MARCO PELOSO — LECTURE 3

IS REHEATING REALLY INSTANTANEOUS ?

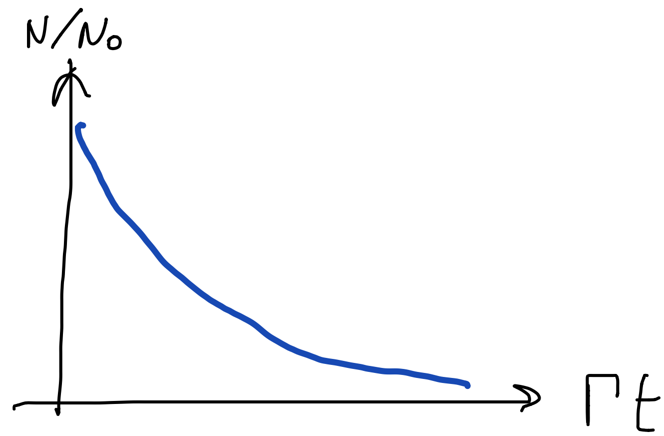
RECALL $H \sim \frac{\rho^{1/2}}{M_p} \sim \frac{1}{t} \Rightarrow \frac{t_{RH}}{t_{END}} \sim \frac{\rho_{END}^{1/2}}{\rho_{RH}^{1/2}} \sim \frac{\rho_{INF}^{1/2}}{T_{RH}^2} \sim \frac{10^{33}\,\mathrm{GeV}^2\, r^{1/2}}{T_{RH}^2}$

FOR $T_{RH} = 10^5\,\mathrm{GeV}$, $\boxed{t_{RH} \sim 10^{22}\, r^{1/2}\, t_{END}}$

FROM $N(t) = N_0\, e^{-\Gamma t}$

WE NOTE THAT

$$\left|\frac{dN}{dt}\right| = N_0\, \Gamma\, e^{-\Gamma t}$$



IS GREATES AT THE INITIAL TIME, SO WE EXPECT THE FORMATION OF A THERMAL BATH AT TIMES $t \ll \Gamma^{-1}$

(ASSUMPTION OF THERMALIZATION OF THE DECAY PRODUCTS FAR FROM TRIVIAL. IT APPEARS TO BE A REASONABLE ASSUMPTION IN PRESENCE OF GAUGE INTERACTIONS

DAVIDSON, SARKAR '00; HARIGAYA, MUKAIDA '13)

ASSUMING NON-INSTANTANEOUS DECAY, BUT INSTANTANEOUS THERMALIZATION OF THE DECAY PRODUCTS, HOW DOES THE TEMPERATURE EVOLVE ?

AND HOW DOES THE GRAVITINO ABUNDANCE ?

THE EVOLUTION OF THE INFLATON + RADIATION ENERGY DENSITY IS GOVERNED BY

$$\begin{cases} \dot{\rho}_\phi + (3H+\Gamma)\rho_\phi = 0 \\ \dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\rho_\phi \\ \rho_\phi + \rho_\gamma = 3M_p^2 H^2 \end{cases}$$

THE INFLATON DECAY, WITH RATE Γ, DECREASES ρ_ϕ & INCREASES ρ_γ

INTERESTED IN EVOLUTION OF ρ_γ AT VERY EARLY TIMES, ASSUMING $\rho_\gamma = 0$ AT THE END OF INFLATION

AT VERY EARLY TIMES, $\rho_\gamma \ll \rho_\phi$ & $\Gamma \ll H$ ← 1/TIME

↑ 1/LIFETIME

WITH THESE APPROXIMATIONS, THE ABOVE SYSTEM BECOMES

$$\begin{cases} \dot{\rho}_\phi + 3H\rho_\phi = 0 \\ \dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\rho_\phi \\ \rho_\phi = 3M_p^2 H^2 \end{cases}$$

THE FIRST & THIRD EQ. GIVE MATTER DOMINATION FROM THE (OSCILLATING) INFLATON

$$\rho_\phi = \frac{\rho_{\phi,in}}{a^3} \; ; \; a = \left(\frac{t}{t_{in}}\right)^{2/3}$$

(NOTICE WE NORMALIZED $a = 1$ AT t_{IN})

CHANGING VARIABLE $\frac{d}{dt} = aH\frac{d}{da}$, 2nd EQ. BECOMES

$$aH\frac{d}{da}\left(a^4\rho_\gamma\right) = a^4\Gamma\rho_\phi$$

$$\Rightarrow \frac{d}{da}\left(a^4 \rho_\gamma\right) = \sqrt{3}\, M_p \Gamma a^3 \sqrt{\rho_\phi} = \sqrt{3}\, M_p \Gamma a^3 \sqrt{\frac{\rho_{\phi,in}}{a^3}}$$

$$= \sqrt{3\rho_{\phi,in}}\, M_p \Gamma a^{3/2} \equiv C a^{3/2}$$

WE INTEGRATE IT TO OBTAIN

$$\int_0^{a^4\rho_\gamma} d\left(a^4\rho_\gamma\right) = C \int_1^a a^{3/2}\, da \rightarrow a^4\rho_\gamma = C\,\frac{2}{5}\left(a^{5/2}-1\right)$$

$$\boxed{\rho_\gamma = \frac{2}{5}\sqrt{3\rho_{\phi,in}}\, M_p \Gamma\, \frac{a^{5/2}-1}{a^4}}$$

FUNCTION OF a STARTS AT 0 ($a_{in}=1$) AND REACHES A MAX AT $a = \left(\frac{8}{3}\right)^{2/5}$, WHERE IT EVALUATES TO 0.35

$$\Rightarrow \rho_{\gamma,MAX} = \frac{2}{5}\sqrt{3\rho_{\phi,in}}\, M_p \Gamma\, 0.35 \simeq 0.24\, \Gamma M_p\, \rho_{\phi,in}^{1/2}$$

→ IF WE INSTEAD ASSUME INSTANTANEOUS REHEATING WHEN $\Gamma = H$, WE HAVE

$$\Gamma = H = \frac{\sqrt{\rho_{\gamma,inst}}}{\sqrt{3}M_p}$$

(THE INFLATON NOW IS ASSUMED TO DECAY AT THIS TIME, PRODUCING A THERMAL BATH WITH ENERGY DENSITY $\rho_{\gamma,inst}$)

$$\rho_{\gamma,inst} = 3 M_p^2 \Gamma^2$$

⇒ THE RATIO BETWEEN THE MAXIMUM ENERGY DENSITY, AND THE ENERGY DENSITY OBTAINED UNDER THE ASSUMPTION OF INSTANTANEOUS DECAY IS

$$\frac{\rho_{\gamma,MAX}}{\rho_{\gamma,inst}} \simeq \frac{0.24\,\Gamma\, M_p\, \rho_{\phi,in}^{1/2}}{3\, M_p^2\, \Gamma^2} = \frac{0.08\,\sqrt{3}\, M_p H_{in}}{M_p \Gamma} \simeq 0.14\,\frac{H_{in}}{\Gamma}$$

THIS RATIO CAN BE >>1 BY MANY ORDERS OF MAGNITUDES

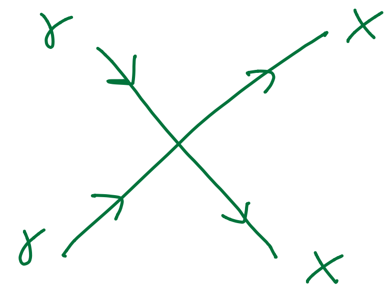
SLIDE 4

$\rho_{\gamma,inst}$ IS THE ENERGY DENSITY AT $H \simeq \Gamma$ WHEN MOST OF THE INFLATON HAS DECAYED, AND THE THERMAL BATH BECOMES DOMINANT. WE SAW THAT THE MAXIMUM ENERGY DENSITY IS ACHIEVED MUCH EARLIER, WHEN THE THERMAL BATH IS STILL SUBDOMINANT. MOREOVER, WE SAW THAT $\rho_{\gamma,MAX} >> \rho_{\gamma,INST}$

GIVEN THIS, WHY IS INSTANTANEOUS REHEATING SO POPULAR?

CONSIDER A SPECIES X COUPLED TO THE THERMAL BATH WITH CROSS SECTION σ_X

$$\frac{dn_X}{dt} + 3Hn_X = \sigma_X n_\gamma^2$$

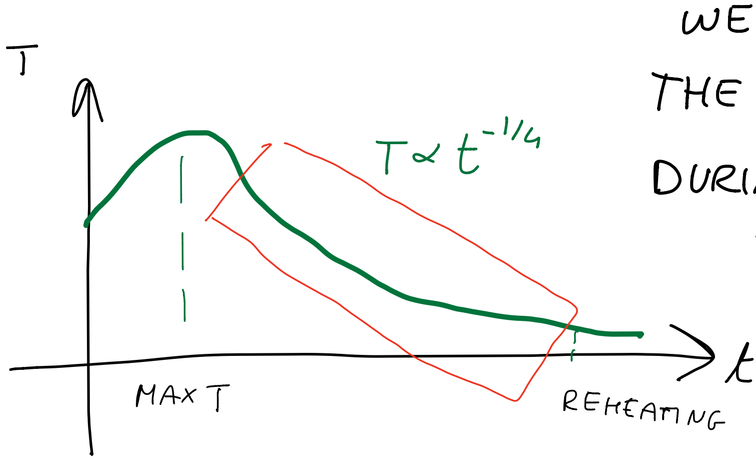


CONSIDER THE RELATIVE ABUNDANCE $r_X = \frac{n_X}{n_\gamma}$

$n_\gamma \propto T^3$

$\Rightarrow \frac{1}{n_\gamma}\frac{dn_\gamma}{dt} = 3\frac{\dot{T}}{T}$

$$\Rightarrow \dot{r}_X + 3\left(H + \frac{\dot{T}}{T}\right) r_X = \sigma_X n_\gamma$$



WE ARE INTERESTED IN THE PRODUCTION OF X DURING THE STAGE BETWEEN T_{MAX} & T_{REH}, WHEN THE THERMAL BATH IS STILL SUBDOMINANT

WE SAW THAT $T \propto t^{-1/4}$ IN THIS STAGE

MATTER DOMINATION FROM OSCILLATING INFLATON

$$H + \frac{\dot{T}}{T} = \frac{2}{3t} + \frac{\frac{d}{dt}\left(t^{-1/4}\right)}{t^{-1/4}} = \frac{2}{3t} - \frac{1}{4t} = \frac{5}{12t}$$

$$\boxed{\text{ASSUME} \quad \sigma_x \propto T^n}$$

$$\Rightarrow \sigma_x n_\gamma \propto T^{n+3} \propto t^{-\frac{n+3}{4}}$$

$$\text{OR} \quad \sigma_x n_\gamma = c\, t^{-\frac{n+3}{4}}$$

A CONSTANT

$$\Rightarrow \dot{r}_x + \frac{5}{4}\frac{r_x}{t} = c\, t^{-\frac{n+3}{4}}$$

CONVENIENT TO DEFINE $\tilde{r}_x \equiv t^{5/4} r_x$

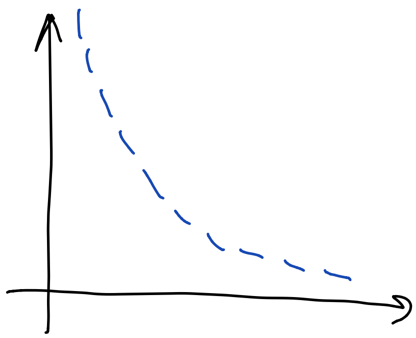
$$\Rightarrow \dot{\tilde{r}}_x = \frac{5}{4} t^{1/4} r_x + t^{5/4}\dot{r}_x = t^{5/4}\left(\dot{r}_x + \frac{5}{4t} r_x\right)$$

$$\Rightarrow \dot{\tilde{r}}_x = c\, t^{\frac{5}{4}}\, t^{-\frac{n+3}{4}} = c\, t^{-\frac{n}{4}+\frac{1}{2}}$$

$$\Rightarrow r_x = t^{-5/4}\int_{t_{END}}^{t} dt'\, c\,(t')^{-\frac{n}{4}+\frac{1}{2}}$$

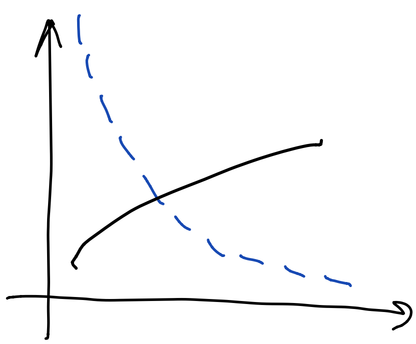
THE EXPONENT INDICATES WHICH REGIME CONTROLS THE PRODUCTION

FOR INSTANCE



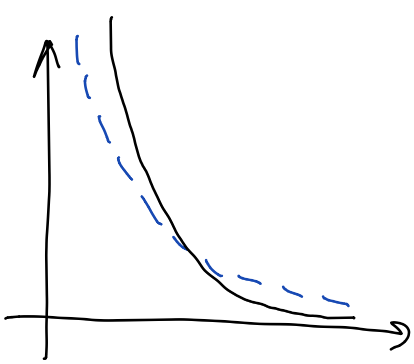
$$\int_{t_{END}}^{t} \frac{dt'}{t'} = \ln \frac{t}{t_{END}}$$

EQUAL CONTRIBUTION FROM EARLY AND FROM LATE TIMES



$$\int_{t_{END}}^{t} dt' \, t'^{\alpha} \quad \text{WITH} \quad \alpha > -1$$

GREATER INTEGRAND AT LATE TIMES $\Rightarrow$ ONLY LATE TIMES CONTRIBUTE



$$\int_{t_{END}}^{t} dt' \, t'^{\alpha} \quad \text{WITH} \quad \alpha < -1$$

ONLY EARLY TIMES CONTRIBUTE

$\Rightarrow$ LATE TIMES DOMINATE FOR $-\frac{n}{4}+\frac{1}{2} > -1$

$\Rightarrow -\frac{n}{4} > -\frac{3}{2} \Rightarrow \boxed{n<6}$

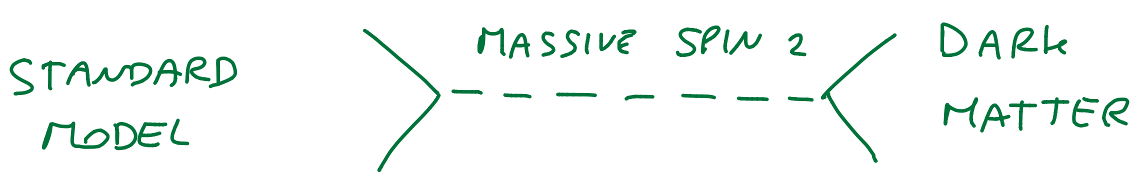
- EXAMPLE : GRAVITINOS $\sigma \simeq \frac{1}{M_p^2} \longrightarrow n=0$

  $\Rightarrow$ PRODUCTION DOMINATED BY LATE LIMES

  $\equiv$ INSTANTANEOUS REHEATING IS A VERY GOOD APPROXIMATION

- EXAMPLE : SPIN-2 PORTAL

ASSUME THAT THE DARK MATTER IS COUPLED TO THE STANDARD MODEL ONLY BY A SPIN-2 PARTICLE OF LARGE MASS $M \gg T_{MAX}$



THE COUPLINGS OF THE SPIN-2 PARTICLE ARE ANALOGOUS TO THAT OF THE GRAVITON

$$\mathcal{L}_{GRAVITON} = \frac{h_{\mu\nu} T^{\mu\nu}}{M_p}$$

$\leftarrow$ ENERGY MOMENTUM ($T^{\mu\nu}$)

$\leftarrow$ MASS SCALE (M_p)

$$\Rightarrow \mathcal{L}_{SPIN\ 2} = \frac{g_{SM}}{\Lambda} H_{\mu\nu} T^{\mu\nu}_{SM} + \frac{g_{DM}}{\Lambda} H_{\mu\nu} T^{\mu\nu}_{DM}$$

THE ABOVE DIAGRAM HAS AMPLITUDE $\sim \frac{g_{SM}}{\Lambda} \underbrace{\frac{1}{\text{MOMENTUM}^2 - M^2}}_{\text{PROPAGATOR}} \frac{g_{DM}}{\Lambda}$

MASS SPIN-2 MEDIATOR

$$\Rightarrow |\text{AMPLITUDE}|^2 \propto \frac{1}{\Lambda^4 M^4}$$

(WE CARE ONLY ABOUT THE SCALING WITH DIMENSIONFUL QUANTITIES ; g_{SM} & g_{DM} ARE DIMENSIONLESS)

THE CROSS SECTION IS $\propto |\text{AMPLITUDE}|^2$ AND IT HAS MASS DIMENSION -2

$$\Rightarrow \sigma \propto \frac{T^6}{\Lambda^4 M^8}$$

THE ONLY POSSIBLE MASS SCALE

$\boxed{n = 6}$

IN THIS CASE EARLY & LATE TIMES CONTRIBUTE IN A COMPARABLE FASHION, AND THE ISTANTANEOUS REHEATING APPROX. GIVES n_x WRONG BY $\mathcal{O}(1)$ FACTOR

HARD TO FIND MODELS WITH $\sigma \sim T^n$ AND $n \geq 6$

THIS IS WHY, TYPICALLY, THE INSTANTANEOUS REHEATING APPROX IS GOOD

SLIDE 5

NONPERTURBATIVE REHEATING

- IN COMPUTING Γ_ϕ WE TREATED THE INFLATON AS A COLLECTION OF INDEPENDENT QUANTA.

- COHERENT OSCILLATIONS $\phi(t) \to$ FASTER DECAY, AT SUFFICIENTLY LARGE COUPLINGS

SHTANOV, TRASCHEN, BRANDENBERGER '94

KOFMAN, LINDE, STAROBINSKY '94 ; '97

EXAMPLE: MASSIVE INFLATON ϕ, OSCILLATING ABOUT MINIMUM OF POTENTIAL, WITH QUARTIC COUPLING TO ANOTHER SCALAR FIELD χ

$$V = \frac{1}{2} m^2 \phi^2 + \frac{g^2}{2} \phi \chi^2$$

NEXT SLIDE, RESULT OF LATTICE SIMULATION (LATTICEEASY: FELDER, TKACHEV). WE WILL SEE

THREE PHASES:

(1) COHERENT INFLATON OSCILLATIONS

(2) χ EXCITATIONS

(3) ϕ EXCITATIONS

SLIDE 6

WHILE LATTICE SIMULATIONS ARE REQUIRED FOR THE FULL DYNAMICS, THE EARLY STAGES, AND THE INITIAL EXCITATIONS OF χ CAN BE OBTAINED ANALYTICALLY

SLIDE 7

OUR GOAL

* DEFINE THE MEANING OF OCCUPATION NUMBER IN THIS CONTEXT
* COMPUTE THE EVOLUTION OF THE OCCUPATION #

PROGRAM

- REVIEW OF STANDARD QFT: QUANTIZATION OF A FREE SCALAR FIELD
- QUANTIZATION OF A FREE SCALAR FIELD ON A TIME-DEPENDENT BACKGROUND