The Naturalness Criterion

't Hooft 1979

- Under what condition is a QFT natural?

G_a

Λ

g_i

$$\Longrightarrow \quad \underbrace{g_i \equiv g_i(G_a)}$$

covariant $\equiv$ respects selection rules

- Imagine $\quad [g_{i_1}] = [F_{i_1}(g_i)] = [F_{i_1}(g_i(G_a))]$

$g_i \neq g_{i_1}$

given the g_i's this offers a way to assess the naturalness of any pattern of this set $\{g_i\}$

1975

$\Rightarrow$ original 't Hooft formulation

a coupling g is naturally small if $g \to 0$ implies the presence of an additional symmetry

Bispinors

$$\bar\sigma\cdot\partial = \bar\sigma^\mu\partial_\mu \qquad \bar\sigma^\mu = (\mathbb{1}, -\sigma^i)$$
$$\sigma\cdot\partial = \sigma^\mu\partial_\mu \qquad \sigma^\mu = (\mathbb{1}, \sigma^i)$$

- **Dirac** $\quad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$$\mathcal{L} = i\psi_L^\dagger i\bar\sigma\cdot\partial\psi_L + i\psi_R^\dagger\sigma\cdot\partial\psi_R - m(\psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L)$$

- **Weyl** $\quad \psi_L$

$$\mathcal{L} = i\psi_L^\dagger\bar\sigma\cdot\partial\psi_L$$

$$\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

- **Majorana**

$$\mathcal{L} = i\psi_L^\dagger\bar\sigma\cdot\partial\psi - m(\psi^T\varepsilon\psi + h.c.)$$

more compactly

$$= i\bar\psi\bar\partial\psi - m(\psi\psi + \bar\psi\bar\psi)$$

- neutrino mass in SM

- bi-spinor notation

$$\ell_a^T \varepsilon \ell_b \, H^a H^b$$

- 4-spinor notation

$$L_a \equiv \begin{pmatrix} \ell_a \\ 0 \end{pmatrix} \qquad C = \begin{pmatrix} \varepsilon & \\ & -\varepsilon \end{pmatrix}$$

$$\ell_a^T \varepsilon \ell_b = L_a^T C L_b$$

Ex 1

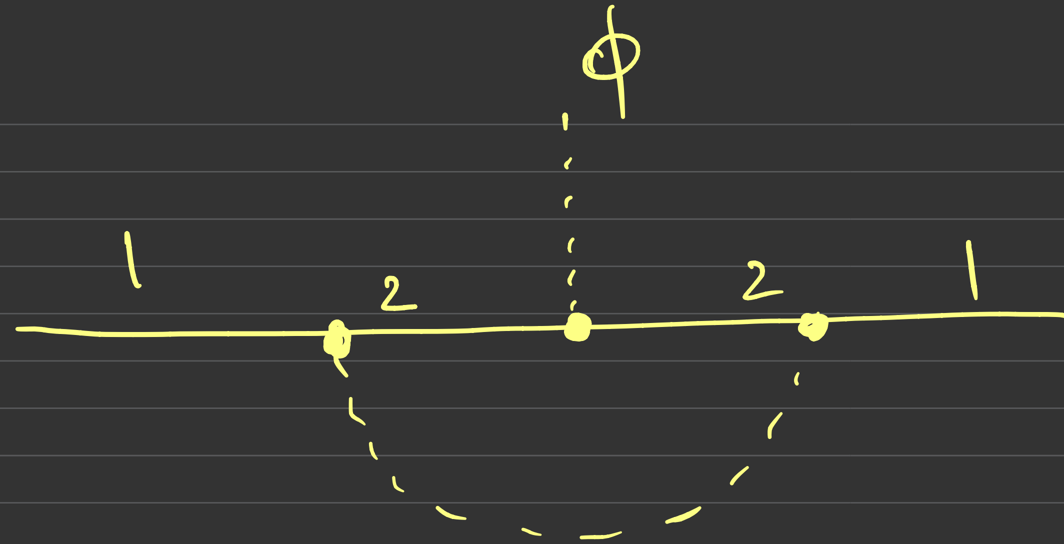
$$\mathcal{L} = (\partial\phi)^2 + i\bar{\nu}_1\bar{\partial}\nu_1 + i\bar{\nu}_2\bar{\partial}\nu_2 + \phi\left(g_{11}\nu_1\nu_1 + g_{12}\nu_1\nu_2 + g_{22}\nu_2\nu_2\right.$$
$$\left. + \ldots\right)$$

- given g_{12} and g_{22}, how small g_{11} naturalness permits?

$U(1)_1 \times U(1)_2 \Longrightarrow$

	ν_1	ν_2	g_{11}	g_{12}	g_{22}
$U(1)_1$	1	0	-2	-1	0
$U(1)_2$	0	1	0	-1	-2

$$\Longrightarrow \quad [g_{11}] = [g_{12}^2\, g_{22}^*]$$



O(1)

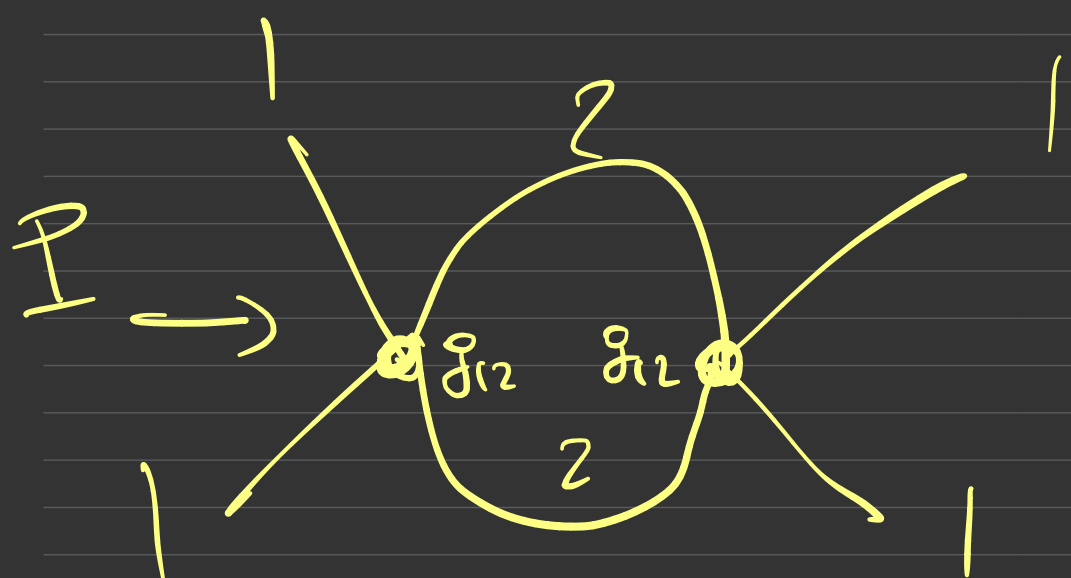
$$\Rightarrow \quad \delta g_{11} = c \, \frac{g_{12}^2 \, g_{22}^*}{16\pi^2} \ln \frac{\Lambda}{p}$$

$$g_{11}(\mu) = g_{11}(\Lambda) + c \, \frac{g_{12}^2 \, g_{22}^*}{16\pi^2} \ln \Lambda/\mu$$

$$|g_{11}| \gtrsim \frac{|g_{12}^2 \, g_{22}|}{16\pi^2} \log \frac{\Lambda}{\mu}$$

Ex. 2 $\mathcal{L} = (\partial\phi_1)^2 + (\partial\phi_2)^2 - g_{11}\phi_1^4 - g_{12}\phi_1^2\phi_2^2 - g_{22}\phi_2^4$

• given g_{12}, g_{22} what about g_{11}?



$$\sim i\,\frac{g_{12}^2}{16\pi^2}\ln\frac{\Lambda}{P}$$

$$|g_{11}(\mu)| \gtrsim \frac{g_{12}^2}{16\pi^2}\ln\frac{\Lambda}{\mu}$$

Symmetries of free field theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 \longrightarrow H = \int \frac{d^3k}{(2\pi)^3} \, \omega_k \, a_k^\dagger a_k$$

$$\sqrt{m^2 + \underline{k}^2}$$

- $Q_k \equiv a_k^\dagger a_k$

$$S = \int d^4p \, \frac{1}{2} \hat{\phi}(-p)(p^2 - m^2)\hat{\phi}(p)$$

$$\hat{\phi}(-p) = \hat{\phi}(p)^*$$

$$\Downarrow$$

$$\hat{\phi}(p) \longrightarrow e^{i\theta(p)} \phi(p) \Rightarrow \theta(-p) = -\theta(p)$$

- $\theta = a_\mu P^\mu + a_{\mu\nu\rho} P^\mu P^\nu P^\rho + \dots$

- $\phi \longrightarrow \left(1 + a_\mu \partial^\mu - a_{\mu\nu\rho} \partial^\mu \partial^\nu \partial^\rho + \dots\right) \phi$

- Noether $\quad T_\mu{}^\lambda \quad T_{\mu\nu\rho}{}^\lambda \quad \dots$

  $\partial_\lambda T_\mu{}^\lambda = 0$

- Conservation $\quad P_\mu \quad P_\mu P_\nu P_\rho \quad \dots$

$\downarrow$

forbids interaction

see Coleman Mandula

- More directly

$$\Gamma(\phi) = \sum_n \int \delta^4(p_1 + \cdots + p_n)\, \Gamma^{(n)}(p_1, \dots, p_n)\, \hat{\phi}(p_1) \cdots \hat{\phi}(p_n)$$

- $\hat{\phi}(p) \to e^{i\theta(p)} \hat{\phi}(p)$ $\quad \theta(p) = -\theta(-p)$ $\quad$ is symmetry

$$\Downarrow$$

$$\boxed{\Gamma^{(n)} = 0 \quad \text{for} \quad n > 2}$$

- $n=2$ $\quad p_1 = -p_2$ $\quad \Gamma(-p_2, p_2)\, e^{i\theta(-p_2) + i\theta(p_2)} = \Gamma(-p_2, p_2)$

- interaction break the symmetry
- couplings $\equiv$ spurions

Ex $\int d^4x \, \lambda \phi^4 \Longrightarrow \int d^4p_1 \cdots d^4p_4 \; \lambda(p_1, \cdots p_4)\, \tilde{\phi}(p_1) \cdots \tilde{\phi}(p_4)$

reality $\quad \lambda(p_1 \cdots p_4) \longrightarrow \lambda \, \delta^4(p_1+p_2+p_3+p_4)$

- formally $\begin{cases} \lambda(p_1 \cdots p_4) \to \lambda(p_1 \cdots p_4)\, e^{-i\theta(p_1) \cdots - i\theta(p_4)} \\ \hat{\phi}(p) \longrightarrow \phi(p)\, e^{i\theta(p)} \end{cases}$

is a symmetry

In our case $\quad g_{11}\phi_1^4 + g_{12}\phi_1^2\phi_2^2 + g_{22}\phi_2^4$

$$g_{12}(p_1,p_2,p_3,p_4) \longrightarrow g_{12}(p_1,p_2,p_3,p_4)\, e^{i\theta_1(p_1)+i\theta_1(p_2)+i\theta_2(p_3)+i\theta_2(p_4)}$$

$$g_{11}(p_1 \cdots p_4) \longrightarrow g_{11}(\cdots)\, e^{-i\theta_1(p_1)-i\theta_1(p_2)-i\theta_1(p_3)-i\theta_1(p_4)}$$

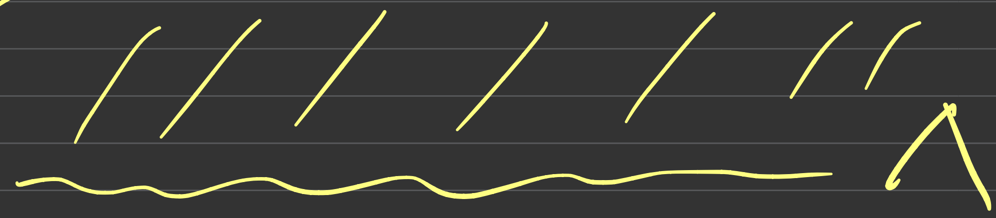
$$[g_{11}(p_1,p_2,p_3,p_4)] = [g_{12}(p_1,p_2,p_5,p_6)\, g_{12}(p_3,p_4,-p_5,-p_6)]$$

$=0$

expect $\quad g_{11} \sim g_{12}^2 \quad$ consistent with 1-loop effects

Ex: mass in $\lambda\phi^4$

$$\mathcal{L} = (\partial\phi)^2 + m^2\phi^2 + g\phi^4$$



• given (g, Λ) what do selection rules tell us on m^2?

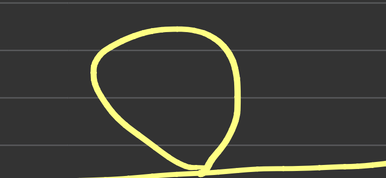
$$\left.\begin{aligned} m^2 &\to m^2(p_1, p_2) \\ g &\to g(p_1, p_2, p_3, p_4) \end{aligned}\right\} \text{high spin symm}$$

$$[m^2(p_1, p_2)] = [g\cdot(p_1, p_2, k, -k)\,\Lambda^2]$$

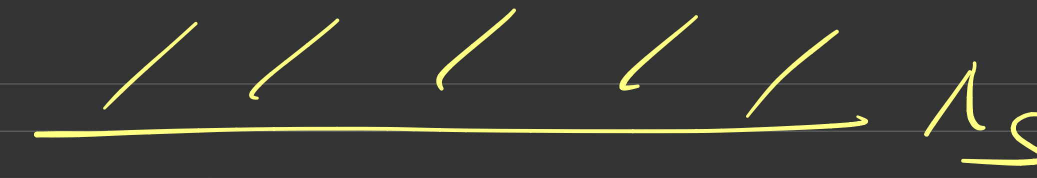
$\Longrightarrow$ expect

$$m^2 \sim g\,\Lambda^2$$

$$\left.\begin{aligned} x &\to \lambda x \\ m^2 &\to \lambda^{-2} m^2 \\ \Lambda^2 &\to \lambda^{-2} \Lambda^2 \end{aligned}\right\} \text{dilations}$$

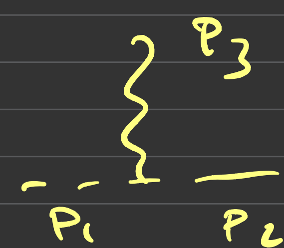
indeed  $\sim \frac{g}{16\pi^2}\int dp^2$

• similar to $m^2_{\pi^+}$



$$\mathcal{L} = |\partial_\mu + i e A_\mu \pi_+|^2$$

$$e \equiv e(p_1, p_2, p_3)$$



$$\left[m^2(p_1, p_2)\right] = \left[e(p_1, p, k)\, e(p_2, -p, -k)\, \Lambda_s^2\right]$$

$$\Rightarrow \quad m^2_{\pi_+} \sim \frac{e^2}{16\pi^2} \cdot \Lambda_S^2$$

is as good an estimate as

$$\omega_{\text{pendulum}} \sim \sqrt{\frac{g}{L}}$$