Cosmological SPT & beyond - 1

M. Pietroni - Parma

GGI School “Lectures on Fundamental Interactions”, Jan. 25-29 2021

(drop the time dependence)

$$\frac{\partial}{\partial \tau}\delta_R(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta_R(\mathbf{x}))v_R^i(\mathbf{x})\right] = 0$$

continuity eq.

$$\frac{\partial}{\partial \tau}v_R^i(\mathbf{x}) + \mathcal{H}v_R^i(\mathbf{x}) + v_R^k(\mathbf{x})\frac{\partial}{\partial x^k}v_R^i(\mathbf{x}) = -\nabla_x^i\phi_R(\mathbf{x}) - J_\sigma^i(\mathbf{x}) - J_1^i(\mathbf{x})$$

Euler eq.

short-distance effects

$$J_\sigma^i(\mathbf{x}) \equiv \frac{1}{1+\delta_R(\mathbf{x})}\frac{\partial}{\partial x^k}\left((1+\delta_R(\mathbf{x}))\sigma_R^{ki}(\mathbf{x})\right)$$

$$J_1^i(\mathbf{x}) \equiv \frac{1}{1+\delta(\mathbf{x})}\left(\langle(1+\delta)\nabla^i\phi\rangle_R(\mathbf{x}) - (1+\delta_R)(\mathbf{x})\nabla^i\phi_R(\mathbf{x})\right)$$

To close the system, we must provide information on the short-distance effects

Buchert, Dominguez, ’05, Pueblas Scoccimarro, ’09, Baumann et al. ’10

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 ....

Single stream approximation

Set $\sigma^{ij} = \omega^{ijk} = \cdots = \nabla \times \mathbf{v} = 0$

...+ no higher moments, no vorticity,...

$\updownarrow$

$$f(\mathbf{x}, \mathbf{p}, \tau) = \bar{\rho}\,(1 + \delta(\mathbf{x}, \tau))\,\delta_D(\mathbf{p} - am\nabla\varphi(\mathbf{x}, \tau))$$

System described by $\delta(\mathbf{x}, \tau)\,, \quad \theta(\mathbf{x}, \tau) \equiv \nabla^2\varphi(\mathbf{x}, \tau) \equiv \nabla \cdot \mathbf{v}(\mathbf{x}, \tau)$

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot ((1 + \delta)\mathbf{v})$$ continuity

$$\frac{\partial\, \mathbf{v}}{\partial\, \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi$$ Euler

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$ Poisson

warning: self-consistent … but ultimately wrong!

Linear order solution

$$\frac{\partial \delta}{\partial \tau} + \theta = 0$$

$$\frac{\partial \theta}{\partial \tau} + \mathcal{H}\theta = -\nabla^2 \phi$$

$$\nabla^2 \phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} = \frac{3}{2}\Omega_M \mathcal{H}^2 \delta$$

linear GR equation for $k \gg \mathcal{H}$

Solution : $\delta^{(1)}(\mathbf{k},\tau) = -\dfrac{\theta^{(1)}(\mathbf{k},\tau)}{\mathcal{H} f_\pm} = \delta(\mathbf{k},\tau_{in}) D_\pm(\tau)$ $\quad (D_\pm(\tau_{in}) = 1)$ $\quad \left(f_\pm \equiv \dfrac{d \ln D_\pm}{d \ln a} \right)$

growing/decaying mode

For EdS (Ω_M=1): $\quad D_\pm = \left(\dfrac{a(\tau)}{a(\tau_{in})} \right)^{f_\pm} \qquad f_+ = 1,\ f_- = -3/2$

Compact notation

$$\varphi_a(\mathbf{k},\eta) = e^{-\eta}\begin{pmatrix} \delta(\mathbf{k},\eta) \\ -\theta(\mathbf{k},\eta)/(\mathcal{H}f) \end{pmatrix}$$

$$\eta \equiv \log D(\tau)/D(\tau_0)$$

The continuity+Euler+Poisson system reads:

$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta))\,\varphi_b(\mathbf{k},\eta) = e^{\eta}\int \frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{q}-\mathbf{p})\gamma_{abc}(\mathbf{k},\mathbf{q},\mathbf{p})\varphi_b(\mathbf{q},\eta)\varphi_c(\mathbf{p},\eta)$$

linear

$$\Omega_{ab}(\eta) = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\frac{\Omega_m(\eta)}{f^2(\eta)} & \frac{3}{2}\frac{\Omega_m(\eta)}{f^2(\eta)} \end{pmatrix}$$

nonlinear

$$\gamma_{112}(\mathbf{k},\mathbf{q},\mathbf{p}) = \gamma_{121}(\mathbf{k},\mathbf{p},\mathbf{q}) = \frac{\mathbf{k}\cdot\mathbf{p}}{2p^2}$$

$$\gamma_{222}(\mathbf{k},\mathbf{q},\mathbf{p}) = \frac{k^2\,\mathbf{q}\cdot\mathbf{p}}{2\,q^2p^2}$$

SPT=Iterative solution

$$\varphi_1^{(1)}(\mathbf{k},\eta) = \varphi_2^{(1)}(\mathbf{k},\eta) \equiv \varphi^{(1)}(\mathbf{k})$$

linear solution

$$g_{ab}(\eta) = \left[\begin{pmatrix} 3/5 & 2/5 \\ 3/5 & 2/5 \end{pmatrix} + e^{-5/2\,\eta} \begin{pmatrix} 2/5 & -2/5 \\ -3/5 & 3/5 \end{pmatrix} \right] \Theta(\eta)$$

linear propagator

$$\varphi_a^{(2)}(\mathbf{k},\eta) = \int_0^\eta ds\, g_{ab}(\eta - s)\, e^s\, I_{\mathbf{k},\mathbf{q},\mathbf{p}} \gamma_{bcd}(\mathbf{k},\mathbf{q},\mathbf{p}) \varphi_c^{(1)}(\mathbf{q},s) \varphi_d^{(1)}(\mathbf{p},s)$$
$$= e^{2\eta} I_{\mathbf{k},\mathbf{q}_1,\mathbf{q}_2} F_a^{(2)}(\mathbf{q}_1,\mathbf{q}_2) \varphi^{(1)}(\mathbf{q}_1) \varphi^{(1)}(\mathbf{q}_2)$$

2nd order solution

$$\left(I_{\mathbf{k},\mathbf{q_1},\cdots,\mathbf{q_n}} \equiv \int \frac{d^3 q_1}{(2\pi)^3} \cdots \frac{d^3 q_n}{(2\pi)^3} \delta_D(\mathbf{k} - \sum_{i=1}^n \mathbf{q_i}) \right)$$

...

$$\varphi_a^{(n)}(\mathbf{k},\eta) = e^{n\eta} I_{\mathbf{k},\mathbf{q}_1,\cdots,\mathbf{q}_n} F_a^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n) \varphi^{(1)}(\mathbf{q}_1) \cdots \varphi^{(1)}(\mathbf{q}_n)$$

nth order solution

If the initial conditions are gaussian, then only correlators involving an even number of initial fields are non-vanishing

tree-level

Power spectrum: $\langle\varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{k}',\eta)\rangle = \langle\varphi_a^{(1)}(\mathbf{k},\eta)\varphi_b^{(1)}(\mathbf{k}',\eta)\rangle$

$$+\langle\varphi_a^{(1)}(\mathbf{k},\eta)\varphi_b^{(3)}(\mathbf{k}',\eta)\rangle + \langle\varphi_a^{(3)}(\mathbf{k},\eta)\varphi_b^{(1)}(\mathbf{k}',\eta)\rangle$$

one-loop

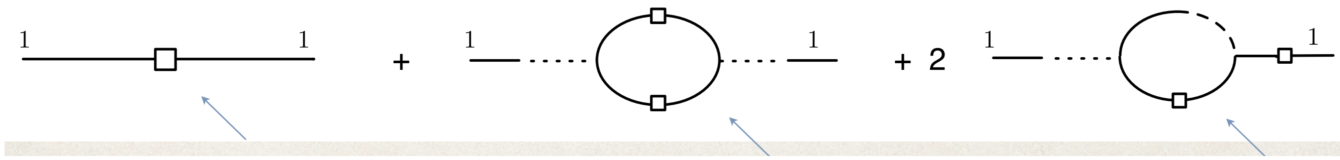
$$+\langle\varphi_a^{(2)}(\mathbf{k},\eta)\varphi_b^{(2)}(\mathbf{k}',\eta)\rangle + O((\varphi^{in})^6)$$

+ ...

Bispectrum: $\langle\varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{k}',\eta)\varphi_c(\mathbf{k}'',\eta)\rangle = \langle\varphi_a^{(2)}(\mathbf{k},\eta)\varphi_b^{(1)}(\mathbf{k}',\eta)\varphi_c^{(1)}(\mathbf{k}'',\eta)\rangle$

tree-level $\quad +2 \text{ permutations} + O((\varphi^{in})^6)$

Computation of the Power Spectrum



$$P(k,\tau) = D^2(\tau) P_{11}(k) + D^4(\tau)\left[P_{22}(k) + P_{13}(k)\right]$$

linear PS: output from CAMB, CLASS, …

$$P_{13}(k) = \frac{k^3 P_{11}(k)}{252\,(2\pi)^2} \int_0^\infty dr P_{11}(kr) \left[\frac{12}{r^2} - 158 + 100r^2 - 42r^4 + \frac{3}{r^3}\left(r^2-1\right)^3 (7r^2+2) \ln\left|\frac{1+r}{1-r}\right|\right]$$

$$P_{22}(k) = \frac{k^3}{98\,(2\pi)^2} \int_0^\infty dr P_{11}(kr) \int_{-1}^1 dx P_{11}\left[k\left(1+r^2-2rx\right)^{1/2}\right] \frac{\left(3r+7x-10rx^2\right)^2}{\left(1+r^2-2rx\right)^2}$$

Cosmology information in $P_{11}(k)$ **and** $D(\tau)$

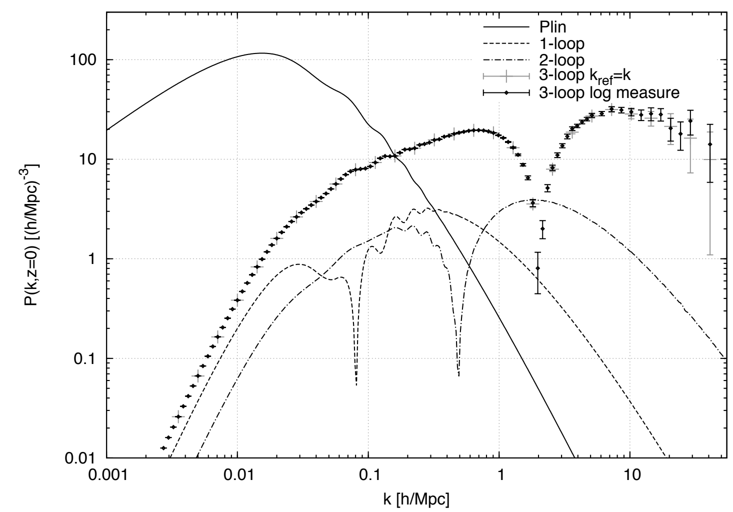
Integrals to be performed numerically for ΛCDM…

FFTLog approach

Simonovic et al 1708.08130

Fourier Transform the PS (with respect to log k)

$$\bar{P}_{\rm lin}(k_n) = \sum_{m=-N/2}^{m=N/2} c_m \, k_n^{\nu + i\eta_m}$$



Log k

(ν is a parameter)

$$\eta_m = \frac{2\pi m}{\log(k_{\rm max}/k_{\rm min})} \qquad c_m = \frac{1}{N}\sum_{l=0}^{N-1} P_{\rm lin}(k_l)\, k_l^{-\nu} k_{\rm min}^{-i\eta_m}\, e^{-2\pi i m l/N}$$

$$P_{22}(k) = 2\int_{\boldsymbol{q}} F_2^2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) P_{\rm lin}(q) P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$F_2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) = \frac{5}{14} + \frac{3k^2}{28q^2} + \frac{3k^2}{28|\boldsymbol{k}-\boldsymbol{q}|^2} - \frac{5q^2}{28|\boldsymbol{k}-\boldsymbol{q}|^2} - \frac{5|\boldsymbol{k}-\boldsymbol{q}|^2}{28q^2} + \frac{k^4}{14|\boldsymbol{k}-\boldsymbol{q}|^2 q^2}$$

The 1-loop integral becomes a combination of

$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_1}|\boldsymbol{k}-\boldsymbol{q}|^{2\nu_2}} \equiv k^{3-2\nu_{12}} \mathsf{I}(\nu_1,\nu_2)$$

with $$\mathsf{I}(\nu_1,\nu_2) = \frac{1}{8\pi^{3/2}} \frac{\Gamma(\frac{3}{2}-\nu_1)\Gamma(\frac{3}{2}-\nu_2)\Gamma(\nu_{12}-\frac{3}{2})}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(3-\nu_{12})}$$

(+ symetries and recursion relations …)

$$\bar{P}_{22}(k) = k^3 \sum_{m_1,m_2} c_{m_1} k^{-2\nu_1} \cdot M_{22}(\nu_1,\nu_2) \cdot c_{m_2} k^{-2\nu_2}$$

Cosmology dependence (PS shape)

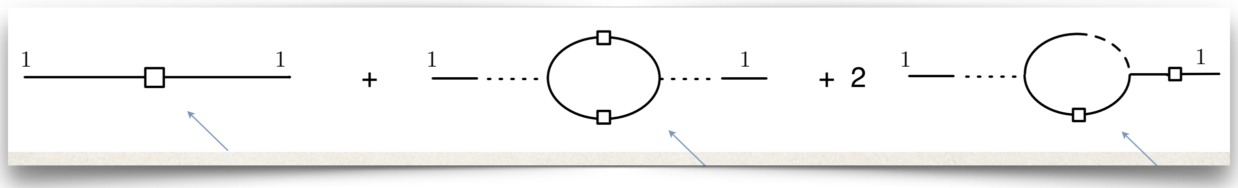
Integrals done once forever

$$M_{22}(\nu_1,\nu_2) = \frac{(\frac{3}{2}-\nu_{12})(\frac{1}{2}-\nu_{12})[\nu_1\nu_2(98\nu_{12}^2-14\nu_{12}+36)-91\nu_{12}^2+3\nu_{12}+58]}{196\,\nu_1(1+\nu_1)(\frac{1}{2}-\nu_1)\,\nu_2(1+\nu_2)(\frac{1}{2}-\nu_2)} \mathsf{I}(\nu_1,\nu_2)$$

Loop integral —> FFT+matrix multiplication: Very Fast!!

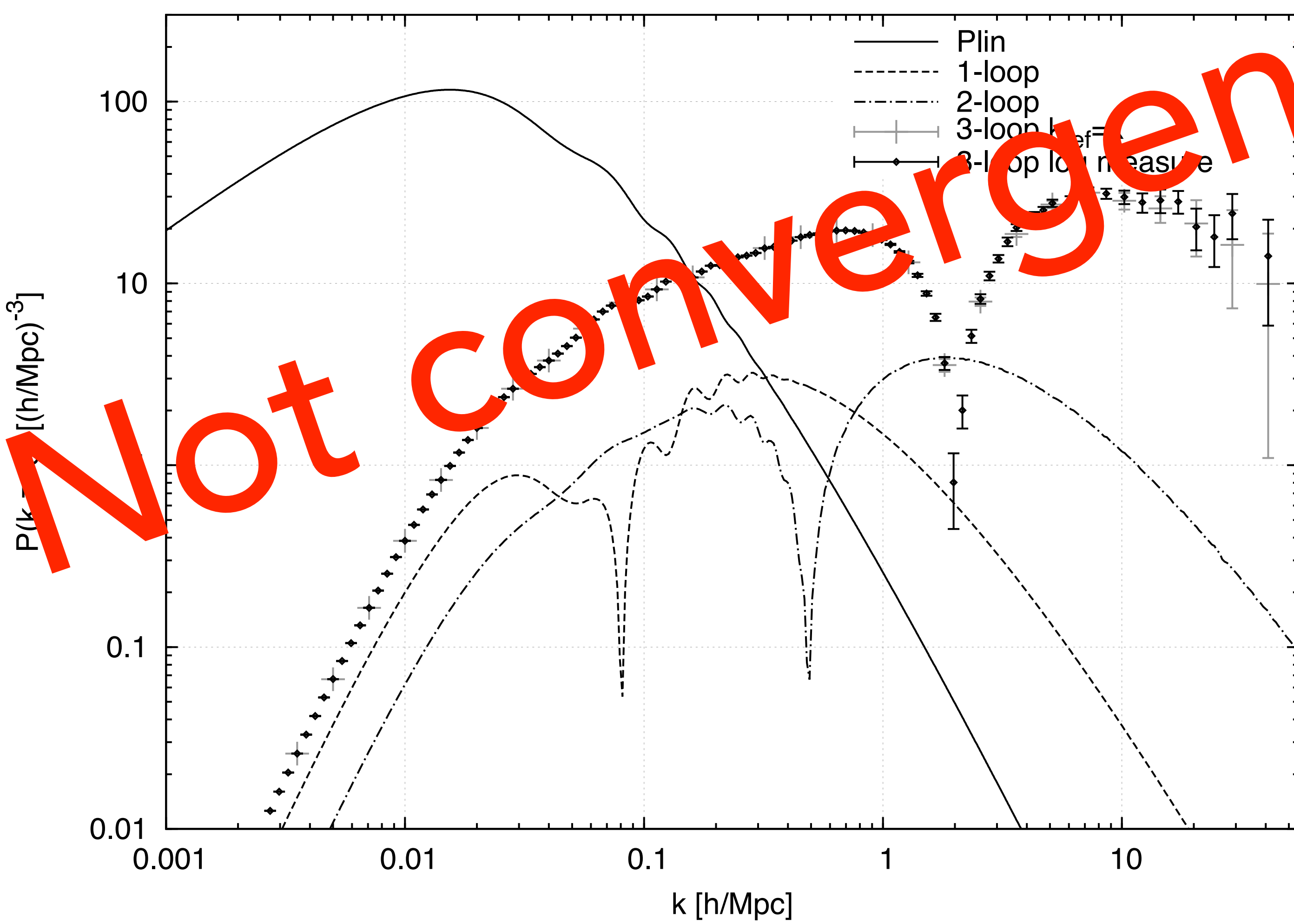
Performance of Standard PT

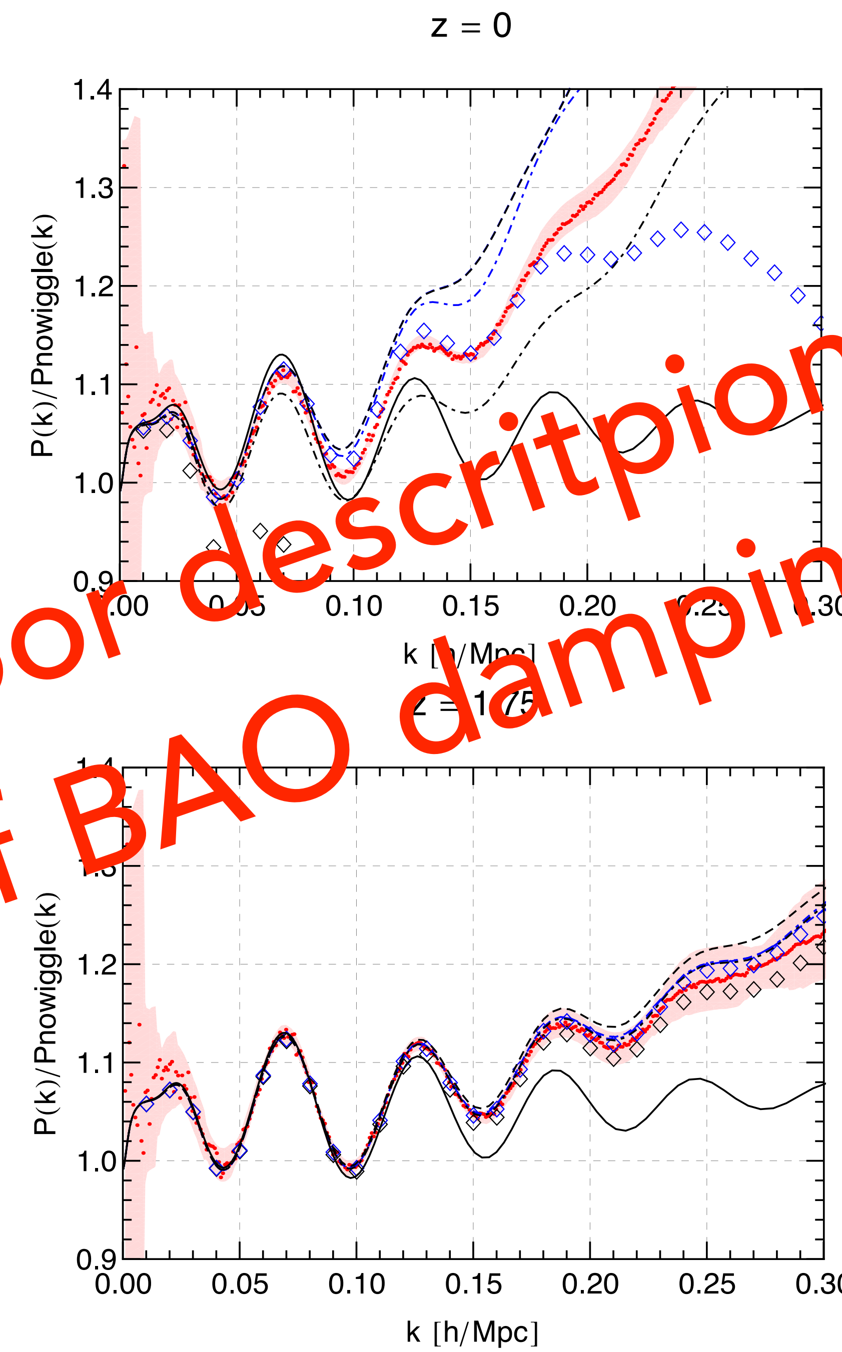
$$P(k,z) = D(z)^2 P^{(1)}(k) + D(z)^4 F^{(1l)}(k) + D(z)^6 F^{(2l)}(k) + \cdots$$



linear

1-loop



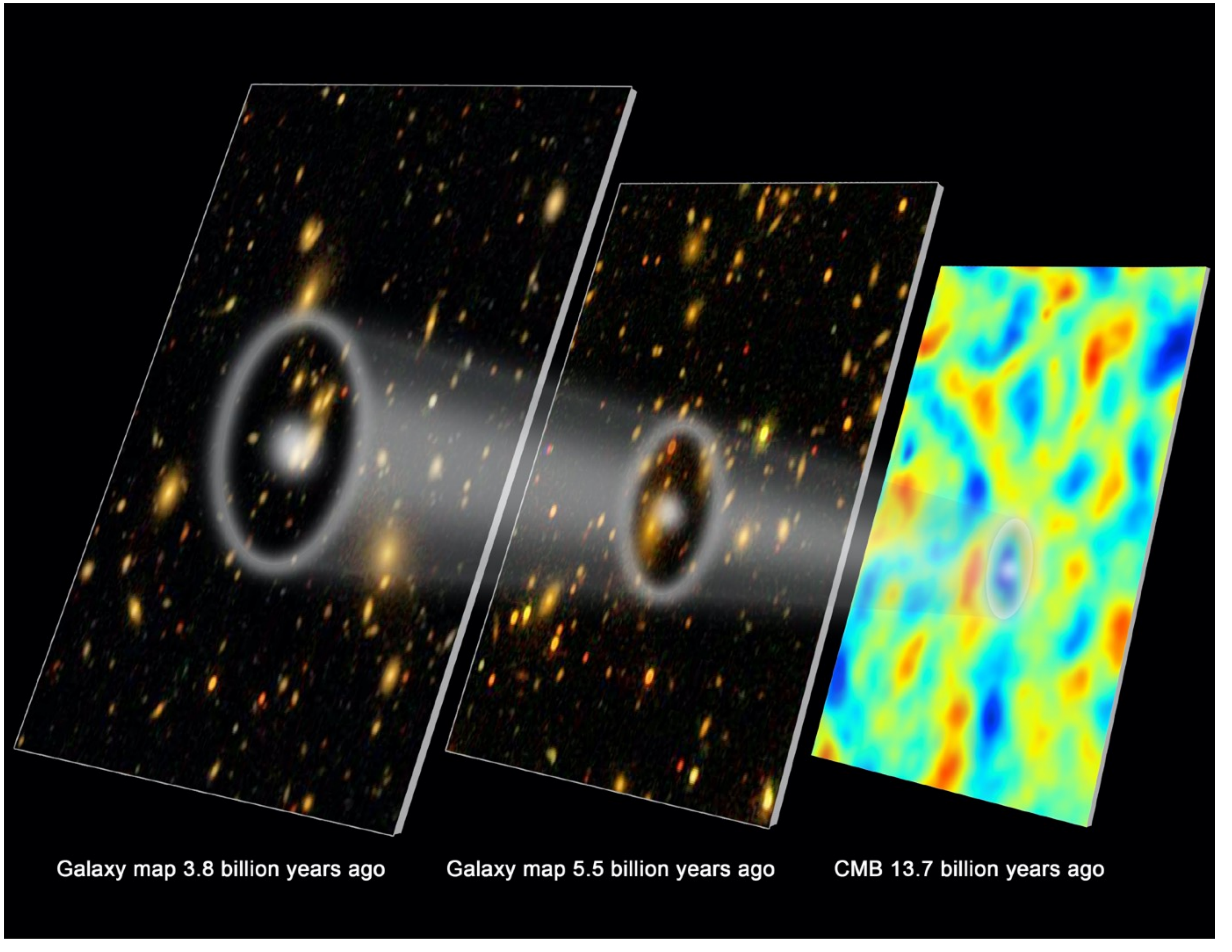


Not convergent !

poor descritpion of BAO damping

Blas et al. 1309.3308

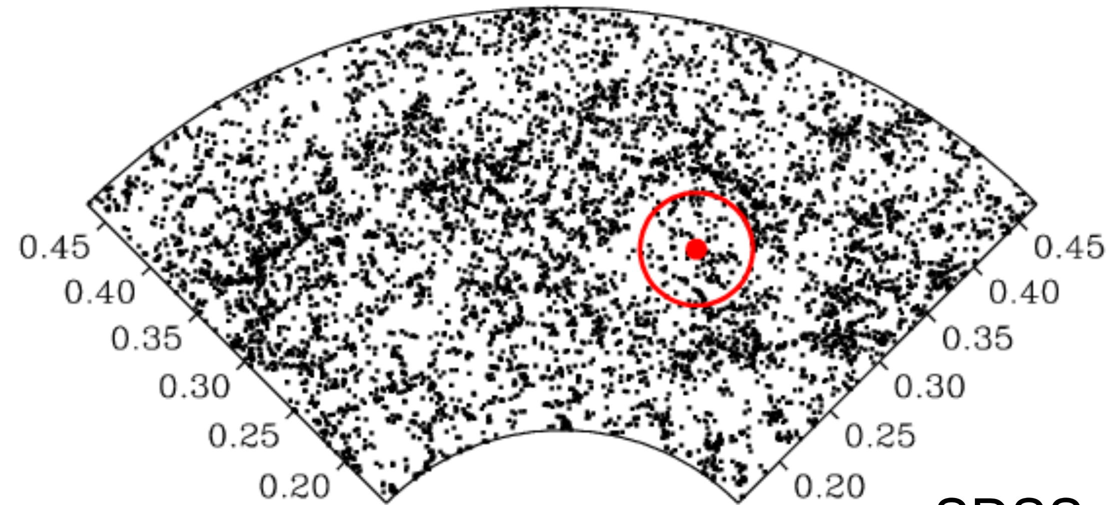
IR effects on Baryonic Acoustic Oscillations



Credit: Eric Huff, the SDSS-III team, and the South Pole Telescope team

$$s = \frac{1}{H_0 \Omega_m^{1/2}} \int_0^{a_*} da \frac{c_s}{(a + a_{eq})^{1/2}} \simeq 110 \mathrm{Mpc}\, h^{-1}$$

comoving sound horizon at recombination



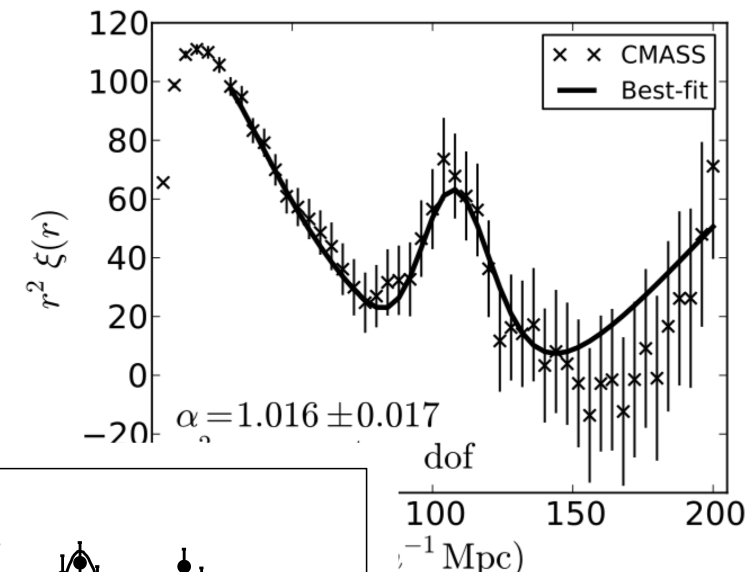
SDSS

same comoving scale seen in CMB anisotropies

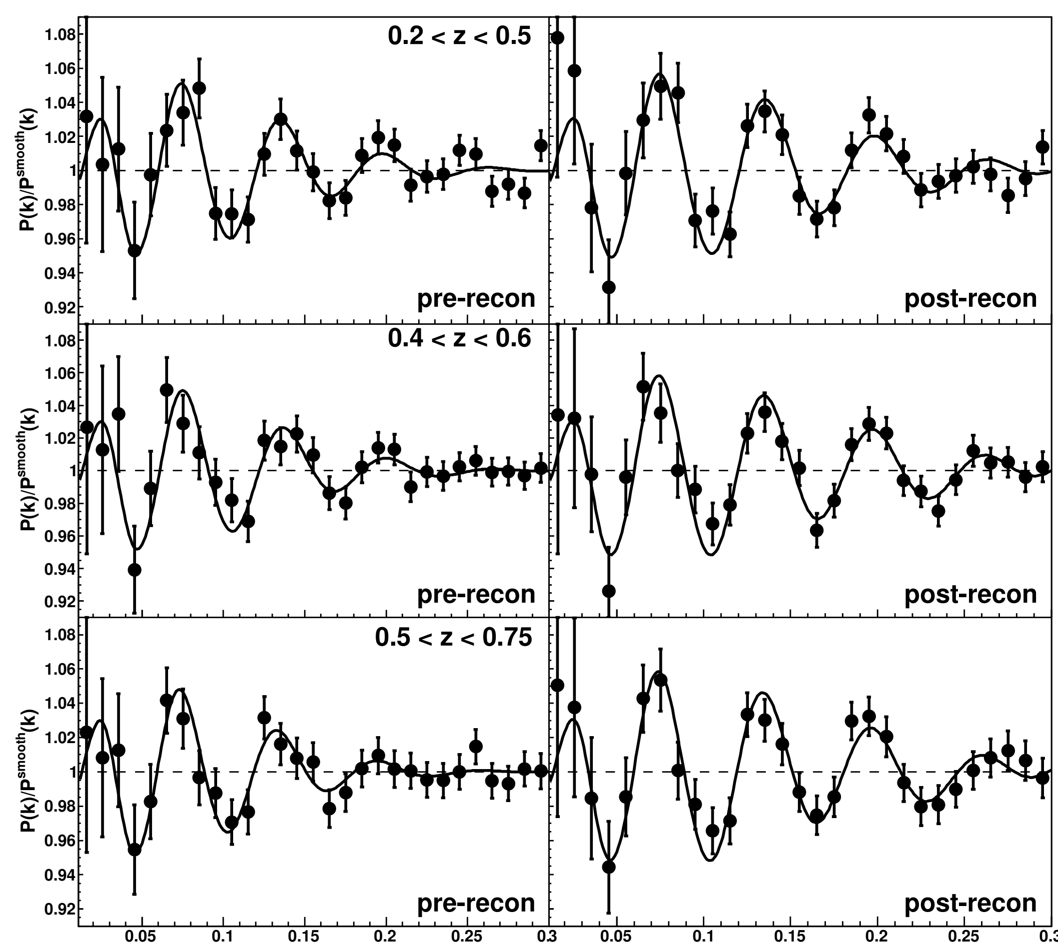
and in LSS at different redshifts.

Map from comoving to (angular and diameter) distances

is cosmology-dependent: STANDARD RULER

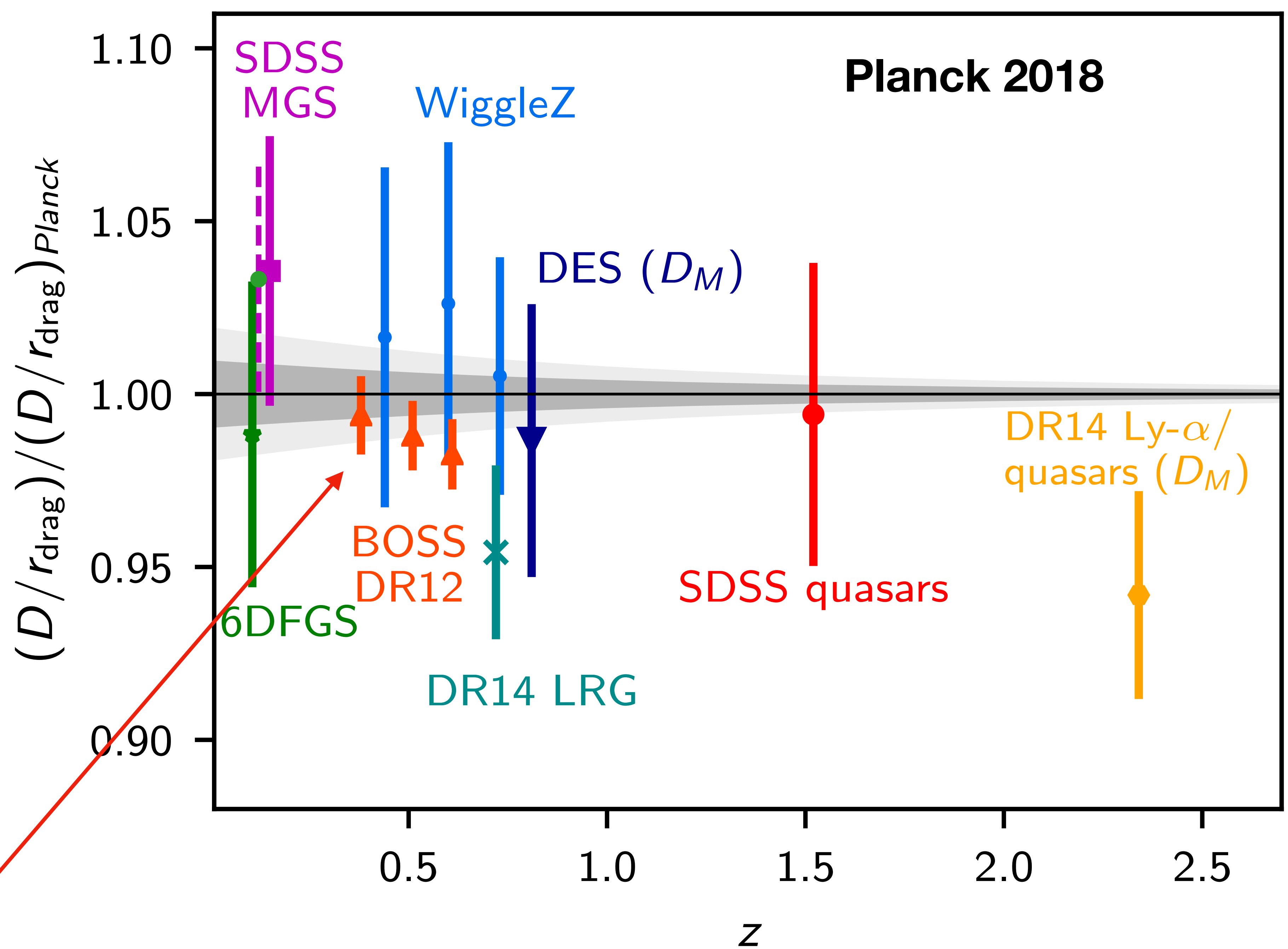


$$k_{BAO} = \frac{2\pi}{s} \simeq 0.057\ h/\mathrm{Mpc}$$



Beutler et al '16

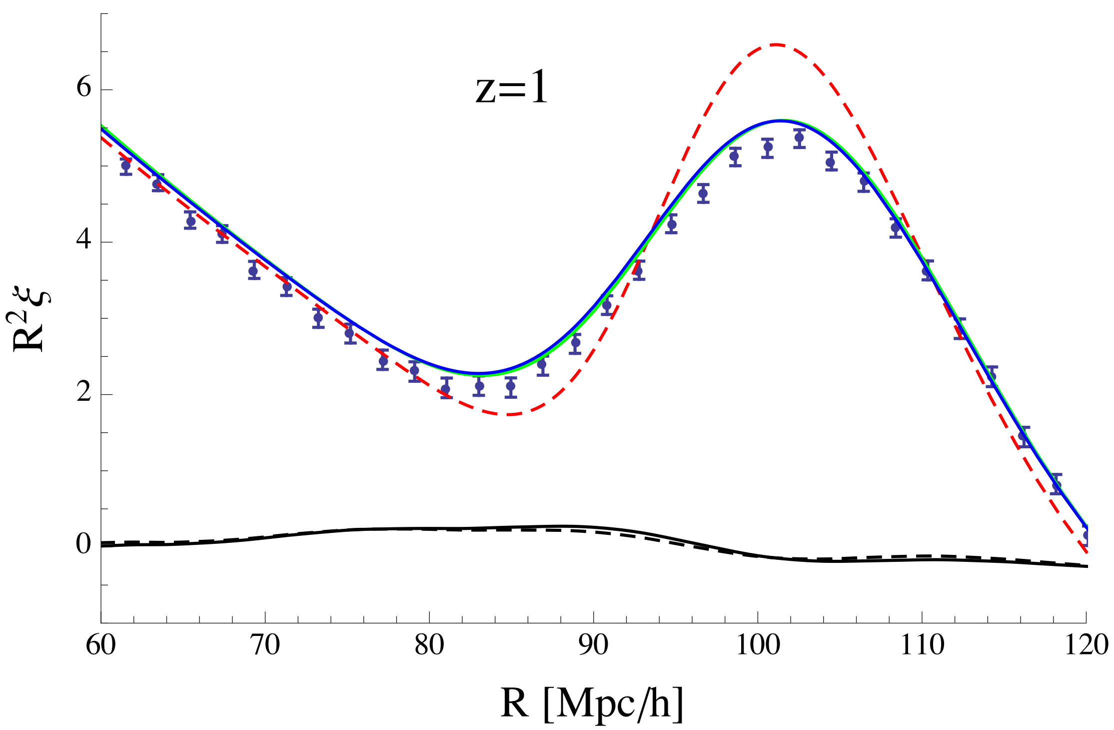
sound horizon from LSS vs. Planck

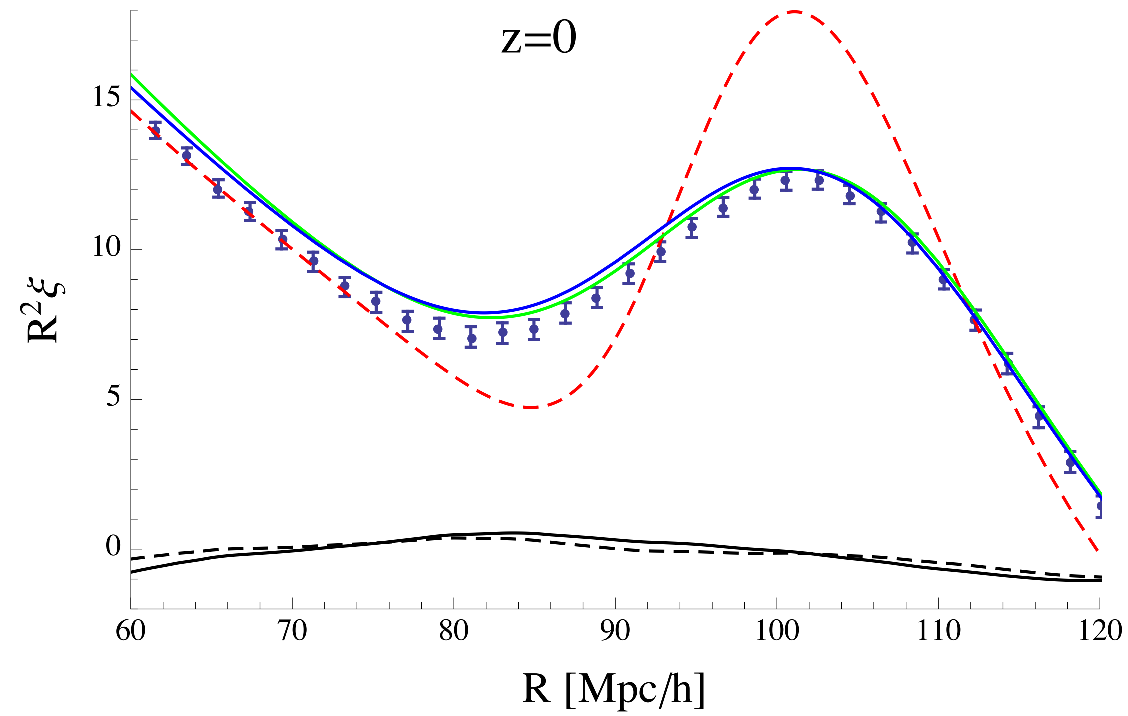


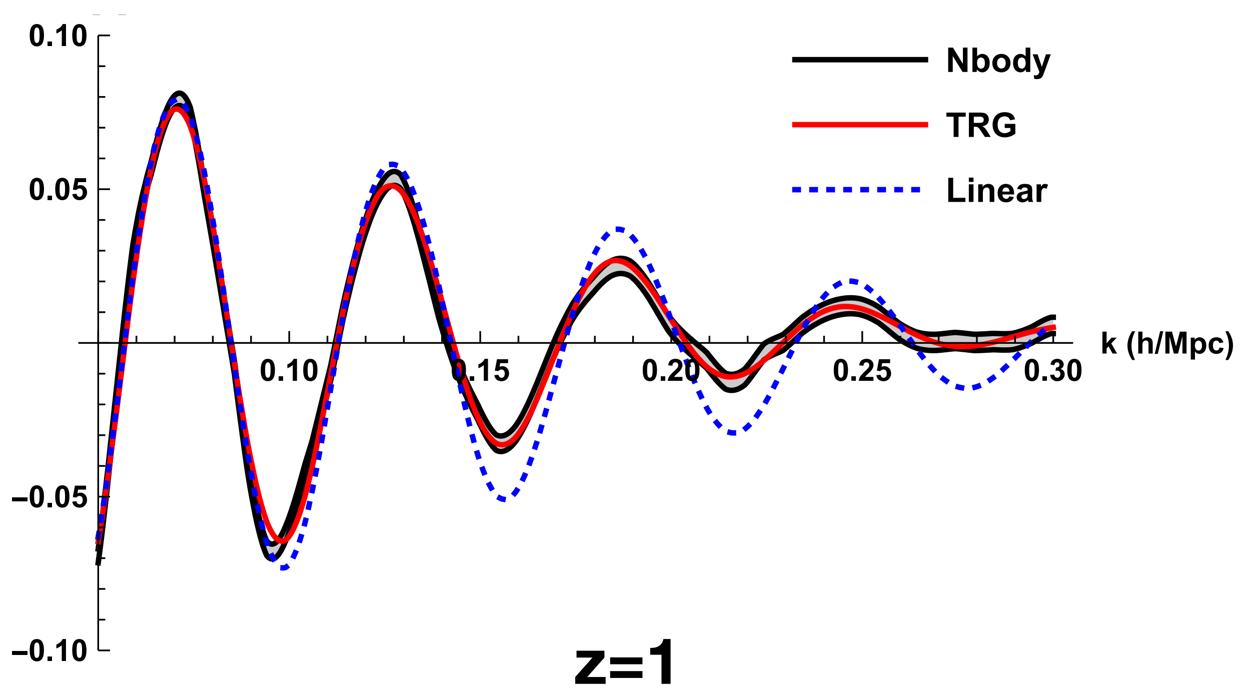
< 2 % error!!

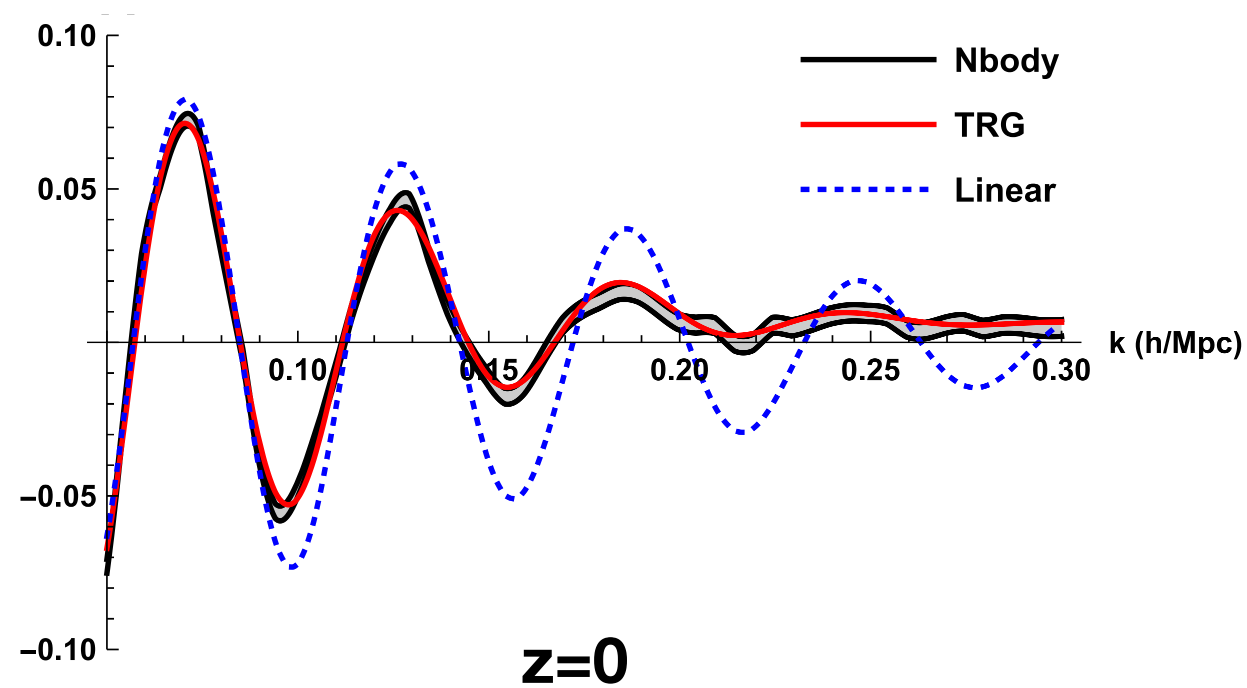
1% error on k_{bao} translates in ~5% error on w_{de}

nonlinear effects on BAO's

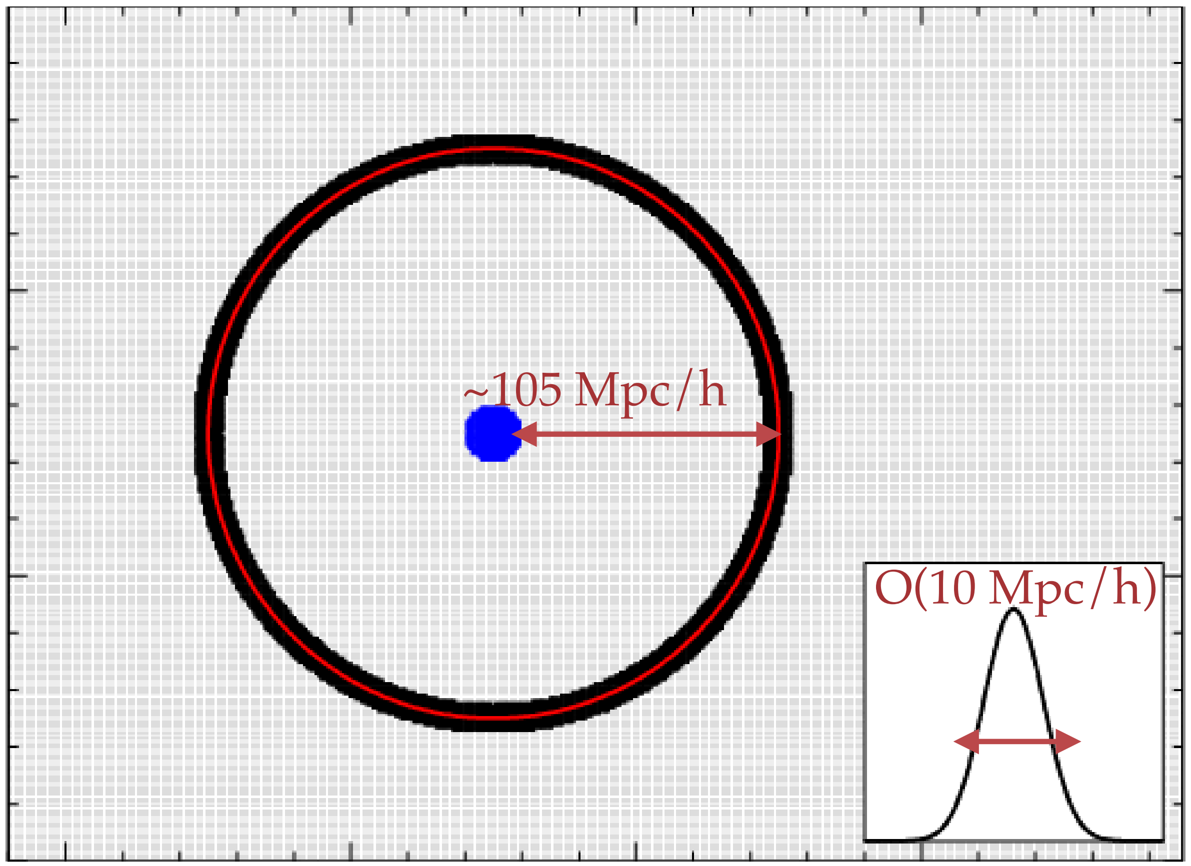


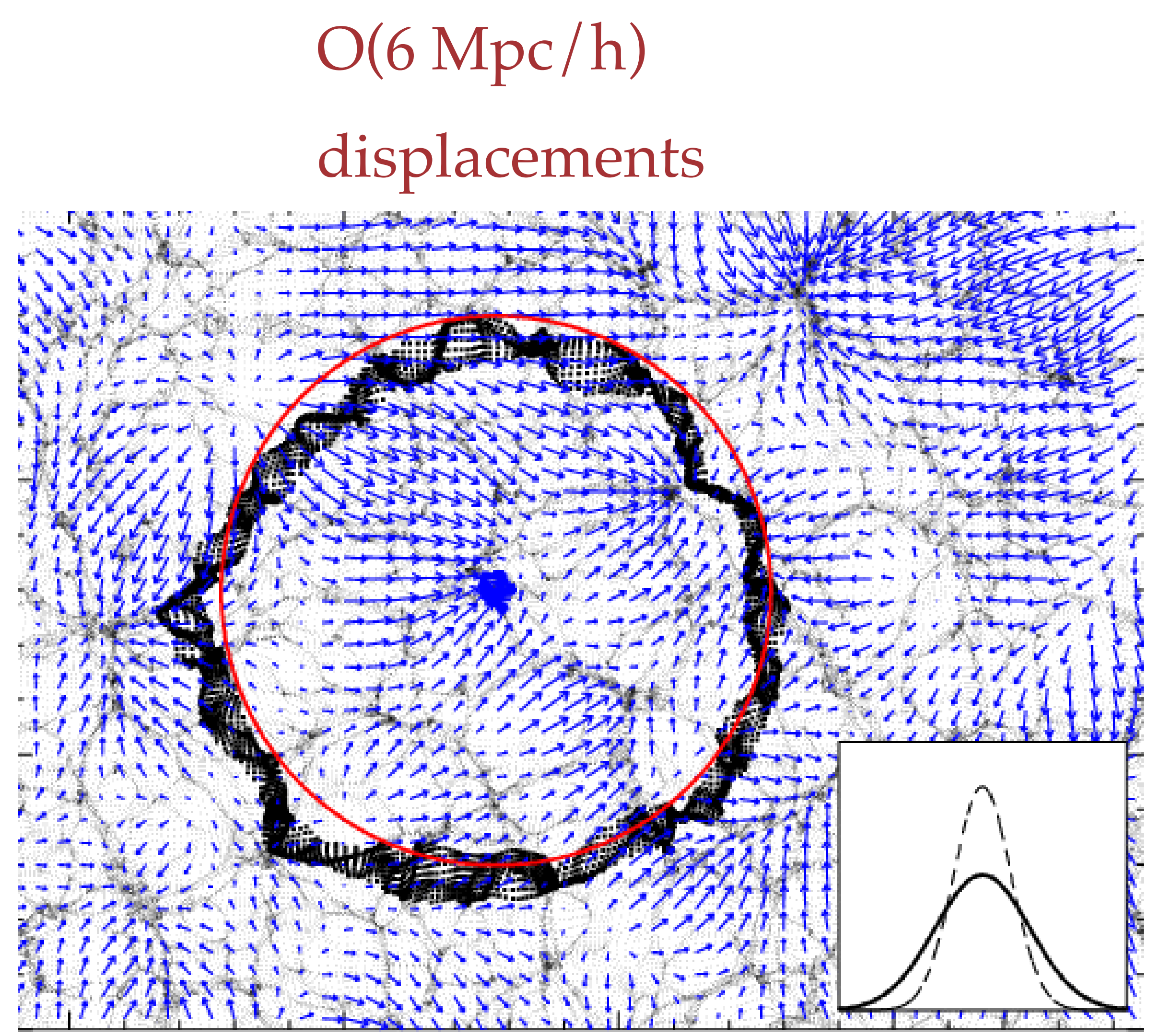






random displacements





Padmanabhan et al 1202.0090

$$\vec{\psi}(\vec{x},\tau) = \int_0^\tau d\tau' \vec{v}(\vec{x},\tau') = \frac{\vec{v}(\vec{x},\tau)}{\mathcal{H}f(\tau)}$$

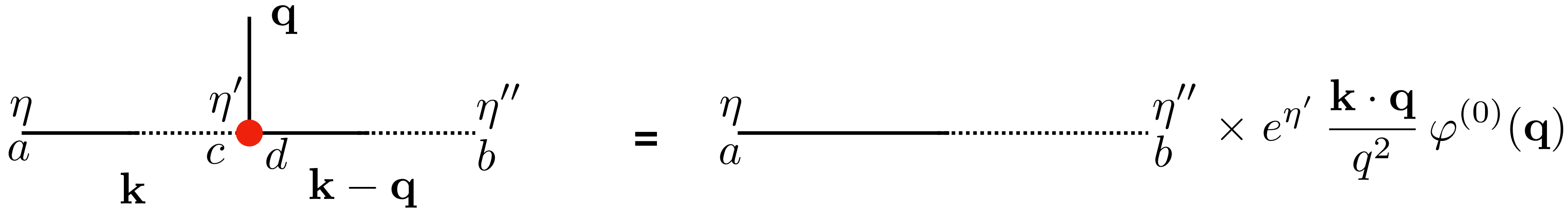
linear displacement

$$\langle\psi^2(0)\rangle = \int \frac{d^3p}{(2\pi)^3}\langle\tilde{\psi}(\mathbf{p})^2\rangle' = \int \frac{d^3p}{(2\pi)^3}\frac{P(p)}{p^2} \simeq (6\,\mathrm{Mpc/h})^2$$

Soft dressing of hard fields

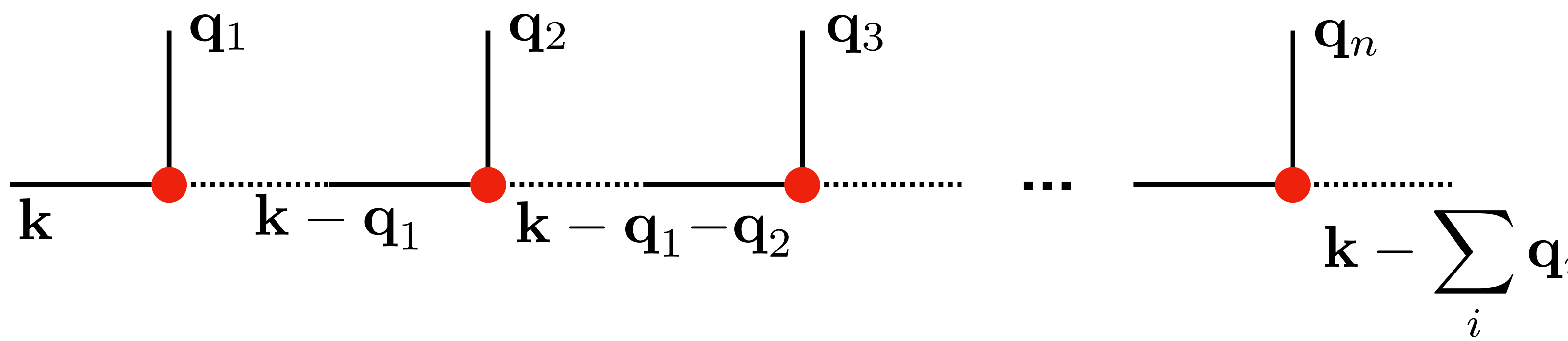
crucial factorization property:

q<<k



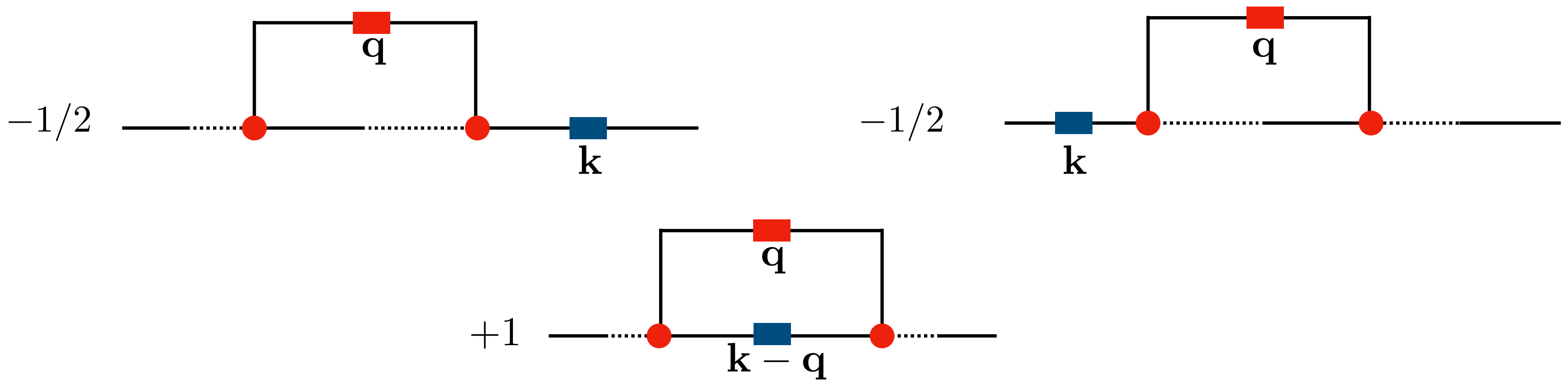
$$\varphi_a^{h,\text{ir}(1)}(\mathbf{k};\eta) = u_a e^{\eta}\int^{\Lambda}\frac{d^3q_1}{(2\pi)^3}\frac{\mathbf{k}\cdot\mathbf{q}_1}{q_1^2}\varphi^{(0)}(\mathbf{q}_1)\varphi^{(0)}(\mathbf{k}-\mathbf{q}_1) \qquad \Lambda \ll k$$

consequence of Equivalence Principle, true non-perturbatively (more later)



$$\varphi_a^{h,\text{ir}(m)}(\mathbf{k};\eta) = u_a\frac{e^{m\eta}}{m!}\int^{\Lambda}\frac{d^3q_1}{(2\pi)^3}\cdots\int^{\Lambda}\frac{d^3q_m}{(2\pi)^3}$$
$$\times\left(\frac{\mathbf{k}\cdot\mathbf{q}_1}{q_1^2}\cdots\frac{\mathbf{k}\cdot\mathbf{q}_m}{q_m^2}\right)\left(\varphi^{(0)}(\mathbf{q}_1)\cdots\varphi^{(0)}(\mathbf{q}_n)\right)\varphi^{(0)}(\mathbf{k}-\sum_{i=1}^{m}\mathbf{q}_i)$$

effect on 1-loop PS



$$P_{ab}^{\mathrm{ir},(1)}(k;\eta) = -u_a u_b e^{2\eta} \int^{\Lambda} \frac{d^3q}{(2\pi)^3} \left(\frac{\mathbf{k}\cdot\mathbf{q}}{q^2}\right)^2 P^{(0)}(q) \left(P^{(0)}(k) - P^{(0)}(|\mathbf{k}-\mathbf{q}|)\right)$$

$$= -u_a u_b e^{2\eta}\, O\left(k^2 \frac{\partial^2 P^{(0)}(k)}{\partial k^2} \sigma_\delta^2(\Lambda)\right)$$

$$u_a \equiv (1,1)$$

$$\sigma_\delta^2(\Lambda) \equiv \int^{\Lambda} \frac{d^3q}{(2\pi)^3} P^{(0)}(q)$$

for a featureless PS: $P^{(0)}(k) \sim k^n$ → $P_{ab}^{\mathrm{ir},(1)}(k;\eta) \sim O(P^{(0)}(k)\,\sigma_\delta^2(\Lambda))$

typical 1-loop correction,
no enhancement w.r.t. other contributions

effect on oscillating component

$$P^{(0)}(k) = P^{(0),\mathrm{nw}}(k) + P^{(0),\mathrm{w}}(k)$$

$$r_d \simeq 110\,\mathrm{Mpc/h}$$

$$P^{(0),\mathrm{w}}(k) = P^{(0),\mathrm{nw}}(k) A(x) \sin(k r_d + \phi(k)) \qquad \left(P^{(0),\mathrm{nw}}(k),\ A(k),\ \phi(k) \quad \text{“smooth”}\right)$$

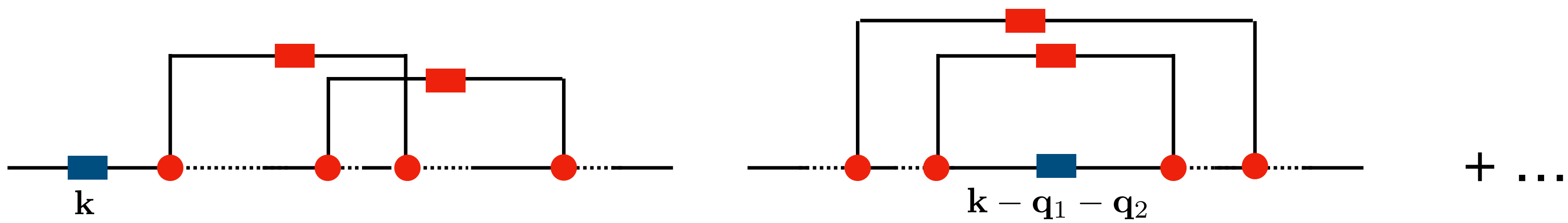
$$k^2 \frac{\partial^2 P^{(0),\mathrm{w}}(k)}{\partial k^2} \sigma_\delta^2(\Lambda) = O\left(k^2 r_d^2\, P^{(0),\mathrm{w}}(k) \sigma_\delta^2(\Lambda)\right)$$

$k^2 r_d^2 \gg 1$ enhancement factor on other 1-loop effects

explicit result:

$$\begin{aligned} P_{ab}^{\mathrm{IR\,w},(1)}(k;\eta) &= -u_a u_b P^{(0),\mathrm{w}}(k) e^{2\eta} \int^\Lambda \frac{d^3q}{(2\pi)^3} \left(\frac{\mathbf{k}\cdot\mathbf{q}}{q^2}\right)^2 P^{(0)}(q)(1 - \cos(q r_d \mu)) \\ &= -u_a u_b P^{(0)\mathrm{w}}(k) \frac{k^2 e^{2\eta}}{6\pi^2} \int^\Lambda dq P^{(0)}(q) \left(1 - j_0(q\, r_d) + 2 j_2(q\, r_d)\right) \\ &\equiv -k^2 \Xi(r_d, \Lambda; \eta) P^{(0),\mathrm{w}}(k)\,, \end{aligned}$$

two-loops



$$P_{ab}^{\mathrm{IR\,w},(2)}(k;\eta) = \frac{k^4}{2}\Xi(r_d,\Lambda;\eta)^2 P^{(0),\mathrm{w}}(k)u_a u_b = -\frac{k^2}{2}\Xi(r_d,\Lambda;\eta)P_{ab}^{\mathrm{IR\,w},(1)}(k;\eta)$$

n+1-loops

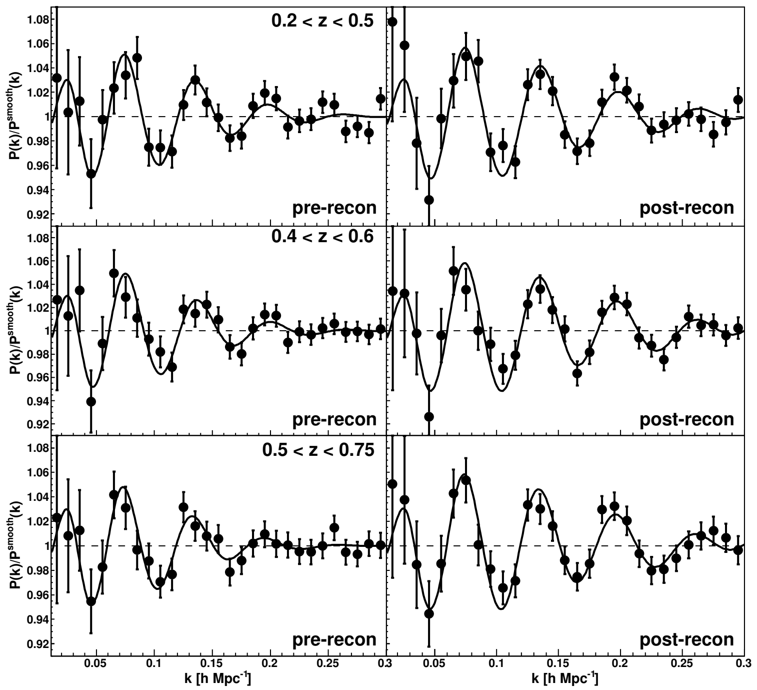
$$P_{ab}^{\mathrm{IR\,w},(n+1)}(k;\eta) = -\frac{k^2}{n+1}\Xi(r_d,\Lambda;\eta)P_{ab}^{\mathrm{IR\,w},(n)}(k;\eta) = \frac{(-k^2)^{n+1}}{n+1!}\Xi(r_d,\Lambda;\eta)^{n+1}P^{(0),\mathrm{w}}(k)u_a u_b$$

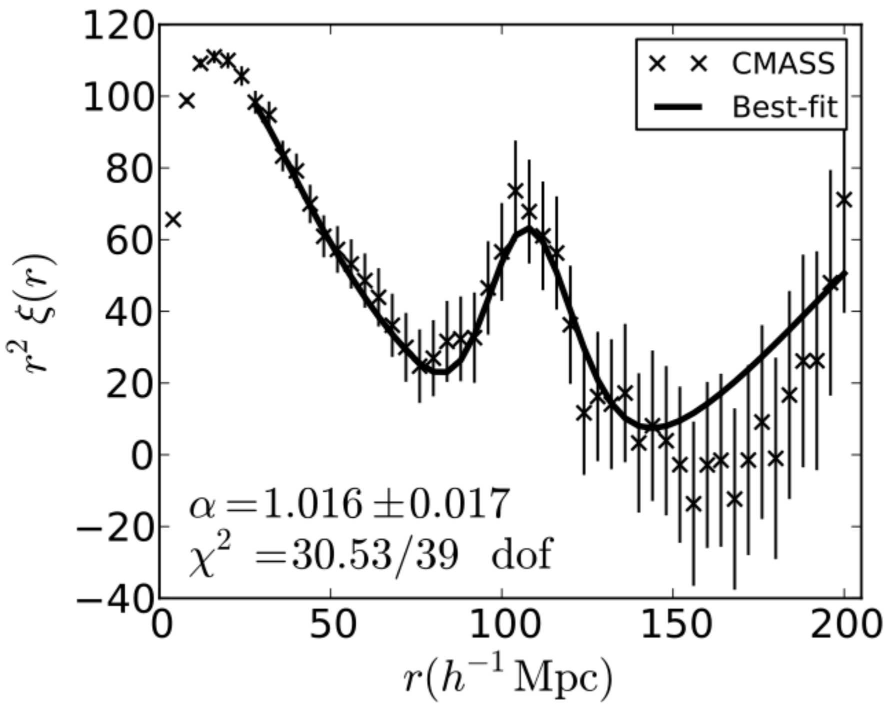
IR-resummed Power Spectrum

$$P_{ab}^{\mathrm{IR,\,LO}}(k;\eta) = e^{-k^2\Xi(r_d,\Lambda;\eta)}P^{(0),\mathrm{w}}(k)u_a u_b + P^{(0),\mathrm{nw}}(k)u_a u_b$$

$$P_{ab}^{\mathrm{ir,\,NLO}}(k;\eta) = P_{ab}^{11}(k;\eta) + e^{-k^2\Xi(r_d,\Lambda;\eta)}P_{ab}^{h(1)\mathrm{w}}(k;\eta)$$
$$+ k^2\Xi(r_d,\Lambda;\eta)P^{(0),\mathrm{w}}(k)u_a u_b - P_{ab}^{h(1)\mathrm{w}}(k;\eta)$$ (avoid double counting)

Different ways to include displacements lead to equivalent results.
Well understood theoretically.
Not a nuisance, but a *calculable physical effect*!!



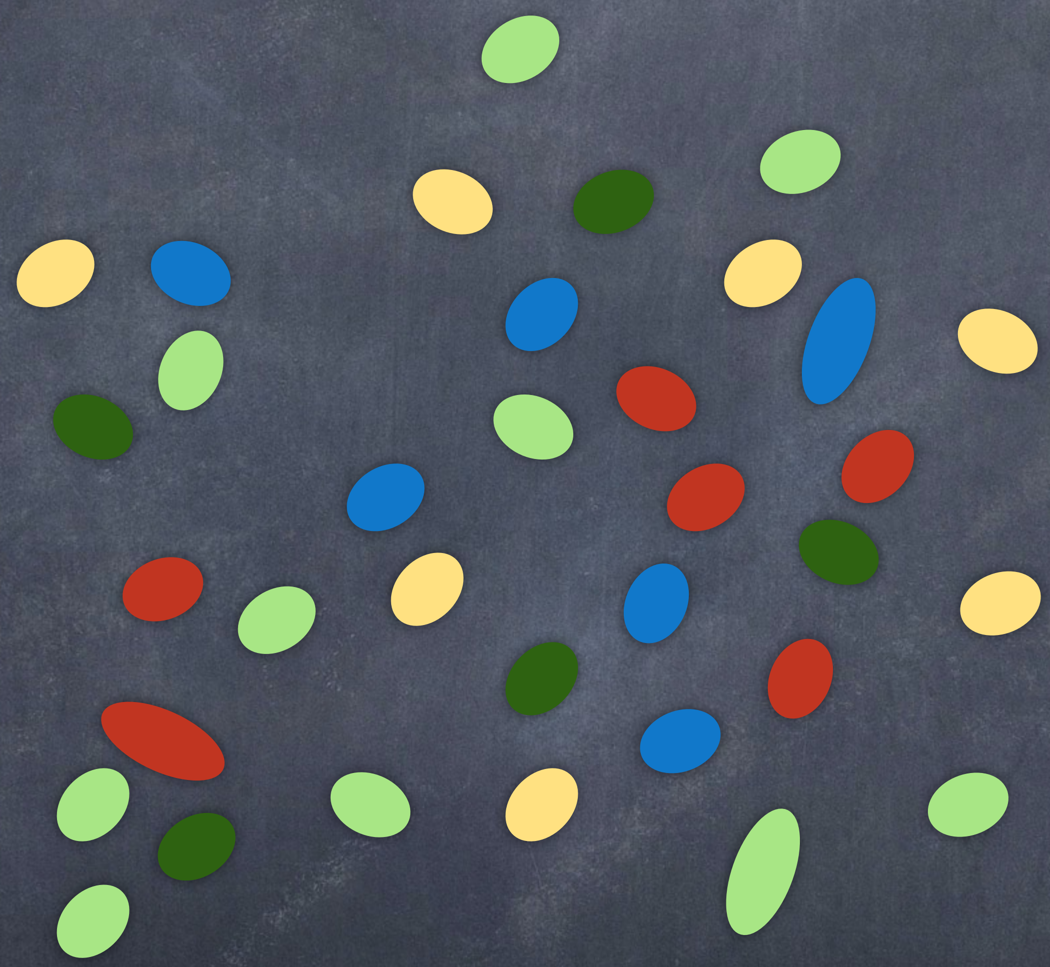


Extended "Galilean" invariance (EP)

$\mathbf{x}$

$\mathbf{p}$

$\nabla\phi$

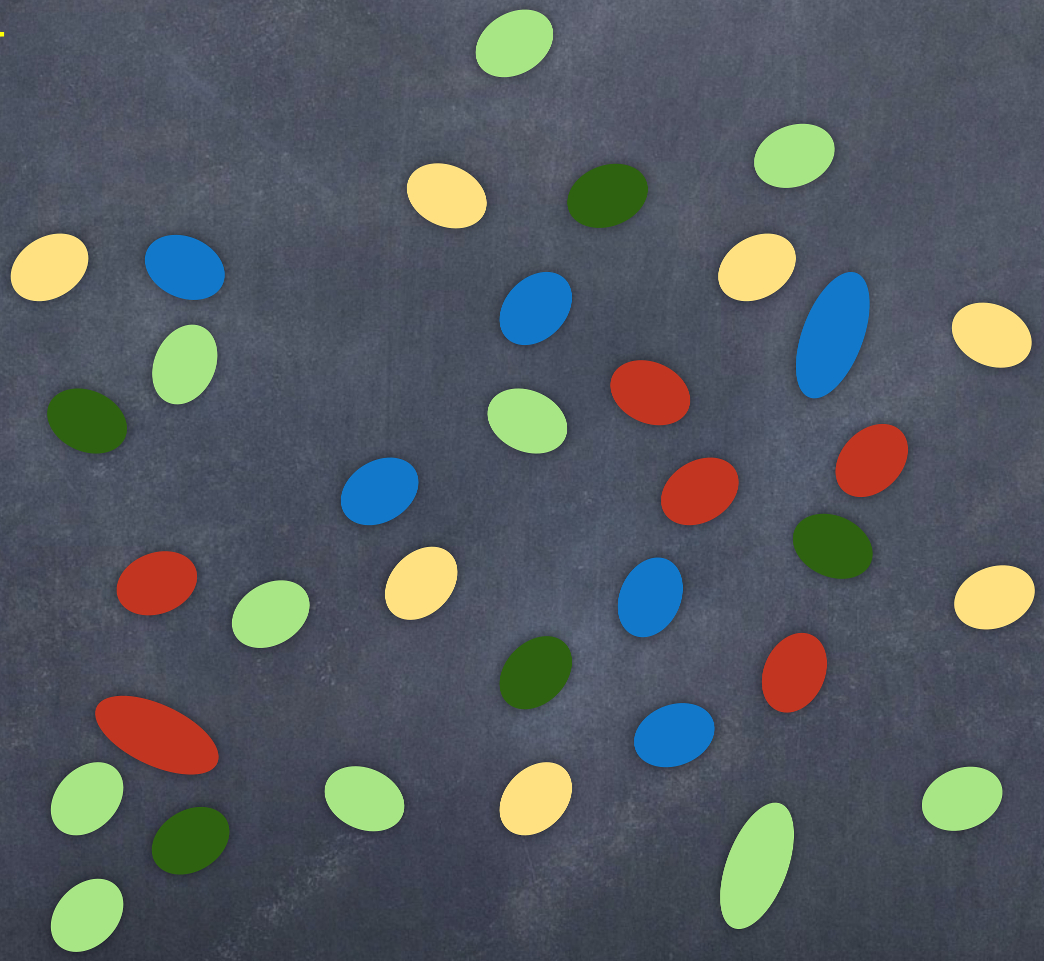


Extended "Galilean" invariance (EP)

$$\mathbf{x} + \mathbf{d}(\tau)$$

$$\mathbf{p} + am\,\dot{\mathbf{d}}(\tau)$$

$$\nabla\phi - \mathcal{H}\dot{\mathbf{d}} - \ddot{\mathbf{d}}$$



- Valid for the Vlasov eq. (no single-stream approx)
- Valid for matter and biased tracers
- Valid in redshift space

Almost uniform displacement

$$\int d^3x e^{i\mathbf{x}\cdot\mathbf{q}}\mathbf{d}(\tau) = \mathbf{d}(\tau)(2\pi)^3\delta_D(\mathbf{q}) \longrightarrow \mathbf{d}(\mathbf{q},\tau)$$

then study the q->0 limit:

$$\mathbf{d}(\mathbf{q},\tau) = \int^{\tau} d\tau'\,\mathbf{v}(\mathbf{q},\tau') = \frac{\mathbf{v}(\mathbf{q},\tau)}{\mathcal{H}f} = i\frac{\mathbf{q}}{q^2}\,\delta_m(\mathbf{q},\tau)$$

linear theory

Consequence of EP: constraints on mode-coupling

ex: continuity equation

$$\partial_\tau \delta(x,\tau) + \vec{\nabla}((1+\delta(x,\tau))\vec{v}(x,\tau)) = 0$$

effect of the (almost uniform) displacement: $\vec{v}(x,\tau) \to \vec{v}(x,\tau) + \dot{\vec{d}}(x,\tau)$

$$\vec{\nabla}((1+\delta(x,\tau))\vec{v}(x,\tau)) \to \vec{\nabla}((1+\delta(x,\tau))\vec{v}(x,\tau)) \boxed{+\dot{\vec{d}}(x,\tau)\cdot\vec{\nabla}\delta(x,\tau)} + \cdots$$

the new term is canceled by $\delta(x,\tau) \to \delta(x - \vec{d}(x,\tau),\tau)$

$$\partial_\tau \delta(x - \vec{d}(x,\tau),\tau) \simeq \partial_\tau \delta(x,\tau) \boxed{-\dot{\vec{d}}(x,\tau)\cdot\vec{\nabla}\delta(x,\tau)} + \cdots$$

Long mode can be absorbed in a coordinate shift

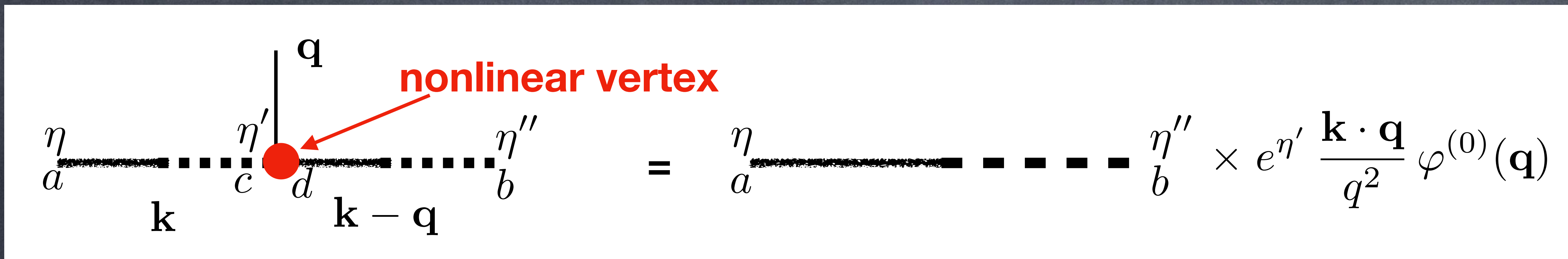
$$\delta(x - \vec{d}(x,\tau),\tau) \simeq \delta(x,\tau) - \vec{d}(x,\tau)\cdot\vec{\nabla}\delta(x,\tau)$$

Fourier Space:

$$\delta_k(\tau) \to \delta_k(\tau) + i\int^{\Lambda} \frac{d^3q}{(2\pi)^3}\,\vec{k}\cdot\vec{d}(q,\tau)\,\delta_{k-q}(\tau)$$
$$= \delta_k(\tau) - \int^{\Lambda} \frac{d^3q}{(2\pi)^3}\,\frac{\vec{k}\cdot\vec{q}}{q^2}\delta_q(\tau)\,\delta_{k-q}(\tau)$$

$$\mathbf{d}(\mathbf{q},\tau) = \int^{\tau} d\tau'\,\mathbf{v}(\mathbf{q},\tau') = \frac{\mathbf{v}(\mathbf{q},\tau)}{\mathcal{H}f} = i\frac{\mathbf{q}}{q^2}\,\delta_m(\mathbf{q},\tau)$$

CONSISTENCY RELATIONS for the LSS



infinite set of Ward Identities (Peloso, MP, ’13)

this induces a contribution to the bispectrum
in the sqeezed limit (k>>q):

$$B_{\alpha\beta\gamma}(q, k_+, k_-; \tau_\alpha, \tau_\beta, \tau_\gamma) \simeq$$

$$\frac{\mathbf{k}\cdot\mathbf{q}}{q^2} P_{\alpha m}(q; \tau_\alpha, \tau_\alpha) \left[\frac{D(\tau_\beta)}{D(\tau_\alpha)} P_{\beta\gamma}(k_-; \tau_\beta, \tau_\gamma) - \frac{D(\tau_\gamma)}{D(\tau_\alpha)} P_{\beta\gamma}(k_+; \tau_\beta, \tau_\gamma) \right] + O\left(\left(\frac{q}{k} \right)^0 \right)$$

$$\mathbf{k}_\pm = \mathbf{k} \pm \frac{\mathbf{q}}{2}$$

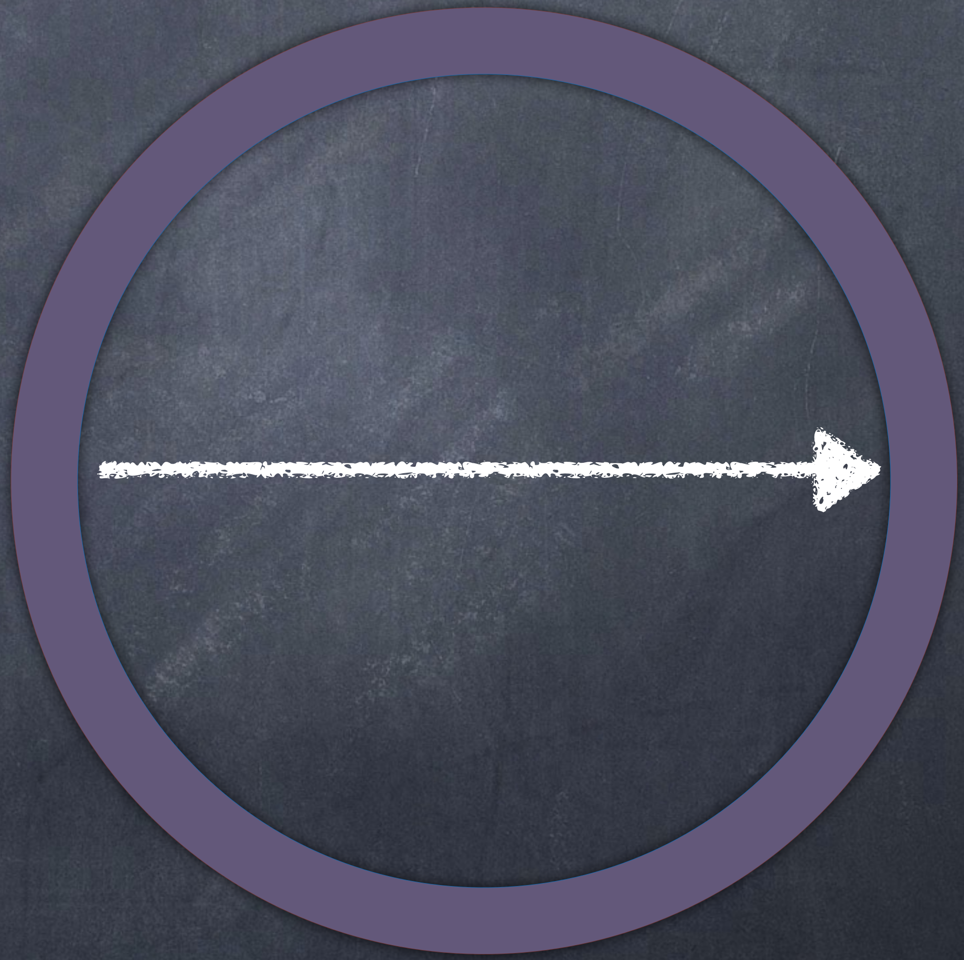
Peloso, MP, ’13; Kehagias, Riotto, ’13;….

Constant displacement: k/q contributions

angular dependence: $\propto \mu$

$\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}$

only if $\mathbf{d}_\beta \neq \mathbf{d}_\gamma$



- unequal times
- non adiabatic initial conditions
- Large scale velocity bias
- EP violation

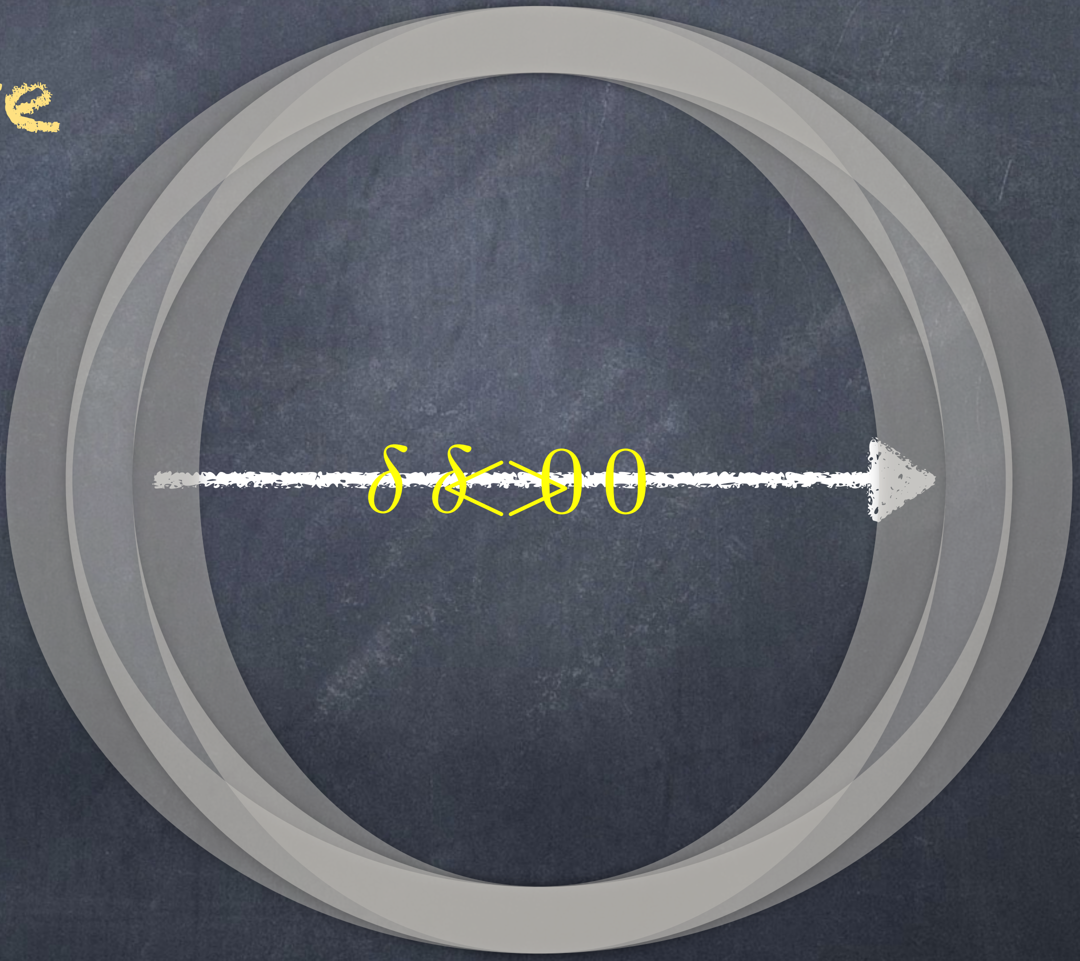
Constant gradient displacement: $O(q^0)$ contributions

also for $\mathbf{d}_\beta = \mathbf{d}_\gamma$

angular dependence: $\propto \mu^2$

- equal times OK
- depends on the derivative of the corr. function

$$\propto \mu^2 \frac{d \log \xi(r)}{d \log r}$$



see also Baldauf et al. '15

Equal-time sqeezed limit (real space)

$$\lim_{q/k\to 0} \frac{B_{\alpha\alpha\alpha}(q,k_+,k_-)}{P_{\alpha\alpha}(q)P_{\alpha\alpha}(k)} = -\frac{\mu^2}{b_\alpha(q)}\frac{d\log P_{\alpha\alpha}(k)}{d\log k} + O\left(\left(\frac{q}{k}\right)^0\right)$$

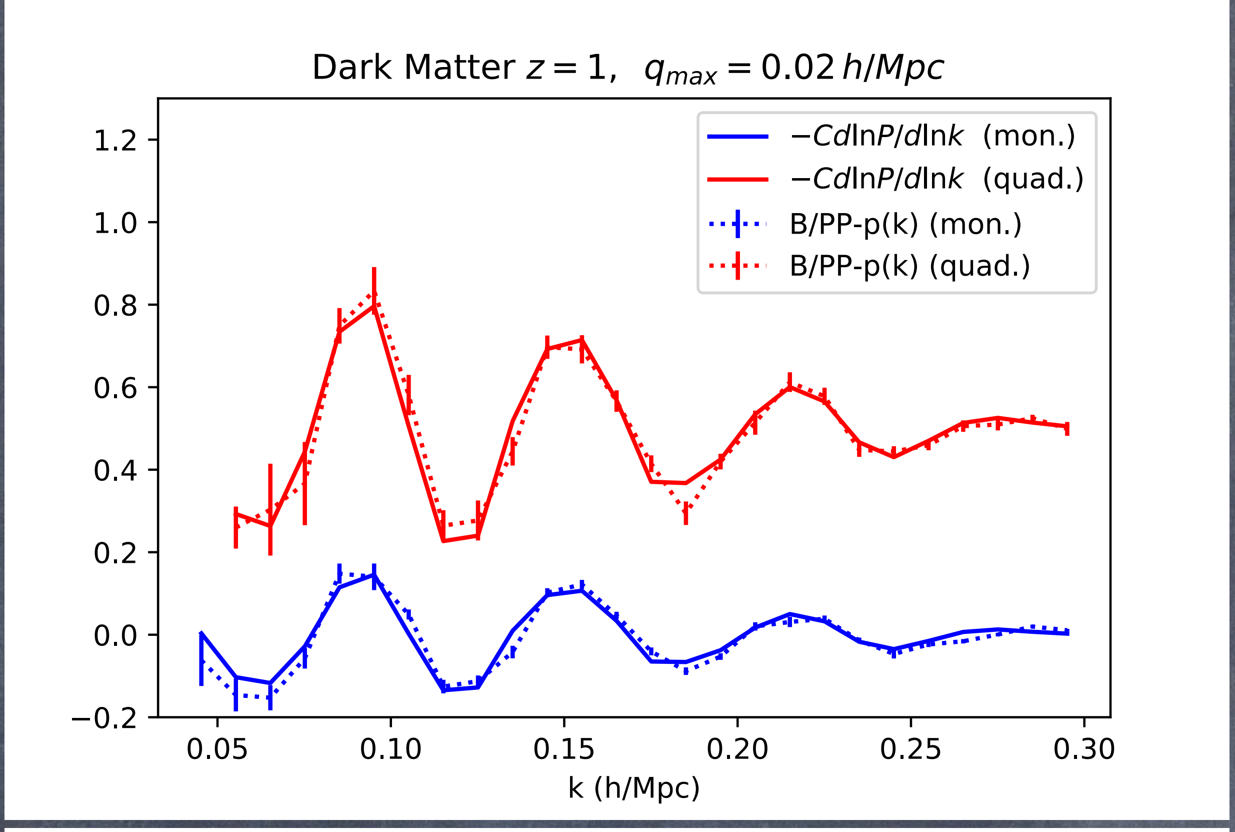
bias: $b_\alpha(q;\tau_\alpha) \equiv \dfrac{P_{\alpha\alpha}(q;\tau_\alpha,\tau_\alpha)}{P_{\alpha m}(q;\tau_\alpha,\tau_\alpha)}$

$$P^0_m(k) = P^{nw}_m(k)(1 + A(k)\sin(kr_s))$$

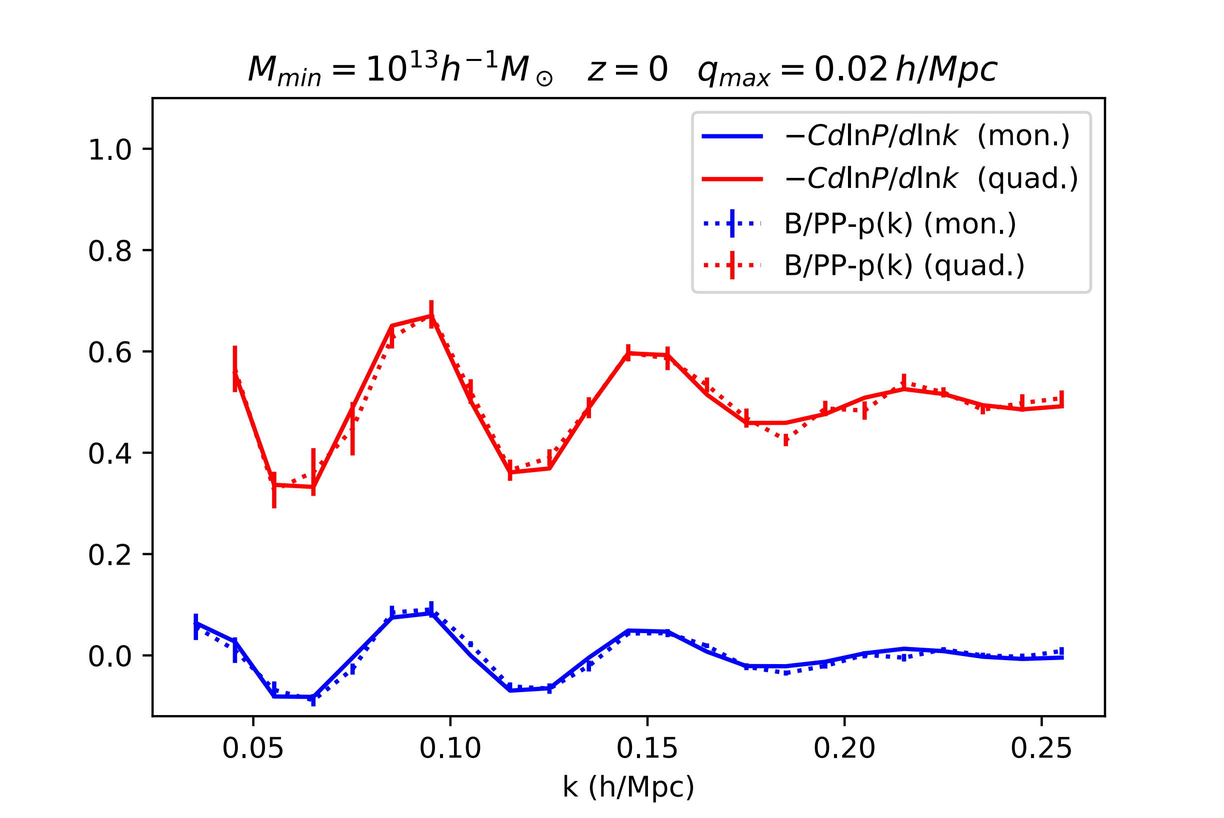
oscillations enhanced by kr_s $\quad \sim \dfrac{kr_s A(k)\cos(kr_s)}{1 + A(k)\sin(kr_s)} + \dfrac{d\log P^{nw}_m(k)}{d\log k}$

unchanged by nonlinearities:
Compare *measured* PS with *measured* BS

Test on simulations

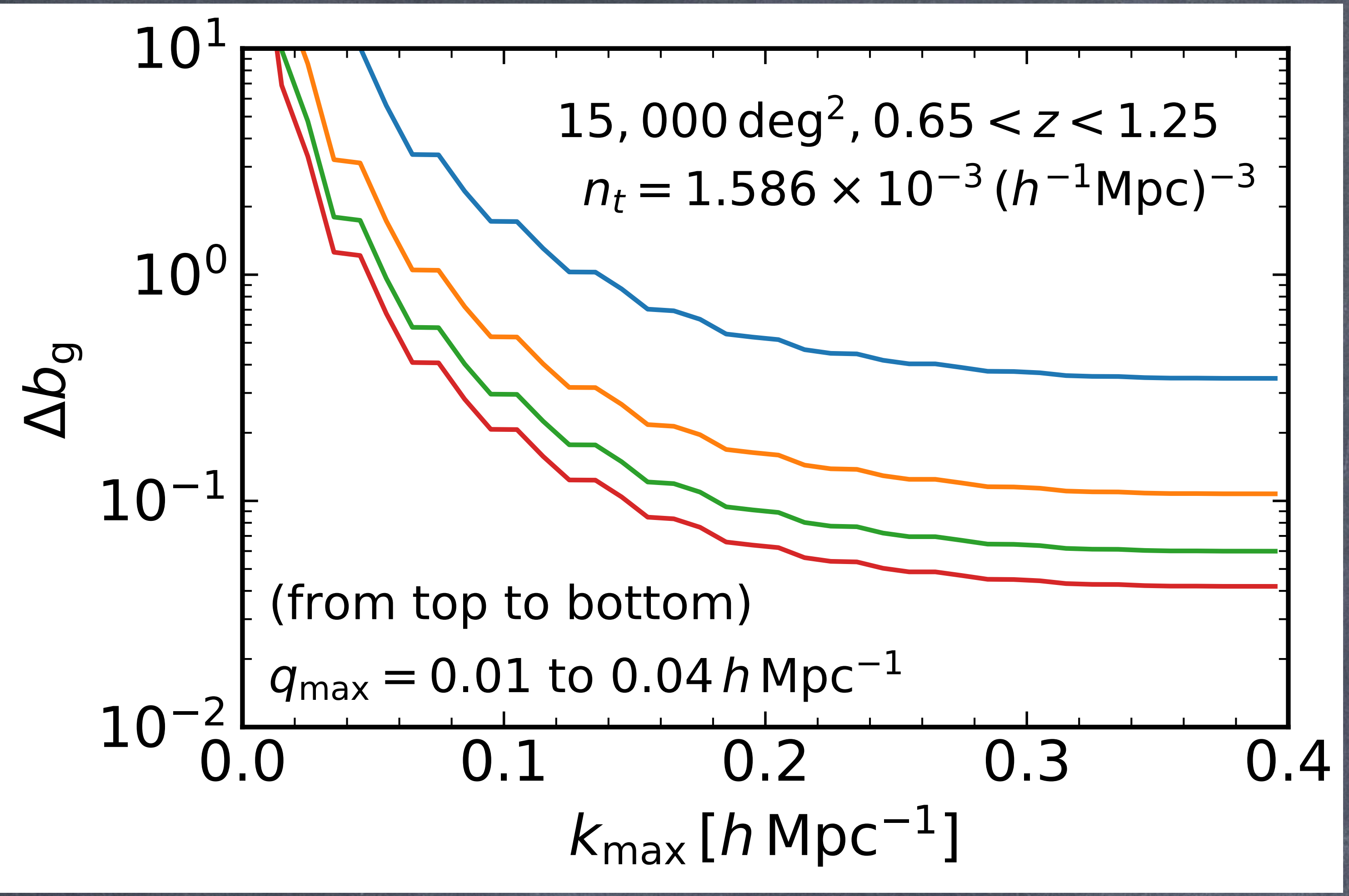


Dark Matter



Halos

forecast on b from a Euclid-like survey



- better that 10% determination of b and f
- constraint independent on SPT, EFT, … Nbody
- constraint independent on the cosmological model!
- main limitation: survey volume