# Lecture 3: An Introduction to Jets

Last lecture, we made a profound assertion, that the process

$$p\,p \to 1\ 2 \cdots n$$

could be decomposed into a partonic cross section

$$\sum_{ij} i\,j \to \underbrace{1\ 2 \cdots}_{\text{few}} \times Br(1 \to \_\_) \times Br(2 \to \_\_)$$

$$\vdots$$

We derived the NWA approximation and stated its assumptions.

We asserted PDFs, though proofs in that case are more delicate.

Crucial to this factorization is an assumption about the measurement function

$$f_{obs}(\text{hadrons}) \simeq f_{obs}(\text{partons from hard process})$$

This is really two kinds of assumption:

- Insensitivity to non-perturbative hadronization effects (hadrons $\simeq$ partons)
- Insensitivity to "underlying event" (ie. beam remnants) (partons $\simeq$ partons from hard process)

Need to worry about whether your measurement is sensitive to soft physics effects or not.

We now want to extend our discussion to jet formation, to understand how these partons fragment into the hadrons we see.

<u>Jets</u> : collimated sprays of hadrons.

g gluon

d down

u up

s strange

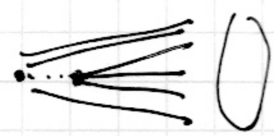
Challenging to distinguish from each other

Spray of $\mathcal{O}(10)$ hadrons initiated by a parton.

$m_{parton} \approx 0$ but $m_{jet} \approx 10\% \cdot E_{jet}$ (not massless!)

c charm

b bottom



Hadronizes into a heavy-flavor hadron to yield a displaced vertex signature.

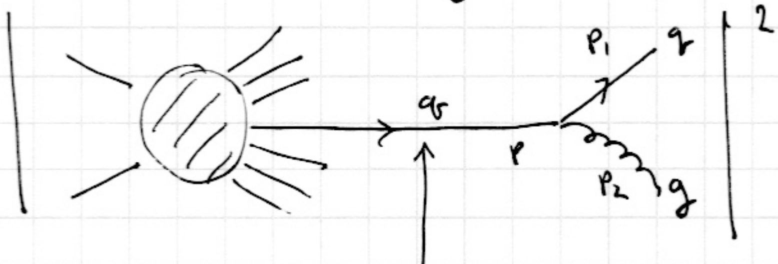
Jets are really two different things:

- A physical phenomenon: emergent features of certain confining gauge theories
- An analysis technique: strategy to interpret hadronic final states.

Of course, these go hand in hand, but worth remembering that physical phenomenon happens, regardless of how you decide how to interpret it.

# Why do jets form?

1) Soft-Collinear Singularities of QCD




Singularity when
propagator goes on shell

$$p = p_1 + p_2 \quad \text{with} \quad p_1^2 = p_2^2 = 0$$

When does $p^2 = 0$?

Collinear limit: $\vec{p}_1 \parallel \vec{p}_2$ (tendancy to get jet-like collimated)

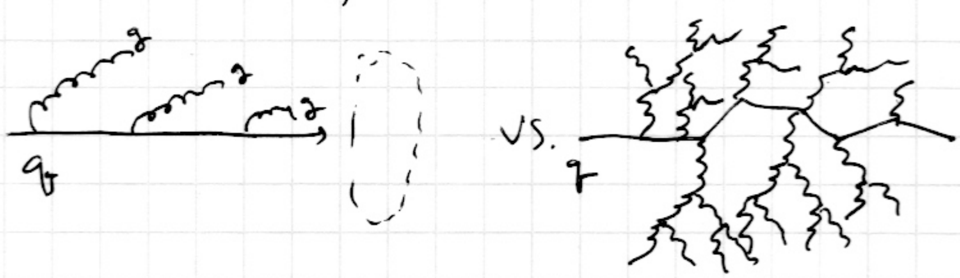
Soft limit: $|\vec{p}_2| \to 0$ (annoyance for precision jet physics)

Note that jet collimation is a <u>perturbative</u> feature.

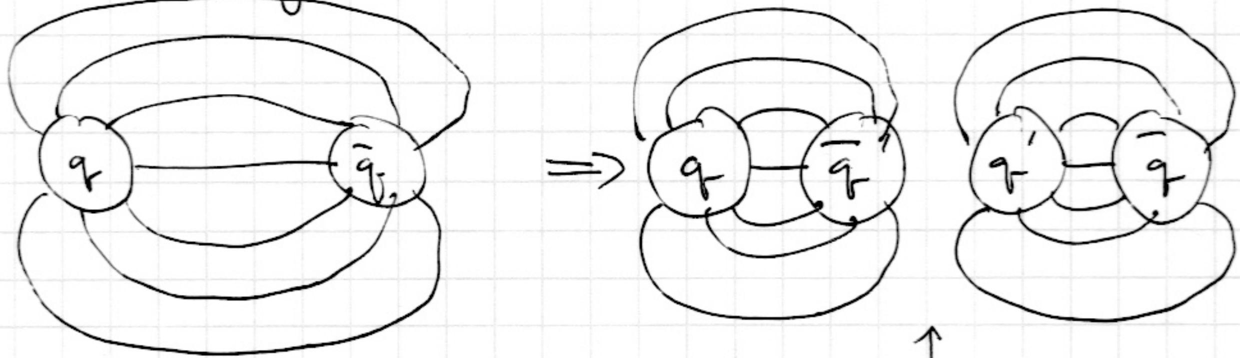
2) Asymtotic Freedom ($\alpha_s$ small at high-enough energies)

$$\alpha_s(M_Z) \simeq 0.12$$

Otherwise, many emissions create ball (instead of cone) of radiation.



3) Color strings break




Confining gluon flux

As you pull apart, eventually favorable to sever flux tube.

If color strings didn't break, you would get lots of excited hadron states instead of jets.

String picture explains a lot, actually.

→ strings break easily: direction of hard quark/gluon $\approx$ direction of jet

→ String has energy density: massless quark/gluon $\Rightarrow$ massive jet

$$m_{jet}^2 \sim \alpha_s R E_{jet}^2$$

→ String severed by $q\bar{q}$ pairs: jets are mostly mesons

$$\text{jet} \approx \underbrace{\pi^\pm \pi^0}_{80\%} + \underbrace{K^\pm K^0}_{15\%} + \underbrace{\text{else}}_{5\%}$$

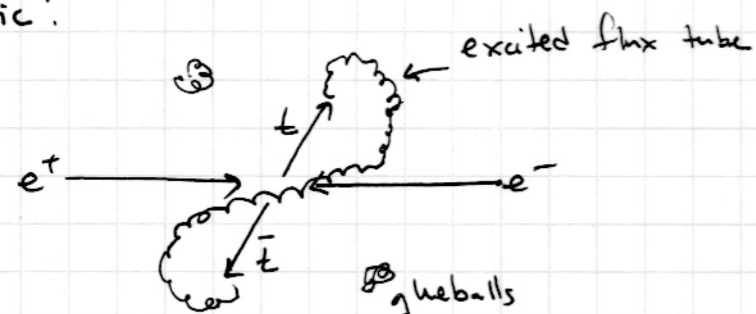
→ String is a color singlet: jets are ambiguous

Where exactly does string break?

color triplet quark
color octet gluon $\Rightarrow$ color singlet hadrons.

Jets are not automatic!

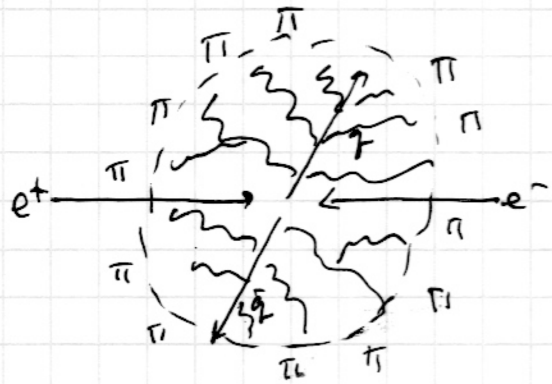
Quirky World
(QCD with only top quarks




Can't break strings!

Only toponium and glueballs.

Quasi-Conformal World
$(\beta \approx 0,\ g \approx 4\pi)$

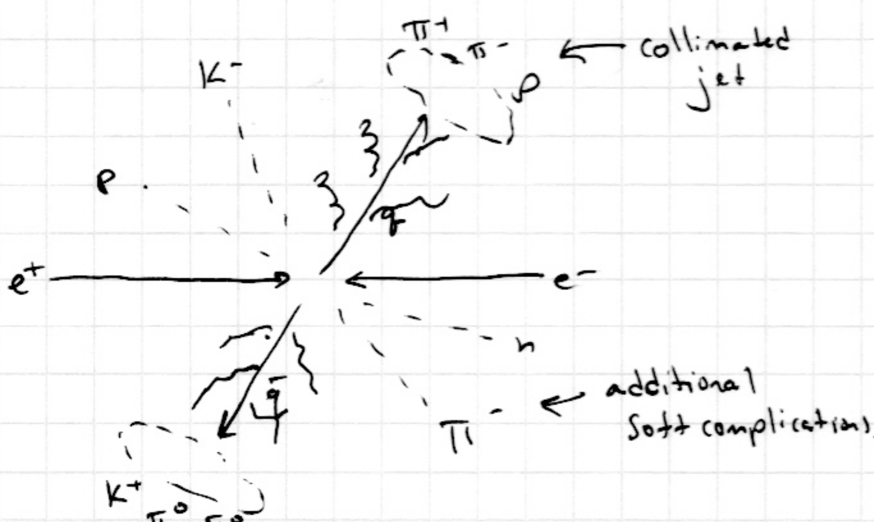



No hierarchies of scales!

Most likely: spherical events

Our World
$(N_c = 3 \quad N_f = 2\text{-}5\ (\text{or } 6))$

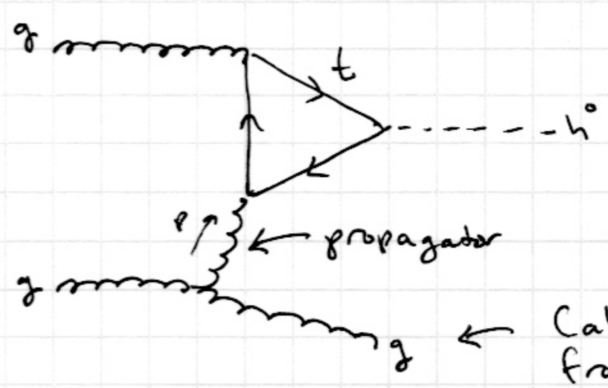
→ Colliner singularity
→ Color coherence
→ Asymptotically free...
... but confining
→ Color strings break
$(m_{u,d} << \Lambda_{QCD})$



Let me emphasize that jets can already be understood as a perturbative phenomenon in QCD. While there are non-perturbative complications, much of jet dynamics is calculable, specifically:

## Initial- and Final-State Radiation.

As a warm-up, let's look at a higher-order radiative correction to Higgs boson production:




← Called ISR since it is radiated from initial state.

In the amplitude, the propagator has a $1/p^2$, so as discussed already, emitted gluon wants to be soft, and/or it want to be emitted in same direction as incoming state.

Just like NWA, simplification when an internal propagator is close to on-shell: factorization!

Details depend on exactly what your measurement function does. Are you inclusive over ISR? Do you measure ISR radiation? Do you veto ISR?

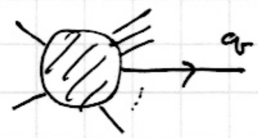
(Or maybe you ask such a detailed question that factorization is violated...)

Exercise: Show that in soft & collinear limit:

$$\sum_{\text{polarization}} \int d\Phi_{n+1} \left| \right.$$


$$\left. \right|^2$$

(like NWA but for massless partons)

$$\simeq \sum_{\text{polarizations}} \int d\Phi_n \left| \right.$$


$$\left. \right|^2$$

$$\times \int_0^1 dz \int_0^{\theta_{max}} d\theta \; \frac{2\alpha_s}{\pi} \; C_F \; \frac{1}{z} \; \frac{1}{\theta}$$

$C_F$: color factor, swap for $C_A$ for gluon case

$\frac{1}{z}$, $\frac{1}{\theta}$: soft / collinear singularities

Energy fraction: $z = \dfrac{E_g}{E_{initial}}$ $\quad z \to 0$ soft limit.


Splitting angle: $\theta = \theta_{qg}$ $\quad \theta \to 0$ collinear limit.

Color factors: $\qquad SU(N_c) \longrightarrow SU(3)$

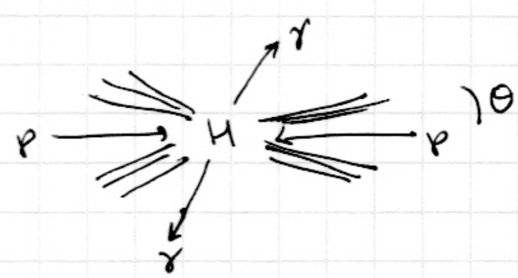
Quark: $\displaystyle\sum_{a,j} t^a_{ij} t^a_{jk} = C_F \, \delta_{ik} \quad C_F = \frac{N_c^2 - 1}{2N_c} \longrightarrow 4/3$

Gluon: $\displaystyle\sum_{a,b} f^{abc} f^{abd} = C_A \, \delta^{cd} \quad C_A = N_c \longrightarrow 3$

In the soft & collinear limits only gluon emissions ($g \to q\bar{q}$ is a subleading effect). We will be able to understand many aspects of perturbative QCD dynamics just from this expression.

(Though not in these lectures, DGLAP evolution comes from only tracking one branch of collinear emissions.)

Returning to ISR in Higgs production...




... preference for ISR to go in direction of initial beams!

Approximate angular distribution of ISR:

$$\frac{d\theta}{\theta} \text{ near } \theta = 0 \qquad \frac{d\theta}{\pi - \theta} \text{ near } \theta = \pi$$

$$\Rightarrow \frac{d\theta}{\sin\theta} \text{ smoothly captures both behaviors.}$$

This peaked behavior is a bit annoying to deal with, so let's do a change of variables:

$$\eta = -\log \tan\frac{\theta}{2} \Rightarrow |d\eta| = \left|\frac{d\theta}{\sin\theta}\right| \quad \text{(a good exercise)}$$

↑ pseudorapidity

A convenient coordinate system since massless QCD emissions have roughly uniform distribution in $(\eta, \phi)$ plane.

For massive particles, better to use true rapidity:

$$y = \frac{1}{2}\log\frac{E + p_z}{E - p_z}$$

This explains coordinate system used in proton-proton colliders.

$$(E, p_x, p_y, p_z) \Rightarrow (p_T, y, \phi, m)$$

Conversion is straightforward:

$$E = \sqrt{p_T^2 + m^2}\cosh y$$

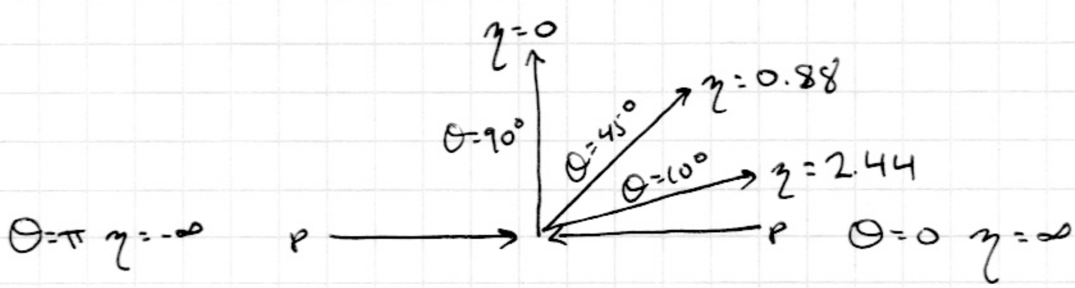
$$p_x = p_T \cos\phi$$

$$p_y = p_T \sin\phi$$

$$p_z = \sqrt{p_T^2 + m^2}\sinh y$$

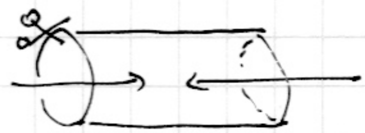
A number of nice features.

- $p_T$ invariant to boosts along beam axis
- $\Delta y$ invariant to boosts along beam axis
- QCD radiation quasi-uniform in $(y, \phi)$ plane.

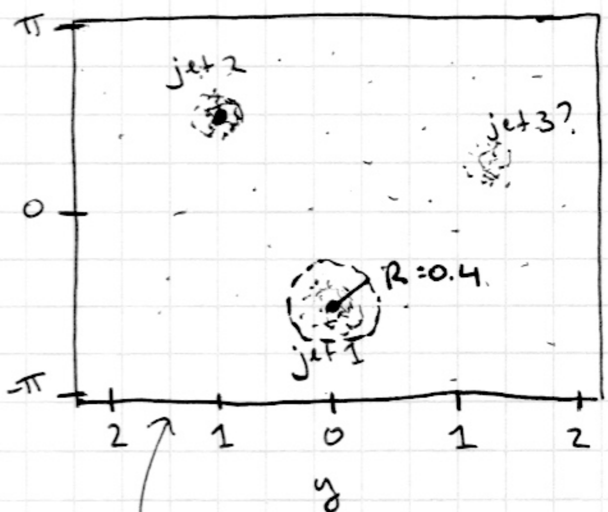


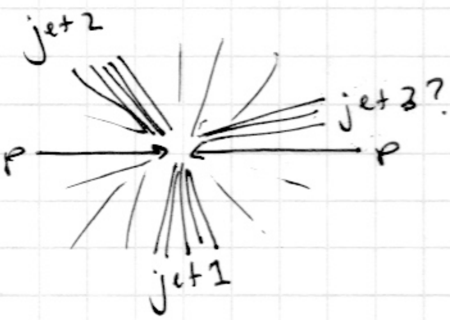

ATLAS / CMS have some sensitivity out to $|\eta| = 5$.

Now, we can draw a picture of a typical dijet event, We typically slice open detector...



.... and unroll in $(y, \phi)$ plane.




(more about jet algorithms later)

uniform haze of ISR/UE | pileup

Collinear singularities explain collimation of radiation along jet directions, and quasi-uniform ISR haze.

One nice feature is that jets are roughly circular even in $(y, \phi)$ plane.

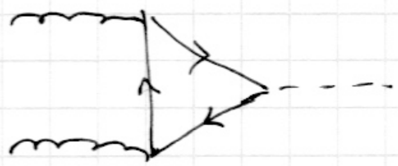
Standard opening angle: $d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$

Using $\sin\theta = \dfrac{1}{\cosh y}$ for massless particles:

$$d\Omega^2 = \frac{1}{\cosh^2 y} \underbrace{\left( dy^2 + d\phi^2 \right)}_{\equiv dR^2}$$

So $\Delta R$ is like opening angle, just rescaled by $\cosh y$.

We will talk much more about jets and jet substructure in the last lecture, but one important fact we will need is that Feynman diagrams are a good approximation to <u>inclusive</u> cross sections.



$\simeq \sigma_{\text{Higgs plus } \underline{\text{anything}}}$

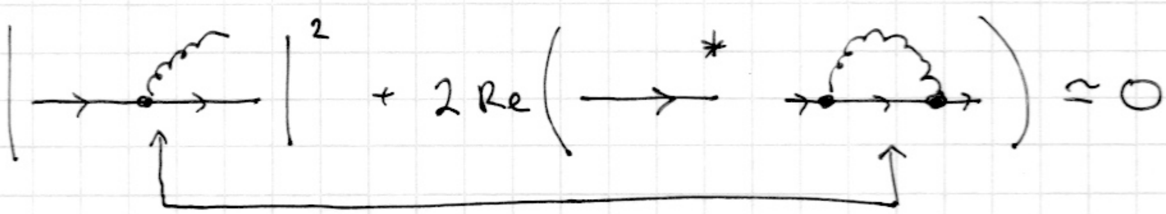
even if it isn't in the diagram!

Why? ISR scales like $\alpha_s \frac{d\theta}{\theta}$, so no real suppression.

perturbative ($\alpha_s$), singular ($\frac{d\theta}{\theta}$)

If you say "Higgs plus nothing else", you are sensitive to singular behavior (exclusive cross sections)

If you say "Higgs plus anything else", you get a cancellation between real and virtual diagrams. Schematically:

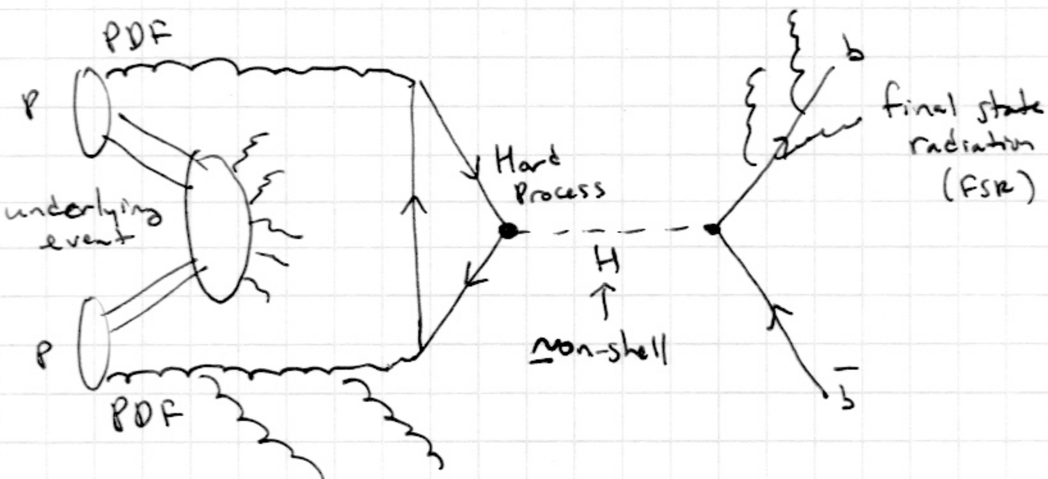



infrared divergences cancel when integrating over phase space.

As we'll discuss more next time, infrared & collinear safe observables are ones that preserve this cancellation of divergences.

Summarizing, with appropriate choices of measurement functions, we can take a factorized approach to QCD and collider physics.

The picture you should have in your brain:




initial state radiation (ISR)

plus NP hadronization.

If you are sufficiently inclusive, you can ignore crosstalk between these subprocesses (up to corrections)

ISR/FSR are governed by <u>perturbative</u> soft/collinear singularity structure. A preference for hard quarks/gluons to emit soft/collimated gluons.

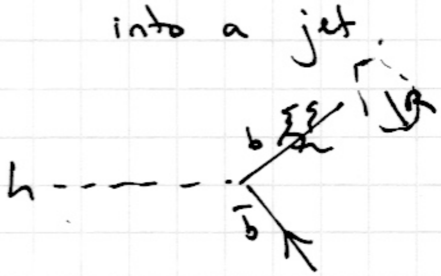
The last ingredient we need are jet algorithms to "undo" FSR and give us a proxy for hard partons.

# Jet Algorithms (very briefly)

We want a proxy for short-distance quark/gluons constructed from measured long-distance hadrons.

We want an algorithm that is quasi-inclusive over FSR emissions, but not so inclusive that you soak up lots of ISR.

Jet definitions are fundamentally ambiguous, but typical algorithms (like anti-$k_t$) collect all radiation within some radius R (in $y$-$\phi$ plane) into a jet.




$m_B < m_{b\text{-jet}}$

just the B-hadron

includes gluon radiation.

Choice of R needs to balance two effects:

Want to include FSR $\Rightarrow \uparrow R$

Want to exclude ISR/UE $\Rightarrow \downarrow R$

At LHC, $R = 0.4$ is typical, such that

$$m_{jet} \simeq 10\% \; p_{T\,jet}$$

Many different algorithms to map

$$\{p_{hadrons}\} \Rightarrow \{p_{jet}\}$$

~massless

massive!

A key requirement for jet algorithms is that they are

Infrared & Collinear Safe

which is a way to ensure that real and virtual divergences cancel in perturbation theory.

In the Tevatron era, there was a lot of confusion about cone algorithm and IRC safety, but SISCone and anti-$k_t$ put these concerns to rest.

Here is a simple IRC-safe algorithm to find one perfectly conical jet of radius $R$. Consider a point $\{y_A, \phi_A\}$ on the $y$-$\phi$ plane. Compute the following

$$\tau_1(y_A, \phi_A) = \sum_{i \in \text{event}} p_{T_i} \min\Big\{ \underset{\substack{\uparrow \\ \text{in jet}}}{R_{iA}^2},\ \underset{\substack{\uparrow \\ \text{out of jet}}}{R^2} \Big\}$$

where $\quad R_{iA}^2 = (y_i - y_A)^2 + (\phi_i - \phi_A)^2$

The jet algorithm is to just minimize $\tau_1$ over $y_A, \phi_A$.

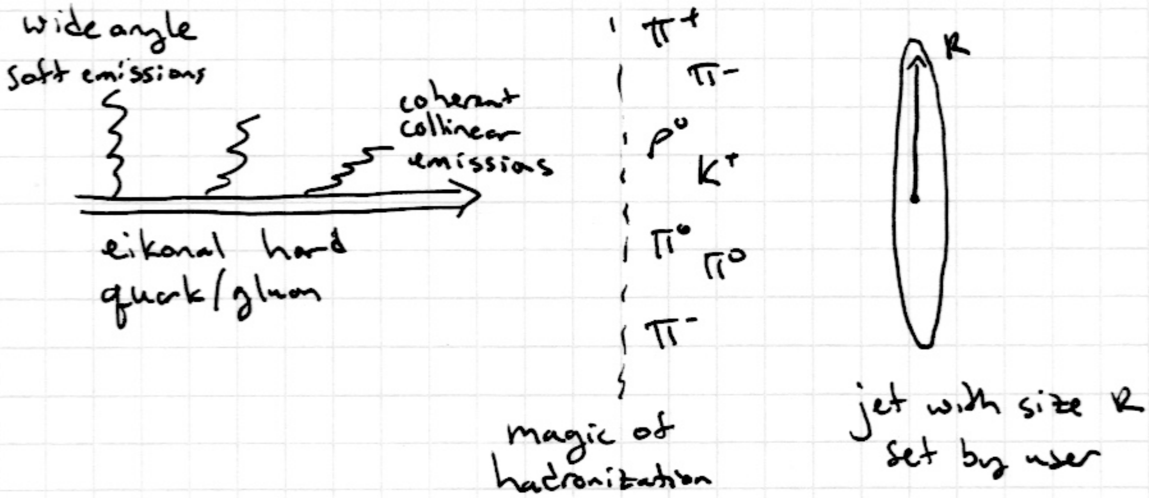
By construction this is IRC safe:

soft $\quad$ Soft limit: $\quad p_{T_i} \to 0 \qquad \tau_1$ unchanged ✓

collinear $\quad$ Collinear limit: $\quad p_{T_i} \to p_{T_i}^{(1)} + p_{T_i}^{(2)}$ and parallel $\qquad \tau_1$ unchanged ✓

Easy exercise to show that jet region is a perfect cone on $y$-$\phi$ plane that aims to minimize $p_T$ outside of jet. (Can extend to multiple jets via XCone algorithm)

With a suitable jet algorithm, we can have a reasonable cartoon of a jet:




Energy flow of partons $\approx$ Energy flow of hadrons (up to $\Lambda_{QCD}$ corrections)

FSR emission probability:

$$dP_{i\to ig} = \frac{2\alpha_s}{\pi} C_i \frac{dz}{z} \frac{d\theta}{\theta}$$

soft gluon — momentum fraction to jet — angle to jet axis

With appropriate choice of observables, substructure of jet factorizes from hard process that produced partons to initiate jet. So we have a kind of NWA for QCD radiation:

$$\sigma_{pp\to \underset{\text{property}}{\text{jet}} + \underset{\text{inclusive}}{X}} \approx \sum_{\text{partons}} \sigma_{pp\to \text{parton}+X} \times \text{“}Br\left(\text{parton} \to \underset{\text{property}}{\text{jet}}\right)\text{”}$$