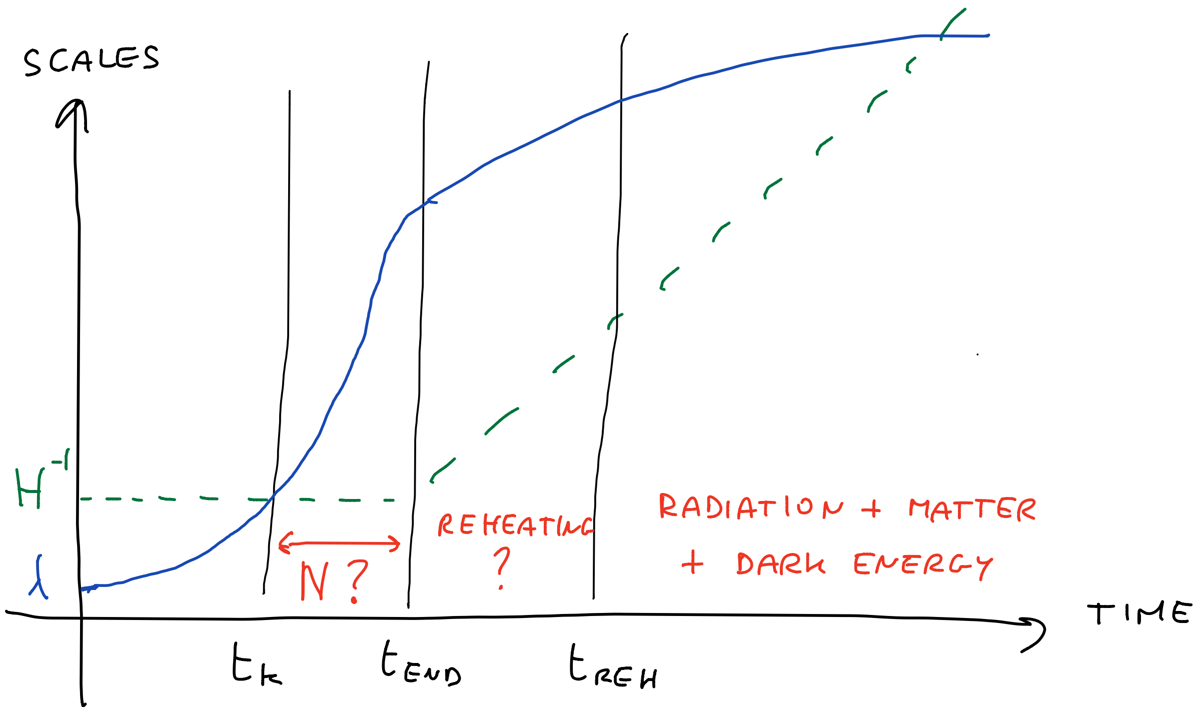
MARCO PELOSO — LECTURE 2

UNCERTAINTY AT REHEATING → UNCERTAINTY IN INFLATIONARY PHENOMENOLOGY

PLANCK PIVOT SCALE $k = 0.05\ \mathrm{Mpc}^{-1}$. AT WHICH E-FOLD N WAS IT PRODUCED?



UNKNOWN INFLATIONARY PARAMETERS

- ρ_k ENERGY DENSITY AT PRODUCTION (≡ HORIZON EXIT)
- ρ_{END} ENERGY DENSITY END INFLATION

UNKNOWN REHEATING PARAMETERS

- ρ_{REH} ENERGY DENSITY WHEN REHEATING COMPLETED
- ω EQUATION OF STATE DURING REHEATING

* HORIZON CROSSING DURING INFLATION $\lambda = \frac{2\pi}{k} a \sim H^{-1}$

$\rightarrow a_k H_k = k$

* CORRESPONDING E-FOLDS $a_k = e^{-N} a_{END}$

* NORMALIZATION SCALE FACTOR AT PRESENT $a_0 = 1$

$$\Rightarrow \quad e^N = \frac{a_{END}}{a_k} = \frac{H_k}{k} \frac{a_{END}}{a_{REH}} \frac{a_{REH}}{a_0}$$

FIRST FACTOR

- HUBBLE RATE AT HORIZON EXIT

$$H_k = \frac{V_k^{1/2}}{\sqrt{3}\, M_P} = 2.4 \cdot 10^{13}\, GeV \left(\frac{V_k^{1/4}}{10^{16}\, GeV} \right)^2$$

- PLANCK PIVOT SCALE $k = 0{,}05\ Mpc^{-1} = 3.2 \cdot 10^{-40}\, GeV$

SECOND FACTOR

- WE SAW $\rho \propto a^{-3(1+w)}$

$$\Rightarrow \quad \frac{a_{END}}{a_{REH}} = \left(\frac{\rho_{REH}}{\rho_{END}} \right)^{\frac{1}{3(1+w)}}$$

THIRD FACTOR

- ENTROPY CONSERVATION $g_{*,s} T^3 a^3 = const.$

WHERE g_{*s} COUNTS THE LIGHT DEGREES OF FREEDOM

(SOME COMPLICATIONS DUE TO FERMI VS. BOSE STATISTICS

$g_{*S,REH} = 106.75$ FOR STANDARD MODEL

$g_{*S,0} = 3.909$ PHOTONS & NEUTRINOS

$$\Rightarrow \frac{a_{REH}}{a_0} = \left(\frac{3.909}{106.75}\right)^{1/3} \frac{2.348 \cdot 10^{-13}\,\mathrm{GeV}}{T_{REH}}$$

- ENERGY DENSITY AT REHEATING $\rho_{REH} = \frac{\pi^2}{30} g_* T_{REH}^4$

$$\Rightarrow \frac{a_{REH}}{a_0} = \frac{1.9 \cdot 10^{-13}\,\mathrm{GeV}}{\rho_{REH}^{1/4}}$$

PUT THE 3 FACTORS TOGETHER, TAKE LOG

$$N \simeq 55.6 + 2 \ln \frac{V_k^{1/4}}{10^{16}\,\mathrm{GeV}} + \ln \frac{10^{16}\,\mathrm{GeV}}{\rho_{END}^{1/4}} + \frac{1-3w}{12(1+w)} \ln \frac{\rho_{REH}}{\rho_{END}}$$

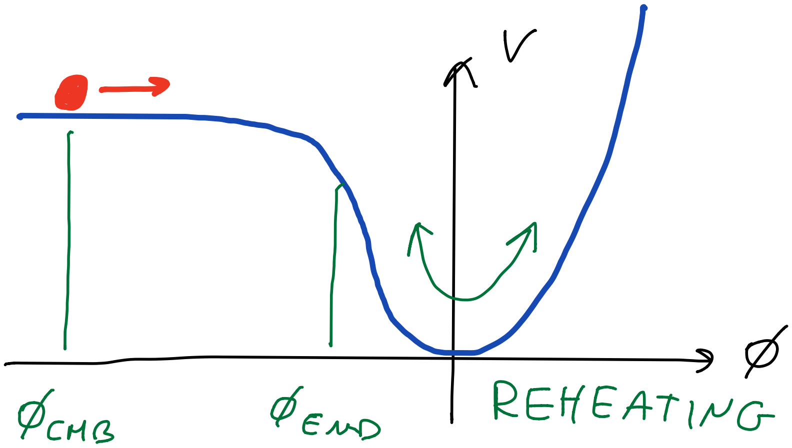
THE LAST TERM ENCODES THE UNCERTAINTY ON N PURELY DUE TO OUR IGNORANCE OF REHEATING

HOW LARGE CAN THIS TERM BE ?

WE EVALUATE THIS TERM FOR

(i) ISTANTANEOUS REHEATING AFTER INFLATION $\rightarrow \Delta N = 0$

(ii) SLOWEST POSSIBLE DECAY $T_{REH} \sim \mathrm{MeV}$



FOR A SLOW DECAY, ϕ PERFORMS MANY OSCILLATIONS ABOUT THE MINIMUM OF THE POTENTIAL BEFORE DECAYING. WHAT IS ω?

$$H^2 = \frac{\rho}{3M_p^2} = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 + V\right)$$

$$\dot{\rho} + 3H(\rho + p) = 0 \rightarrow \left(\frac{1}{2}\dot{\varphi}^2 + V\right)^{\bullet} + 3H\left(\frac{1}{2}\dot{\varphi}^2 + V + \frac{1}{2}\dot{\varphi}^2 - V\right) = 0$$

$$\dot{\varphi}\ddot{\varphi} + V'\dot{\varphi} + 3H\dot{\varphi}^2 = 0$$

TAYLOR EXPAND $V(\varphi)$ ABOUT MINIMUM. SHIFT φ SO THAT MINIMUM AT $\varphi = 0$; $V = 0$ AT MINIMUM

$\Rightarrow V = \frac{1}{2}m^2\varphi^2 +$ HIGHER ORDER   ASSUME $\boxed{V = \frac{1}{2}m^2\varphi^2}$

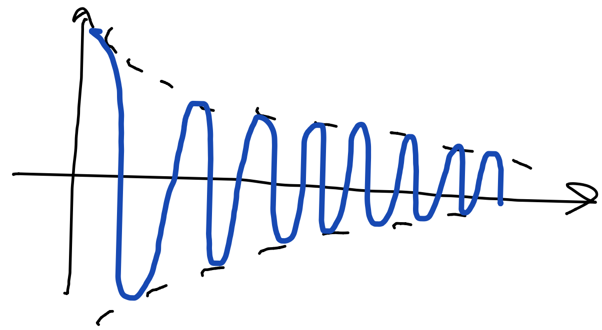
THE EQUATIONS ARE

$$\boxed{\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0 \\ H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right) \end{cases}}$$

← OSCILLATOR WITH FRICTION ($a = -\gamma v$)

← FRICTION PROPORTIONAL TO ENERGY DENSITY OF THE OSCILLATOR

ANSATZ : OSCILLATIONS
WITH ADIABATICALLY
VARYING AMPLITUDE



TO FIND THE EVOLUTION, WE STUDY THE RELATION BETWEEN THE AVERAGE KINETIC ENERGY AND THE AVERAGE POTENTIAL ENERGY, NEGLECTING THE VARIATION OF THE AMPLITUDE DURING ONE OSCILLATION

VIRIAL THEOREM : $\langle E_k \rangle = \langle V \rangle$

$$\Rightarrow \langle \text{PRESSURE} \rangle = \left\langle \frac{1}{2}\dot{\varphi}^2 - V \right\rangle = 0$$

COHERENT INFLATON OSCILLATIONS $\equiv$ MATTER
$\omega = 0$

$$\Rightarrow \phi = \frac{\Phi_0}{mt} \sin(mt) + \mathcal{O}\left(\frac{1}{t^2}\right) \Rightarrow \rho_\phi \propto \left(\frac{\Phi_0}{mt}\right)^2 \propto \frac{1}{a^3} \quad \checkmark$$

RECALL $a \sim t^{2/3}$
IN MATTER DOMINATION

BACK TO $\Delta N = \frac{1-3w}{12(1+w)} \ln \frac{\rho_{REH}}{\rho_{END}}$

$$\underset{\substack{\uparrow\\ \text{SLOW ROLL}}}{\rho_{END}^{1/4} \simeq} \rho_h^{1/4} \underset{\substack{\uparrow\\ \text{SEEN EARLIER}}}{\simeq} 3\cdot 10^{16}\, GeV^{1/4}\, r^{1/4}$$

TAKING $\rho_{END}^{1/4} \simeq 10^{16}$ GeV & $\rho_{REH} \sim T_{REH}^4 \sim MeV^4$

& $w=0 \longrightarrow \Delta N = -15$

AS COMPARED TO $\Delta N = 0$ FOR ISTANTANEOUS REHEATING.

VERY LARGE UNCERTAINTY, THAT IMPACTS OUR PREDICTIONS FOR ANY GIVEN INFLATIONARY POTENTIAL

$\boxed{\text{SLIDE 2}}$

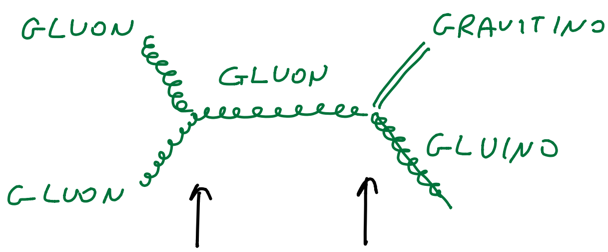
ESTIMATES IN PERTURBATIVE REHEATING

GOAL: OBTAIN QUICK ANSWERS → APPLICATION TO GRAVITINO PROBLEM

THIS IS ALL WE NEED FOR THE ESTIMATES

- NATURAL UNITS $\hbar = c = k_B = 1$
  - $1 = \hbar c = 200\,\mathrm{MeV\,fm} \Rightarrow 1\,\mathrm{m} \simeq \frac{5\cdot 10^{15}}{\mathrm{GeV}}$
  - $1 = c = 3\cdot 10^8\,\mathrm{m\,s^{-1}} \Rightarrow 1\,\mathrm{s} \simeq 3\cdot 10^8\,\mathrm{m} \simeq \frac{10^{24}}{\mathrm{GeV}}$
- IN A THERMAL BATH, # DENSITY OF RELATIVISTIC PARTICLES $N_x \sim g_x T^3$
- $N_{\gamma,0} \sim 400\,\mathrm{cm}^{-3}$
- $H^2 = \frac{\rho}{3M_P^2}$; $\rho \sim T^4$
- $H \sim 1/t$ IN MATTER/RADIATION UNIVERSE
- $t_0 \sim 10^{10}\,\mathrm{yrs} \sim \pi\, 10^{17}\,\mathrm{s}$
- CROSS SECTION $[\sigma] = -2$; OBTAIN IT FROM COUPLINGS^2, ENERGIES $\sim T$, AND MASSES (IF GREATER THAN T)
- Γ = RATE FOR A PROCESS TO OCCUR ; FOR RELATIVISTIC PARTICLES $\Gamma \sim \sigma N v \sim \sigma T^3$
- IN TIME $t \sim H^{-1}$, THE NUMBER OF PROCESSES $\int \Gamma\, dt \sim \frac{\Gamma}{H}$ OCCURS $\Rightarrow$ PROCESS OCCURS IF $\Gamma > H$

THE GRAVITINO PROBLEM



DOMINANT PRODUCTION PROCESS

STRONG INTERACTION $g \sim 1$

GRAVITATIONAL INTERACTION $g \sim \frac{1}{M_P}$

$\Rightarrow \quad \sigma \sim \frac{1}{M_P^2}$

$\Rightarrow \Gamma \sim \frac{T^3}{M_P^2}$ FROM A THERMAL BATH

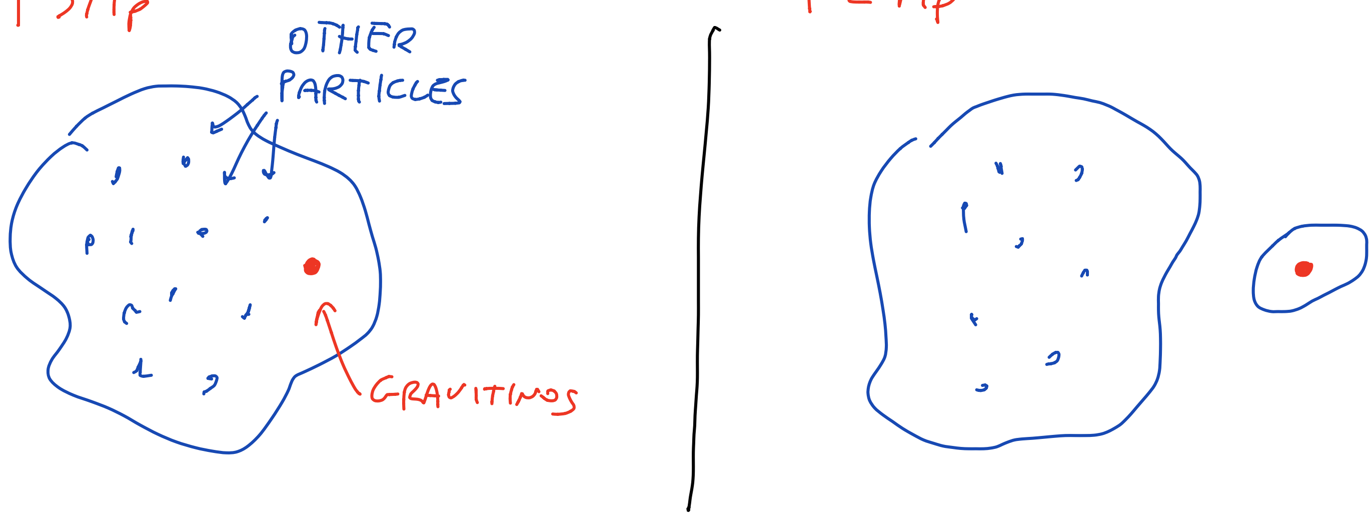
ASSUME NO INFLATION, $T_{\text{INITIAL}} \simeq \infty$

AS WE WROTE, PRODUCTION EFFICIENT FOR $\Gamma > H$

$$\Rightarrow \quad \frac{T^3}{M_P^2} > \frac{T^2}{M_P} \quad \Rightarrow \quad T > M_P$$

GRAVITINOS WERE IN THERMAL CONTACT WITH THE OTHER PARTICLES FOR $T > M_P$. THEN THEY DECOUPLED

$T > M_P$ $\qquad\qquad$ $T < M_P$



- AS TEMPERATURE COOLS, ALL OTHER PARTICLES ANNIHILATE INTO PHOTONS ⇒ # PARTICLES INITIALLY ≃ NUMBER OF PHOTONS TODAY

- THIS DID NOT HAPPENED TO GRAVITINOS, SINCE DECOUPLED

$$\Rightarrow \frac{\#\ \text{GRAVITINOS TODAY}}{\#\ \text{PHOTONS TODAY}} = \frac{\#\ \text{GRAVITINO STATES}}{\#\ \text{STATES OF ALL OTHER PARTICLES}} \simeq 10^{-2}$$

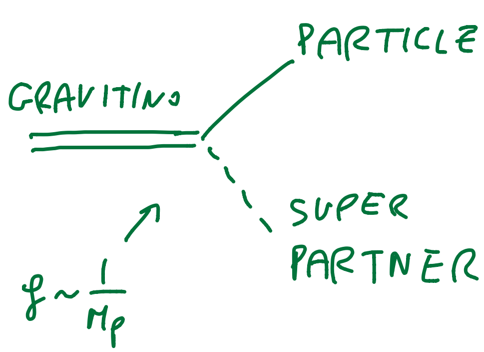
$$\Rightarrow \boxed{n_{3/2,0} \simeq cm^{-3}}$$

COMPARE WITH $H_0 \sim \frac{1}{\Pi\, 10^{17} s} \sim \frac{1}{10^{26} m} \sim \frac{1}{10^{28} cm}$

$$\Rightarrow \rho_0 \sim 3 H_0^2 M_P^2 \sim 3\cdot 10^{-56} cm^{-2} \times 10^{36} GeV^2 \sim 3\cdot 10^{-20} cm^{-2} \times 5\cdot 10^{15} m^{-1} GeV$$
$$\sim 10^{-6} cm^{-3} GeV$$

FROM $m_{3/2}\, n_{3/2,0} < \rho_0 \Rightarrow \boxed{m_{3/2} \lesssim keV}$ PAGELS PRIMACK 1982

- THE ABOVE LIMIT HOLDS IF THE GRAVITINO IS STABLE. IF IT IS UNSTABLE, IT MUST DECAY BEFORE BBN OR DECAY PRODUCTS DISSOCIATE LIGHT NUCLEI FORMED AT BBN.



$\Gamma_{DECAY} \propto \frac{1}{M_P^2}$; RECALL Γ HAS THE DIMENSIONS OF AN ENERGY ($\Gamma_{DECAY} \sim \text{LIFETIME}^{-1}$)

SO THE PROPORTIONALITY COEFFICIENT MUST BE AN ENERGY^3

THE ONLY QUANTITY AT OUR DISPOSAL THAT HAS DIMENSIONS OF AN ENERGY IS THE GRAVITINO MASS

$$\Rightarrow \Gamma_{DECAY} \sim \frac{m_{3/2}^3}{M_P^2}$$

THE DECAY HAPPENS WHEN $H = \Gamma$, SO THE TEMPERATURE OF THE THERMAL BATH AT THE DECAY SATISFIES

$$\frac{T_{DECAY}^2}{M_P} = \frac{m_{3/2}^3}{M_P^2} \Rightarrow m_{3/2} \sim \left(M_P\, T_{decay}^2\right)^{1/3}$$

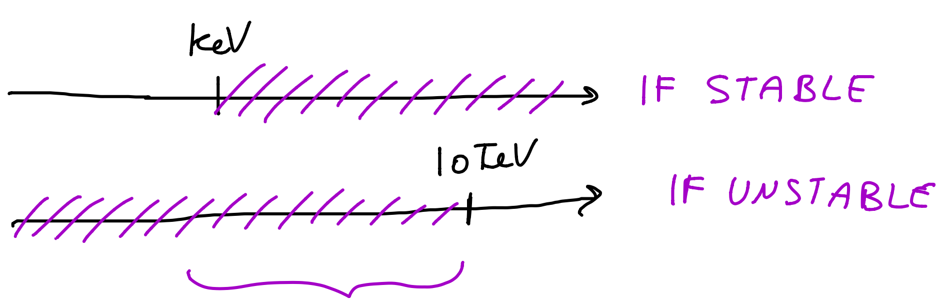
NUCLEAR REACTIONS HAVE TYPICAL ENERGIES $\sim$ MeV SO BBN STARTS AT $T_{BBN} \sim$ MeV. WE REQUIRE THAT $T_{decay} >$ MeV

$$\Rightarrow m_{3/2} \gtrsim \left(M_P \cdot \mathrm{MeV}^2\right)^{1/3} \simeq 10^{\frac{18-2\cdot 3}{3}}\ \mathrm{GeV} \sim 10^4\ \mathrm{GeV}$$

$\boxed{m_{3/2} \gtrsim 10\ \mathrm{TeV}}$ IF GRAVITINOS UNSTABLE

WEINBERG '82

- TO SUMMARIZE



RANGE $\mathrm{keV} \lesssim m_{3/2} \lesssim 10\ \mathrm{TeV}$

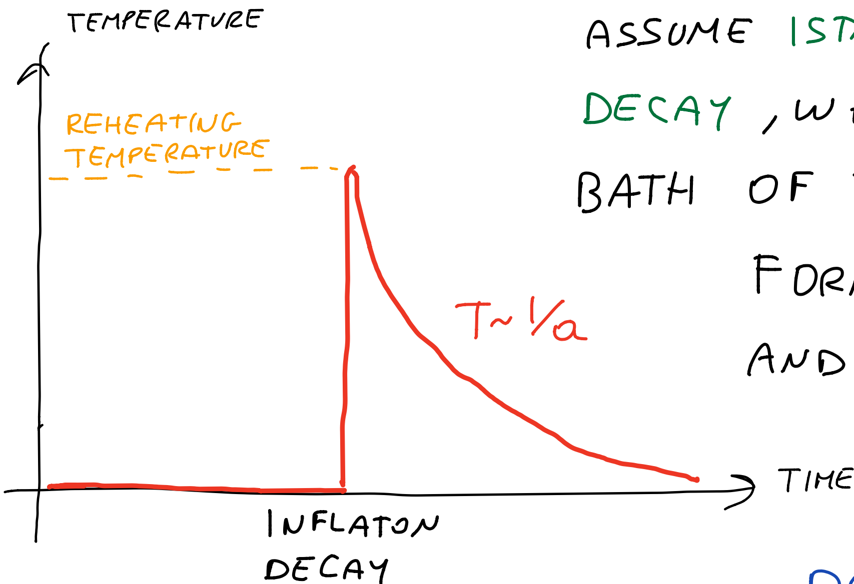
EXCLUDED !

THE RESULTS WE DERIVED ASSUME THERMAL ABUNDANCE

IN PARTICULAR, WE ASSUMED $T_{INITIAL} > M_P \simeq 10^{18}\,GeV$

SO THAT GRAVITINOS INITIALLY PRODUCED.

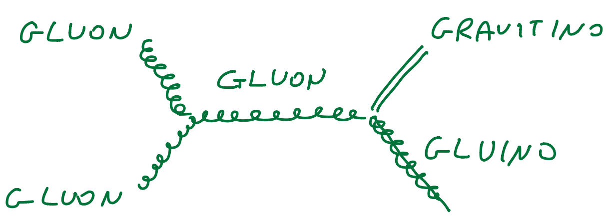
IF INFLATION, PRE-EXISTING GRAVITINOS DILUTED AWAY
HOWEVER, REPOPULATED AT REHEATING. HOW MANY ?



ASSUME ISTANTANEOUS INFLATON DECAY, WHERE A THERMAL BATH OF TEMPERATURE T_{RH} IS FORMED. RECALL $\rho_{RAD} \propto a^{-4}$ AND $\rho_{RAD} \propto T^4$

$\Downarrow$

TEMPERATURE DECREASES AS $T \sim \frac{1}{a} \sim \frac{1}{t^{1/2}}$ AFTER THERMAL BATH IS FORMED



RECALL $\Gamma \sim \frac{T^3}{M_P^2}$

- IN THE TIME $t \sim H^{-1}$, THE NUMBER OF PROCESSES $\int \Gamma dt \sim \frac{\Gamma}{H}$ OCCURS.

$\frac{\Gamma}{H} \sim \frac{T^3}{M_P^2} \frac{M_P}{T^2} = \frac{T}{M_P}$, MAXIMUM PRODUCTION WHEN $T \sim T_{RH}$

ONE GRAVITINO PER PROCESS $\Rightarrow$ $\boxed{n_{3/2} \sim \frac{T_{RH}}{M_P} n_\gamma}$

NANOPOULOS
OLIVE
SREDNICKI '83

INVERSE PROCESS NEGLIGIBLE FOR $T_{RH} \ll M_P$

SLIDE 3