BSM Lecture 2

$SU(3)_c \otimes \boxed{SU(2)_W \otimes U(1)_Y}$ chiral theory

$A_\mu^{a=1..8}$ $\quad W^\pm_\mu, Z_\mu \quad \gamma_\mu$

gluons

"messengers". spin $= 1$

"Matter" spin $= \frac{1}{2}$ quarks, leptons

generation index

?

$$Q^{A=1,2,3}_{L\alpha} \equiv \begin{pmatrix} u \\ d \end{pmatrix}^A_L \quad u^A_R \quad d^A_R \qquad L^A_{L\alpha} \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}^A_L \quad \nu^A_R \quad e^A_R$$

$\alpha = 1,2$

$SU(2)_W$ index

$$H_\alpha = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \qquad \boxed{\langle H_\alpha \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}}$$

$$g^u_{AB} H_\alpha \bar{Q}^{\alpha A}_L u^B_R + g^d_{AB} H^{+\beta} \epsilon_{\beta\alpha} \bar{Q}^{\alpha A}_L d^B_R$$

$$+ g^\nu_{AB} H_\alpha \bar{L}^{\alpha A}_L \nu^B_R + g^e_{AB} H^{+\beta} \epsilon_{\beta\alpha} \bar{L}^{\alpha A}_L e^B_R$$

?

$\nu_R =$ gauge-neutral

(absolutely) symmetries

$U_B(1) \leftarrow$ baryon number.

quarks $\to e^{i\alpha}$ quarks.

$U_L(1) \leftarrow$ lepton number

lepton $\to e^{i\beta}$ lepton.

But global symmetries can be explicitly broken.

$\cancel{B+L}$ by anomaly.

$B-L$ is not broken by anomaly within SM

L - number $U(1)_L$

$$\boxed{\nu_R \to e^{i\alpha} \nu_R}$$

charge conjugation

$$\boxed{M_R \nu_R^T C \nu_R}$$

$$\boxed{M_R,\ g^{\nu}_{AB} = ?}$$

Dirac ... Real Majorana

$\psi_j \leftarrow$ Spinor

$$\boxed{\psi_j = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}}$$

Majorana Basis

$\boxed{\gamma_\mu \leftarrow \text{real}}$

$\boxed{\gamma_0 \quad \vec{\gamma}}$

$\boxed{\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3}$

Majorana invariant

$$(\psi^T \gamma_0 \psi) = \text{invariant.}$$

$$M_M \psi^T \gamma^0 \psi = M_M \psi^T \gamma^0 \left(\frac{1+i\gamma_5}{2} + \frac{1-i\gamma_5}{2}\right)\psi =$$

$$\boxed{\psi_\pm = \frac{1 \pm i\gamma_5}{2}\psi} = M_M \psi^T \gamma^0 (\psi_+ + \psi_-)$$

$$\boxed{\psi = \psi_+ + \psi_-}$$

$$\left(\frac{1 \pm i\gamma_5}{2}\right)^2 = \frac{1 \pm i\gamma_5}{2}$$

$$H = \rho(x) \underbrace{\begin{pmatrix} \cos\theta\, e^{i\alpha} \\ \sin\theta\, e^{+i\beta} \end{pmatrix}}$$

$$\rho(x) = \sqrt{H^\dagger H}$$

$$H = (v + h(x)) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$g_\psi H \bar{\psi}_L \psi_R = g_\psi (v + h(x)) \bar{\psi}_L \psi_R =$$

$$= \underbrace{g_\psi v}_{m_\psi} \bar{\psi}_L \psi_R + g_\psi h(x) \bar{\psi}_L \psi_R$$

$$g_\psi = \frac{m_\psi}{v}$$

$$\boxed{G_H = \frac{1}{v^2}}$$

Maxwell $\quad U(1) \quad \boxed{A_\mu \to A_\mu + \partial_\mu \alpha}$

$$\boxed{\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial^\nu \left(\partial^\mu F_{\mu\nu} = J_\nu \right) \longrightarrow \partial_\nu J^\nu = 0$$

$$\boxed{\partial_\mu A^\mu = 0} \qquad \partial_\mu A^{\prime\mu} \neq 0$$

$$A^\mu = A^{\prime\mu} + \partial_\mu \alpha$$

$$\Box \alpha = -\partial_\mu A^{\prime\mu}$$

$$A^\mu \to A^\mu + \partial_\mu \alpha \qquad \partial_\mu A^\mu = 0$$

$$\boxed{\Box \alpha = 0}$$

$$\boxed{\Box A_\mu = J_\mu}$$

2 - degrees of freedom.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} \tilde{A}_\mu \tilde{A}^\mu + \tilde{A}_\mu J^\mu$$

Proca field

$$\boxed{\cancel{\tilde{A}_\mu \to \tilde{A}_\mu + \partial_\mu \alpha}}$$

$$\partial^\nu \left(\partial^\mu F_{\mu\nu} + m^2 \tilde{A}_\nu = J_\nu \right)$$

$$\boxed{\partial_\nu \tilde{A}^\nu = \frac{1}{m} \partial_\nu J^\nu}$$

$$3 = 2 + 1$$

Proca Maxwell longitudinal

$$\tilde{A}^\nu = A^\nu + \partial^\nu \phi$$

$$A^\nu \to A^\nu + \partial^\nu \alpha(x)$$

$$\phi \to \phi - \alpha(x)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m^2}{2}\chi^2$$

one real scalar

$m \to 0$ continuous.

Motivation for BSM

(1) Gravity. Theory we have is

SM + GR

Embedding in Gravity
or UV-completion . . .

(2) Unification

(3) Puzzles in SM

* Naturalness puzzles.
* Some become sharper in Cosmological context

(4) Observational data

* Example Dark Matter.

$\to$ Dark energy.

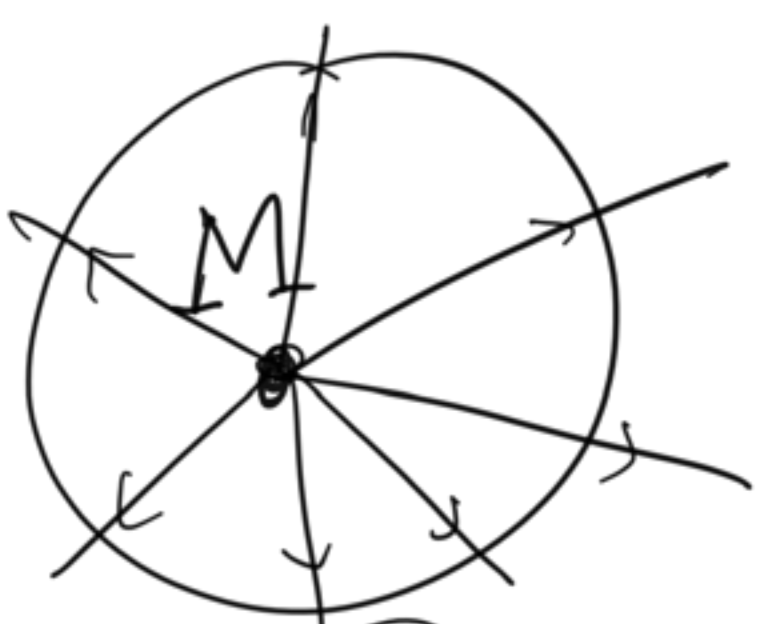
B+L

Gravity.

What is gravity?

Newtonian gravity

$$V(r) = -G_N \frac{m_1 m_2}{r}$$



$$\phi_N = -G_N \frac{M}{r}$$

$$\phi_N m = V(r)$$

probe.

Einstein gravity is a

completion of Newton in two directions:

(1) Relativistic (fast moving sources)

(2) Nonlinear (self-gravitating gravity)

Einstein also introduces a geometric meaning to gravity: gravity is a dynamical metric

$$\eta_{\mu\nu} = \begin{pmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

$$\boxed{g_{\mu\nu}(x) = \eta_{\mu\nu} + \delta g_{\mu\nu}(x)}$$

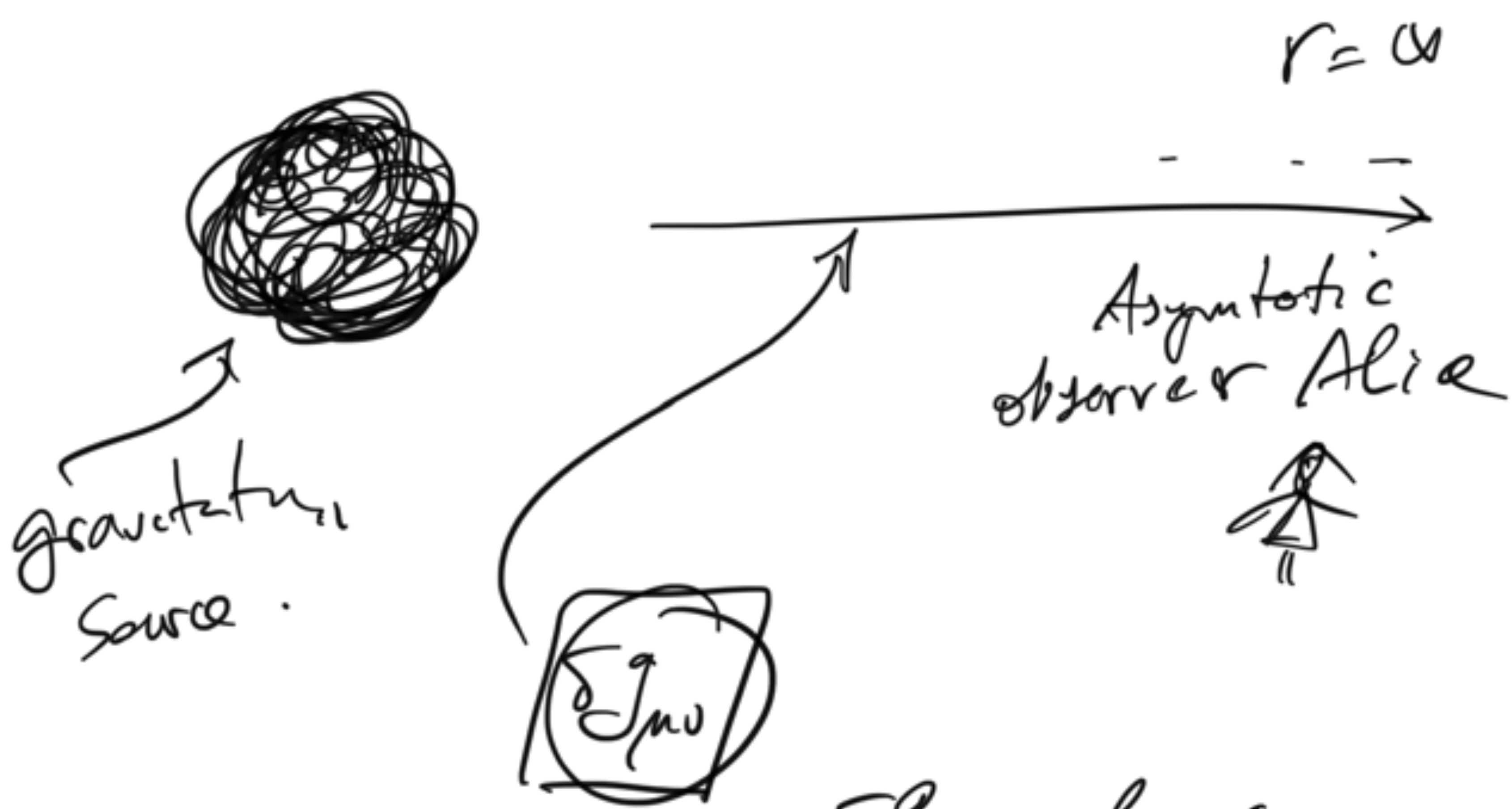
excitation of graviton

vacuum

$$|\delta g_{\mu\nu}| \ll 1$$

Picture: Asymptotic vacuum = flat Minkowski space

= flat Minkowski space.



$$\delta g_{\mu\nu} \equiv h_{\mu\nu}(x)$$

$$\underline{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x)}$$

weak field regime $|h_{\mu\nu}| \ll 1$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

"top-down": linearize this theory.

"bottom-up": Construct an effective theory of $h_{\mu\nu}(x)$ (field which is massless and spin = 2).

$$\phi\, T_\mu^\mu \qquad \boxed{h_{\mu\nu} T^{\mu\nu}}$$

$$A(\partial T_{\mu\nu})$$

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linear

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \equiv \frac{1}{2}\mathcal{E}_{\mu\nu} \qquad h^\mu_\mu = \eta^{\mu\nu} h_{\mu\nu} \equiv h$$

$$-\frac{1}{2}\partial_\mu\partial_\nu h$$

$$\mathcal{E}_{\mu\nu} = \Box h_{\mu\nu} - \eta_{\mu\nu}\Box h - \partial_\mu \cancel{\partial^\alpha h_{\alpha\nu}} -$$

$$- \partial_\nu \cancel{\partial^\alpha h_{\alpha\mu}} + \partial_\mu \cancel{\partial_\nu h} + \eta_{\mu\nu}\partial^\alpha\partial^\beta h_{\alpha\beta}$$

$$-\frac{1}{2}\partial_\nu\partial_\mu h \qquad \frac{1}{2}\eta_{\mu\nu}\partial^\alpha\partial_\alpha h$$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\xi_\nu(x) \qquad \delta h = 2\partial^\alpha \xi_\alpha$$

$$\delta\mathcal{E}_{\mu\nu} = \Box(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu) - \eta_{\mu\nu}\Box 2\partial^\alpha\xi_\alpha$$

$$-\partial_\mu\partial^\alpha(\partial_\alpha\xi_\nu + \partial_\nu\xi_\alpha) - \partial_\nu\partial^\alpha(\partial_\alpha\xi_\mu + \partial_\mu\xi_\alpha) +$$

$$+\partial_\mu\partial_\nu 2\partial^\alpha\xi_\alpha + \eta_{\mu\nu}\partial^\alpha\partial^\beta(\partial_\alpha\xi_\beta + \partial_\beta\xi_\alpha) =$$

$$= 0$$

$$\boxed{\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h = 0} \leftarrow \boxed{\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h}$$

$$h'_{\mu\nu} \to h_{\mu\nu} = h'_{\mu\nu} + \underline{\partial_\mu\xi_\nu + \partial_\nu\xi_\mu}$$

$$\partial^\mu(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu) - \frac{1}{2}\partial_\nu(2\partial^\alpha\xi_\alpha) =$$

$$= \Box\xi_\nu$$

$$\boxed{\Box\xi_\nu = \partial^\mu h'_{\mu\nu} - \frac{1}{2}\partial_\nu h' \equiv J_\nu}$$

$$\boxed{10-4=6}$$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \bar{\xi}_\nu + \partial_\nu \bar{\xi}_\mu$$

$$\boxed{\Box \bar{\xi}_\mu = 0}$$

$$6-4=2$$

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h\right) = 16\pi G_N T_{\mu\nu} \equiv \mathcal{T}_{\mu\nu}$$

$$\Box(h - 2h) = \mathcal{T} \qquad \mathcal{T} \equiv \mathcal{T}_\mu^{\ \mu}$$

$$\Box h = -\mathcal{T}$$

$$\boxed{\Box h_{\mu\nu} = 16\pi G_N \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right)}$$

M $\qquad$ $\boxed{T_{00} = M\,\delta(\vec{r})}$

$$h_{00} = \phi_N$$

$$\vec{\nabla}\vec{\nabla}$$

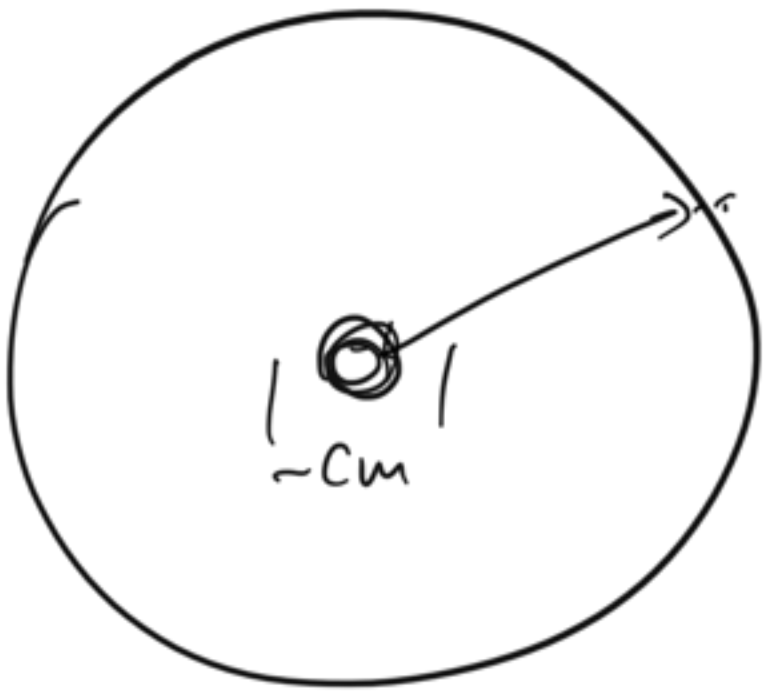
$$\Delta \phi_N = \bullet\, \delta(\vec{r})\, M G_N$$

$$h_{00} \propto \underline{M G_N}$$

r

$r_g \equiv 2 G_N M$

$\boxed{h_{00} \sim \frac{r_g}{r}}$ $\quad$ $\boxed{r \gg r_g}$



$\boxed{h_{00} \lesssim 10^{-8}}$

Question:

$B+L \leftarrow$ Global symmetry.

$\text{Quarks} \to e^{i\frac{\alpha}{3}}\, \text{Quarks}$

$\text{Leptons} \to e^{i\alpha}\, \text{Leptons}$

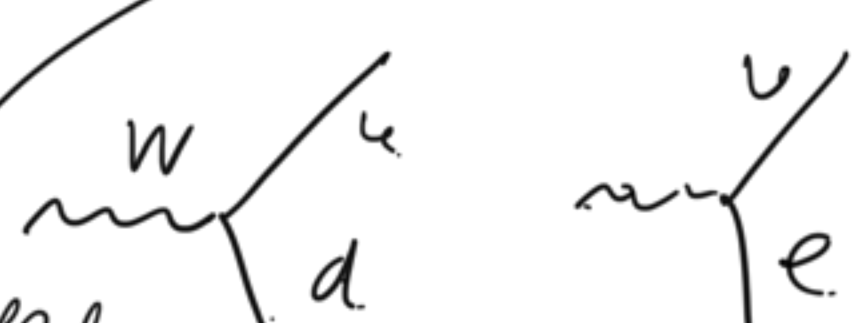
$SU(3)_c \otimes \underbrace{SU(2)_W}_{} \otimes U(1)_Y$

$Q_{L j\alpha} = \begin{pmatrix} u \\ d \end{pmatrix}_L^{j} \quad u_{Rj} \quad d_{Rj} \qquad L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \nu_R \; ? \quad e_R$

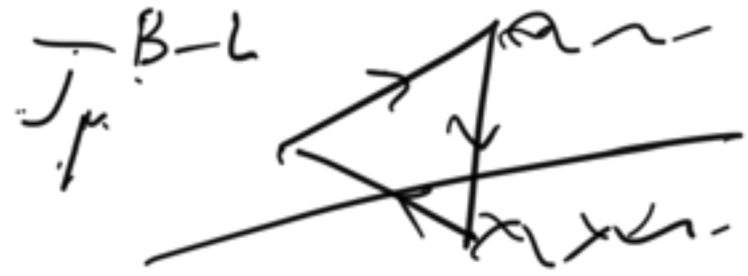
$j = 1, 2, 3 \quad SU(3)_c$

$\alpha = 1, 2 \leftarrow SU(2)_w$

3 doublets. $\quad$ doublet

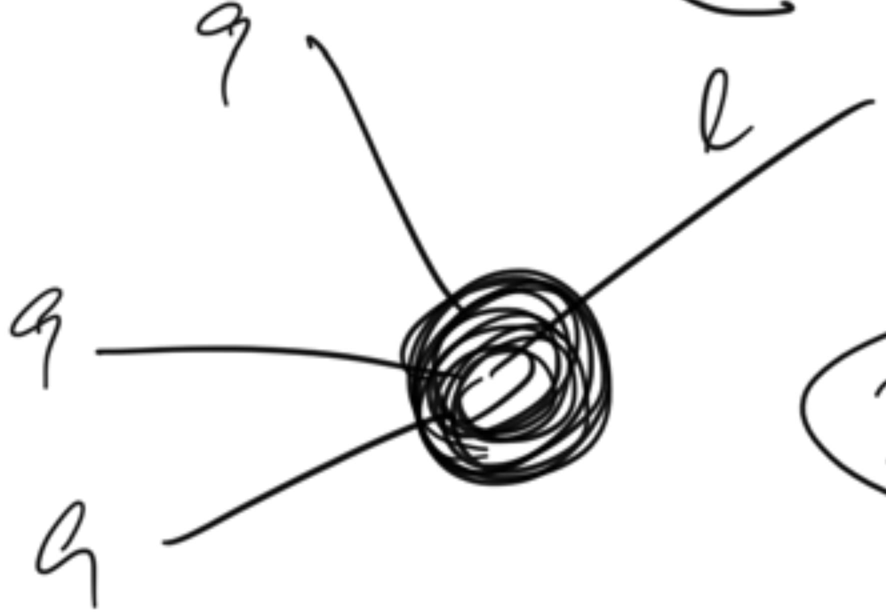


$3+1 = \text{dim}?$



$= 0.$

Sphalerons



$$e^{-\frac{1}{g_W^2}}$$

B+L

B−L

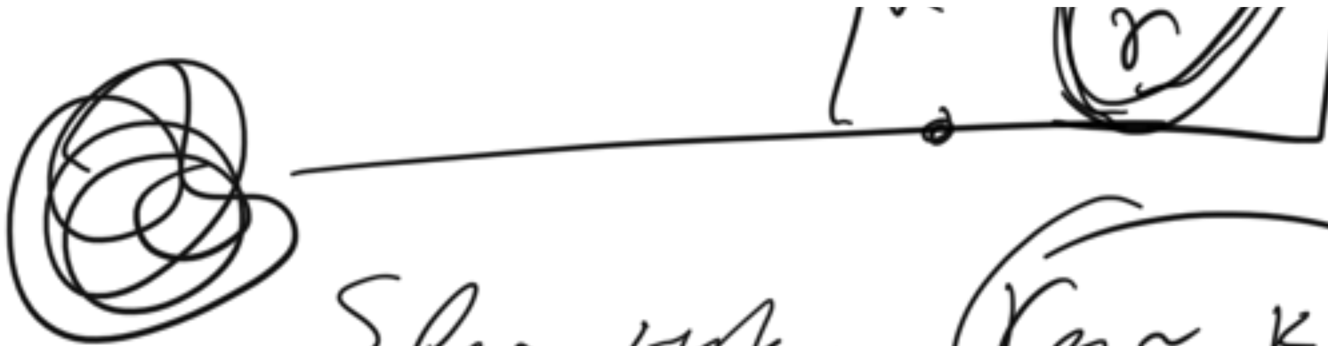
Question from a Student

(1) Linear gravity

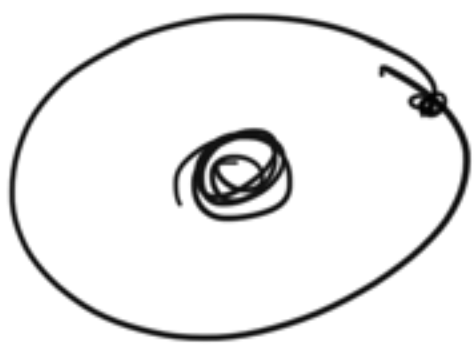
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$|h_{\mu\nu}| \ll 1$$

$$h \sim \frac{r_g}{\ }$$



Solar system $r_g \sim$ km



$$\left(\frac{r_g}{r}\right)^2$$

$$r_g \sim G_N M$$

$c=1$

$$\boxed{\hat{A}_\mu(x) = \sum_{\vec{k}\, e} \sqrt{\frac{\hbar}{2V\omega_k}}\left[e^{-i\vec{k}\vec{x}}\,\hat{a}_{\vec{k}e}\,\varepsilon^e_\mu + h.c\right]}$$

moment polariz

Quantum-field $\longrightarrow \sqrt{\frac{\hbar}{V}}\,\hat{a}$

$$\hat{n} = \hat{a}^+\hat{a}$$

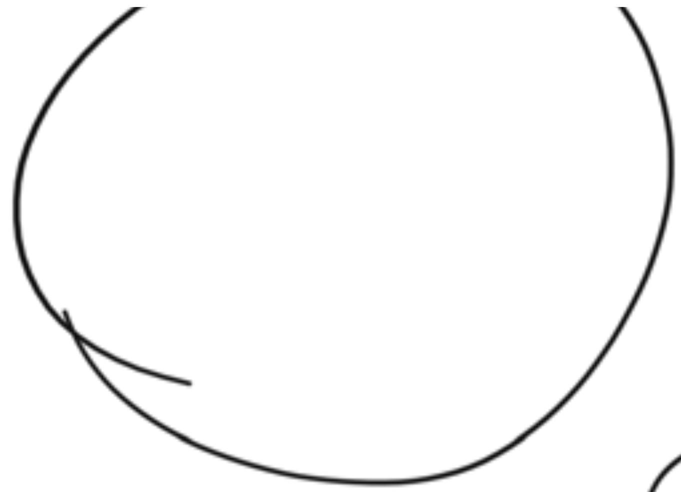
Classical $= \langle N|\widehat{QF}|N\rangle$

Class field $= \sqrt{\frac{\hbar\, N}{V}}$

large N

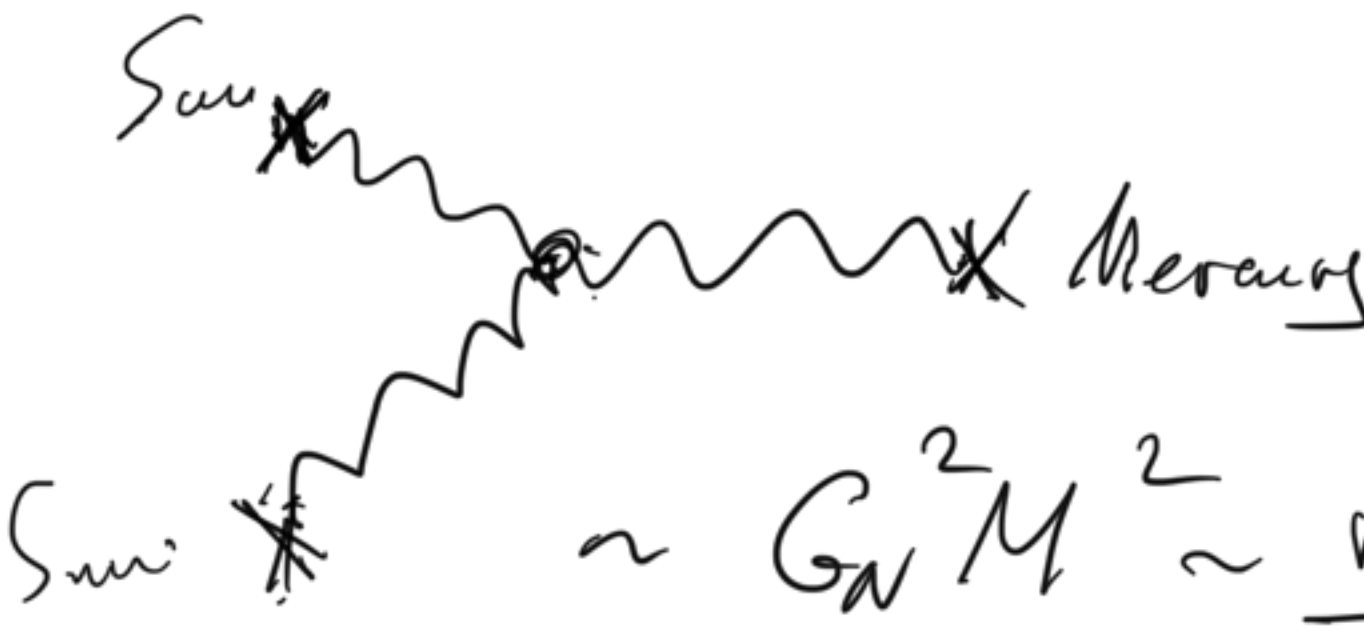
$\hat{a} = c$

$$\boxed{h \sim 10^{-8}}$$



$h \sim 1$

$h \sim 0.1$



$$\sim \frac{G_N^2 M^2}{r^2} \sim \frac{r_s^2}{r^2}$$

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + \partial_{[\alpha}\Omega_{\beta\gamma]}$$

$$\boxed{F = \partial_{[\mu} C_{\alpha\beta\gamma]}}$$

$$F^2$$

$$\partial^\mu F_{\mu\alpha\beta\gamma} = 0$$

$$\boxed{F_{\mu\alpha\beta\gamma} = \epsilon_{\mu\alpha\beta\gamma} \cdot \text{Const}}$$

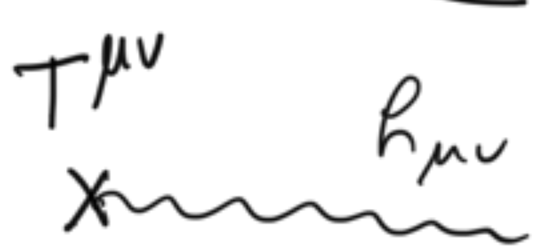
BSM Lecture 3

weak field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad T \equiv T_\mu{}^\mu$$

$$\Box h_{\mu\nu} = 16\pi G_N \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right)$$

responce from gravity



$$h_{\mu\nu} = \frac{1}{\Box} \underline{\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right)} 16\pi G_N$$

$$h_{\mu\nu}(x) = \int d^4x' \, G_{\mu\nu,\alpha\beta}(x-x') \, T^{\alpha\beta}(x') \, 16\pi G_N$$

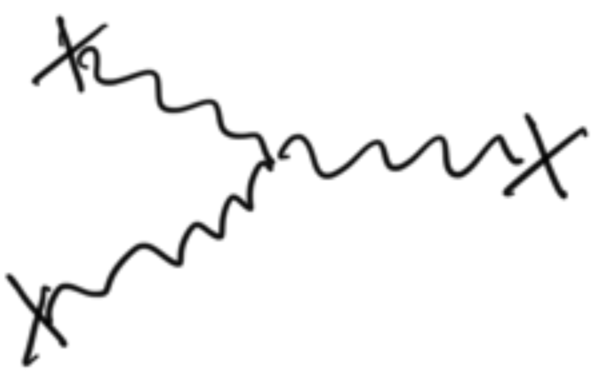
$$G_{\mu\nu,\alpha\beta}(x-x') = \frac{\frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}\right) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}{\Box + i\varepsilon}$$

$\tau^{\mu\nu}$ probe

$$S = \int d^4x \, h_{\mu\nu}(x) \, \tau^{\mu\nu}(x)$$

Linear in G_N

$$\underline{S = \int d^4x \, d^4x' \, \tau^{\mu\nu}(x) \, G_{\mu\nu,\alpha\beta}(x-x') \, T^{\alpha\beta}(x') \, 16\pi G_N}$$



L $\quad N_L \gg 1$

$\sim \frac{1}{N_L}$

$$\hat{\Phi} = \sqrt{\frac{\hbar}{V}}\,\hat{a}$$

$$n = \frac{N}{V}$$

$\downarrow$

$$\Phi_{cl} = \sqrt{\frac{\hbar \langle N \rangle}{V}} = \sqrt{\hbar n}$$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \leftarrow$ dimensionless

$\hbar = 1$

$[\text{Mass}]^2$

$$S = \int d^4x \, \frac{1}{G_N} h^{\mu\nu} \mathcal{E}_{\mu\nu} + h_{\mu\nu}\left(T^{\mu\nu} + \tau^{\mu\nu} + \dots\right)$$

$$S_{\text{Bose}} = \int d^4x \, (\partial \text{Boson})^2$$

$[\text{Boson}] = [\text{mass}]$. $\quad c = 1$

$$L_p^2 \equiv \underline{\hbar G_N} \qquad L_p \sim 10^{-33}\,\text{cm}$$

$$\hbar \to 0 \qquad L_p \to 0$$

$$M_P \equiv \frac{\hbar}{L_p} = \frac{\hbar}{\sqrt{\hbar G_N}} = \sqrt{\frac{\hbar}{G_N}}$$

$$M_p \sim 10^{-4.5}\,g \sim 10^{19}\,\text{GeV}$$

$\hbar \equiv 1 \qquad M_P \to \infty$

$G_N \to 0$

$[\text{mass}]^4$

$$S = \int d^4x \; \hat{h}^{\mu\nu} \mathcal{E}_{\mu\nu} + \frac{\hat{h}_{\mu\nu}}{M_P}\left(T^{\mu\nu} + \tau^{\mu\nu} + \dots\right)$$

$[\hat{h}_{\mu\nu}] = [\text{mass}]$

$$h_{\mu\nu} \Rightarrow \frac{\hat{h}_{\mu\nu}}{M_P}$$

$$\boxed{\frac{\hat{h}_{\mu\nu}}{M_P} T^{\mu\nu}}$$

$$\frac{\hat{h}}{M_P}\partial^2 \hat{h}^2$$

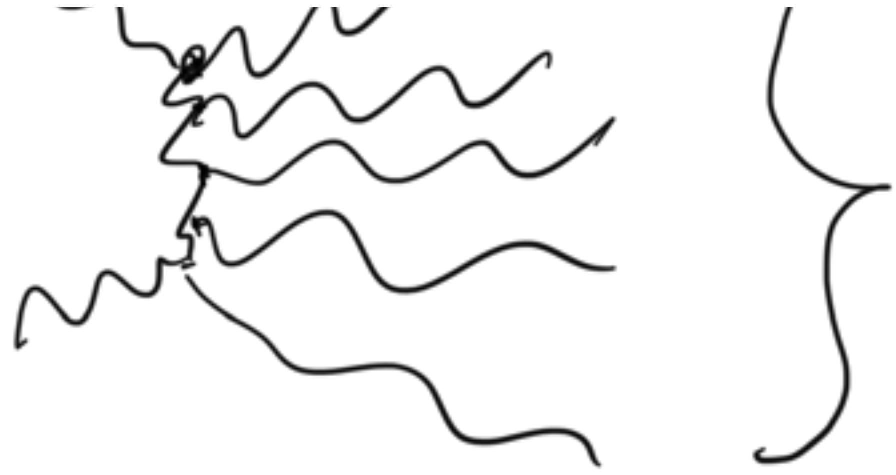
$$\left(\frac{\hat{h}}{M_P}\right)^n \partial^2 \hat{h}^2$$

momentum transfer

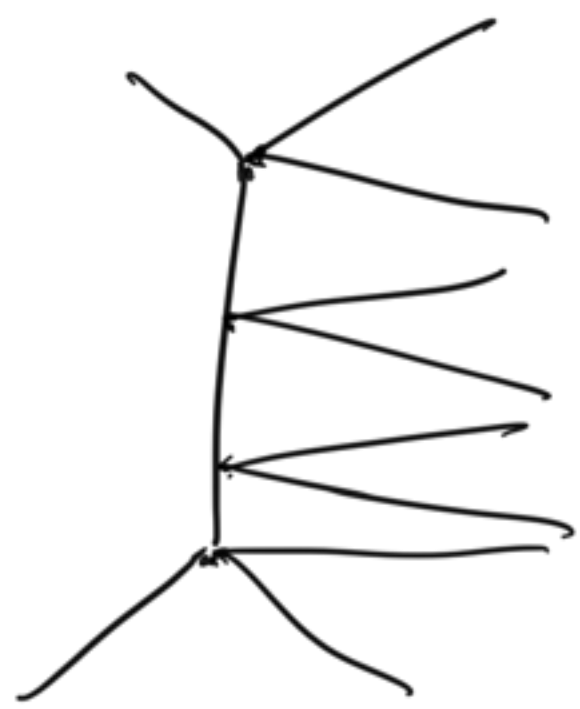
$$\boxed{\alpha_{gr} = \frac{q^2}{M_P^2}}$$

$q \sim M_P \quad \alpha_{gr} \sim 1$

breakdown of perturbation theory in α_{gr}.



$\alpha \phi^4$

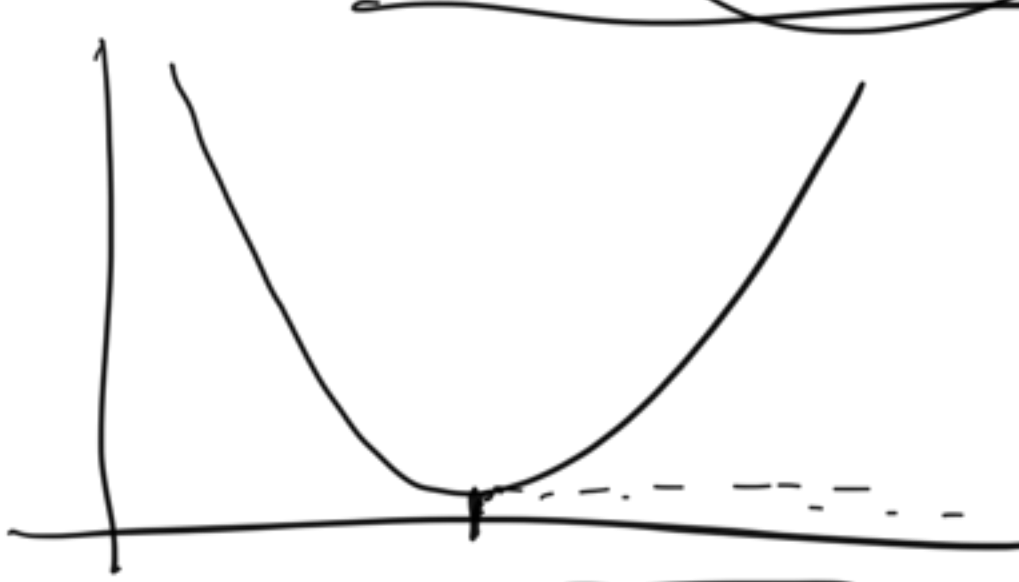


α^n $\quad \sim n!$

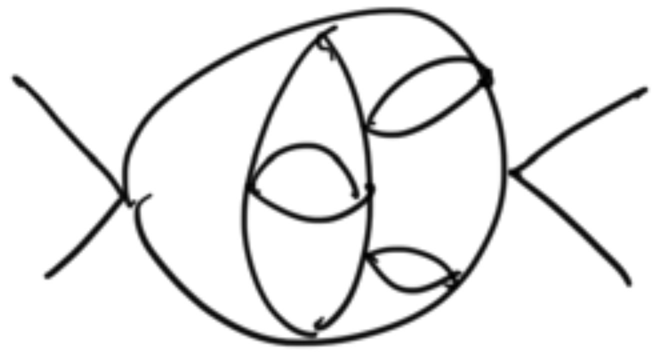
$$\sigma \sim \alpha^n n!$$

$n+1$

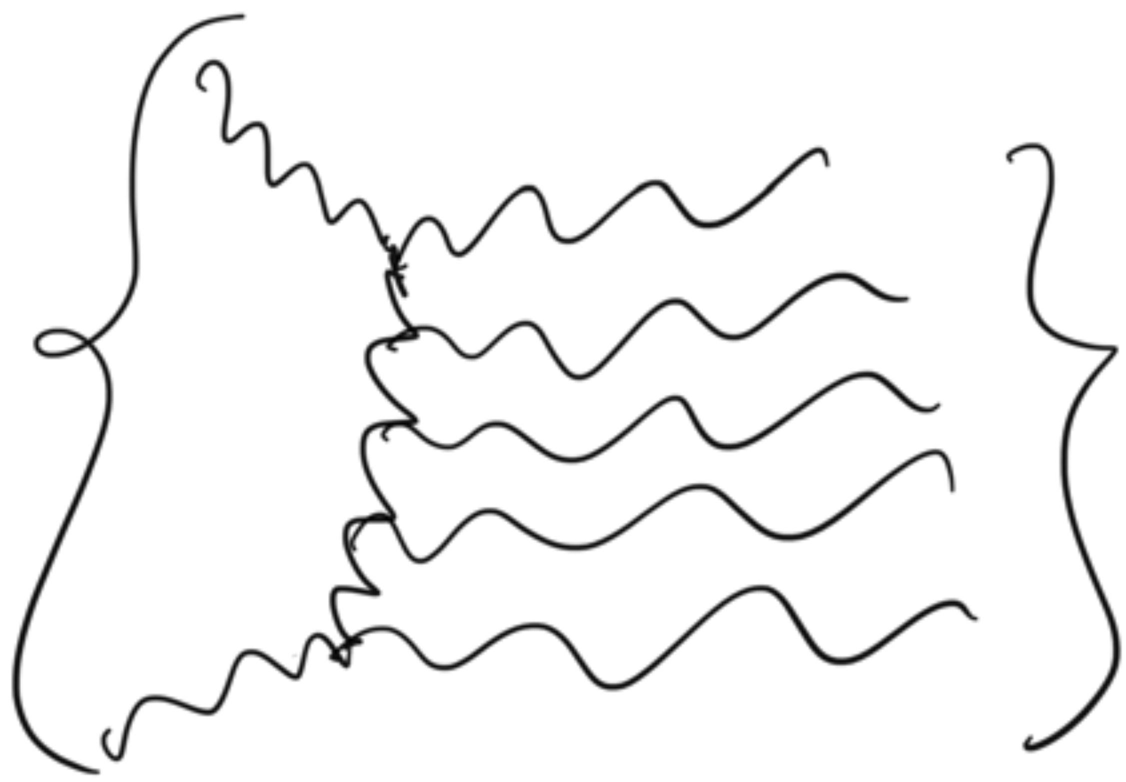
n



$$\boxed{n \simeq \frac{1}{\alpha}}$$



$n \gtrsim \frac{1}{\alpha}$



classical

α^n

$$r_g \sim M G_N \sim \frac{M}{M_p^2}$$

$\frac{r_g}{r}$ $\boxed{r \sim r_g}$

$h_{00} \sim 1$ $h_{00} \sim \frac{r_g}{r}$

$\hat{h}_{00} \sim M_P$

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$d\theta^2 + \sin^2\theta \, d\phi^2$

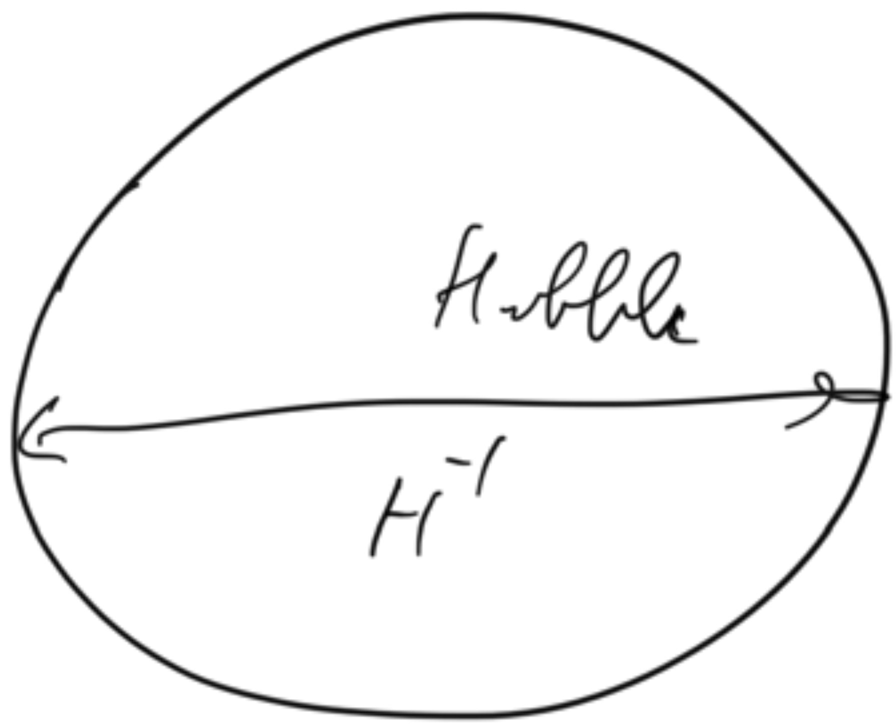
$$\boxed{\left(\frac{\hat{h}}{M_p}\right)^n \partial^2 \hat{h}^2}$$

$\partial \to \frac{1}{r}$

$\partial \to \frac{1}{r_g}$

$\langle \hat{h} \rangle \sim M_p$

$r_g \gg L_p = \frac{1}{M_p}$



$r_g^{max} \sim H^{-1}$

Strong field $\langle \hat{h} \rangle \sim M_p$

weak coupling $\left(\frac{\hat{h}}{M_p}\right)^4 \partial^2 \hat{h}^2 \ll M_p^4$

$$\alpha_{gr} \ll 1$$

$$\alpha_{gr} \sim \frac{1}{M_p^2 r_g^2}$$

$r_g \sim \text{cm}$ $\quad \boxed{\alpha_{gr} \sim 10^{-66}}$

$$N_{gr} \sim \frac{1}{\alpha_{gr}} \sim 10^{66}$$

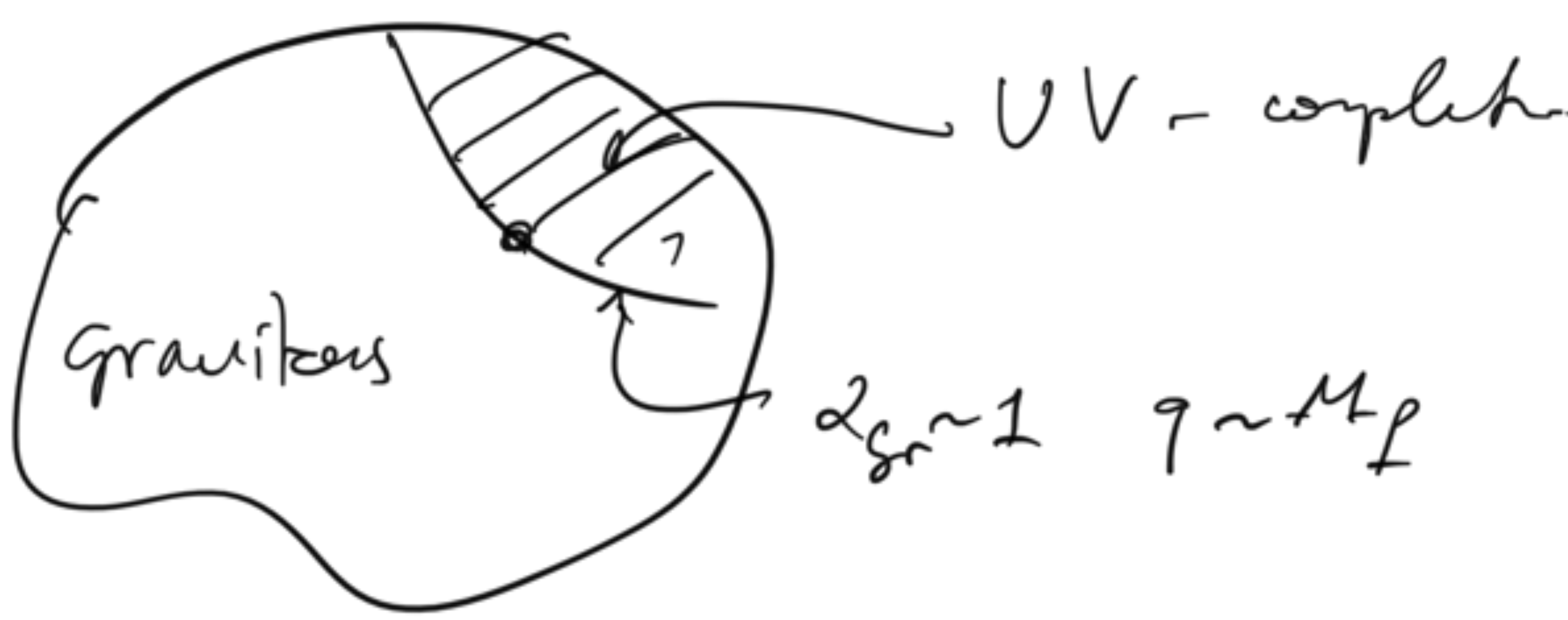
a particle of mass m

Length scales:

① Compton wavelength

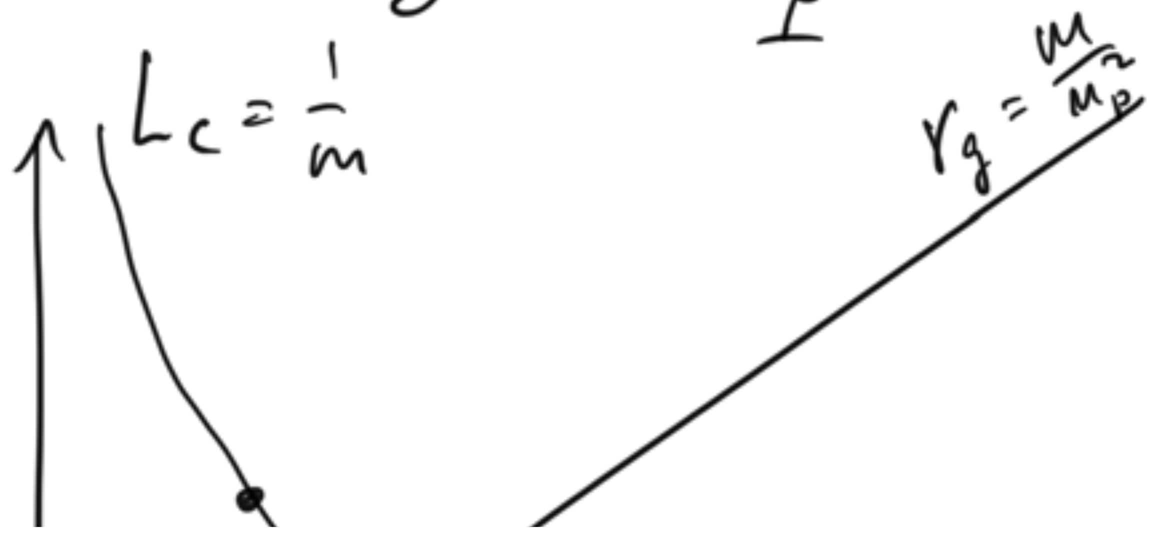
$$L_c \equiv \frac{1}{m}$$

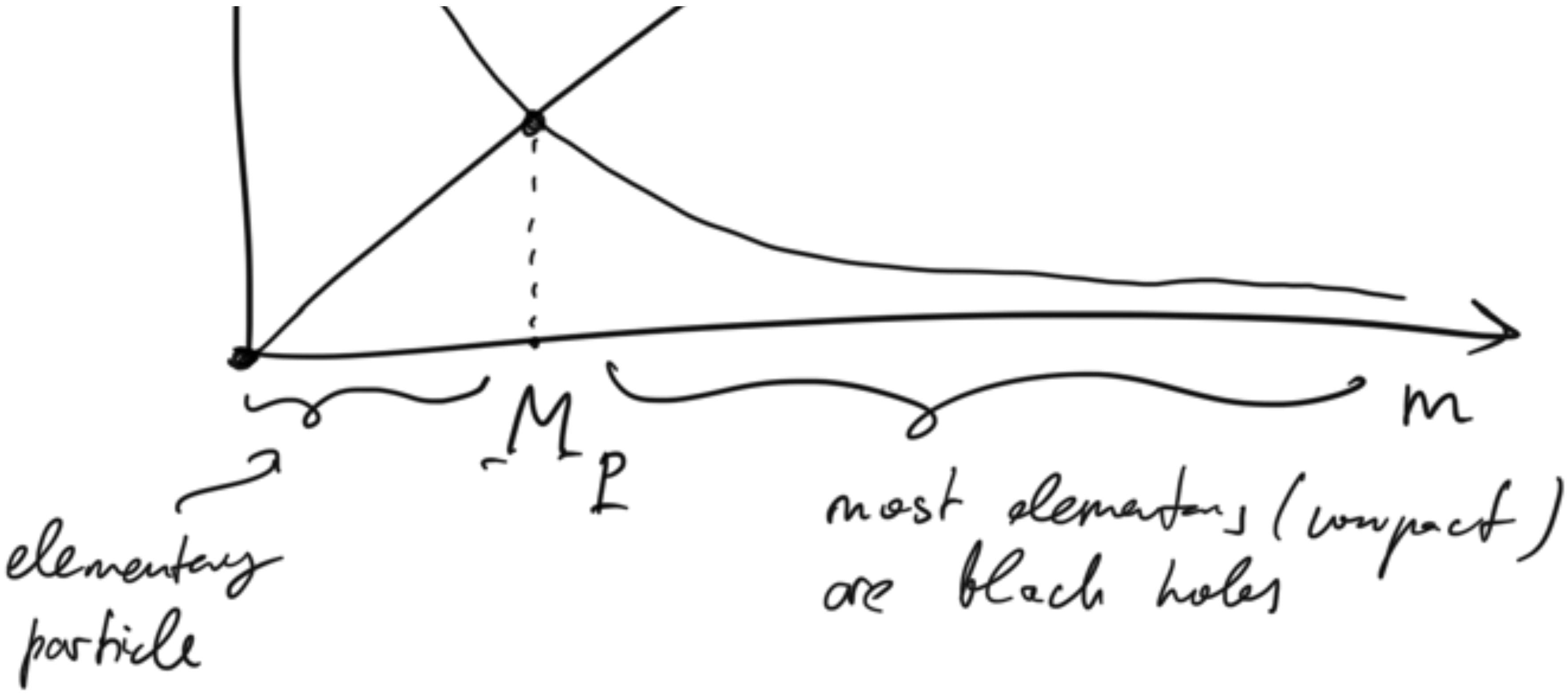
② Gravity length $L_P = \frac{1}{M_P}$

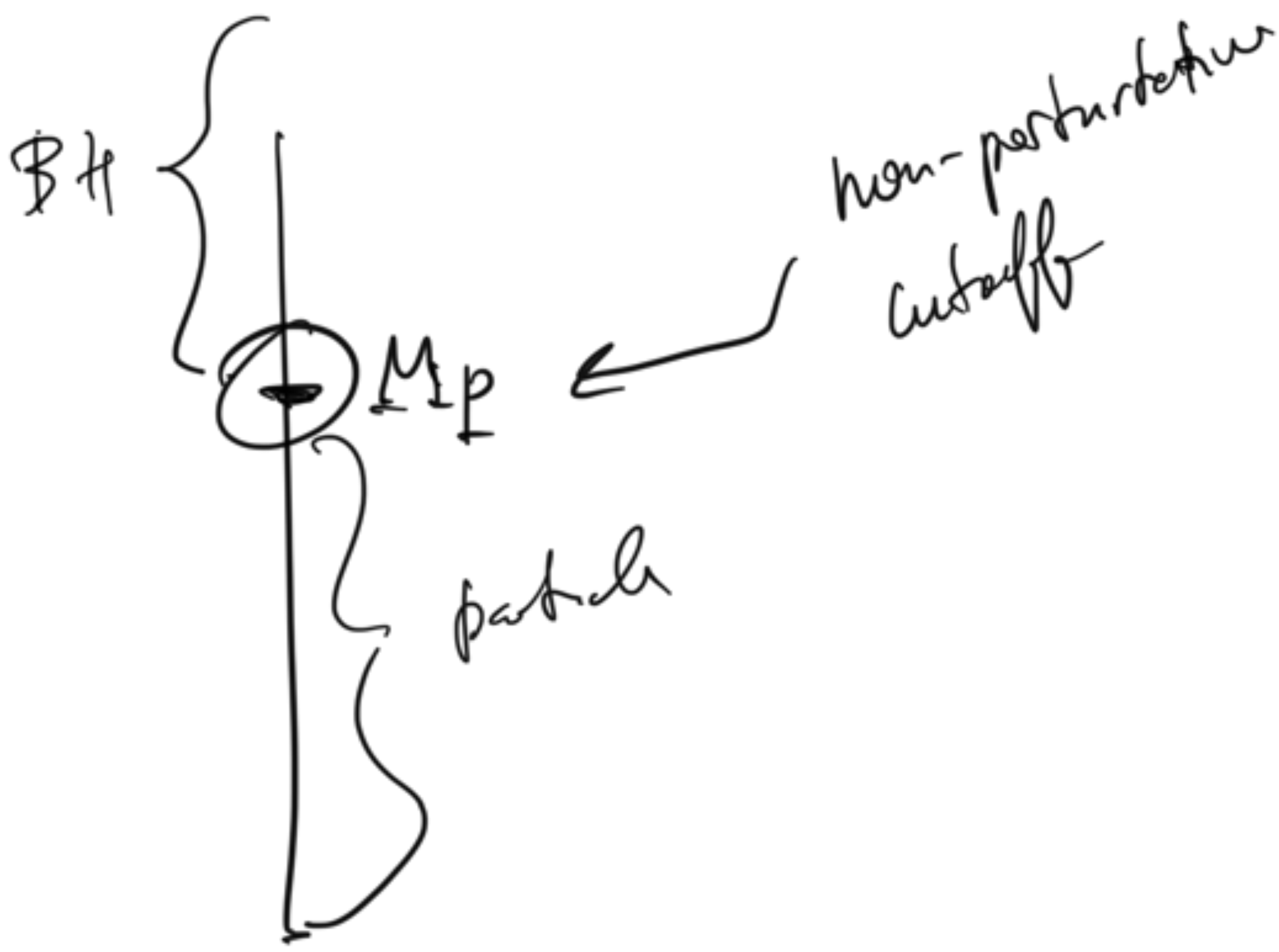


③ Particle-characteristic length.

$$r_g = \frac{m}{M_P^2} \leftarrow \text{classical.}$$







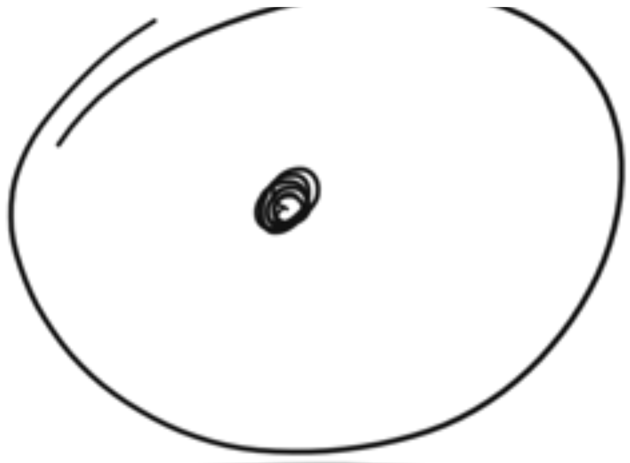
M

$$r_g = 2 M G_N$$

$$r_g \sim \frac{M}{M_P^2}$$

weak field $\quad h \sim \frac{r_g}{r} \ll 1$

weak coupling $\quad \alpha_g \ll 1$



M

$\phi_N \sim \frac{r_g}{r}\,\frac{\phi}{M_p}$

$\boxed{\Phi = M_p \frac{r_g}{r}}$

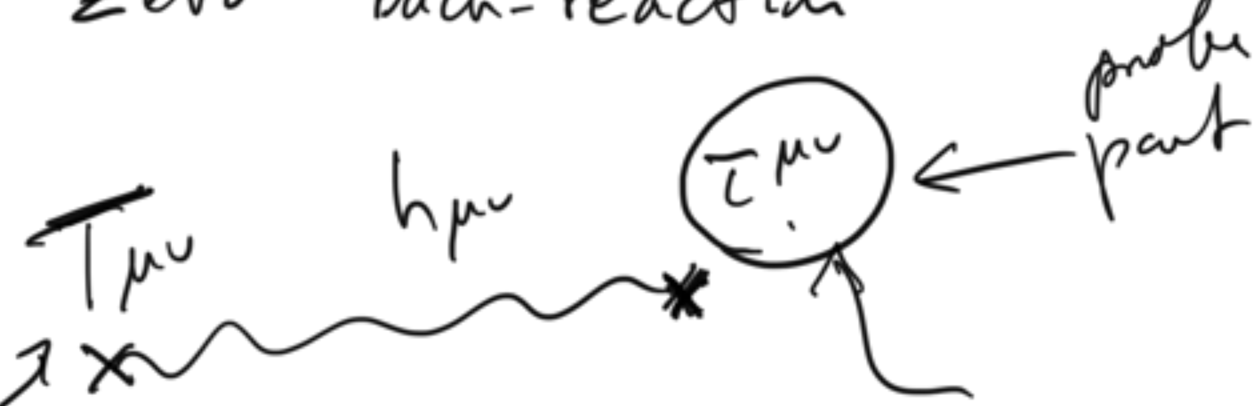
$\int d^3x \, \vec{\nabla}\Phi \vec{\nabla}\Phi$

$\boxed{N \sim \left(\frac{M}{M_p}\right)^2}$

m_e^{-1}

Semi-classical limit

zero back-reaction



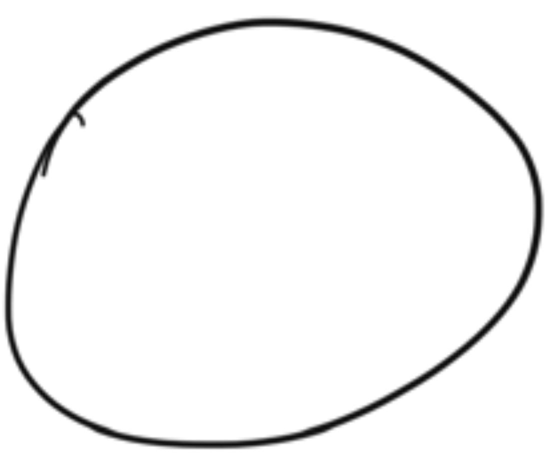
$\boxed{\hbar = 1}$ $\quad G_N \to 0 \quad M \to \infty$

M

$$r_g = 2G_N M = \text{fixed}$$

$$G_N \to 0 \Rightarrow \boxed{M_p \to \infty \qquad L_p \to 0}$$

Hawking radiation.



$|\leftarrow r_g|$

$$\boxed{T_H = \frac{1}{r_g}}$$

$$\boxed{\dot{M} = -T^2 \cdot N_{sp}}$$

$$\boxed{M \gg M_p}$$