FLAVOUR

* An overview (how flavour arises in SM)
* A few theoretical tools
* CKM matrix
* Anomalies
* NP considerations

OVERVIEW + SM SITUATION

QFT = special relativity + quantum mechanics

$\mathcal{L}$ → gauge group [symmetries]
→ field content with representations
→ symmetry breaking

→ renormalisable → mathematical requirement
→ physical requirement

⇝ SM description → successful
→ BUT structure contrived
large # of parameters
hierarchical

EXPLANATION

FLAVOUR

SM STRUCTURE

- Gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Matter fields (fermions) distinguish between chiralities

$$\psi_L = P_L \psi = \frac{1-\gamma_5}{2}\psi \qquad \xrightarrow[\text{limit}]{\text{massless}} \qquad \overset{S}{\longleftarrow}\!\!\longrightarrow p \quad h = -1/2$$

$$\psi_R = P_R \psi = \frac{1+\gamma_5}{2}\psi \qquad\qquad \overset{S}{\longrightarrow}\!\!\longrightarrow p \quad h = +1/2$$

$$\underbrace{\qquad\qquad\qquad}_{\text{chiralities}} \qquad\qquad \underbrace{\qquad\qquad}_{\text{helicities}}$$

$(\text{colour}, L)_Y$

$\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} = Q_{iL}$	$(3,2)_{1/6}$		$i = 1,2,3$ generation index
u_{iR}	$(3,1)_{+2/3}$		
d_{iR}	$(3,1)_{-1/3}$		
$\begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} = L_{iL}$	$(1,2)_{-1/2}$		
e_{iR}	$(1,1)_1$		no ν_R

- Symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$

related to the dynamics of the Higgs field

$$\phi = (1,2)_{1/2} \qquad \tilde{\Phi} = i\sigma_2 \phi^* : (1,\bar{2})_{-1/2}$$

LAGRANGIAN

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Dirac} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{Dirac} = \sum_\psi i\bar{\psi} \not{D} \psi = \sum_\psi i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R$$

no mass term $\bar{\psi}_L \psi_R$

$$D_\mu \psi = (\partial_\mu - i g_s G^A_\mu T^A - i g W^a_\mu \tau^a - i g' y B_\mu)\psi$$

$$\mathcal{L}_{Higgs} = D_\mu \phi^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

* triggers the EWSB
* gives mass to W, Z

ϕ 4 dof $\longrightarrow$ 3 into the longitudinal polarisation of W & Z

$\longrightarrow$ 1 remaining dof H boson

$\longrightarrow$ express $\underbrace{W_1, W_2, W_3}_{SU(2)_L}$ $\quad \underbrace{B_\mu}_{U(1)_Y}$ in terms of $W^\pm, Z^\mu, A$

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu) \qquad m_W = g\frac{v}{2}$$

$$Z^0_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g W^3_\mu - g' B_\mu) \qquad m_Z = \sqrt{g^2+g'^2}\,\frac{v}{2}$$

$$A^0_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g' W^3_\mu + g B_\mu) \qquad \underline{\underline{m_A = 0}}$$

$U(1)_{em}$

$$D_\mu \psi = \left(\partial_\mu - \frac{ig}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-)\right.$$

$$\ldots (T^3 \ldots Q) \ldots A_\mu Q)\psi$$

$$-\frac{ig}{\cos\theta_W} \quad t_\mu(1-\sin\theta_W Q) \cdots$$

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} \qquad Q = \tau^3 + Y \qquad \tau^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$\mathcal{L}_{Dirac}$ interaction between fermions & the physical gauge bosons

$$\mathcal{L}_{Dirac} = \bar{L}_L (i\not{D}) L_L + \bar{e}_R (i\not{D}) e_R + \bar{Q}_L (i\not{D}) Q_L + \bar{u}_R (i\not{D}) u_R + \bar{d}_R (i\not{D}) d_R$$

$$+ g(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z^0 J^\mu_Z) + e A_\mu J^\mu_{em}$$

$$J^\mu_{W^+} = \frac{1}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L)$$

$$J^\mu_{W^-} = \frac{1}{\sqrt{2}}(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L)$$

charged currents involving only left handed fermions

$$J^\mu_Z = \frac{1}{\cos\theta_W}\Big[\bar{\nu}_L\gamma^\mu\left(\tfrac{1}{2}\right)\nu_L + \bar{e}_L\gamma^\mu\left(-\tfrac{1}{2}+\sin^2\theta_W\right)e_L + \bar{e}_R\gamma^\mu\left(\sin^2\theta_W\right)e_R$$
$$+ \bar{u}_L\gamma^\mu\left(\tfrac{1}{2}-\tfrac{2}{3}\sin^2\theta_W\right)u_L + \bar{u}_R\gamma^\mu\left(-\tfrac{2}{3}\sin^2\theta_W\right)u_R$$
$$+ \bar{d}_L\gamma^\mu\left(-\tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta_W\right)d_L + \bar{d}_R\gamma^\mu\left(\tfrac{1}{3}\sin^2\theta_W\right)d_R\Big]$$

neutral currents with L & R

$$J^\mu_{em} = \bar{e}\gamma^\mu(-1)e + \bar{u}\gamma^\mu\left(\tfrac{2}{3}\right)u + \bar{d}\gamma^\mu\left(-\tfrac{1}{3}\right)d$$

same coupling L R

→ for a single generation

$i\bar{\psi}^j \not{D} \psi^j$ → in this expression generation indices implicit

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Expressions are correct for physical gauge bosons

but what about the physical dof fermions

→ mass eigenstates of the fermions?

MASS EIGENSTATES FOR FERMIONS

- $\mathcal{L}_{YUKAWA}$ after EWSB contains the term

$$\mathcal{L}_{YUKAWA} = -Y_e^{ij}\,\bar{L}^i \phi\, e_R^j - Y_D^{ij}\,\bar{Q}_L^i \phi\, d_R^j - Y_U^{ij}\,\bar{Q}_L^i \tilde{\phi}\, u_R^j + hc$$

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \longrightarrow -\bar{e}_L^i M_E^{ij} e_R^j - \bar{d}_L^i M_D^{ij} d_R^j - \bar{u}_L^i M_U^{ij} u_R^j + hc$$

→ "Diagonalise" Singular Value Decomposition

$$M_x^{ij} = \frac{Y_x^{ij} v}{\sqrt{2}} \qquad M_x = V_{xL}\, m_x\, V_{xR}^\dagger$$

diagonal eigenvalues

unitary matrices

$$m_x = (m_{x1}, m_{x2}, m_{x3})$$

u	u, c, t
d	d, s, b
e	e, μ, τ

- If we redefine $\begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \to V_{u\,L,R} \times \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}$

same for $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ $\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$ (idem for neutrinos)

* kinetic Dirac term $\bar{\Psi} i \not{\partial} \Psi$, J^μ_Z , J^μ_A

$$\bar{\Psi}_{xL} \gamma^\mu \Psi_{xL} \qquad \bar{\Psi}_{xR} \gamma^\mu \Psi_{xR}$$

$$\Psi_x = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \text{ or } \begin{pmatrix} d \\ s \\ b \end{pmatrix} \text{ or } \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$\bar{\Psi}_{xL} \cancel{V^\dagger_{xL}} \gamma_\mu \cancel{V_{xL}} \Psi_{xL} = \bar{\Psi}_{xL} \gamma_\mu \Psi_{xL}$$

same for R

$$J^\mu_{W^+} = \frac{1}{\sqrt{2}} \Big[\bar{\nu}^i_L \gamma^\mu \overbrace{(V^\dagger_{\nu_L} V_{e_L})}^{\neq 1}{}^{ij} e^j_L + \bar{u}^i_L \gamma^\mu \underbrace{(V^\dagger_{u_L} V_{d_L})}_{\neq 1}{}^{ij} d^j_L \Big]$$

$\longrightarrow$ PMNS matrix: Pontecorvo – Maki – Nakagawa – Sakata

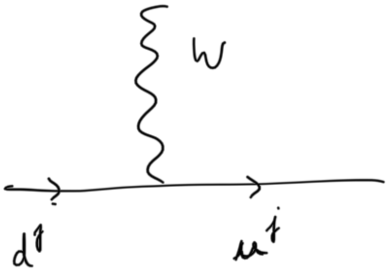
$\longrightarrow$ CKM matrix: Cabibbo Kobayashi Maskawa

if there is a misalignment between V_{uL} & V_{dL}

$\longrightarrow$ PMNS structure depends on assumptions on the mass generation mechanism

$\longrightarrow$ focus on the CKM matrix

CHARGED CURRENTS



$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \Big[\bar{u}_{Li} V_{ij} \gamma^\mu d_{Lj} W^+_\mu + \bar{d}_{Lj} V^*_{ij} \gamma^\mu u_{Li} W^-_\mu \Big]$$

i, j generation indices

h.c. $\rightarrow$ V for W^+, V^* for W^-

for the CKM matrix

$\rightarrow$ V is not necessarily the identity

$\rightarrow$ left handed fermions are involved

- C, P, T discrete symmetries

  $\rightarrow$ P cannot be satisfied (L and not R fermions)

  $\rightarrow$ what about CP ?

$$CP \, \mathcal{L}_W \, CP \Rightarrow \frac{g}{\sqrt{2}} \Big[\bar{d}_{Lj} V_{ij} \gamma^\mu u_{Li} W_\mu^- + \bar{u}_{Li} V_{ij}^* \gamma^\mu d_{Lj} W_\mu^+ \Big]$$

identical to $\mathcal{L}_W$ if $V_{ij} = V_{ij}^*$

$\rightsquigarrow$ invariance under CP holds only if V_{CKM} is real

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HOW MANY PARAMETERS TO DESCRIBE V_{CKM} ?

$\rightarrow$ how many sources of CP violation are contained?

SIMPLE VERSION

- V_{CKM} is $N \times N$ unitary complex matrix

  * $2N^2$ real parameters $V_{ij} = r_{ij} e^{i\theta_{ij}}$ modules phase

* relation of orthogonality involving the product of the same row / column

  N relations 1 condition each

* relation of orthogonality involving the product of two different rows / columns

  $\frac{N(N-1)}{2}$ relation 2 conditions each

* $2N^2 - N - 2 \times \frac{N(N-1)}{2} = N^2$ real parameters

- How many moduli & phases are inside these N^2 parameters?

  * the moduli can be obtained by considering a real orthogonal matrix

    N^2 real parameters

    N relation with same row / column (1 condition)

    $\frac{N(N-1)}{2}$ $\neq$ row / column (1 condition)

  * $N^2 - N - \frac{N(N-1)}{2} = \frac{N(N-1)}{2}$ moduli

- $\frac{N(N+1)}{2}$ phases? Yes, but there is still a freedom in among the phases, we can remove $(2N-1)$ phases

  $$\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2} \text{ phases}$$

- N - 2) 1 modules No CP violation

$\}$ 0 phase

$$V = \begin{bmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix}$$ Cabibbo matrix

- $N = 3$ $\begin{cases} 3 \text{ moduli} \\ 1 \text{ phase} \end{cases}$ CP violation is possible

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MORE COMPLICATED VERSION

- Can be obtained at the level of the Yukawa

Very large flavour symmetry group

Yukawa of the quark fields set to zero → large flavour group

$$U(3)_Q \otimes U(3)_U \otimes U(3)_D$$

$$\sim \underbrace{SU(3)_Q \otimes SU(3)_U \otimes SU(3)_D} \otimes U(1)_B \otimes U(1)_{Y_q} \otimes U(1)_{PQ}$$

- $SU(3)_{QDU}$: $\begin{pmatrix} u \\ c \\ t \end{pmatrix}_R \rightarrow V_U \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$

- $U(1)_B$: global phase redefinition associated with baryon number

$$\psi_{L,R} \rightarrow e^{i\alpha/3} \psi_{L,R} \qquad \psi = u, d, s, c, b, t$$

- $U(1)_{Y_q}$: ______________ quark hypercharge

$$\psi_{L,R} \rightarrow e^{i\beta Y_\psi} \psi_{L,R}$$

Y_ψ hypercharge for $\psi_{L,R}$

• $U(1)_{PQ}$: ———————— d_R - type quarks
[Peccei Quinn]
like

→ The Yukawas break down these

$$U(3)_Q \otimes U(3)_U \otimes U(3)_D \to U(1)_B$$

- real parameters $3 \times 3 \to 0$
- imag parameters $3 \times 6 \to 1$

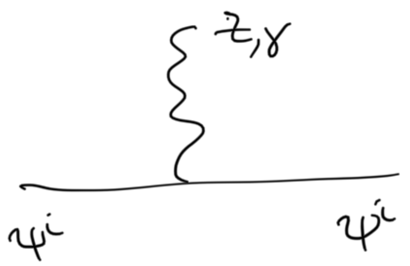
trapped by Y_u, Y_d containing 2×9 real
2×9 imag

→ the physical parameters remaining

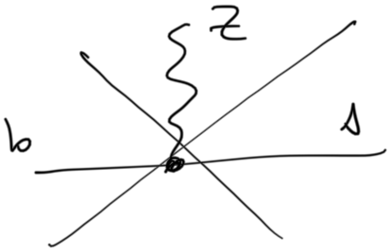
- $2 \times 9 - (9 - 0)$ real parameters : 6 quark masses
3 CKM parameters
- $2 \times 9 - (18 - 1)$ imag parameters : 1 phase CKM

→ 1 single source of CP violation in the quark sector
related to the CKM matrix
arising in the charged current.

NEUTRAL CURRENTS



remain inside the generation



no flavour changing neutral currents in the SM at the tree level

Suppression in the SM

- loop process $\frac{g^2}{(4\pi)^2}$

- If we neglect the mass difference $m_u = m_c = m_t$

$$A = \# \times \underbrace{(V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us})}_{=0 \text{ because of unitarity of the CKM matrix}}$$

GIM mechanism leads to a suppression

- Means that

$$\underline{A = \sum_{j=u,c,t} V_{jb}^* V_{js} \, f\left(\frac{m_j^2}{m_W^2}\right)}$$

* $f\left(\frac{m_c^2}{m_W^2}\right) \propto \frac{m_c^2}{m_W^2}$ for $m_c \ll m_W$

* decoupling does not work for m_t

$\longrightarrow$ suppression + top contribution is the most relevant

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FLAVOUR

- Large number of parameters set to arbitrary values in the SM
    - * fermion masses [6]
    - * mixing matrix between the generations (CKM (3+1) + PMNS) (3+1)

→ much larger number

gauge sector g_s, g, g'

Higgs sector μ, λ

→ How to understand better the source of the arbitrariness?

→ We need first to know exactly the pattern that we want to reproduce in the SM

- charged current
    - → CP violation (1 source for fermion sector)
    - → constraint on NP models

- neutral currents
    - → very much suppressed
    - → sensitivity to high energies

→ tools / CKM matrix / anomalies