Axions

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1 Quick Intro

The QCD axion, or simply the axion, is a hypothetical 0^- particle arising in the Peccei-Quinn generic mechanism to ease the strong CP problem. Furthermore, axions are candidates for the cold dark matter of the Universe. Axions have been searched in a small number of experiments and constrained with astrophysical and cosmological arguments, but not yet found. Many different theoretical realisations have been proposed, which relate axion physics with other theories beyond the standard model. After some years of abandon and despair, the interest in axions is growing strong again, and many new experiments have been proposed and will be built in the next few years. In this lectures, we will introduce the axion in the context of the strong CP problem and study its phenomenological consequences in astrophysics and dark matter, as well as its experimental signatures.

The notes were born to provide a closer recollection of the first lecture I gave in the 2016 Invisibles school (SISSA Trieste, 5-9 July 2016). They were later used in the TAE 2017[1] and in the ICCUB school[2] in 2017. They are growing bigger in the GGI school[3] in 2019. The few hours I had assigned were unpurposely stretched to the limit, and yet my humble talents did not amount to much when trying to cover all aspects of this exciting field. Because of this reason, but mostly because is always advisable to complement any lecture with different approaches to the problem, I list here other pedagogical readings that I encourage to get acquainted with. I was very lucky to enjoy the lectures of the 1st Joint ILIAS-CAST-CERN Training back in 2005 [1], which produced excellent lecture notes. The very same Roberto Peccei taught on Axions and the strong CP problem [2], Pierre Sikivie

[1]Benasque, Spain, 3-16 September 2017. http://benasque.org/2017tae/
[2]Barcelona, 23-26 October 2017 http://icc.ub.edu/congress/ICCUBschool/
[3]Florence, 14-18 Jan 2019 http://webtheory.sns.it/ggilectures2019/

on Axion Cosmology [3] and Georg Raffelt on Astrophysical Axion bounds [4]. As an easy and enjoyable read for the moments where everything seems uphill, I also recommend [5]. For a thorough review on Axions with a well fed collection of references, see the review of J. E. Kim and G. P. Carosi [6] (take a deep breath before). I am always available for requests on deeper readings on specific aspects and updated experimental proposals. Use me. Likewise, if you have corrections to these notes, please drop me a line and get a free coffee, beer or dinner (depending on the correction).

2 Strong CP ~~problem~~ hint

The strong CP problem is a conceptual issue with QCD being the theory of strong interactions in the standard model of particle physics. From a theoretical point of view we expect that such an SU(3) gauge theory coupled to massive quarks is "generically" CP violating, and yet there is no sign of CP violation in the strong interactions. Sure, the heart of the matter is on what we mean by generically.

When $\mathrm{SU}(3)_c$ was proposed as a theory of the strong interactions, one of the designer's choice was CP conservation, which was already a clear constraint from the experimental point of view. The low energy theory of $\mathrm{SU}(3)_c$ had, however, a mysterious problem: Weinberg's $\mathrm{U}(1)_A$ "missing meson" problem. Its resolution by 't Hooft triggered the recongnition of the strong CP problem and thus is our starting point for these lectures.

2.1 $\mathrm{U}(1)_A$ missing meson problem

Consider QCD with 2 quark flavours, u, d in a vector notation $q = (u, d)$

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + i\bar{q}\not{D}q - (\bar{q}_L m_q e^{i\theta_Y} q_R + \text{h.c.}) \tag{1}$$

where m_q is a diagonal mass matrix with the m_u, m_d masses and θ_Y a common phase. Note that in the SM, quark masses come from Yukawa couplings of the Higgs and Yukawa matrices are allowed to be completely general, so we expect θ_Y to be there in a general case.

In the $m_q \to 0$ limit, the quark phase transformations

$$q_R \to e^{i(\alpha_0+\vec{\alpha}\cdot\sigma)}q_R \quad ; \quad q_L \to e^{-i(\alpha_0+\vec{\alpha}\cdot\sigma)}q_L, \quad \text{or} \quad q \to e^{i\gamma_5(\alpha_0+\vec{\alpha}\cdot\vec{\sigma})}q \tag{2}$$

are a four-parameter $(\alpha_0, \vec{\alpha} = (\alpha_1, \alpha_2\alpha_3))$ symmetry of the Lagrangian,

$$U(2)_A = U(1)_A \otimes SU(2)_A. \tag{3}$$

The U(1) part shifts a common phase of u_R and d_R quarks at the same time and opposite for LH, while the SU(2) part shifts phases differently for u and d flavours. The symmetry is explicitly violated by the quark masses, but since they are much smaller than QCD energy scales we can think about them as a perturbation. Note that a $U(1)_A$ transformation can be used to reabsorb θ_Y in the quark fields, and thus it should have unobservable effects.

When QCD grows strong at low energies, this symmetry becomes spontaneously broken by the quark condensate $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -v^3$. According to the Goldstone theorem, a global symmetry spontaneously broken implies the existence of Nambu-Golstone bosons (NGB), massless particles that appear in the low energy effective theory. The NGB's have quantum numbers of the symmetry generators which are spontaneously broken. A 4-parameter symmetry implies 4 Goldstone bosons, which are associated with the η' and the 3 pions π^0, π^+, π^-. Since the symmetry is not perfect, i.e. it is violated by the small mass terms, the NGBs become massive, and are usually called pseudo-NGB's. Let us compute the spectrum of mesons.

Meson masses can be computed by promoting the phases of quarks $\alpha_0, \vec{\alpha}$ to NGB fields

$$\alpha_0 \to \frac{\Pi_0(x^\mu)}{f_0} = \theta_0(x^\mu) \quad \vec{\alpha} \to \frac{\vec{\Pi}(x^\mu)}{f_\pi} \equiv \vec{\theta}(x^\mu) \tag{4}$$

where f_0, f_π are energy scales related to $\Lambda_{\rm QCD}$. For simplicity in the exposition I will tend to take $f_0 = f_\pi = f$ when it does not compromise the main points under discussion [4]

We define Goldstone-less quarks $\tilde{q}$

$$q_R = e^{+i(\theta_0+\vec{\theta}_\pi\cdot\vec{\sigma})/2}\tilde{q}_R \quad ; \quad q_L = e^{-i(\theta_0+\vec{\theta}_\pi\cdot\vec{\sigma})/2}\tilde{q}_L \quad \text{or} \quad q = e^{i\gamma_5(\theta_0+\vec{\theta}_\pi\cdot\vec{\sigma})/2}\tilde{q} \tag{5}$$

Note that after this redefinition, $U(2)_A$ transformations will appear as global shifts of the $\theta_0(x), \vec{\theta}(x)$ fields.

Under this redefinition, the quark mass term in the Lagrangian leads to a potential for the NGB's when subject to the quark condensate, Let us first consider the charged sector $\theta^\pm = (\theta_1 \pm i\theta_2)/2$. We get

$$\bar{q}_L m_q q_R + \text{h.c.} \to -(m_u+m_d)v^3\cos(\sqrt{\theta_-\theta_+}) = (m_u+m_d)v^3 + \frac{(m_u+m_d)v^3}{2f_\pi^2}\Pi^-\Pi^+ + ..., \tag{6}$$

from which we can directly read the charged pion mass [5]

$$m_{\pi^+}^2 = \frac{(m_u+m_d)v^3}{f_\pi^2}. \tag{7}$$

.

In the neutral sector Π^0, Π_3 we have ($\theta_3 = \Pi_3/f_\pi$, appears with σ_3)

$$\bar{q}_L m_q q_R + \text{h.c.} \to -m_u v^3\cos(\theta_0+\theta_3) - m_d v^3\cos(\theta_0-\theta_3), \tag{8}$$

[4] Note a slight abuse of notation, in which θ_λ is a constant (and so will be $\theta_{\rm QCD}, \theta_{\rm SM}$) later but $\theta_0, \theta_{1,2,3}$ are fields. I am open to change of notation proposals.

[5] This assumes the kinetic terms for $\Pi_{0,1,2,3}$ are canonically normalised. The kinetic term for the Goldstones is generated by the quark-gluon-condensation non-perturbatively, it cannot be derived straightforwardly from the QCD Lagrangian.

which gives mass to two linear combinations of Π_0, Π_3, one of which has to be the neutral pion π^0 and the other something related with η' (because of quantum numbers). Expanding around $\Pi_0 = \Pi_3 = 0$ to quadratic order, the mass matrix for the neutral mesons is

$$\frac{\partial^2 V}{\partial \Pi_i \partial \Pi_j} = \frac{v^3}{f^2} \begin{pmatrix} m_u + m_d & (m_u - m_d)\beta \\ (m_u - m_d)\beta & (m_u + m_d)\beta^2 \end{pmatrix}, \tag{9}$$

where $\beta = f_\pi / f_0$. The sum of the squared masses is related to the charged pion mass (7)

$$m_{\eta'}^2 + m_{\pi^0}^2 = m_{\pi^+}^2 (1 + \beta^2). \tag{10}$$

The observed pion and η' masses,

$$m_{\pi^+}^2 :: m_{\pi^0}^2 :: m_{\eta'}^2 = (134.9766(6))^2 :: (139.57018(35))^2 :: (957.78(6))^2 \quad \text{MeV}^2 \tag{11}$$

would require $\beta^2 \simeq m_{\eta'}^2 / m_\pi^2 \simeq 50$ which is not justified from the theoretical point of view. With all we have said, we would naively expect $f_0 \sim f_\pi$, $\beta \sim 1$, because why should be so different if QCD confinement cannot distinguish u, d except for their small mass difference?

Therefore, the theory as it is predicts a pNGB with similar mass to the neutral pion, which is not observed in nature. Sometimes it is called, Weinberg's missing $U(1)_A$ meson.

The puzzle stays when including the strange quark, which forces us to consider $\mathrm{U}(3)_A$ and has three neutral mesons that have to be associated with π^0, η, η'.

2.2 QCD instantons solve the issue ...

The divergence of the $\mathrm{U}(1)_A$ current $j_A^\mu = \bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d$ gets a contribution from the triangle loop diagram

$$\partial_\mu j_A^\mu = -2m_u \bar{u} i\gamma_5 u - 2m_d \bar{d} i\gamma_5 d + 4\frac{\alpha_s}{8\pi} G_{\mu\nu}^a \widetilde{G}_a^{\mu\nu} \tag{12}$$

where $\widetilde{G}_a^{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} G_a^{\alpha\beta}/2$ is the gluon field strength. The current is not conserved even for non-zero masses $\partial_\mu j_A^\mu \neq 0$. In other words, $\mathrm{U}(1)_A$ is not a symmetry even when m_u, m_d are zero. The factor of 4 comes from the 2 flavours and two chiralities that run in the loop. The first two terms, proportional to quark masses, simply state that quark masses violate the symmetry too, but this we already knew.

Generically, an axial phase transformation of one quark (SU(3) fermionic triplet)

$$u \to e^{i\alpha\gamma_5} u \tag{13}$$

implies that the current associated has a triangle anomaly

$$\partial_\mu (\bar{u}\gamma^\mu\gamma_5 u) = ... + 2\frac{\alpha_s}{8\pi} G_{\mu\nu}^a \widetilde{G}_a^{\mu\nu}. \tag{14}$$

Transformations along the Π_3 direction, i.e. along σ_3 =diag{1,-1} are not colour anomalous because the u and d parts cancel out. But the $U(1)_A$ part is proportional to the identity in flavour space and all the quarks contribute the same.

Physical effects of the $G\widetilde{G}$ term were neglected in early times because it turns out to be a total derivative,

$$G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a = \partial_\mu K^\mu \tag{15}$$

with

$$K^\mu = \epsilon^{\mu\nu\alpha\beta} A_{a\nu}\left(F_{a\alpha\beta} - \frac{g_s}{3} f_{abc} A_{b\alpha} A_{c\beta}\right) \tag{16}$$

(note that it is not gauge-invariant).

By partial integration all its effects are defined by field configurations at infinity, which shall not contribute to local processes if gluon fields go to zero at infinity $A^a_\mu \to 0$. But Gerard 't Hooft realised that there are actually topologically non-trivial field configurations, called instantons, that contribute to this operator, and thus it cannot be neglected. These instantons are strongly related to the structure of the QCD vacuum, which is displayed in appendix A.

But we can continue with the lectures without the fine points of instantons so I will skip the discussion as much as I can. The important points I cannot avoid to list are the following:

- The term violates P and T, or equivalently, P and CP
- A $G\widetilde{G}$ term must be admitted in our Lagrangian (1), because it is compatible with all symmetries of the SM gauge group and instanton configurations contribute to it. Thus, we are led to consider

  $$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\bar{q}\not{D}q - (\bar{q}_L m_q e^{i\theta_\lambda} q_R + \text{h.c.}) - \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \theta_{\text{QCD}}. \tag{17}$$

  where θ_{QCD} is a parameter to be determined. In appendix A we show that it appears as the phase that determines one of the possible vacua of QCD.
- $G\widetilde{G}$ violates the $U(1)_A$ symmetry explicitly (even if only at the quantum level, i.e. after the triangle radiative correction is included) so we expect that it will generate a mass term for the Π_0 field, just like m_u generated mass for the $\theta_0 + \theta_3$ combination.

When an infinitesimal $U(1)_A$ transformation(redefinition) like (2) with parameter α is performed on quark terms, the Lagrangian gets a new piece

$$\delta\mathcal{L} = \alpha \partial_\mu j^\mu_A \tag{18}$$

that, of course, vanishes if $U(1)_A$ would be a symmetry with its current conserved. This leads to

$$\delta\mathcal{L} = \alpha\partial_\mu j^\mu_A = -2m_u\bar{u}i\alpha\gamma_5 u - 2m_d\bar{d}\alpha\gamma_5 d + \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \times 4\alpha. \tag{19}$$

The first two terms are the phase shifts of quark masses, expected because $U(1)_A$ transformations do not leave invariant mass terms, and the last redefines(shifts) $\theta_{\rm QCD}$. Fukujita showed that it appears from the transformation of the measure in the path integral [].

Therefore, note that when performing these transformations, we are effective shifting a phase from θ_λ to $\theta_{\rm QCD}$. For instance, we can rotate θ_λ away from the quark mass term. Then, the combination

$$\theta_{\rm SM} = \theta_{\rm QCD} + 2\theta_Y \tag{20}$$

appears multiplying the $G\widetilde{G}$ term in the Lagrangian. Only this combination (often denoted called $\bar{\theta}$) is thus physical and all the CP violation observables are going to depend on it. The $G\widetilde{G}$ term will solve the missing meson problem, but it brings with it CP violation that we thought was absent in QCD.

Note that $\theta_{\rm SM}$ it is a sum of two phases which in principle have a different origin: θ_Y originates as the common phase of the Yukawa couplings and $\theta_{\rm QCD}$ defines the QCD vacuum. Therefore, in principle we shall not expect any cancellation. Moreover, the only CP violating phase observed so far, which appears in the CKM and has a similar origin than θ_Y, it is O(1) ($\gamma \sim 60$ degrees).

Let us now discuss some important details of how the eta' mass has to be generated. We aim at guessing the contribution to the meson potential due to the new $G\widetilde{G}$ term. There are three points to consider:

- The spacetime integral of $G\widetilde{G}$ is a very special object. It only cares about special field configurations with non-trivial topology like instantons (see appendix A). Indeed, it turns out to be an integer,

  $$\int d^4x \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a = n, \tag{21}$$

  corresponding to the change of winding number ("Pontryagin index") of the gluon field configuration integrated. For the moment we only want to highlight that n is an integer and thus any quantity that depends on $\theta_{\rm SM}$ must be 2π-periodic

  $$\theta_{\rm SM} \equiv \theta_{\rm SM} + 2\pi. \tag{22}$$

  In particular, the energy density (or effective potential) dependence on for $\theta_{\rm SM}$ (Euclidean path integral)

  $$e^{-\int d^4x_E V[\theta]} = \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}] - i\theta\int d^4x_E \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a} \tag{23}$$

  will satisfy $V(\theta) = V(\theta + 2\pi)$.

- The effective potential has its absolute minimum[6] at $\theta_{\rm QCD} = 0$

$$
\begin{aligned}
e^{-\int d^4x_E V[\theta]} &= \left| \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}] - i\theta \int d^4x_E \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a} \right| && (24)\\
&\le \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}]} \left| e^{-i\theta \int d^4x_E \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a} \right| && (25)\\
&\le \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}]} = e^{-\int d^4x_E V[0]} && (26)
\end{aligned}
$$

  so $V[0] \le V[\theta]$.

- The VEV of the Π^0 field also contributes to CP violation. When we define Goldstone-less quarks in (5) we are effectively doing a position dependent $\mathrm{U}(1)_A$ transformation with parameter $\theta_0(x)/2$. This produces the term in the Lagrangian

$$
\mathcal{L} = \frac{\theta_0}{2}\partial_\mu j^\mu_A \ni \frac{\alpha_s}{8\pi} G\widetilde{G} \times 2\theta_0. \tag{27}
$$

  which adds the dynamical field $2\theta_0(x)$ to the theta-angle.

With these considerations in mind, we can write a new contribution to the meson potential, which reads now,

$$
V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 - \theta_{\rm SM}). \tag{28}
$$

We have modelled the effect from QCD instantons as $-\Lambda^4\cos(2\theta_0 - \theta_{\rm SM})$ with Λ an energy scale related with non-perturbative QCD to be determined from the eta' mass. The reasons are: 1) the induced term has to have its minimum at whatever value multiplies $\frac{\alpha_s}{8\pi}G\widetilde{G}$ in the Lagrangian, i.e. $2\theta_0 - \theta_{\rm SM}$, 2) it has to give a large mass $\sim$GeV to η' so it must have a non-zero second derivative at the minimum and 3) it must be periodic in $2\theta_0 - \theta_{\rm SM}$. The cosine form is a simple choice which at this point I decided[7]. The potential can be computed analytically in the so-called dilute-instanton-gas-approximation (DIGA) to give precisely this form, but this calculation is only physically justified at high temperatures (when multi-instanton configurations do not interact). Introducing the cosine gave me the excuse to tell you about all this, but I will actually only use the position of the minimum and the fact that its second derivative is large (because eta' is much more massive than π^0) so forgive me for the liberty.

[6] In this proof we have assumed that the Euclidean action at $\theta = 0$, $S_E[A^a_\mu]$ is real, which is the case when the $G\widetilde{G}$ term is the only source of CP violation. Since the EW sector of the SM has also a phase in the CKM matrix, this is not exactly true, but quantitatively irrelevant for these lectures. However, the CP-interested student will shall remember this.

[7] In principle, this term can be computed in lattice QCD but we still do not have the adequate algorithms to sample non positive definite path integrals. People is working on it, though.

The meson mass matrix is now (taking $f_0 = f_\pi$),

$$= \frac{v^3}{f^2} \begin{pmatrix} m_u + m_d & m_u - m_d \\ m_u - m_d & m_u + m_d \end{pmatrix} + 4\Lambda^4 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{29}$$

which allows to fit the large hierarchy between η' and π^0 masses (and have $m_{\pi^0} = m_{\pi^\pm}$ up to corrections). Essentially, eta' takes its mass from the new term and pions from chiral symmetry breaking by quark masses and the quark condensate,

$$m_{\pi^0}^2 = \frac{(m_u + m_d)v^3}{f^2} \quad ; \quad m_{\eta'}^2 \simeq 4\Lambda^4 + \mathcal{O}(m_q v^3). \tag{30}$$

2.3 ... but create the strong CP problem

Let us turn into CP violation. The meson potential allows also to identify formally the VEVs of the θ_0 and θ_3, which behave as CP violating phases.

Note also that the potential also reflects the fact that under $\mathrm{U}(1)_A$ transformations, which are now shifts of the θ_0 field ($\theta_0 \to \theta_0 + \alpha_0$) allow us to move the θ_{SM} phase from the QCD instanton term to quark mass terms, but they do not allow to redefine it away. Also, we learn that, if any of the quark masses is zero, one could reabsorb θ_{SM} inside θ_0 and θ_3 and thus CP violation will be absent.

Before minimising the meson potential V, note that if $\theta_{\mathrm{SM}} = 0$, every of the three terms of the potential can be made minimum by $\theta_0 = \theta_3 = 0$. Since all the phases are zero, the theory is of course CP-conserving. Considering now a small value of θ_{SM}, and a perturbation the CP conserving solution (expand the cosines at second order). Minimisation of V leads to a linear system for the VEVs of θ_0, θ_3 which solves to

$$2\theta_0 - \theta_{\mathrm{SM}} \sim -\frac{m_u m_d}{(m_u + m_d)^2} \frac{m_{\pi^0}^2}{m_{\eta'}^2} \theta_{\mathrm{SM}}, \tag{31}$$

$$\theta_0 + \theta_3 \sim \frac{m_d}{(m_u + m_d)} \theta_{\mathrm{SM}}, \tag{32}$$

$$\theta_0 - \theta_3 \sim \frac{m_u}{(m_u + m_d)} \theta_{\mathrm{SM}}. \tag{33}$$

Two things are again obvious here. First, $\theta_{\mathrm{SM}} = 0$ implies CP conservation. Second, the effects of θ_{SM} on CP violation also disappear in the case that any of the masses is zero, for instance $m_u \to 0$. CP violation appears multiplied by quark masses, if $m_u = 0$ it has to be proportional to m_d but the phase $\theta_d = \theta_0 - \theta_3 \to 0$ and $\theta_0 \to 0$ too.

We could have advanced this by noting that a phase redefinition of the u quark alone can shift θ_{SM} to zero in the theta-term of the Lagrangian. If $m_u = 0$ this redefinition only shifts the theta-term, so the theta-term must have no physical consequences. In its absence, the θ_0, θ_3 VEVs can not either violate CP. For all we know, there is no massless quark in SM, but the fact that u and d have small masses, suppresses a bit CP violation observables.

We have also noted that your honest P,T violation comes from $2\theta_0 - \theta_{\rm SM}$. Note that this is suppressed with respecto the "natural value" $\sim \theta_{\rm SM}$ by the small factor $m^2_{\pi^0}/m^2_{\eta'} \sim 1/50$. Indeed the new term in the potential wants to minimise the combination $2\theta_0 - \theta_{\rm SM}$, which is precisely the source of CP violation. However, it cannot do it completely because of the quark mass terms. The final value is a compromise between minimising both. Here, θ_0, which would be essentially the η' field (mass eigenstate) uses the instanton potential to adapt its VEV to cancel $\theta_{\rm SM}$ and it does it to a certain level. This is in a sense a dynamical mechanism that screens the effects of $\theta_{\rm SM}$.

A most discused CP violating observable arising from $\theta_{\rm SM}$ is the neutron electric dipole moment (NEDM). Its calculation is a bit cumbersome so instead I will quote the results

$$d_n = \frac{g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2 m_N}\log\left(\frac{m_N}{m_\pi}\right) \sim 4.5\times 10^{-15}{\rm ecm} \tag{34}$$

where CP violation enters in the CP violating pion-nucleon coupling $\bar{g}_{\pi NN} \sim -\theta_{\rm SM} m_u m_d/(m_u+m_d)$, as it comes from $2\theta_0 - \theta_{\rm SM}$.

The last attempt to measure the NEDM reported an upper limit

$$d_n < 3\times 10^{-26}{\rm ecm} \tag{35}$$

which implies the amazingly stringent constraint

$$\theta_{\rm SM} < 0.7\times 10^{-11}. \tag{36}$$

The fact that $\theta_{\rm SM}$ is that small while on general grounds we could expect it to be O(loop correction times m_u suppression) is dubbed the strong CP problem. It is actually not a technical problem, because $\theta_{\rm SM}$ does not receive large radiative corrections in the SM. It could just be that nature chose small $\theta_Y, \theta_{\rm QCD}$ or a fine tuning among them.

However, small numbers like this could very well have a dynamical origin hinting at new dynamics and new physics. In this lectures we will discuss a very elegant mechanism to cancel (almost) completely the effect of $\theta_{\rm SM}$ the Peccei-Quinn mechanism based on the axion. The most appealing aspect in my opinion is that the dynamics required is already build in the SM, concretely in the strong interactions. As a timely side-effect, it turns out that the mechanism provides a very intriguing cold dark matter candidate, and a hint for a new (high) energy scale in nature.

2.4 A new degree of freedom: Axion solution

Each of the terms of the meson potential

$$V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 - \theta_{\rm SM}) \tag{37}$$

has the tendency to minimise a given combination of CP violating VEVs and phases. Unfortunately, we have three terms in the potential and only two degrees of freedom so the minimisation of them all at a time has to find a compromise, which is generically CP violating. This suggests a possible explanation to the shocking absence of NEDM: could there be a new meson-like degree of freedom?

If we include a new meson-like *without introducing new terms in the potential* we will have three degrees of freedom to minimise three terms, each of which depends on three different linear combinations of our degrees of freedom. The system has now enough freedom to set all-three combinations to zero, i.e. to go to the CP conserving absolute minimum dynamically. Peccei and Quinn argued in a different way, but it all boils down to the above argument.

The simplest axion realisation involves a new meson-like field ϕ, that will be called axion. The important pieces of its Lagrangian are just 2: a kinetic term and an anomalous coupling to gluons, just like the theta-term

$$\mathcal{L}_\phi \in \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \frac{\phi}{f_\phi}. \tag{38}$$

The energy scale f_ϕ is called axion decay constant and will play a very important role in phenomenology. We can define $\theta_\phi(x) = \phi(x)/f_\phi$. At low energies, below QCD confinement, the axion appears in the instanton contribution to the potential

$$V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 + \theta_\phi - \theta_{\rm SM}) \tag{39}$$

The minimum of the potential is given by $\partial_{\theta_0} V = \partial_{\theta_3} V = \partial_{\theta_\phi} V = 0$, which is equivalent to

$$\theta_0 + \theta_3 = 0 \tag{40}$$
$$\theta_0 - \theta_3 = 0 \tag{41}$$
$$2\theta_0 + \theta_\phi - \theta_{\rm SM} = 0 \tag{42}$$

i.e.

$$\theta_0 = \theta_3 = 0 \quad ; \quad \theta_\phi = \theta_{\rm SM} \tag{43}$$

the axion VEV is adjusted *by the QCD potential* to cancel any possible value of $\theta_{\rm SM}$, and thus any effect of CP violation.

Another way to look at the absence of CP violation in the presence of axions is that we can redefine $\theta_a \to \theta_\phi - \theta_{\rm SM}$ at the Lagrangian level. This completely wipes out any dependence on $\theta_{\rm SM}$ and thus of CP violation. In other words, the presence of such axion field makes $\theta_{\rm SM}$ unphyiscal (this was closer to the thinking of Peccei and Quinn).

Yet another saying you will hear: the axion promotes $\theta_{\rm SM}$ to a dynamical variable, i.e. the role of $\theta_{\rm SM}$ in the SM is now played by the axion field $\theta_\phi(x)$. This dynamical variable can now respond to the QCD potential, adjusting its VEV to cancel $\theta_{\rm SM}$.

What Peccei and Quinn missed was to realise that θ_ϕ, as a dynamical field, has particle excitations: axions. This was realised very fast by Weinberg and Wilczek independently, which worked out their properties. The minimal version presented here is called the hadronic axion and turns out remarkably predictive.

2.4.1 Axion mass and mixings

The meson mass (squared) matrix is now, using $\beta = f/2f_a$ in the (π_3, Π^0, ϕ) basis

$$[m^2] = \begin{pmatrix} m_u + m_d & m_u - m_d & 0 \\ m_u - m_d & m_u + m_d & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{v^3}{f^2} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \beta \\ 0 & \beta & \beta^2 \end{pmatrix} \frac{4\Lambda^4}{f^2}. \tag{44}$$

Integrating out the heavy particle by setting $\eta'(x) = \Pi^0(x) + \beta\phi(x) = 0$ at the tree-level. i.e. we will use $\theta_0 = -\theta_\phi/2$ and forget about the new term $\propto \Lambda^4$.

$$V \sim -m_u v^3 \cos(\theta_3 - \theta_\phi/2) - m_d v^3 \cos(\theta_3 + \theta_\phi/2) \tag{45}$$

The system becomes 2x2 and one easily finds the mass eigenstates,

$$\pi^0 = \pi_3 + \varphi_{a\pi}\phi \quad ; \quad m_\pi^2 = \frac{(m_u + m_d)v^3}{f^2}, \tag{46}$$

$$a = \phi - \varphi_{a\pi}\pi_3 \quad ; \quad m_a^2 = \frac{m_u m_d v^3}{(m_u + m_d) f_a^2} = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f^2}{f_a^2}, \tag{47}$$

where we have redefined $f_a = f_\phi$ and the pion-axion mixing angle is

$$\varphi_{a\pi} = \frac{m_d - m_u}{2(m_u + m_d)} \frac{f}{f_a}. \tag{48}$$

People uses the word axion for both ϕ and a. Here we will reserve the symbol a for the physical mass eigenstate. Note that, in a sense, a takes some features of Weinberg's missing meson, like becoming massless in the m_u limit and its mixing to the π, proportional to $m_u - m_d$. This mixing angle allows to compute axion couplings to nucleons and self-couplings by simply taking standard expressions for axion-less theories and substituing

$$\pi_3(x) \to \pi^0(x) - \varphi_{a\pi}\phi(x) \sim \pi^0(x) - \varphi_{a\pi} a(x). \tag{49}$$

Note that the larger the axion decay constant f_a is, the smaller the axion mass and the weaker its interactions with photons, hadrons, etc. Soon we will be forced to consider $f_a > 10^9$ GeV, so that axions will be indeed low mass and weakly interacting.

We should not forget that axions mix also with Π_0. The mass eigenstate $\eta' = \Pi^0(x) + \beta\phi(x) \sim \Pi^0(x) + \beta a(x) + \mathcal{O}(\beta^2)$ implies

$$\Pi_0(x) \simeq \eta'(x) + \varphi_{a\eta} a(x) = \eta'(x) - \beta a(x). \tag{50}$$

Note that we can treat the mixing with the Π_0 and π_3 directions as independendent because β and the $\pi_3 - \eta'$ (not discussed) mixings are small.

2.5 Axion couplings

2.5.1 Coupling to photons

We can now compute axion couplings with SM particles in this simple model. We will start with the most important coupling to photons. Even if axions would not couple to photons directly, they would inherit a coupling from their mixing with Π^0 and π_3. These anomalous couplings follow from the divergence of the $\mathrm{U}(1)_A$ and third generator of $\mathrm{SU}(2)_A$,

$$\begin{aligned}\mathcal{L} \ni \; & [6\left(\frac{2}{3}\right)^2 + 6\left(\frac{1}{3}\right)^2]\frac{\Pi^0}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} + [6\left(\frac{2}{3}\right)^2 - 6\left(\frac{1}{3}\right)^2]\frac{\pi_3}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} & (51)\\ = \; & \frac{10}{3}\frac{\Pi^0}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} + 2\frac{\pi_3}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} & (52)\end{aligned}$$

These follow from the same anomaly equations that we used for the colour anomaly, but applied to the EM anomaly. Note that $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ is a total derivative, irrelevant if present alone in the lagrangian, but not harmless if multiplied by a Goldstone field like $F_{\mu\nu}\widetilde{F}^{\mu\nu}a(x)$.

Now use $\Pi_0 \to \eta' - \beta a$ and $\pi_3 = \pi^0 - \varphi_{a\phi} a$, the axion coupling is

$$\left[-\frac{10}{3} - 2\frac{m_d - m_u}{2(m_u + m_d)}\right]\frac{a}{2f_a}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} = -\frac{2}{3}\frac{4m_d + m_u}{m_u + m_d}\frac{a}{f_a}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} \tag{53}$$

A recent world-averaged value of the up to down quark masses $z \sim 0.48$ gives

$$-2.02 \times \frac{a}{f_a}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} \tag{54}$$

but including corrections and strange mass mixing gives $-1.92(4)$ for the coefficient.

In KSVZ models, the axion can have also electromagnetic anomaly and many quarks contributing to both em and colour anomaly. In this case, the so-called (by me) $C_{a\gamma}$ coefficient is

$$C_{a\gamma} = \frac{\mathcal{E}}{\mathcal{N}} - 1.92 \tag{55}$$

A QCD vacuum and instantons

This section is under construction.

For more information about the topological properties of instantons and the QCD vacuum in the context of axions, one can see [2] and references therein. A classical reference is [8]. A nice review by Forkel can be found in [7].

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