Cosmological SPT & beyond - 2

M. Pietroni - Parma

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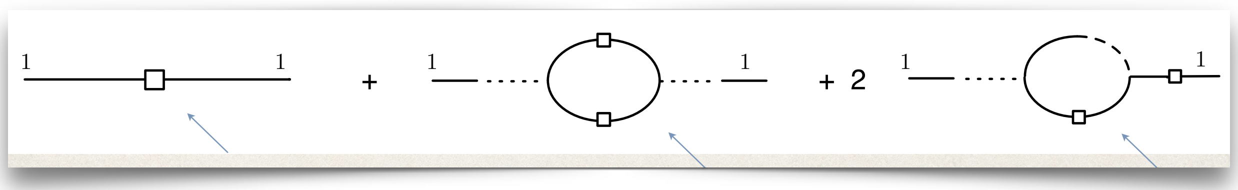
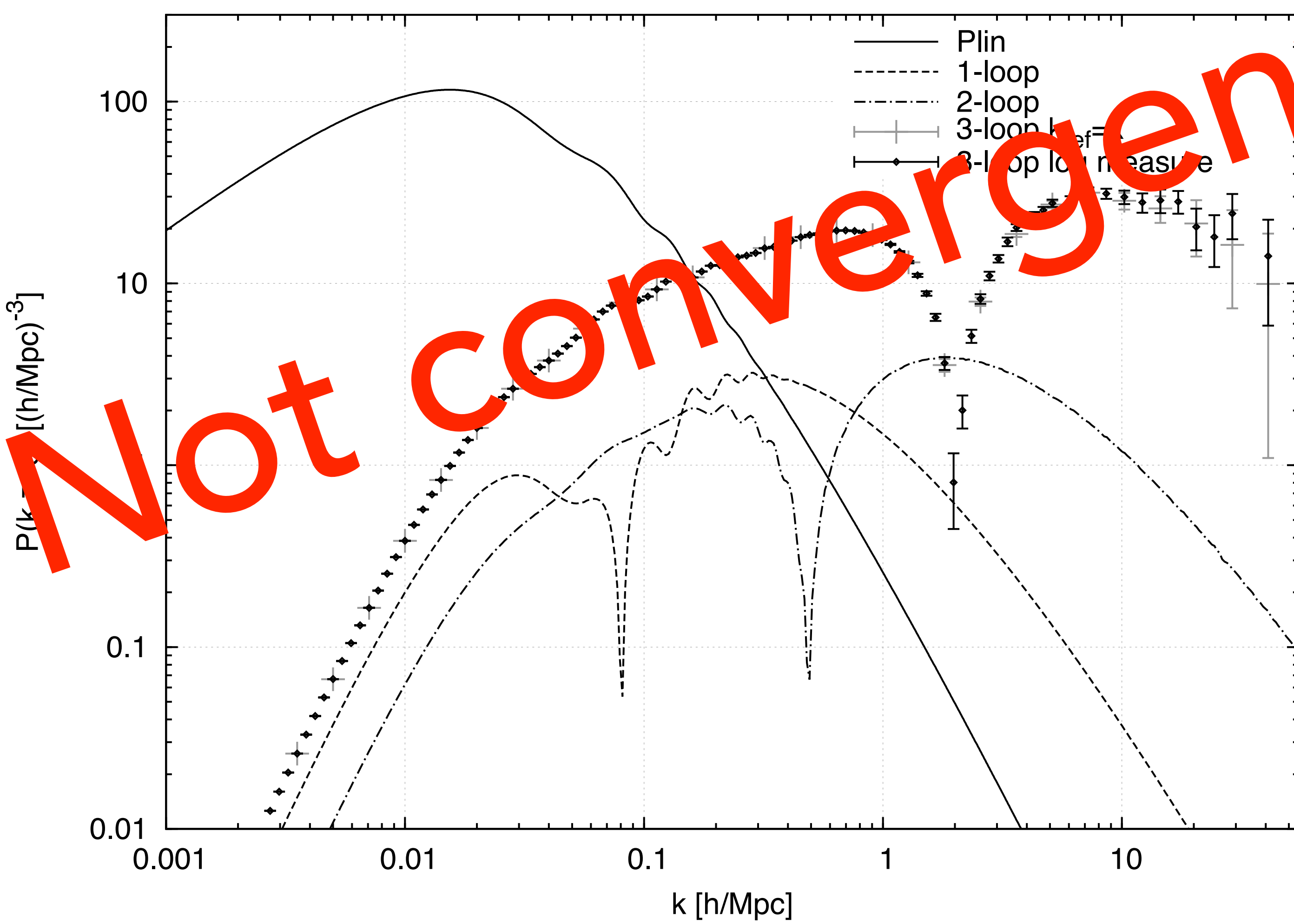
Performance of Standard PT

$$P(k,z) = D(z)^2 P^{(1)}(k) + D(z)^4 F^{(1l)}(k) + D(z)^6 F^{(2l)}(k) + \cdots$$

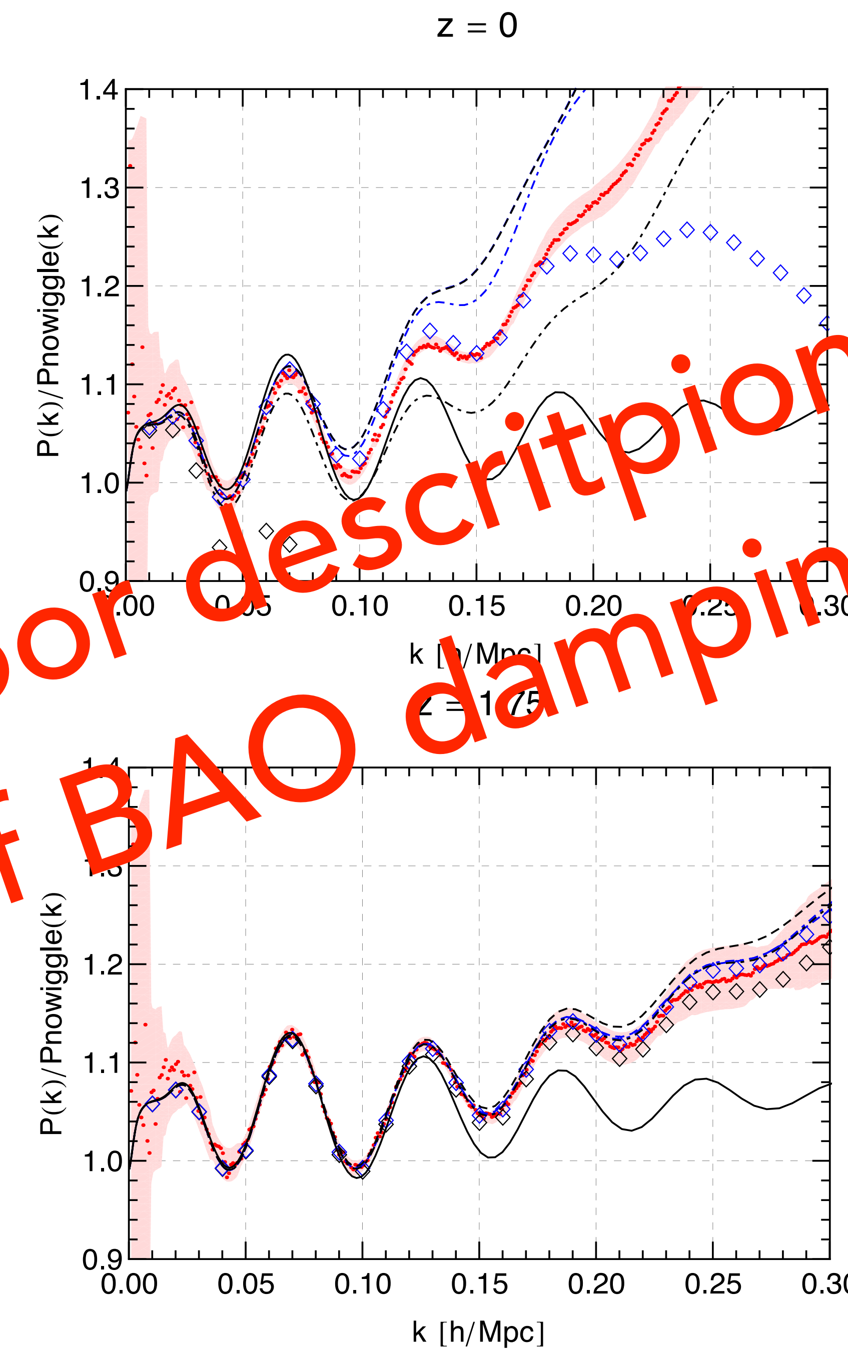


linear

1-loop



Not convergent !



poor descritpion of BAO damping

Blas et al. 1309.3308

MODE COUPLING

Linear Response Function

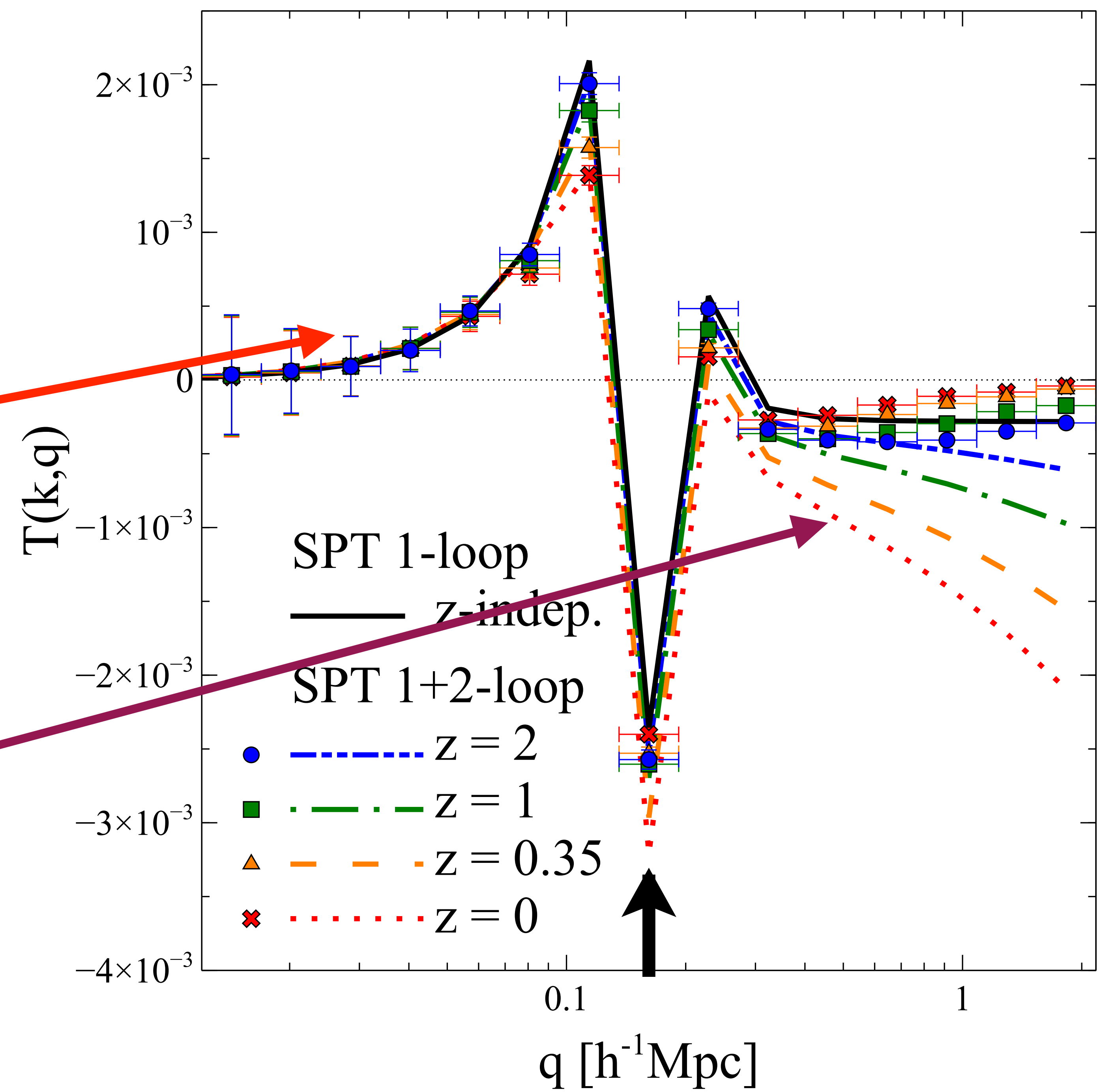
$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

IR: "Galilean" invariance (EP)

$K(k,q;z) \sim q^3$

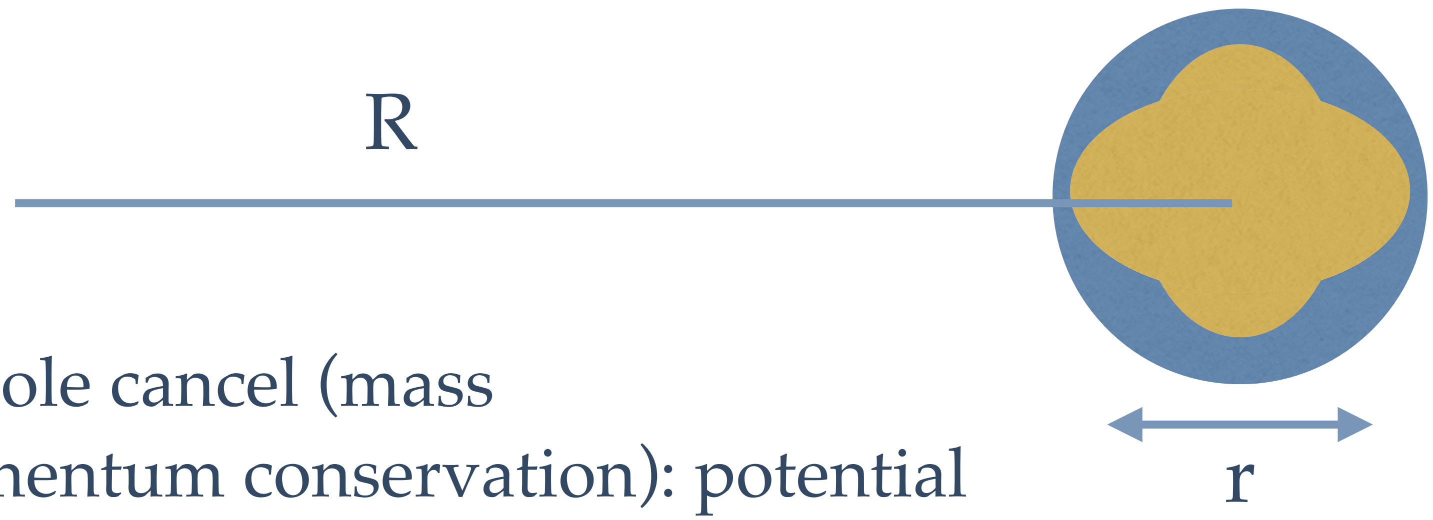
UV: SPT over predicts the effect of small scales on intermediate ones

Nishimichi, Bernardeau, Taruya 1411.2970



The effect of short scales (UV)

Effect of an isolated small density profile ~ r at large distance R(>>r)



Monopole and dipole cancel (mass conservation+momentum conservation): potential generated by quadrupole:

$$\phi(R) \propto r^2/R^3 \qquad \delta(R) \propto \nabla^2\phi(R) \rightarrow \delta(k) \propto k^2 r^2 \qquad P(k) \propto k^4$$

PT exhibits the k^4 decoupling of small scales q (k << q)

However, virialized structures decouple more efficiently than k^4 (Peebles '80, Baumann et al 1004.2488, Blas et al 1408.2995).
Highly nonlinear scales decouple.
This effect is missed by the single stream approximation.

Effective approaches to the UV

- Perturbation Theory (even after resummations) fails at short scales due to non-convergent series and multistreaming
- General idea: take the UV physics from N-body simulations and use PT only for the large and intermediate scales

(drop the time dependence)

$$\frac{\partial}{\partial\tau}\delta_R(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta_R(\mathbf{x}))v_R^i(\mathbf{x})\right] = 0$$

continuity eq.

$$\frac{\partial}{\partial\tau}v_R^i(\mathbf{x}) + \mathcal{H}v_R^i(\mathbf{x}) + v_R^k(\mathbf{x})\frac{\partial}{\partial x^k}v_R^i(\mathbf{x}) = -\nabla_x^i\phi_R(\mathbf{x}) - J_\sigma^i(\mathbf{x}) - J_1^i(\mathbf{x})$$

Euler eq.

short-distance effects

$$J_\sigma^i(\mathbf{x}) \equiv \frac{1}{1+\delta_R(\mathbf{x})}\frac{\partial}{\partial x^k}\left((1+\delta_R(\mathbf{x}))\sigma_R^{ki}(\mathbf{x})\right)$$

$$J_1^i(\mathbf{x}) \equiv \frac{1}{1+\delta(\mathbf{x})}\left(\langle(1+\delta)\nabla^i\phi\rangle_R(\mathbf{x}) - (1+\delta_R)(\mathbf{x})\nabla^i\phi_R(\mathbf{x})\right)$$

To close the system, we must provide information on the short-distance effects

Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 ....

EXACT TIME-EVOLUTION

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b^R(\mathbf{k},\eta) = e^\eta I_{\mathbf{k};\mathbf{q}_1,\mathbf{q}_2}\gamma_{abc}(\mathbf{q}_1,\mathbf{q}_2)\varphi_b^R(\mathbf{q}_1,\eta)\varphi_c^R(\mathbf{q}_2,\eta) - h_a^R(\mathbf{k},\eta)$$

$$P_{ab}^R(k) = \langle \varphi_a^R(\mathbf{k})\varphi_b^R(-\mathbf{k})\rangle'$$

$$B_{abc}^R(q_1,q_2,q_3) = \langle \varphi_a^R(\mathbf{q}_1)\varphi_b^R(\mathbf{q}_2)\varphi_c^R(\mathbf{q}_3)\rangle'$$

$$h_a^R(\mathbf{k},\eta) \equiv -i\,\frac{k^i J_R^i(\mathbf{k},\eta)}{\mathcal{H}^2 f^2} e^{-\eta}\delta_{a2}$$

$$\partial_\eta P_{ab}^R(k) = \Big[-\Omega_{ac}P_{cb}^R(k) \;\; + (a \leftrightarrow b)\Big],$$

Linear PT

single stream

(vorticity treated perturbatively)

fully non-linear, equal-time correlators

need:

1) consistent truncations
2) measurement of UV correlators
3) IR resummation

UV INFORMATION

Need input on the UV "sources"

$$J^i_\sigma(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\frac{\partial}{\partial x^k}\left(n(\mathbf{x})\sigma^{ki}(\mathbf{x})\right)$$

$$J^i_1(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\left(\langle n_{mic}\nabla^i\phi_{mic}\rangle(\mathbf{x}) - n(\mathbf{x})\nabla^i\phi(\mathbf{x})\right)$$

Measure them from N-body simulations

(MP, Mangano, Saviano, Viel 1108.5203, Manzotti, Peloso, MP, Viel, Villaescusa-Navarro 1407.1342)

EFToLSS: Expand in terms of long wavelength fields + power law expansion in momentum, with arbitrary coefficients to be fitted

(Carrasco, Hertzberg, Senatore, 1206.2926 ….)

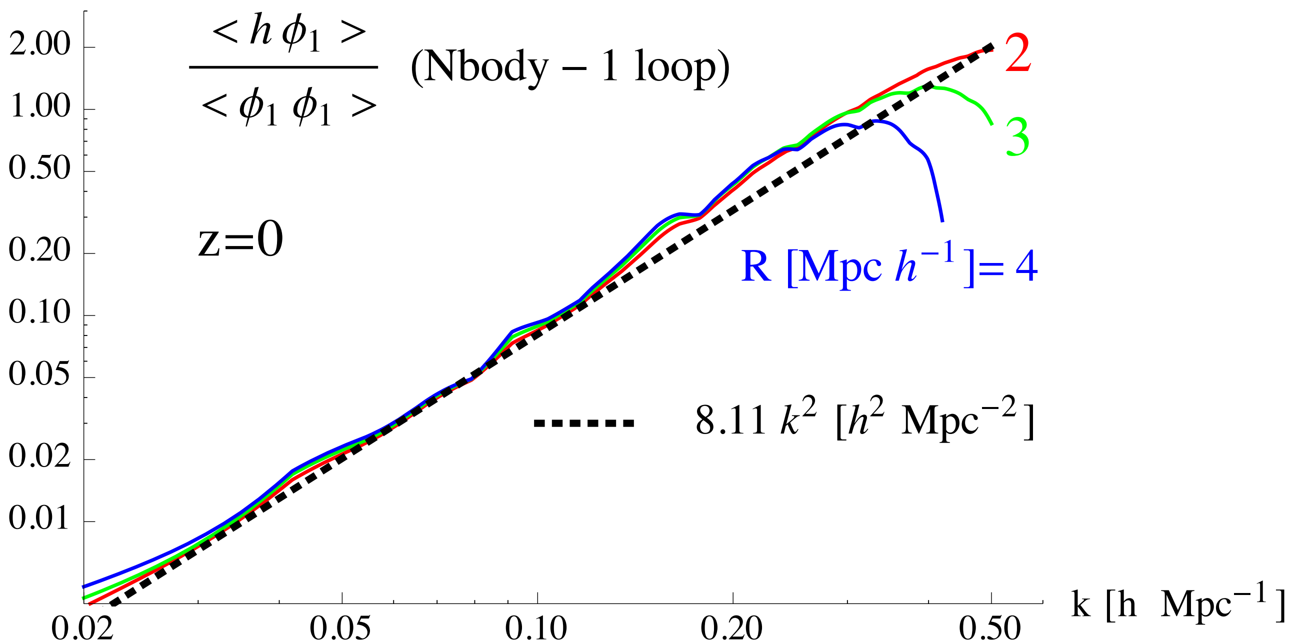
Compute them from first principles. Shell-crossing!

1+1 dim attempts

(Mc Quinn, White, 1502.07389; Taruya, Colombi, 1701.09088; Rampf, Frisch, 1705.08456; McDonald, Vlah, 1709.02834, Pajer, van der Woude, 1710.01736, MP, 1804.09140)

UV CORRELATORS FROM N-BODY

scale-dependence

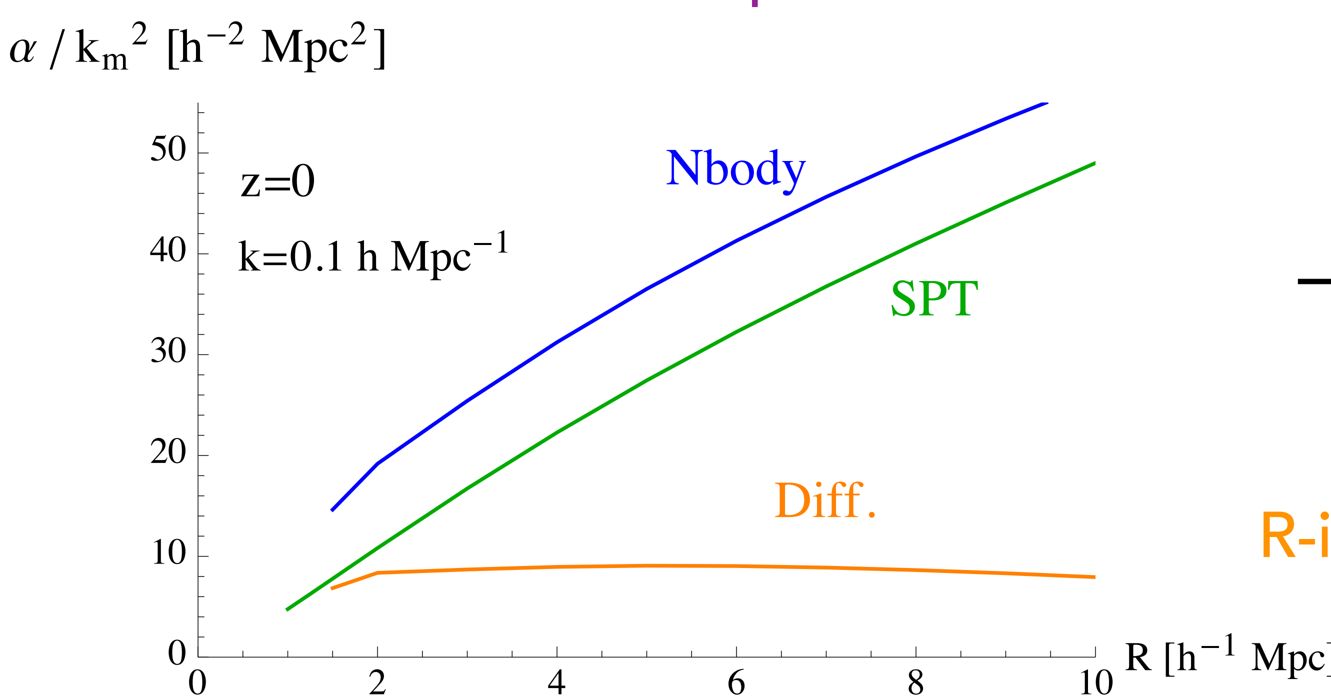


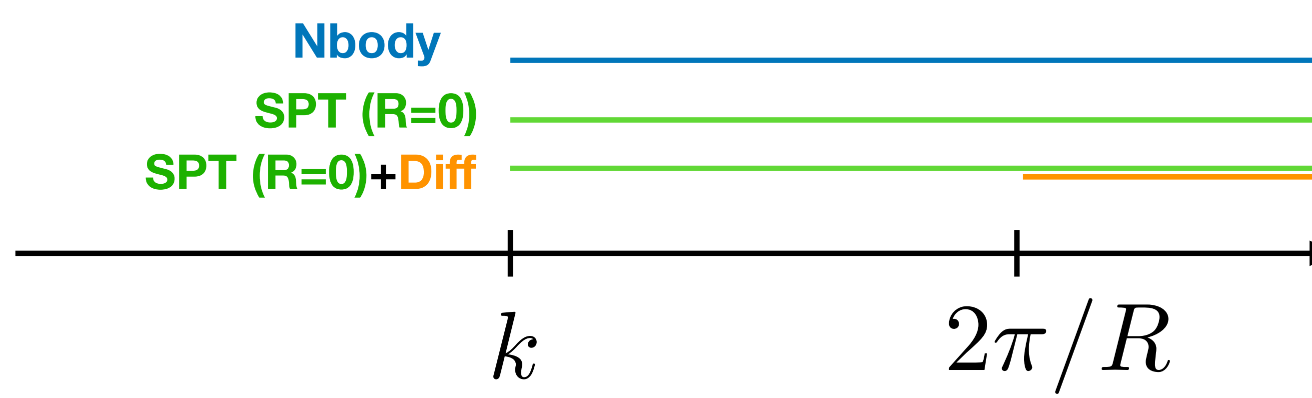
Parameterize the correlator as:

$$\langle h_a^R(\mathbf{k})\varphi_b^R(-\mathbf{k})\rangle' = \alpha^R(\eta)\frac{k^2}{k_m^2}P_{1b}^R(k;\eta)\,\delta_{a2}$$

nonlinear PS

UV-cutoff dependence





R-independent plateau

Relation with EFToLSS

Baumann et al 1004.2488
Carrasco et al 1206.2926
...

$$J^i_1 + J^i_\sigma$$

$$\dot\rho_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0 \ ,$$

$$\dot v_l^i + H v_l^i + \frac{1}{a} v_l^j \partial_j v_l^i + \frac{1}{a}\partial_i \phi_l = -\frac{1}{a\rho_l}\partial_j \left[\tau^{ij}\right]_\Lambda \ .$$

$$\langle \left[\tau^{ij}\right]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha}\delta^{ij}\partial_k v_l^k - \frac{3}{4}\frac{c_{sv}^2}{Ha}\left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3}\delta^{ij}\partial_k v_l^k\right)\right] + \Delta\tau^{ij} + \dots \ .$$

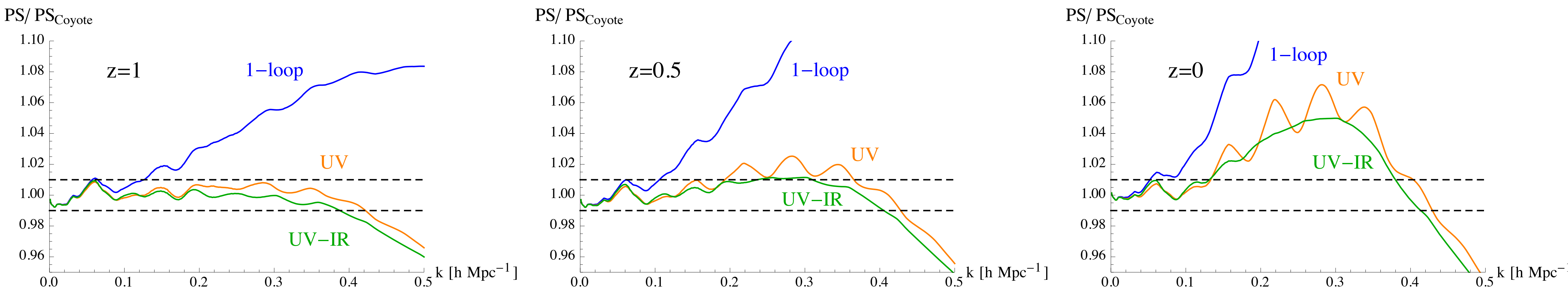
derivative expansion, or expansion in k/k_nl

coefficients should be scale independent, nice results for simple power law linear PS

"MINIMAL" SETTING AND PERFORMANCE

$P^{nw}(k)$ 1-loop SPT + UV source

$P^{w}(k)$ 1-loop SPT + IR resummation+ UV source



Noda, Peloso, M.P. 1705.01475

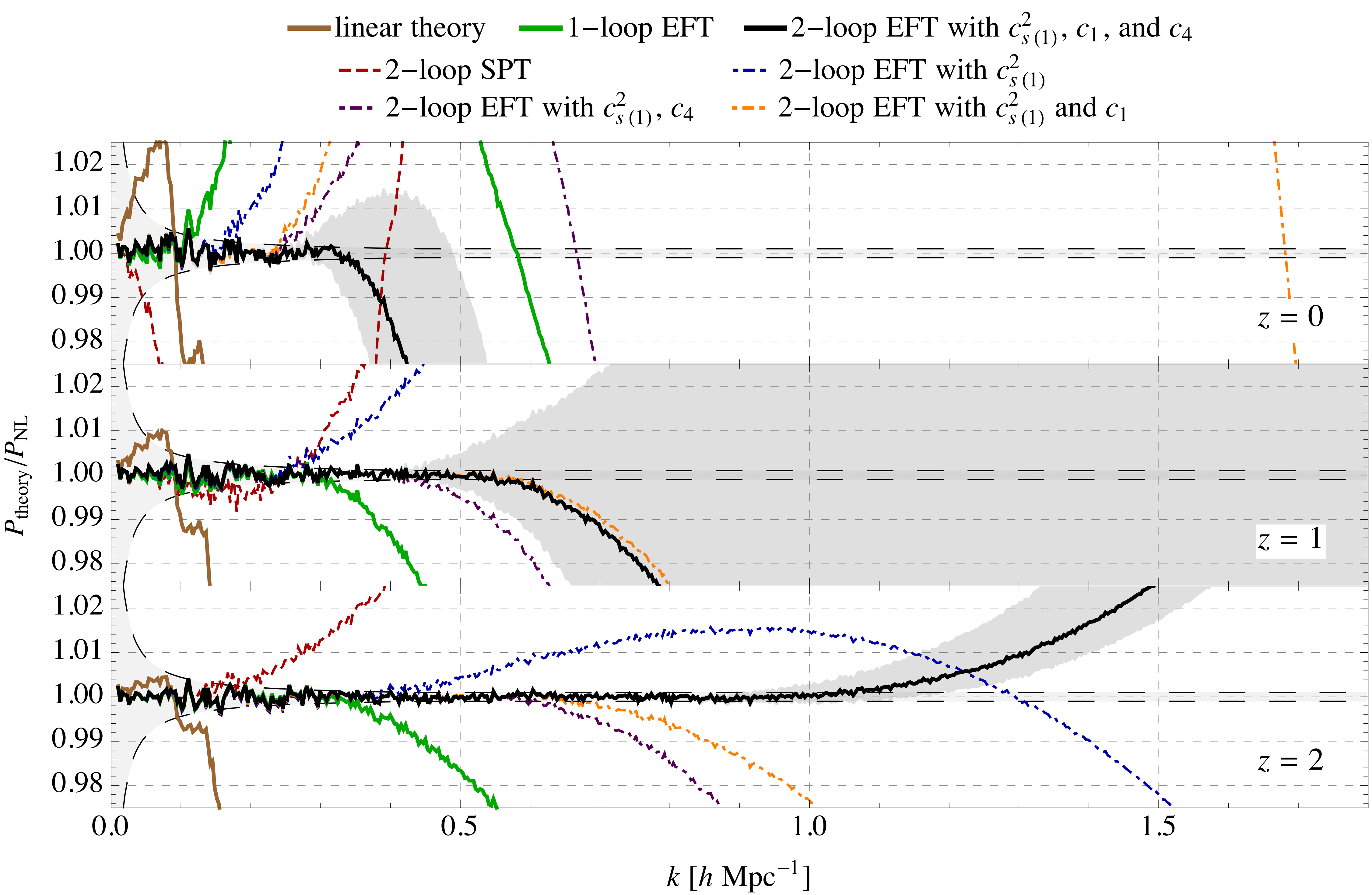
Broad band: k_{max} ~ 0.4 h/Mpc @ z=1 —> ~0.1 @ z=0 (go to 2-loop…)

no fitting on the PS!! (results comparable to EFToLSS @ 1-loop)

BAO residuals: ok at all redshifts

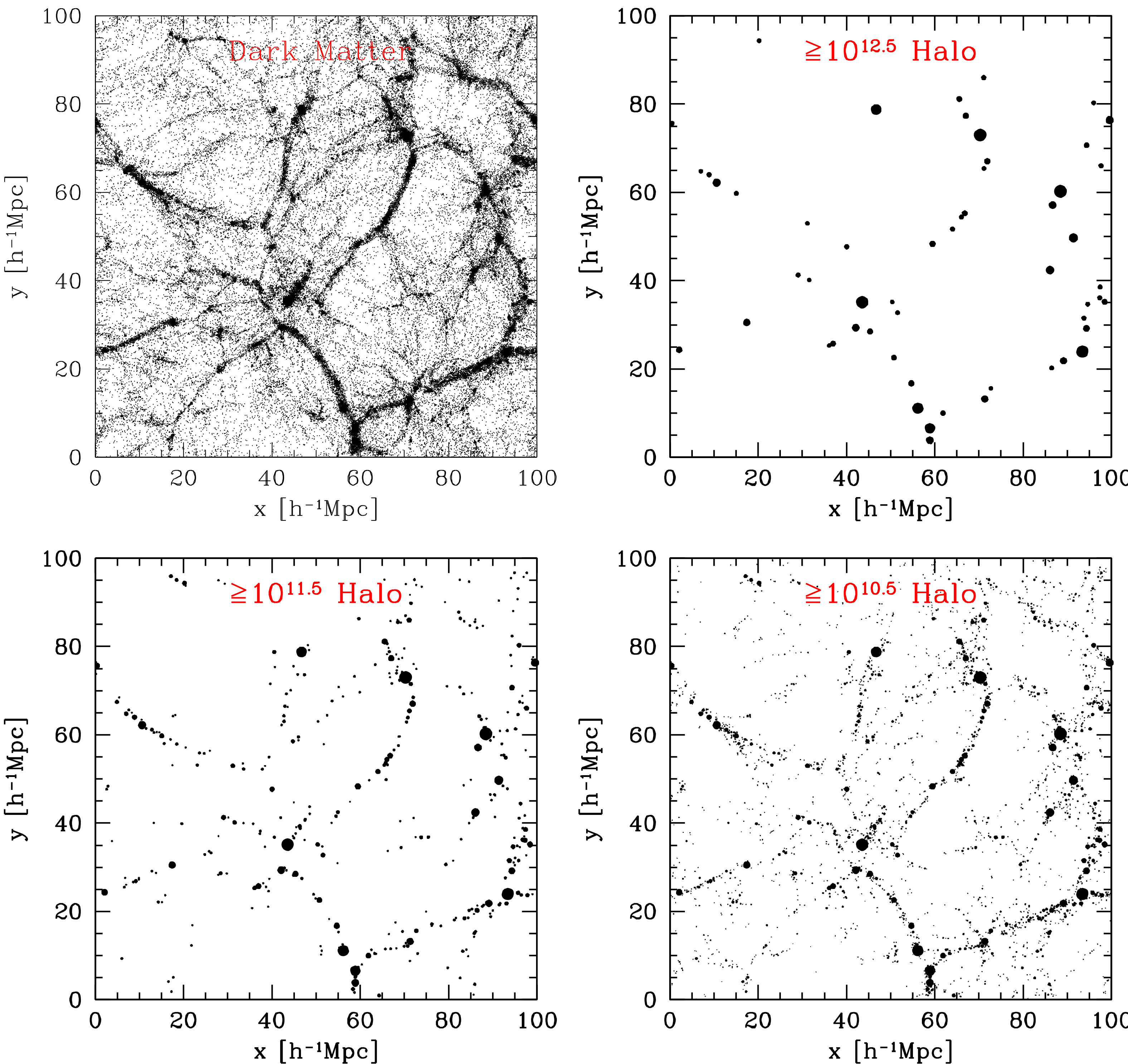
next order: 2-loop PT + $\langle J\,\delta\delta\rangle$ correlators

PERFORMANCE OF THE EFT OF LSS



Foreman, Perrier, Senatore, 1507.0532

Bias



BIAS: distribution of Galaxy and DM Halos
is a nonlinear and non local function of the DM one.

$$\delta_g = \mathcal{F}[\delta_{DM}, (\nabla_i \nabla_j \Phi)^2, \cdots]$$

Zhang, Y. et al, ApJ 706, 747, (09)

Review by Desjacques, Jeong, Schmidt, 1611.09787

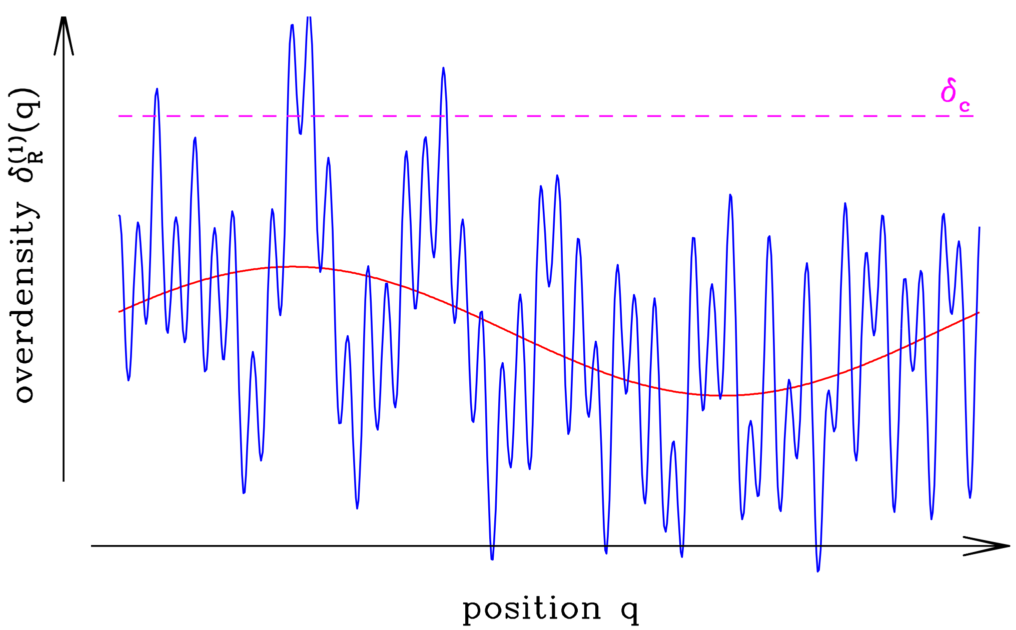
The Perturbative Bias Expansion

$$\delta_g(\mathbf{x},\tau) = \mathcal{F}[\delta; (\partial_i\partial_j\Phi)^2; \epsilon; \cdots] \longrightarrow \sum_{\mathcal{O}} b_{\mathcal{O}}(\tau)\, \mathcal{O}(\mathbf{x},\tau)$$

non-linear, nonlocal, stochastic

bias parameters

statistical fields describing the galaxies' environment



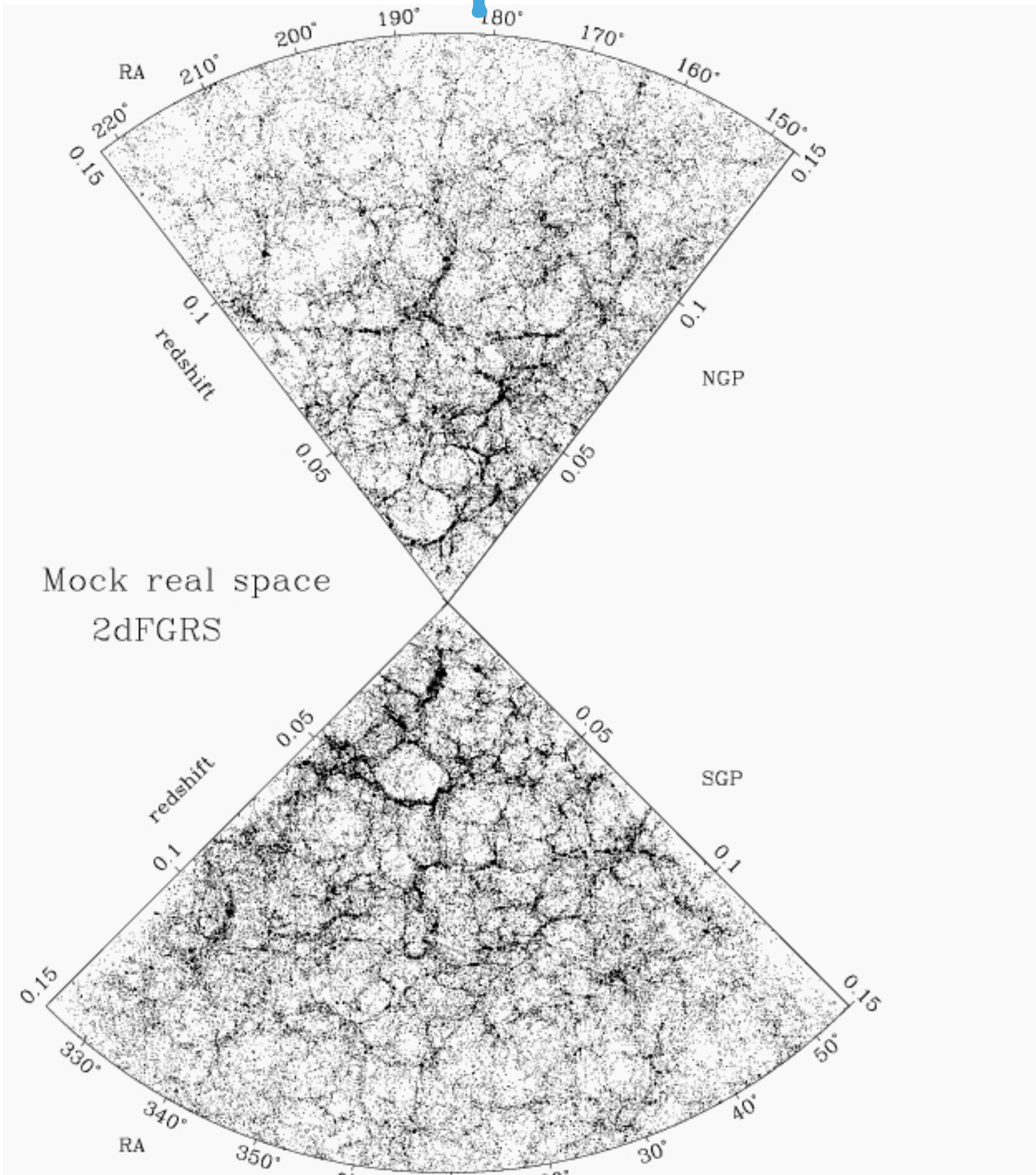
Effect of a long-wavelength perturbation on the density of local tracers (galaxies, halos…)

$$\delta_g = b_1[\delta] + b_{\nabla^2\delta}[\nabla^2\delta] + [\varepsilon]$$
$$+ \frac{1}{2}b_2[\delta^2] + b_{K^2}[(K_{ij})^2] + [\varepsilon_\delta\delta]$$
$$+ \ldots$$

Local: 2 derivatives of Φ
Higher derivative
Stochastic

$K_{ij} = (\partial_i\partial_j/\nabla^2 - \delta_{ij}/3)\delta$ **Tidal field**

real-space

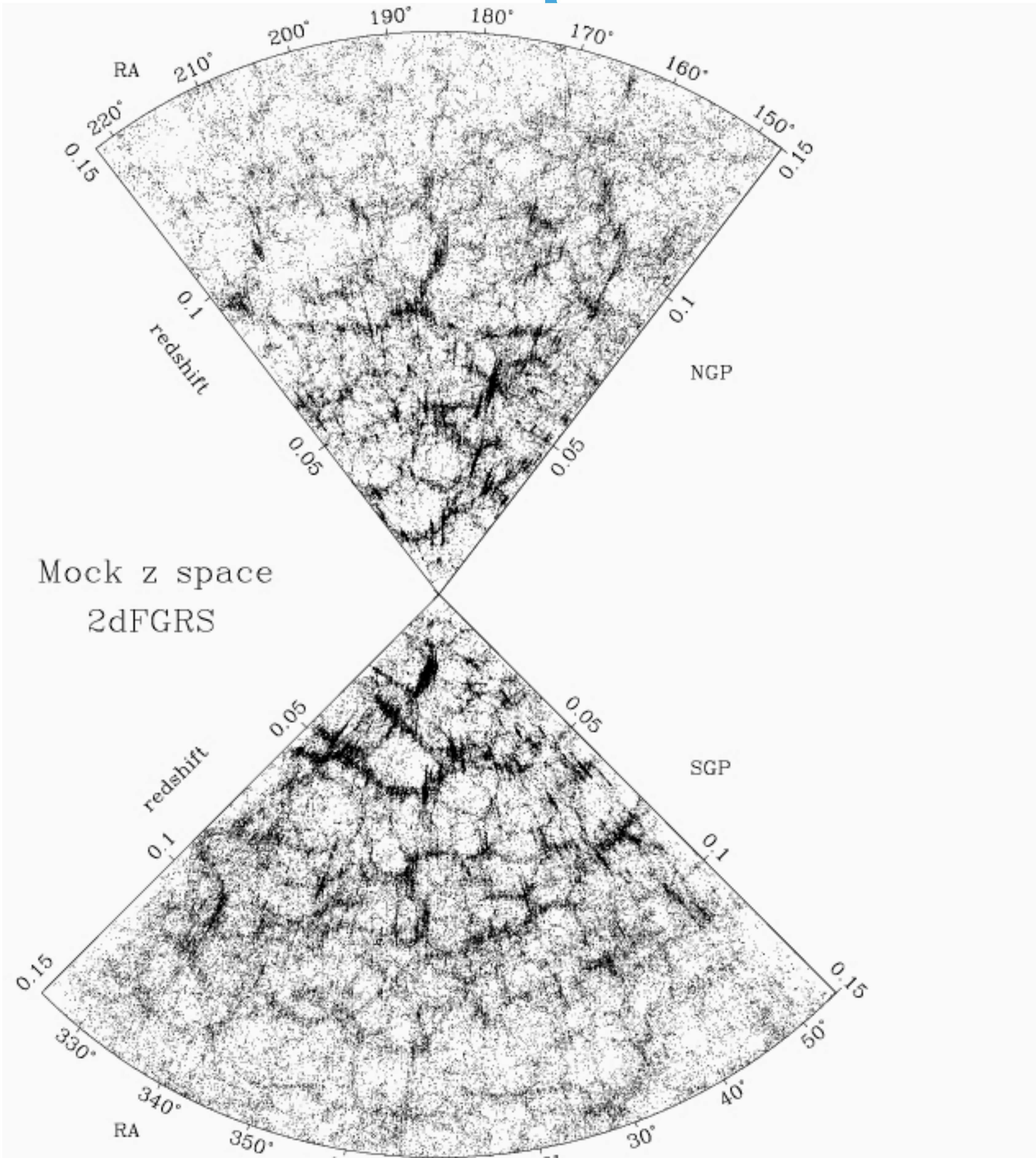


RA
220°
210°
200°
190°
180°
170°
160°
150°
0.15
0.1
0.05
redshift
NGP
Mock real space
2dFGRS
SGP
330°
340°
350°
30°
40°
50°

redshift-space

large scale:
Kaiser

small scale:
FoG



IR-UV mixing in redshift space

Real to redshift space mapping:

$$\vec{x}_n \to \vec{s}_n = \vec{x}_n + \frac{p_n^z}{a\mathcal{H}m}\hat{z}$$

(plane parallel approx.)

$$\delta_D(\vec{k}) + \delta_s(\vec{k}) = \int \frac{d^3\vec{x}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[1 + \delta(\vec{x})\right] \exp\left[ik_z v_z(\vec{x})/\mathcal{H}\right]$$

see Scoccimarro '04,

$\langle \delta_s(\vec{x})\,\delta_s(\vec{y})\rangle$ gets contributions from terms like $\langle \delta(\vec{x})\,\delta(\vec{y}) v_z^2(\vec{y})\rangle \sim \langle \delta(\vec{x})\,\delta(\vec{y})\rangle \langle v_z^2\rangle$

even at large $|\vec{x} - \vec{y}|$

short scale effect

Large scales feel short ones!!

Problems for PT even at very large scales

Models for RSD

$$\mu = \hat{k} \cdot \hat{z}$$

L-Kaiser: $P_g^{\mathrm{S}}(k,\mu) = (1 + f\mu^2)^2 P_g(k)$

NL-Kaiser:

$$P_g^{\mathrm{S}}(k,\mu) = P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)$$

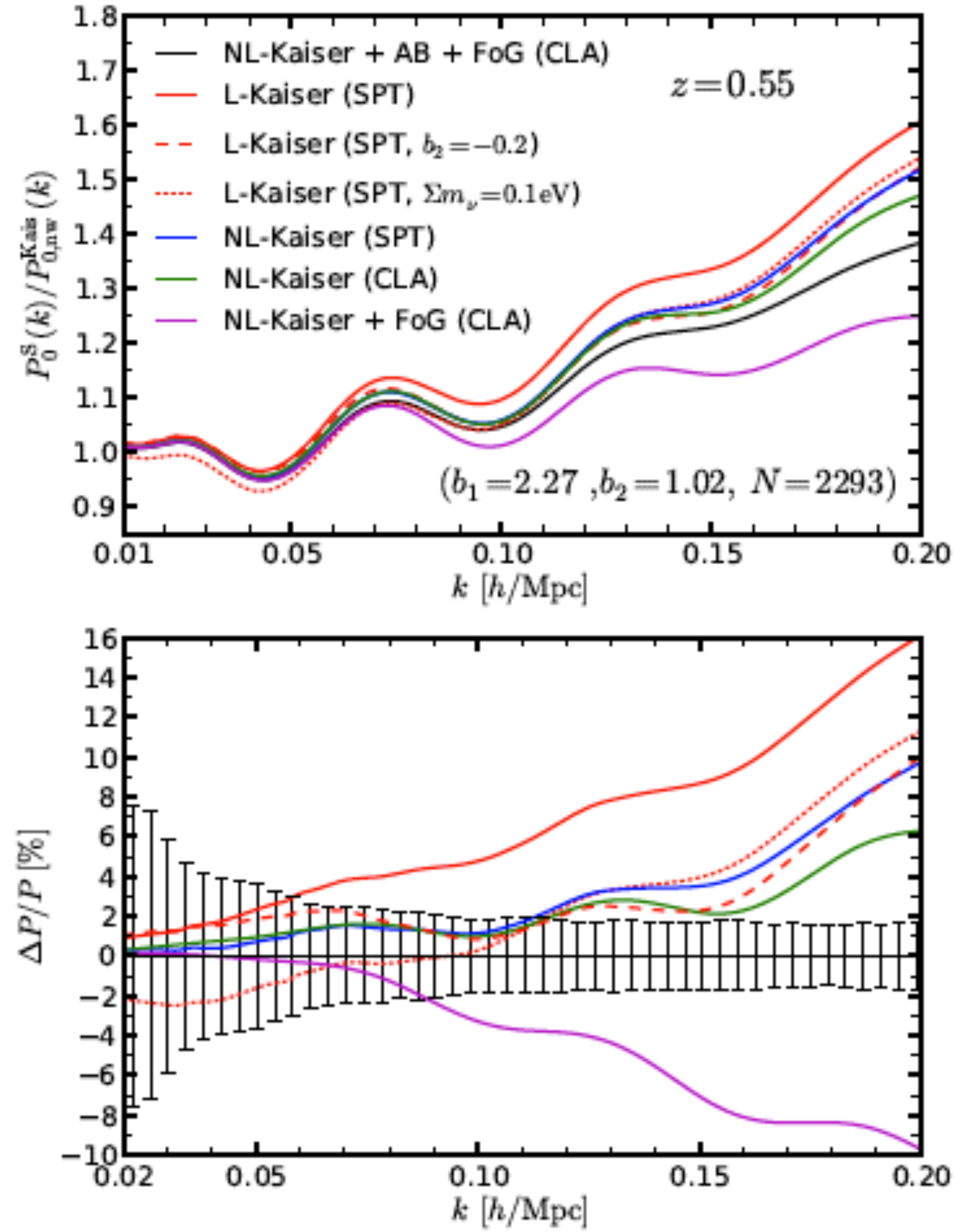
NL-Kaiser +FoG:

$$P_g^{\mathrm{S}}(k,\mu) = \exp\left(-f^2\sigma_{\mathrm{V}}^2 k^2 \mu^2\right) \times \left[P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)\right]$$

NL-Kaiser +AB+FoG:

(Taruya, Nishimichi, Saito 1006.0699)

$$P_g^{\mathrm{S}}(k,\mu) = \exp\left(-f^2\sigma_{\mathrm{V}}^2 k^2 \mu^2\right) \times \left[P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + b_1^3 A(k,\mu;\beta) + b_1^4 B(k,\mu;\beta)\right],$$



Zhao et al (BOSS) 1211.3741

Putting all together…

D'Amico et al. 1909.05271
Ivanov et al. 1909.05277
Colas et al. 1909.07951

$$P_{g,\ell}(k) = P^{\rm tree}_{g,\ell}(k) + P^{1-\rm loop}_{g,\ell}(k) + P^{\rm noise}_{g,\ell}(k) + P^{\rm ctr}_{g,\ell}(k)$$

$$P^{\rm tree}_{g}(k,\mu) = (b_1 + f\mu^2)^2 P_{\rm lin}(k)$$

RSD linear (Kaiser)

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2$$

$$P^{\rm noise}_{g,0}(k) = P_{\rm shot}\,, \qquad P^{\rm noise}_{g,2}(k) = 0$$

$$P^{\rm ctr,LO}_{\ell}(k) \equiv -2\,c_\ell^2\,k^2\,P_{\rm lin}(k)\,, \qquad \ell = 0,2$$

EFT counterterms in redshift space

$$P^{\rm ctr,NLO}(k,\mu) \equiv \tilde{c}\,k^4\,\mu^4\,f^4\,(b_1 + f\mu^2)^2 P_{\rm lin}(k)$$

RSD beyond Kaiser

+ IR resummation!!

Putting all together…

D'Amico et al. 1909.05271
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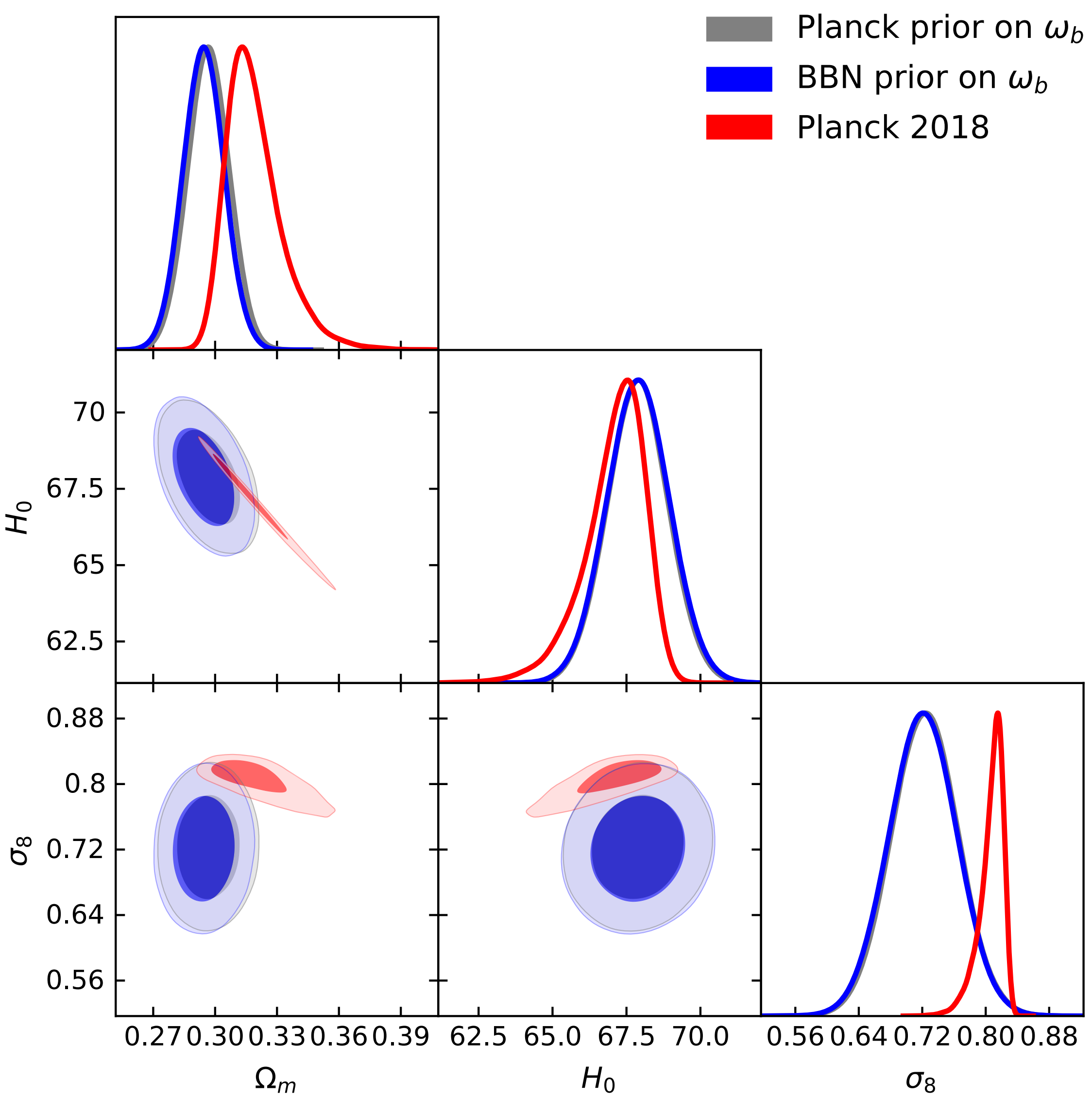

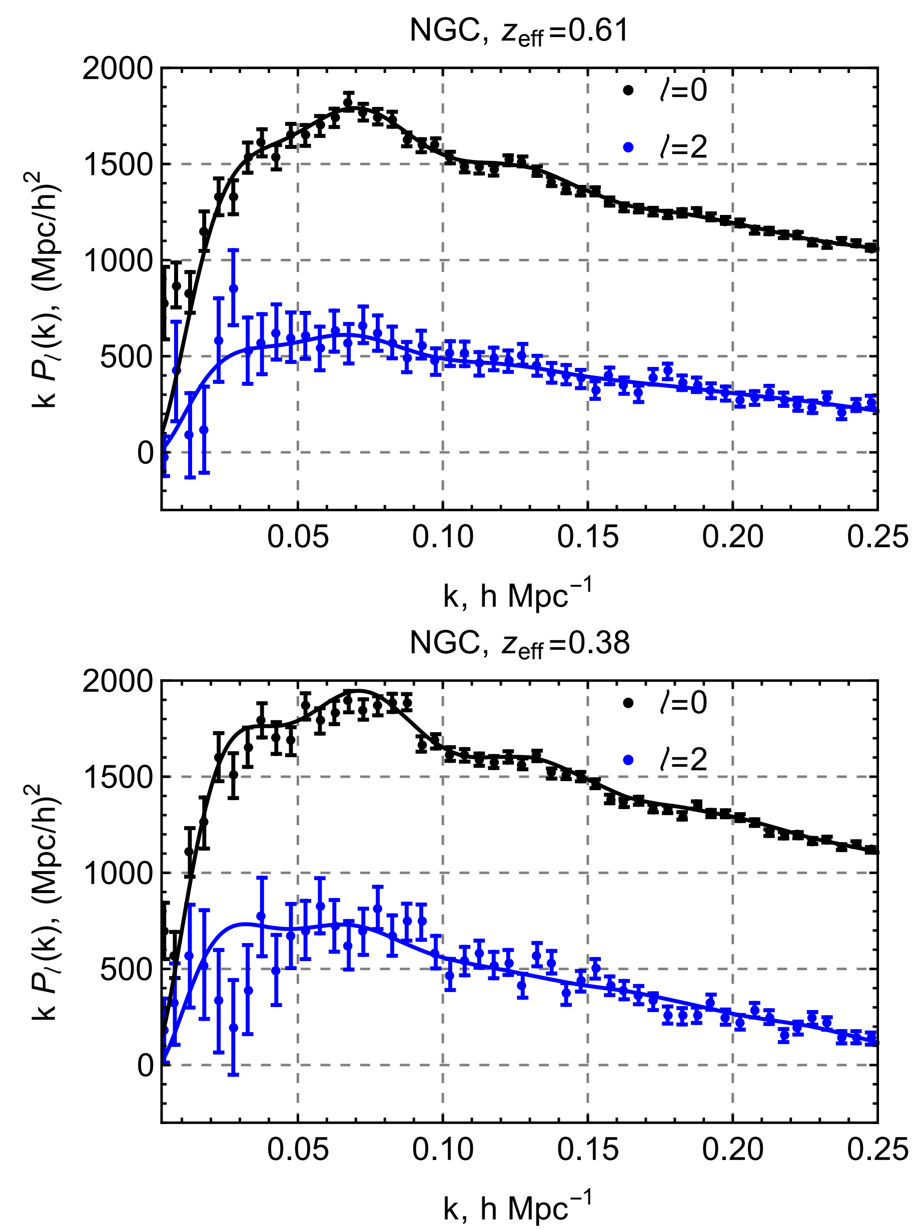

Constraints on (some) cosmological parameters already comparable with Planck