Scattering Amplitudes in Effective Gravitational Theories

Stavros Mougiakakos

Supervisor: Pierre Vanhove

Based on:

- N. E. J Bjerrum-Bohr, John F. Donoghue, Pierre Vanhove; arXiv : 1309.0804v2
- N. E. J. Bjerrum-Bohr, Poul H. Damgaard, Guido Festuccia, Ludovic Plante, Pierre Vanhove; arXiv : 1806.04920v2
- M. Levi, S. Mougiakakos and M. Vieira; arXiv : 1912.06276
- Upcoming work with P. Vanhove

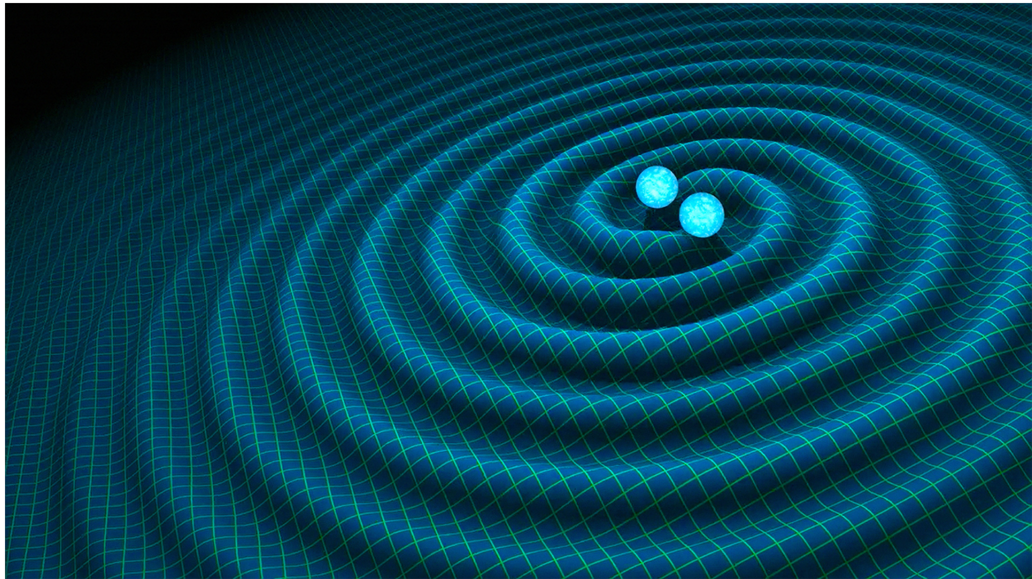
Journee des theses, 3 December 2019, IPhT-CEA

Observational Window on gravity

The detection of gravitational waves (GW150914) has opened a new window on the physics of our universe:

- For the first time detection and test of GR in the strong gravity coupling regime
- For the first time dynamics of Black holes

(not just static object curving space-time)



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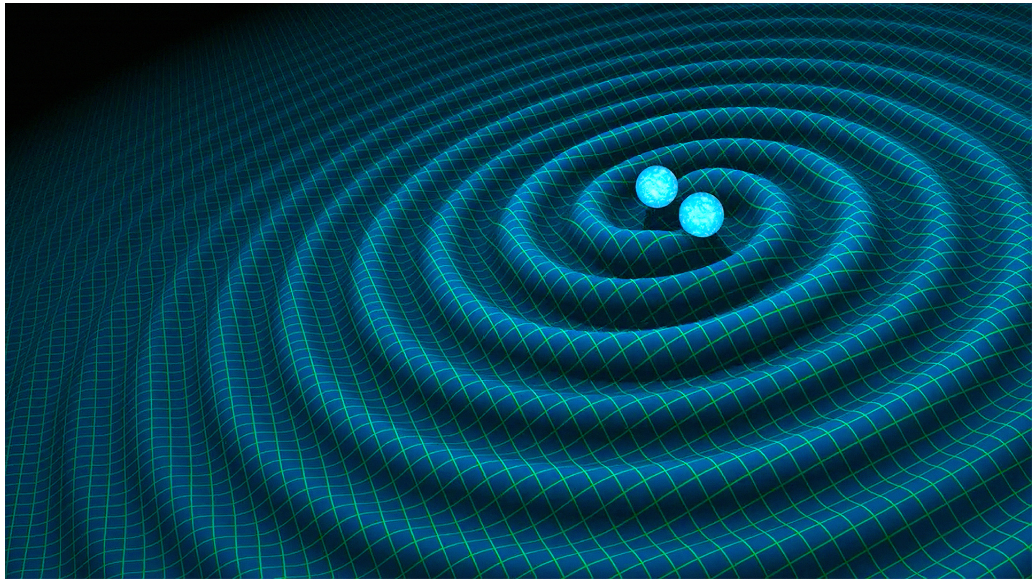

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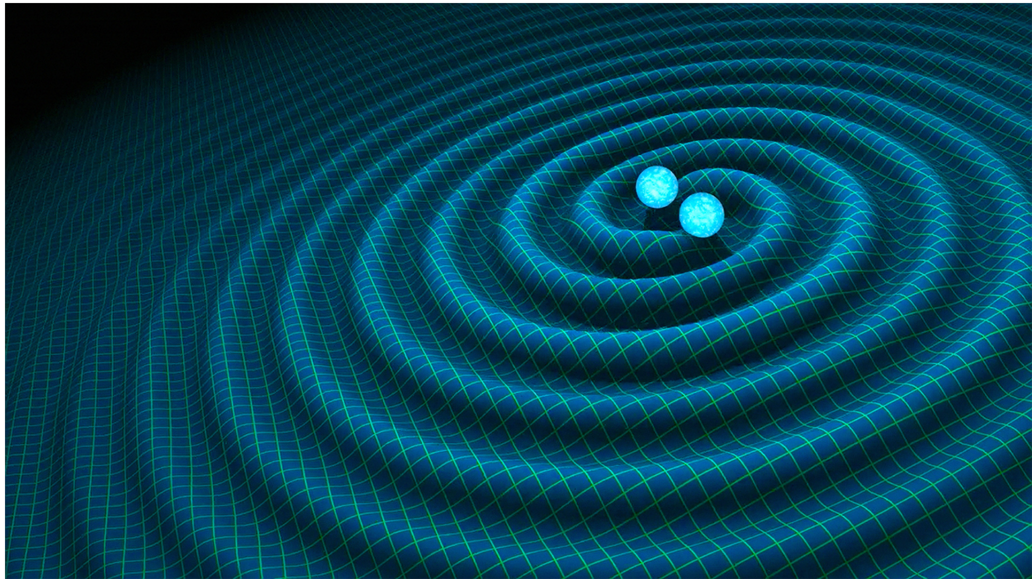
What can we learn from the gravitational wave signals?



Gravity

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<u>BUT</u> Problem with quantization of General Relativity → **Non Renormalisable QFT**

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Is there a way out???

Quantum Gravity

↙

<u>UV Completion of Gravity</u>

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- Main candidate: **Superstring Theory**
- Exciting field of research with many questions left to be answered

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Quantum Gravity as an EFT

John F. Donoghue; arXiv : gr-qc/9512024v1

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Quantum Gravity as an EFT

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What is the EFT logic?

Effective Field Theory/ Non-renormalizability is not a problem

<u>Top</u>

↓

<u>Down</u>

<u>If we know the high energy theory:</u>

- Integrate out the high energy degrees of freedom
- The final theory contains the low energy degrees of freedom and the "loss" of the heavy dof's is compensated by additional interactions suppressed by the scale of separation.

Effective Field Theory/
Non-renormalizability is not a problem

<u>Top</u>
↓
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<u>Up?</u>
↑
<u>Bottom</u>

<u>If we **don’t** know the high energy theory:</u>

- We construct a theory with all the operators allowed by the symmetries of the theory suppressed by the energy scale that we are considering

Effective Field Theory/ Non-renormalizability is not a problem

Toy model: $$\mathcal{L}(\phi) = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

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$$\phi(k) \equiv \begin{cases} \phi(k) & |k| < b\Lambda \\ 0 & |k| \geq b\Lambda \end{cases} \quad , \quad \hat{\phi}(k) \equiv \begin{cases} \phi(k) & b\Lambda \leq |k| < \Lambda \\ 0 & \text{Otherwise} \end{cases}$$

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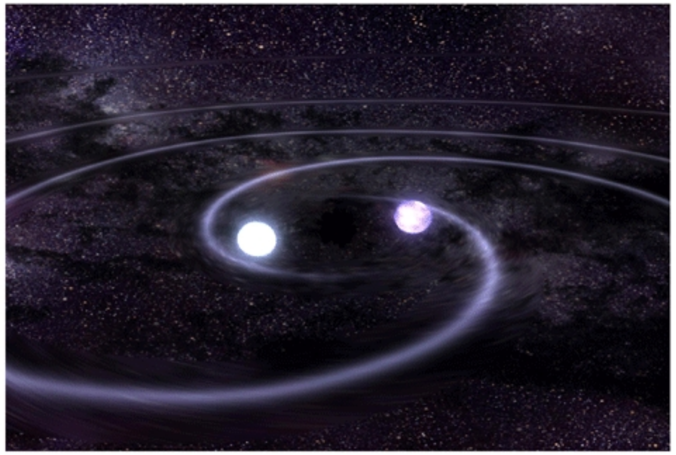
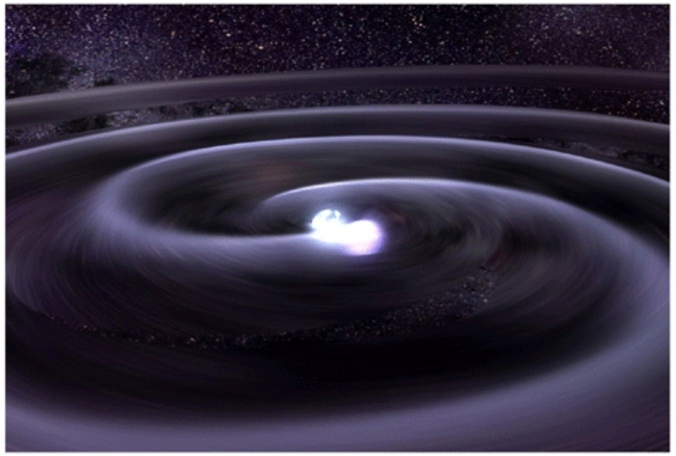
$\downarrow$

$$\mathcal{L}_{\text{eff}}(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\left(m^2 + \rho\right)\phi^2 + \frac{1}{4!}(\lambda + \zeta)\phi^4 + \frac{1}{4}\eta\phi^2(\partial_\mu \phi)^2 + \kappa\phi^6 + \ldots$$

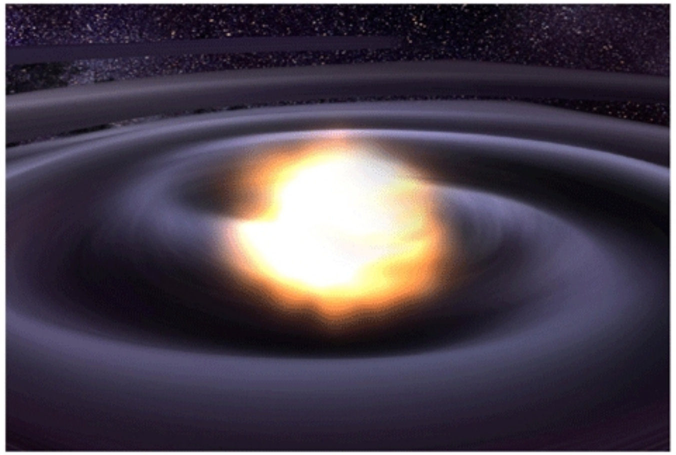
2-body problem



Inspiral

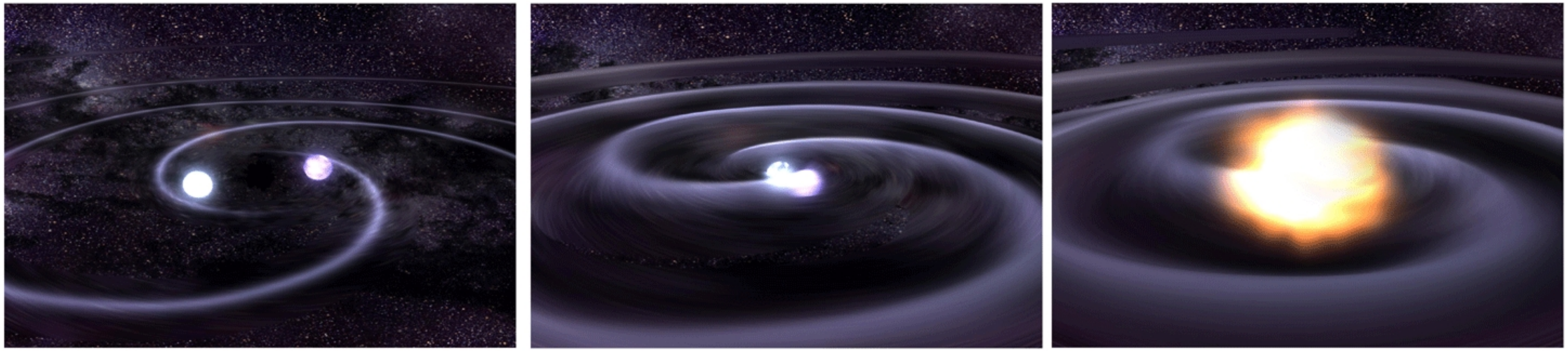


Merger



Ringdown

2-body problem



Inspiral Merger Ringdown

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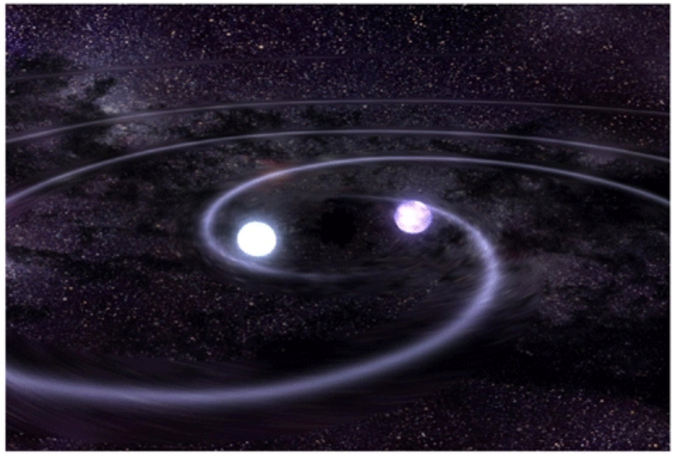
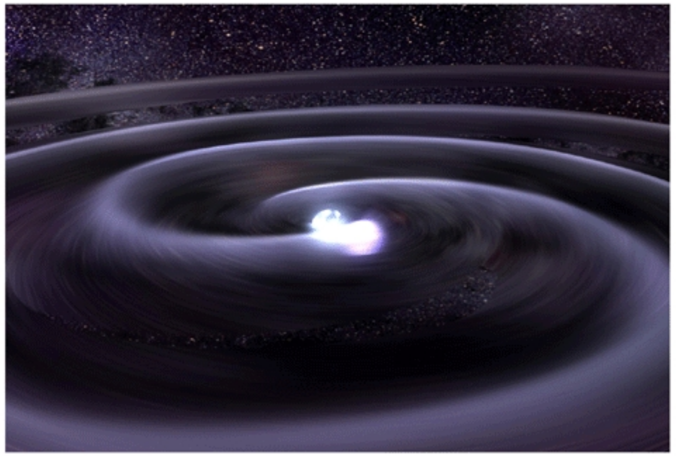
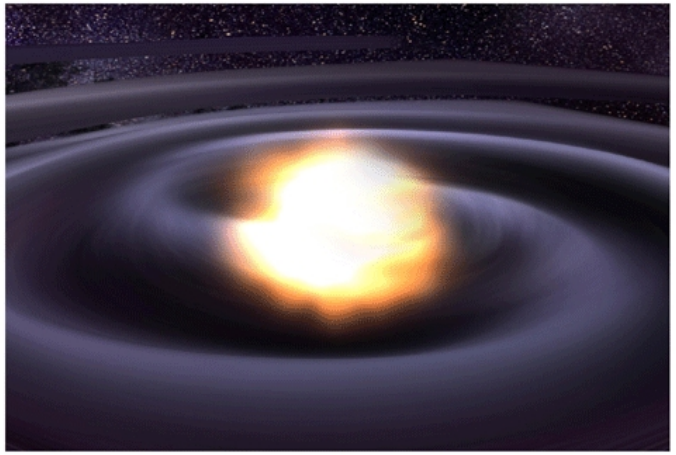

Inspiral **Merger** **Ringdown**

A lot of work has already been done in the analytical solution in perturbative GR.

T. Damour, L. Blanchet, A. Buonanno et al.

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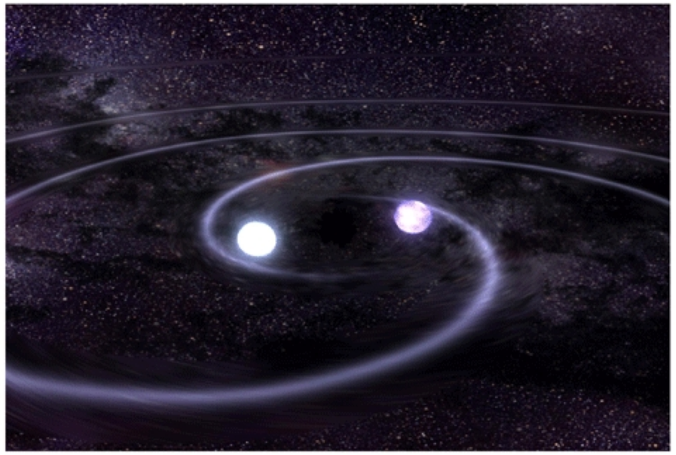

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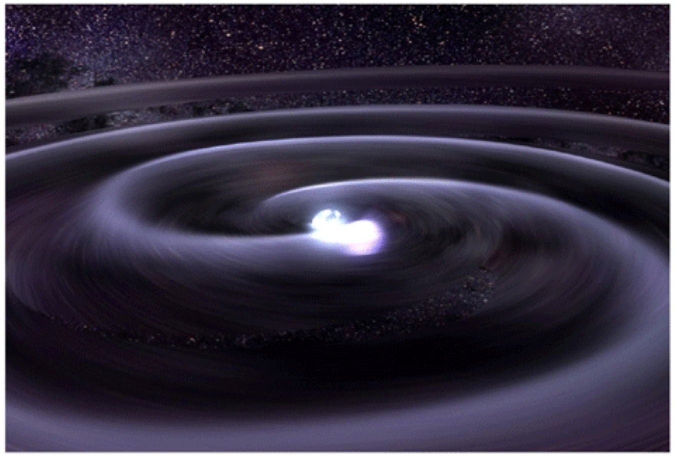

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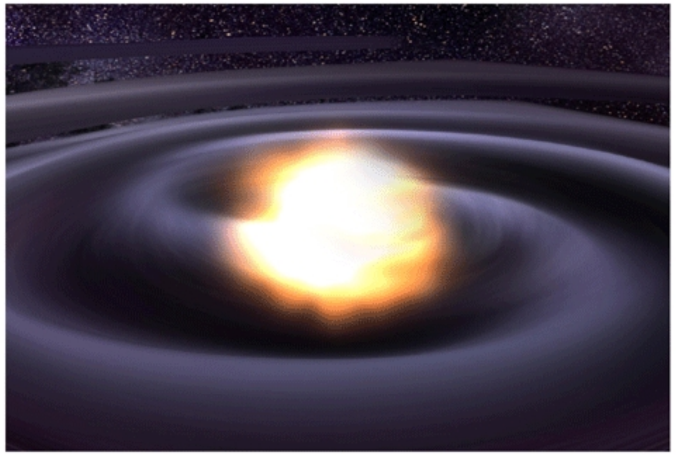

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Can the particle physics community contribute to this problem?

Particle Physicist's Point of View on the gravitational 2-body problem

↙ ↘

EFT of Post-Newtonian Gravity

W.D Goldberger, I. Rothstein, R. Porto, M. Levi et al

- Classical computation
- Takes advantage of QFT toolbox
- Non relativistic computation
- Can deal effectively with spin effects
- State of the art: 4-PN without spins

  4,5-PN(S^3)

Scattering Amplitudes and Post-Minkowskian

Z. Bern, N. Arkani-Hamed, P. Vanhove, N.E.J. Bjerrum-Bohr, J. Donoghue, D. Kosower et al.

- Quantum computation
- Takes advantage of modern methods for on-shell scattering amplitudes (BCJ relations/double copy)
- Fully relativistic computation
- Active work for inclusion of spin effects

  arXiv:1709.04891, arXiv:1812.08752
- State of the art: 3-PM without spins

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where $$S_{\text{pp}} = \int d\sigma\left[-m\sqrt{u^2} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{SI}}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}\left(y^\mu\right)\right]\right]$$

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and $$L_{\text{SI}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n-1}}D_{\mu_{2n}}\cdots D_{\mu_3}\frac{E_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\ldots S^{\mu_{2n-1}}S^{\mu_{2n}}$$
$$+\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{C_{BS^{2n+1}}}{m^{2n}}D_{\mu_{2n+1}}\cdots D_{\mu_3}\frac{B_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\ldots S^{\mu_{2n-1}}S^{\mu_{2n}}S^{\mu_{2n+1}}$$

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We can make a Kaluza-Klein reduction over the time dimension to decompose the metric, as we are working in the NR limit and the orbital modes are instantaneous at leading order:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \equiv e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij}dx^i dx^j$$

EFT of Post-Newtonian Gravity

Finally, we are left with the 3 NRG fields and we can compute the Feynman-like rules for the couplings of the worldlines and the metric fields in order to integrate out the orbital field modes and obtain the EFT of the composite particle.

$$e^{iS_{\text{eff}}[\tilde{g}_{\mu\nu},(y_c)^\mu,(e_c)^\mu_A]} \equiv \int \mathcal{D}H_{\mu\nu}\, e^{iS_{\text{eff}}[\bar{g}_{\mu\nu},y_1^\mu,y_2^\mu,e_1{}^\mu{}_A,e_2{}^\mu{}_A]}$$

Length scales: $r_s \sim Gm$ $\quad r_s \sim rv^2$ $\quad Gm/r \sim v^2$

When we include spin: $S \lesssim m^2 \sim r_s^2$

EFT of Post-Newtonian Gravity

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}$	$+\ \dots)$	G^1
2PM		$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ \dots)$	G^2
3PM			$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ \dots)$	G^3
4PM				$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ \dots)$	G^4
5PM					$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ \dots)$	G^5
6PM						$(1$	$+\ v^2$	$+\ v^4$	$+\ \dots)$	G^6
							$\vdots$			

EFT of Post-Newtonian Gravity

State of the art computation

- Spin-less:4 PN arXiv:1607.04252.
- Spin-less:5 PN- static piece arXiv:1902.11180
- Spin-Orbit at NNLO (3.5 PN) arXiv:1506.05794
- Spin-Orbit at NNLO (4 PN) arXiv:1506.05794
- Cubic in Spin interactions at NLO (4.5 PN)-M. Levi, S.M. et all arXiv : 1912.06276

Scattering Amplitudes and Post-Minkowskian expansion

John F. Donoghue; arXiv : gr-qc/9512024v1

- Non-renormalizability can be handled order by order
- Long-range infra-red contributions can be calculated in this framework since they depend only on the structure of the low-energy fields and the classical background

$$\mathcal{L}_g = \sqrt{g}\left\{\frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \mathcal{O}(R^3)\right\}$$

Scattering Amplitudes and Post-Minkowskian expansion

Loops give classical contributions!

$$\mathcal{M}_{n-loop} = \frac{M^3}{\hbar\sqrt{-s}^3}(r_S\sqrt{-s})^{n+1}\sum_{k=0}^{n}\alpha_k(\lambda\sqrt{-s})^k$$

where $\lambda = \frac{\hbar}{M}$

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WHY?!

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$$\downarrow$$

When we have massless+massive particles, the powercounting gives us the above.

Scattering Amplitudes and Post-Minkowskian expansion

The problem of getting a Post-Minkowskian expansion of the potential comes down to computing Loop diagrams.

$$V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{n=1}^{\infty} \left(\frac{G}{|\boldsymbol{r}|} \right)^n c_n(\boldsymbol{p}^2)$$

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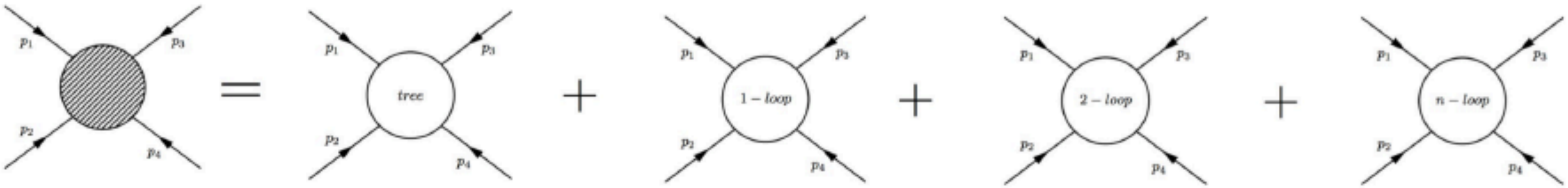
WHICH TOPOLOGIES ARE WE LOOKING FOR?

Scattering Amplitudes and Post-Minkowskian expansion



$$\frac{Gm_1m_2}{r}$$

$$\frac{G^2m_1^2m_2}{r^2},\ \frac{G^2m_1m_2^2}{r^2} \qquad \frac{\hbar G^2 m_1 m_2}{r^3}$$

$$\frac{G^3m_1^3m_2}{r^3},\ \frac{G^3m_1^2m_2^2}{r^3},\ \frac{G^3m_1m_2^3}{r^3} \qquad \frac{\hbar G^3m_1^2m_2}{r^4}\ \ \frac{\hbar G^3m_1m_2^2}{r^4} \qquad \frac{\hbar^2 G^3 m_1 m_2}{r^5}$$

In general:

$$\frac{G^{n+1}m_1m_2}{r^{n+1}}\sum_{k=0}^{n} m_1^k m_2^{n-k}$$

$$\frac{\hbar G^{n+1}m_1m_2}{r^{n+2}}\sum_{k=0}^{n-1} m_1^k m_2^{n-1-k}$$

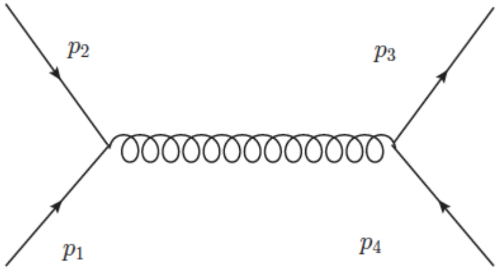
$$\frac{\hbar^2 G^{n+1}m_1m_2}{r^{n+3}}\sum_{k=0}^{n-2} m_1^k m_2^{n-2-k}$$

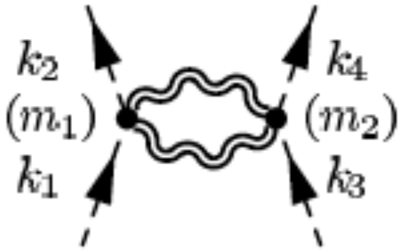
$$\cdots$$

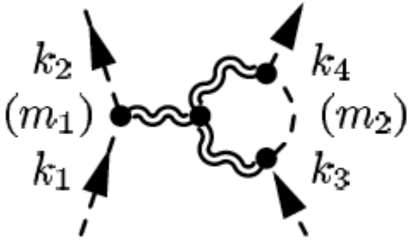
$$\frac{\hbar^n G^{n+1}m_1m_2}{r^{2n+1}}$$

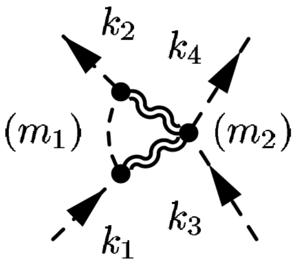
Scattering Amplitudes and Post-Minkowskian expansion

<u>**Classical terms**</u>

- Tree-level: 

- 1-loop: 

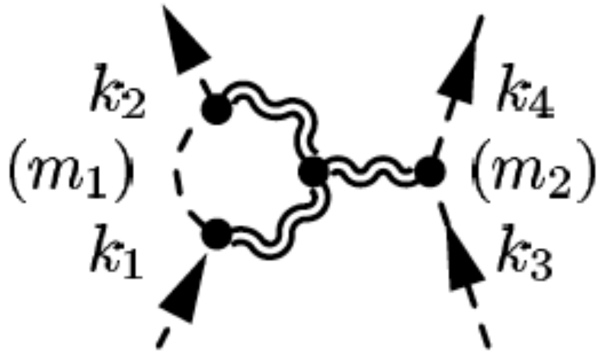


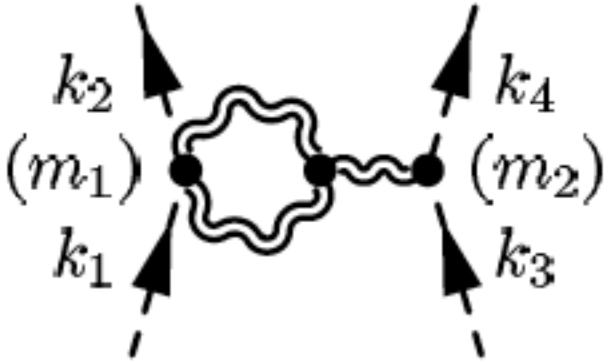


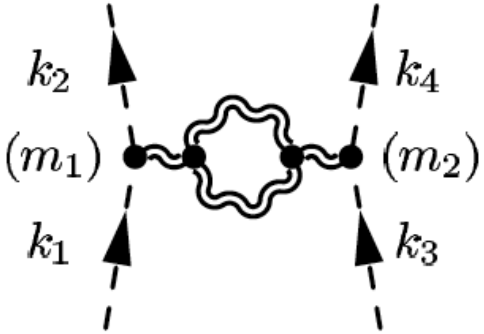
Scattering Amplitudes and Post-Minkowskian expansion

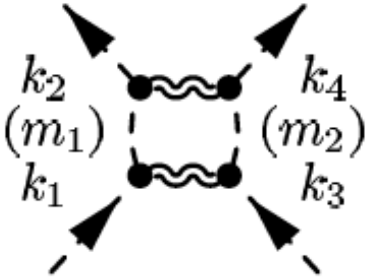
Quantum contributions

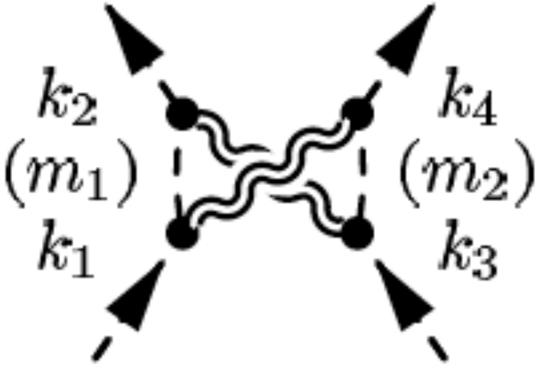
- 1-loop:





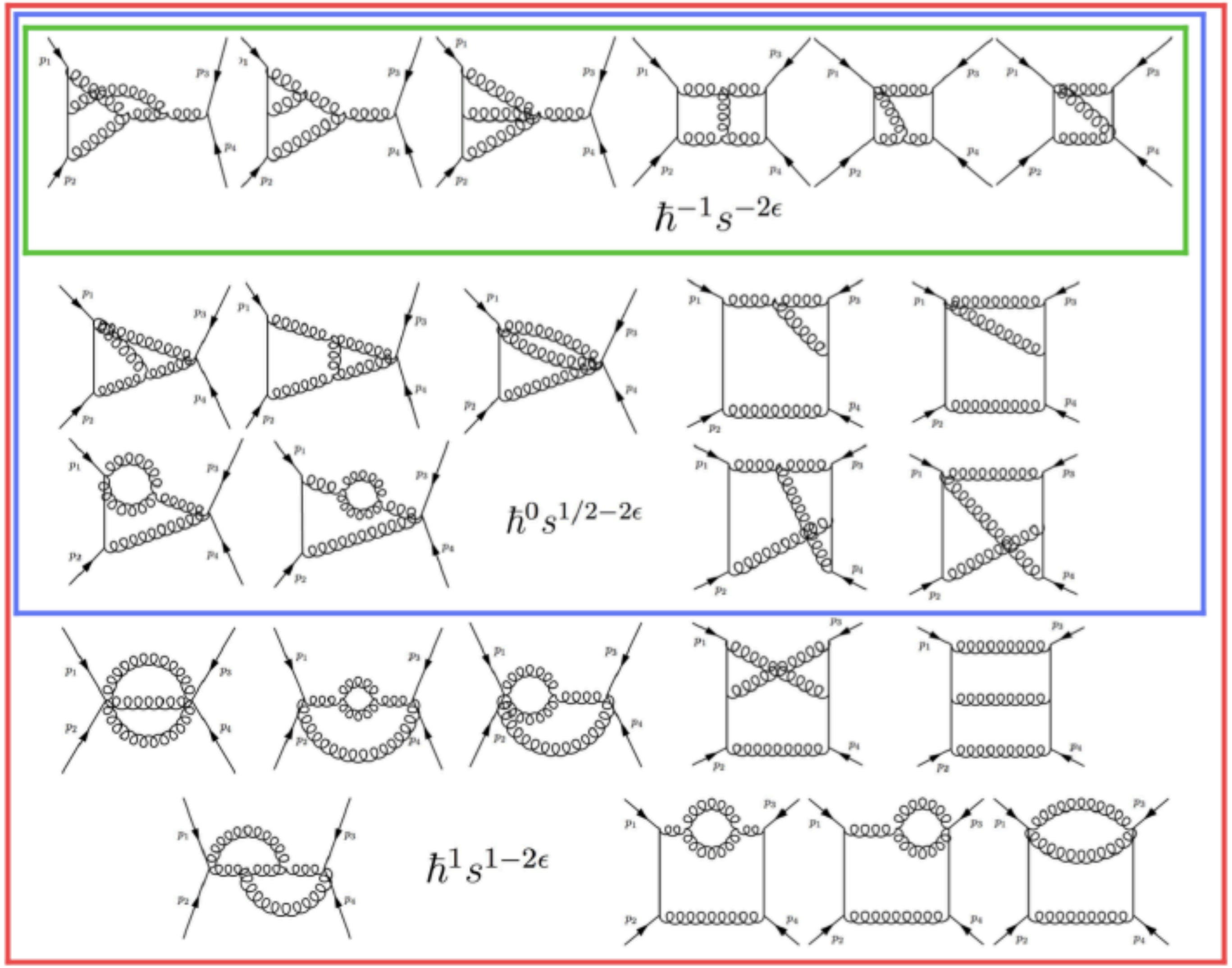






Scattering Amplitudes and Post-Minkowskian expansion

•2-loops:
(both classical and quantum contributions)



Scattering Amplitudes and Post-Minkowskian expansion

We know which diagrams we are looking for <u>and</u> what contributions they give.

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BUT graviton amplitudes are really hard!!

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DOUBLE COPY!!

$$\boxed{\text{gravity} \sim (\text{gauge theory}) \times (\text{gauge theory}).}$$

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$$\text{gravity} \sim (\text{gauge theory}) \times (\text{gauge theory}).$$

KLT relation of string amplitudes $\rightarrow$ BCJ relations/ double copy

spinor helicity formalism

Generalized Unitarity

Scattering Amplitudes and Post-Minkowskian expansion

METRIC PROBLEM: perfect playground to utilize our tools because we know the solution

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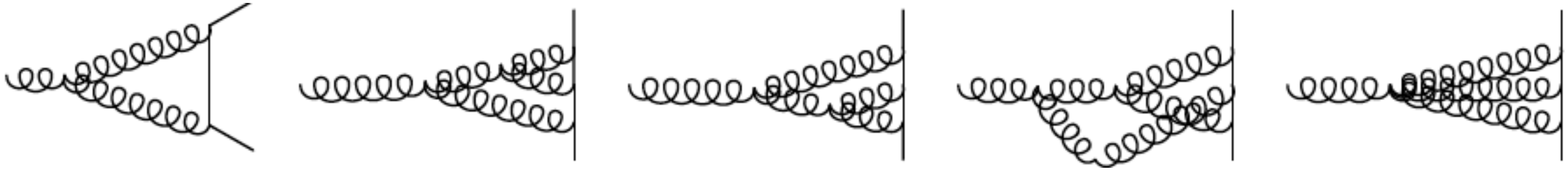
Nevertheless, the problem is slightly easier because we have good control of the contributing topologies. We can use the Feynman rules and compare with the already known solution of Schwarzschild (Kerr) metric.



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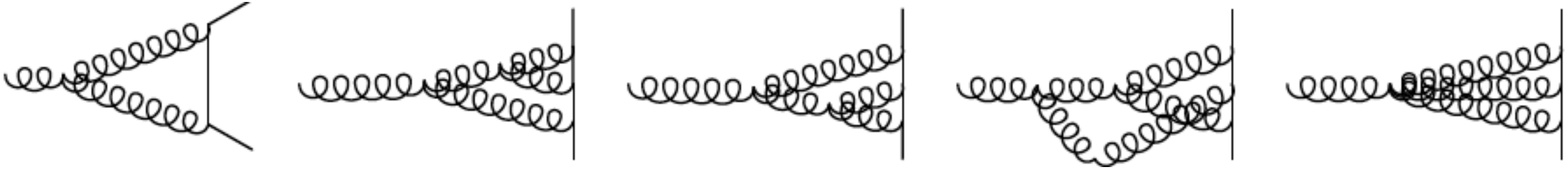

Aim: exploit higher loop effects (currently at 3-loops)- [to appear on arXiv]

Scattering Amplitudes and Post-Minkowskian expansion

	0PN		1PN		2PN		3PN		4PN		5PN		6PN		7PN			
1PM	$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	v^{10}	$+$	v^{12}	$+$	v^{14}	$+$	$\dots)$	G^1
2PM			$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	v^{10}	$+$	v^{12}	$+$	$\dots)$	G^2
3PM					$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	v^{10}	$+$	$\dots)$	G^3
4PM							$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	$\dots)$	G^4
5PM									$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	$\dots)$	G^5
6PM											$(1$	$+$	v^2	$+$	v^4	$+$	$\dots)$	G^6
												$\vdots$						

Scattering Amplitudes and Post-Minkowskian expansion

<u>State of the art computation</u>

- Spin-less: 3 PM (2 loops) arXiv:1908.01493v1.
- Spin effects: 2 PM (1 loop) arXiv:1812.08752v3

Outline

<u>EFT of Post-Newtonian Gravity</u>

By extending the computation to higher PN orders and spins:

- Dissipative effects entering at 5 PN. Information about microphysics of internal structure
- Extension to radiation modes and non conservative sector
- Issues of IR divergences
- <u>Supplementary computation</u> to the Post-Minkowskian expansion

Outline

Scattering Amplitudes and Post-Minkowskian expansion

By utilizing the powerfull **on-shell techniques** (spinor-helicity formalism, generalized unitarity, BCJ relations etc), we can calculate both classical and quantum corrections:

- Higher order calculation of Post-Minkowskian expansion (needed for LIGO)
- Constraints on the EFT extensions of GR
- Inclusion of spin effects
- Consistency check using metric computation [to appear P. Vanhove, S. M.]
- Long-range quantum effects
- Supplementary computation to the Post-Newtonian expansion

Thank you for your attention