The Standard Model as an Effective Field Theory

SM is defined by its gauge symmetry and its field content

gauge fields	$SU(3) \times SU(2) \times U(1)_Y \times \text{diffs}$
Higgs field	$H = (\mathbf{1}, \mathbf{2}, 1)$
spinors	$q_L\,,\quad u_R\,,\quad d_R\,,\quad \ell_L\,,\quad e_R \quad \times \quad 3\ \text{families}$ $(\mathbf{3},\mathbf{2},1/3)\quad(\mathbf{3},\mathbf{1},4/3)\quad(\mathbf{3},\mathbf{1},-2/3)\quad(\mathbf{1},\mathbf{2},-1)\quad(\mathbf{1},\mathbf{1},-2)$

Write most general EFT lagrangian as expansion in inverse powers of microphysics cut-off $\Lambda_{UV} = 1/a$

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{kin} + g A_\mu \bar{F} \gamma_\mu F + Y_{ij} \bar{F}_i H F_j + \lambda (H^\dagger H)^2$$

d=4

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + gA_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda(H^\dagger H)^2$$

d=4

$$+ \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$

$$+ \frac{c_{ijkl}}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell + \frac{c_{ij}}{\Lambda_{UV}} \bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots$$

$$+ \ldots$$

d>4

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$$+ \ldots$$

d>4

$\Lambda_{UV} \gg$ TeV (pointlike limit) nicely accounts for ‘what we see’

$$+\theta\,\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu} \qquad \theta \lesssim 10^{-10}$$

d=4

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{kin} + g A_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda(H^\dagger H)^2$$

d=4

$$+ \quad \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$

$$+ \quad \frac{c_{ijkl}}{\Lambda^2_{UV}} \bar{F}_i F_j \bar{F}_k F_\ell \;+\; \frac{c_{ij}}{\Lambda_{UV}} \bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots$$

$$+ \quad \ldots$$

d>4

$\Lambda_{UV} \gg$ TeV (pointlike limit) nicely accounts for ‘what we see’

$$+\, c_2\, \Lambda_{UV}^2\; H^\dagger H \qquad c_2 \simeq 0.008 \left(\frac{\mathrm{TeV}}{\Lambda_{UV}}\right)^2$$

d=2

$$+\, \theta\, \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} \qquad \theta \lesssim 10^{-10}$$

d=4

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$$+ \quad \ldots$$

d>4

$\Lambda_{UV} \gg$ TeV (pointlike limit) nicely accounts for ‘what we see’

$$+\, c_0\, \Lambda_{UV}^4 \sqrt{g} \qquad c_0 \sim -10^{-60} \left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^4$$

d=0

$$+\, c_2\, \Lambda_{UV}^2\, H^\dagger H \qquad c_2 \simeq 0.008 \left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^2$$

d=2

$$+\, \theta\, \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} \qquad \theta \lesssim 10^{-10}$$

d=4

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + g A_\mu \bar{F} \gamma_\mu F + Y_{ij} \bar{F}_i H F_j + \lambda (H^\dagger H)^2$$

d=4

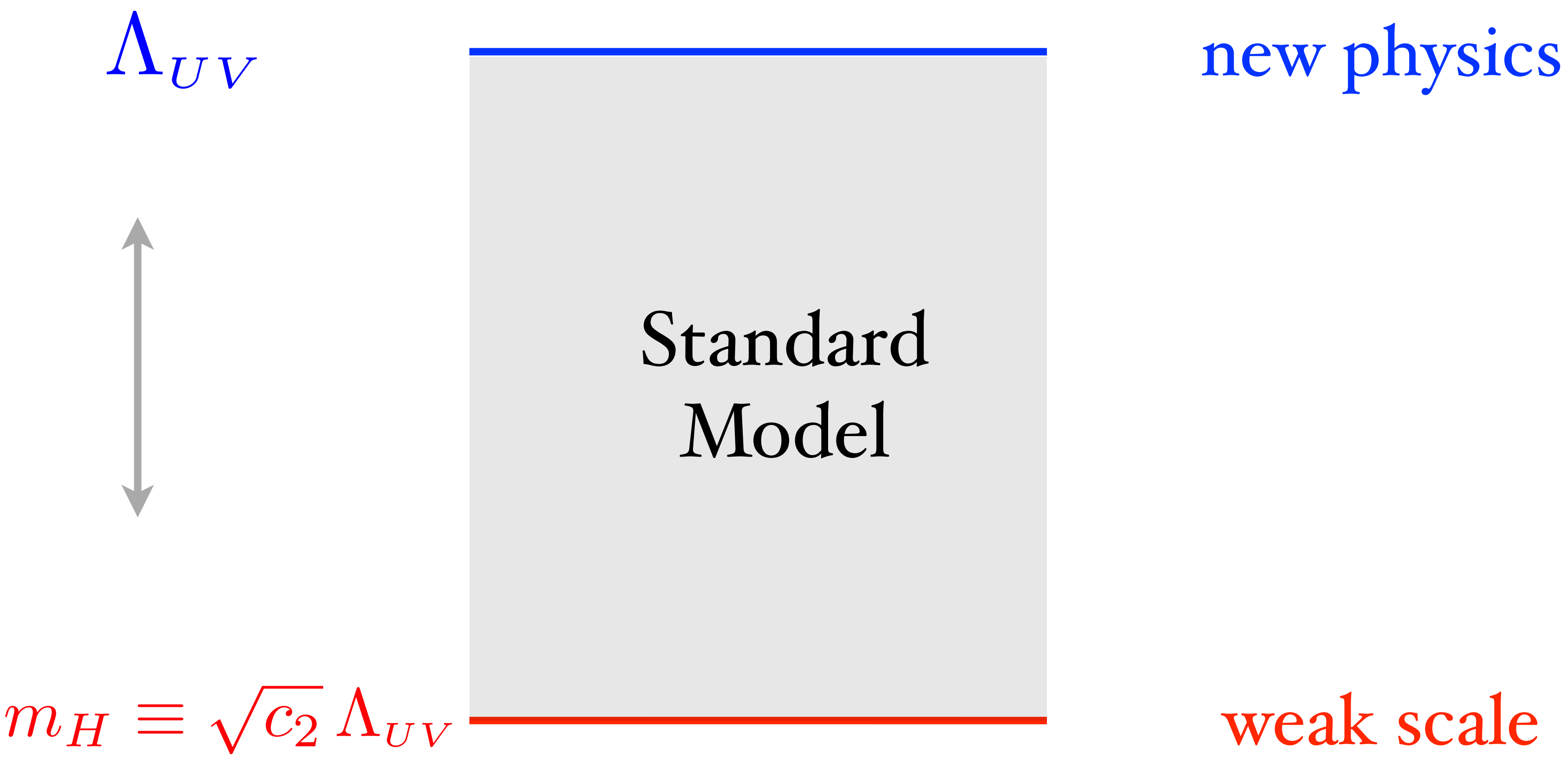
$$\begin{aligned} &+ \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H \\ &+ \frac{c_{ijkl}}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell + \frac{c_{ij}}{\Lambda_{UV}} \bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots \\ &+ \ldots \end{aligned}$$

d>4

$\Lambda_{UV} \gg \text{TeV}$ (pointlike limit) nicely accounts for 'what we see'

If

$$\Lambda_{UV} \gg m_H$$



some basic features of physical reality beautifully explained

by the magic of $\mathcal{L}_{d=4}$

The magic of $\mathcal{L}_{d=4}$

$$\mathcal{L}_4 = -\frac{1}{4g_3^2}G_{\mu\nu}^2 - \frac{1}{4g_2^2}W_{\mu\nu}^2 - \frac{1}{4g_Y^2}B_{\mu\nu}^2 + |D_\mu H|^2 + V(H)$$

$$+\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + \bar{e}_R \not{D} e_R$$

$$+Y_u^{ij}\bar{q}_L^i H^\dagger u_R^j + Y_d^{ij}\bar{q}_L^i H d_R^j + Y_e^{ij}\bar{\ell}_L^i H e_R^j$$

- $U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$ broken only by $Y_{u,d,e}$
- only $\bar{f} f$ terms → fermion number conserved
- can always redefine Y_e to make it diagonal

$U(1)_{L_1} \times U(1)_{L_2} \times U(1)_{L_3} \times U(1)_B$ and massless ν's emerge just accidentally

but in nice qualitative agreement with observations

$$q_L^{\,i}\,,\quad u_R^{\,i}\,,\quad d_R^{\,i}\,,\quad \ell_L^{\,i}\,,\quad e_R^{\,i} \qquad\qquad i = 1, 2, 3$$

$(\mathbf{3}, \mathbf{2}, 1/3)$ $(\mathbf{3}, \mathbf{1}, 4/3)$ $(\mathbf{3}, \mathbf{1}, -2/3)$ $(\mathbf{1}, \mathbf{2}, -1)$ $(\mathbf{1}, \mathbf{1}, -2)$

$$q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

$$q_L^i \to U_q^{ij}\, q_L^j$$

$$u_R^i \to U_u^{ij}\, u_R^j$$

$$d_R^i \to U_d^{ij}\, d_R^j$$

$$\ell_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

$$\ell_L^i \to U_\ell^{ij}\, \ell_L^j$$

$$e_R^i \to U_e^{ij}\, e_R^j$$

$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$ ≡ Flavor Symmetry

The magic of $\mathcal{L}_{d=4}$

$$\mathcal{L}_4 = -\frac{1}{4g_3^2}G_{\mu\nu}^2 - \frac{1}{4g_2^2}W_{\mu\nu}^2 - \frac{1}{4g_Y^2}B_{\mu\nu}^2 + |D_\mu H|^2 + V(H)$$

$$+\,\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + \bar{e}_R \not{D} e_R$$

$$+Y_u^{ij}\bar{q}_L^i H^\dagger u_R^j + Y_d^{ij}\bar{q}_L^i H d_R^j + Y_e^{ij}\bar{\ell}_L^i H e_R^j$$

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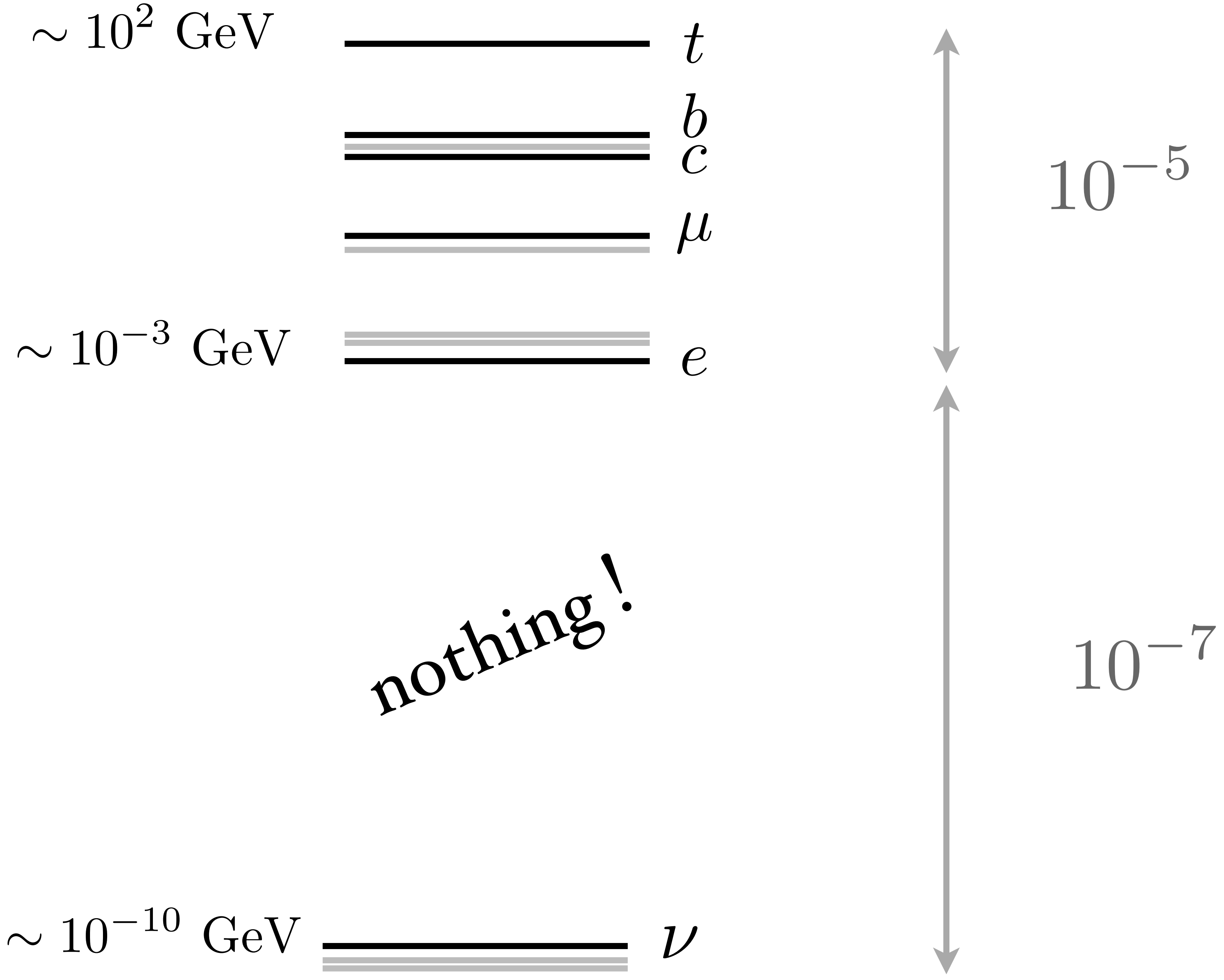
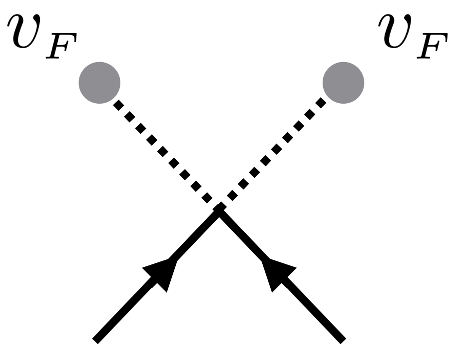


Data seem to speak for a qualitatively different origin for the $\boldsymbol{\nu}$ mass

The next order in the $1/\Lambda_{UV}$ expansion offers indeed such source



$$\mathcal{L}_{d=5} = \frac{b_{ij}}{\Lambda_{UV}}\, \ell_i^a C \ell_j^b\, H_a H_b$$

⇒

$$m^\nu_{ij} = b_{ij}\, \frac{v_F^2}{\Lambda_{UV}}$$

taking grossly $m_\nu \sim 0.1\,\mathrm{eV}$ ⇒ $\Lambda_{UV} \sim 10^{14}\,\mathrm{GeV} \times |b_{ij}|$

for $|b_{ij}| = O(1)$ this is not too far from unification scale

$$U(1)_{L_1} \times U(1)_{L_2} \times U(1)_{L_3} \times U(1)_B$$

✦ baryon number conserved: the proton is stable (...and life possible)

proton lifetime from SuperKamiokande Collab.

$$\tau(p \to e^+\pi^0) > 1.6 \times 10^{34}\,\text{yrs} \qquad 90\%\,CL$$

✦ individual lepton number conserved

$$\left.\begin{array}{c} \mu \to e\gamma \\ \tau \to \mu\,\gamma \\ \mu^- \to e^- e^+ e^- \\ \ldots \end{array}\right]$$

forbidden

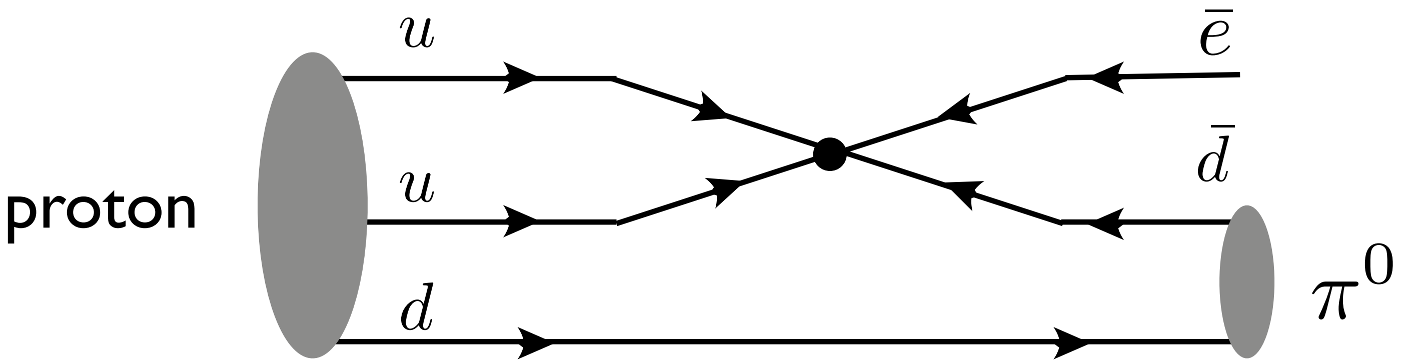
Ex. MEG experiment

$$\mathrm{Br}(\mu \to e\,\gamma) < 4.2 \times 10^{-13} \qquad 90\%\,CL$$

but all these effects are generated by terms in $\mathcal{L}_6$

$$\mathcal{L}_6 \supset \frac{\kappa_{uude}}{\Lambda_{UV}^2} \left(u_R^\alpha \, C \, d_R^\beta\right) \left(u_R^\gamma \, C \, e_R\right) \, \epsilon^{\alpha\beta\gamma}$$

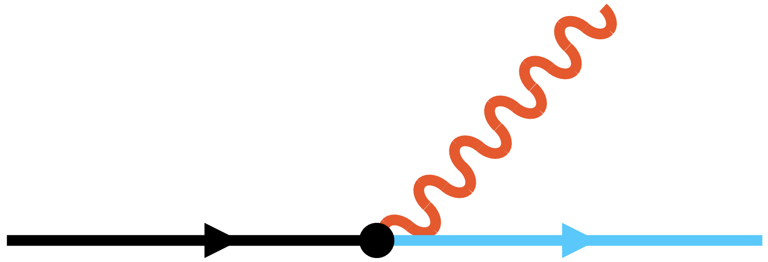
B+L violation: proton decay



$$p \to e^+ \pi^0$$

$$\Lambda_{UV} > \sqrt{\kappa_{uude}} \; 10^{15} \text{ GeV}$$

$$\mathcal{L}_6 \supset \sqrt{y_e y_\mu} \, \frac{c_{e\mu}}{\Lambda_{UV}^2} \left(\bar{\ell}_e \sigma_{\rho\sigma} \mu \, H\right) B^{\rho\sigma}$$



$$\text{Br}(\mu \to e \, \gamma)\Big|_{exp} \implies \Lambda_{UV} \gtrsim \sqrt{c_{e\mu}} \, 300 \text{ TeV}$$

The remarkable ‘flavor’ of $\mathcal{L}_4$ (GIM mechanism)

$$Y_u^{ij}\,\bar{q}_L^i H^\dagger u_R^j + Y_d^{ij}\,\bar{q}_L^i H d_R^j \xrightarrow{\text{flavor rotations}} \left[\begin{array}{l} Y_u = D_u \\ Y_d = V D_d \\ \\ Y_u = V^\dagger D_u \\ Y_d = D_d \end{array}\right.$$

$$D_u = \mathrm{diag}(y_u,\, y_c,\, y_t)$$

$$D_d = \mathrm{diag}(y_d,\, y_s,\, y_b)$$

Flavor violation purely in the interplay between u- and d-type quarks

If either D_u or D_d degenerate, then V can be eliminated

Flavor Changing Neutral Currents

- tree: none
- loop: depend on the mass (differences)
  in the other charge sector

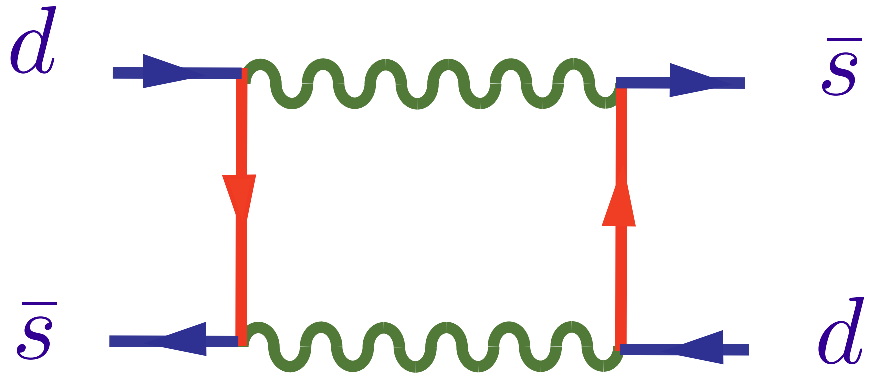
CP violation

- $V \neq V^*$ only source of CP violation
- V physical up to quark phase rotations $V^{jk} \to e^{i(\theta_u^j - \theta_d^k)}\, V^{jk}$
- truly physical CP violation $J_{ijk\ell} \equiv \mathrm{Im}\left\{V_{ij}V_{kj}^*V_{k\ell}V_{i\ell}^*\right\}$
- unitarity $V_{ij}V_{ik}^* = \delta_{jk}$
  - 2-families $J_{ijk\ell} = 0$
  - 3-families $J_{ijk\ell} \equiv J \quad i \neq k,\ j \neq \ell$
- Jarlskog invariant $J = \mathrm{Im}\left\{V_{ud}V_{td}^*V_{tb}V_{ub}^*\right\} < |V_{td}V_{ub}| \lesssim 3 \times 10^{-5}$
- if any two quarks of the same charge degenerate, J unphysical

Consequences

Ex: $K\bar{K}-$ mixing

$$\mathcal{A}_{\Delta S=2} \; =$$



$\mathrm{Re}(\mathcal{A}_{\Delta S=2})$ ➡ $$\frac{\Delta m_K}{m_K} \sim \frac{\alpha_W}{4\pi}\left(\frac{f_K}{v_F}\right)^2 \left(\frac{m_c}{m_W}\right)^2 \sin^2\theta_c \cos^2\theta_c$$

$\mathrm{Im}(\mathcal{A}_{\Delta S=2})$ ➡ $$\epsilon_K = \frac{\mathrm{Im}(\mathcal{A}_{\Delta S=2})}{\mathrm{Re}(\mathcal{A}_{\Delta S=2})} \sim \frac{J}{\cos^2\theta_c \sin^2\theta_c} \sim 10^{-3}$$

Electric dipole moments

state with definite

$\vec{J} \cdot \vec{J} = j(j+1)$

$$\langle \Psi(J) | \, \vec{D} \, | \Psi(J) \rangle \ =$$

Electric dipole moments

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$\vec{J} \cdot \vec{J} = j(j+1)$

$$\langle \Psi(J) | \, \vec{D} \, | \Psi(J) \rangle \quad = \quad c \, \langle \Psi(J) | \, \vec{J} \, | \Psi(J) \rangle$$

Electric dipole moments

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T-even

T-odd

Electric dipole moments

state with definite

$$\vec{J}\cdot\vec{J} = j(j+1)$$

$$\langle\Psi(J)|\,\vec{D}\,|\Psi(J)\rangle \;=\; c\,\langle\Psi(J)|\,\vec{J}\,|\Psi(J)\rangle$$

T-even

T-odd

T-odd

Electric dipole moments

state with definite

$$\vec{J}\cdot\vec{J} = j(j+1)$$

$$\underbrace{\langle\Psi(J)|\,\vec{D}\,|\Psi(J)\rangle}_{\text{T-even}} = \underbrace{\boxed{c}}_{\text{T-odd}}\,\underbrace{\langle\Psi(J)|\,\vec{J}\,|\Psi(J)\rangle}_{\text{T-odd}}$$

non-vanishing edm ⟷ broken T $\xleftrightarrow{\text{CPT}}$ broken CP

Electric dipole moments

state with definite
$\vec{J}\cdot\vec{J}=j(j+1)$

$$\langle\Psi(J)|\,\underset{\substack{\downarrow\\ \text{T-even}}}{\vec{D}}\,|\Psi(J)\rangle \;=\; \underset{\substack{\downarrow\\ \text{T-odd}}}{c}\,\langle\Psi(J)|\,\underset{\substack{\downarrow\\ \text{T-odd}}}{\vec{J}}\,|\Psi(J)\rangle$$

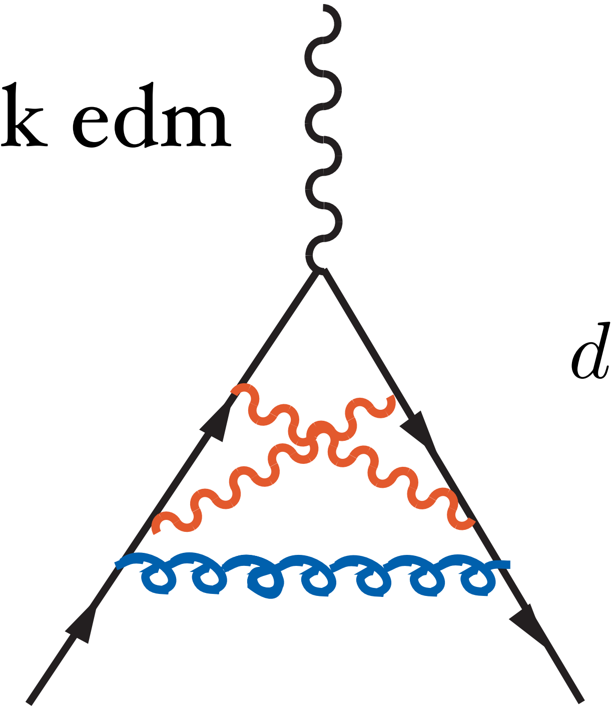
non-vanishing edm $\longleftrightarrow$ broken T $\xleftrightarrow{\text{CPT}}$ broken CP

Experimentally

- neutron $d_n < 10^{-26}\, e\,\mathrm{cm}$ nEDM 2020
- electron $d_e < 10^{-29}\, e\,\mathrm{cm}$ ACME 2018

$\mathcal{L}_4$ contribution to edms: J and QCD vacuum angle θ_{QCD}

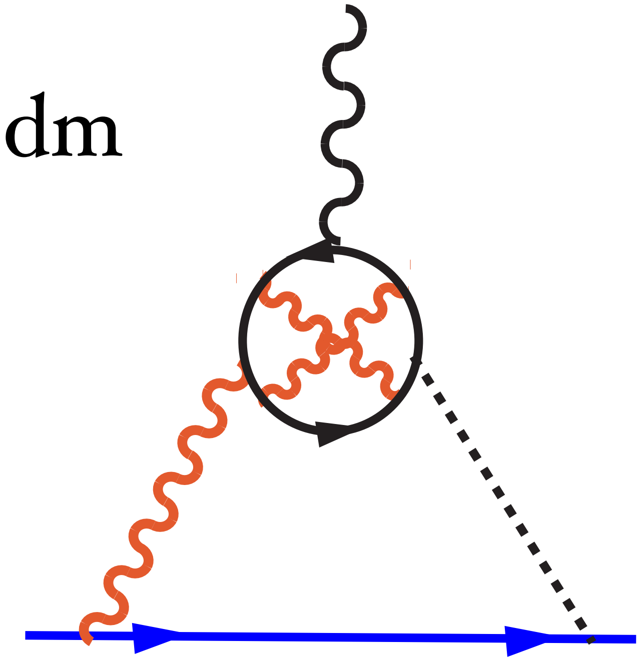
down quark edm



$$d_d \sim e \frac{\alpha_s}{4\pi} \left(\frac{\alpha_W}{4\pi}\right)^2 \frac{m_d}{m_W^2} \frac{m_c^2}{m_W^2} J \sim 10^{-34}\, e\,\text{cm}$$

Czarnecki, Krause 1997

electron edm

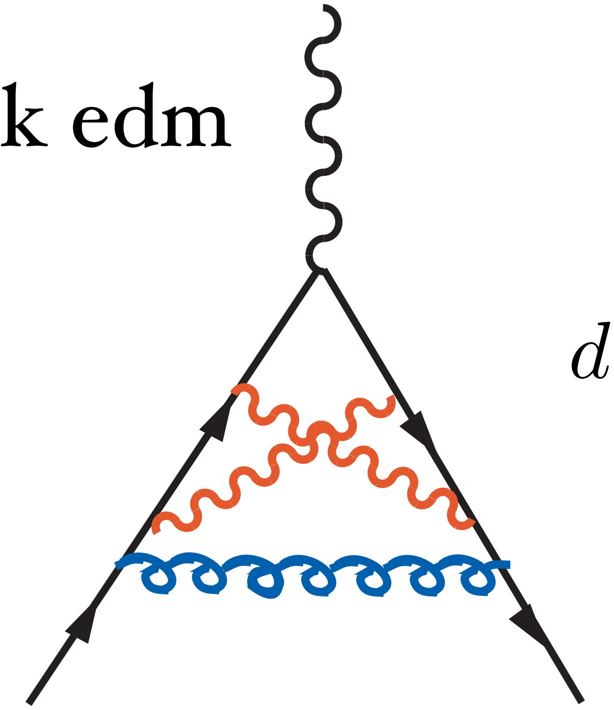


$$d_e \sim 10^{-38}\, e\,\text{cm}$$

Khriplovich, Pospelov 1991

$\mathcal{L}_4$ contribution to edms: J and QCD vacuum angle θ_{QCD}

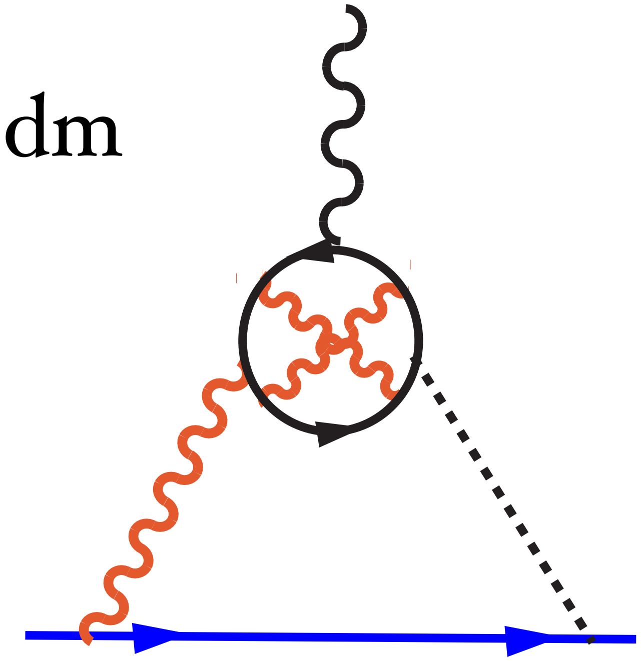
down quark edm



$$d_d \sim e\,\frac{\alpha_s}{4\pi}\left(\frac{\alpha_W}{4\pi}\right)^2 \frac{m_d}{m_W^2}\,\frac{m_c^2}{m_W^2}\,J \sim 10^{-34}\,e\,\mathrm{cm}$$

Czarnecki, Krause 1997

electron edm



$$d_e \sim 10^{-38}\,e\,\mathrm{cm}$$

Khriplovich, Pospelov 1991

While $\mathcal{L}_6$ contributes to all these processes at tree level

$$\mathcal{L}_6 \supset \frac{c_{\Delta S=2}}{\Lambda_{UV}^2} (\bar{d}\gamma^\mu s)^2$$

$$\Lambda_{UV} > \sqrt{\mathrm{Re}(c_{\Delta S=2})} \times 10^6 \text{ GeV}$$

$$\Lambda_{UV} > \sqrt{\mathrm{Im}(c_{\Delta S=2})} \times 10^7 \text{ GeV}$$

$$\mathcal{L}_6 \supset \frac{c_e y_e}{\Lambda_{UV}^2} (\bar{\ell}_L \sigma^\mu e_R H) \, B_{\mu\nu}$$

$$\Lambda_{UV} > \sqrt{\mathrm{Im}(c_e)} \times 10^6 \text{ GeV}$$

QCD vacuum angle θ_{QCD} → neutron edm

$$d_n \sim \theta_{QCD} \times 10^{-16}\, e\,\mathrm{cm}$$

$$\theta_{QCD} \lesssim 10^{-10}$$

hard to understand given CP violation in CKM
seems just small by accident

This is the Strong CP Problem

The Strong CP problem looks like a stain on the magic of $\mathcal{L}_4$

However this stain is remarkably mitigated by the existence of a dynamical solution, entailing the existence of an ultralight scalar, the axion, and compatible with a fundamental new physics scale f_a many orders of magnitude above the weak scale ($f_a \sim 10^{10} - 10^{12}\,\mathrm{GeV}$)

More magic of $\mathcal{L}_4$: custodial symmetry

$$\mathbf{H} = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad \mathbf{H} \xrightarrow{SU(2)_L} \hat{U}\mathbf{H}$$

$$\mathbf{H} = (\,\mathrm{Re}\,H^+,\, \mathrm{Im}\,H^+,\, \mathrm{Re}\,H^0,\, \mathrm{Im}\,H^0\,)$$

4 of $O(4) \sim SU(2)_L \times SU(2)_R$

$$\mathbf{\Phi} \equiv \begin{pmatrix} H^{0*} & H^+ \\ -H^{+*} & H^0 \end{pmatrix} \qquad \mathbf{\Phi} \xrightarrow{O(4)} \hat{U}_L \mathbf{\Phi} \hat{U}_R^\dagger$$

$$D_\mu \mathbf{\Phi} = \partial_\mu \mathbf{\Phi} + i g_2 T_L^A W_\mu^A \mathbf{\Phi} - i g_Y \mathbf{\Phi} T_R^3 B_\mu$$

$$\mathcal{L}_{Higgs} = \frac{1}{2}\mathrm{Tr}(D_\mu \mathbf{\Phi}^\dagger D_\mu \mathbf{\Phi}) - \frac{m^2}{2}\mathrm{Tr}(\mathbf{\Phi}^\dagger \mathbf{\Phi}) - \frac{\lambda}{4}\left[\mathrm{Tr}(\mathbf{\Phi}^\dagger \mathbf{\Phi})\right]^2$$

- hypercharge Y acts like T_R^3 ⇒ g_Y explicitly breaks $SU(2)_R \rightarrow U(1)_Y$
- $O(4)$ is only broken by hypercharge and other small effects

$$\langle \mathbf{\Phi} \rangle = \left(\begin{array}{cc} \langle H^{0*} \rangle & \langle H^{+} \rangle \\ -\langle H^{+*} \rangle & \langle H^{0} \rangle \end{array} \right) = \left(\begin{array}{cc} v_F & 0 \\ 0 & v_F \end{array} \right)$$

$\mathbf{\Phi} \quad \longrightarrow \quad \hat{U}\,\mathbf{\Phi}\hat{U}^{\dagger}$ is a residual approx symmetry: $SU(2)_c$ (custodial)

$$\langle \mathbf{\Phi} \rangle = \begin{pmatrix} \langle H^{0*} \rangle & \langle H^{+} \rangle \\ -\langle H^{+*} \rangle & \langle H^{0} \rangle \end{pmatrix} = \begin{pmatrix} v_F & 0 \\ 0 & v_F \end{pmatrix}$$

$\mathbf{\Phi} \longrightarrow \hat{U}\mathbf{\Phi}\hat{U}^{\dagger}$ is a residual approx symmetry: $SU(2)_c$ (custodial)

$(W_\mu^1, W_\mu^2, W_\mu^3)$ form a triplet under $SU(2)_c$

$$\mathcal{L}_{mass} = \frac{v_F^2}{4} \begin{pmatrix} W_\mu^1 & W_\mu^2 & W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g_2^2 & & & \\ & g_2^2 & & \\ & & g_2^2 & g_2 g_Y \\ & & g_2 g_Y & g_Y^2 \end{pmatrix} \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$m_Z^2 = \frac{v_F^2}{2}(g_2^2 + g_Y^2) = \frac{m_W^2}{\cos^2\theta_W}$$

$$\boxed{\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1}$$

- $SU(2)_C$ is also an *accidental* symmetry

$$\mathcal{L}^{d=6} = \frac{1}{\Lambda^2}(\mathbf{H}^\dagger D_\mu \mathbf{H})(\mathbf{H}^\dagger D^\mu \mathbf{H}) \quad \Rightarrow \quad \delta\rho \sim \frac{v_F^2}{\Lambda^2}$$

Electroweak Precision Tests
(LEP/SLC/Tevatron) ⇒ $\delta\rho_{BSM} \lesssim 10^{-3}$ ⇒ $\Lambda \gtrsim 10\,\mathrm{TeV}$

- It is remarkable how the hypothesis $\Lambda_{UV} \gg$ 1TeV, the ***desert***, very simply explains many structural aspects of particle physics

- This encourages us to try and understand how can m_H be naturally made hierarchically separated from Λ_{UV}

- It is remarkable how the hypothesis $\Lambda_{UV} \gg 1\text{TeV}$, the ***desert***, very simply explains many structural aspects of particle physics

- This encourages us to try and understand how can m_H be naturally made hierarchically separated from Λ_{UV}

... to our great frustration we find we cannot !

$$+ m_H^2 \, H^\dagger H$$

d<4

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{kin} + g A_\mu \bar{F} \gamma_\mu F + Y_{ij} \bar{F}_i H F_j + \lambda (H^\dagger H)^2$$

d=4

$$\begin{aligned} &+ \quad \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H \\ &+ \quad \frac{c_{ijkl}}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell \;+\; \frac{c_{ij}}{\Lambda_{UV}^2} \bar{F}_i \sigma_{\mu\nu} F_j H G^{\mu\nu} + \ldots \\ &+ \quad \ldots \end{aligned}$$

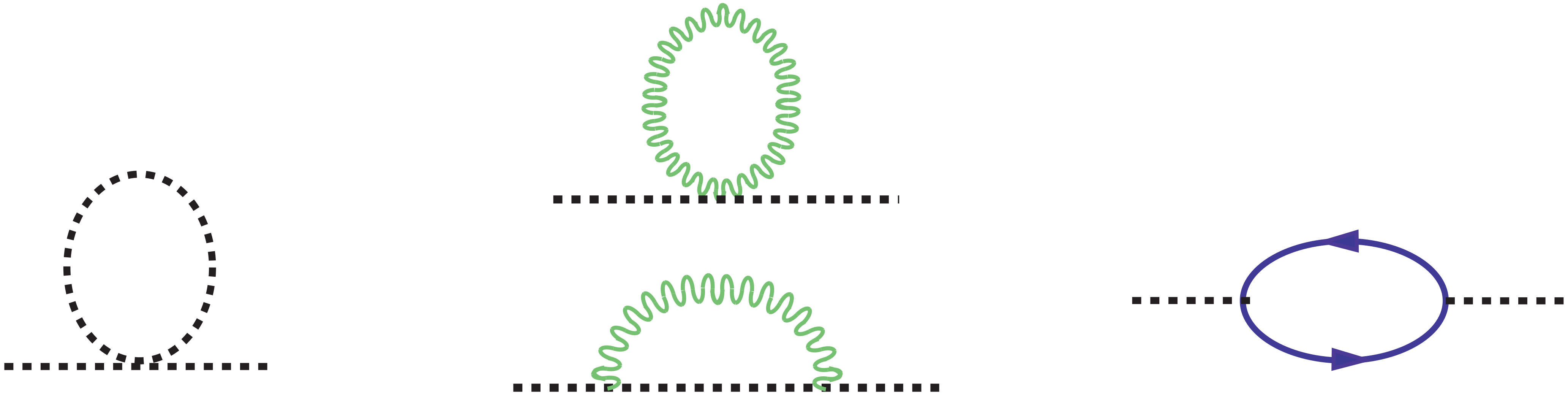
d>4

$$m_H^2 \;=\; c_2 \, \Lambda_{UV}^2$$

$$\Lambda_{UV} = 10^6 \,\mathrm{GeV} \;\Rightarrow\; c_2 \sim 10^{-8}$$

$$\Lambda_{UV} = 10^{15} \,\mathrm{GeV} \;\Rightarrow\; c_2 \sim 10^{-26}$$

How plausible is such a tremendously small c ?

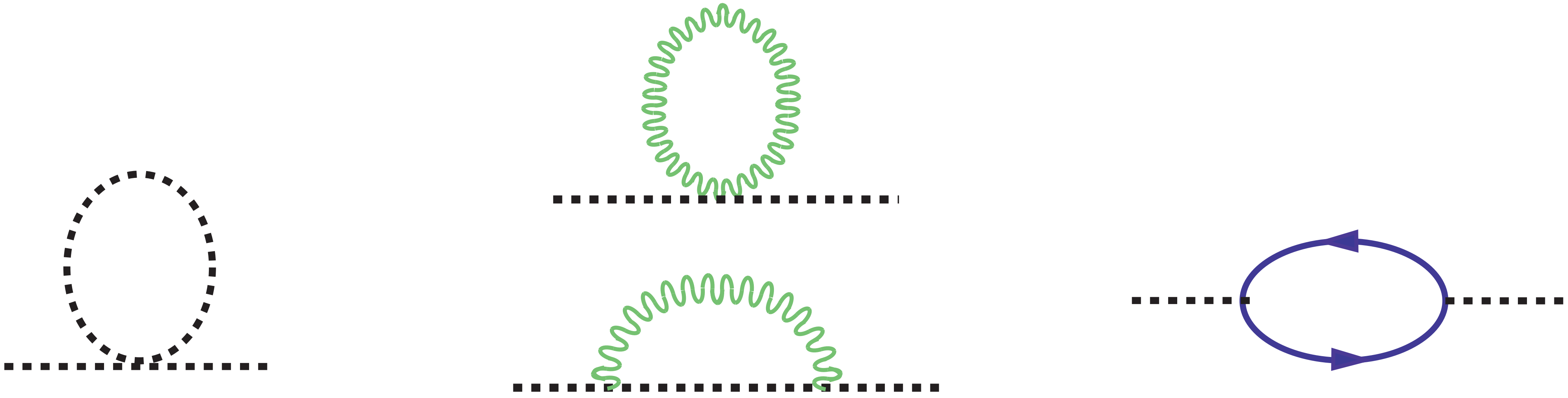


$$\delta m_H^2 = +\frac{3\lambda}{2(2\pi)^4}\int^{\Lambda_{UV}}\frac{d^4p}{p^2} \qquad +\frac{9g_W^2}{8(2\pi)^4}\int^{\Lambda_{UV}}\frac{d^4p}{p^2} \qquad -\frac{3y_t^2}{(2\pi)^4}\int^{\Lambda_{UV}}\frac{d^4p}{p^2}$$

$$= -\frac{3\lambda}{16\pi^2}\Lambda_{UV}^2 + \frac{9g_2^2}{64\pi^2}\Lambda_{UV}^2 - \frac{3y_t^2}{8\pi^2}\Lambda_{UV}^2$$

$$\delta m_H^2 \lesssim m_H^2|_{exp} \quad \Rightarrow \quad \Lambda_{UV} \lesssim 500\,\mathrm{GeV}$$

It seem we have a problem understanding $m_H \ll \Lambda_{UV}$!



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$\delta m_H^2 \lesssim m_H^2|_{exp}$ ➡ $\Lambda_{UV} \lesssim 500\,\mathrm{GeV}$

It seem we have a problem understanding $m_H \ll \Lambda_{UV}$!

Notice $\delta m_H^2 \sim \frac{y_t^2}{8\pi^2} \Lambda_{UV}^2$ fully fixed by symmetries

higher spin symm

dilatation symm

very much like the frequency of pendulum $\omega = c\sqrt{\frac{g}{L}}$

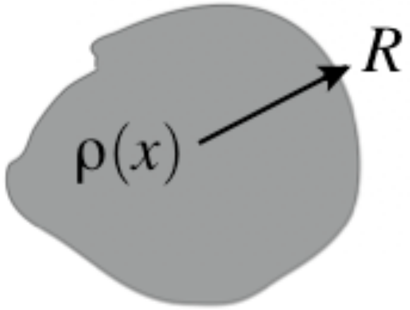
Galileo would surely have gasped had he found $c = 10^{-20}$

But why didn’t our ancestors worry about the electron mass?

But why didn't our ancestors worry about the electron mass?

....well, actually at a certain point they did

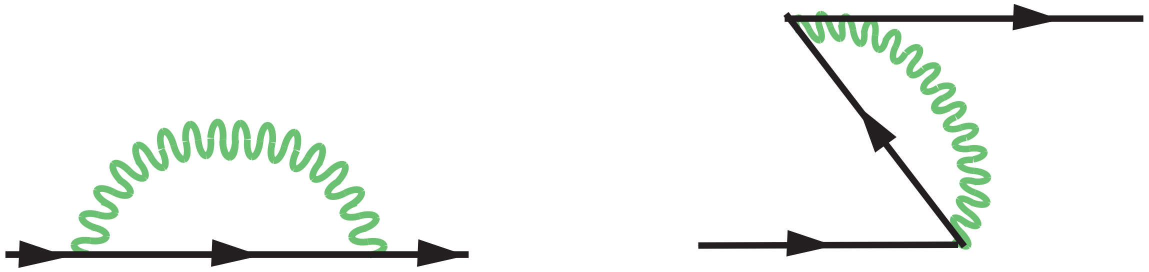
naive classical picture of electron



$$E \sim \frac{e^2}{R}$$

relativity $$m = E \sim \frac{e^2}{R} \xrightarrow{R \to 0} \infty$$

puzzle solved
by QED



$$\Delta m_e = +\frac{e^2}{16\pi^2}\Lambda \quad - \frac{e^2}{16\pi^2}\Lambda \quad = 0$$

The reason for this cancellation is chiral symmetry

$$\left.\begin{array}{l} \psi_L \to \psi_L \, e^{-i\theta} \\ \psi_R \to \psi_R \, e^{i\theta} \\ m_e \to m_e \, e^{i2\theta} \end{array}\right]$$

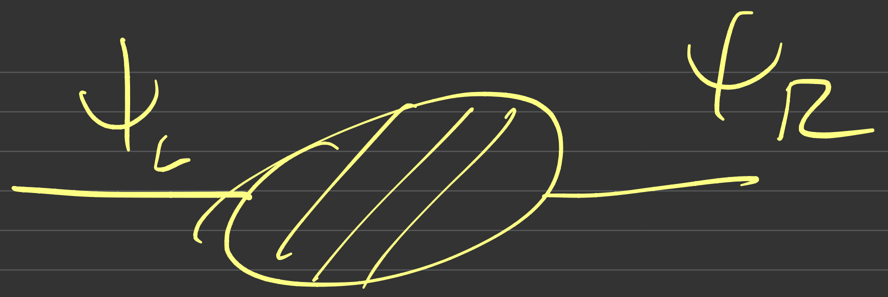
$$\Delta m_e \sim m_e \frac{e^2}{(2\pi)^4} \int \frac{d^4p}{(p^2)^2}$$

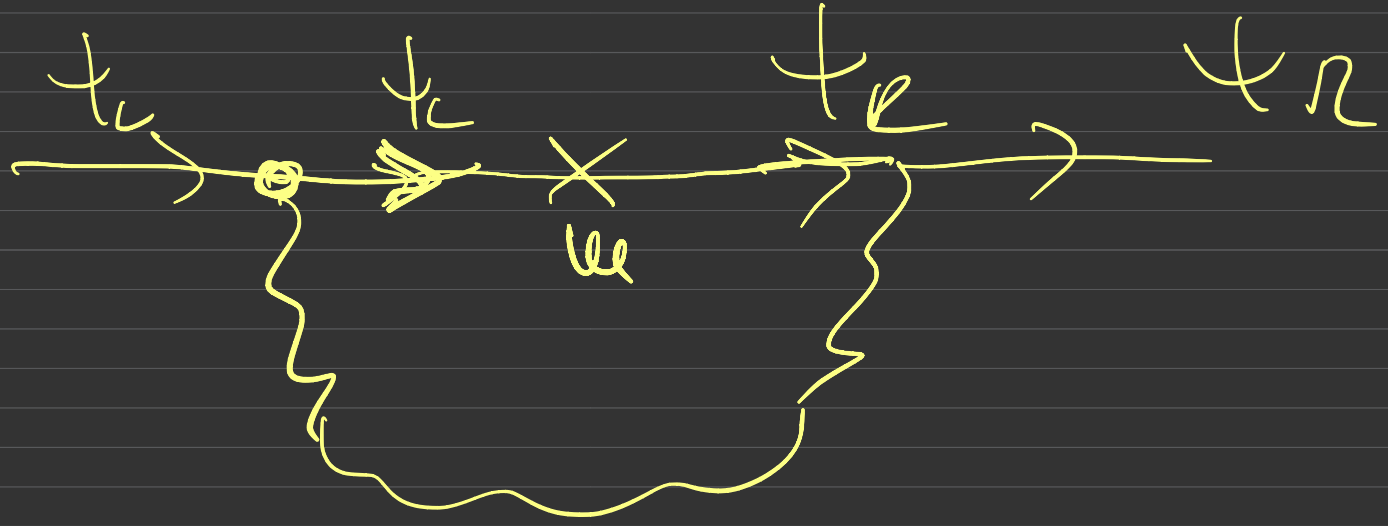
Fermion mass is only multiplicatively renormalized
no additive, possibly large, contribution

- $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ $\qquad D_\mu \equiv \partial_\mu - i e A_\mu$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \psi_L^\dagger \bar{\sigma} \cdot D \psi_L + i \underline{\underline{\psi_R^\dagger \sigma D}} \psi_R - \underline{m \psi_L^\dagger \psi_R} - m^* \psi_R^\dagger \psi_L$$

$$U(1)_A \longrightarrow \boxed{\begin{aligned} \psi_L &\to e^{-i\theta} \psi_L \\ \psi_R &\to e^{i\theta} \psi_R \\ m &\longrightarrow m e^{2i\theta} \\ e &\longrightarrow e \end{aligned}} \Longrightarrow \delta m_e \Big|_{\text{loops}} \propto m_e$$


ψ_L
ψ_R


ψ_L
ψ_L
ψ_R
ψ_R
m

- EFT perspective : must assume $\mathcal{O}_{\Delta>4}$ satisfy this approx symm

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\psi_L^\dagger\bar{\sigma}\cdot D\psi_L + i\psi_R^\dagger\sigma D\psi_R - m\psi_L^\dagger\psi_R - m^*\psi_R^\dagger\psi_L +$$

$$+\frac{g_5}{\Lambda}\underline{\psi_L^\dagger}\sigma_{\mu\nu}\underline{\psi_R}F^{\mu\nu} + \frac{g_6}{\Lambda^2}(\psi_L^\dagger\bar{\sigma}_\mu\psi_L)(\psi_R^\dagger\sigma^\mu\psi_R) + \dots$$

$U(1)_A$: $\frac{g_5}{\Lambda} \to e^{2i\theta}\frac{g_5}{\Lambda}$

$g_5 \sim \frac{m_e}{\Lambda}$

$$\delta m_e \sim \frac{1}{16\pi^2}\frac{g_5}{\Lambda}\int\frac{d^4p}{p^2}$$

$$\approx \frac{1}{16\pi^2}g_5\cdot\Lambda$$

Λ ⟶ approximate $U(1)_A$ could be accidental

m

Ex

- EFT above Λ $U(1)_A$ is gauged
- $U_A(1)$ broken by fermion condensate $\langle T_L^\dagger T_R \rangle \sim \Lambda^3$

$$\mathcal{L}_{mass} = \frac{1}{\Lambda_*^2} T_L^\dagger T_R \, e_R^\dagger e_L$$

$$\Longrightarrow \quad m_e \approx \frac{\Lambda^3}{\Lambda_*^2} \ll \Lambda$$

And what about the photon?

Shouldn’t we worry for the origin of his vanishing mass?

And what about the photon?

Shouldn’t we worry for the origin of his vanishing mass?

No!

as long as $2 \neq 3$

$$\mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu}^2 + M^2 A_\mu A^\mu$$

$$\mathcal{L}_2 = -\frac{1}{4} F_{\mu\nu} + \left(\underline{\partial_\mu \pi} + M A_\mu\right)^2$$

$$-\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + M^2 G^a_\mu G^{a\,\mu}$$

$\Sigma(x) = 3 \times 3$ unitary

$$\Sigma \xrightarrow{SU(3)\,\text{color}} \qquad \Sigma \longrightarrow U\,\Sigma$$

$U = SU(3)$ gauge rotation

$$D_\mu \Sigma = \partial_\mu \Sigma + i g_s G^a_\mu \lambda^a \Sigma$$

$$D_\mu \Sigma \to U D_\mu \Sigma$$ (covariant derivative !)

$$(D_\mu \Sigma)^\dagger \longrightarrow (D_\mu \Sigma)^\dagger U^\dagger$$

$$\Rightarrow \mathrm{Tr}\,(D_\mu \Sigma)^\dagger (D^\mu \Sigma) \equiv \text{gauge invariant}$$

$$\Rightarrow S = \int -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \frac{M^2}{g^2} \mathrm{Tr}(D_\mu \Sigma)^\dagger (D^\mu \Sigma)$$

$\equiv$ equivalent to simply adding a gluon mass M

- Indeed using $\Sigma \to U \Sigma$ under $SU(3)_{\text{color}}$

I can choose the gauge $\Sigma = \mathbb{1}$ (unitary gauge)

$$\Rightarrow S = \int -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{M^2}{g^2} \operatorname{Tr} g^2 G_\mu G^\mu$$

$$G_\mu \equiv G^a_\mu \lambda^a$$

- What does the theory of a massive gluon have of dramatically different? To analyze that let us hunt for the new effects, by making the known ones as small as possible this can be done by zooming on the limit

$$g \longrightarrow 0$$

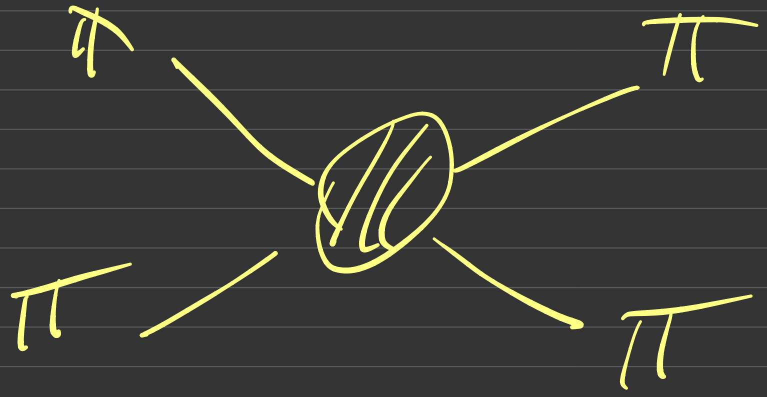
$$f = \frac{M}{g} = \text{fixed}$$

. in this limit

$$S \Rightarrow \int \underbrace{-\frac{1}{4}\left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a\right)^2}_{\text{free gluon}} + \underbrace{f^2 \,\mathrm{Tr}\, \partial_\mu \Sigma^+ \partial^\mu \Sigma}_{\text{non-linear } \sigma \text{ model}}$$

- $\Sigma = e^{i\hat{\pi}/f} \qquad \hat{\pi} \equiv \pi_a \lambda_a$

$$\Rightarrow f^2 \,\mathrm{Tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right) \sim \partial\pi\,\partial\pi + \frac{(\pi\,\partial\pi)^2}{f^2} + \dots$$



$$\sim \frac{E^2}{f^2}$$

- π-scattering is strong at $E \sim 4\pi f$

- π ~ longitudinal polarization of massive gluon