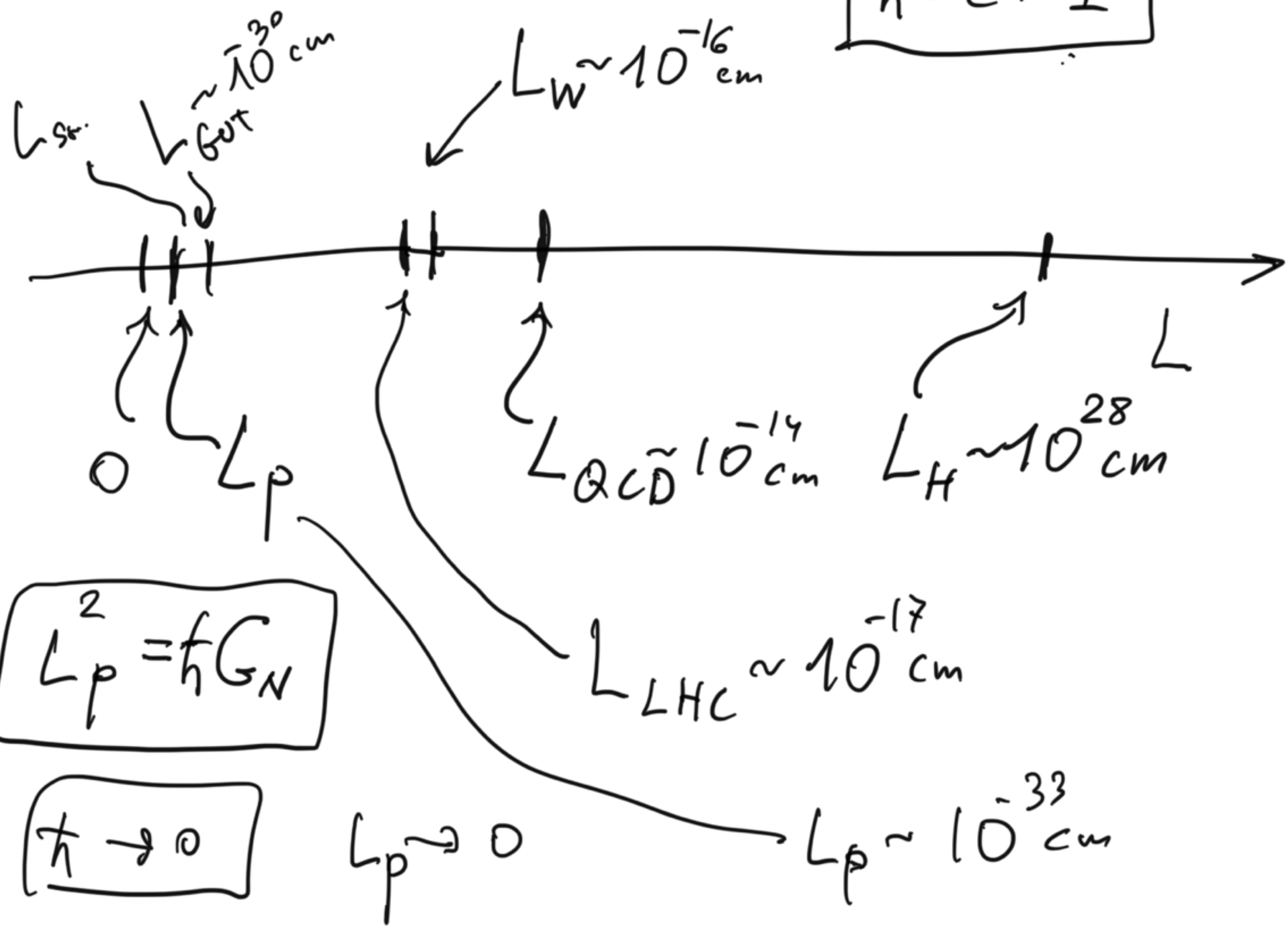
BSM Lecture 1.

$\boxed{\hbar = c = 1}$



$$\boxed{L_p^2 = \hbar G_N}$$

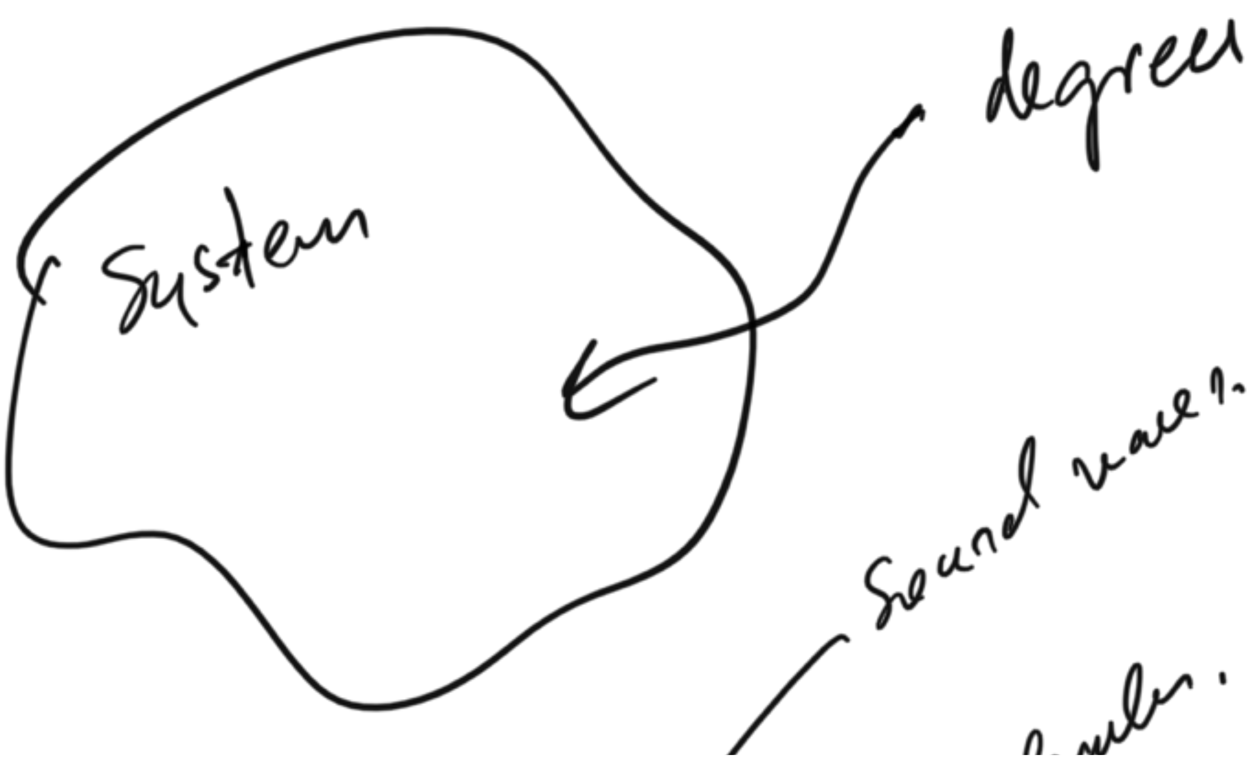
$\boxed{\hbar \to 0}$ $\quad L_p \to 0$

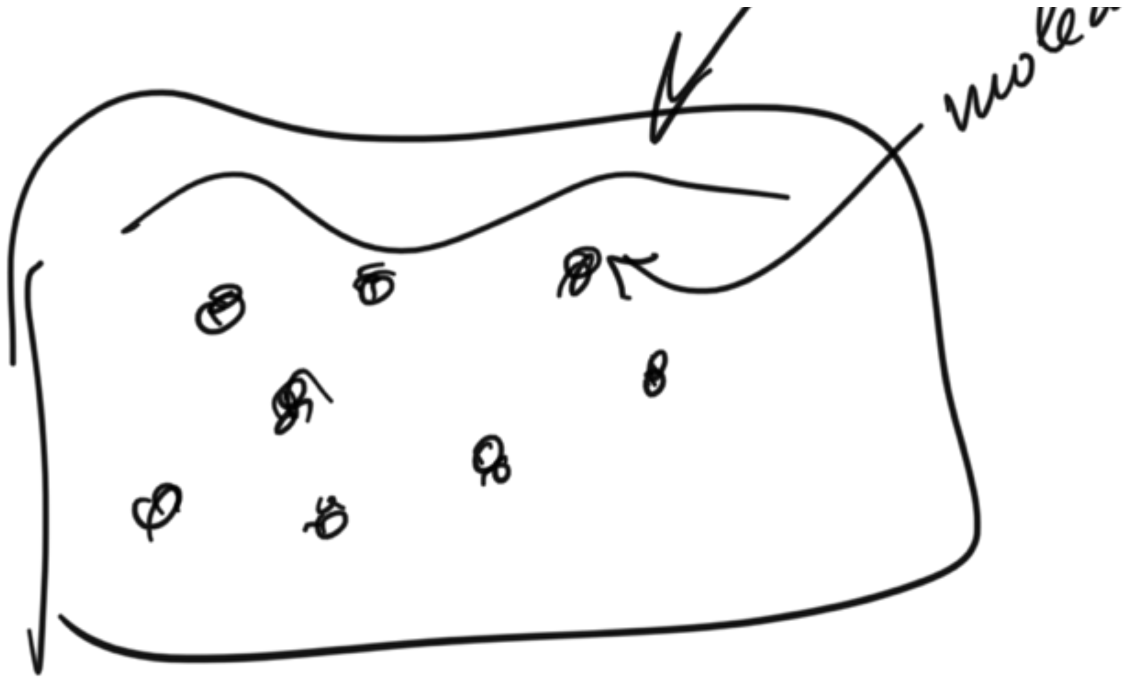
$G_N \to 0.$

Elementarity $\neq$ short length-scale

Framework $\longrightarrow$ QFT, EFT

degrees of freedom





QFT $\longrightarrow$ quantum oscillators

$$\hat{a}_j , \hat{a}_j^{+} \qquad [a_j , \hat{a}^{+}_{j'}] = \delta_{jj'}$$

label (identity)

In QFT $\quad \hat{\phi}(x) = \sum_j \hat{a}_j$

$\boxed{\text{local QF}}$

momentum eigenmode.
spin polarizations
internal quantum numbers.

$$\hat{\phi}(x) = \sum_j \psi_j(x) \hat{a}_j + h.c.$$

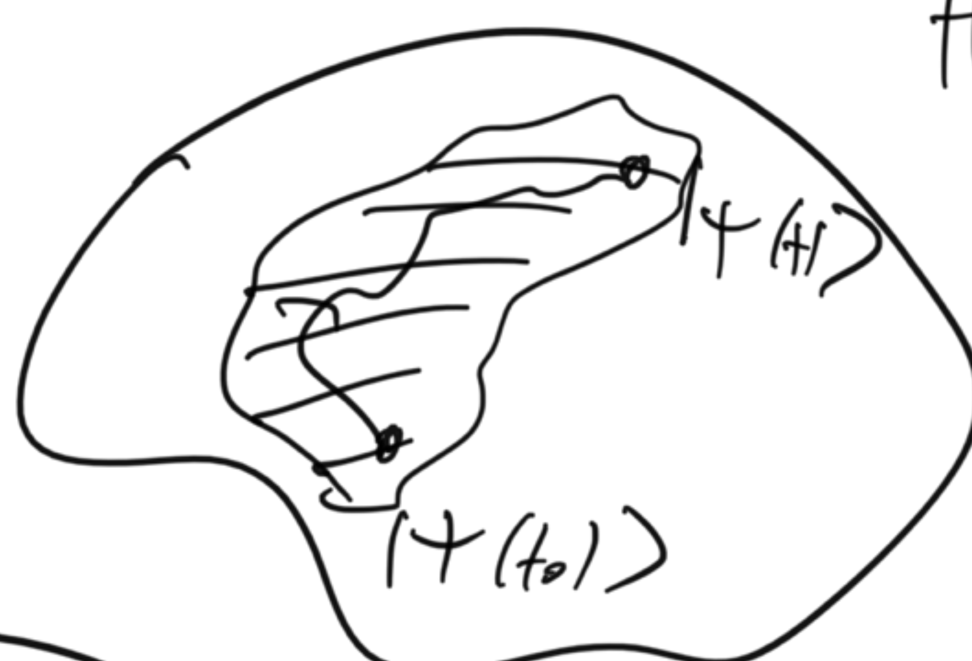
complete set of eigenfunctions

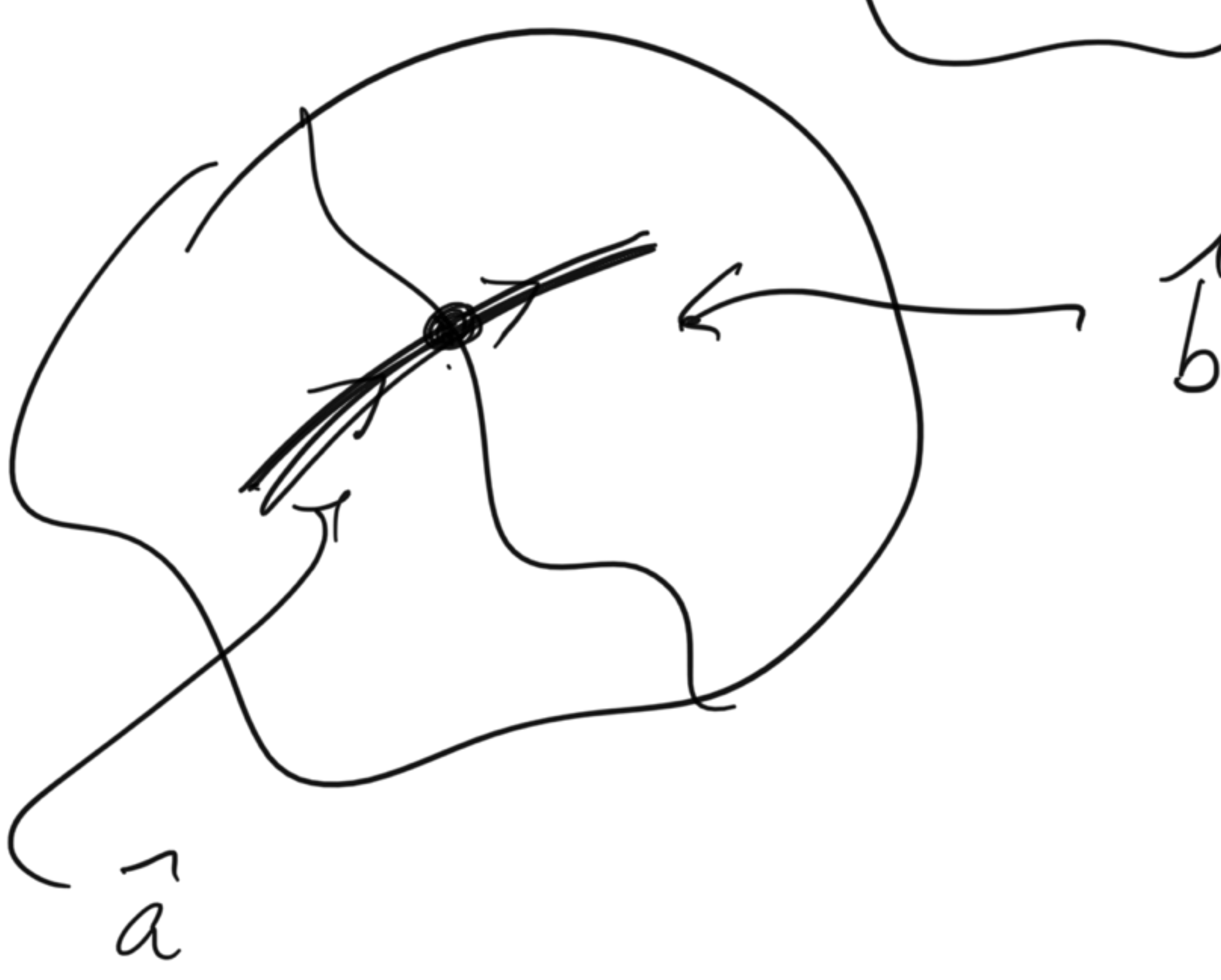
Which are good degrees of freedom?

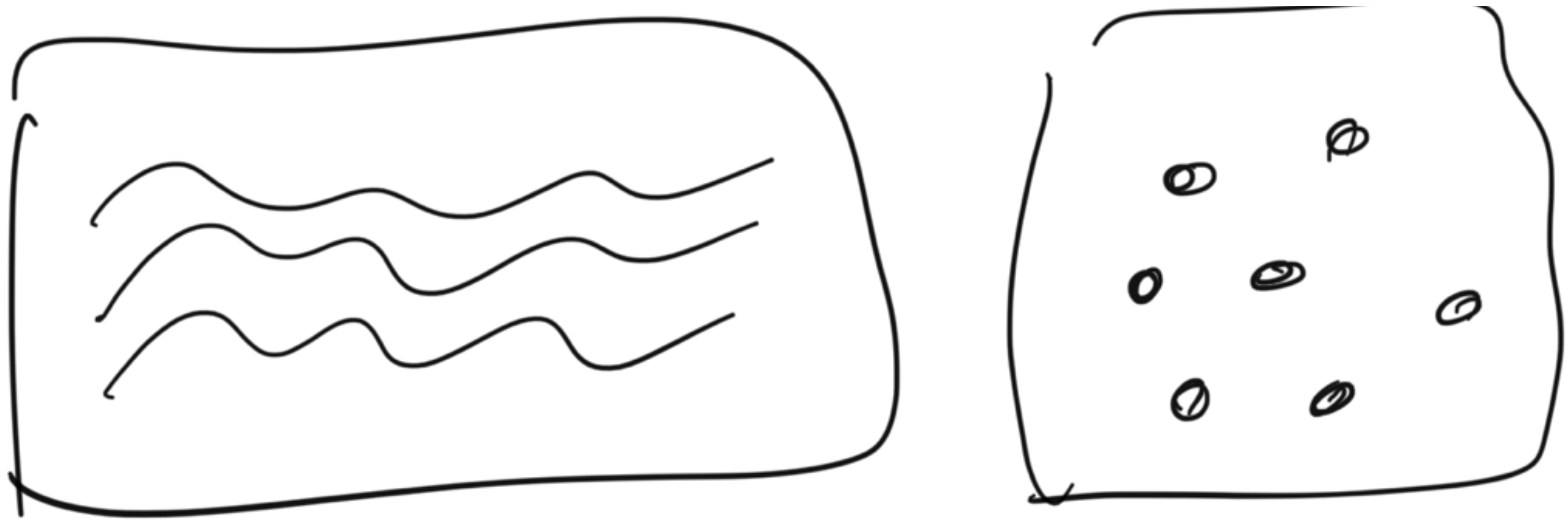
In our classification there are <u>weakly interacting</u> degrees of freedom

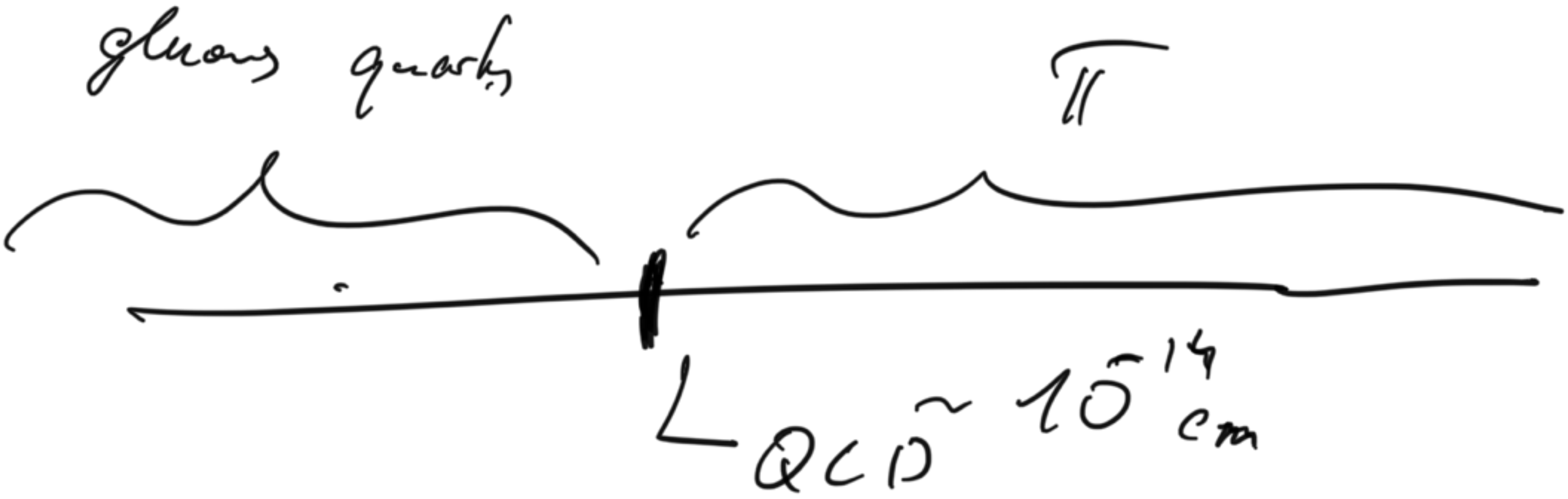
$$\hat{H} = \underbrace{\sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j}_{\text{"Free" Hamiltonian}} + \text{higher order terms}$$

$$\underbrace{\boxed{\alpha_{jkmn}\, \omega\, \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_m \hat{a}_n} + \dots}_{\hat{H}_{int}}$$









For weakly-interacting degrees of freedom, we can define a dimensionless coupling constant

$\equiv \alpha \leftarrow$ controls interaction among canonically normalised QF

$\alpha \ll 1$

we can work in perturbative expansion in α

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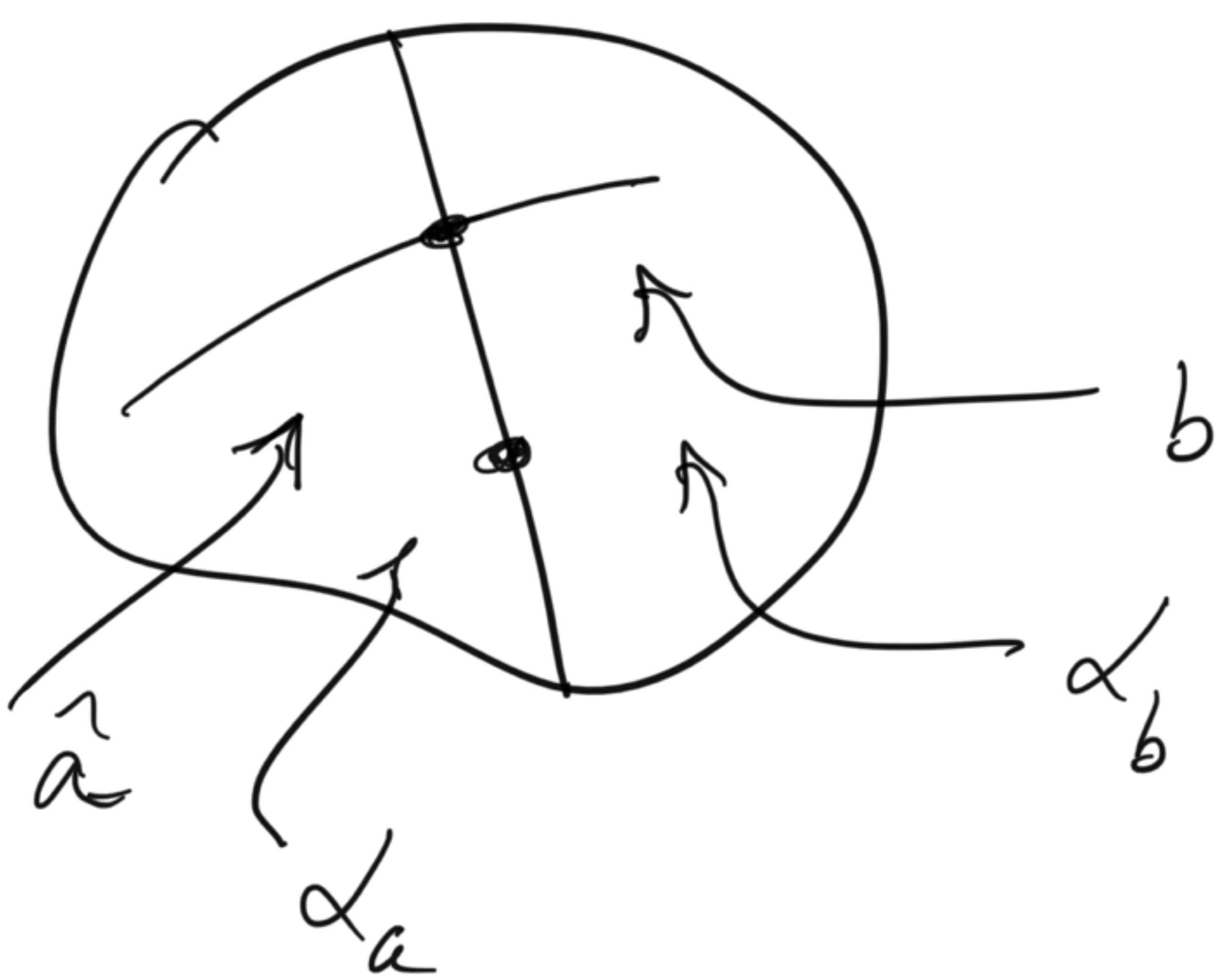
n-loop order

$$\boxed{n \sim \frac{1}{\alpha}}$$

$n \ll \frac{1}{\alpha} \leftarrow$ perturbative expansion is OK



SM extremely successfull description of nature

① No experimental data indicating

inconsistency of SM

(Incompleteness, yes!)

(2) Theoretical potential "inconsistencies" set in at extremely short scales.

QFT building block of SM:

SM describes three interactions

Strong, weak, electromagnetic.

They have messengers (they are mediated by messengers) <u>spin-1</u>.

Fields (theories) of spin ≥ 1 exhibit gauge redundancy.

(Even pseudoscalars can be reformulated in gauge redundant way).

For example, Maxwell theory

γ_μ $\qquad F_{\mu\nu} = \partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu$

$$\underline{\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}} + \gamma_\nu J^\nu$$

exhibit gauge redundancy:

$$\gamma_\mu \to \underline{\gamma_\mu} + \partial_\mu \alpha(x)$$

arbitrary.

$$\partial^\mu F_{\mu\nu} = J_\nu$$

$$\hookrightarrow \partial_\nu J^\nu = 0$$

The gauge redundancy of SM is effectively described as

$$SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$$

local (gauge) symmetry.

$A_\mu^{a=1,2,\dots 8}$ gluons

$W_\mu^\pm, Z_\mu \leftarrow$ massive

$\gamma_\mu \leftarrow$ massless photon.

(momentum) at the fundamental level. → $\boxed{m_\gamma \lesssim 10^{-14-17}\,\text{ev}}$

However, $SU(3)_c$ exhibits a mass gap due to confinement.

Three main phases of spin-1 gauge theory:

(1) Coulomb phase. γ_μ

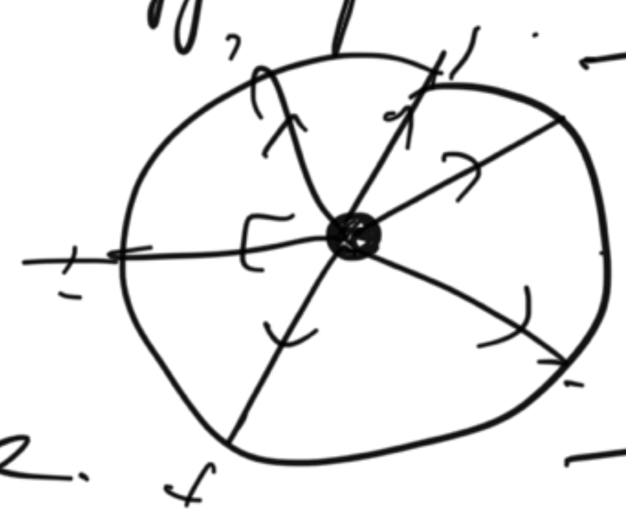
$J^\mu \quad \gamma_\mu \quad J^\mu$ $\qquad V(r) \propto \frac{1}{r}$

(2) Higgs (Proca) phase

$J^\mu \quad Z_\mu \quad J^\mu$ $\qquad V(r) \propto \frac{e^{-m_Z r}}{r}$

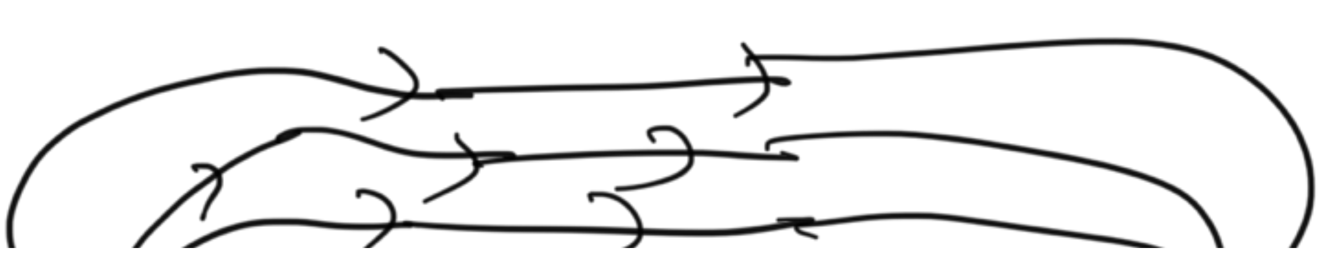
Yukawa type potential

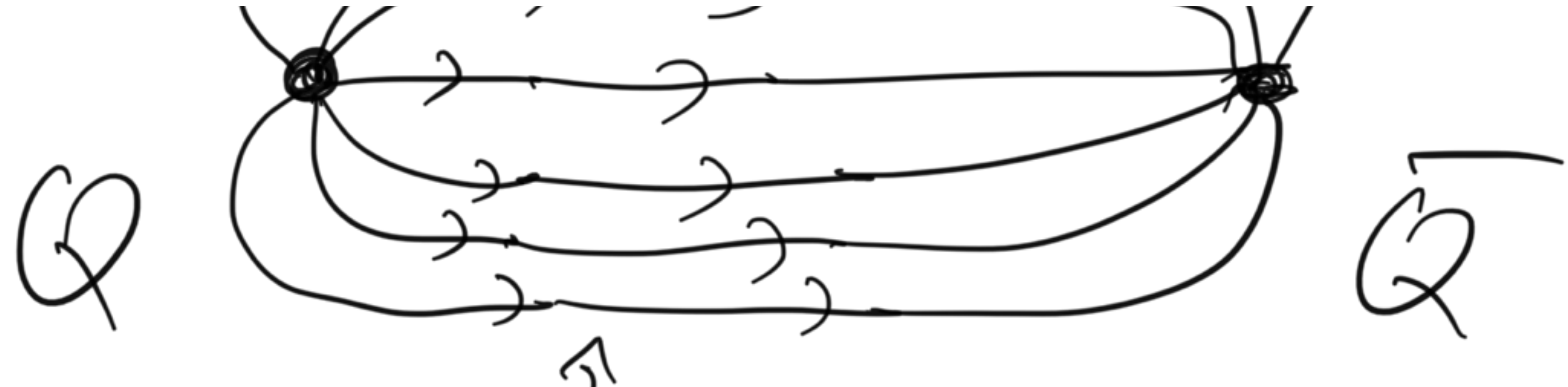
$m_Z^{-1} \sim 10^{-16}\,\text{cm.}$



m_γ^{-1}

(3) Confining phase.





gluon (color) flux.

Next ingredient "matter": $\boxed{\text{spin} = \frac{1}{2}}$

$SU(2)_W \otimes U(1)_Y$

$\hookrightarrow L$

ψ_L $\quad$ $\psi_R \to \psi_L^c$

?

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L\alpha} \quad \begin{matrix} u_R \leftrightarrow u_L^c \\ d_R \leftrightarrow d_L^c \end{matrix} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{matrix} \nu_{eR} \leftrightarrow \nu_{eL}^c \\ e_R \leftrightarrow e_L^c \end{matrix}$$

$$\begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{matrix} c_R \\ s_R \end{matrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{matrix} \nu_{\mu R} \; ? \\ \mu_R \end{matrix}$$

$$\begin{pmatrix} t \\ b \end{pmatrix}_L \quad \begin{matrix} t_R \\ b_R \end{matrix} \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \begin{matrix} \nu_{\tau R} \; ? \\ \tau_R \end{matrix}$$

$\boxed{A, B = 1, 2, 3}$

SU(2)

Higgs field $H_\alpha = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$ $\alpha = 1, 2$

$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$ $v > 0$

$$\boxed{v \sim 100 \text{ GeV}}$$

flavor generation index $A = 1, 2, 3$

?

$$Q^A_{L\alpha} \equiv \begin{pmatrix} u \\ d \end{pmatrix}^A_L \quad u^A_R \quad d^A_R \quad L_\alpha \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}^A_L \quad \nu^A_R \quad e^A_R$$

$$\boxed{\begin{aligned} &g^u_{AB} H_\alpha \bar{Q}^{\alpha A}_L u^B_R + g^d_{AB} H^{+\beta} \epsilon_{\beta\alpha} \bar{Q}^{\alpha A}_L d^B_R \\ &g^\nu_{AB} H_\alpha \bar{L}^{\alpha A}_L \nu^B_R + g^e_{AB} H^{+\beta} \epsilon_{\beta\alpha} \bar{L}^{\alpha A}_L e^B_R \end{aligned}}$$

$$\langle H_\alpha \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} + \begin{pmatrix} h(x) \\ 0 \end{pmatrix} \qquad \boxed{H = \rho(x) \begin{pmatrix} \cos\theta_{(x)} e^{i\alpha_{(x)}} \\ \sin\theta_{(x)} e^{i\beta_{(x)}} \end{pmatrix}}$$

$$H = \rho(x) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \langle \rho(x) \rangle = v$$

$$\boxed{\rho(x) = v + h(x)}$$

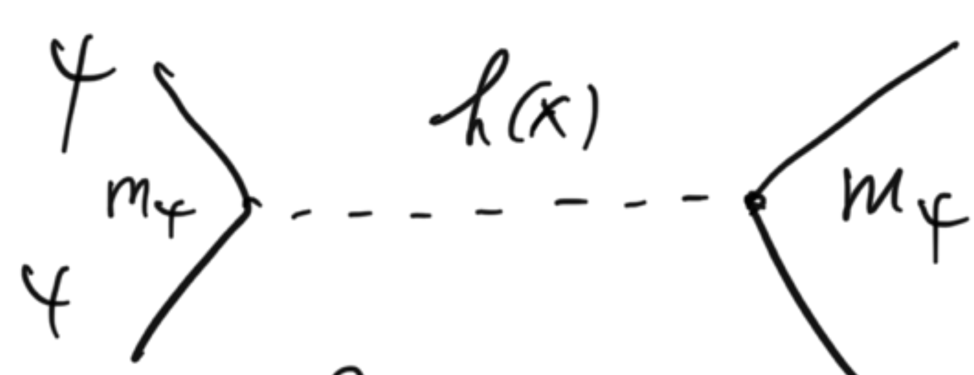
Higgs boson.

$$\overbrace{g^{\psi}_{AB}}^{m_{AB}}(v + h(x))\bar{\psi}^{A}_{L}\psi^{B}_{R} =$$

$m^{\psi}_{AB} \equiv g^{\psi}_{AB} v$

$g^{\psi}_{AB} = \dfrac{m^{\psi}_{AB}}{v}$

$$= m^{\psi}_{AB}\bar{\psi}^{A}_{L}\psi^{B}_{R} + h(x)\left(\frac{m^{\psi}_{AB}}{v}\right)\bar{\psi}^{A}_{L}\psi^{B}_{R}$$



$$\boxed{V(r) \propto \frac{e^{-m_h r}}{r} \cdot \frac{m_\psi^2}{v^2}}$$

$$\boxed{G_H = \frac{1}{v^2}}$$

effective "Newtons" constant

of " Higgs"

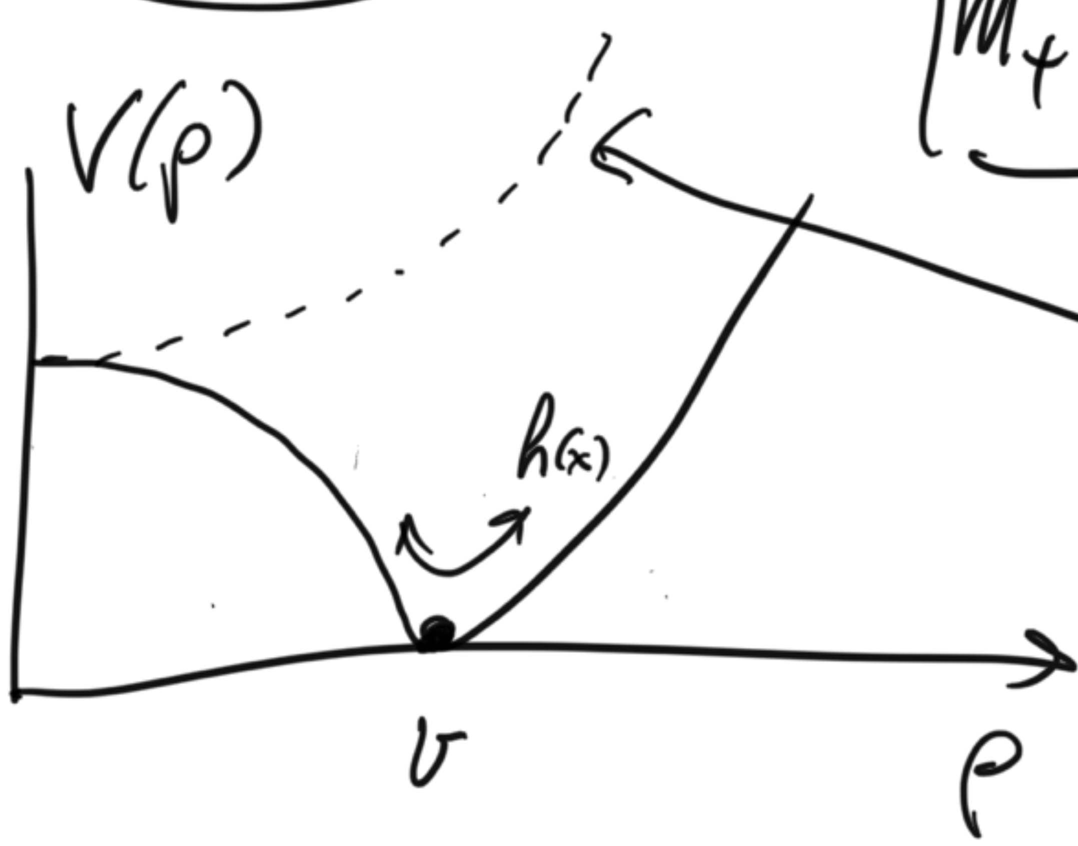
$M_p \sim 10^{19}\,\text{GeV}$

$$G_N \equiv L_p^2 = \frac{1}{M_p^2}$$

$$\boxed{\frac{G_N}{G_H} = \left(\frac{v}{M_p}\right)^2 \sim 10^{-34}}$$

$m_h = 0$

$$\boxed{m_\psi = g_\psi v}$$



$T \gg v$

$$\boxed{E = \sqrt{m_\psi^2 + |\vec{p}|^2}}$$

$$\boxed{|\vec{p}| \gg m_\psi}$$

Spin ≥ 1

$$H = \underline{v} + h(x)$$

$$V(H) = \frac{\lambda^2}{2}\left(H^\dagger H - v^2\right)^2 =$$

$$= -\underline{\underline{\lambda^2 v^2}}\, H^\dagger H + \frac{\lambda^2}{2}\left(H^\dagger H\right)^2 + \frac{\lambda^2}{2} v^4$$

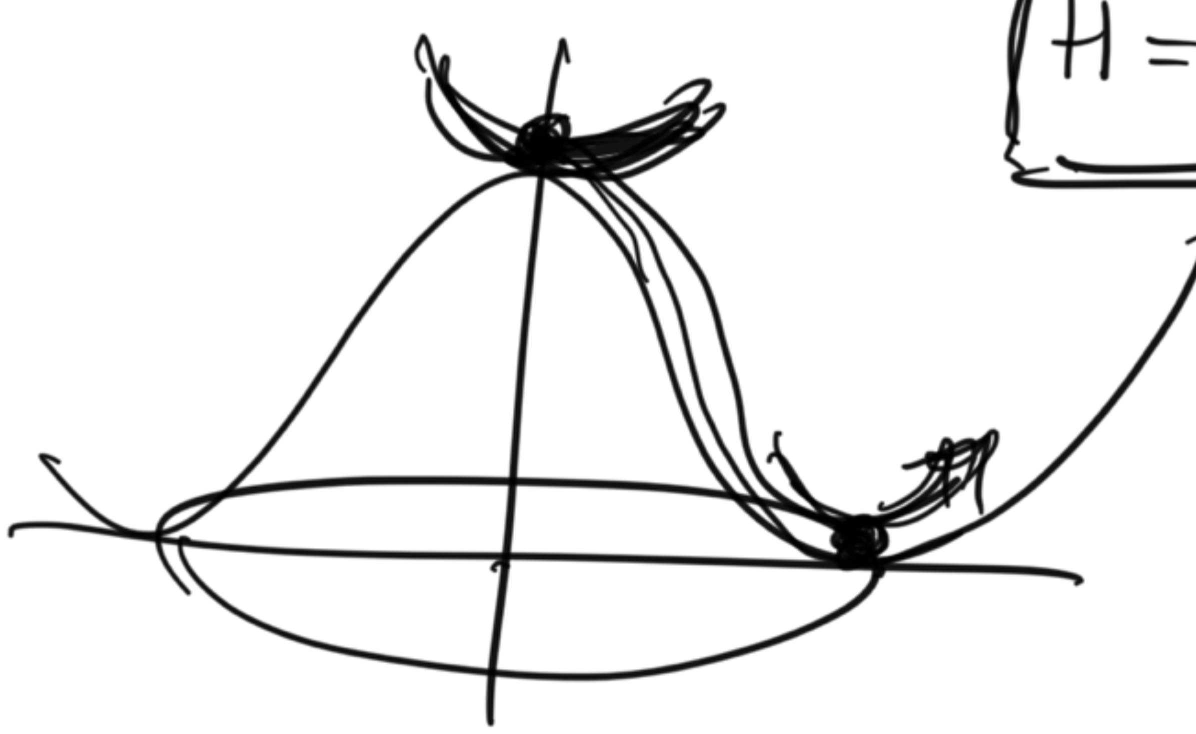
$$|0\rangle \qquad |1\rangle \qquad |2\rangle$$

$$a^\dagger |0\rangle = |1\rangle$$

$$\boxed{\langle H \rangle = 0} \qquad \underline{\delta H} = h$$

$$m^2 \qquad\qquad \langle H \rangle = v$$

$$\boxed{H = v + h(x)}$$



$$\underline{\left(\Box + m^2\right) h = 0}$$

$$h \Rightarrow h_{\vec{k}}(t)$$

$$\ddot{h}_{\vec{k}}(t) + k^2 h_{\vec{k}}(t) + m^2 h_{\vec{k}}(t) = 0$$

ordinary $\quad m^2 > 0$

tachyon. $\quad m^2 < 0$

$$\boxed{|k| < |m|}$$

$$\partial^\mu F_{\mu\nu} = J_\nu$$

Spin-1

$$F_{\mu\nu} = \partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu$$

$$\gamma_\mu \to \gamma_\mu + \partial_\mu \alpha(x) \qquad \boxed{\partial_\mu \gamma^\mu = 0}$$

$$\partial^\nu \left(\boxed{\partial^\mu F_{\mu\nu} + m^2 \gamma_\nu = J_\nu} \right)$$

Proca

$$\boxed{\partial_\nu \gamma^\nu = \frac{1}{m^2} \partial_\nu J^\nu}$$

$$\gamma_\nu^{(P)} = \gamma_\nu^{(M)} + \partial_\nu \phi$$

$$\boxed{SU(3)_c \otimes SU(2)_w \otimes U(1)_Y} \subset SU(5)$$

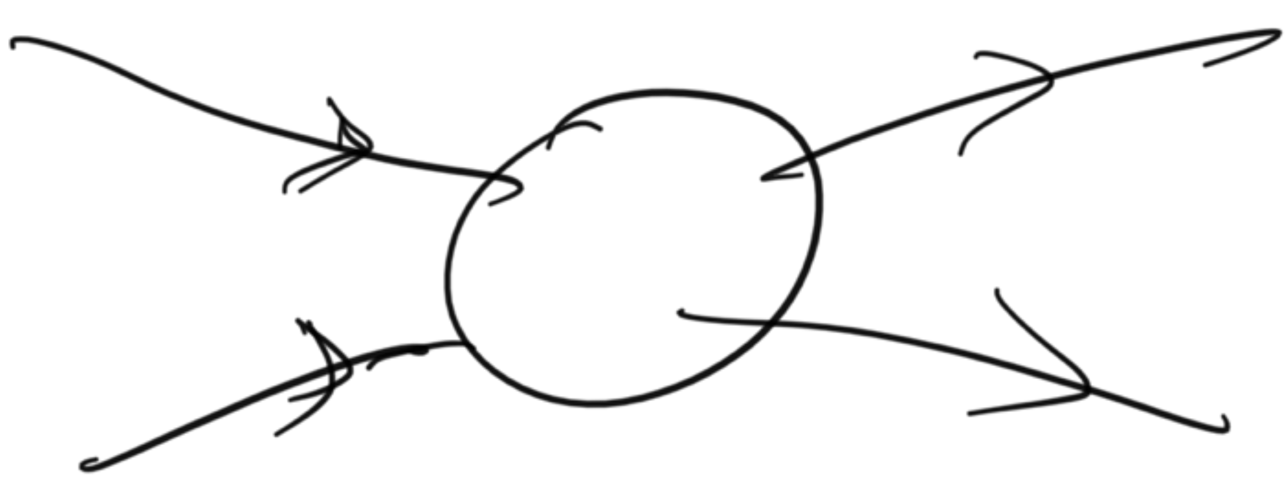
$$\boxed{\gamma_\mu \gamma^\mu m^2}$$

$$\boxed{T > 0} \qquad \cancel{\text{Poincare}}$$

$$\hat{a}_k \longrightarrow \boxed{\hat{\phi}(x)}$$

$$\boxed{\hat{H} = \sum \hat{a}^\dagger \hat{a} + \cdots}$$

S-matrix



$|i\rangle$ $t=-\infty$ $\qquad$ $|f\rangle$ $t=+\infty$

$$S_{fi} = \boxed{\langle f | \hat{S} | i \rangle}$$

$h \quad T^3$

$|0\rangle$ $n \sim 1$

$|n\rangle$

$g_\psi H \bar{\psi}_L \psi_R$

$\psi_L \to e^{i\alpha_L} \psi_R$

$\psi_R \to e^{i\alpha_R} \psi_R$

vector.

$\psi_L \to e^{i\alpha} \psi_L$

$\psi_R \to e^{i\alpha} \psi_R$

axial

chiral

$\psi_L \to e^{-i\alpha} \psi_L$

$\psi_R \to e^{i\alpha} \psi_R$

$m \bar{\psi}_L \psi_R \to m e^{2i\alpha} \bar{\psi}_L \psi_R$.

$$\boxed{m_q \ll \Lambda_{QCD}}$$

$\eta' \to$ anomaly

$m_u = ?$

PQ $U(1)$

$\Phi \bar{\psi}_L \psi_R$ $\langle \phi \rangle \neq 0$

$$\psi_L \to e^{i\alpha} \psi_L$$

$$\psi_R \to e^{-i\alpha} \psi_R$$

$$\Phi \to e^{i2\alpha} \phi$$

$g_u H \bar{u}_c \psi_R$

$$g_u = 0$$

$$\boxed{m_q < \Lambda_{QCD}}$$

$$m_t \gg \Lambda_{QCD}$$

$$m_\psi \bar{\psi}_L \psi_R \rightarrow g_\psi H \bar{\psi}_L \psi_R =$$

$$\langle H \rangle = v \qquad = g_\psi v \bar{\psi}_L \psi_R +$$

$$H = v + h(x) \qquad g_\psi h(x) \bar{\psi} \psi$$