Tutorial 1: Neutrino and Astroparticle Physics

Hints of the existence of sterile neutrino have been reported by experiments such as LSND, MiniBooNE, a reanalysis of reactor neutrino data and more recently by the DANSS and NEOS collaborations. Consider a nearly-sterile neutrino ν_2 with mass $m_2 = 1$ eV. The massive basis ν_1, ν_2 is related to the flavour basis of the electron, ν_e, and sterile neutrino, ν_s, as

$$\begin{pmatrix} \nu_e \\ \nu_s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} ,$$

with $\sin^2 2\theta = 0.1$. The mass m_1 is negligible.

(a) At $t = 0$ a $\bar{\nu}_e$ is produced with energy $E \gg m_1, m_2$. Compute the oscillation probability to sterile neutrinos at a later time t in vacuum and then the survival probability $P(\bar{\nu}_e \to \bar{\nu}_e)$.

(b) Why does the presence of a third mass squared difference imply the existence of sterile neutrinos?

In a very dense environment, with constant electron and neutron densities N_e and N_n, matter effects need to be taken into account in oscillations. The Hamiltonian in matter in the flavour basis is

$$\mathcal{H}^m = \begin{pmatrix} \frac{\Delta m^2}{2E}\sin^2\theta + A & \frac{\Delta m^2}{2E}\sin\theta\cos\theta \\ \frac{\Delta m^2}{2E}\sin\theta\cos\theta & \frac{\Delta m^2}{2E}\cos^2\theta \end{pmatrix} ,$$

with $\Delta m^2 = m_2^2 - m_1^2$. $A \equiv \sqrt{2} G_F (N_e - N_n/2)$ is the matter potential.

(d) Find the two eigenvalues of the Hamiltonian, E_{Am} and E_{Bm}, and the mixing angle in matter, i.e. the mixing angle between the flavour states and the eigenstates of $\mathcal{H}^m$, giving an expression for $\sin^2 2\theta_m$ or $\tan 2\theta_m$.

(e) For $\Delta m^2 > 0$, discuss θ_m in the case in which $A \gg \Delta m^2/(2E)$ and in which A is negligible. What happens for $A = \Delta m^2 \cos 2\theta/(2E)$?

(f) Where can you find an environment in which such oscillations with $E \sim \mathcal{O}(1)$ MeV could be put on resonance for the mass squared difference considered above ($\Delta m^2 \sim \mathcal{O}(1)$ eV^2)?

Consider now the full 3+1 neutrino mixing case, in which there are 4 massive states and 4 flavour states, ν_e, ν_μ, ν_τ, ν_s:

$$|\nu_a\rangle = U_{ai} |\nu_a\rangle ,$$

with a indicating the flavour states and i the massive ones.

(g) Under the assumption that Δm_{12}^2 and Δm_{13}^2 are negligible compared with Δm_{14}^2, compute the appearance oscillation probability in MiniBooNE or LSND $P(\nu_\mu \to \nu_e)$.

(h) MiniBooNE and LSND have reported hints of $P(\nu_\mu \to \nu_e) \sim$ few 0.001. Why strong constraints on U_{e4} and $U_{\mu 4}$ at the level of 0.1 lead to a tension between data?

(i) Could reactor neutrino experiments check this probability by looking for $P(\bar{\nu}_e \to \bar{\nu}_\mu)$?

Solution

(a) i) The state at a time t, $|\nu, t\rangle$, is given by the solution of the evolution equation and is expressed as

$$\begin{aligned} |\nu, t\rangle &= \exp(-i\hat{H}t)|\nu_a\rangle \\ &= \cos\theta\ \exp(-iE_1 t)|\nu_1\rangle + \sin\theta\ \exp(-iE_2 t)|\nu_2\rangle\ . \end{aligned}$$

ii) The probability $P(\nu_e \to \nu_s, t)$ is:

$$\begin{aligned} P(\nu_e \to \nu_s, t) &= |\langle \nu_s|\nu, t\rangle|^2 \\ &= \Big|\Big(-\sin\theta\langle\nu_1| + \cos\theta\langle\nu_2|\Big)\exp(-i\hat{H}t)\Big(\cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle\Big)\Big|^2 \\ &= \cos^2\theta\sin^2\theta\,|\exp(-iE_1 t) - \exp(-iE_2 t)|^2 \\ &\odot \text{ Use the relativistic approximation } E_2 - E_1 = \tfrac{(m_2^2 - m_1^2)}{2E}. \\ &\simeq \sin^2(2\theta)\sin^2\tfrac{m_2^2 t}{4E}\ . \end{aligned}$$

(b) With 3 mass squared differences, one needs 4 massive neutrinos and correspondigly 4 flavour states. From the invisibles width decay we know that only three of these can be active, therefore the 4th one is a sterile neutrino.

(d) The eigenvalues of the Hamiltonian in matter are

$$E_{A,Bm} = \frac{1}{2}\left(\frac{\Delta m^2}{2E} + A \pm \sqrt{\Big(\frac{\Delta m^2}{2E} - A\Big)^2 + \Big(\frac{\Delta m^2}{2E}\Big)^2 \sin^2 2\theta}\right)\ .$$

The mixing angle in matter can be found to be:

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - A}\ .$$

(e) If $A \gg \Delta m^2/(2E)$, the mixing angle in matter is strongly suppressed.
If A is negligible, one recovers the vacuum case.
If the condition $A = \Delta m^2 \cos 2\theta/(2E)$ is satisfied, there is maximal mixing $\theta_m = \pi/4$.

(f) As in the Earth neutrino oscillations are on resonance for energies in the GeV range and $\Delta m^2 \sim 10^{-3}\ \mathrm{eV}^2$, densities of a factor of $10^6 - 10^7$ higher are needed. These could be found in supernovae.
Alternatively, one can compute the density which corresponds to the resonance condition.

(g) By using the unitarity of the mixing matrix, the probability can be computed to be $P(\nu_\mu \to \nu_e) \simeq 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\frac{\Delta m_{14}^2 t}{4E}$.

(h) Because the appearance probability depends on their product and therefore is constrained to be smaller than what observed.

(i) First of all it should be noted that the two probabilities are the CPT conjugate of each other and therefore are equal. So searching for $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ tests the probability under consideration.
However, reactor neutrinos have a typical energy in the MeV range and therefore the neutrinos do not have enough energy to produce a muon and CC interactions cannot take place.