[Neutrino and Astroparticle Physics]

Neutrinoless double beta decay.

Consider an extension to the Standard model in which sterile neutrinos are added. The massive basis ν_1, ν_4 is related to the flavour basis of the active, ν_a, and sterile neutrino, ν_s, as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} . \tag{1}$$

We are interested in case in which $m_4 \gg m_1, m_2, m_3$. Assume that these massive neutrinos are Majorana particles.

Their existence can be tested in neutrinoless double beta decay.

(a) What is the contribution of the sterile neutrinos to the effective Majorana mass parameter (assume that the sterile neutrino mass is $m_4 \ll 100$ MeV)?

(b) Choose now some reference values for $m_4 = 1\text{eV}$ and $|U_{e4}|^2 = 0.02$. For which type of neutrino mass spectrum (NH, IH, QD) can there be a complete cancellation with the standard light neutrino contribution? [Note: Use the following values for the neutrino parameters: $\sin^2\theta_{12} = 0.3$, $\cos^2\theta_{13} \simeq 1$, $\sin^2\theta_{13} = 0.1$, $\Delta m^2_\odot = 8 \times 10^{-5}$ eV^2 and $\Delta m^2_A = 2.5 \times 10^{-3}$ eV^2. Alternatively, you can refer to the expected values for $|\langle m \rangle|$ presented in the lecture notes.]

(c) If, instead $M \gg 100$ MeV, what will be the dependence of the decay rate on $M \simeq m_{\text{heavy}}$? [Hint: Recall the form of the fermionic propagator and require lepton number violation.]

Neutrino masses.

(a) Discuss the relation between the existence of Majorana neutrinos and the fundamental symmetries of elementary particles.

(b) Show that $P_L(\nu_L)^c = 0$ where $P_L = (1 - \gamma_5)/2$ and the other symbols have the usual meaning.

(c) Discuss why the Majorana mass term

$$\mathcal{L}_M = -\frac{1}{2}\nu_L^T C m_M \nu_L + \text{h.c.}$$

is forbidden and show that it breaks lepton number by two units.

(d) Consider a mass term which, in the simplest case of one active (e.g. ν_e) and one sterile neutrino, in this basis reads

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} ,$$

with $m_D \ll M$. What is the dependence of the light neutrino mass on the mixing angle between the flavour and the mass eigenstates, $\sin^2\theta \simeq \tan^2\theta$?

(e) Consider neutrinoless double beta decay. If $M \ll 100$ MeV, (where 100 MeV is the typical momentum exchanged in the process), what is the total contribution due to both the heavy and light mass states to the effective Majorana mass parameter?

Extend the see-saw type I to the inverse-see-saw by including one additional sterile neutrino N_c. Now the mass matrix in the ν_a, N_b, N_c basis reads

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_N \\ 0 & M_N & \mu \end{pmatrix} .$$

(f) Compute the resulting masses in the limit of $M_N \gg m_D \gg \mu$.

Consider now neutrino masses generated radiatively. Take a model in which a scalar particle ϕ of mass 10 keV and a sterile neutrino N_R, singlet of the Standard model gauge group, with a Majorana mass $M = 10$ MeV, are introduced. The coupling and mass terms for ϕ and N_R are

$$-\mathcal{L} = m^2 \phi^\dagger \phi + \frac{1}{2} M N_R^T C N_R + g \bar{\nu}_L N_R \phi + \text{h.c.} ,$$

where ν_L indicates an active left-handed neutrino. C is the charge-conjugation matrix.

(g) If the scalar ϕ does not get a vacuum expectation value, is a Dirac mass term for the neutrino being generated?

(h) A neutrino mass can emerge at the loop-level. Discuss the symmetries of the interaction term $g \bar{\nu}_L N_R \phi$, specifically with respect to a leptonic $U(1)$. Note that there are two possible choices for the assignment of the leptonic numbers to N_R and ϕ. Discuss both cases. What terms do you need to add to the Lagrangian to have non-vanishing masses?

(i*) Based on the discussion above, write down a estimate for the neutrino masses and compare with a direct computation of the neutrino mass loop.

Solution

Neutrinoless double beta decay.

(a) The contribution to the effective Majorana mass parameter is

$$\langle m \rangle_4 = m_4 U_{e4}^2$$

(b) For these values of the parameters we have:

$$\langle m \rangle_4 = 20 \text{ meV}$$

We need to consider each mass spectrum separately, using the fact that $U_{e1} = \cos\theta_{12}\cos\theta_{13}$, $U_{e2} = \sin\theta_{12}\cos\theta_{13}e^{i\alpha_{21}/2}$, $U_{e3} = \sin\theta_{13}e^{i\alpha_{31}/2}$.

NH case ($m_1 \ll m_2 \ll m_3$): $m_1 \sim 0$, $m_2 \simeq \sqrt{\Delta m_\odot^2}$, $m_3 \simeq \sqrt{\Delta m_A^2}$

$$\langle m \rangle_{ee} \simeq \sqrt{\Delta m_\odot^2}\sin^2\theta_{12}\cos^2\theta_{13} + \sqrt{\Delta m_A^2}\sin^2\theta_{13}e^{i\alpha_{31}} \simeq 1 - 5 \text{ meV}$$

In this case there cannot be a cancellation.

IH case ($m_3 \ll m_1 \simeq m_2$): $m_3 \sim 0$, $m_1 \simeq m_2 \simeq \sqrt{\Delta m_A^2}$

$$\langle m \rangle_{ee} \simeq \sqrt{\Delta m_A^2}(\sin^2\theta_{12} + \cos^2\theta_{12}e^{i\alpha_{21}})\cos^2\theta_{13} \simeq 20 - 50 \text{ meV}$$

In this case there can be some partial cancellation between the light neutrino mass and the sterile neutrino contributions.

QD case ($m_1 \sim m_2 \sim m_3 \equiv m_0$): The QD case is a good approximation for the neutrino masses as far as $m_1 > 0.1$ eV. And we can neglect the term proportional to $\sin^2\theta_{13}$.

$$\langle m \rangle_{ee} \simeq m_0(\sin^2\theta_{12} + \cos^2\theta_{12}e^{i\alpha_{21}}) \simeq 40 - 100 \text{ meV}$$

Also in this case there cannot be a cancellation.

(c) If $M \gg 100$ MeV, the decay rate will depend on $1/M^2$.

Neutrino masses.

(a) The Majorana condition breaks lepton number and any other $U(1)$ symmetry by two units, therefore if neutrinos are Majorana particles, the lepton number is not a conserved symmetry of elementary particles.

(b) $P_L(\nu_L)^c = \frac{1-\gamma_5}{2}C\bar{\nu_L}^T = \frac{1-\gamma_5}{2}C\frac{1+\gamma_5}{2}\bar{\nu}^T = \frac{1-\gamma_5}{2}\frac{1+\gamma_5}{2}C\bar{\nu}^T = 0$.

(c) This mass term breaks $SU(2)$ as it behaves as a triplet.

Under a $U(1)_{\text{lepton}}$ transformation we have $\nu_L \to e^{i\alpha}\nu_L$ and the mass term transforms as

$$\mathcal{L} \to e^{2i\alpha}\mathcal{L}$$

.

(d) By diagonalising the mass matrix, the mass eigenvalues are found to be

$$m_{\text{light}} \simeq -\frac{m_D^2}{M} , \qquad m_{\text{heavy}} \simeq M .$$

The mixing angle is given by $\tan 2\theta = \frac{2m_D}{M}$.

And it follows that

$$m_{\text{light}} \simeq -\sin^2\theta m_{\text{heavy}}$$

(e) If $M \ll 100$ MeV, the total contribution will be 0 as it corresponds to the ee term in $\mathcal{M}$.

(f) Diagonalising the mass matrix, one finds that $m_a \simeq -m_b \simeq \sqrt{M_N^2 + m_D^2}$ and $m_c \simeq \mu \frac{m_D^2}{M_N^2}$.

(g) No.

(h) Under a leptonic $U(1)$, the leptonic numbers can be assigned as

	$U(1)_{L1}$	$U(1)_{L2}$
ν_L	-1	-1
N_R	-1	0
ϕ	0	-1

$U(1)_{L1}$ is broken by the Majorana mass term. $U(1)_{L2}$ is conserved by the Lagrangian implying that neutrino masses are not allowed and will not be generated by this coupling. We need to introduce a term $\mu^2\phi^2 + \text{h.c.}$ which breaks $U(1)_{L2}$.

(i) Based on the breaking of the various lepton numbers, one can expect the neutrino mass to be

$$m_\nu \sim \frac{g^2}{16\pi^2}\frac{\mu^2}{M} \ .$$

The detailed computation can be found in Ref. hep-ph/0612228.