MARCO PELOSO LECTURE 1

INFLATION & REHEATING

→ INFLATION

(MOTIVATIONS, SIMPLEST REALIZATION, PRIMORDIAL PERTURBATIONS)

→ PRODUCTION DURING REHEATING

(WHILE THE INFLATON IS DECAYING)

→ PRODUCTION DURING INFLATION

(SIGNATURES IN CMB & IN GRAVITATIONAL WAVES)

INFLATION

• EXPANSION IN STANDARD COSMOLOGY

DISTANCES $\propto a(t)$ SCALE FACTOR

$H \equiv \frac{\dot{a}}{a}$ HUBBLE RATE

NATURAL UNITS
$c = \hbar = k_B = 1$

$M_P \equiv \frac{1}{\sqrt{8\pi G_N}}$ (REDUCED) PLANCK MASS

PERFECT FLUID ↗ ρ ENERGY DENSITY

PERFECT FLUID ↘ $P \equiv w\rho$ PRESSURE

w EQUATION OF STATE

FRIEDMANN EQUATION $H^2 = \frac{1}{3M_P^2}\rho$ (*)

LOCAL CONSERVATION EQ. $\dot{\rho} + 3H(\rho + P) = 0$

SOLVED BY $\boxed{\rho \propto a^{-3(1+w)}}$ $\boxed{a \propto t^{\frac{2}{3(1+w)}}}$

MATTER DOMINATION: $P = 0 \Rightarrow w = 0$ ↗ $a \propto t^{2/3}$, $H = \frac{\dot{a}}{a} = \frac{2}{3t}$

MATTER DOMINATION: $P = 0 \Rightarrow w = 0$ ↘ $\rho \propto a^{-3}$

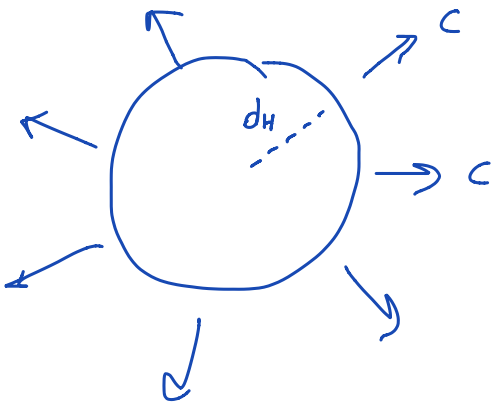
RADIATION DOMINATION: $P = \frac{\rho}{3} \Rightarrow w = \frac{1}{3}$ ↗ $a \propto t^{1/2}$, $H = \frac{1}{2t}$

RADIATION DOMINATION: $P = \frac{\rho}{3} \Rightarrow w = \frac{1}{3}$ ↘ $\rho \propto a^{-4}$

• HORIZON PROBLEM

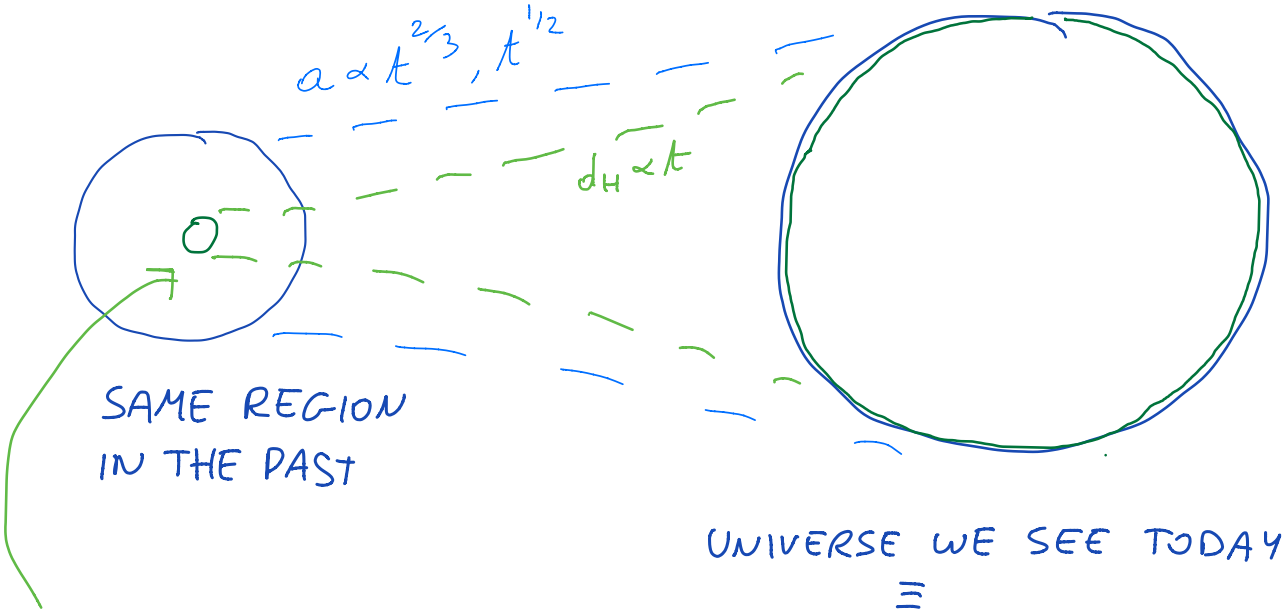
REGION IN CAUSAL CONNECTION

$$d_H \sim ct \sim \frac{1}{H} \qquad (c=1)$$



HORIZON GROWS FASTER

THAN SCALE FACTOR



SAME REGION
IN THE PAST

SIZE HORIZON
IN THE PAST

UNIVERSE WE SEE TODAY
≡
HORIZON TODAY

THE SKY WE SEE TODAY IS COMPOSED OF MANY REGIONS THAT WERE CAUSALLY DISCONNECTED IN THE PAST

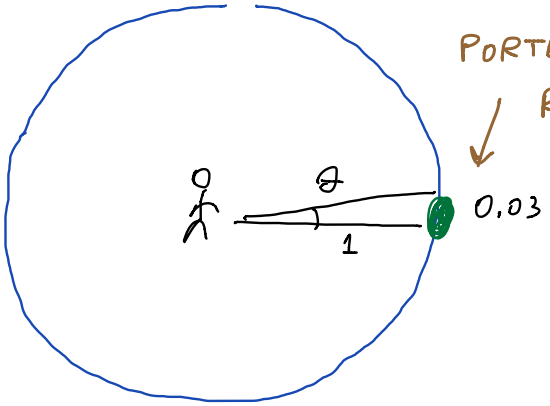
EXAMPLE : CMB EMITTED WHEN $a_{CMB} \sim 10^{-3} a_0$

↑ SUFFIX 0 MEANS TODAY

MATTER DOMINATION IN THIS PERIOD

$$a \propto t^{2/3} \Rightarrow t \propto a^{3/2} \qquad \frac{\text{HORIZON}}{\text{PHYSICAL SIZES}} \propto \frac{t}{a} \propto a^{1/2}$$

$$\Rightarrow \left.\frac{d_H}{a}\right|_{CMB} \Big/ \left.\frac{d_H}{a}\right|_0 = \sqrt{\frac{a_{CMB}}{a_0}} \sim 10^{-3/2} \sim 0.03$$



PORTION OF MY SKY OCCUPIED BY A REGION THAT WAS CAUSALLY CONNECTED WHEN CMB WAS EMITTED

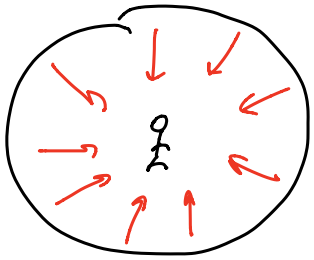
$\theta \approx \arcsin 0{,}03 \approx 0{,}03$ rad

$\approx 1{,}7°$

REGIONS SEPARATED BY MORE THAN 1.7° WHERE NOT IN CAUSAL CONNECTION WHEN CMB WAS EMITTED.

THERE IS NO REASON WHY THEY SHOULD HAVE THE SAME TEMPERATURE

ON THE OTHER HAND, CMB RADIATION ARRIVES WITH THE SAME TEMPERATURE $\simeq 2.7$ K FROM ALL DIRECTIONS



PROBLEM BECAUSE $\frac{d_H}{a}$ INCREASES

$$\frac{d_H}{a} \propto \frac{t}{a} \propto \frac{1}{aH} = \frac{1}{a\frac{\dot{a}}{a}} = \frac{1}{\dot{a}} \leftarrow \text{DECREASES}$$

PROBLEM BECAUSE $\ddot{a} < 0$ IN A MATTER + RADIATION UNIVERSE

RECALL $a \propto t^{\frac{2}{3(1+w)}}$. DECELERATION MEANS a SLOWER THAN t

$\equiv \frac{2}{3(1+w)} < 1 \Rightarrow 2 < 3+3w \Rightarrow \boxed{w > -\frac{1}{3}}$

FLATNESS PROBLEM

FRIEDMANN EQUATION WRITTEN ABOVE VALID FOR FLAT UNIVERSE ($k=0$). UNIVERSE COULD BE CLOSED ($k=1$) OR OPEN ($k=-1$)

$$\frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{1}{3M_p^2}\left(\frac{\rho_M^{(w)}}{a^3} + \frac{\rho_R^{(w)}}{a^4}\right)$$

↑

AS UNIVERSE EXPANDS, THIS TERM BECOMES DOMINANT.
THIS TERM IS $< 0.5\%$ TODAY $\Rightarrow$ MUST HAVE BEEN $\leq 10^{-18}$ WHEN UNIVERSE $\sim$ 1S OLD

PROBLEM BECAUSE ENERGY DENSITY DILUTED FASTER

$\rho \propto \frac{1}{a^p}$ WITH $p > 2$

RECALL $\rho \propto \frac{1}{a^{3(1+w)}}$

PROBLEM BECAUSE $3(1+w) > 2 \Rightarrow 3w+3 > 2 \Rightarrow \boxed{w > \frac{1}{3}}$

BOTH HORIZON & FLATNESS PROBLEM ORIGINATE BECAUSE MATTER & RADIATION HAVE $w > -\frac{1}{3}$, LEADING TO DECELERATED EXPANSION

• INFLATION

PROBLEMS SOLVED IF UNIVERSE UNDERWENT A PHASE OF ACCELERATED EXPANSION, $\ddot{a} > 0$. THIS MUST HAVE HAPPENED BEFORE BIG-BANG-NUCLEOSYNTHESIS (BBN), $t < 1s$.

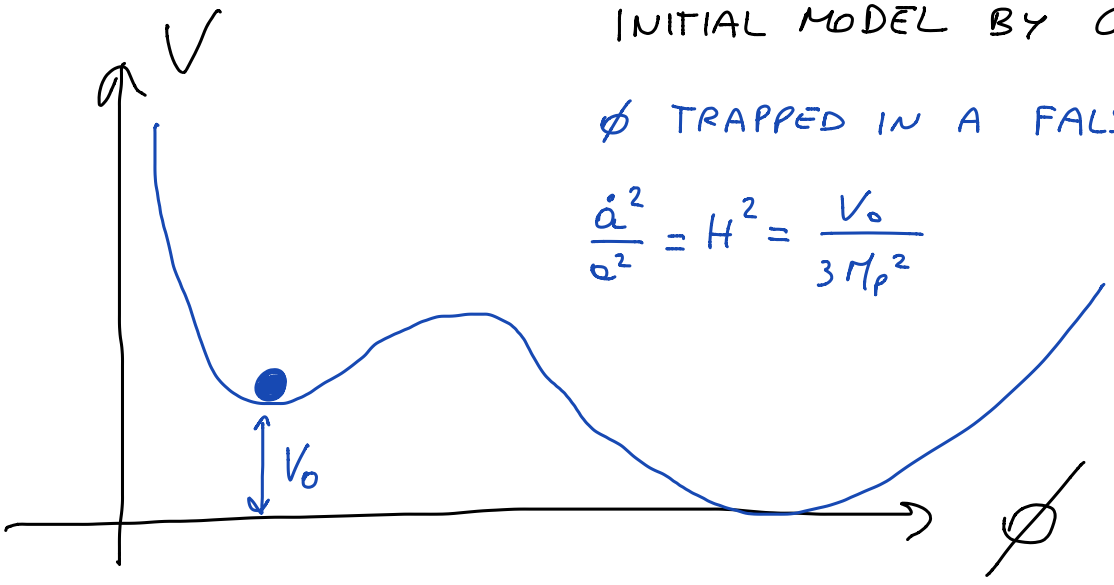
GUTH, 1981

* DURING INFLATION, A SINGLE CAUSALLY CONNECTED REGION GROWS SO MUCH AS TO COVER ALL THE SKY WE SEE TODAY

* DURING INFATION $H^2 = -\frac{k}{a^2} + \frac{\rho_x}{3M_p^2}$ $\leftarrow w < -\frac{1}{3}$

ρ_x DECREASES MORE SLOWLY THAN RADIATION, BECOMES MUCH GREATER THAN CURVATURE, AND THEN IT DECAYS INTO MATTER & RADIATION

WE CALL THIS SOURCE "INFLATON FIELD". WE ASSUME IT IS A SCALAR FIELD (NO EVIDENCE FOR SPIN) AND WE INDICATE IT BY ϕ



INITIAL MODEL BY GUTH:

ϕ TRAPPED IN A FALSE VACUUM

$\frac{\dot{a}^2}{a^2} = H^2 = \frac{V_0}{3M_p^2}$

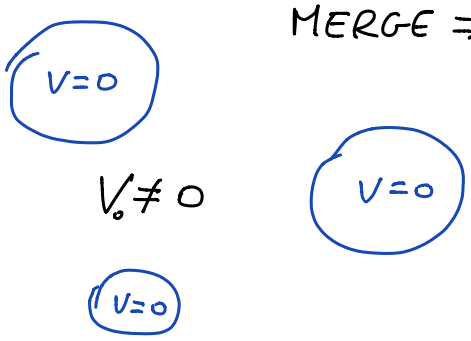
$\dot{a} = \sqrt{\frac{V_0}{3\pi\rho^2}}\, a \Rightarrow a \propto e^{\sqrt{\frac{V_0}{3\pi\rho^2}}\, t}$ EXPONENTIAL GROWTH

(de Sitter geometry)

PBH: FALSE VACUUM MUST DECAY TO PRODUCE THE PRESENT UNIVERSE

↗ $\Gamma > H$ → TOO LITTLE INFLATION (decays happens too soon)

↘ $\Gamma < H$ → DECAY OCCURS AT SAPARATE PLACES IN THE UNIVERSE, BUT SPACE IN BETWEEN KEEPS INFLATING, AND THESE REGIONS DO NOT MERGE ⇒ UNIVERSE DOES NOT REHEAT

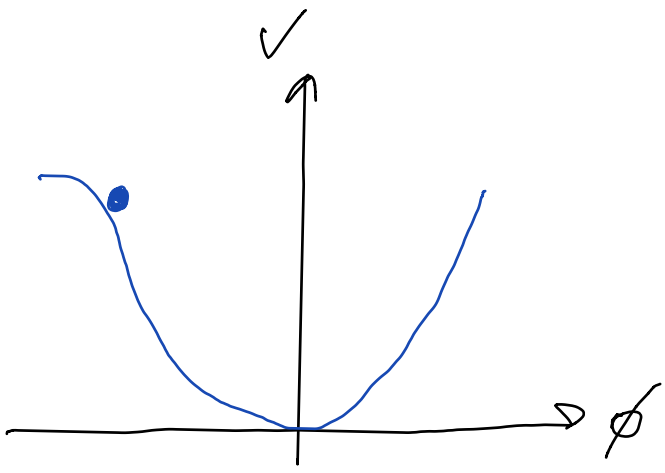


(REHEATING SHOULD HAPPEN WHEN BUBBLES COLLIDE)

- SLOW ROLL INFLATION

LINDE 1982

ALBRECHT, STEINHARDT 1982



$\rho = \frac{1}{2}\dot{\phi}^2 + V$

$p = \frac{1}{2}\dot{\phi}^2 - V$

SIMILAR TO VACUUM ENERGY IF $\dot{\phi}^2 << V$

ACCELERATION : $\omega = \frac{\frac{1}{2}\dot{\varphi}^2 - V}{\frac{1}{2}\dot{\varphi}^2 + V} < -\frac{1}{3} \Rightarrow \frac{3}{2}\dot{\varphi}^2 - 3V < -\frac{\dot{\varphi}^2}{2} - V$

$\Rightarrow 2\dot{\varphi}^2 < 2V \Rightarrow \boxed{\dot{\varphi}^2 < V}$

SLOW ROLL PARAMETERS

$$\varepsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 << 1 \quad , \quad \eta \equiv \frac{M_P^2}{2}\frac{V''}{V} << 1$$

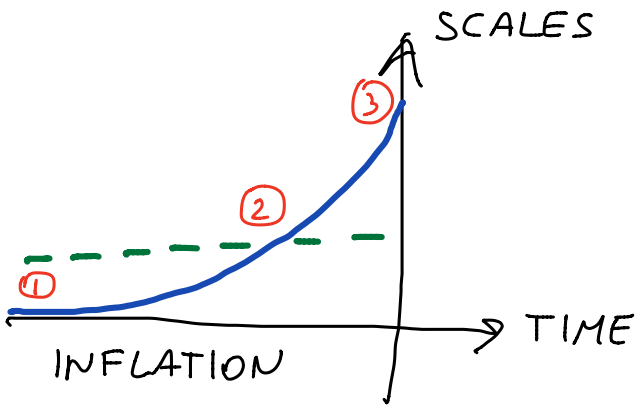
BYPRODUCT : MECHANISM FOR PRIMORDIAL PERTURBATIONS

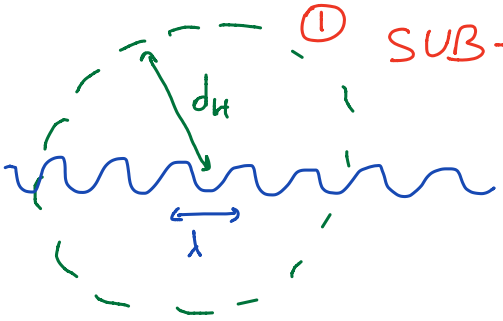
$\delta\phi, \delta\rho$

- $\frac{\delta\rho}{\rho} \simeq \mathcal{S}$ (SCALAR) DENSITY PERTURBATIONS
- $h_+, h_\times$ (TENSOR) GRAVITATIONAL WAVES

CONSIDER A PERTURBATION ON OUR SKY AND TRACE IT BACK DURING INFLATION

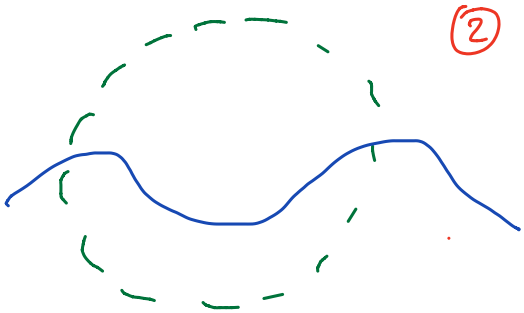
"HUBBLE HORIZON $d_H = H^{-1} \simeq$ const.
WAVELENGTH $\lambda \propto a \simeq e^{Ht}$





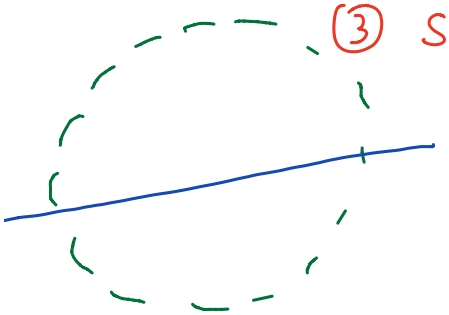
① SUB-HORIZON

QUANTUM EVOLUTION AS IN MINKOWSKI. NEGLIGIBLE EFFECT FROM GEOMETRY



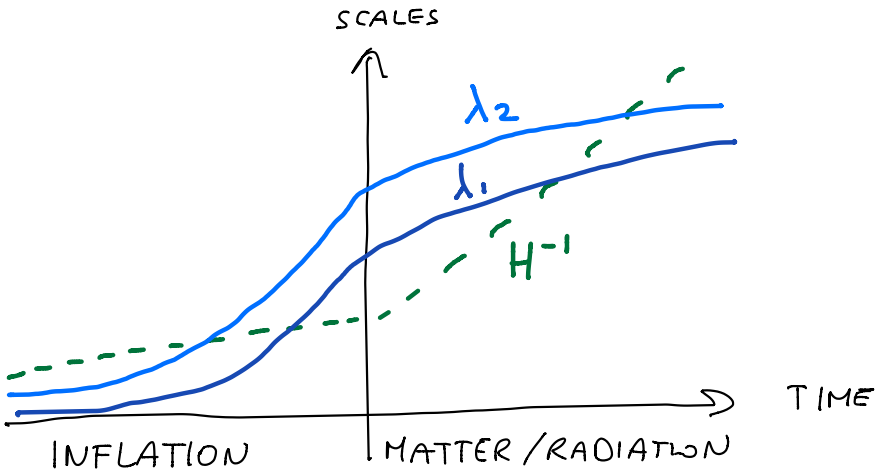
② HORIZON CROSSING

THIS IS WHEN THE MODE IS IMPRINTED. PERTURBATION PROBES CONDITION OF THE UNIVERSE AT THIS MOMENT



③ SUPER-HORIZON

MODE SEEN AS CONSTANT IN EACH HORIZON PATCH. FROZEN (NO EVOLUTION IN TIME) DUE TO CAUSALITY



MODES OF GREATER WAVELENGTH ($\lambda_2 > \lambda_1$) CROSS THE HORIZON EARLIER. THEY PROBE EARLIER TIMES OF INFLATION.

DEPARTURE FROM dS (CONSTANT V)

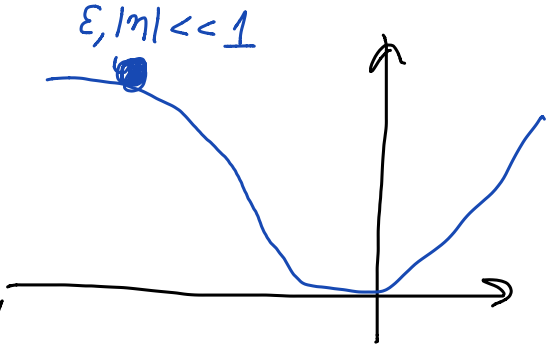
FROM $\varepsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$, $\eta \equiv M_p^2 \frac{V''}{V}$

• 3 IMPORTANT PROPERTIES OF PRIMORDIAL PERTURBATIONS

1) NEAR SCALE INVARIANCE

MODES OF ≠ SIZE HAVE NEARLY THE SAME POWER,

$P_s \propto \lambda^{1-n_s}$ $\quad 1-n_s \simeq 6\varepsilon - 2\eta$

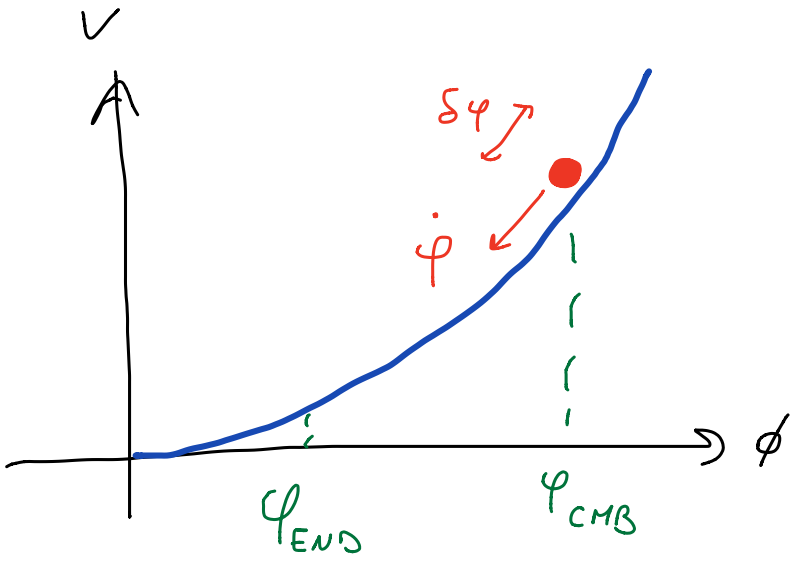


MODES OF ≠ SIZE LEAVE THE HORIZON AT ≠ TIMES, WHEN ϕ HAS MOVED DO A ≠ LOCATION IN V

BUT, FOR SLOW ROLL, $V(\phi)$ HAS CHANGED VERY LITTLE, SO NEARLY SAME POWER AT ALL SCALES

2) GW << DENSITY PERTURBATIONS

INFLATION PREDICTS $\quad r \equiv \frac{P_{GW}}{P_s} = 16\,\varepsilon << 1$



INFLATON IS A CLOCK, MEASURING THE TIME TO THE END OF INFLATION

SMALLER / GREATER ENERGY DENSITY ≡ EARLY / LATE CLOCK

SCALAR PERTURBATIONS $\equiv \delta\varphi \equiv$ PERTURBATIONS OF THE CLOCK

$$\zeta \sim \frac{\delta\rho}{\dot{\rho}} \propto \frac{\delta\varphi}{\dot{\varphi}}$$

$\dot{\varphi}$ ↖ BACKGROUND CLOCK

THE SLOWER THE BACKGROUND CLOCK ($\equiv$ SMALL $\dot{\varphi} \equiv$ SMALL ε)

THE GREATER THE EFFECT OF $\delta\varphi$ ON ζ

$$\Rightarrow P_\zeta \propto \frac{1}{\varepsilon} \Rightarrow r = \frac{P_{GW}}{P_\zeta} \propto \varepsilon$$

SLIDE 1

(3) HIGHLY GAUSSIAN, $\langle \zeta^3 \rangle \ll \langle \zeta^2 \rangle^{3/2}$

SMALL DEVIATIONS FROM GAUSSIANITY, PARAMETRIZED AS

$$\zeta = \zeta_g + f_{NL} * \zeta_g^2$$

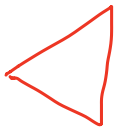
↑ GAUSSIAN

↑ CONVOLUTION IN MOMENTUM SPACE $\neq$ FUNCTIONAL DEPENDENCES GIVE $\neq$ SHAPES

$|f_{NL,\,LOCAL}| \lesssim 10$

(RECALL $\zeta \sim 10^{-5}$)

$|f_{NL,\,EQUILATERAL}| \lesssim 100$



FREE FIELDS ARE GAUSSIAN

$$\langle \phi\phi\phi\phi \rangle = \langle \overbrace{\phi\phi}\,\overbrace{\phi\phi} \rangle + \langle \overbrace{\phi\phi\phi\phi} \rangle + \langle \overbrace{\phi\phi\phi\phi} \rangle$$

$$= 3 \langle \phi\phi \rangle \langle \phi\phi \rangle$$

$$\langle \phi\phi\phi \rangle = 0$$

....

$\Rightarrow$ NON-GAUSSIANITY IS A MEASURE OF ϕ INTERACTIONS

- GRAVITATIONAL INTERACTIONS ARE WEAK
- SELF INTERACTIONS PROPORTIONAL TO DERIVATIVES OF THE POTENTIAL

$$V = V_0 + \frac{V''}{2}\delta\varphi^2 + \frac{V'''}{6}\delta\varphi^3 + \frac{V^{(iv)}}{24}\delta\varphi^4 + \dots$$

SMALL DUE TO SLOW ROLL

$\therefore$ LARGE NON-GAUSSIANITY POSSIBLE FROM INTERACTIONS WITH OTHER FIELDS, AS WE WILL SEE LATER IN THESE LECTURES.

REHEATING AFTER INFLATION

DURATION OF INFLATION MEASURED IN e-FOLDS $a = a_{END} e^{-N}$. AS WE WILL SEE, N IS MODEL-DEPENDENT. IN MANY MODELS $N \sim 60$.

$\Rightarrow$ THE NUMBER DENSITY OF ANY SPECIES THAT MIGHT HAVE BEEN PRESENT AT THE START OF INFLATION GETS DILUTED BY $\frac{1}{\text{VOLUME}} = \frac{1}{a^3} \sim e^{-180}$

$\Rightarrow$ ALL MATTER & RADIATION IN THE PRESENT UNIVERSE PRODUCED DURING REHEATING

↗ EITHER BY THE DECAY OF THE INFLATOR

↘ OR BY THE DECAY OF SOME OTHER LIGHT SCALAR FIELD AMPLIFIED DURING INFLATION THAT CAME TO DOMINATE OVER THE INFLATON AT REHEATING (LESS STANDARD)

REHEATING $\equiv$ ALL PROCESSES FROM THE END OF INFLATION TO THE ESTABLISHMENT OF A DOMINANT THERMAL BATH

WE REQUIRE

- FORMATION OF A THERMAL BATH, WITH $T \gtrsim 2\,\mathrm{MeV}$ (REQUIRED FOR BBN)
- PRODUCTION OF DARK MATTER $\left.\frac{\rho_{CDM}}{\rho_\gamma}\right|_{BBN} \simeq 10^{-6}$
- PRODUCTION OF BARYON ASYMMETRY $\frac{n_b}{n_\gamma} \sim 6 \cdot 10^{-10}$
- NO OVERPRODUCTION OF GRAVITINOS, TOPOLOGICAL DEFECTS, ... (MODEL DEPENDENT)

MOST UNKNOWN COSMOLOGICAL PERIOD !

CRUCIAL UNKNOWNS:

→ SCALE OF INFLATION (WHEN REHEATING STARTS)

→ NATURE & INTERACTIONS OF THE INFLATON: PERTURBATIVE (SLOW) VS. NON-PERTURBATIVE INFLATON DECAY; EQUATION OF STATE DURING REHEATING

→ MECHANISM FOR BARYOGENESIS (THERMAL? NON-THERMAL?)

→ NATURE & GENERATION OF DARK MATTER (FREEZE-OUT? FREEZE-IN? NON-THERMAL?)