Beyond
the
Standard Model

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Behind
the
Standard Model

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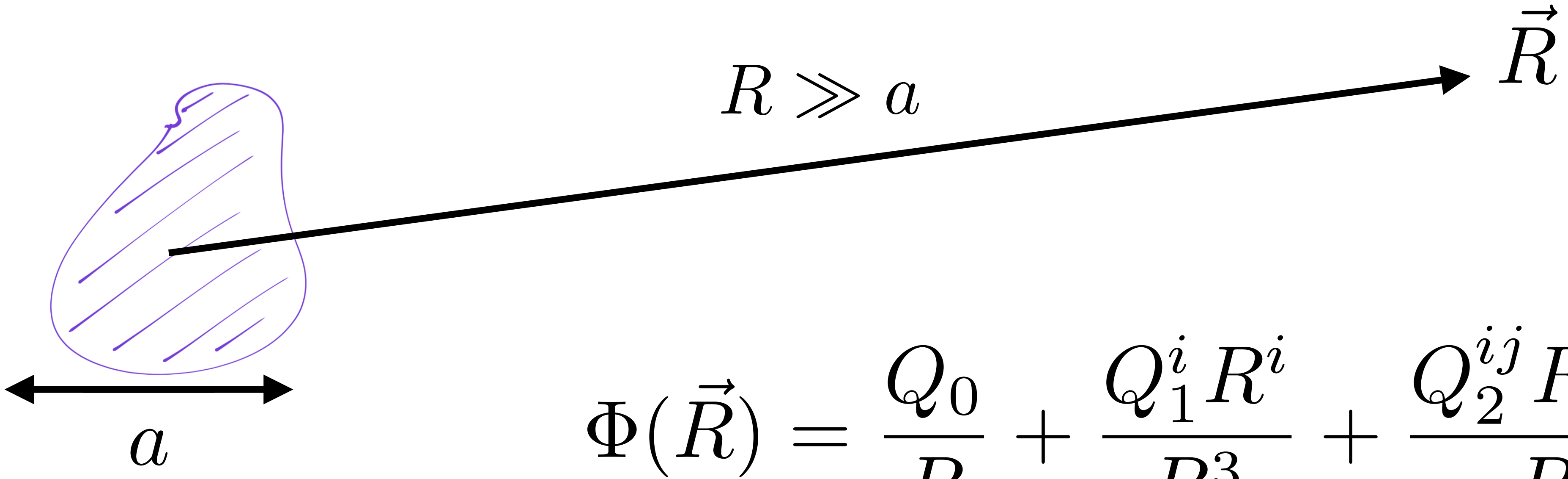
- ✦ Effective Field Theory Ideology
- ✦ The Standard Model as an EFT
- ✦ BSM and the Hierarchy Paradox

Universality and Reductionism

Of lengths and energies

$$\overbrace{E \sim \rlap{$\underbrace{\phantom{p \sim \frac{1}{\lambda}}}_{\text{quantum mechanics}}$}p}^{\text{relativity}} \sim \frac{1}{\lambda}$$

Example: electrostatic potential at large distance



$$\Phi(\vec{R}) = \frac{Q_0}{R} + \frac{Q_1^i R^i}{R^3} + \frac{Q_2^{ij} R^i R^j}{R^5} + \ldots$$

$1/R$ $\quad a/R^2 \quad$ a^2/R^3

n-multipole contribution is of relative size $\left(\frac{a}{R}\right)^n$

at fixed accuracy

$R \to$ large: fewer multipoles needed ➔ Universality

$R \to$ small: more multipoles needed ➔ Reductionism

$R \sim a$ expansion breaks down: ∞ number of parameters needed

The case of electrostatic potential is emblematic but perhaps too simple as it is indeed ...static and classical

however the same logic carries over to dynamical situations, both classically and quantum mechanically

Example: classical fluids

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$\Delta x \lesssim L_{coll}\,, \quad \Delta t \lesssim \tau_{coll}$ point particle description

$\Delta x \gg L_{coll}\,, \quad \Delta t \gg \tau_{coll}$ hydrodynamic description

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perfect fluid
leading long distance description

$$\rho\left(\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}p$$

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$$\rho \equiv \rho_0\,\bar{\rho}$$

$$p \equiv \rho_0 v_s^2\,\bar{p}$$

$$\eta \equiv \rho_0 v_s^2 \tau_{coll}\,\bar{\eta}$$

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$$\rho \equiv \rho_0\,\bar{\rho}$$

$$p \equiv \rho_0 v_s^2\,\bar{p}$$

$$\eta \equiv \rho_0 v_s^2 \tau_{coll}\,\bar{\eta}$$

$$\rho\frac{\partial v}{\partial t} \gg \eta\nabla^2 v$$

$$\Downarrow$$

$$\omega \gg \tau_{coll}\,\omega^2 \;\Rightarrow\; \boxed{\omega \ll \frac{1}{\tau_{coll}}}$$

Example: classical fluids

$\Delta x \lesssim L_{coll}\,, \quad \Delta t \lesssim \tau_{coll}$ point particle description

$\Delta x \gg L_{coll}\,, \quad \Delta t \gg \tau_{coll}$ hydrodynamic description

perfect fluid
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viscosity
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$$\eta \equiv \rho_0 v_s^2 \tau_{coll}\,\bar{\eta}$$

$$\rho\frac{\partial v}{\partial t} \gg \eta\nabla^2 v$$

$$\rho\omega \gg \eta k^2 = \eta\omega^2/v_s^2$$

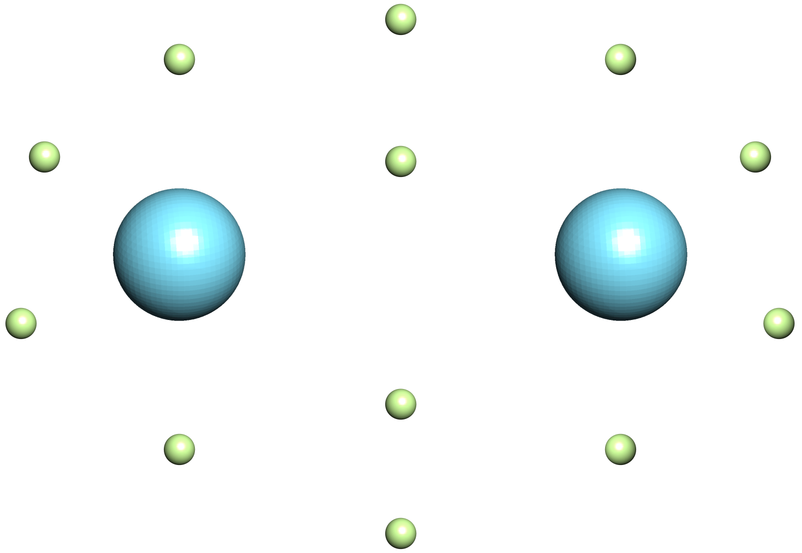
$$\omega \gg \tau_{coll}\,\omega^2$$

$$\boxed{\omega \ll \frac{1}{\tau_{coll}}}$$

Example: molecules in Born-Oppenheimer approximation

electrons move faster than nucleons

$m_e \ll m_N \qquad \omega_e \gg \omega_N$



Schroedinger eq. solved in two steps

- Find electron spectrum treating $\vec{X}_N$ as parameters
- Solve nucleon motion in resulting effective potential

$$H_{eff} = \frac{P_1^2}{2m_N} + \frac{P_2^2}{2m_N} + V_{eff}(|X_1 - X_2|, n)$$

nucleon dynamics refined in systematic expansion in $\frac{\omega_N}{\omega_e} \sim \left(\frac{m_e}{m_N}\right)^{\frac{1}{2}}$

In the path integral approach effective descriptions can be viewed as arising by ***integrating out*** the fast degrees of freedom

$$\{q\} \,\equiv\, \{q_{fast},\, q_{slow}\}$$

$$\int Dq\, e^{iS[q]} \,\equiv \int Dq_{slow}\, Dq_{fast}\, e^{iS[q_{slow}, q_{fast}]} \,= \int Dq_{slow}\, e^{iS_{eff}[q_{slow}]}$$

$$\langle\, q_{slow}(t_1)\, \ldots\, q_{slow}(t_N)\,\rangle \,=$$

$$\int Dq\, q_{slow}(t_1)\, \ldots\, q_{slow}(t_N)\, e^{iS[q]} \,= \int Dq_{slow}\, q_{slow}(t_1)\, \ldots\, q_{slow}(t_N)\, e^{iS_{eff}[q_{slow}]}$$

- Effective long distance descriptions are ubiquitous

- Their universality is the very reason we can do physics

- In fact we expect all theories of nature to appear sooner or later as just effective ones

- Notice: until a few decades ago there was the notion that Quantum Field Theory had to be fundamental not just effective. The crucial role in that view was played by renormalizable QFTs

- The modern view, however, especially after Wilson, is that QFT should also be viewed as effective, like all else.

Effective Quantum Field Theory

Any QFT is but an effective description
characterized by a short distance cut-off

$$\Lambda \equiv \frac{1}{\tau} \equiv \frac{1}{L}$$

Like in all other cases, short distance effects are
controlled by an infinite, but systematic, expansion
in powers of $L = 1/\Lambda$

Example: a theory with just one scalar field φ

Lagrangian is organized in series in inverse powers of Λ:
close analogy with multipole expansion

$$\mathcal{L} = \partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2 + \lambda_4\varphi^4 \qquad \Lambda^{\geq 0}$$

$$+ \frac{\lambda_6}{\Lambda^2}\varphi^6 + \frac{\eta_4}{\Lambda^2}\varphi^2\partial_\mu\varphi\partial^\mu\varphi \qquad \Lambda^{-2}$$

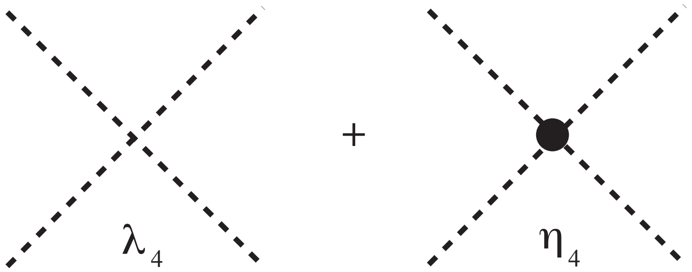
$$+ \frac{\lambda_8}{\Lambda^4}\varphi^8 + \frac{\eta_6}{\Lambda^4}(\partial_\mu\varphi\partial^\mu\varphi)^2 + \cdots \qquad \Lambda^{-4}$$

$$+ \cdots \qquad \Lambda^{\leq -4}$$

- $\lambda_4, \lambda_6, \eta_6, \ldots$ expected to be < O(1)
- must assume $m^2 \ll \Lambda^2$ otherwise no long wavelength quanta

Scattering amplitudes at $E \ll \Lambda$

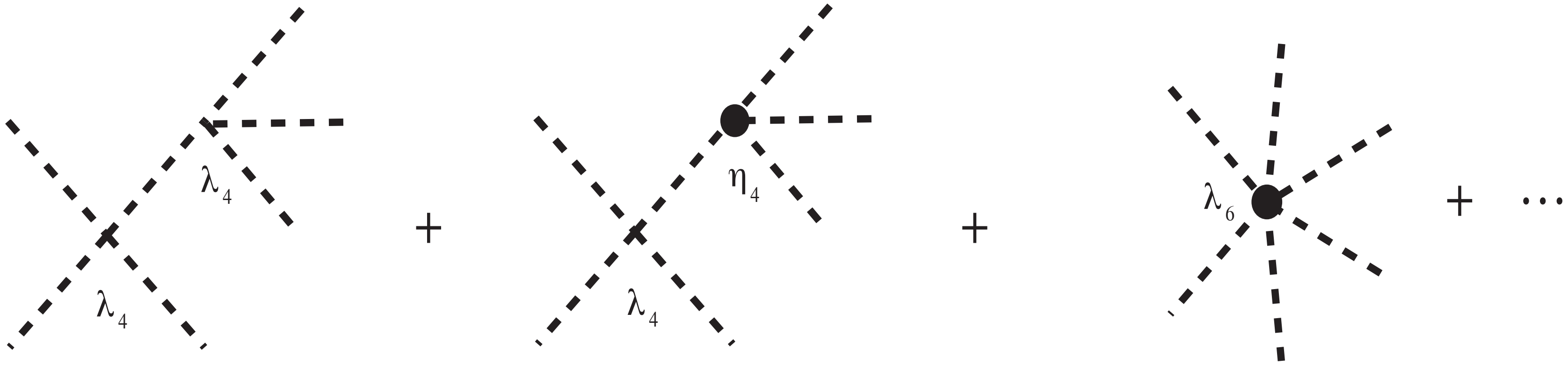
$$\mathcal{A}_{2\to 2} =$$



$$+ \dots$$

$$= \lambda_4 + \eta_4 \frac{E^2}{\Lambda^2} + \dots \xrightarrow{E \to 0} \lambda_4$$

$$\mathcal{A}_{2\to 4} =$$



$$= \frac{1}{E^2}\left\{ \lambda_4^2 + \lambda_4 \eta_4 \frac{E^2}{\Lambda^2} + \lambda_6 \frac{E^2}{\Lambda^2} + \dots \right\}$$

at low energy only lowest dimension coupling matters

the infinite set of couplings with negative mass dimension is irrelevant !

E << Λ

$\Downarrow$

$$\boxed{\mathcal{L} \;=\; \partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2 + \lambda_4\varphi^4}$$

$$\xcancel{\begin{array}{l} + \dfrac{\lambda_6}{\Lambda^2}\varphi^6 + \dfrac{\eta_4}{\Lambda^2}\varphi^2\partial_\mu\varphi\partial^\mu\varphi \\ + \dfrac{\lambda_8}{\Lambda^4}\varphi^8 + \dfrac{\eta_6}{\Lambda^4}(\partial_\mu\varphi\partial^\mu\varphi)^2 + \cdots \\ + \cdots \end{array}}$$

‘Renormalizable
Lagrangian’

Long distance physics described by renormalizable truncation **!**

The same conclusion holds for theories with gauge bosons and fermions

In general $$\mathcal{L} = \sum_i g_i \, \mathcal{O}_i \qquad [g_i] = 4 - [\mathcal{O}_i]$$

relative contribution to amplitudes $$\frac{\delta \mathcal{A}}{\mathcal{A}} \sim g_i \, E^{[\mathcal{O}_i]-4}$$

Ex: scalar mass term $m^2\varphi^2$

$[\varphi^2] = 2$ $$\frac{\delta \mathcal{A}}{\mathcal{A}} \sim \frac{m^2}{E^2}$$

$[g_i] > 0$ relevant at small E

$[g_i] = 0$ relevant at all E

renormalizable

$[g_i] < 0$ irrelevant at small E

non-renormalizable

- The same conclusion holds also when considering loop diagrams, (but proof is more technical)

Weinberg 1980
Polchinski 1984

- Loops simply add an additional mild log E dependence

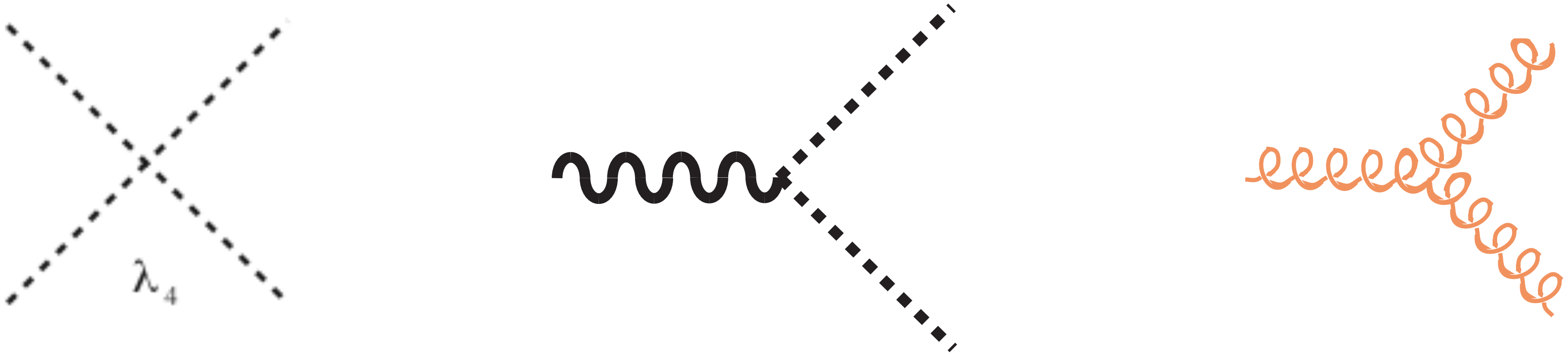
$$\prod_i^N g_i \, E^{[\mathcal{O}_i]-4} \longrightarrow \prod_i^N g_i \, E^{[\mathcal{O}_i]-4} \left(1 + a_1 \ln E/\mu + a_2 (\ln E/\mu)^2 + \dots\right)$$

In conclusion, at sufficiently low energy

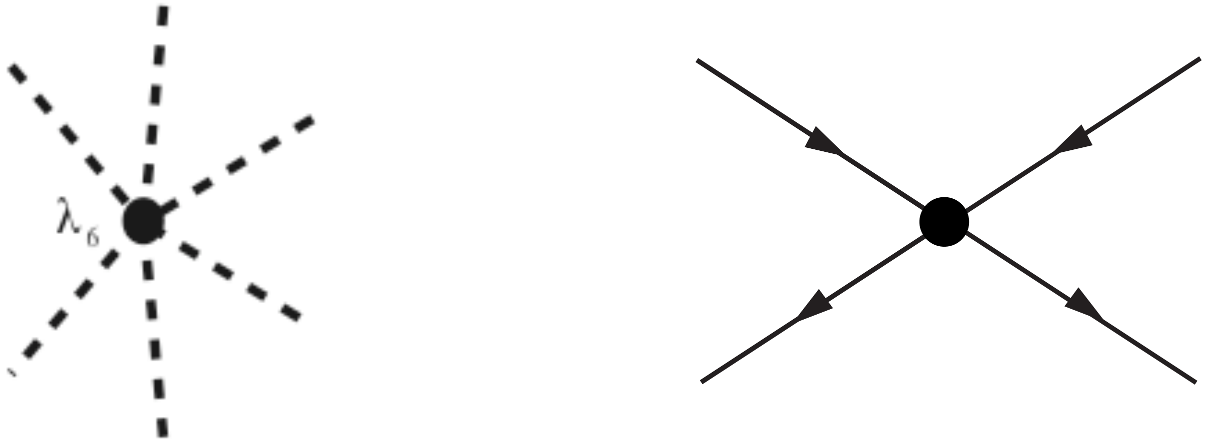
- any quantum field theory is well described by a renormalizable lagrangian

- effects from all the other (infinitely many) couplings are suppressed by powers of E/Λ

the ‘renormalizable’ terms (dimension 4 or less)
fully describe elementary (pointlike) particles



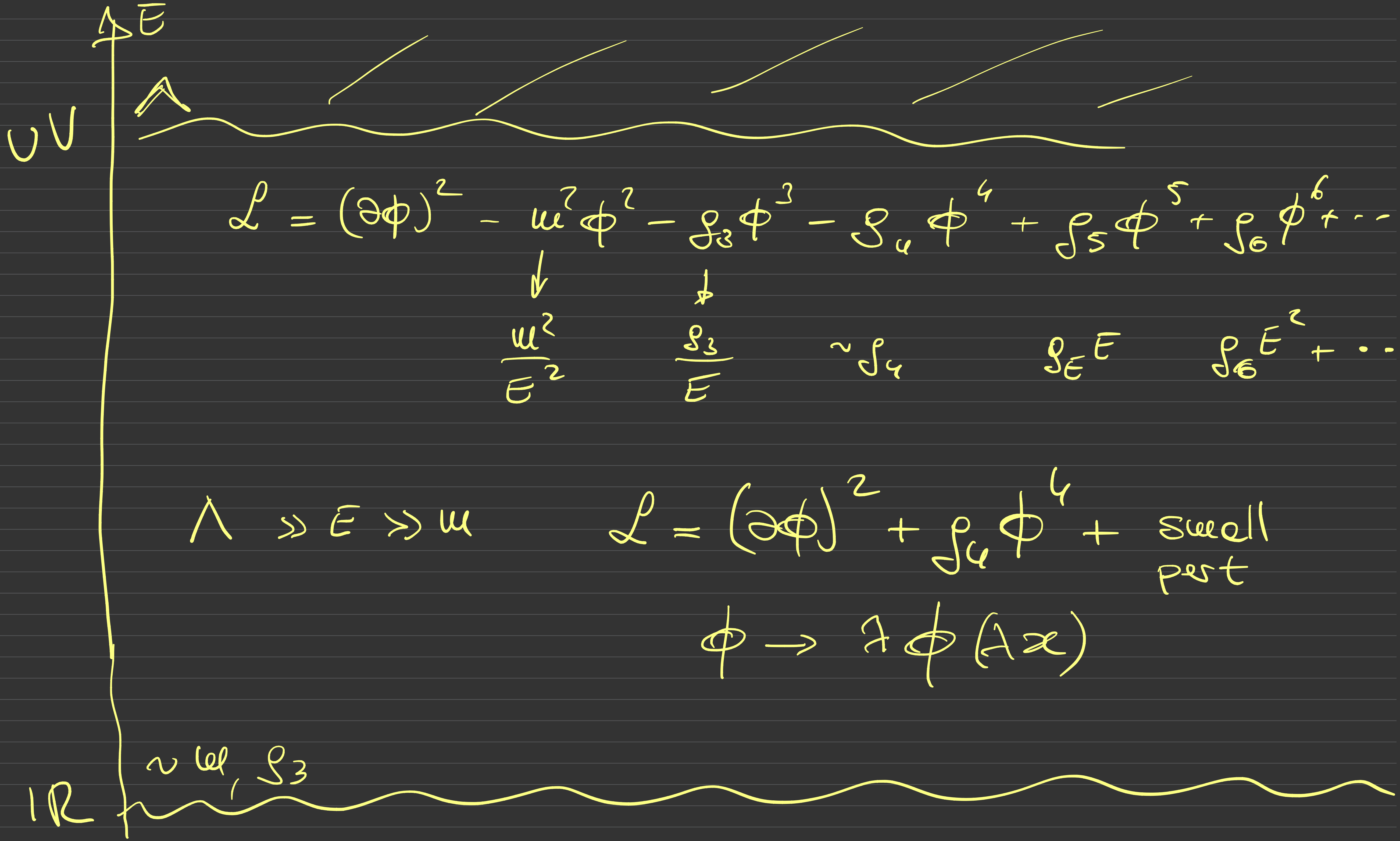
‘non-renormalizable’ terms (dimension 5 or more)
describe inner structure of particles



$$E \sim \Lambda$$

needed to directly probe structure

wawelengths $\sim \dfrac{1}{\Lambda}$



$$\mathcal{L} = (\partial\phi)^2 - m^2\phi^2 - g_3\phi^3 - g_4\phi^4 + g_5\phi^5 + g_6\phi^6 + \cdots$$

$$\downarrow \qquad \downarrow$$

$$\frac{m^2}{E^2} \qquad \frac{g_3}{E} \qquad \sim g_4 \qquad g_5 E \qquad g_6 E^2 + \cdots$$

$$\Lambda \gg E \gg m \qquad \mathcal{L} = (\partial\phi)^2 + g_4\phi^4 + \text{small pert}$$

$$\phi \to \lambda\phi(Ax)$$

u, d, s

Ex QCD

$\Lambda = m_W$

$$\mathcal{L} = \mathrm{Tr}\left(\underbrace{\partial_\mu G_\nu - \partial_\nu G_\mu}_{} + i g_s [G_\mu, G_\nu]\right)^2 + i\bar{q}\gamma^\mu(\partial_\mu - i g_s G_\mu) q$$

$$+ \cdots$$

$$\simeq G_{\mu\nu} G^{\mu\nu} + i\bar{q}\not{\partial} q + g_s(\qquad)$$

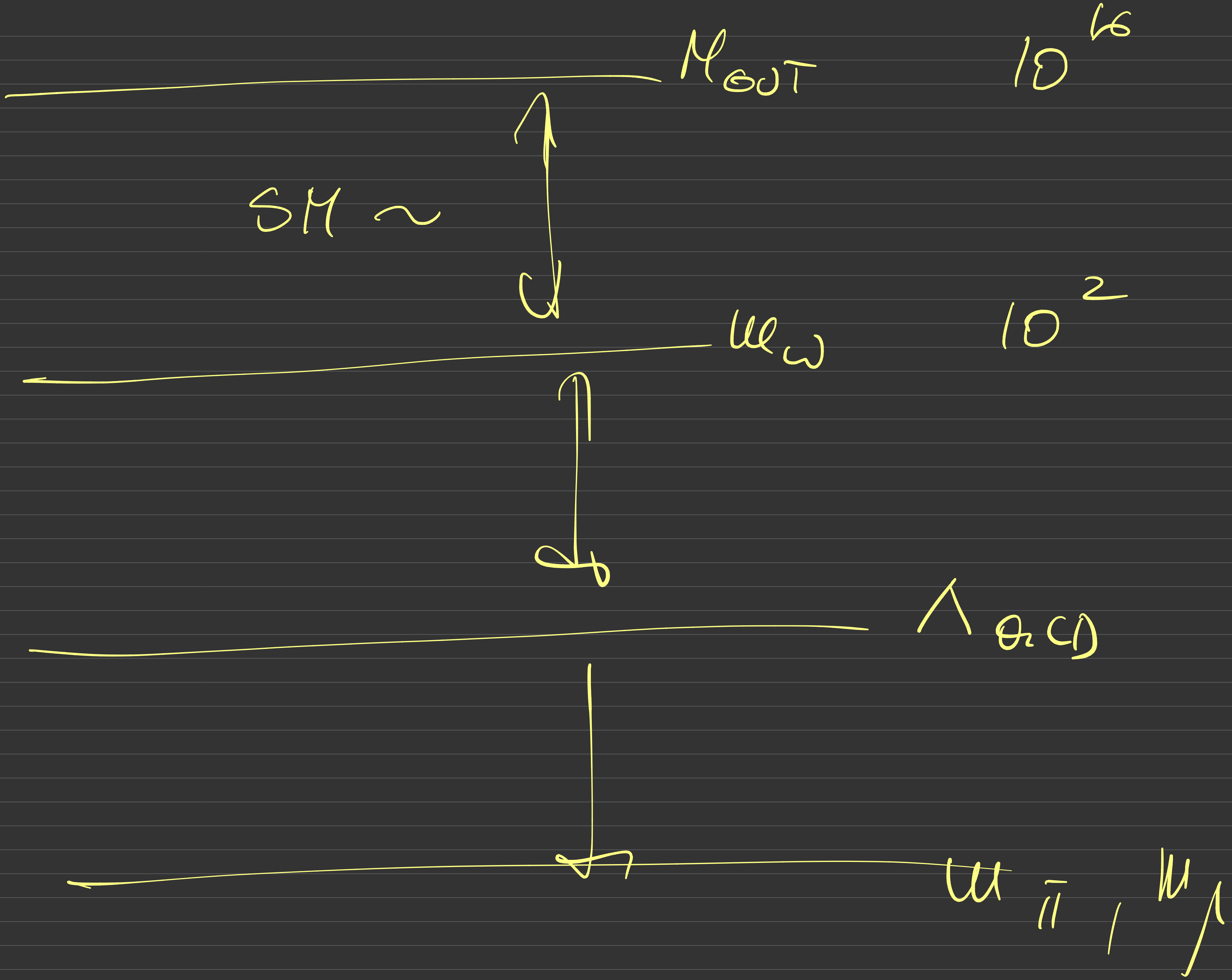
$$\bar{g}_s(E) \sim 4\pi$$

Λ_{QCD}

$$\downarrow$$

$$\mathcal{L} = (\partial\pi)^2 + \frac{1}{\Lambda_{QCD}^2}(\pi\partial\pi)^2 + \cdots$$

m_π


M_{GUT}
10^{16}
SM ~
m_{EW}
10^{2}
Λ_{QCD}
m_{π}, m_{ν}

Conformal Field Theories $\equiv$ CFT

Picture

Λ UV

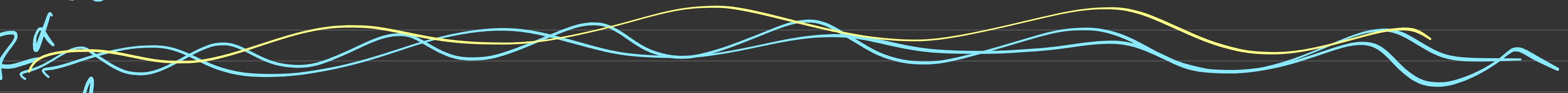
$$S = S_{CFT} + \boxed{\sum_{\substack{\text{relevant}\\ \alpha}} g_{R\alpha} O_{R\alpha}} + \cancel{\sum_{\substack{\text{irrelevant}\\ a}} g_{Ia} O_{Ia}}$$

$\Delta_{R\alpha} < 4$ $\Delta_{Ia} > 4$

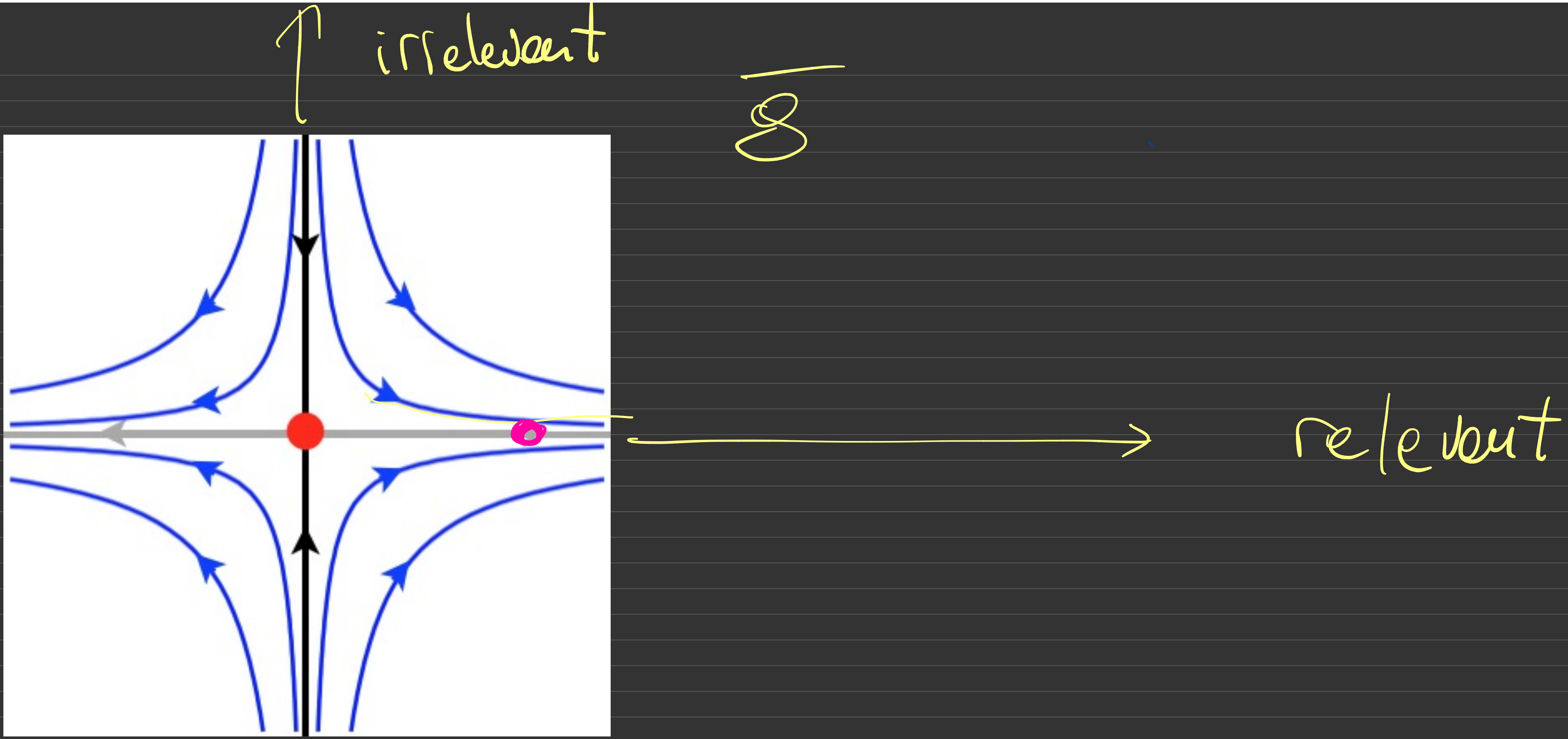
$$\bar g_{R\alpha} = g_{R\alpha} E^{\Delta_{R\alpha}-4}$$

↓ $E^{\Delta_{Ia}-4}$

$\bar g_{R\alpha} \sim O(1)$



↓


irrelevant
$\bar{g}$
relevant

The crucial role of symmetries

- mostly concerned with global symmetries (possibly approximate)

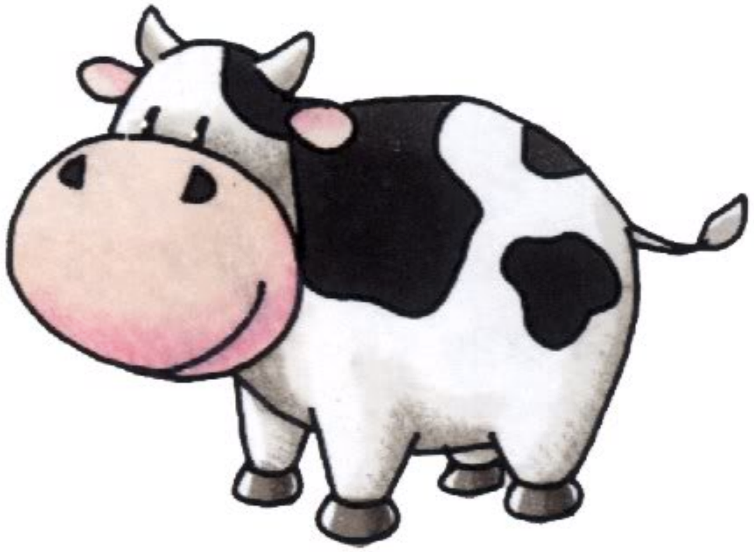
- local (gauge) symmetries are another story: these correspond to redundancies in the parametrization of the field variables; their main role is to reduce the number of physical degrees of freedom (Ex. photon field has 4 components, but carries only 2 physical polarizations)

Accidental Symmetries

- The IR relevance of just a finite number of parameters implies a great structural simplification

- this often entails the effective occurrence in the long distance dynamics of additional, accidental, symmetries

Long Distance Physics: Simplicity & Accidental Symmetries



Long Distance Physics: Simplicity & Accidental Symmetries

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accidental

-

$SO(3)$

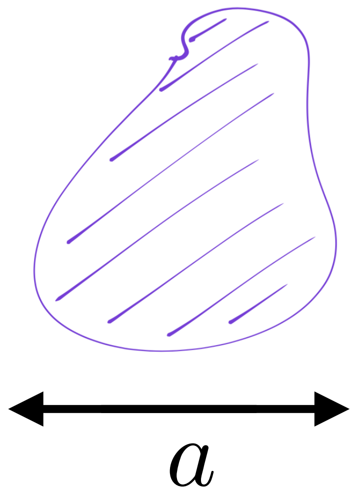
Long Distance Physics: Simplicity & Accidental Symmetries

accidental

•

$SO(3)$

Ex.: electrostatic potential at large distance



$$\xrightarrow{R \gg a}$$

$$1/R \qquad a/R^2 \qquad a^2/R^3$$

$$\Phi(R) = \frac{Q_0}{R} + \frac{\vec{Q}_1 \cdot \vec{R}}{R^3} + \frac{Q_2^{ij} R_i R_j}{R^5} + \ldots$$

$$SO(3) \supset SO(2) \supset \varnothing$$

Accidental symmetries of renormalizable Lagrangians

Ex: parity in QED

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^\mu D_\mu\psi + \bar{\psi}(m_1 + i\gamma_5 m_2)\psi + \frac{a}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$(m_1 + i\gamma_5 m_2) \quad \to \quad m = \sqrt{m_1^2 + m_2^2}$ by chiral rotation $\psi \quad \longrightarrow \quad e^{i\beta\gamma_5}\psi$

$F_{\mu\nu}\tilde{F}^{\mu\nu}$ = total derivative

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$F_{\mu\nu}\tilde{F}^{\mu\nu}$ = total derivative

dim 6 operator
violates parity

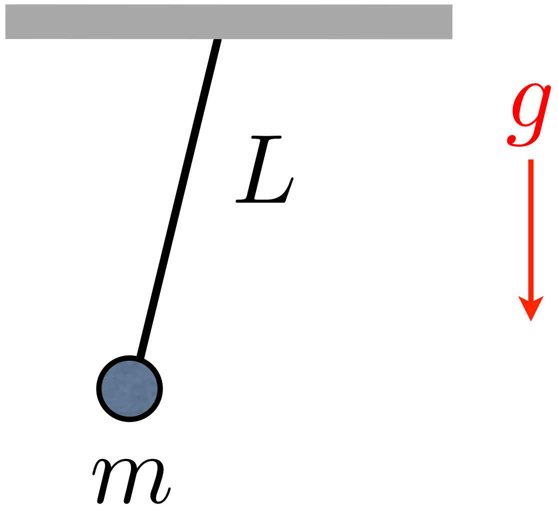
$$O_{\not P} = \frac{1}{\Lambda^2}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma_{\mu}\gamma_5\psi)$$

Symmetries, dimensional analysis and selection rules

it is always possible to imagine *symmetry* tranformations of the parameters describing a physical system

the dependence of physical observables on such parameters is dictated by covariance under such *symmetries*

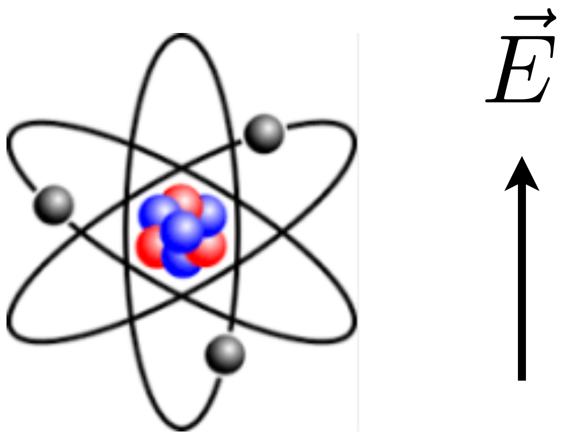
Ex.: classical pendulum



$$\left[\begin{array}{l} D_x : \vec{x} \to \lambda_x \vec{x} \\ D_t : t \to \lambda_t t \\ \\ L \to \lambda_x L \\ g \to \lambda_x \lambda_t^{-2} g \\ m \to m \end{array}\right.$$

$$\omega = \# \sqrt{\frac{g}{L}}$$

Ex: atom in external electric field

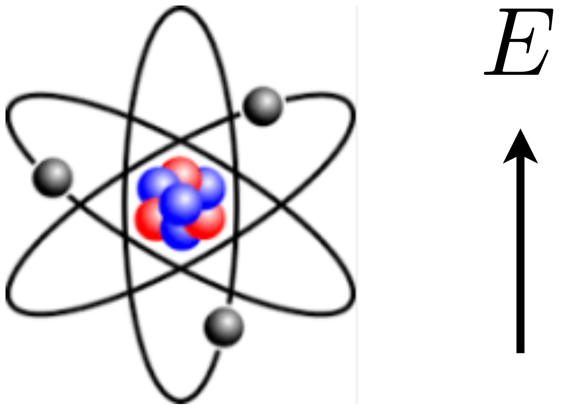


$$|\Psi_0\rangle \xrightarrow{\text{slowly turn on } \vec{E}} |\Psi_0(\vec{E})\rangle$$

electric
dipole

$$\langle\Psi_0(\vec{E})|d_j|\Psi_0(\vec{E})\rangle \;=\; ?$$

Ex: atom in external electric field



$$|\Psi_0\rangle \xrightarrow{\text{slowly turn on } \vec{E}} |\Psi_0(\vec{E})\rangle$$

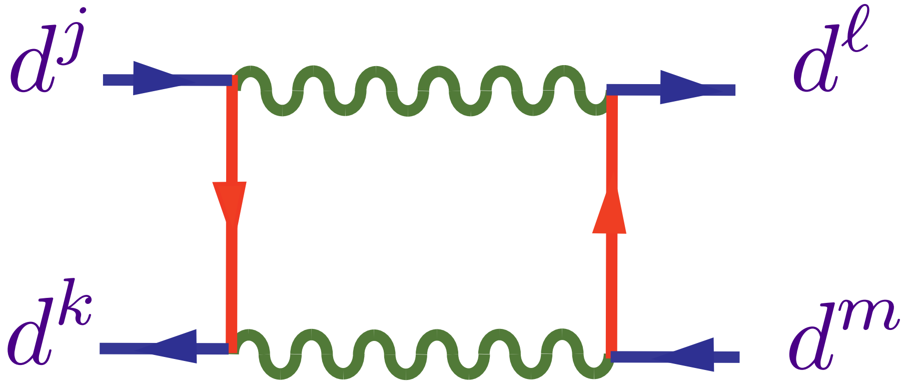
electric
dipole

$$\langle\Psi_0(\vec{E})|d_j|\Psi_0(\vec{E})\rangle \overset{O(3)}{=} E_j\, f(|\vec{E}|)$$

Ex: Flavor Changing Neutral Currents & CKM matrix

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \left(\bar{u}^i_L \gamma^\mu d^j_L \, W^+_\mu \, V^{ij} + \text{h.c.} \right)$$

$$\text{flavor ‘symmetry’} \quad \left[\begin{array}{lcl} u^j_L & \rightarrow & e^{i\theta^j_u} \, u^j_L \\ d^k_L & \rightarrow & e^{i\theta^k_d} \, d^k_L \\ V^{jk} & \rightarrow & e^{i(\theta^j_u - \theta^k_d)} \, V^{jk} \end{array} \right.$$



$$\propto \; V^{ik*} \, V^{ij} \, V^{nm*} \, V^{n\ell}$$

In general, given couplings $\{\lambda_a\}$, an observable $\mathcal{O}$ is given by

$$\langle \mathcal{O} \rangle = \sum_a \langle \mathcal{O} \rangle_a \qquad\qquad \langle \mathcal{O} \rangle_a = c_a \, \underbrace{\lambda_1^{n_{1a}} \ldots \lambda_N^{n_{Na}}}_{\text{symm. \& dim.}}$$

$O(1)$ coeff.

If $\quad |\langle \mathcal{O} \rangle_{exp}| \ll \max |\langle \mathcal{O} \rangle_a| \quad$ it seems we are missing something

✦ we overlooked a ***symmetry*** implying cancellations among the $\langle \mathcal{O} \rangle_a$

✦ there is a ***fine-tuning*** of parameters not related to symmetry

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If $\quad |\langle \mathcal{O} \rangle_{exp}| \ll \max |\langle \mathcal{O} \rangle_a| \quad$ it seems we are missing something

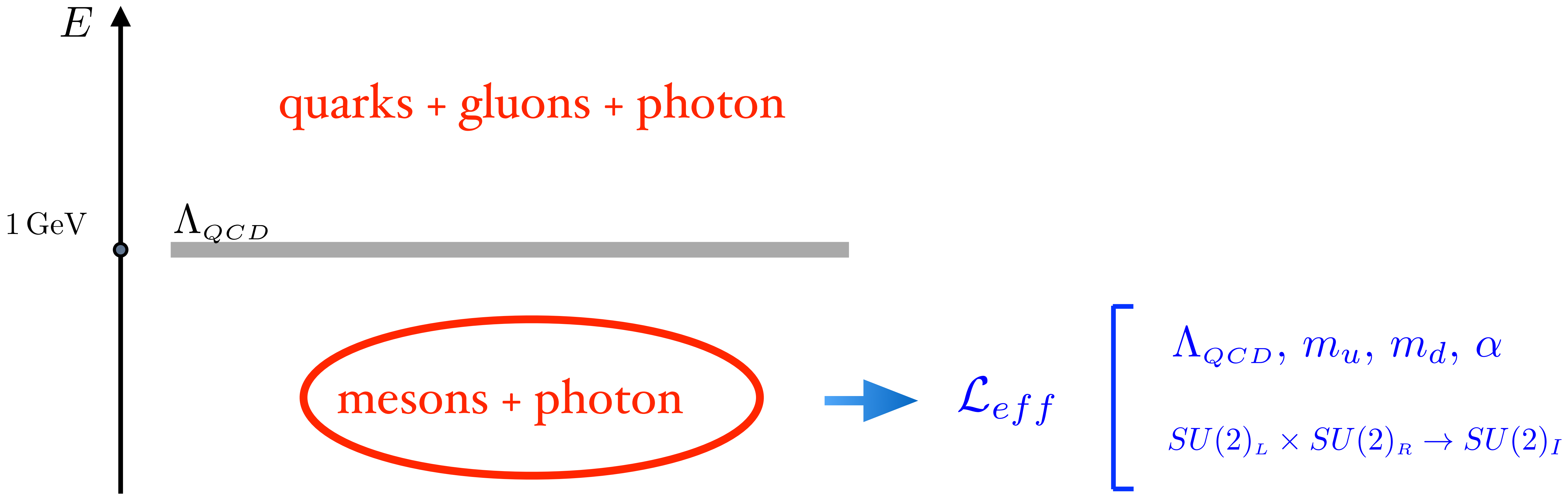
natural

✦ we overlooked a ***symmetry*** implying cancellations among the $\langle \mathcal{O} \rangle_a$

✦ there is a ***fine-tuning*** of parameters not related to symmetry

un-natural

Example of Naturalness: pion mass



quarks + gluons + photon

mesons + photon

$\mathcal{L}_{eff}$

$\Lambda_{QCD},\ m_u,\ m_d,\ \alpha$

$SU(2)_L \times SU(2)_R \to SU(2)_I$

- $U \equiv e^{i\hat{\pi}} \to V_L U V_R^\dagger = e^{i\hat{\pi}'} \qquad \hat{\pi} = \pi_a \sigma_a$

- $M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \to V_R M_q V_L^\dagger$

- $D_\mu U = \partial_\mu U - ieA_\mu(Q_L U + U Q_R) \qquad Q_L = -Q_R = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}$

$eQ_L \to V_L e Q_L V_L^\dagger \qquad eQ_R \to V_R e Q_R V_R^\dagger$

- dilations: match mass dimensions

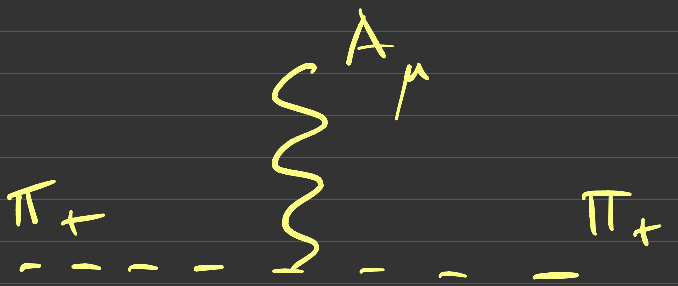
$$\mathcal{L}_{eff} \;=\; c_1 \frac{\Lambda^2_{QCD}}{16\pi^2} \Big\{ \mathrm{Tr}(\partial U \partial U^\dagger) \;+\; 2c_2\, \Lambda_{QCD} \mathrm{Tr}(M_q U + \mathrm{h.c.})$$

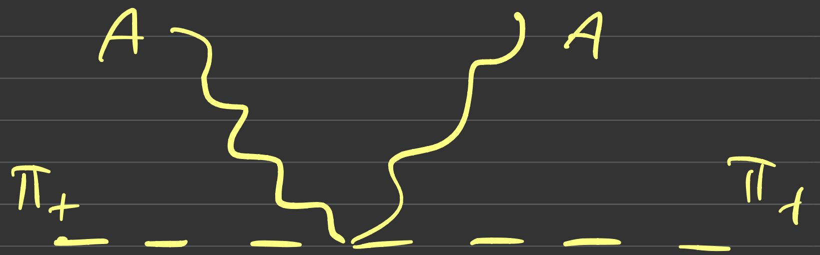
$$- \, c_3\, \Lambda_{QCD} \mathrm{Tr}\left[(M_q U)^2 + \mathrm{h.c.} \right] \;-\; 2c_4\, \Lambda^2_{QCD} \frac{e^2}{16\pi^2}\, \mathrm{Tr}(Q_L U Q_R U^\dagger) + \ldots \Big\}$$

leading $$m^2_{\pi^+} = m^2_{\pi^0} \;=\; c_2\, \Lambda_{QCD}(m_u + m_d)$$

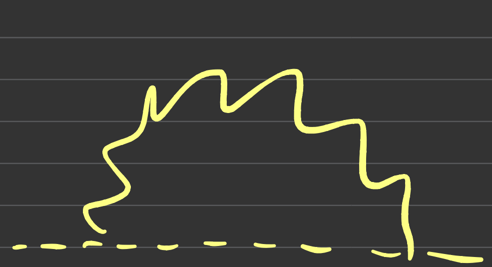
sub-leading $$m^2_{\pi^+} - m^2_{\pi^0} \;=\; c_3\,(m_u - m_d)^2 \;+\; c_4\, \frac{e^2}{16\pi^2}\, \Lambda^2_{QCD}$$

- $m^2_{\pi^+} - m^2_{\pi_0}$ correctly estimated by <u>naive</u> 1-loop computation within EFT





none for π_0



$+$

$$\Longrightarrow \delta m^2_{\pi^+}$$

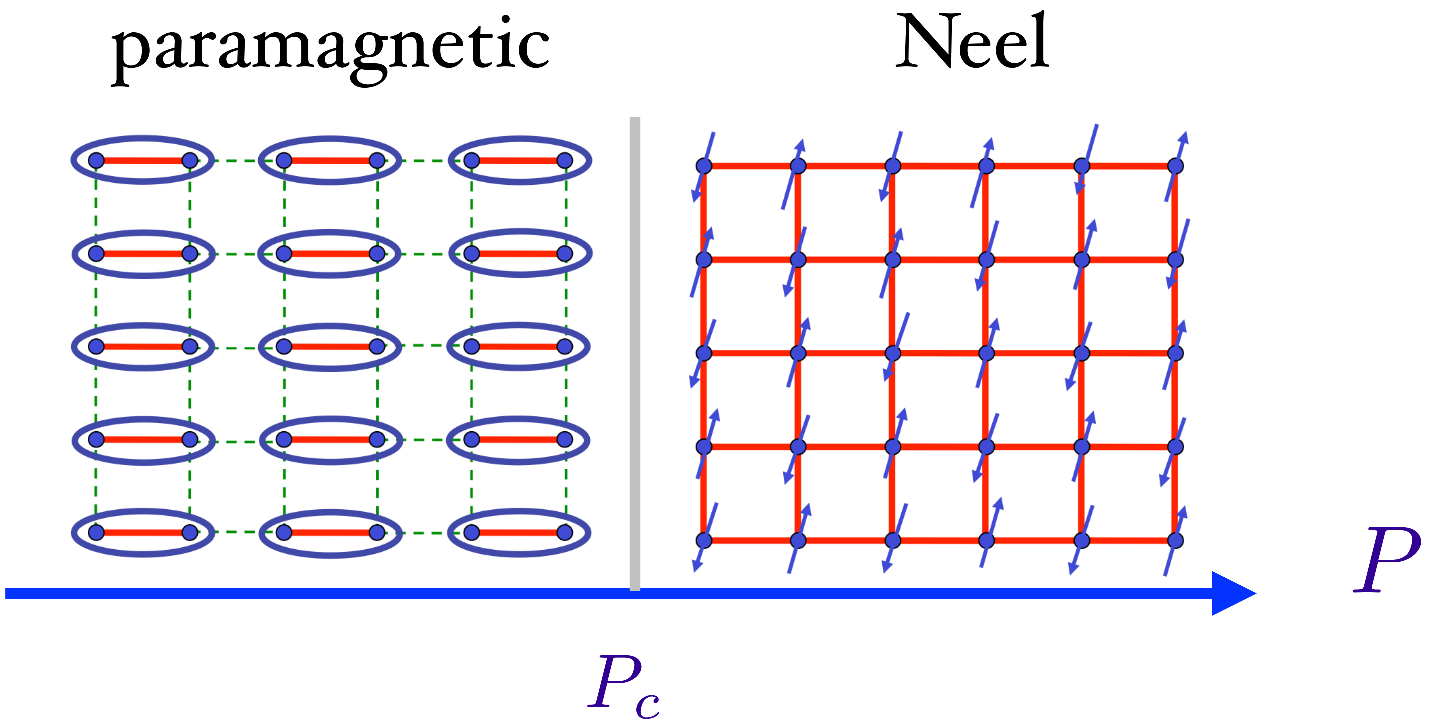
$$\simeq e^2 \int \frac{d^4p}{16\pi^4} \frac{1}{p^2} \sim \frac{e^2}{16\pi^2} \int d^2p \sim \frac{e^2}{16\pi^2} \Lambda^2_{QCD}$$

In the normal (lab) practice extreme fine tunings are associated to the existence of a large set of options (a landscape) from which we can pick

Ex. quantum criticality in anti-ferromagnet (TlCuCl3)

paramagnetic

Sachdev ’09



$$\vec{\varphi} \equiv (-1)^{x+y} \vec{S}(x,y)$$

$$V(\vec{\varphi}) = m^2(P)\vec{\varphi}\cdot\vec{\varphi} + \lambda(\vec{\varphi}\cdot\vec{\varphi})^2 \qquad m^2(P) = m_0^2\left(1 - \frac{P}{P_c}\right)$$

Can undo ***natural*** expectation from atomic physics by ***tuning*** the pressure at a ***critical*** value in a ***landscape*** of possibilities