Asymptotic safety and Conformal Standard Model

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Asymptotic safety

Renormalisation group equations

In the quantum field theory the couplings change with energy ("run") due to renormalisation group equations:

$$k\frac{\partial g_i(k)}{\partial k} = \beta_i\left(\{g_i(k)\}\right), \tag{1}$$

where β functions are calculated for a given theory. Standard possibilities:

- Landau pole: $g \to \infty$ for some μ_0. Example: QED.
- $g \to \infty$ for $\mu \to \infty$. Example: Square of Higgs mass.
- asymptotic freedom, $\lim_{\mu\to\infty} g = 0, \forall_i \beta_i(g^*) = 0$. Theory has a UV fixed point. Example: QCD.

Non-standard possibilities

Non standard possibilities are:

- Asymptotic safety, $\lim_{\mu \to \infty} g \neq 0, \forall_i \beta_i(g^*) = 0$. Theory has a UV fixed point. Example: Weinberg hypothesis: Gravity.
- Oscillating g. Theory has a limit cycle. Quantum mechanics: $-g/r^2$ potential.

Fixed points

Fixed point for a given coupling can be:

- repulsive. Example: QED
- attractive. Example: QCD

For repulsive fixed point there is only one IR value of a parameter, which will result in asymptotic safe (free) theory! For attractive fixed point there is a range of allowed parameters.

Standard Model with gravitational corrections

For the Standard Model beta one can calculate the gravitational corrections:

$$\beta(g_j) = \beta_{SM}(g_j) + \beta_{grav}(g_j, k), \tag{2}$$

where due to universal nature of gravitational interactions the β_{grav} are given by:

$$\beta_{grav}(g_j, k) = \frac{a_j k^2}{M_P^2 + 2\xi k^2} g_j, \tag{3}$$

with $\xi \approx 0.024$. The a_j are unknown parameters, however they can be calculated. Then, depending on a sign of a_j, we have repelling/attracting fixed point at 0 in the perturbative region of couplings.

Standard Model with gravitational corrections: Higgs mass

The Higgs mass (self coupling) was calculated by Mikhail Shaposhnikov and Christof Wetterlich using this approach. They obtained the correct value

$$m_H = 126\mathrm{GeV}$$

two years before the detection.

Conformal Standard Model

Conformal Standard Model

- Sterile complex (real) scalar ϕ coupled to Higgs doublet:

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) + (\partial_\mu \phi^\star \partial^\mu \phi) - V(H, \phi). \quad (4)$$

$$V(H,\phi) = -m_1^2 H^\dagger H - m_2^2 \phi^\star \phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\phi^\star \phi)^2 + 2\lambda_3 (H^\dagger H) \phi^\star \phi. \quad (5)$$

- The scalar particles are combined from two mass states:

$$m_1^2 = \lambda_1 v_H^2 + \lambda_3 v_\phi^2, \qquad m_2^2 = \lambda_3 v_H^2 + \lambda_2 v_\phi^2, \quad (6)$$

and the ligher is identified with Higgs particle.

Conformal Standard Model (2)

Conformal Standard Model also includes right handed neutrinos coupled to ϕ with the coupling y_M:

$$\mathcal{L} \ni \frac{1}{2} Y^M_{ji} \phi N^{j\alpha} N^i_\alpha, \tag{7}$$

where $Y^M_{ij} = y_M \delta_{ij}$. To resolve the baryogenesis problem via resonant leptogenesis the right handed neutrinos have to be unstable:

$$M_N > y_M v_\phi / \sqrt{2}. \tag{8}$$

Furthermore this model can resolve the SM problems like: inflation candidates, triviality, hierarchy problem and has dark matter candidates.

The running of couplings

We run following couplings: g_1, g_2, g_3 (Gauge couplings),y_t (top Yukawa coupling), $\lambda_1, \lambda_2, \lambda_3, y_M$. The CSM beta functions are $\hat{\beta} = 16\pi^2\beta$:

$$
\begin{aligned}
\hat{\beta}_{g_1} &= \tfrac{41}{6} g_1^3, \\
\hat{\beta}_{g_2} &= -\tfrac{19}{6} g_2^3, \\
\hat{\beta}_{g_3} &= -7 g_3^3, \\
\hat{\beta}_{y_t} &= y_t \left(\tfrac{9}{2} y_t^2 - 8 g_3^2 - \tfrac{9}{4} g_2^2 - \tfrac{17}{12} g_1^2 \right), \\
\hat{\beta}_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_3^2 - 3\lambda_1 \left(3g_2^2 + g_1^2 - 4y_t^2 \right) \\
&\quad + \tfrac{9}{8} g_2^4 + \tfrac{3}{4} g_2^2 g_1^2 + \tfrac{3}{8} g_1^4 - 6 y_t^4, \\
\hat{\beta}_{\lambda_2} &= \left(20\lambda_2^2 + 8\lambda_3^2 + 6\lambda_2 y_M^2 - 3 y_M^4 \right), \\
\hat{\beta}_{\lambda_3} &= \tfrac{1}{2}\lambda_3 \left[24\lambda_1 + 16\lambda_2 + 16\lambda_3 \right. \\
&\quad \left. - \left(9g_2^2 + 3g_1^2 \right) + 6 y_M^2 + 12 y_t^2 \right], \\
\hat{\beta}_{y_M} &= \tfrac{5}{2} y_M^3.
\end{aligned}
\tag{9}
$$

Conditions for low energy coupling values and a_i values

We impose two conditions:

- absence of Landau poles
- $\lambda_1(\mu) > 0, \quad \lambda_2(\mu) > 0, \quad \lambda_3(\mu) > -\sqrt{\lambda_2(\mu)\lambda_1(\mu)}.$

Furthermore we take:

$$a_{g_i} = a_{y_M} = -1, a_{y_t} = -0.5, a_{\lambda_1} = +3 \tag{10}$$

and

$$a_{\lambda_2} = \pm 3, a_{\lambda_3} = \pm 3, \tag{11}$$

hence we have four possibilities.

Values of parameters

Coefficient: $a_{\lambda_3} = +3$

For $a_{\lambda_3} = +3$, we get that: $\lambda_3 = 0$. So SM and ϕ decouple.

Coefficient: $a_{\lambda_2} = a_{\lambda_3} = -3$, set of allowed couplings $\lambda_2, \lambda_3, y_M$

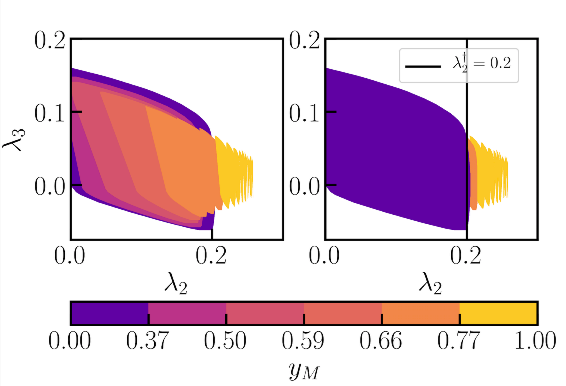

Figure 1: Maximal (left) and minimal (right) $y_M(\lambda_3, \lambda_2)$, $a_{\lambda_2} = -3, a_{\lambda_3} = -3$

Coefficient: $a_{\lambda_2} = a_{\lambda_3} = -3$, λ_1

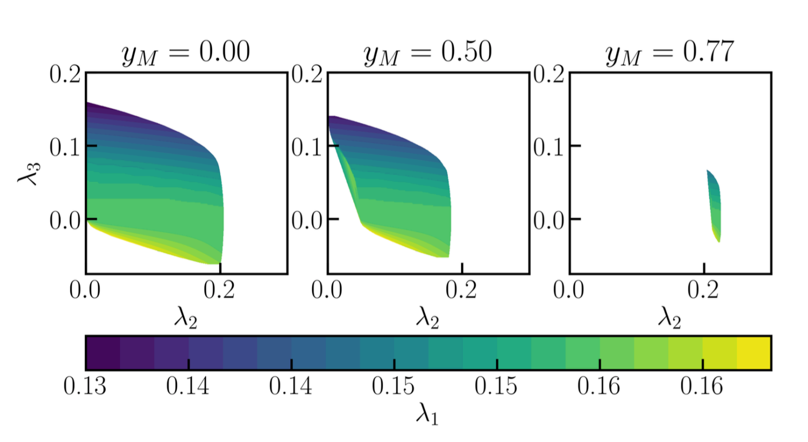

Figure 2: Plot of $\lambda_1(\lambda_2, \lambda_3, y_M)$

Second scalar particle mass

With the tree level relations:

$$m_1^2 = \lambda_1 v_H^2 + \lambda_3 v_\phi^2, \tag{12}$$
$$m_2^2 = \lambda_2 v_\phi^2 + \lambda_3 v_H^2, \tag{13}$$

and with m_1 taken as Higgs mass, $v_\phi = 226$ GeV we are able to constrain the second scalar mass as:

$$270 \text{ GeV} < m_2 < 328 \text{ GeV}. \tag{14}$$

Further restrictions

We can also constraint the neutrino mass with the leptogenesis condition($M_N > y_M v_\phi/\sqrt{2}$):

$$M_N = 342^{+41}_{-41}\ \mathrm{GeV} \tag{15}$$

Moreover for $y_M = 0.0$ we found out that:

$$m_2 = 160^{+103}_{-100}\ \mathrm{GeV}, \tag{16}$$

so classically it is stable.

Summary

Summary and further work

Take home message:

- Standard Model supplemented by the gravitational corrections can be a fundamental theory, yet not a complete one
- Applying the gravitational corrections can give the quantitive predictions for new particles, which can be tested in near future

Further work:

- The remaining a_i's have to be calculated
- The (higher)-loop corrections have to be taken into account

Thank you for your attention

Talk based on article: arxiv.org/abs/1810.08461

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Backup slides

Conditions on m_2

One can parametrize the discrepancies from SM as:

$$\tan\beta = \frac{\lambda_0 - \lambda_1}{\lambda_3}\frac{v_H}{v_\phi}. \tag{17}$$

- $|\tan\beta| < 0.35$. At μ_0,
- Global stability condition of the potential: $\lambda_3(\mu_0) < \sqrt{\lambda_2(\mu_0)\lambda_1(\mu_0)}$,
- un-stability condition for the second particle: $m_2 > 2m_1$.

Coefficient: $a_{\lambda_3} = +3$(2)

For $a_{\lambda_3} = +3$, we get that: $\lambda_3 = 0$. So SM and ϕ decouple.

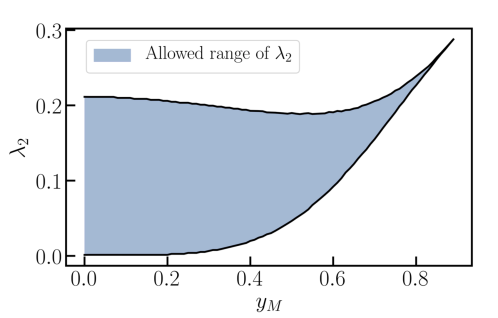


Figure 3: λ_2 dependence on y_M

The case when $a_{\lambda_2} = +3$ follows the lower bound of the plot.

Restrictions from $a_{\lambda_2=+3}, a_{\lambda_3} = -3$

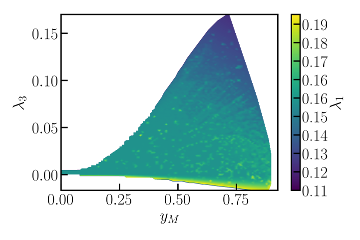
We found that there are only two sets of parameters satisfying the imposed conditions

$$y_M = 0.84, m_2 = 275, v_\phi = 538, M_N = 319, \tag{18}$$

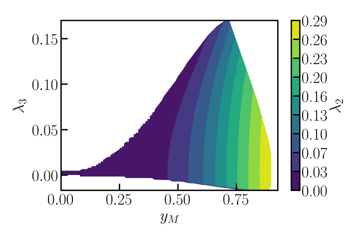
and

$$y_M = 0.85, m_2 = 296, v_\phi = 574, M_N = 345. \tag{19}$$

Coefficients $a_{\lambda_3} = -3, a_{\lambda_2} = +3$



(a) λ_1 dependence on λ_3, y_M



(b) λ_2 dependence on λ_3, y_M