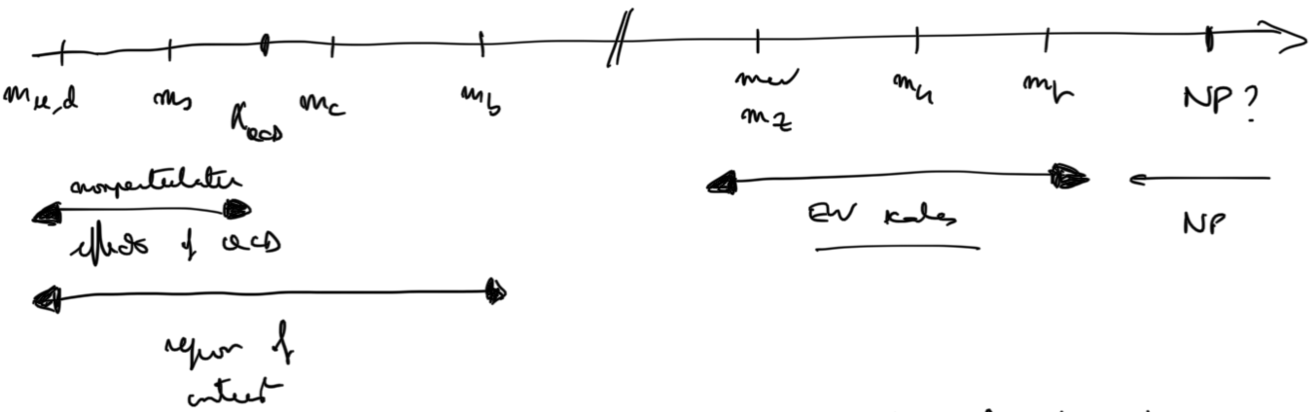
THEORETICAL TOOLS FOR FLAVOUR

* Effective Hamiltonian
* Hadronic matrix elements
* Heavy Quark Effective Theory

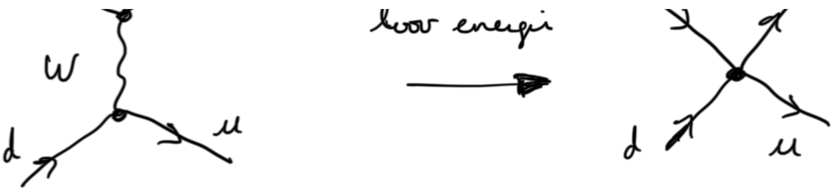
EFFECTIVE HAMILTONIAN



for flavour studies (concerning the external states)

Flavour process under all scales
* weak interactions
* strong interaction
* NP ?

- Separate the scales to simplify the problem
  → Fermi-style description of weak interactions



SM (EW)

$$\to V_{ud} V_{cs}^* \frac{G_F}{\sqrt{2}} \cancel{\frac{m_W^2}{m_W^2 - p_W^2}} \; \bar{u} \gamma_\mu (1-\gamma_5) d \; \bar{s} \gamma^\mu (1-\gamma_5) c$$

$$\longrightarrow \underbrace{V_{ud} V_{cs}^* \frac{G_F}{\sqrt{2}}}_{\text{number, short-distance physics}} \times \underbrace{\bar{u} \gamma_\mu (1-\gamma_5) d \; \bar{s} \gamma^\mu (1-\gamma_5) c}_{\text{4-quark operator with light fields remaining } \mathcal{O}}$$

$$A(B \to f) = \underbrace{C}_{\substack{\text{short dist} \\ \text{Wilson coeff}}} \times \underbrace{\langle f | \mathcal{O} | B \rangle}_{\substack{\text{long dist} \\ \text{Matrix element}}}$$

μ

→ integrate out the dof corresponding to massive or energetic modes. (t, W, Z, H) (plus photons)

↳ Wilson coefficients

EXAMPLE OF COMPUTATION WITHIN EFF HAM APPROACH

starting from the 4-quark operator

→ what is the impact of QCD corrections

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^{*} V_{ud} \left[C_1 \, Q_1 + C_2 \, Q_2 \right]$$

NEW → $Q_1 = (\bar{b}_\alpha c_\beta)_{V-A} \; (\bar{u}_\beta d_\alpha)_{V-A}$ $\qquad (\bar{b} c)_{V-A} = \bar{b}\gamma_\mu (1-\gamma_5) c$

LO → $Q_2 = (\bar{b}_\alpha c_\alpha)_{V-A} \; (\bar{u}_\beta d_\beta)_{V-A}$ $\qquad \alpha\beta$ colour indices

In the absence of QCD $\quad C_1 = 0 \quad C_2 = 1$

how is it changed with QCD corrections?

- These coefficients are short-distance contributions
  - → consider any convenient external state to make a computation in order to determine the values of C_1 & C_2
  - → take free quarks (neglecting their masses and assuming that they're off-shell $p^2 < 0$)

$$A_{SM} = \frac{G_F}{\sqrt{2}} V_{cb}^{*} V_{ud} \left[M_2 + \frac{3}{\;} \frac{\alpha_s}{\;} \ln \frac{M_W^2}{\;} \; M_2 \right|$$

$$\ldots \quad N_C\, 4\pi \qquad -p^2$$

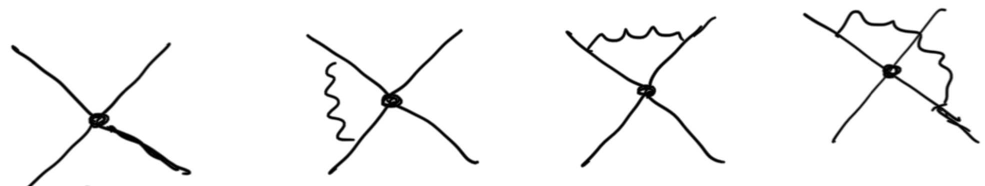
$$- \frac{3\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2}\, M_1 \Big]$$

$$M_1 = \langle Q_1 \rangle^{\mathrm{LO}} = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$

$$M_2 = \langle Q_2 \rangle^{\mathrm{LO}} = \underbrace{(\bar{b}_\alpha c_\alpha)_{V-A} (\bar{u}_\alpha d_\alpha)_{V-A}}_{\text{not the fields}}$$

but the solutions to Dirac equation
(spinors)

- In the effective Hamiltonian



Q_2

Q_1

$$\langle Q_1 \rangle^{(0)} = M_1 + \frac{3}{N_C} \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) M_1 - \frac{3\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) M_2$$

$$\langle Q_2 \rangle^{(0)} = M_2 + \frac{3}{N_C} \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) M_2 - \frac{3\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) M_1$$

$1/\varepsilon \rightarrow$ dimensional regularisation / $\overline{MS}$ scheme

* Additional divergences compared to the SM expression
→ RGE

* Only low scale involved in the logs
→ MATCHING

MATCHING

Identify the results of the high-energy theory (SM)

and the low-energy theory (Eff Ham)

$\longrightarrow$ tune the values of the Wilson Coefficients

$$A^{(SM)}_{full} = \frac{G_F}{\sqrt{2}} V^*_{cb} V_{ud} \left[C_1(\mu) \langle Q_1(\mu)\rangle + C_2(\mu) \langle Q_2(\mu)\rangle \right]$$

$$\begin{cases} C_1(\mu) = -\dfrac{3\alpha_s(\mu)}{4\pi} \ln \dfrac{M_W^2}{\mu^2} & + O(\alpha_s^2) \\ C_2(\mu) = 1 + \dfrac{3\alpha_s(\mu)}{4\pi} \ln \dfrac{M_W^2}{\mu^2} \end{cases}$$

$$\underbrace{\int_{-p^2}^{M_W^2} \frac{dk^2}{k^2}}_{SM} = \underbrace{\int_{\mu^2}^{M_W^2} \frac{dk^2}{k^2}}_{C_i} + \underbrace{\int_{-p^2}^{\mu^2} \frac{dk^2}{k^2}}_{\langle Q_i\rangle}$$

$\swarrow$ can be reused

even considering other external states (hadrons)

$\longrightarrow$ Wilson coefficients involve:

$$\frac{\alpha_s(\mu)}{4\pi} \ln\left(\frac{M_W^2}{\mu^2}\right)$$

$\mu = O(M_W)$ small (no problem)

$\mu = O(m_b, m_c)$

large logarithms $\longleftarrow$ $\alpha_s(\mu) \nearrow$ $\ln\left(\frac{M_W^2}{\mu^2}\right) \nearrow$

μ ,

→ potentially problematic at low scale μ

RGE

- Renormalisation procedure

$$\langle Q_i \rangle^{(0)} = Z_{ij} \langle Q_j \rangle$$

$$Z_{ij} = 1 + \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} \begin{pmatrix} 3/N_c & -3 \\ -3 & 3/N_c \end{pmatrix}$$

$$Q_\pm = \frac{Q_2 \pm Q_1}{2} \qquad C_\pm = C_2 \pm C_1$$

$$Q_\pm^{(0)} = Z_\pm \, Q_\pm \qquad \boxed{C_\pm^{(0)} = Z_\pm^{-1} C_\pm}$$

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[C_+ Q_+ + C_- Q_- \right]$$

(can be written as bare (0) or renormalised)

$$C_\pm(\mu) = C_\pm^{(0)} Z_\pm(\alpha_s(\mu))$$

RGE:
$$\frac{d C_\pm(\mu)}{d\ln\mu} = \gamma_\pm(\mu)\, C_\pm(\mu)$$

$$\gamma_\pm(\mu) = \frac{1}{\ldots} \frac{dZ_\pm}{\ldots} = \pm \frac{\alpha_s(\mu)}{\ldots} \frac{6(N_c \mp 1)}{\ldots}$$

$\epsilon_\pm \quad d\ln\mu \qquad 4\pi \qquad N_c$

$$C_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{\gamma_\pm^{(0)}}{\beta_0}} C_\pm(M_W)$$

$$\gamma_\pm^{(0)} = \frac{6(N_c \mp 1)}{N_c} \qquad \beta_0 = \frac{11N_c - 2N_f}{3}$$

$$C_+(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{6/23}$$

$$C_-(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{-12/23}$$

and using $C_\pm(M_W) = 1 + O(\alpha_s)$

$$\rightarrow \quad \frac{\alpha_s(M_W)}{\alpha_s(\mu)} = \frac{1}{1 + \frac{\beta_0 \alpha_s(\mu)}{4\pi} \ln\left(\frac{M_W^2}{\mu^2}\right)}$$

$\rightarrow$ sum of all logs $\alpha_s^n(\mu) \ln^n\left(\frac{M_W^2}{\mu^2}\right)$

leading logarithms

$\rightarrow$ going to higher orders in RGE

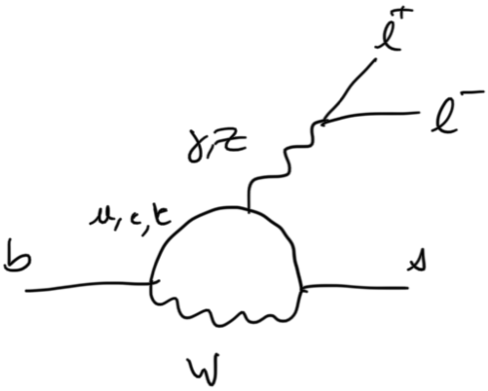
$\rightarrow$ resum next-to leading logs $\alpha_s^n(\mu) \ln^{n-1}\left(\frac{M_W^2}{\mu^2}\right)$

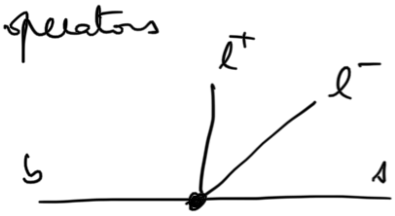
EFFECTIVE HAMILTONIAN

Matching & RGE $\rightarrow$ computing the short-distance part

Remain with matrix elements of higher dimension operators
involving only light degrees of freedom
⊕ "gluons" in the non-perturbative regime

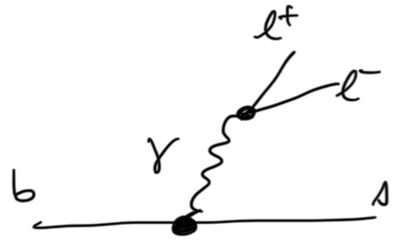
- Effective hamiltonian is built to describe the interaction for given incoming and outgoing states and sometimes more complicated than just one or two four quark operators



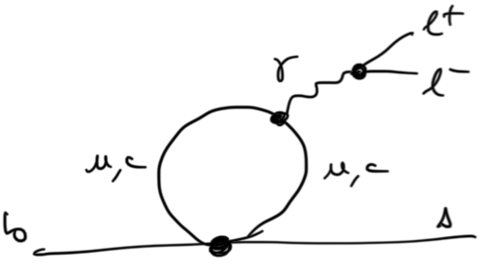


$\mathcal{O}$ $\bar{s}\gamma_\mu(1-\gamma_5)b$ $\bar{l}\gamma^\mu l$

$\bar{l}\gamma_\mu\gamma_5 l$



$\mathcal{O} = \bar{s}\sigma_{\mu\nu}(1+\gamma_5)b F^{\mu\nu}$



$\mathcal{O}$.

HADRONIC MATRIX ELEMENTS

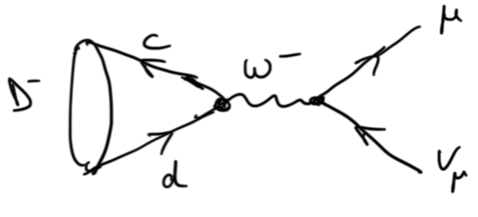
- $\langle f | \mathcal{O} | B \rangle$ at least to consider (see before the case where you have also to take into account contributions from the propagation of light degrees of freedom)

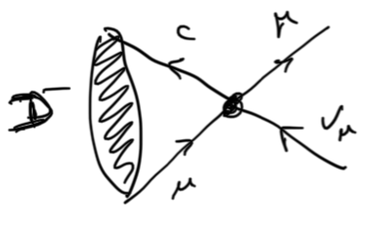
→ $\langle f | \mathcal{O} | B \rangle$ is easier to tackle if there are not too many hadrons involved

→ precision physics

LEPTONIC DECAY OF A HADRON

$D^- \to \mu^- \bar{\nu}_\mu$





strong interaction

free (4. we

part neglect em interaction)

$\langle \mu \nu | \mathcal{H} | D^- \rangle$

$$\propto G_F V_{cd} \underbrace{\bar{u}_{(\mu)} \gamma_\rho (1-\gamma_5) v_{(\nu)}}_{\text{free dirac solutions}} \underbrace{\langle 0 | \bar{c} \gamma_\rho (1-\gamma_5) d | D^- \rangle}_{\text{hadronic matrix element}}$$

$$\langle 0 | \bar{c} \gamma_\rho (\cancel{1}-\gamma_5) d | D^- \rangle = -i f_D (P_D)_\rho$$

Parity invariance of strong interaction

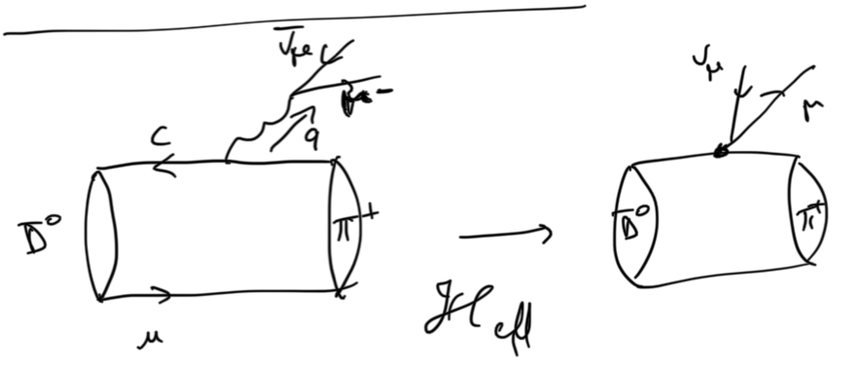
decay constant of the D-meson

$$Br(D^- \to \mu^- \bar{\nu}_\mu) = \frac{G_F^2 m_D^2 m_\mu^2}{8\pi} \left(1 - \frac{m_\mu^2}{m_D^2}\right)^2 |V_{cd}|^2 f_D^2 \tau_D$$

helicity suppression — decay constant

[up to em corrections]

SEMILEPTONIC DECAYS



... in terms

The decay amplitude can be expressed in terms of a single unknown quantity

$$\langle \pi^+ | \bar{c} \gamma_\mu (1-\gamma_5) d | \bar{D}^0 \rangle = \langle \pi^+ | \bar{c} \gamma_\mu d | \bar{D}^0 \rangle$$

P invariance of strong interaction $\qquad q = p_D - p_\pi$

$$= f_+(q^2)(p_D + p_\pi)_\mu + (f_0 - f_+)(q^2)\frac{m_D^2 - m_\pi^2}{q^2}(p_D - p_\pi)_\mu$$

f_+, f_0 are form factors functions of q^2

→ this decomposition in form factors depends on the quantum numbers of the current and the external hadrons

$$\frac{d\Gamma(\bar{D}^0 \to \pi^+ \mu^- \bar{\nu})}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{24\pi^3} \times \frac{(q^2 - m_\mu^2)}{q^4 m_D^2} \sqrt{E_\pi^2 - m_\pi^2}$$
$$\times \left[\left(1 + \frac{m_\mu^2}{2q^2}\right) m_D^2 (E_\pi^2 - m_\pi^2) |f_+(q^2)|^2 + \frac{3 m_\mu^2}{8 q^2} (m_D^2 - m_\pi^2)^2 |f_0(q^2)|^2 \right]$$

WHAT DO WE KNOW ABOUT THE FORM FACTORS ?

- Some aspects of the analytic structure of the form factors that can be obtained on general grounds

- Unitarity of the S matrix

$$S_{\beta\alpha} = \langle \beta_{out} | \alpha_{in} \rangle \qquad S = \mathbb{1} + iT$$

transition matrix

$$\langle \beta | iT | \alpha \rangle = (2\pi)^4 \, \delta\left(\sum p_\alpha - \sum p_\beta\right) \cdot i\, A(\alpha \to \beta)$$

$$S^\dagger S = \mathbb{1} \quad \Rightarrow \quad T - T^\dagger = i T^\dagger T$$

$$\Rightarrow -i\left[A(\alpha\to\beta) - [A(\alpha\to\beta)]^*\right]$$

$$= \sum_f A^*(\beta \to f)\, A(\alpha \to f)$$

Form factors acquire an imaginary part when you can insert (real) intermediate states inbetween

↓

dependent on q^2

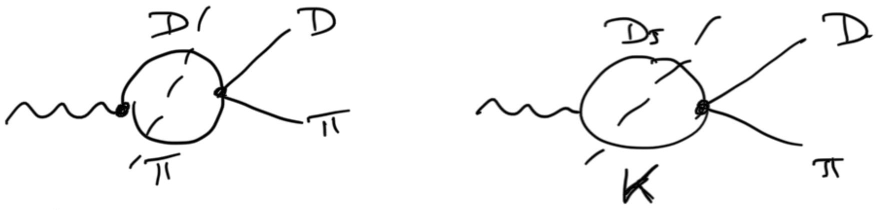
- $D \to \pi l \nu$ form factors

$t = q^2$ between m_l^2 and $t_- = (m_D - m_\pi)^2$

of real only

$t \geq (m_D + m_\pi)^2 = t_+$

" of imaginary part



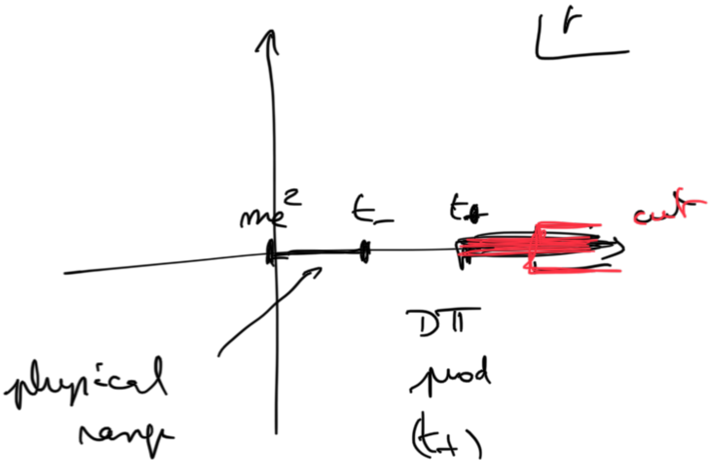
$q^2 \nearrow$: 1^{st} cut then 2^{nd} cut

- Form factors with a simple analytic structure

  * poles for narrow resonances

  * cuts along the real axis when new channels open

  * analytic everywhere else $t = q^2$



- Extrapolation not necessarily easy with this analytic structure

  → propose an alternative mapping

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_- - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_- - t_0}}$$

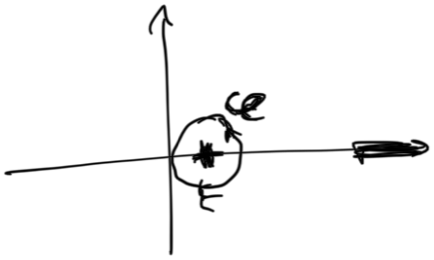
t_0 is arbitrary

the plane in t $\longrightarrow$ disc of radius 1

the cut (right hand) $\longrightarrow$ circumference $|z| = 1$

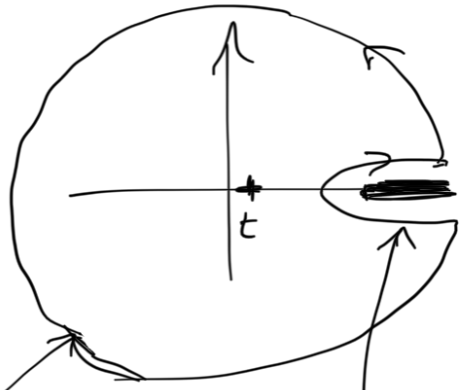
series in z with better convergence properties than in t

* dispersion relations connection between values of f in very different energy regions



[Cauchy theorem]

$$f(t) = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{f(z)}{z-t}\, dz$$



f physical region

$\longleftrightarrow$ Im f (potential intermediate states)

f at very large $|t|$ $\quad \propto \quad$ $\mathrm{Im}\, f(t)$

$+$ f at large $|t|$

- Strong phases from two body interaction / rescattering

  $\rightarrow$ even under CP

  Weak phases from electroweak part of the SM

→ odd under CP

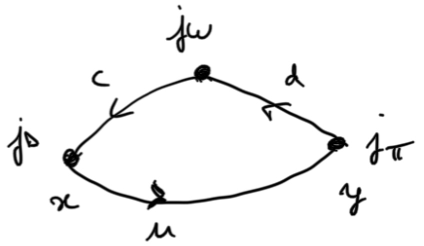
MORE HADRONS? No if you want very accurate info

* decay constant $1\,H \to 0\,H$
* form factors $1\,H \to 1\,H$
* bag parameters (more soon ...) $1\,H \to 1\,H$

HANDLES ON FORM FACTORS & DECAY CONSTANTS

discretise QCD 4D grid on a box

• lattice QCD



at large separation

you select D-meson

π contributions

Euclidean metric (no difference r, x, y, z)

→ no easy access to rescattering

→ large q^2 taken away by the lepton pair

final meson is almost at rest

(no recoil)

• light-cone sum rules

→ low q^2, π is emitted with large energy

(large recoil)

$\longrightarrow$ it flies "almost" along a light-like direction

almost collinear quarks

$\rightsquigarrow$ repartition of these quarks into meson

(light cone distribution amplitude)

- Effective theories (Heavy Quark Effective Theories)

$B \to \pi$ form factors

$m_{u,d,s} < \Lambda_{QCD} < m_b$ Expansion $\frac{\Lambda_{QCD}}{m_b}$

allows one to relate various form factors

separate

$\underbrace{\text{gluons } O(\Lambda_{QCD})}_{\text{reduced hadronic matrix elements}}$ $\underbrace{\text{gluons } O(m_b)}_{\text{perturbative}}$

in terms of a subset of reduced matrix elements

... all form factors