Inflation and the Theory of Cosmological Perturbations

Exercises

The standard Big Bang model and its shortcomings

1. **Friedmann equations**

   Using the FLRW metric and Einstein equations, find the Friedmann equations.

2. **Matter-Radiation Transition**

   Calculate the epoch of matter-radiation equality as a function of redshift.

3. **Flatness Problem**

   Consider an FLRW model dominated by a perfect fluid with equation of state $P = w\rho$, for $w = \text{const}$. Show that

   $$\frac{\mathrm{d}\Omega}{\mathrm{d}N} = (1 + 3w)\Omega(\Omega - 1)$$

   and discuss the stability of the solution $\Omega = 1$.

The inflationary cosmology

1. **Photons in deSitter**

   Show that photons are not produced during inflation.

2. **E-folds and reheating temperature**

   Using entropy conservation and assuming instantaneous reheating at temperature $T_{\rm rh}$, show that we need a number of e-folds of

   $$N = 46 + \log\left[\frac{T_{\rm rh}}{10^{10}\mathrm{GeV}}\right] + \frac{1}{2}\log|\Omega_i - 1| \tag{0.1}$$

   to solve the flatness problem.

3. **Slow-roll inflation I**

   Consider a model of inflation with a potential $V(\phi) \propto \phi^n$. Calculate slow roll-parameters for this class of models. Requiring a minimum number of e-foldings of

$N \geq 60$, what is the minimum inflaton field excursion $\Delta\phi$ between the beginning and the end of inflation?

Cosmological perturbations

1. **Slow-roll inflation II**

   Using the results from the previous exercise, calculate the scalar spectral index n_s and the tensor-to-scalar ratio r for $V(\phi) \propto \phi^n$ as a function of n and ΔN_λ (the number of e-folds between horizon exit of the wavelength λ and the end of inflation). Discuss allowed values of n given the current CMB constraints ($50 < \Delta N_\lambda < 60$).

2. **Lyth bound**

   Find a relation that connects the maximum inflaton field excursion $\Delta\phi$ during inflation to the tensor-to-scalar ratio r.

3. **Scale of Inflation**

   Find a relation that connects the energy scale of inflation to the tensor-to-scalar ratio r.

4. **Symmetries and inflationary N-point correlators**

   Using symmetry arguments, show that the N-point function of ζ in Fourier space in a generic model of inflation is of the form

   $$\langle \zeta_{\vec{k}_1} \dots \zeta_{\vec{k}_N} \rangle = (2\pi)^3 \delta_D \left(\sum_{i=1}^{N} \vec{k}_i \right) F(\vec{k}_i) \tag{0.2}$$

   with F a homogeneous function of the momenta of degree $-3(N-1)$.

5. **Magnetic field during inflation**

   Find the suitable function $I(t)$ such that the Lagrangian $I(t)F_{\mu\nu}F^{\mu\nu}$ leads to the quantum-mechanical generation of a magnetic field during a de Sitter epoch.