An On-Shell Approach to Gauge Anomalies in an EFT

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Work in progress with C. Csaki and O. Telem

Chiral Anomaly

- Quantum violation of a classical symmetry
- A QFT is given by its Lagrangian and path integral measure
- Specifically, the axial current is classically conserved for massless fermions

$$\mathcal{L} = -\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\alpha\mu\nu} - \overline{\psi}\not{D}\psi \qquad J_5^{\mu} = iq\,\overline{\psi}\gamma^{\mu}\gamma_5\psi$$

- We can most easily see the anomaly with the Fujikawa path integral method, in the case of a single U(1) gauge field…

A Change in the Measure

- Applying an infinitesimal axial transformation,

$$\int \mathcal{D}\psi\mathcal{D}\overline{\psi}e^{-i\bar{\psi}\not{D}\psi} = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}e^{-i\bar{\psi}\not{D}\psi}\left[1 + i\int \mathrm{d}^4x\epsilon(x)\left(\mathrm{a}(x) + \partial_\mu J_5^\mu(x)\right) + \ldots\right]$$

$$\mathrm{a}(x) = -\frac{q^3}{16\pi^2}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}(x)F_{\rho\sigma}(x)$$

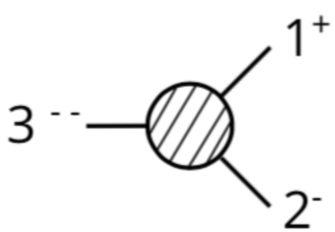
- The axial current is not conserved when gauge field is present
- This topological quantity is related to the difference in the number of left- and right-handed fermion zero modes (and hence the breaking of chiral symmetry)
- Triangle diagrams motivate the on-shell realization

Gauge Anomalies

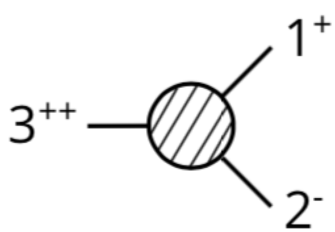
- If the current is gauged, the change to the fermion measure will break gauge invariance
- Alternatively, it is meaningless to couple a gauge field to an unconserved current
- The Standard Model has fermions in anomaly free representations
- This means the charges are such that all U(1),SU(2), and SU(3) anomalies cancel

On-Shell (Massless) Amplitudes

- Forget fields, forget gauge symmetry
- Use only locality and unitarity (later)
- Particles are representations of the little group (the stabilizer of the particle momentum in the Lorentz group), which is U(1)* for massless particles
- Scattering amplitudes should carry the correct “helicity weight” of each external particle



$$= \frac{\langle 23\rangle^2}{\langle 12\rangle}$$



$$= \frac{[13]^2}{[12]}$$

- The angle and square brackets are the usual Weyl spinors, fundamental representations of the little group

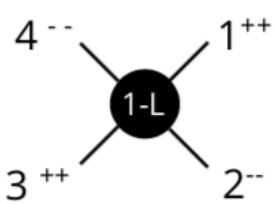
Locality for Points

- The little group fixes the three points
- We use factorization to find higher point amplitudes
- The residue on poles (that field theory tells us means that an internal propagator goes on-shell), is the product of left and right sub-amplitudes
- Chen, Huang, and McGady (2014) consider a theory of massless spin-1 particles in the adjoint of SU(N)* and a single fundamental (anomalous) representation of fermions
- The tree-level, colour-ordered 4-gluon amplitude is Parke-Taylor

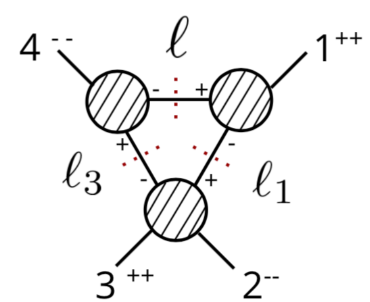
$$\frac{\langle 24\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

Unitarity for Lööps

- We calculate loops by making “unitarity cuts”, sort of…
- The product of sub-amplitudes allows us to compute the coefficients of standard box, triangle, and bubble integrals (Passarino-Veltman)
- Details of calculation in Forde (2007)
- TL;DR: Be clever about parameterizing loop momenta



$$= A^{\mathrm{PT}}\left[c_4 I_4\left(s_{12}, s_{14}\right) + \left(c_3^{(12)} + c_3^{(34)}\right) I_3\left(s_{12}\right)\right.$$



$$\left. + \left(c_3^{(14)} + c_3^{(23)}\right) I_3\left(s_{14}\right) + c_2^{(12)} I_2\left(s_{12}\right) + c_2^{(14)} I_2\left(s_{14}\right) + R(s_{12}, s_{13}, s_{14})\right]$$

Locality Fights Back

- Going to the u-pole one sees

$$\lim_{s_{13}\to 0} A^{1-\text{loop}} = \lim_{s_{13}\to 0} A^{\text{PT}} \left[\sum c_i I_i + R\right] = \lim_{s_{13}\to 0} A^{\text{PT}} \left[-\frac{s_{12}}{s_{13}} + R\right]$$

- To eliminate this pole, respecting colour-ordered cyclicity, the rational term is fixed to be

$$R = \frac{s_{12} - s_{14}}{2 s_{13}}$$

- However, now the s- and t-poles have changed with nothing to account for them – anomaly!

$$\operatorname*{Res}_{s_{12}\to 0} A^{\text{PT}} R = \frac{[13]^2 \langle 24\rangle^2}{2 s_{14}}$$

A Spontaneously Broken Gauge Theory

- D'Hoker and Farhi (1984) introduce a theory that is anomaly-free (the sum of the charges must be zero)

$$\mathcal{L} = -\frac{1}{4} F_{1\mu\nu}^2 - \frac{1}{2}\,\mathrm{Tr}\, F_{2\mu\nu}^2 + \sum_{i=1}^{N} \left[i\overline{\psi}^i_L \not{D}^i_L \psi^i_L + i\overline{\psi}^i_R \not{D}^i_R \psi^i_R \right]$$

$$D^i_{L;\mu} = \partial_\mu + i g_2 A_\mu + i g_1 y_i B_\mu \qquad D^i_{R;\mu} = \partial_\mu + i g_1 \left(y_i + \tau_3\right) B_\mu$$

- They Higgs the theory down to a $U(1)_V$, and use an order one Yukawa to send the last fermion to the Higgs scale
- Finally, they integrate out the Higgs and the (charged) fermion

The Effective Action

$$\mathcal{S}_{\text{EFT}} = \underbrace{\mathcal{S}_{\text{nl}\sigma\text{m}}(A_\mu, B_\mu, \psi^i, U)} + \Gamma_{\text{WZW}}(U) + \int d^4x\, g_1\, y_N\, B_\mu J^\mu_{\text{GW}}$$

Fixed by the nonlinear realization of SU(2)xU(1)

$U \in \frac{\text{SU}(2)_L \times U(1)_R}{\text{U}(1)_V}$ is the Goldstone matrix

- Where the Wess-Zumino-Witten term cancels the non-perturbative SU(2) anomaly, and the Goldstone-Wilczek current

$$J^\mu_{\text{GW}} = \frac{1}{24\pi^2}\epsilon^{\mu\alpha\beta\gamma}\text{Tr}\left[U^\dagger D_\alpha U U^\dagger D_\beta U U^\dagger D_\gamma U - i\frac{3g_2}{2}F_{2\alpha\beta}D_\gamma U U^\dagger - i\frac{3g_1}{2}F_{1\alpha\beta}\tau_3 U^\dagger D_\gamma U \right.$$
$$\left. -g_2^2\left(A_\alpha F_{2\beta\gamma} - \frac{1}{2}ig_2 A_\alpha A_\beta A_\gamma\right)\right]$$

- The top line cancels the anomaly while the bottom pseudo-Chern-Simons (pCS) term shifts the would-be-anomaly between the gauge groups

This is Fine

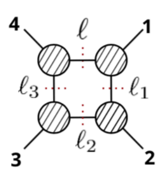
- Preskill (1991) argues that these anomaly canceling terms are gauge artifacts
- Going to unitary gauge, the U fields are set to zero (alternatively, their mass is sent to infinity with this gauge choice)
- Therefore, there is no physical difference between a theory with and without these terms
- An anomalous EFT with massive gauge bosons is consistent (it must be since it came from a consistent theory in the UV)

On-Shell and Massive

- This is where we come in to the story
- We repeat Chen, Huang, and McGady's calculation for a massless fermion loop with four massive massive gauge boson legs
- The little group is now SU(2) and spinors carry extra indices (Arkani-Hamed, Huang, and Huang, 2017)

$$p_i^\mu \bar{\sigma}_\mu^{\dot{\alpha}\alpha} = |\mathbf{i}\rangle^{\dot{\alpha}}{}_I \, [\mathbf{i}|^{I\alpha}$$

- Required to carry these indices through to the coefficients of the scalar integrals



$$= \frac{1}{m^4} \langle \ell \mathbf{1} \rangle [\mathbf{1} \ell_1] \times \langle \ell_1 \mathbf{2} \rangle [\mathbf{2} \ell_2] \times \times \langle \ell_2 \mathbf{3} \rangle [\mathbf{3} \ell_3] \times \langle \ell_3 \mathbf{4} \rangle [\mathbf{4} \ell]$$

$$= \frac{1}{m^4} \langle \mathbf{4} | \ell | \mathbf{1}] \langle \mathbf{1} | \ell_1 | \mathbf{2}] \langle \mathbf{2} | \ell_2 | \mathbf{3}] \langle \mathbf{3} | \ell_3 | \mathbf{4}]$$

Massless limit

- We have completed the calculation of the coefficients
- Checked that the massless limit is reproduced

$$\langle \mathbf{1}| \to m\frac{\langle n_1|}{\langle n_1 1\rangle}\,, \quad |\mathbf{1}] \to |1] \qquad \langle \mathbf{2}| \to \langle 2|\,, \quad |\mathbf{2}] \to m\frac{|n_2]}{[2 n_2]}$$

$$\langle \mathbf{3}| \to m\frac{\langle n_3|}{\langle n_3 3\rangle}\,, \quad |\mathbf{3}] \to |3] \qquad \langle \mathbf{4}| \to \langle 4|\,, \quad |\mathbf{4}] \to m\frac{|n_4]}{[4 n_4]}$$

- The reference spinors reflect the reemergence of gauge symmetry in this limit
- The operations of going to a pole and taking the massless limit do not commute

Checking the u-pole

- Next need to check the residue on the u-pole
- Possible that there is no pole
- Most likely, will have to add a rational term
- May not correct the s- and t-poles, or may correspond to factorizations with a pCS vertex (expect them to emerge at 1-loop)
- In any case, field theory and Preskill tell us that a consistent computation should be possible
- Finally, try to figure out what goes wrong in the massless limit

Summary

- Anomalous symmetries are broken at the quantum level
- These symmetries cannot be gauged
- Unless they are symmetries already broken by a Higgs mechanism in an EFT
- Then you can imagine there are fermions above the cut-off that cancel the anomaly in the high energy limit
- We trade gauge invariance for on-shell locality and unitarity
- A tension arises for massless anomalous theories
- Should see it resolved when the theory is a consistent EFT