3},1)_{-y+1/2} \;\; L(1,2)_{-3y} \;\; e(1,1)_{5y+1/2} \;\; \nu^c(1,1)_{3y-1/2}$$

(An important hint for "algebraic" Unification?)

The unification way: $SU(5)$

A unique "embedding" of $SU_{3,2,1}$ into SU_5

$$\left(\begin{array}{c|c} \frac{1}{2}\lambda^i_{3\times 3} & \\ \hline & \frac{1}{2}\sigma^a_{2\times 2} \end{array}\right) \qquad Y \propto \left(\begin{array}{c|c} 1_{3\times 3} & \\ \hline & -\frac{3}{2}1_{2\times 2} \end{array}\right)$$

The particle content follows in the simplest reps

$$\bar{5} = \begin{pmatrix} d^c{}_1 \\ d^c{}_2 \\ d^c{}_3 \\ e^- \\ -\nu_e \end{pmatrix} \qquad 10 = \begin{pmatrix} 0 & u^c{}_3 & -u^c{}_2 & -u_1 & -d_1 \\ & 0 & u^c{}_1 & -u_2 & -d_2 \\ & & 0 & -u_3 & -d_3 \\ & & & 0 & -e^c \\ & & & & 0 \end{pmatrix}$$

?

with all quantum numbers fixed (including hypercharge)

Neutrino masses

Known to be nonzero since about 1990

Yet vanishing in the SM because of an accidental symmetry: L-conservation

$$(A, Z) \rightarrow (A, Z+1) + e + \bar{\nu}$$

Accidental symmetries are not exact

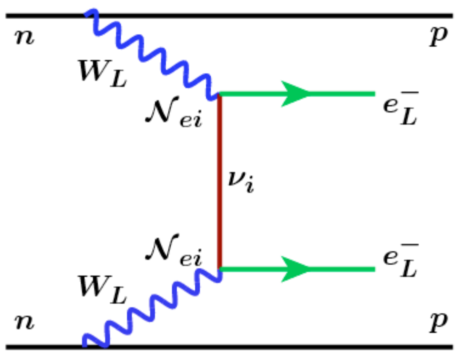
$$\Downarrow$$

$$\Delta\mathcal{L} = \frac{(LH)(LH)}{M}$$

Neutrinos are massive and of Majorana type ($\nu = \bar{\nu}$)

Should observe: $(A, Z) \rightarrow (A, Z+2) + 2e$

So far $\tau(2\beta^{0\nu}) \gtrsim 10^{25}\ years$



Neutrino oscillations

(in the standard 3-neutrino framework)

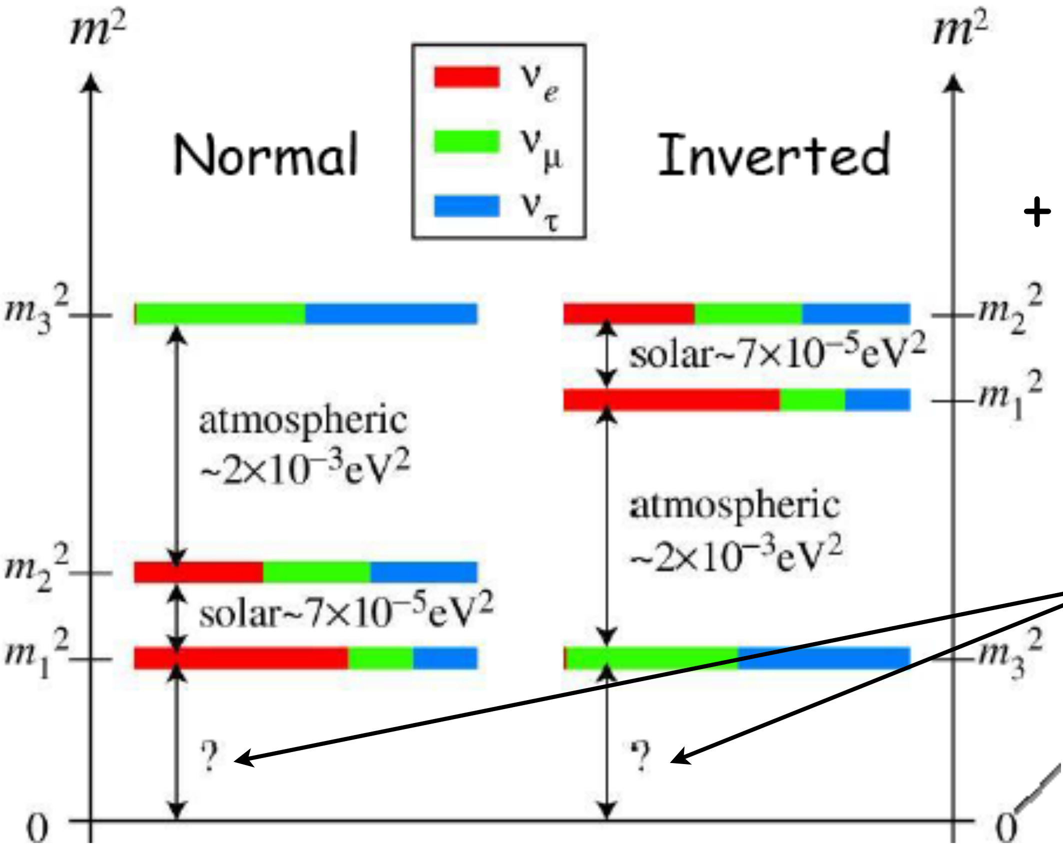
$$|\nu_{l_\alpha}> \;\; \underset{t=0}{\vdash} \overset{}{\underset{R=ct}{\text{———————}}} \underset{t}{\dashv} \;\; |\nu_{l_\beta}> \qquad l_{\alpha,\beta} = e, \mu, \tau$$

?

$$|\nu_i> = V_{i\alpha}|\nu_\alpha>$$

$$\alpha = e, \mu, \tau$$

$$<\nu_\beta|\nu_\alpha, t> = \sum_{i=1,2,3} V^*_{i\beta} V_{i\alpha} e^{-i\frac{m_i^2 t}{2p}}$$



$$V(\theta_{12}, \theta_{23}, \theta_{13}, \phi)$$

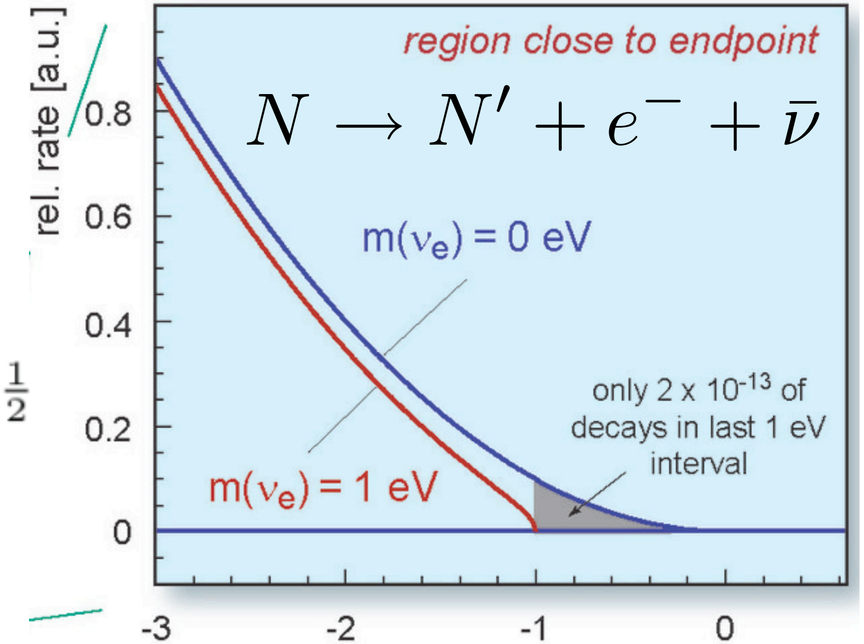
+ 2 more phases if ν's are Majorana, not affecting oscillations

?

the absolute scale yet unknown

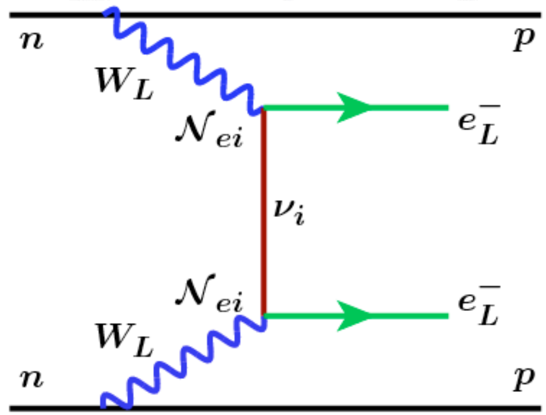
?

3 ways to be sensitive to the absolute ν-mass scale



1- beta-decay endpoint

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$



2- neutrino-less ββ-decay

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

3 - cosmology (large scale structures)

$$\Sigma = m_1 + m_2 + m_3$$

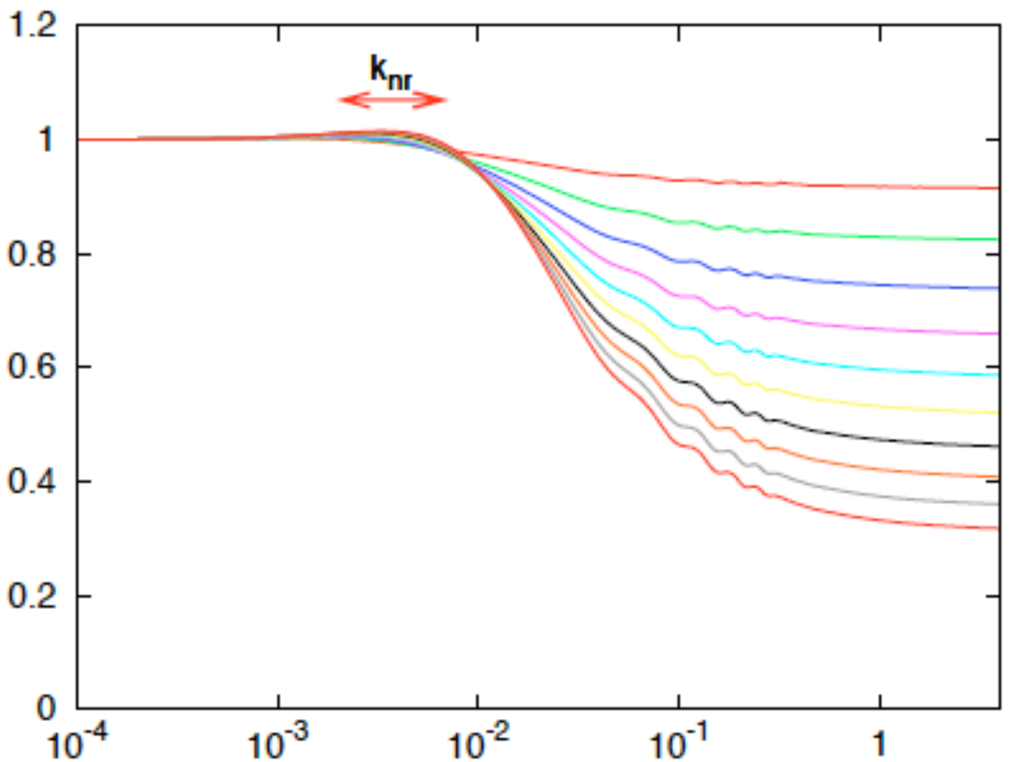
Power spectrum of large scale structures

$$\delta \equiv \frac{\rho(\mathbf{r}) - \bar{\rho}}{\bar{\rho}} \quad \overset{\text{Fourier tr}}{\longrightarrow} \quad \delta(\mathbf{k})$$

$$\xi(\mathbf{r}) \equiv < \delta(\mathbf{x}+\mathbf{r})\delta(\mathbf{x}) > \quad \overset{\text{Fourier tr}}{\longrightarrow} \quad |\delta(\mathbf{k})|^2 \equiv P(\mathbf{k})$$

the neutrino fluid influences $P_m(k)$ by gravitational interactions

$$\frac{P_m(k)^{\Sigma \neq 0}}{P_m(k)^{\Sigma = 0}}$$



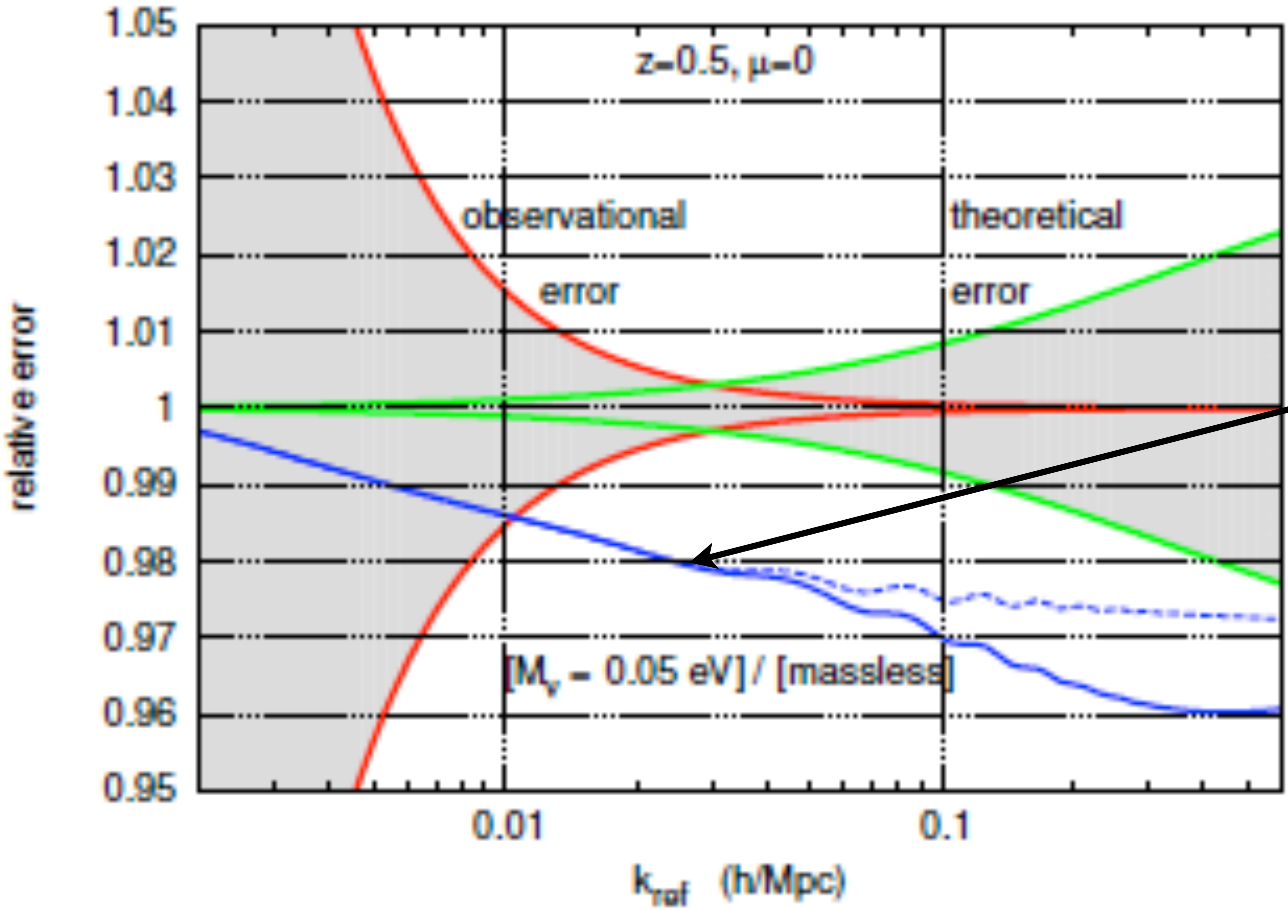
$$\Sigma = m_1 + m_2 + m_3$$

$$k(h/Mpc)$$

Lesgourgues, Pastor 2006

Power spectrum of large scale structures

Power spectrum $P(k)/P_{massless\ \nu}(k)$



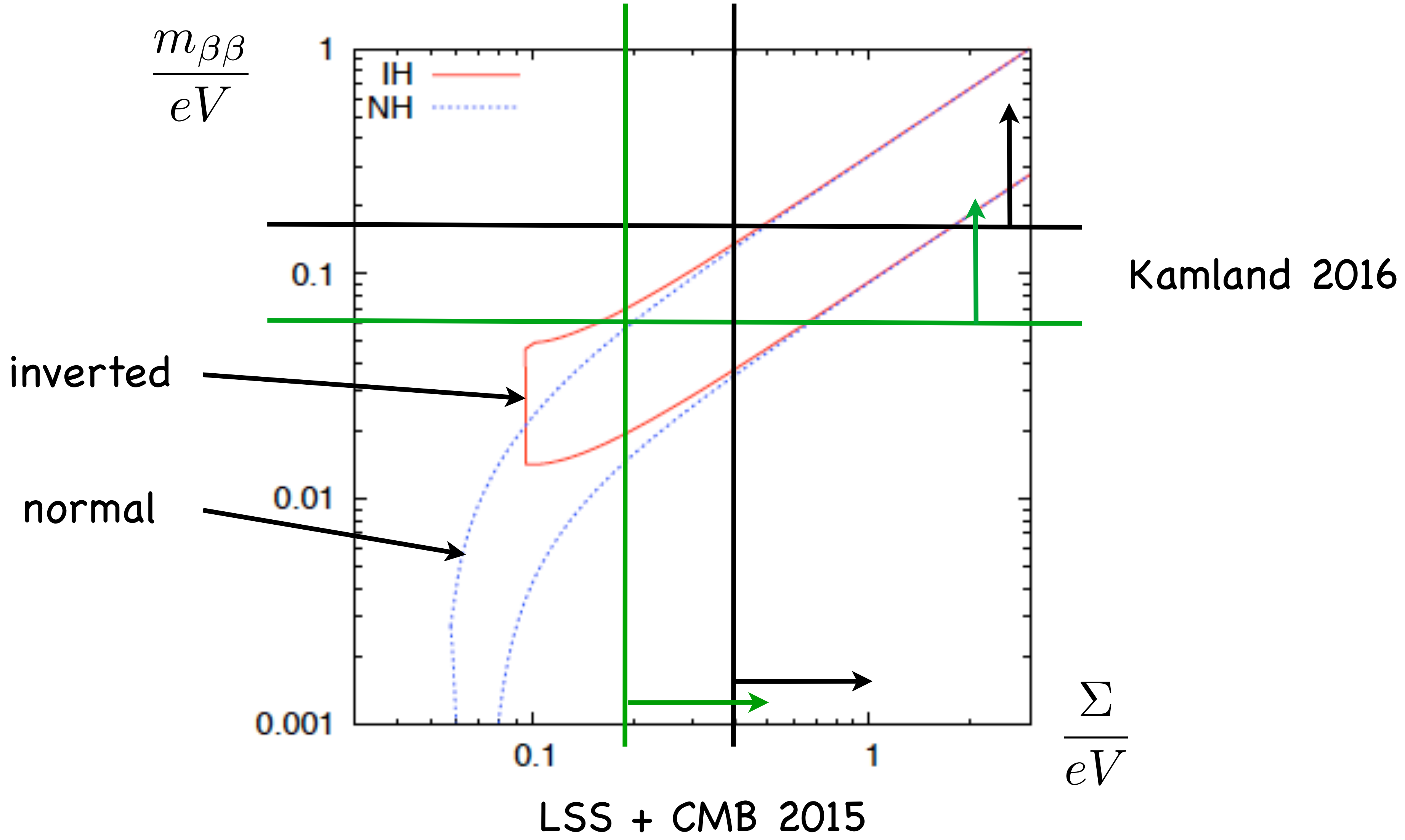
ratio between
with (ν massive) and
without (ν massless)
"free streaming"

Lesgourgues et al, 2103

- Determination with future large-scale structure observations (Euclid) at $2-5\sigma$ depending on control of (mildy) non-linear physics

▶ Not independent on "priors" but still highly significant

current bounds (with uncertainties)



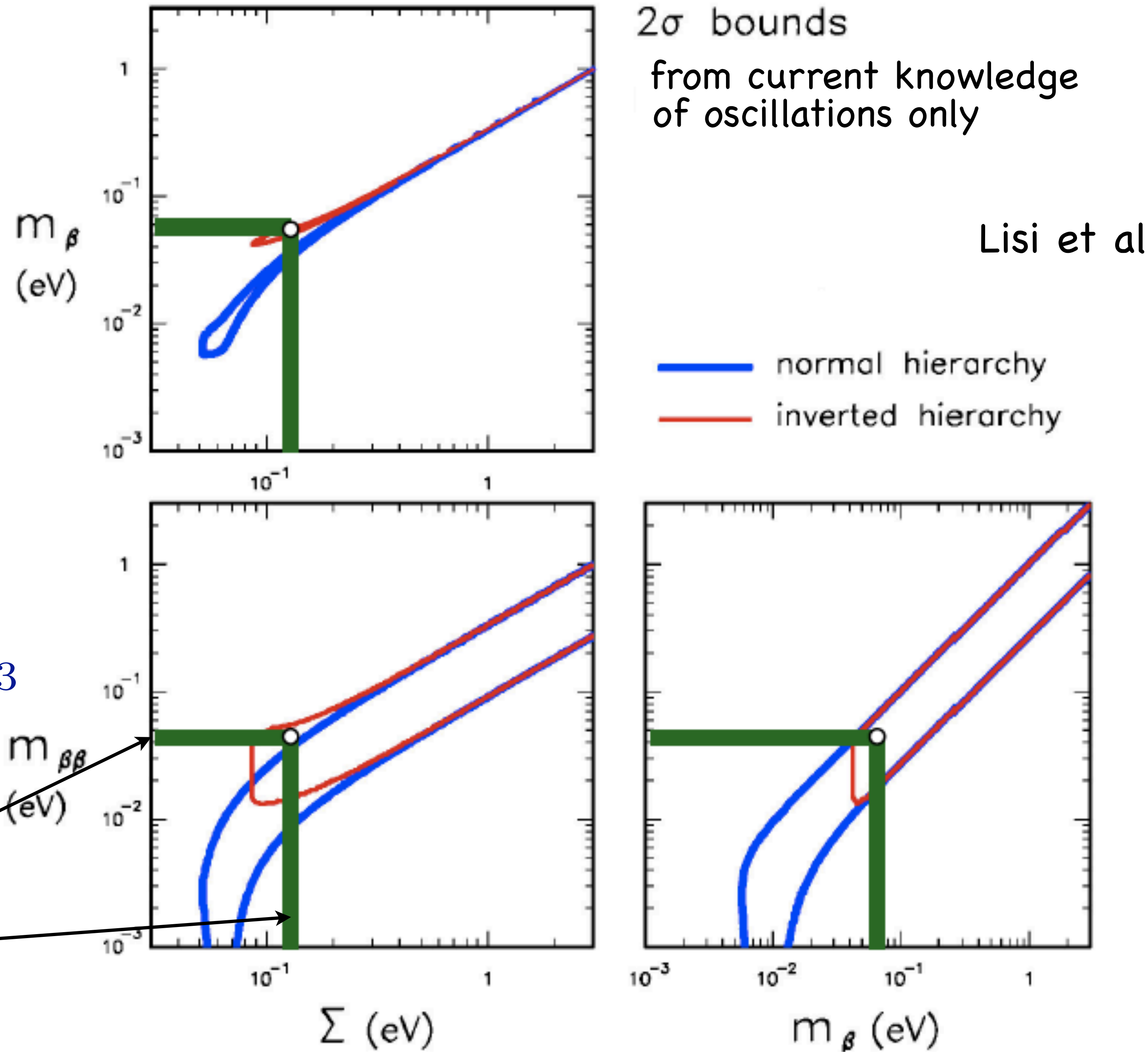
green = optimistic black = realistic/pessimistic

Key neutrino measurements

m_β
beta-decay endpoint

$m_{\beta\beta}$
neutrino-less $\beta\beta$ decay

$\Sigma = m_1 + m_2 + m_3$
large scale structures



hypothetical measurements

2. Why $\theta \lesssim 10^{-10}$? $\theta G_{\mu\nu}\tilde{G}^{\mu\nu}$

How do we know that $\theta \lesssim 10^{-10}$?

$\theta G_{\mu\nu}\tilde{G}^{\mu\nu}$ is T-odd and (almost) the only source of T-violation in the SM

	$\vec{\mu}\cdot\vec{B}$	$\vec{d}\cdot\vec{E}$
T	$+$	$-$

$$|\vec{\mu_N}| = 2\cdot 10^{-14} e\cdot cm$$
$$|\vec{d_N}| \approx \theta\cdot 10^{-15} e\cdot cm$$
$$|\vec{d_N}|_{exp} < 3\cdot 10^{-26} e\cdot cm$$

$\Rightarrow$ Make θ a dynamical field forced in its cosmological history to relax to 0 (almost) and (possibly) appear as DM

A quick introduction to axions

2 field theory results that you should know:

1. In spite of being a 4-divergence $\mathcal{L}_\theta = \theta \; G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$ is physical

In a non-abelian case, there are pure gauge configurations that give a non-vanishing contribution to $S[A^a_\mu]$ at infinity

Crucial to solve the " η" problem in QCD

2. Due to the triangle anomaly

$$J_{\mu 5} = \bar{q}\gamma_\mu\gamma_5 q \qquad \partial_\mu J_{\mu 5} = \frac{\alpha_S N}{8\pi} G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

In fact, by a chiral transformation that makes M_q physical in

$$\mathcal{L}_M = \bar{q}_R M_q q + h.c.$$

$$\int d^4x \mathcal{L} \rightarrow Arg\; det M_q \int d^4x \partial_\mu J_{\mu 5}$$ so that

$\theta_{eff} = \theta + Arg\; det M_q$ is the physical combination

?

(out of $U(N)_L \times U(N)_R$ only $U(1)_A$ anomalous)

A quick introduction to axions

To solve the strong CP problem:

Embed the chiral symmetry into an exact classical

U(1)-symmetry (PQ) spontaneously broken at a scale f_a

Classical examples:

DFS $\mathcal{L} = \lambda S H_u H_d + Y_u \bar{Q} H_u u + Y_d \bar{Q} H_d d + Y_e \bar{Q} H_d e$

KSVZ $\mathcal{L} = \lambda S \bar{T} T + \bar{T} \gamma^\mu D_\mu T$ with T a new QCD triplet

The axion a(x) is the corresponding (pseudo)GB

A simplified laboratory

Consider a gauged $U(1)_A$

	f_L	f_R	ϕ
Q	1	0	-1

$$\mathcal{L}_Y = g_Y \phi \bar{f}_L f_R + h.c. \qquad \phi = (v+h)e^{i\frac{a}{v}}$$

$$J_\mu = \bar{f}_L \gamma_\mu f_L - v\partial_\mu a$$

Naively $\quad \partial_\mu J_\mu = -i g_Y v \bar{f}\gamma_5 f - v\partial_\mu^2 a = 0$

However, because of the anomaly, under a gauge transformation

$$a \to a + v\epsilon \qquad \delta\mathcal{L} = \epsilon\partial_\mu J_\mu = \frac{g^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \neq 0$$

unless one adds to the Lagrangian

$$\Delta\mathcal{L} = \frac{a}{v}\frac{g^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

so that $\quad \delta(\mathcal{L} + \Delta\mathcal{L}) = 0$

The axion Lagrangian

$$\mathcal{L}_a = -\frac{1}{2}|\partial_\mu a|^2 + \frac{\partial_\mu a}{f_a} J_\mu^{PQ} + \frac{a}{f_a}\frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

?

to keep the formal $U(1)_{PQ}$ invariance: $a \to a + \alpha f_a$

Useful to make the transformation to get rid of

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 \frac{a}{f_a} Q_a} q \qquad Q_a = \frac{1}{2}\mathbf{1}$$

$$\mathcal{L}_a \to \frac{\partial_\mu a}{f_a}(J_\mu^{PQ} - \bar{q}\gamma_\mu\gamma_5 Q_a q) - (\bar{q}_L \tilde{M}_q(a) q_R + h.c.)$$

$$\tilde{M}_q(a) = e^{i\frac{a}{f_a}Q_a} M_q e^{i\frac{a}{f_a}Q_a} \qquad < \bar{q}_L q_R >= B f_\pi^2 e^{i\Pi/f_\pi}$$

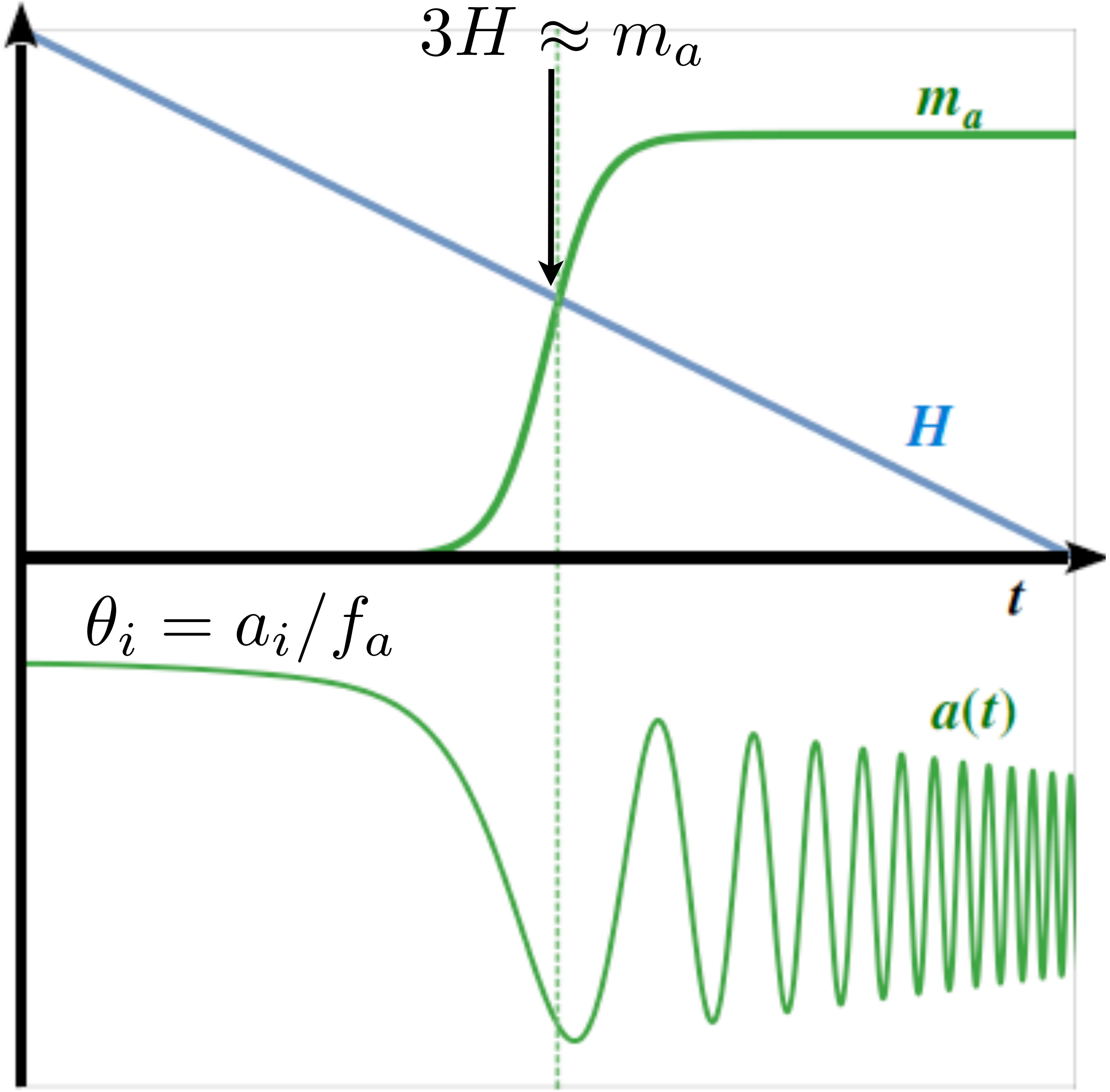
$$V(a, \pi_0) \quad \Rightarrow$$

$< a >= 0$ ⇒ no CPV

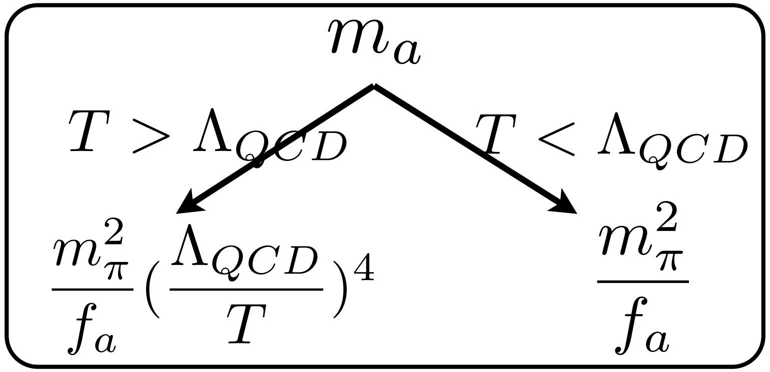
$$m_a^2 = \frac{m_u m_d}{m_u + m_d}\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

?

Relic abundance of the QCD axion



$$H = T^2/M_{Pl}$$



$$\rho_a = m_a^2 a^2 \propto T^3 \propto 1/R^3$$

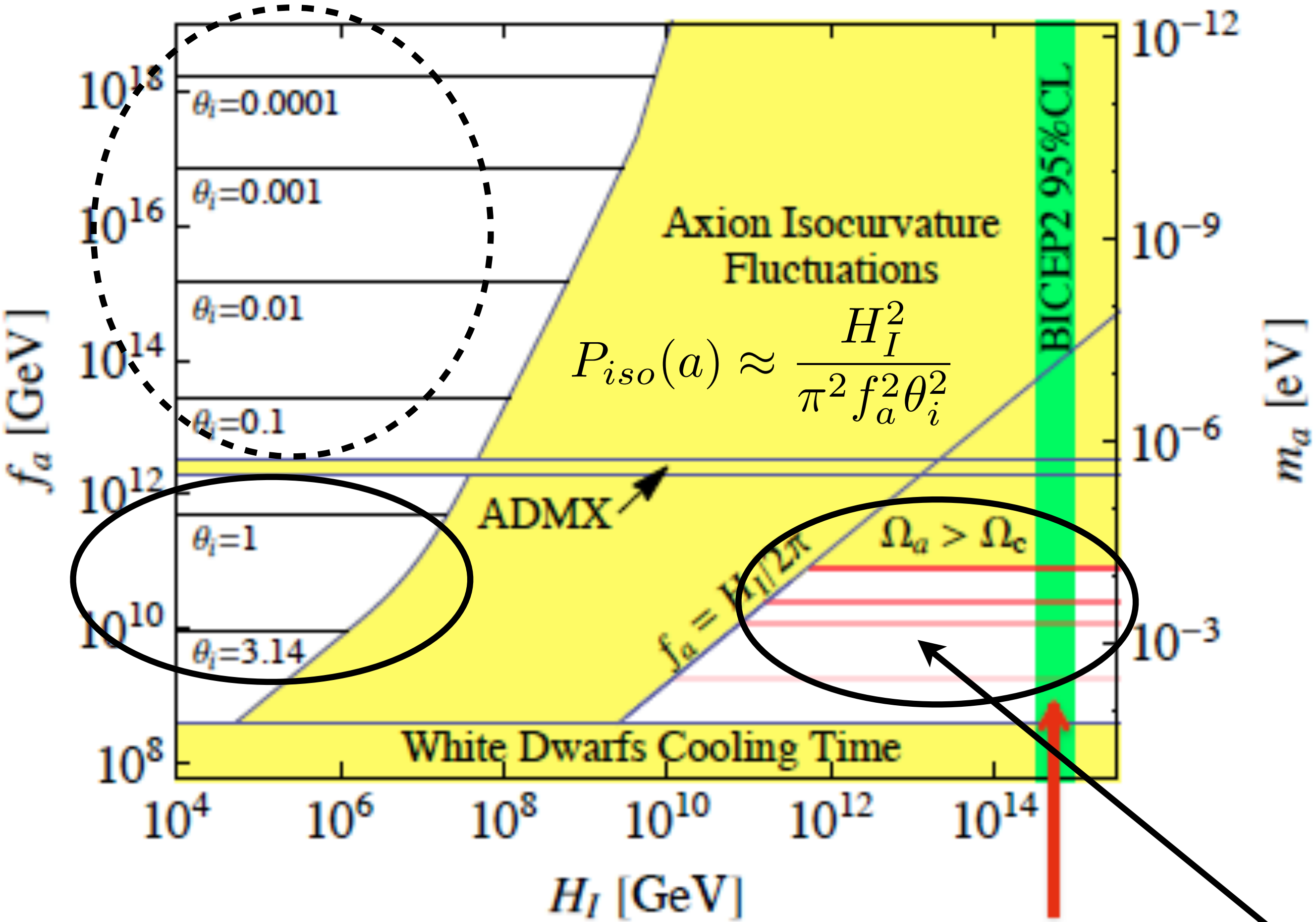
i.e. cold Dark Matter

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

?

QCD Axions in cosmology

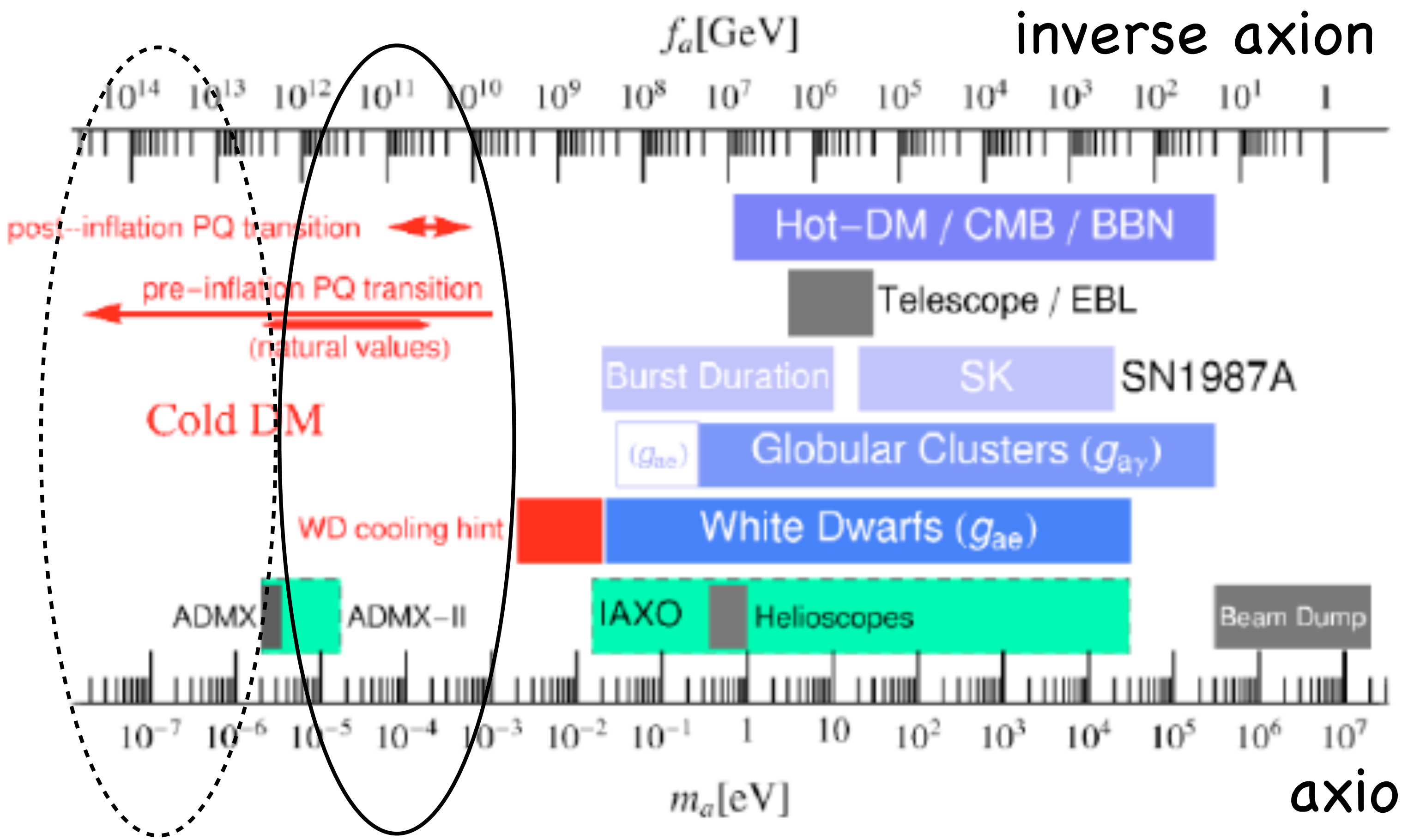
$m_a f_a \approx 10^{-4}\ eV \cdot 10^{11} GeV$



$$\Omega_a h^2 \approx 0.16\left(\frac{m_a}{10^{-5} eV}\right)^{-1.18} \theta_i^2 \quad \theta_i = \frac{a_i}{f_a} \qquad \theta_i^2 = \frac{\pi^2}{3}$$

(Axion Like Particles: m and f unrelated)

The dynamical field, a, is the "axion"



Olive et al, 2104

and is very intensively searched for

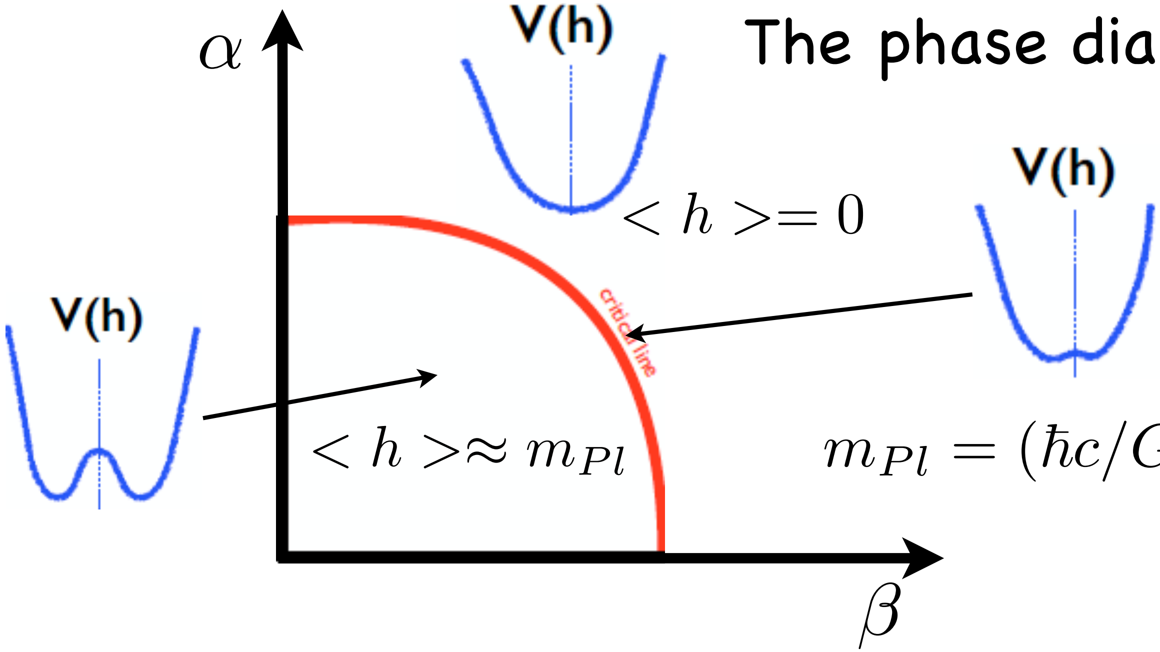
(with the most interesting region still unaccessible)

The "hierarchy" problem

Can we calculate the Higgs mass? NOT in the SM

If we try: $V(h) = m^2(\alpha, \beta)|h|^2 + \lambda |h|^4$

The phase diagram of the SM



$m_{Pl} = (\hbar c / G_N)^{1/2} \approx 10^{19}\ GeV$

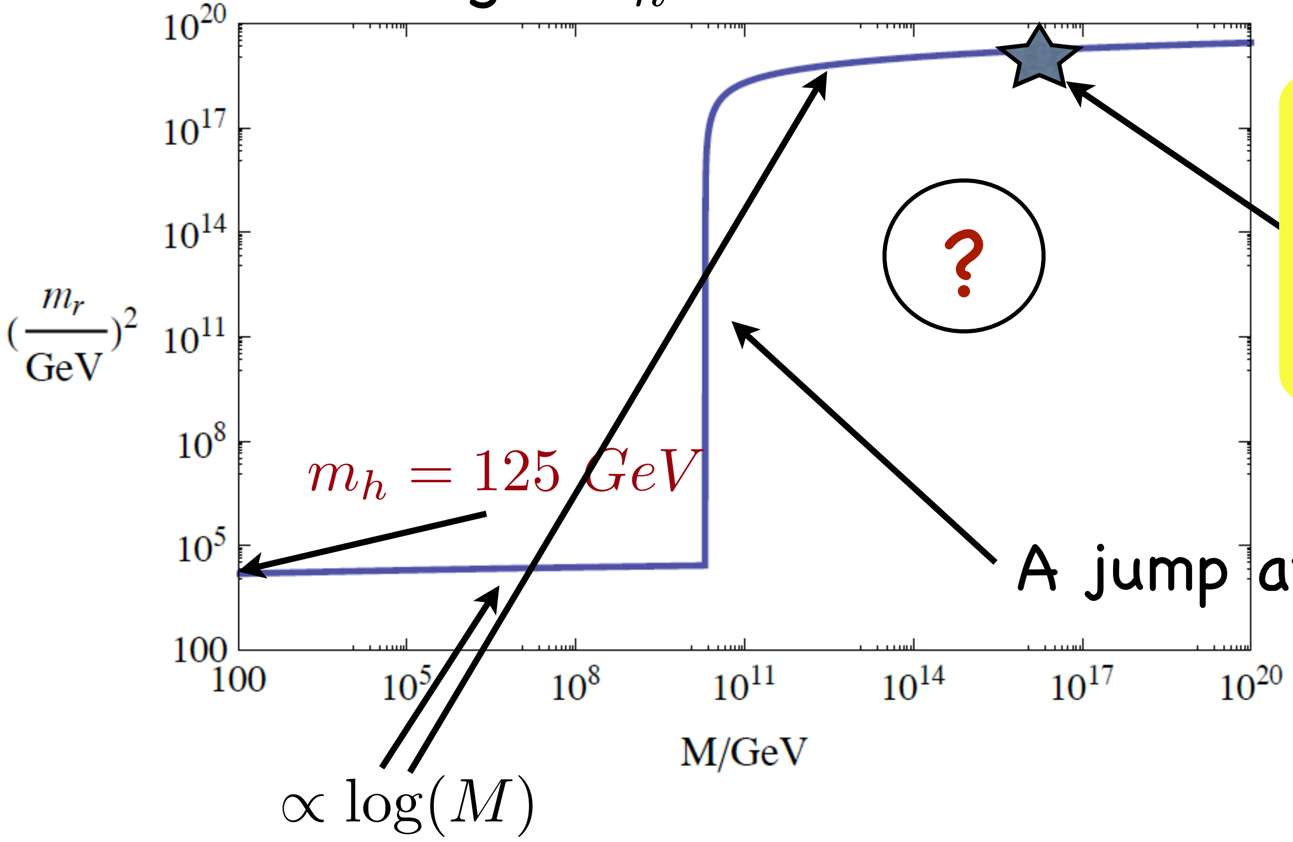
To get <h> = 175 GeV, as observed, we have to live very very close to the critical line

But we don't have knobs!

The Higgs naturalness problem illustrated in another way

Take the SM + a particle of mass $M_H = 10^{10}\ GeV$
and coupling λ_H to the Higgs boson

The running m_h^2 versus the scale M



"fine tuning"

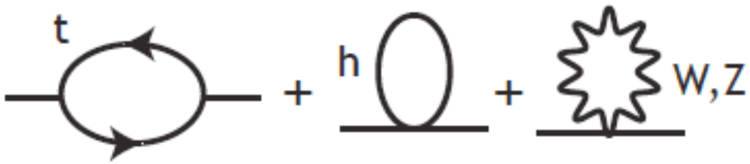
m_h depends on a very precise initial condition of order $O(m_h^2/m_H^2)$ at some short distance

A jump at M_H of size $\frac{(\lambda_H M_H)^2}{16\pi^2}$

The hierarchy problem, once again

Can we compute the Higgs mass/vev in terms of some fundamental dynamics?

NOT in the SM



$$\delta m_h^2 = \frac{3y_t^2}{4\pi^2}\Lambda_t^2 - \frac{9g^2}{32\pi^2}\Lambda_g^2 - \frac{3g'^2}{32\pi^2}\Lambda_{g'}^2 + \dots$$

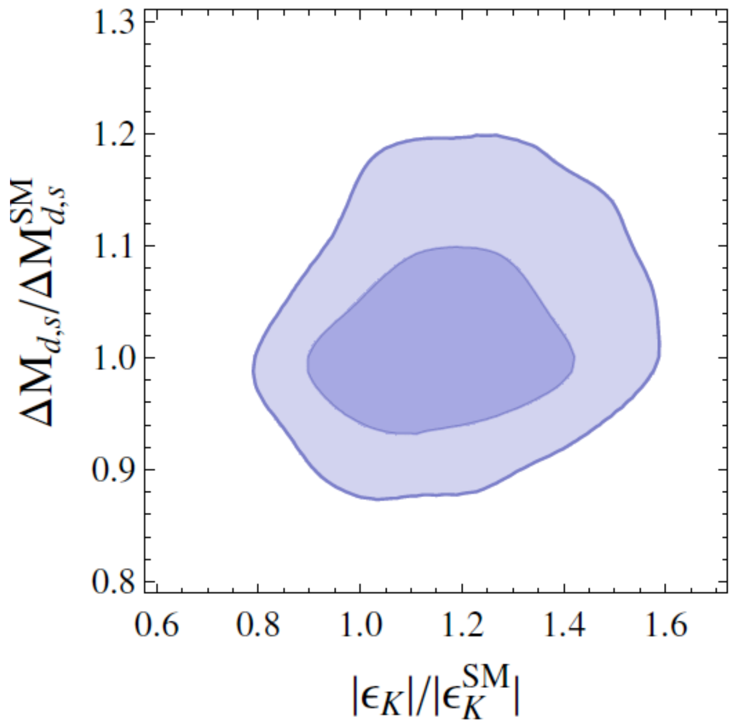
$\Lambda_t \lesssim 0.4\sqrt{\Delta}\ TeV$ $\Lambda_g \lesssim 1.1\sqrt{\Delta}\ TeV$ $\Lambda_{g'} \lesssim 3.7\sqrt{\Delta}\ TeV$

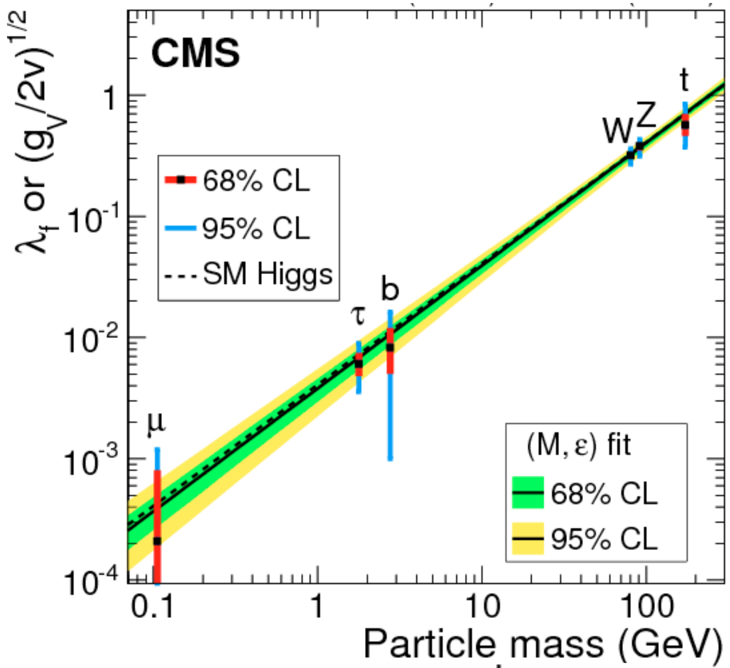
$1/\Delta$ = amount of tuning

⇒ Look for a top "partner" (coloured, S=0 or 1/2) with a mass not far from 1 TeV

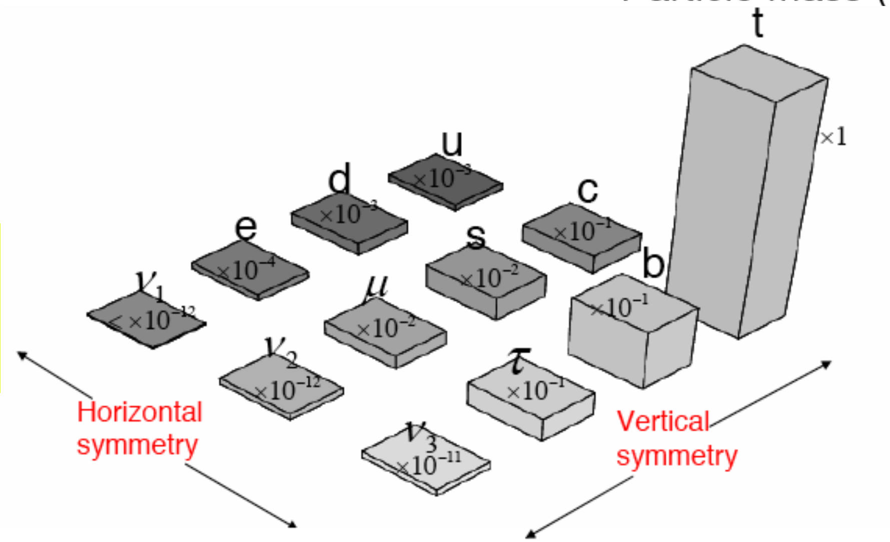
The flavour paradox

Yukawa couplings: a piece of physical reality





as opposed to:



?!?!?

Summary of lectures I and II

The Standard Model is NOT a complete story

Pictures that go Beyond the SM are not lacking,
but - fair to say - we don't know which one is right

The very nature of Particle Physics and the current
uncertain situation REQUIRE
highly diverse frontiers of research

Can an understanding of short distance physics
ever be produced deeper than the SM one?

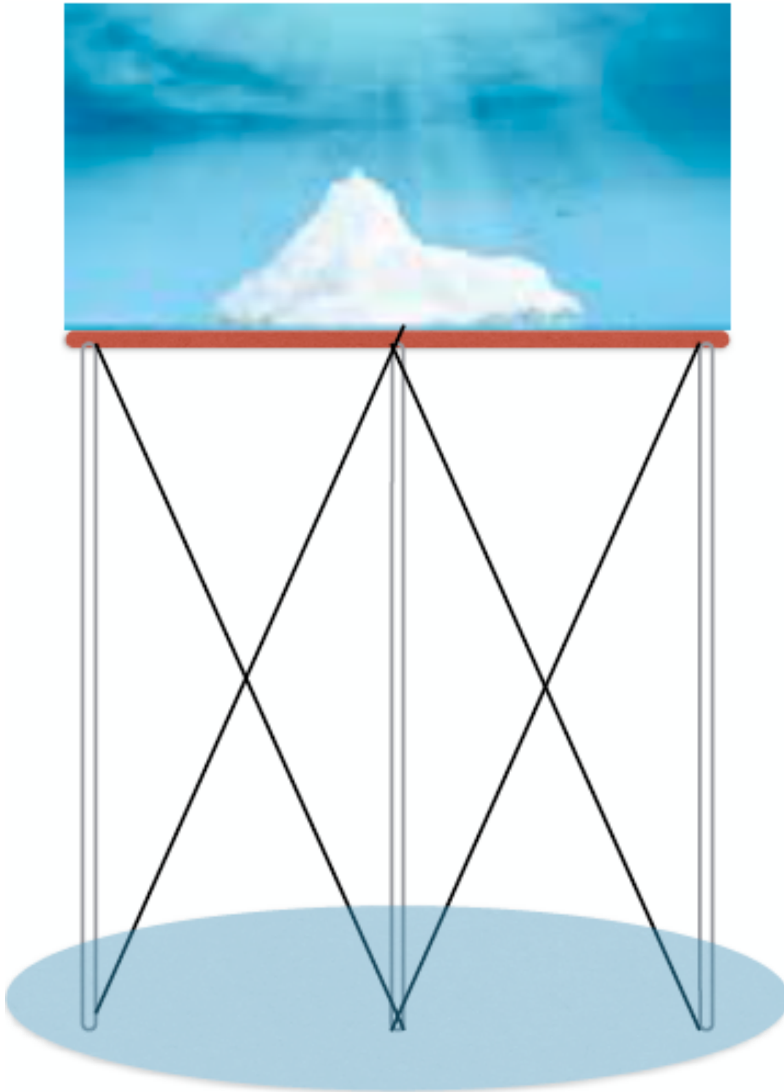
Could such a putative theory not include the SM
as a relevant limit?

The SM as an emerging iceberg

What there is under the water?

BSM in the multi TeV region...

BSM in the multi TeV region...



... or the SM extended up to E >> TeVs?

For question time

vacuum stability

$$V(\varphi) = \mu^2|\varphi|^2 + \lambda|\varphi|^4$$

$$\frac{d\lambda}{d\log Q} = \frac{3}{2\pi^2}\left[\lambda^2 + \frac{1}{2}\lambda y_t^2 - \frac{1}{4}y_t^4 + \cdots\right]$$

$$m_W = gv/\sqrt{2}$$
$$m_H = 2\sqrt{\lambda}v$$
$$m_t = y_t v$$

With current values of $m_H,\ m_t,\ \alpha_S, \ldots$

$$\lambda(\approx 10^{11}\ GeV) < 0$$

⇒ A second minimum of V at $\phi \gtrsim 10^{11}\ GeV$
to which v should tunnel in a very long time (>> t_{Univ})

- Is there a real meta-stability at $\phi < M_{Pl}$?
- Any experimental implication?
- Connection to inflation?
- Is it a problem?

Landau poles

$$\frac{dg_1^2}{dt} = \frac{41}{40} g_1^4 \quad \Rightarrow \text{ a Landau pole at } \Lambda_1$$

- the problem not cured by including other couplings
- can it be cured by gravity? Yes, since $\Lambda_1 > M_{Pl}$, if gravity important at $E \lesssim M_{Pl}$
- what if gravity softened enough, so that it becomes irrelevant? (How is hard to tell, but...)

- need $SU(3) \times SU(2) \times U(1)$ fully immersed in a non-abelian group

$$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$$

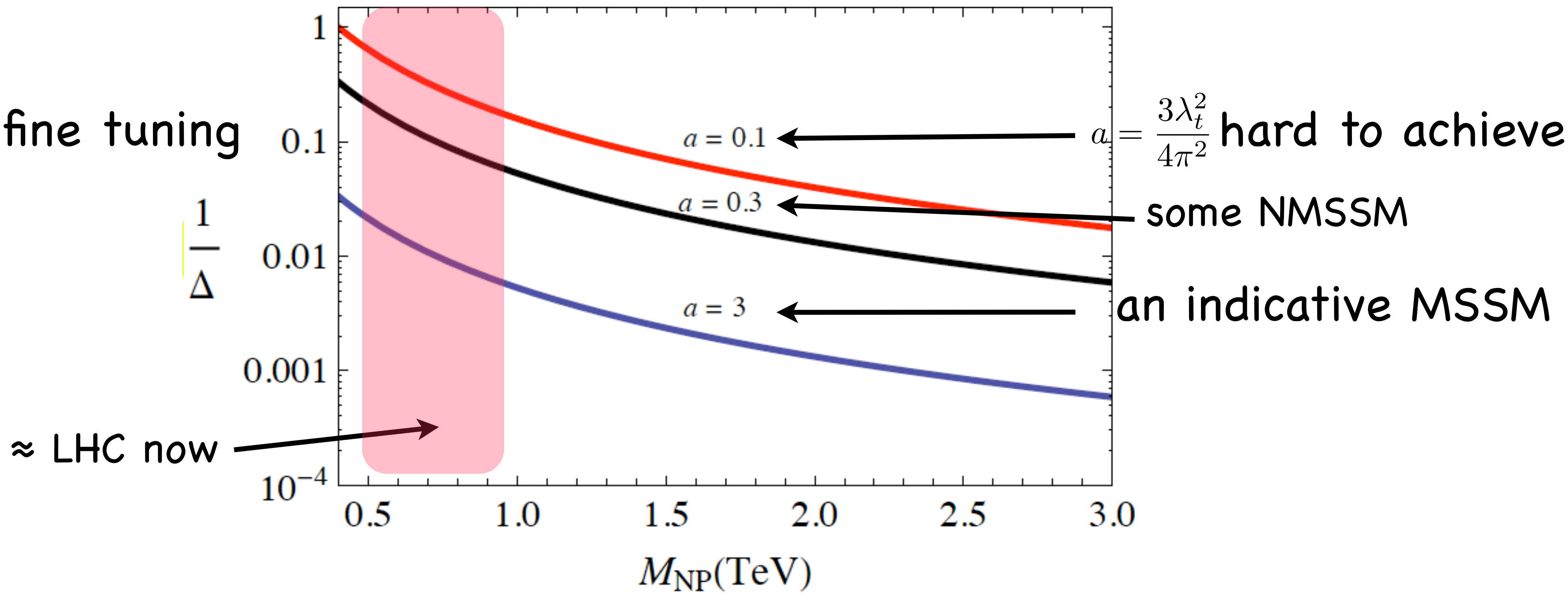
$$SU(3)_c \times SU(3)_L \times SU(3)_R$$

which requires heavier scales than v

How dramatic is the "little hierarchy problem"?

$$\Delta \equiv \frac{\delta m_h^2}{m_h^2} \approx a \frac{M_{NP}^2}{m_h^2}$$

model dependent



- Things do not work the way they were originally thought
- Not a serious problem at a fundamental level

LHC-13 TeV

A self-critical Higgs vev

1. A Goldstone boson ϕ of a U(1) broken at a scale f
2. A U(1)-breaking coupling of ϕ to H
   (that keeps $\phi \to \phi + 2n\pi f$)
3. A breaking of $\phi \to \phi + 2n\pi f$ controlled by a small mass parameter m entering the Higgs mass term

$$V = -f^2|S|^2 + |S|^4 + \rho(H)\frac{S+S^+}{f} + (\Lambda^2 - m\phi)|H|^2 + \lambda|H|^4 + m\Lambda^2\phi$$

$S = se^{-i\phi/f}$

Λ = UV cutoff

V is a natural potential

Minimizing $V(H,\phi)$

$$V = \rho(H)\cos\phi/f + (\Lambda^2 - m\phi)|H|^2 + \lambda|H|^4 + m\Lambda^2\phi$$

$$\rho(H) = \cancel{\rho_0} + \rho_1 \frac{H}{v_F} + \rho_2\left(\frac{H}{v_F}\right)^2 + \dots \qquad v_F^4 > \rho_{1,2}$$

(non trivial)

$$\frac{\partial V}{\partial h} = 0 \;\Rightarrow h^2 \approx \frac{\Lambda^2 - m\phi}{\lambda} > 0$$

$$\frac{\partial V}{\partial \phi} = 0 \;\Rightarrow h \approx v_F \frac{\Lambda^2 m f}{\rho_1}$$

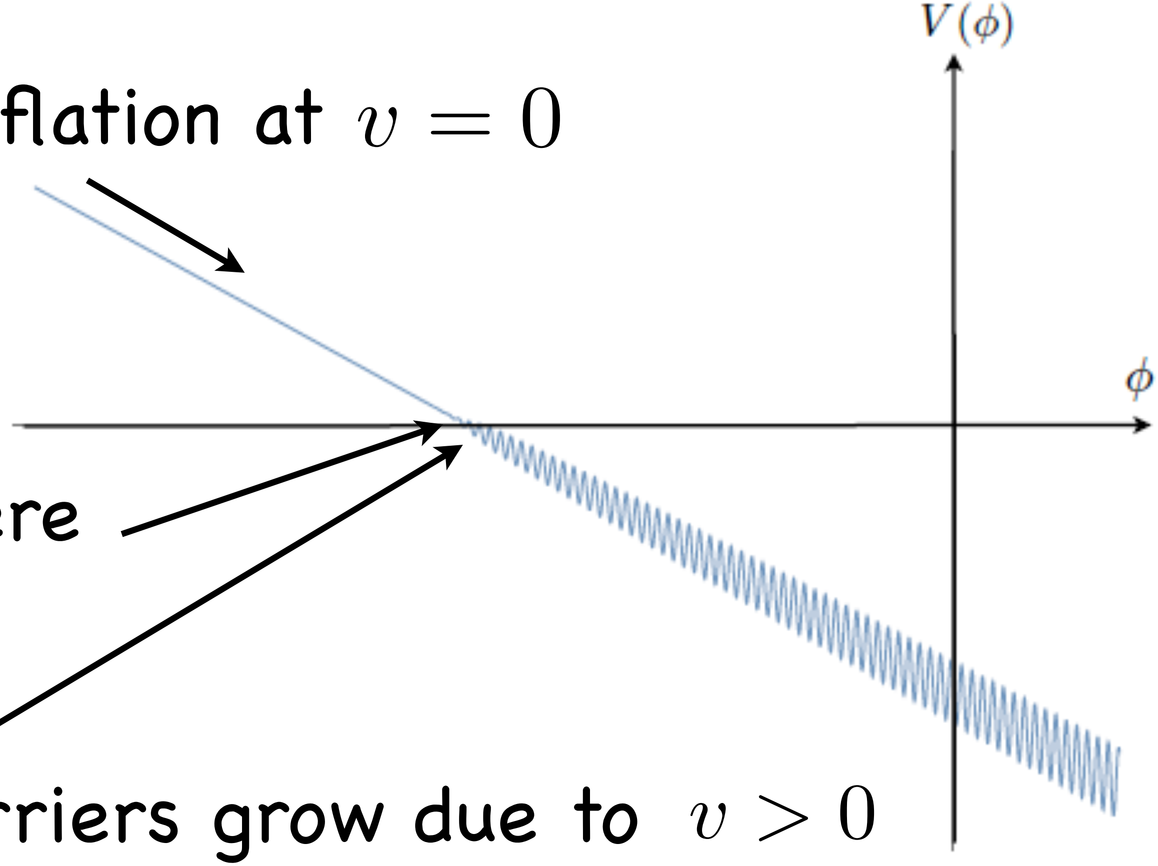
$h = v_F$ natural = moving Λ, m, f, ρ_1 by O(1)
h changes by O(1)

$$m = \frac{\rho_1}{\Lambda^2 f} \lesssim \frac{v_F^4}{\Lambda^2 f} \qquad \phi \approx \frac{\Lambda^2}{m} \gtrsim \frac{\Lambda^4 f}{v_F^4}$$

historical evolution of ϕ (and of v)

(under suitable conditions: e.g. a very very long inflation period)

ϕ slow-rolls during inflation at $v = 0$

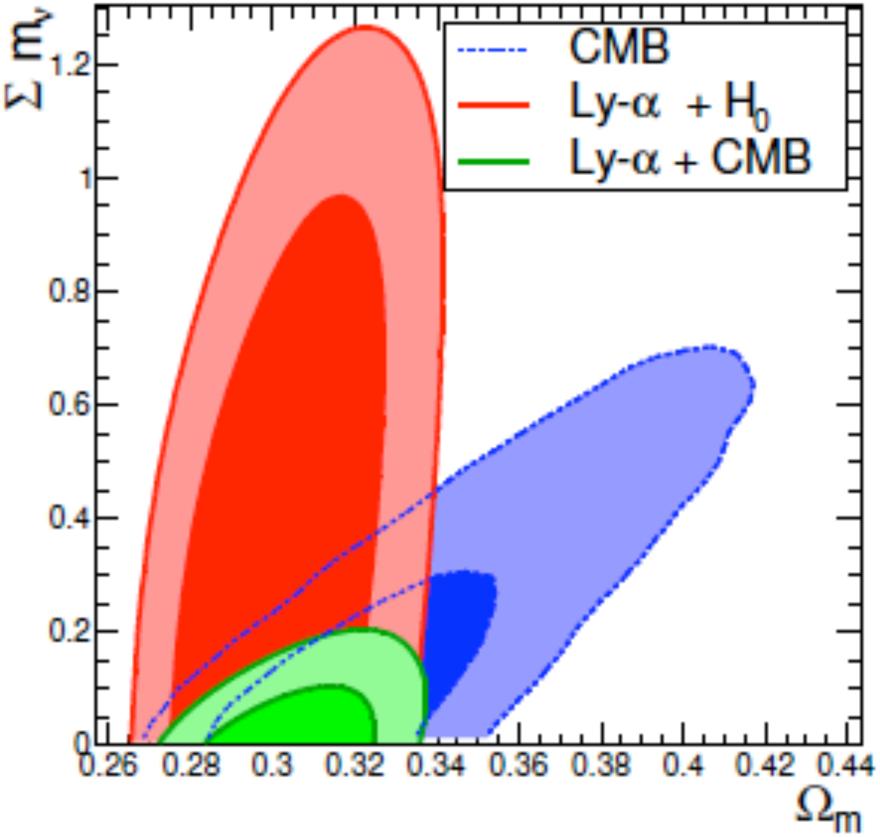


until it hits value where m_h^2 crosses zero

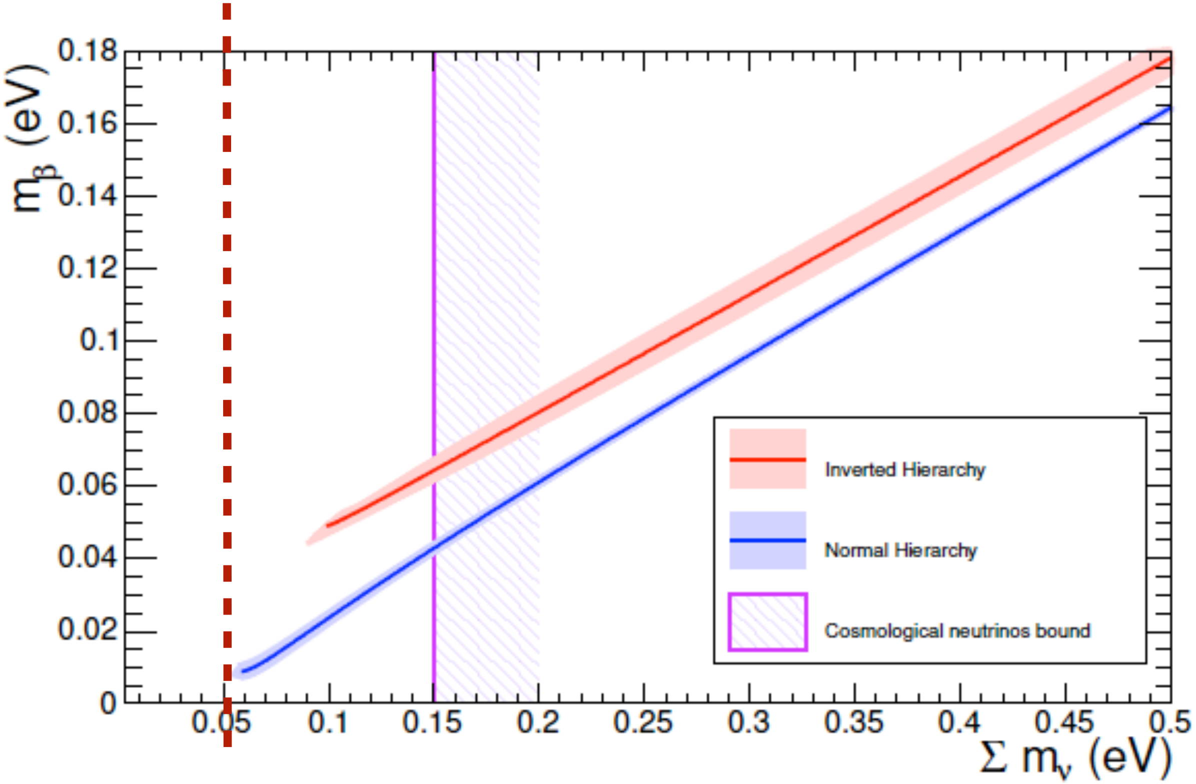
rolling stops when barriers grow due to $v > 0$

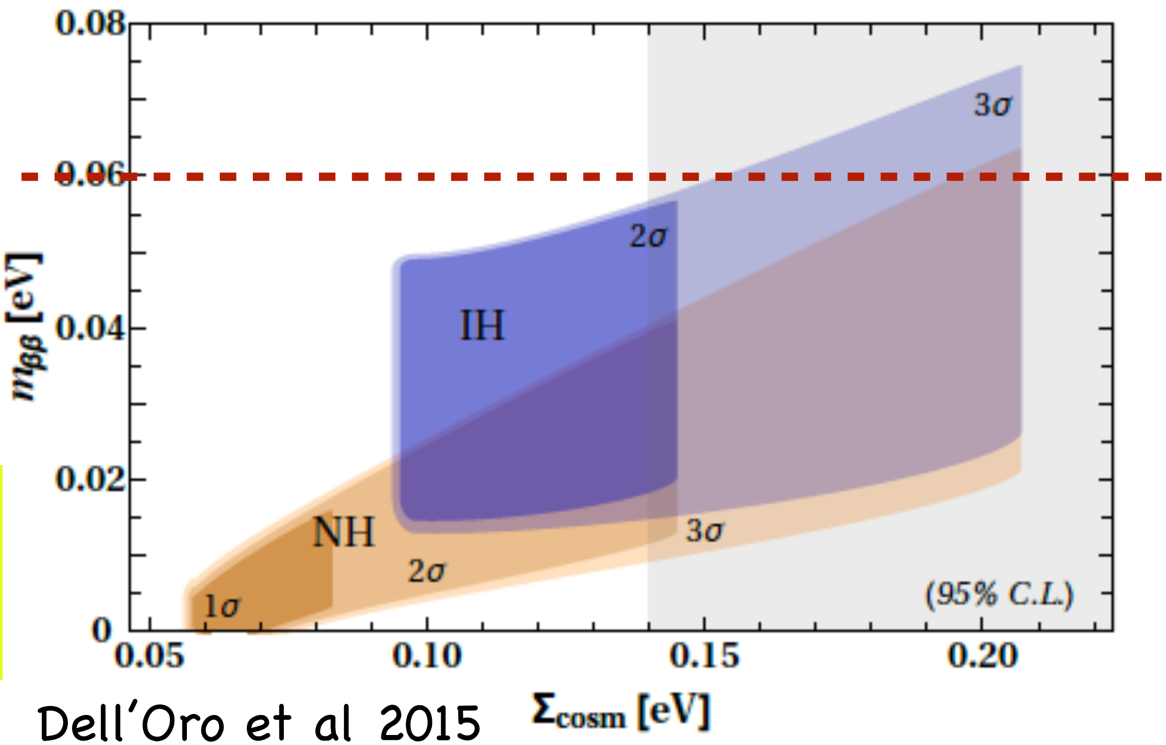
experimental consequences:??

Σm_ν determination



Palanque-Delabroullle et al 2015





Dell'Oro et al 2015

(a recent result from KamLAND)

$$m_{\beta\beta} < 0.06 \div 0.16 \ eV$$