BSM Lecture 4

Semi-classical limit $\hbar = 1$

$M \to \infty \quad G_N \to 0 \quad R = \text{fixed}$

$(M_p \to \infty)$

$S_{BH} = \left(\frac{M}{M_p}\right)^2 \to \infty$ fixed (rigid geometry)

This is also a limit of zero back-reaction

In this limit certain properties are understood exactly.

E.g. thermality of Hawking radiation

Therefore for finite $S_{BH} \gg 1$ the back-reaction (at least for initial time) must be small.

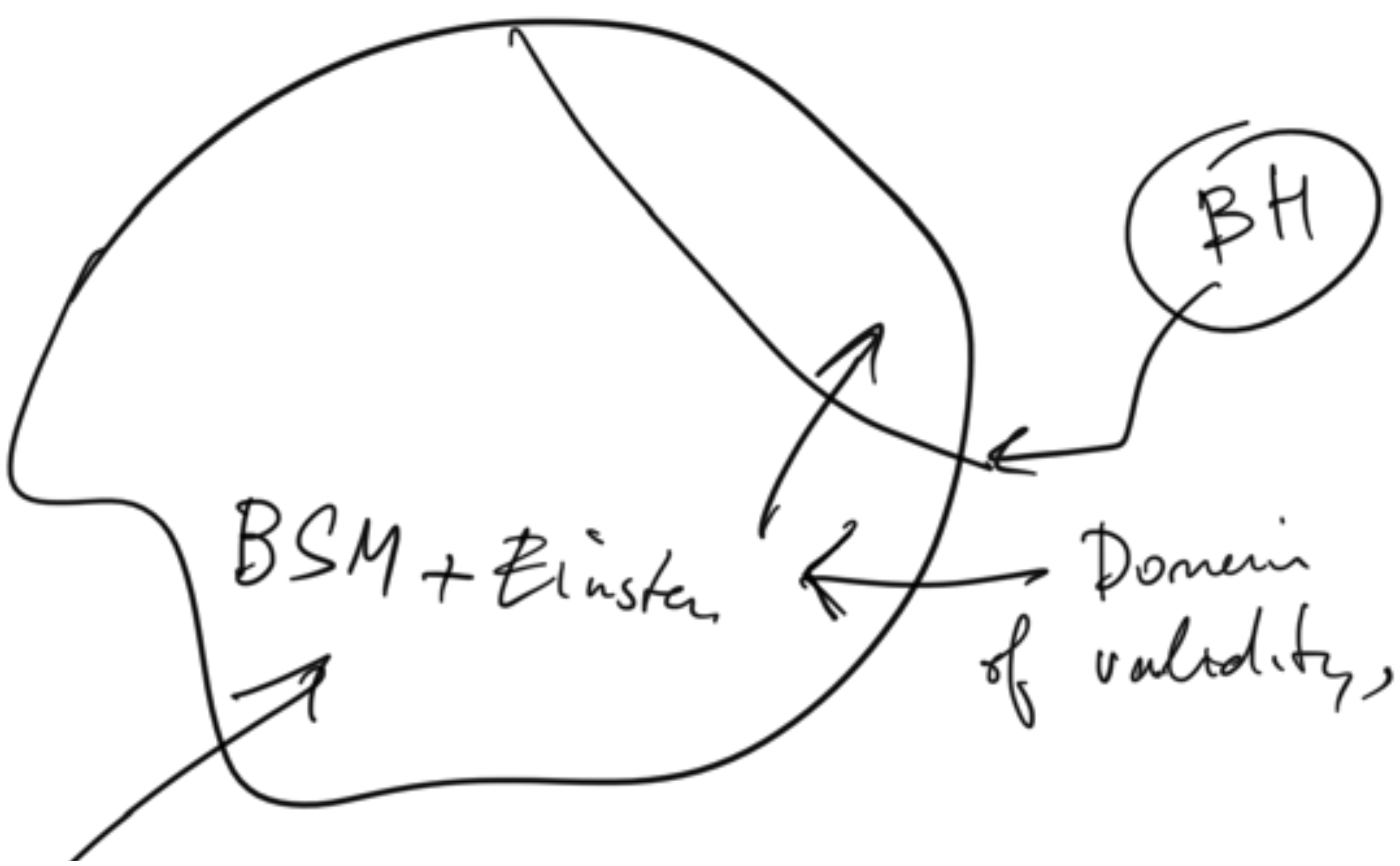
Correspondingly, the correction to properties (like in semi-classical [illegible] properties ([illegible]

Cumil are exact) are expected to be small, as long as we are in the regime of weakly interacting Einstein gravity.

R = finite $\qquad S_{BH} = \infty$.

$$\delta \sim \frac{1}{S_{BH}}$$

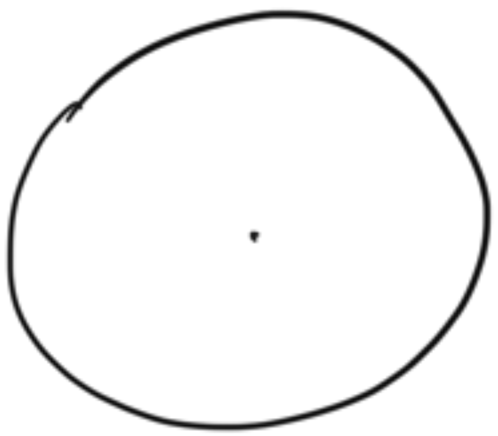
as long as $\alpha_{gr}\left(\frac{1}{R}\right) \ll 1$



Example: Bound on the number of particle species N_{sp}

L $\quad m \lesssim \frac{1}{L}$

active in processes with momentum-transfer $q \gtrsim \frac{1}{L}$



$R \gg L_p$

$S_{BH} \gg 1 \quad S_{BH} \sim \frac{R^2}{L_p^2}$

$K - R$

$$T = \frac{1}{R} = \frac{M_p^2}{M}$$

$$\dot{M} = -T^2 N_{sp}$$

democratic in species

$M = M_p^2 \quad \dot{M} = M_p^2 \left(\dot{T}\right.$

$$\frac{\dot{T}}{T^2} = N_{sp} \cdot \frac{T^2}{M_P^2}$$

$$\frac{\dot{T}}{T^2}$$ = parameter of thermality of semi-classicality.

T

$$\frac{\dot{T}}{T^2} \ll 1$$

we can speak about T reliably

but the notion of thermality breaks down for

$$\frac{\dot{T}}{T^2} \sim 1$$

Thus, weak coupling Einstein description breaks down.

$$\frac{\dot{T}_*}{} = N_{sp} \frac{T_*^2}{} \sim 1$$

T_*^2 M_p^2

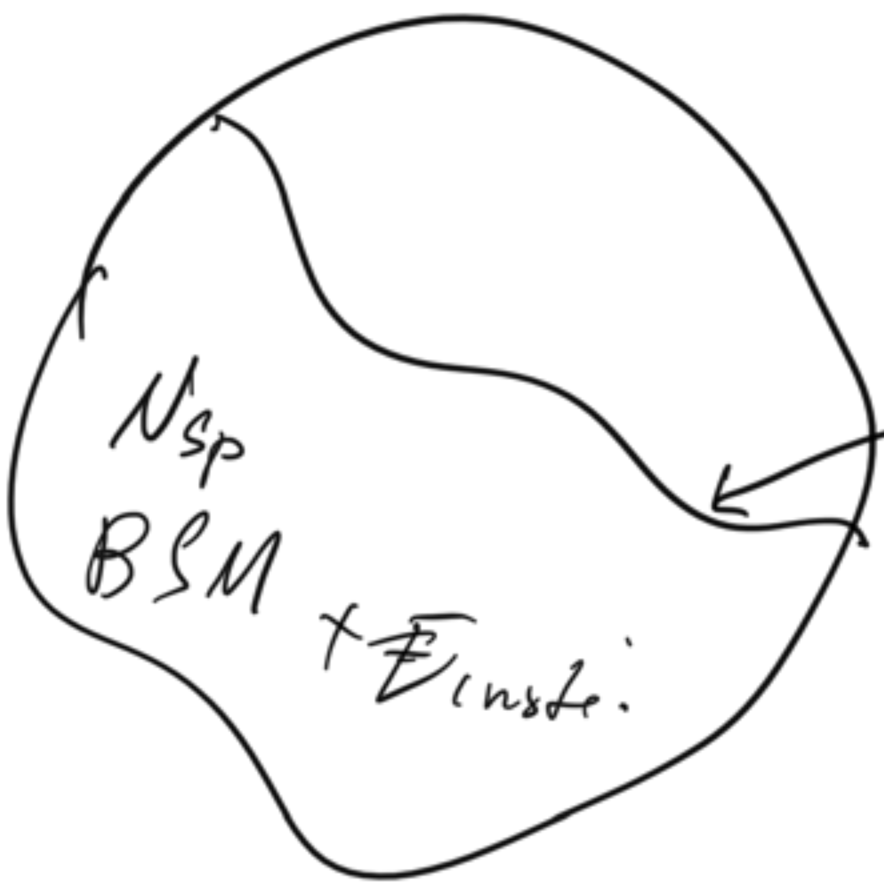
cutoff temperature.

$$\boxed{T_* = \frac{M_p}{\sqrt{N_{sp}}} \equiv M_* \equiv L_*^{-1}}$$

cutoff of weak Einstein gravity

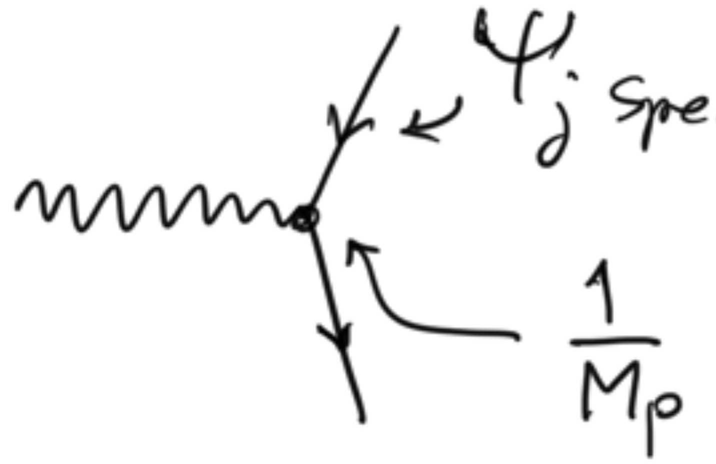
$$R_* = M_*^{-1} \gg L_p$$

Collective effect is too strong.



$$M_* \equiv \frac{M_p}{\sqrt{N_{sp}}}$$

Let us explain collective effect.



$$\frac{\hat{h}_{\mu\nu}}{M_P} T^{\mu\nu}$$

$j = 1 \dots N_{sp}$

$$\alpha_{gr} = \frac{q^2}{M_P^2}$$

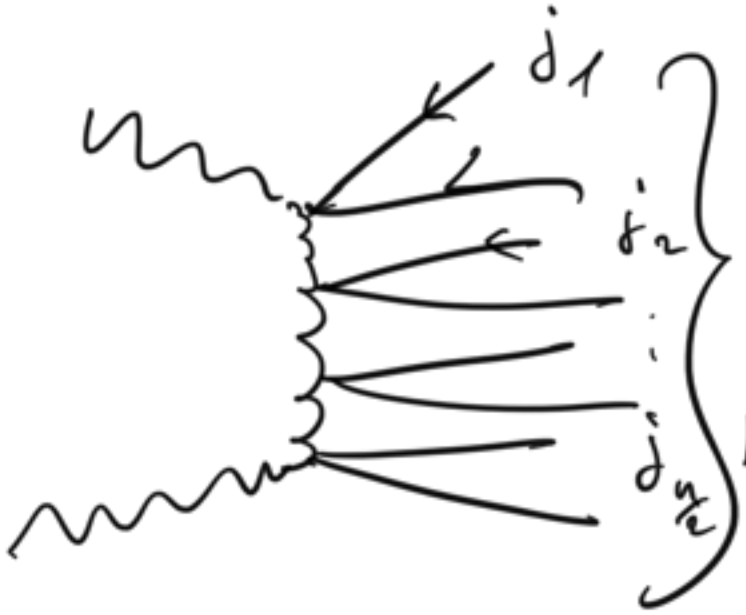
$\times N_{sp}$

Species coupling λ (gravitational 't Hooft coupling)

$$\boxed{\lambda \equiv \alpha_{gr} N_{sp}}$$

$\alpha_{gr} N_{sp}$

$$\lambda^{\ell} \equiv (\alpha_{gr} N_{sp})^{\ell}$$



$n \sim N_{sp}$

saturon

activity domain

$R \ll m^{-1}$

$R \gg m^{-1}$

$\sim e^{-\frac{m}{T}}$

$$R_{BH} \gtrsim M_*^{-1} + \boxed{q < M_*}$$

Hierarch problem puzzle

Naturalness "problem" "puzzle"

$$G_H = \frac{1}{v^2} \gg G_N = \frac{1}{M_P^2} \; ?$$

UV-sensitivity of the Higgs mass (VEV) to the cutoff of the theory.

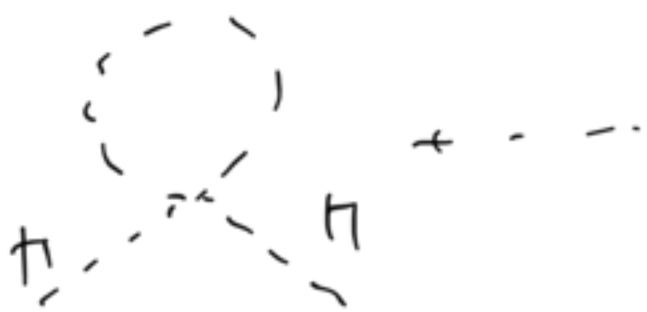
$$\mathcal{L} = -\underline{\underline{m_H^2}}\, H^\dagger H - \lambda^2 (H^\dagger H)^2$$

$$+\; g^\psi H \psi \psi \;+\; \text{gauge} \ldots$$

$\delta m_H^2 =$

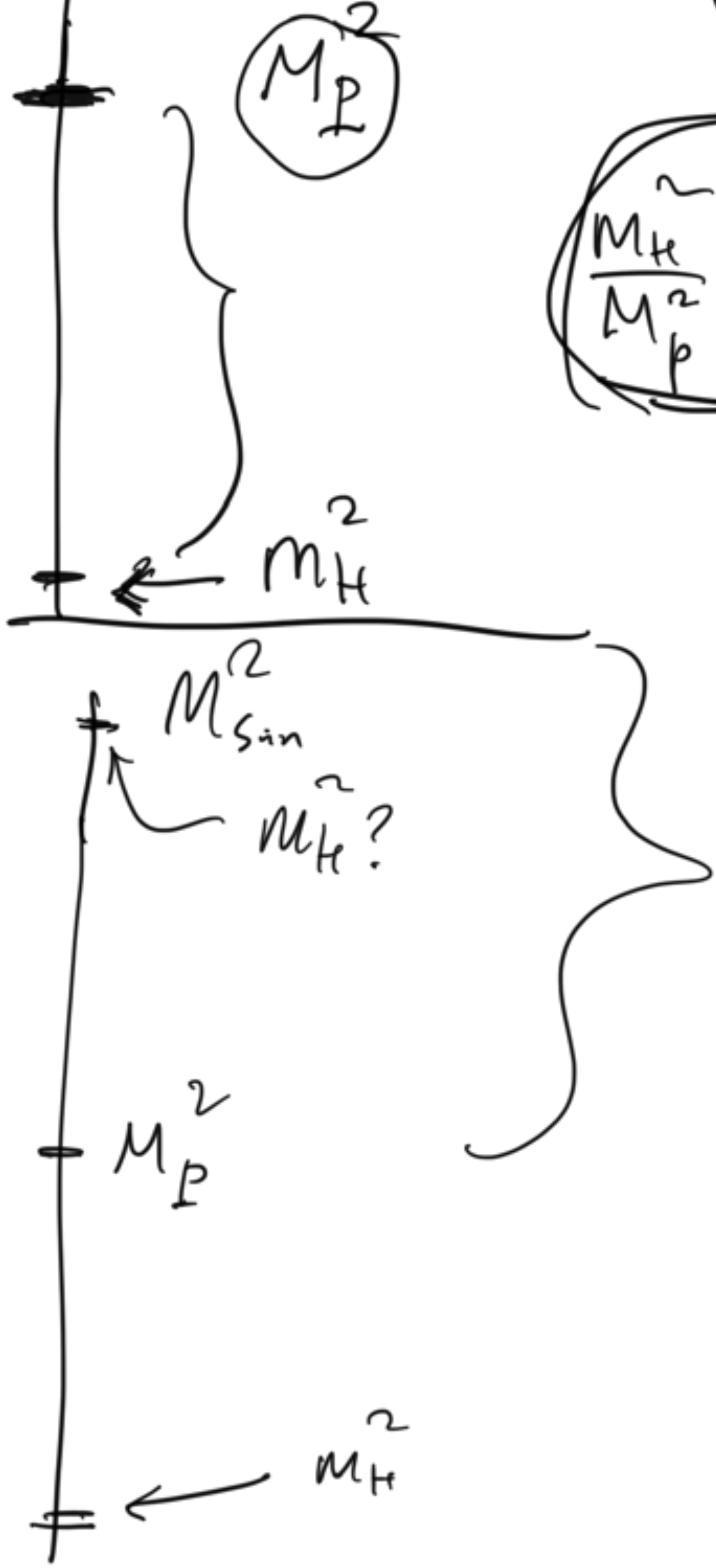
$+$

$+$



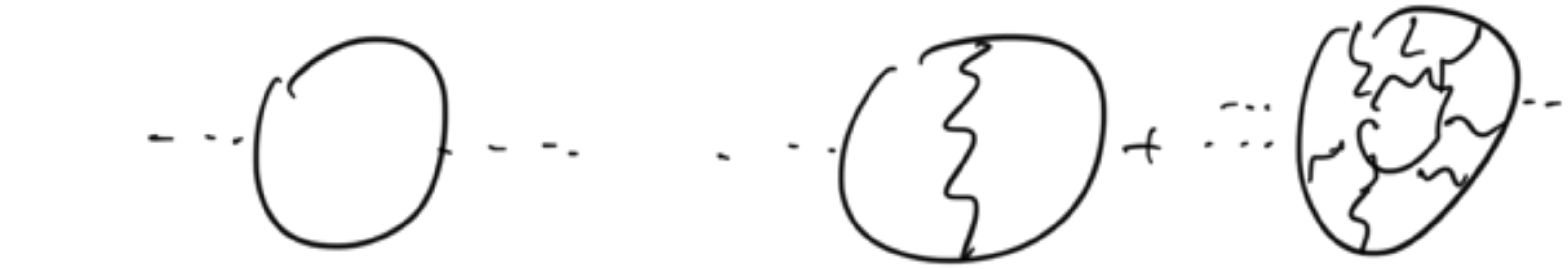
$+ \ldots$

$$\delta m_H^2 \propto M_*^2$$



$N_{sp} \sim 1$

$$\boxed{\frac{m_H^2}{M_P^2} \sim 10^{-34}}$$



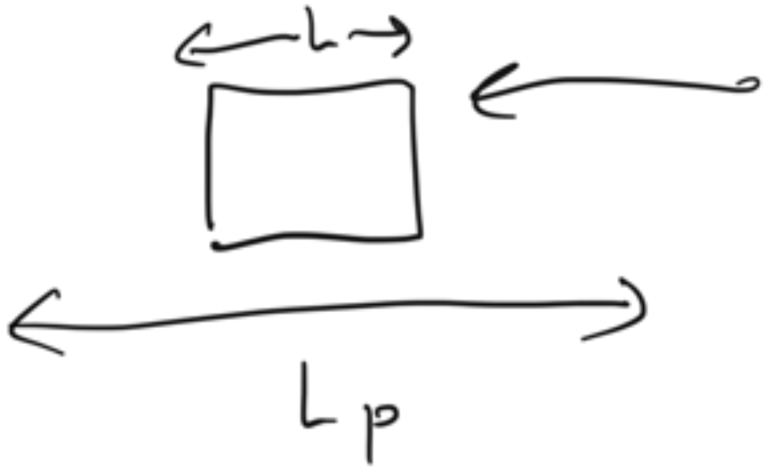
$$\boxed{\delta m_H^2 \lesssim M_p^2}$$

$$q \gg \frac{1}{L}$$

$$E \gg \frac{1}{L}$$

$$L \ll L_p$$

$$\boxed{E \gg \frac{1}{L}}$$

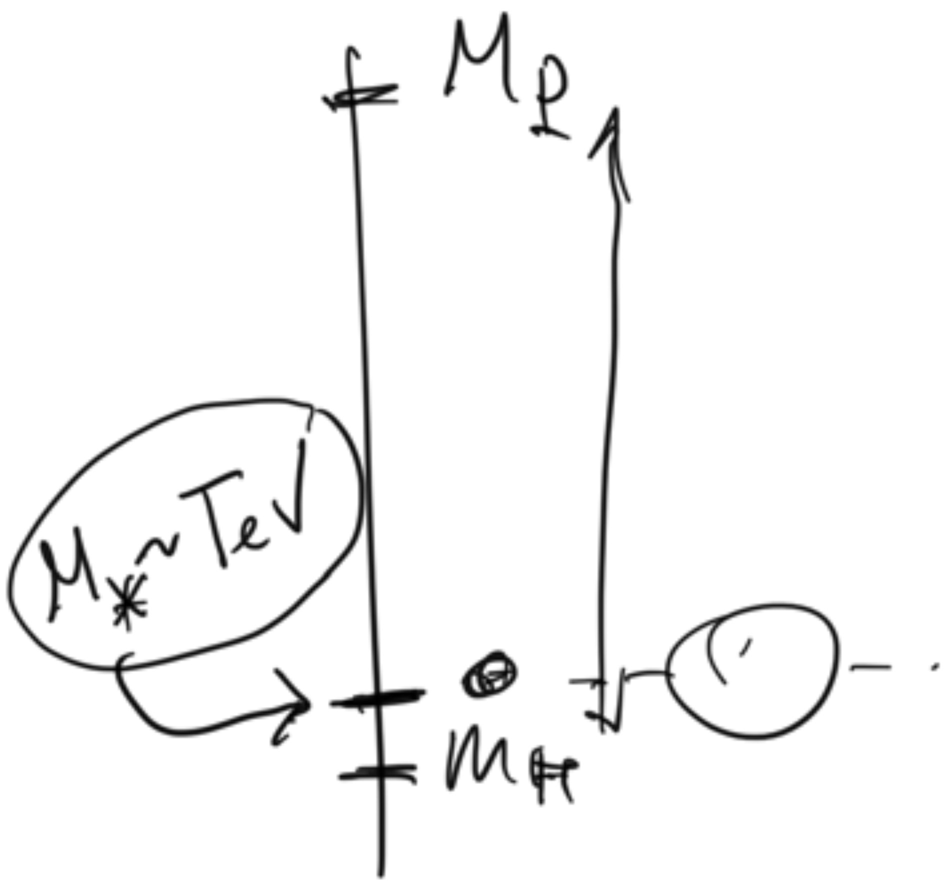


$$r_s \sim E L_p^2 \gg L_p$$

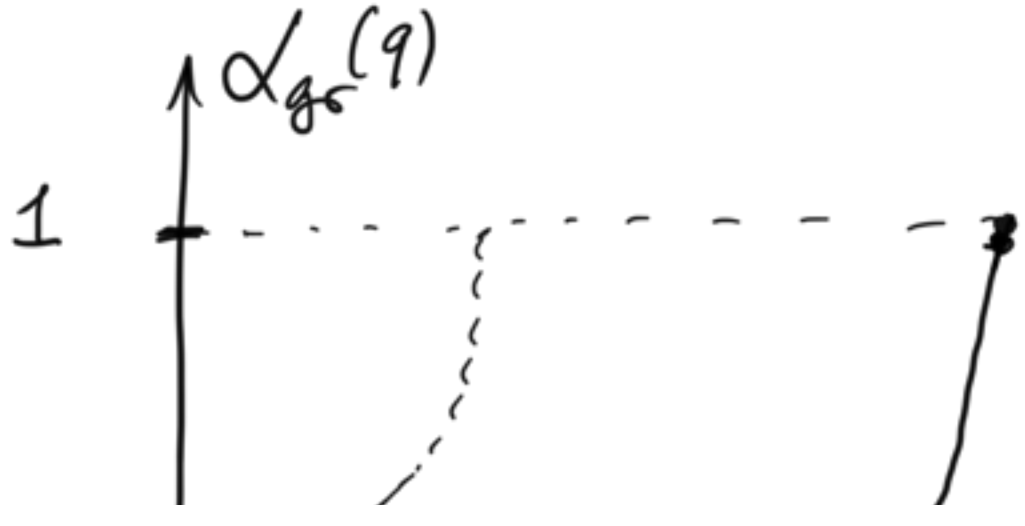
$$m\,\bar{\psi}_L \psi_R$$

$$\boxed{m\bar{\psi}\psi} \qquad \boxed{\psi \to e^{i\alpha\gamma_5}\psi}$$

$$\delta m \propto m$$



$$M_* = \frac{M_P}{\sqrt{N_{sp}}}$$

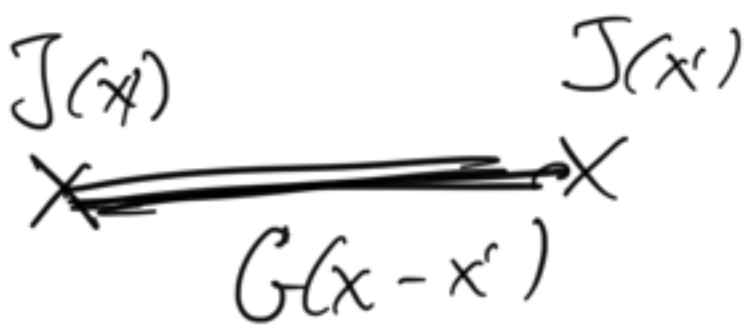


$$S = \int d^4x \, d^4x' \, T^{\mu\nu}(x) \underbrace{H_{\mu\nu}(x) \, H_{\alpha\beta}(x')}_{\boxed{G_{\mu\nu,\alpha\beta}(x-x')}} T^{\alpha\beta}(x')$$

Källen-Lehmann

$$\boxed{G(p) = \int_0^\infty dm^2 \, \frac{\rho(m^2)}{p^2 - m^2 + i\varepsilon}}$$

$J(x)\,\hat{\Phi}(x)$



$$G_{\mu\nu,\alpha\beta}(p) = \int_0^\infty dm^2 \, \frac{\rho(m)_{\mu\nu,\alpha\beta}}{p^2 - m^2 + i\varepsilon}$$

$$\boxed{\rho(m=0)_{\mu\nu,\alpha\beta} = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}\right)-\frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}$$

$$\boxed{\rho(m\neq 0)_{\mu\nu,\alpha\beta} = ?}$$

$$\boxed{\text{Spin}=2 \qquad m\neq 0}$$

not gauge

$$\mathcal{L} = h^{\mu\nu}\mathcal{E}_{\mu\nu} + m^2\left(h_{\mu\nu}h^{\mu\nu}-h^2\right)$$

$$\boxed{h^{\mu\nu}\to h^{\mu\nu}+\partial^\mu\xi^\nu+\partial^\nu\xi^\mu}$$

Fierz-Pauli

$$\Box h_{\mu\nu}-\eta_{\mu\nu}\Box h-\partial_\mu\underline{\partial^\alpha h_{\alpha\nu}}-\partial_\nu\partial^\alpha h_{\alpha\mu}$$
$$+\partial_\mu\partial_\nu h+\eta_{\mu\nu}\partial^\alpha\partial^\beta h_{\alpha\beta}$$
$$+m^2\left(h_{\mu\nu}-\eta_{\mu\nu}h\right) = T_{\mu\nu}$$

$$\partial^\mu \left(\ldots\ldots = T_{\mu\nu} \right)$$

$$\boxed{10-4=6}$$

$$\boxed{\partial^\mu h_{\mu\nu} = \partial_\nu h} \leftarrow \text{constraint.}$$

$$\Box h_{\mu\nu} - \cancel{\eta_{\mu\nu}\Box h} - \cancel{\partial_\mu\partial_\nu h} - \partial_\nu\partial_\mu h$$

$$+ \cancel{\partial_\mu\partial_\nu h} + \cancel{\eta_{\mu\nu}\Box h} +$$

$$+ m^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = T_{\mu\nu}$$

$$\Box h - \Box h - 3m^2 h = T$$

$$\boxed{h = \frac{-1}{3m^2} T} \qquad \boxed{6-1=5}$$

$$\Box h_{\mu\nu} - \partial_\mu\partial_\nu h + m^2 \underline{(h_{\mu\nu} - \eta_{\mu\nu} h)} = T_{\mu\nu}$$

$$\underline{(\Box + m^2)} h_{\mu\nu} = T_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2}\right) T$$

$$h_{\mu\nu} = \frac{1}{\Box + m^2}\left(T_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{m^2}\right) T\right)$$

$$T^{\mu\nu} h_{\mu\nu} = \underline{T^{\mu\nu} \frac{1}{\Box + m^2}\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T\right)}$$

$$T^{\mu\nu} G^{(m)}_{\mu\nu,\alpha\beta} T^{\alpha\beta} =$$

$m \to 0$ $\longrightarrow$ $T^{\mu\nu} \frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T\right)$

$m = 0$ $\quad T^{\mu\nu} \frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right)$

extra polarization

$$\boxed{\begin{aligned} \overbrace{G_{\mu\nu,\alpha\beta}(p) = \frac{1}{M_P^2} \frac{\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2 + i\varepsilon}}^{G_N} \\ + \frac{1}{M_P^2} \sum_{m^2 \neq 0} C_2^{(m)} \frac{\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}) - \frac{1}{3}\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2 - m^2 + i\varepsilon} \end{aligned}}$$

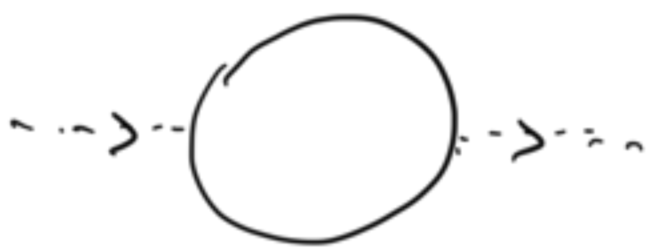
$$\boxed{+ \frac{1}{M_p^2} \sum_{m^2 \geq 0} C_0^{(m)} \frac{\eta_{\mu\nu} \eta_{\alpha\beta}}{p^2 - m^2 + i\varepsilon}}$$

$$\boxed{T_{\mu\nu} = \delta_\mu^0 \delta_\nu^0 M \delta^3(\vec{x})}$$

$$\tau_{\mu\nu} = \delta_\mu^0 \delta_\nu^0 \mu \delta^3(\vec{x} - \vec{r})$$

$$\boxed{\begin{aligned} V(r) = &- \frac{M\mu}{r} G_N - \sum_{m \neq 0} G_2^{(m)} \frac{e^{-mr}}{r} \\ &- \frac{Mm}{r} G_0^{(0)} - \sum_{m \neq 0} G_0^{(m)} \frac{e^{-mr}}{r} \end{aligned}}$$

$$\boxed{m \lesssim r^{-1}}$$



$$H \to e^{i\alpha} H$$

$$\boxed{m_H^2 H^\dagger H}$$

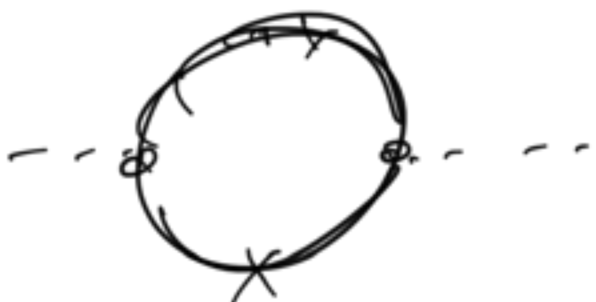
$$\mathcal{L} = \partial_\mu H^\dagger \partial^\mu H - \cancel{m_H^2 H^\dagger H} + \delta m_H^2 H^\dagger H$$

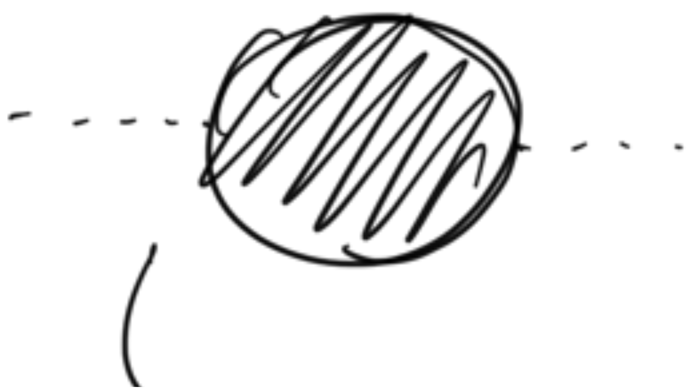
$$\boxed{H \to H + \text{const}} \qquad - \lambda^2 (H^\dagger H)^2$$

$$\delta m_H^2 \sim M_*^2 (\lambda^2 + \dots) + g^4 H \bar{\Psi} \Psi$$

$$\mathcal{L} = \partial_\mu H \partial^\mu H + (\partial_\mu H \partial^\mu H)^2$$

$$+ \cdot$$

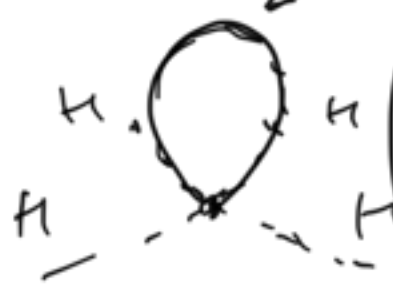




$$\frac{d^4 p}{p^2}$$

$\rightarrow \partial_\mu H \partial^\mu H$

$\delta m_H^2 H^2$



$$\frac{d^4 p}{p^2}$$

$$m_e \bar{\Psi}\Psi \rightarrow m_e \bar{\Psi}_L \Psi_R + h.c$$

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi = i\bar{\Psi}_L\not{\partial}\Psi_L + i\bar{\Psi}_R\not{\partial}\Psi_R$$

$\Psi_L \qquad \Psi_R \rightarrow \Psi_L^c$

chiral symmetry.

$$\gamma_5 \Psi_L = \Psi_L$$
$$\gamma_5 \Psi_R = -\Psi_R$$

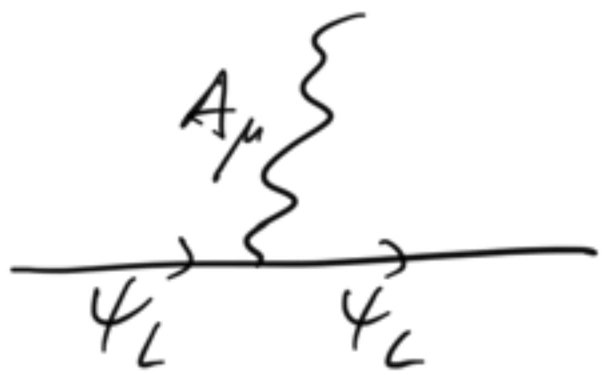
$$\Psi \rightarrow e^{i\alpha\gamma_5}\Psi$$

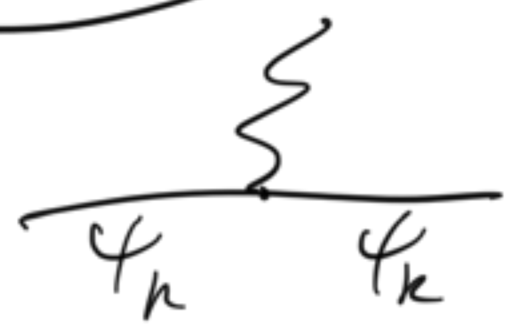
$$\Psi_L \rightarrow e^{i\alpha}\Psi_L$$
$$\Psi_R \rightarrow e^{-i\alpha}\Psi_R$$

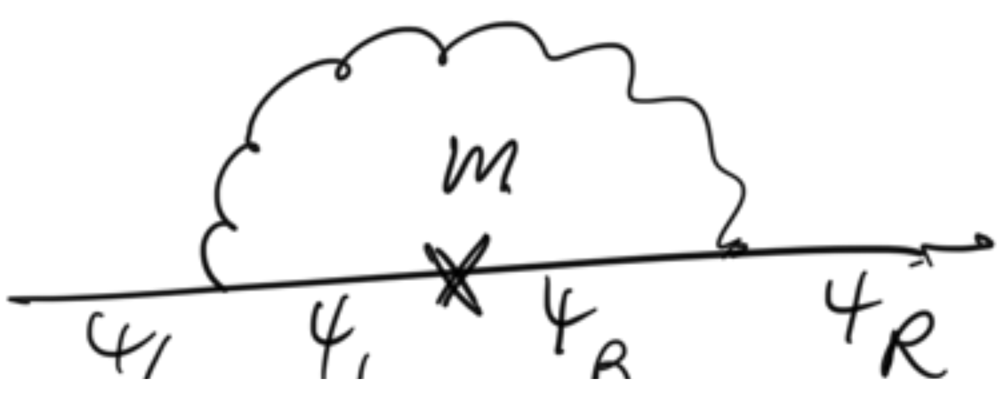
$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi$$

$$m\bar{\psi}_L\psi_R + h.c.$$

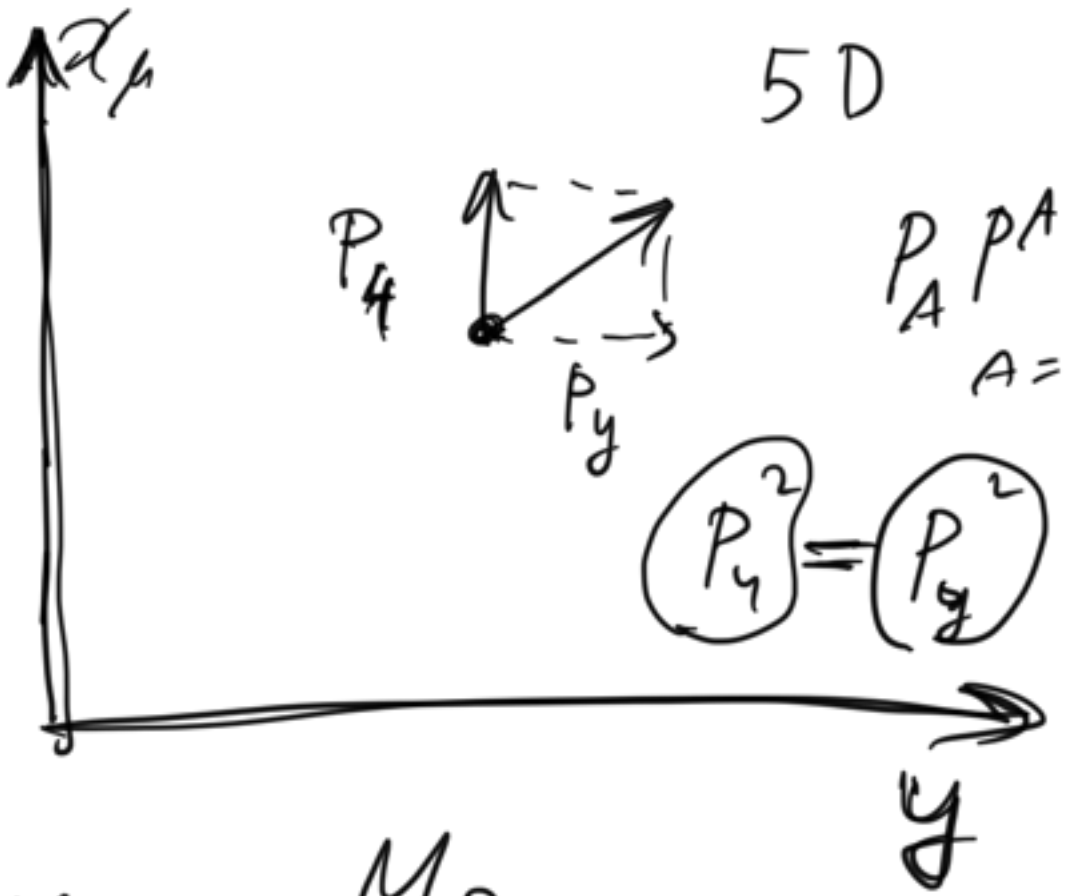
$$i\bar{\psi}\not{\partial}\psi \rightarrow \bar{\psi}_L\gamma^\mu\psi_L A^\mu + \bar{\psi}_R\gamma^\mu\psi_R A^\mu$$







$$\delta m_e \propto m_e .$$



$$P_A P^A = 0$$

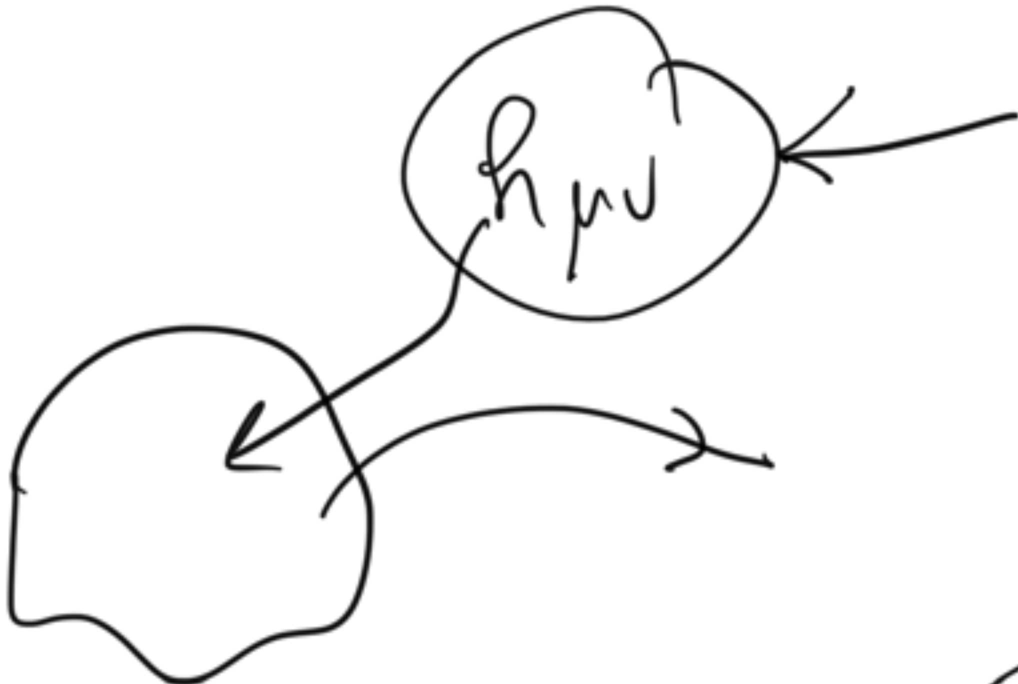
$A = 0, 1, 2, 3, y$

$$P_4^2 = P_y^2$$

$$M_* = \frac{M_P}{\sqrt{N_{sp}}} \longleftarrow \text{KK}$$

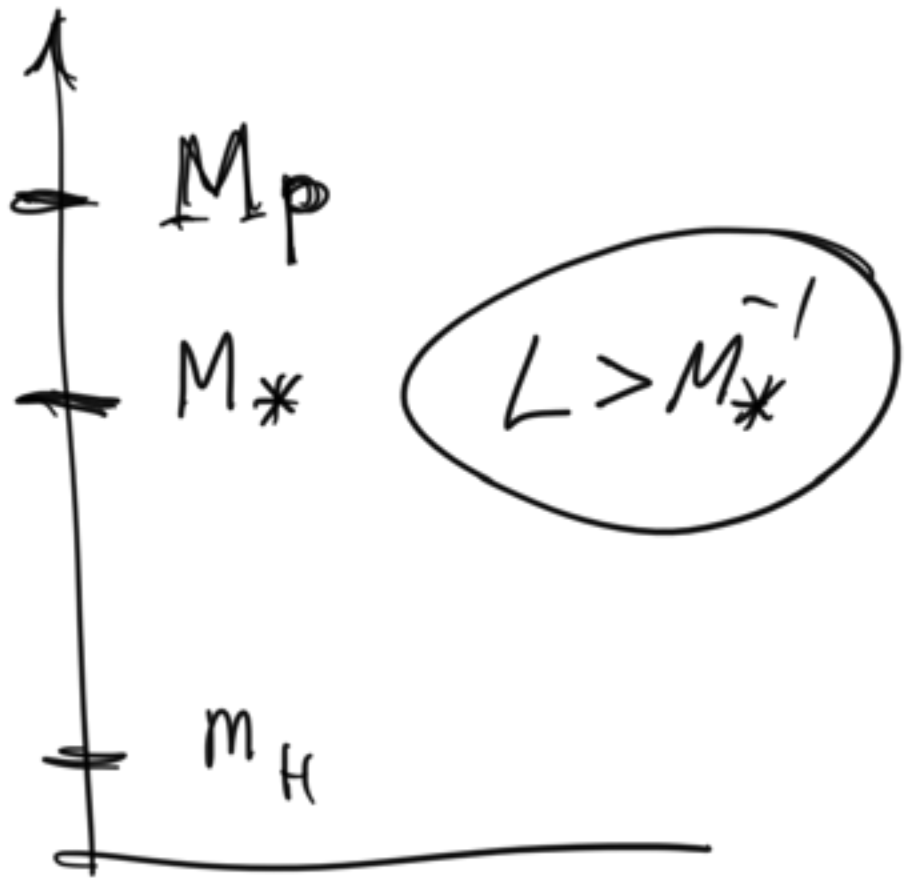
$$m < M_*$$

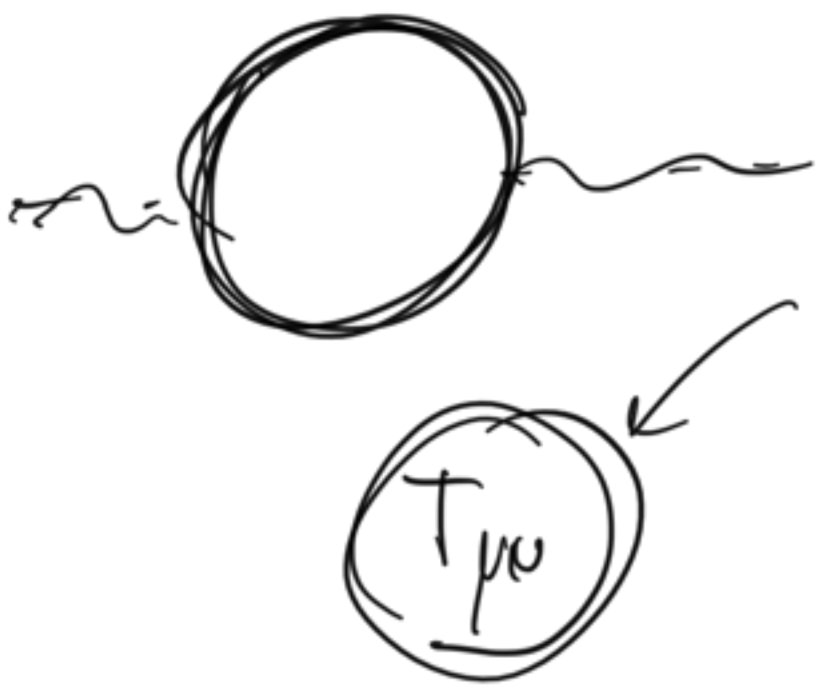
$$\bigcirc = \sum$$



$$\geq 0$$

$$G_{\mu\nu,\alpha\beta} = \int_0^\infty dm^2 \, \frac{\rho(m^2)_{\mu\nu,\alpha\beta}}{p^2 - m^2 + i\varepsilon}$$





Spin = 2   Spin = 1   2

scalar

$$h^{FP}_{\mu\nu} = h^{E}_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$$

$$+ \ 1\,\eta_{\mu\nu}\pi + 1\,\partial_\mu\partial_\nu\pi$$

5 $\quad$ 2 $\quad$ $6 \quad 3 \quad \frac{1}{m^2}$

m

$5 = 2 + 2 + 1$

$\partial_\mu T^{\mu\nu} = 0$

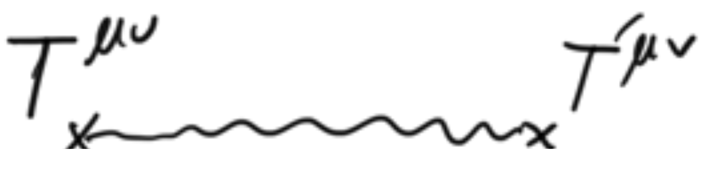
vDVZ

$\mathcal{E}h + m\,\ell + hT^{\mu\nu}$

$m \to 0$

BSM Lecture 5

Källén-Lehman spectral representation for generalized graviton propagator.



$T \equiv T_\mu^\mu$

$$T^{\mu\nu} \underline{G_{\mu\nu,\alpha\beta}} T'^{\alpha\beta} = \frac{1}{M_p^2} \Bigg\{ \overbrace{\frac{T_{\mu\nu} T'^{\mu\nu} - \frac{1}{2} T T'}{p^2 + i\varepsilon}}^{\text{Einstein}}$$

$$+ \sum_{m^2 > 0} \frac{T_{\mu\nu} T'^{\mu\nu} - \frac{1}{3} T T'}{p^2 - m^2 + i\varepsilon} C_2^{(m)} + \sum_{m^2 \geq 0} \frac{T_{\mu\nu} T'^{\mu\nu}}{p^2 - m^2 + i\varepsilon} C_0^{(m)} \Bigg\}$$

$$\boxed{C_2^{(m)},\ C_0^{(m)} > 0}$$

$$T_{\mu\nu} \longrightarrow M_1 \qquad T'_{\mu\nu} \to M_2$$

$$V(r) = -\frac{1}{M_p^2} \frac{M_1 M_2}{r} \Bigg\{ 1 + \frac{4}{3} \sum_{m^2 > 0}^{\infty} C_2^{(m)} e^{-mr}$$

$$+ 2 \sum_{m=0}^{\infty} C_0^{(m)} e^{-mr} \Bigg\}$$

<u>Equivalence principle.</u>

Higgs gravity $\qquad H_0 = \underbrace{v}_{\langle H_0 \rangle} + h_H(x)$

$$g_\psi H_0 \bar{\psi}_L \psi_R \to \underbrace{g_\psi v}_{\text{"}m_\psi\text{"}} \bar{\psi}_L \psi_R + g_\psi h_H \bar{\psi}_L \psi_R$$

$$m_\psi \equiv g_\psi v$$

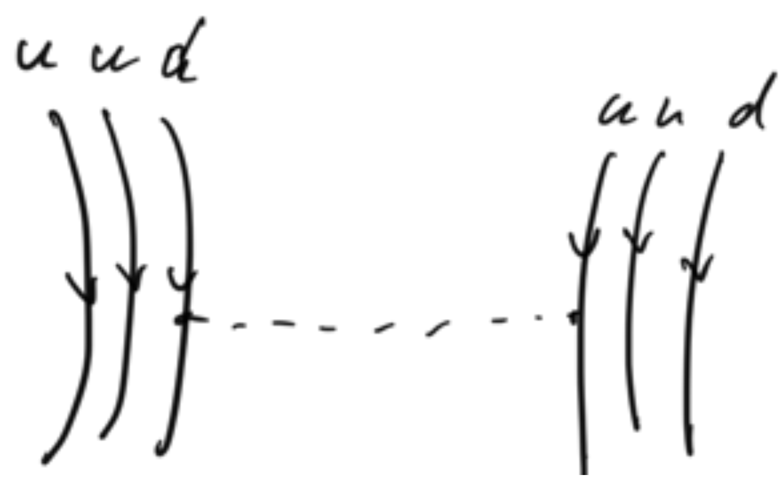
$$\frac{m_\psi}{v} h_H \bar{\psi}_L \psi_R \qquad\qquad \frac{m_u}{v} \quad \frac{m_d}{v}$$

$$m_H = \lambda v \qquad\qquad \boxed{\lambda \to 0}$$

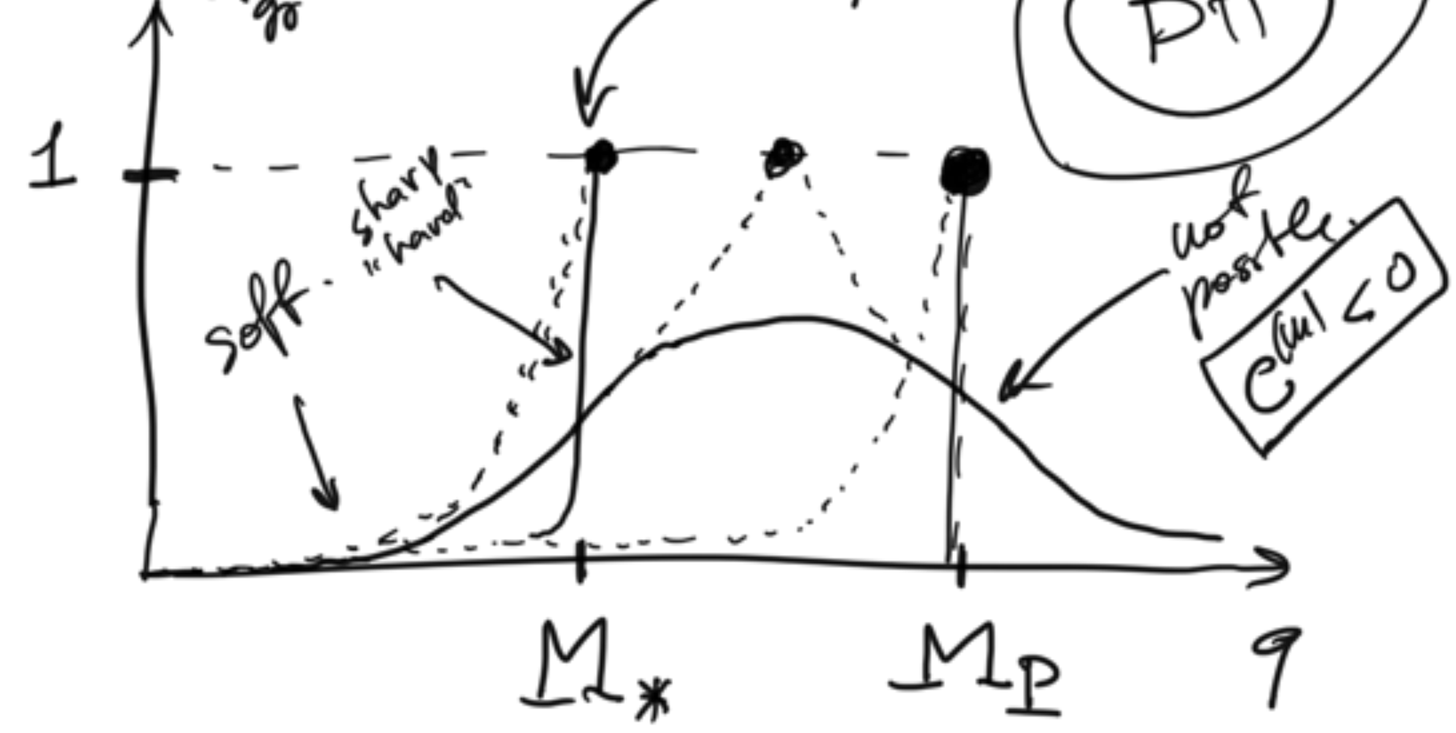
$$V_H(r) = -\frac{m_\psi^2}{v^2} \frac{e^{-m_H r}}{r}$$

proto·
neutron

$$M_{proto} \gg 3 M_q$$



$$M_* = \frac{M_P}{\sqrt{N_{sp}}}$$

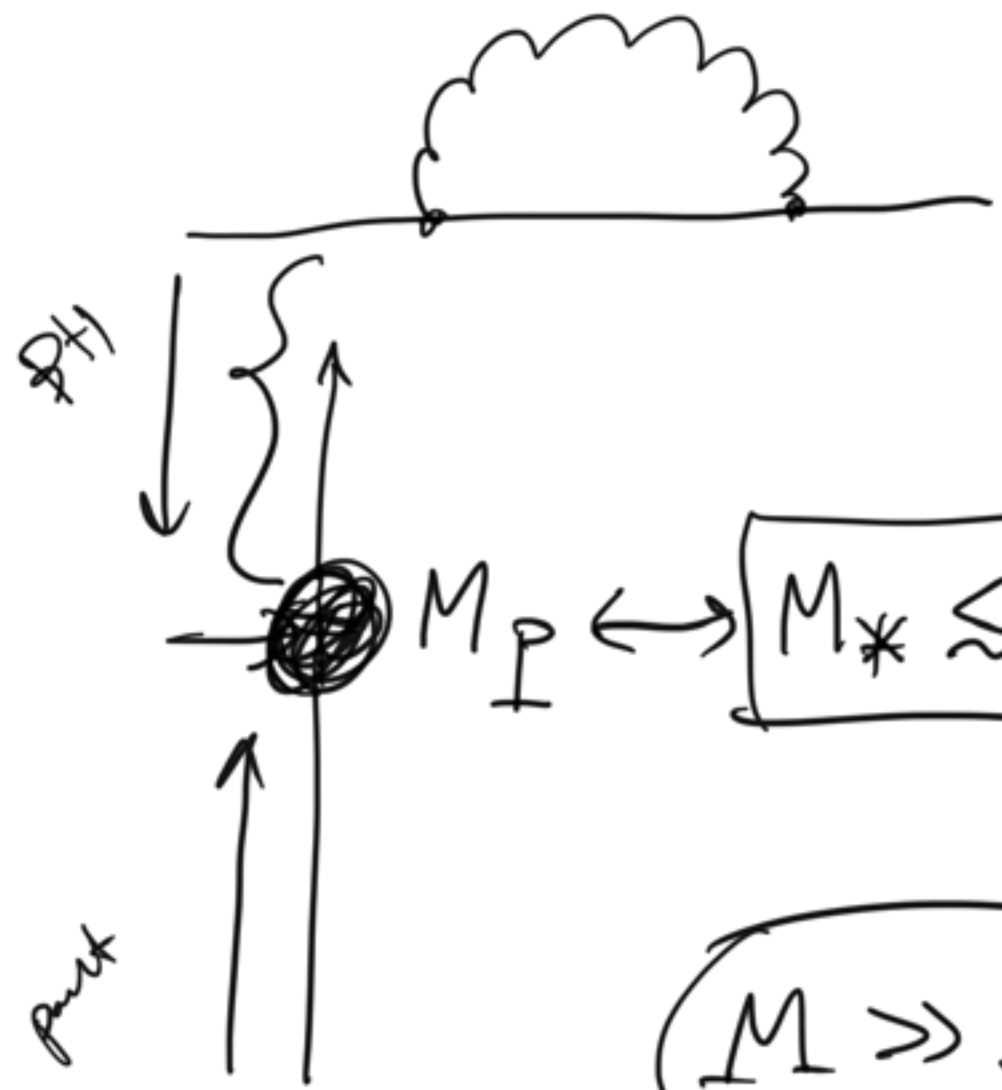


$$\boxed{q \gg M_P}$$

try

this center of mass energy

$$\sqrt{s} \gg M_P$$



$M_P \longleftrightarrow \boxed{M_* \lesssim M_P}$

$M \gg M_P$

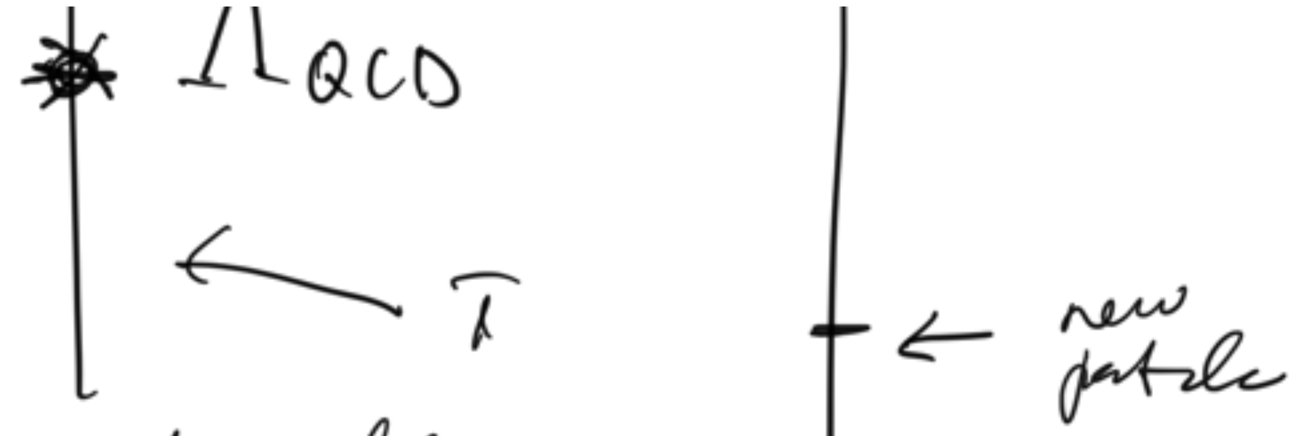
$s = \left(\frac{M}{M_P}\right)^2 \gg 1$

$\frac{1}{s}$

$\boxed{M \longrightarrow M_P}$

quantum "BH" ?

gluons quarks



For hierarchy problem

$$\boxed{M_* \sim \text{TeV}} \longrightarrow \boxed{N_{sp} \sim 10^{32}}$$

LHC

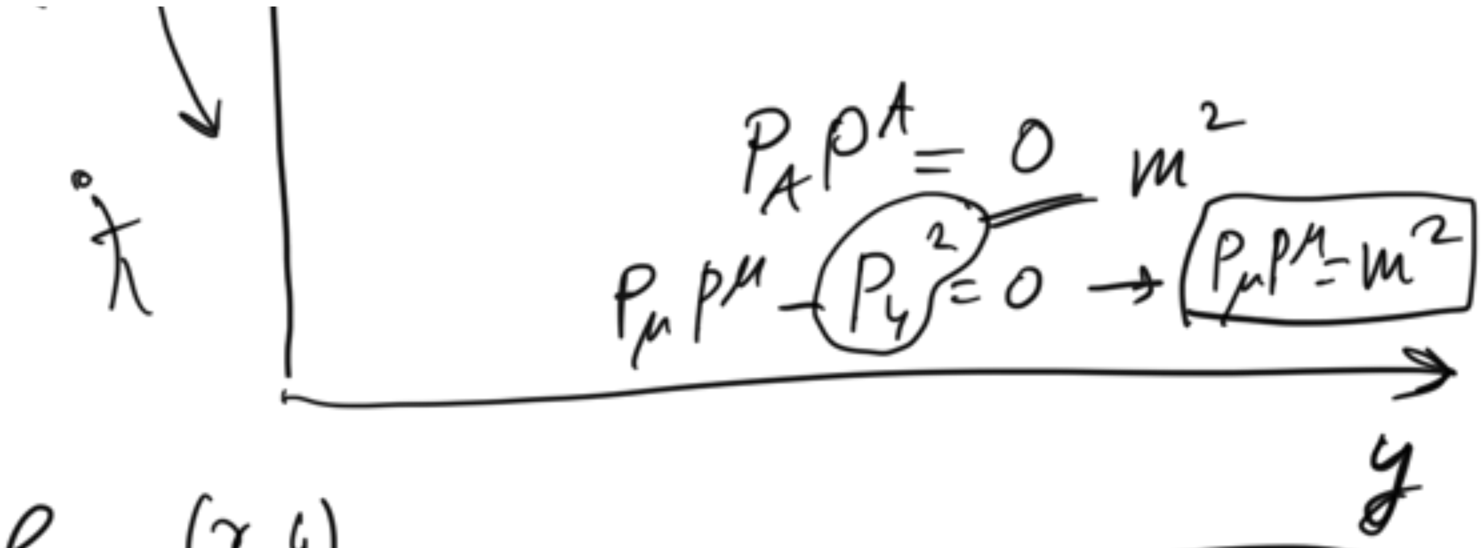
absolute upper bound

$A = 0, 1 \ldots 5$

$X_A \equiv (x_\mu, y)$

Extra dimensions (Large)

$x_\mu \quad \mu = 0,1,2,3 \quad x_5 \equiv y$



$$P_A P^A = 0 \quad m^2$$

$$P_\mu P^\mu - P_y^2 = 0 \rightarrow \boxed{P_\mu P^\mu = m^2}$$

$$h_{AB}(x,y)$$

$$\Box h_{AB} - \eta_{AB}\Box h - \partial_A \partial^C h_{CB} - \partial_B \partial^C h_{CA} + \partial_A \partial_B h + \eta_{AB}\partial^C \partial^D h_{CD} = 0$$

$$h_{AB} \rightarrow h_{AB} + \partial_A \xi_B + \partial_B \xi_A \qquad \begin{matrix}\mu\nu \\ \mu 5 \\ 55\end{matrix}$$

gauge parameter $\boxed{\xi_B(x_\mu, y)}$

(x_μ, y)

$$h_{AB} = h_{\mu\nu} + h_{\mu 5} + h_{55}$$

complete basis $b^{(m)}(y)$ dynamic tensor   vector   scalar.

$1 = \partial y$

$$h_{\mu\nu}(x,y) = \sum b^{(m)}(y)\, \hat{h}^{(m)}(x)$$

$m=0$ Einstein... [illegible]

$$h_{\mu\nu}(x,y) = \sum_m b(y)\, h_{\mu\nu}(x)$$

$m \neq 0$ Fierz-P.

$$h_{\mu 5}(x,y) = \sum b^{(m)\prime}(y)\, A_\mu^{(m)}(x)$$

$m=0$ Maxwell. $m \neq 0$

$$h_{55}(x,y) = \sum b^{(m)\prime\prime}(y)\, \phi^{(m)}(x)$$

$m=0$ scalar. $m \neq 0$

$$\boxed{b^{(m)\prime\prime}(y) = -m^2 b^{(m)}(y)} \qquad \boxed{b^{(m)}(y) = e^{imy}}$$

$$\boxed{\bar{A}_\mu^{(m)} \equiv A_\mu^{(m)} + \frac{1}{2}\partial_\mu \phi^{(m)}_{(m)}}$$

$$\left(\mathcal{E}\hat{h}^{(m)}\right)_{\mu\nu} - m^2\Big(\hat{h}^{(m)}_{\mu\nu} - \eta_{\mu\nu}\hat{h}^{(m)} + \partial_\mu \bar{A}_\nu^{(m)} + \partial_\nu \bar{A}_\mu^{(m)}$$
$$- \eta_{\mu\nu}\, 2\, \partial^\alpha \bar{A}_\alpha^{(m)}\Big) = 0$$

Fierz-Pauli

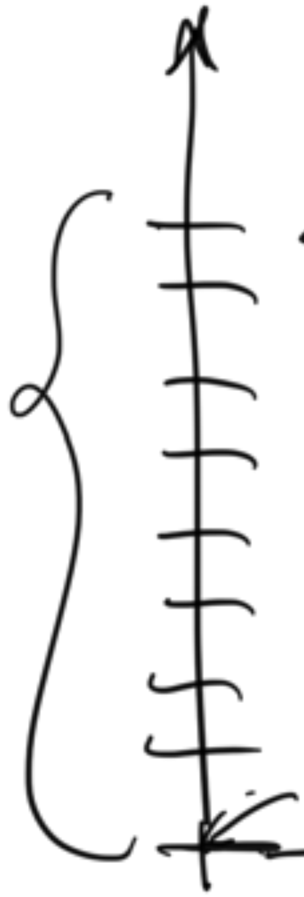
$$\boxed{\begin{aligned} \hat{h}^{(m)}_{\mu\nu} &\to \hat{h}^{(m)}_{\mu\nu} + \partial_\mu \xi_\nu^{(m)} + \partial_\nu \xi_\mu^{(m)} \\ \bar{A}_\nu^{(m)} &\to A_\nu^{(m)} - \xi_\nu^{(m)} \end{aligned}}$$

Stückelberg field

- Stückelberg field.

$$\partial^\mu F^{(m)}_{\mu\nu} + \underbrace{\partial^\mu\left(\hat h^{(m)}_{\mu\nu} - \eta_{\mu\nu}\hat h^{(m)}\right)}=0$$

Maxwell-like with a sum.

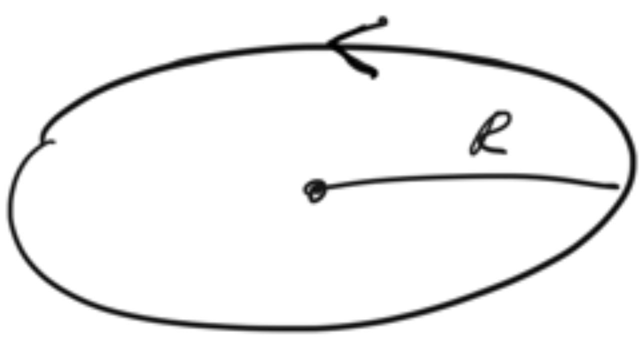


$m \rightarrow \boxed{5 = 2+2+1} \leftarrow$ Fierz-Pauli

KK tower.

Einst. Maxwell scalar.

$m=0 \rightarrow 5 = 2+2+1$



R

y

$$\boxed{m = \frac{n}{R}}$$

Goldstone.

$$\text{Poincaré}_5 \rightarrow \text{Poincaré}_4 \otimes U(1)$$

$$\boxed{y \to y + \text{const.}}$$

$$S = \int dy\, d^4x\, M_*^3 \left(h^{AB} \mathcal{E}_{AB} \right)$$

$$\to \int d^4x\, M_p^2 \left(h^{\mu\nu} \mathcal{E}_{\mu\nu} \right) + \dots$$

$$\boxed{M_p^2 = \int dy\, M_*^3 = M_*^2 \left(M_* \, {}^{2\pi} R \right)}$$

$$\boxed{M_p^2 = M_*^2 N_{KK}}$$

$$m = \frac{n}{R}$$

N_{Sp}

LII

$$M_P^2 = M_*^2 \underbrace{\left(M_*^d V^{(d)}\right)}$$

$$\boxed{N_{sp} \sim 10^{32}}$$

For addressing hierarchy puzzle.

$$\boxed{M_W^{\oplus} \sim e^{-\frac{1}{g^2}} M_P}$$

Tower of "BH"s

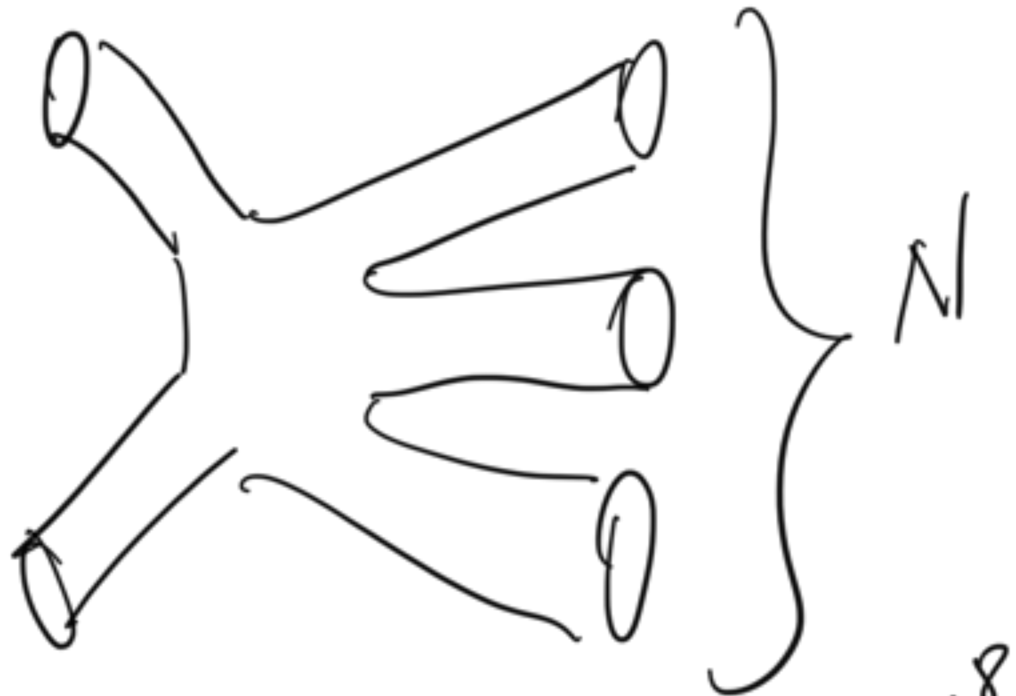
$$\boxed{M_* \sim TeV}$$

predictions are resonance. "quantum BH"

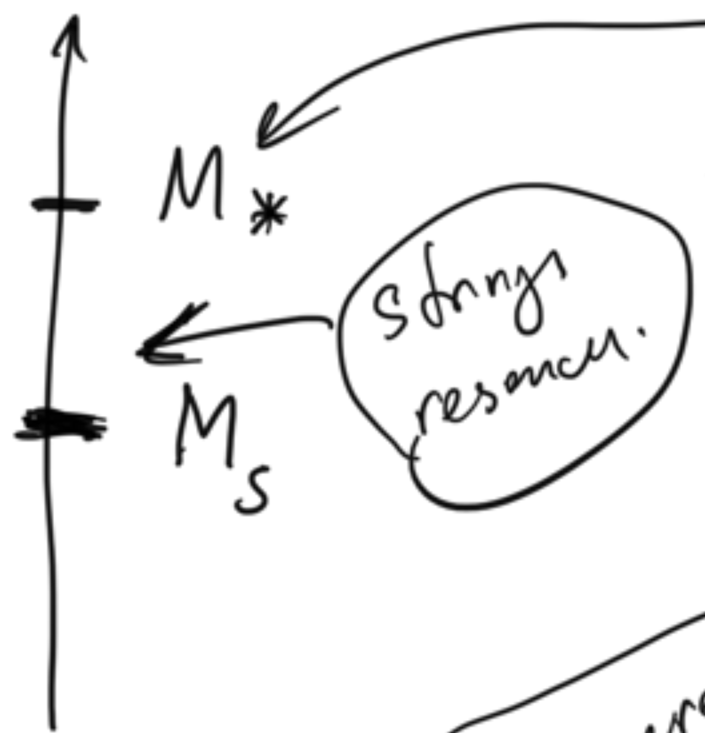
heavier resonance $\boxed{M \gg M_*}$

must be longer lived

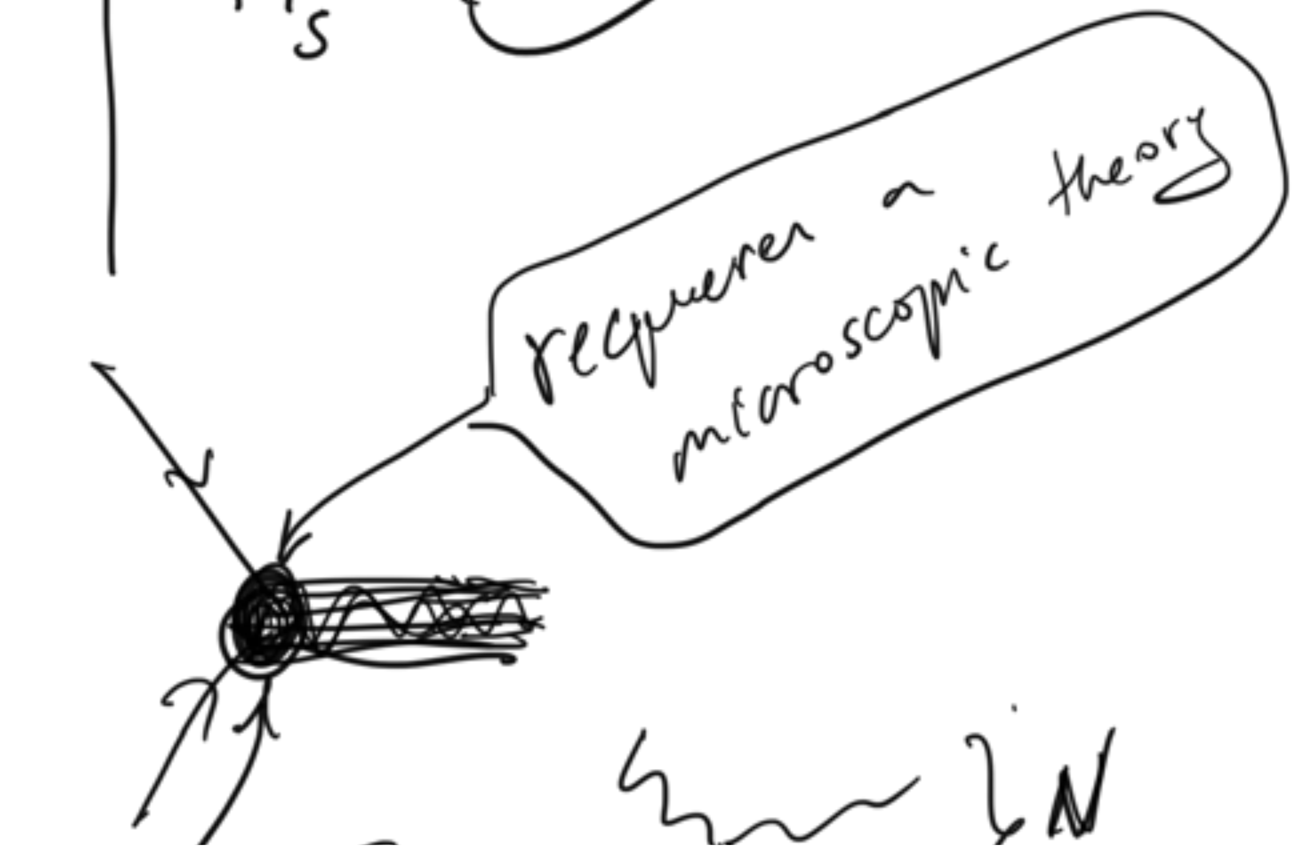
and decay into more number of softer quanta.



$$M_{(10)}^8 = \frac{1}{g_s^2} M_s^8$$



$g_s \ll 1$



$S \sim 1$

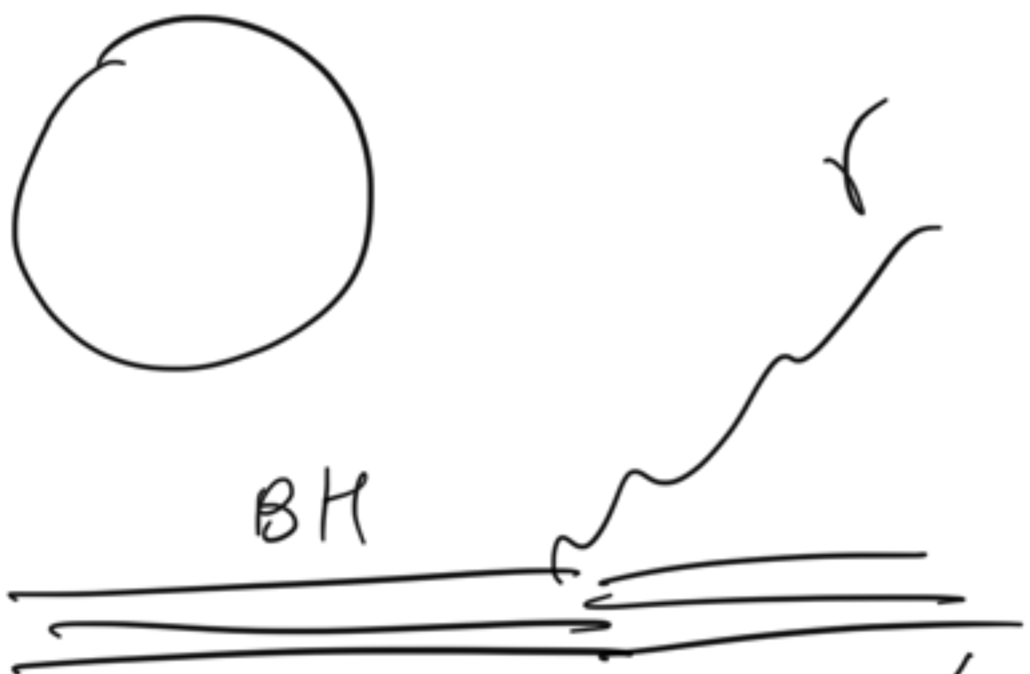
e^{-S}

$N \sim S$

$\frac{1}{S}$

$N \gg 1$

$\frac{1}{N}$



$\dot{M} \sim T^2 N$