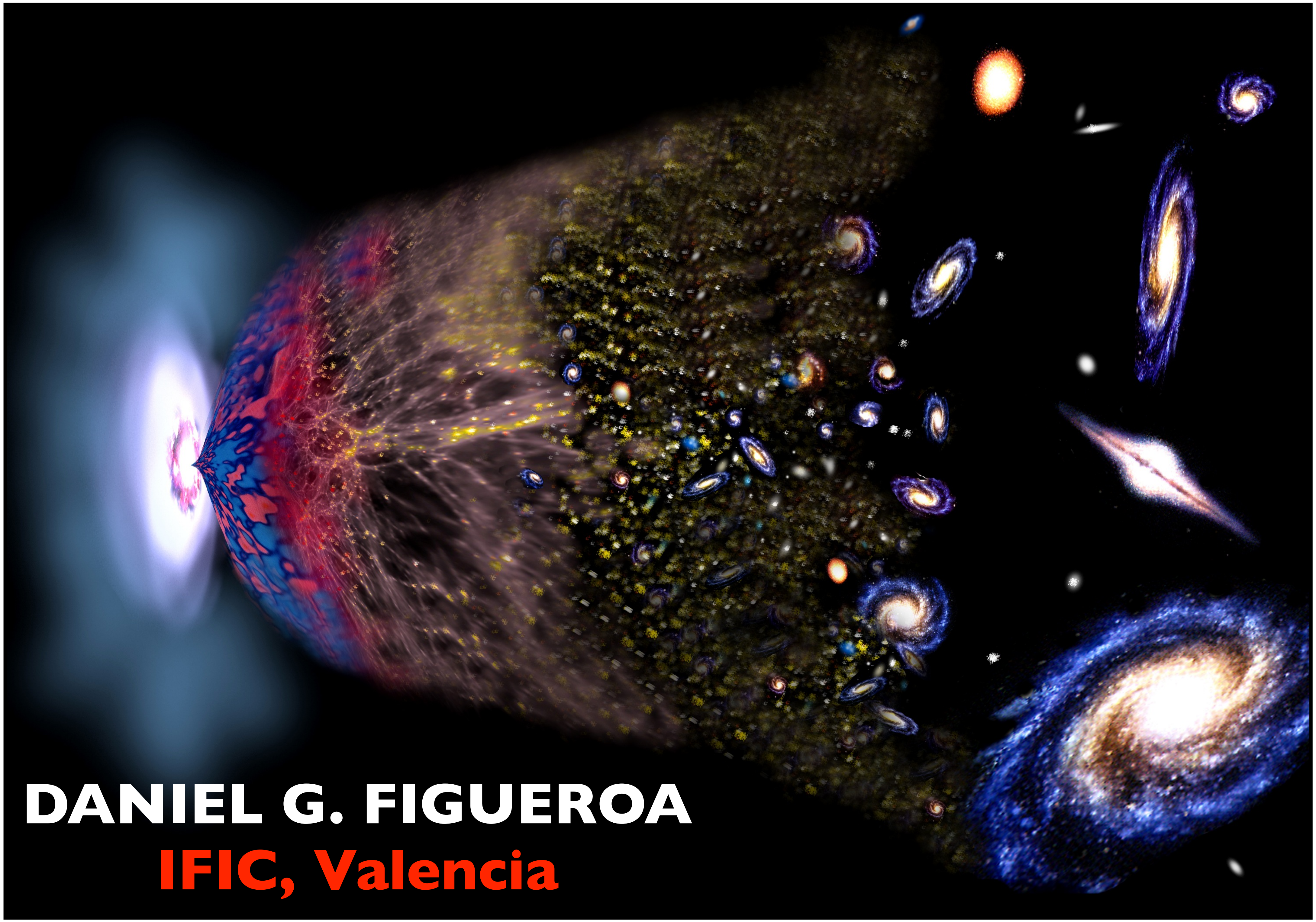
GRAVITATIONAL WAVE
— BACKGROUNDS —



DANIEL G. FIGUEROA
IFIC, Valencia

GGI LECTURES ON THE THEORY OF FUNDAMENTAL INTERACTIONS — 2022 Program (3rd week)

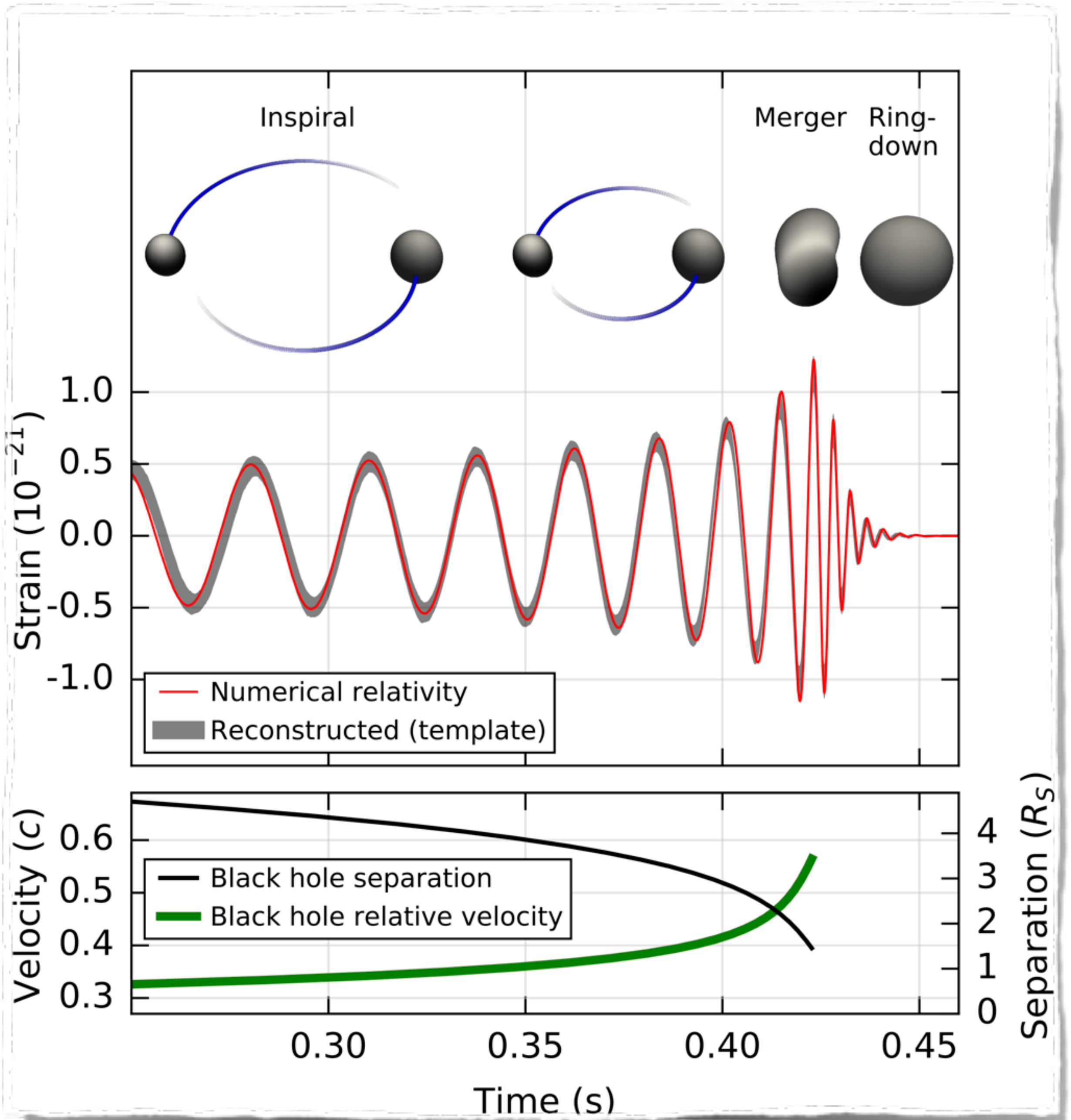
MOTIVATION

(just to warm up)

Let us celebrate …

These are special times !

Let us celebrate …

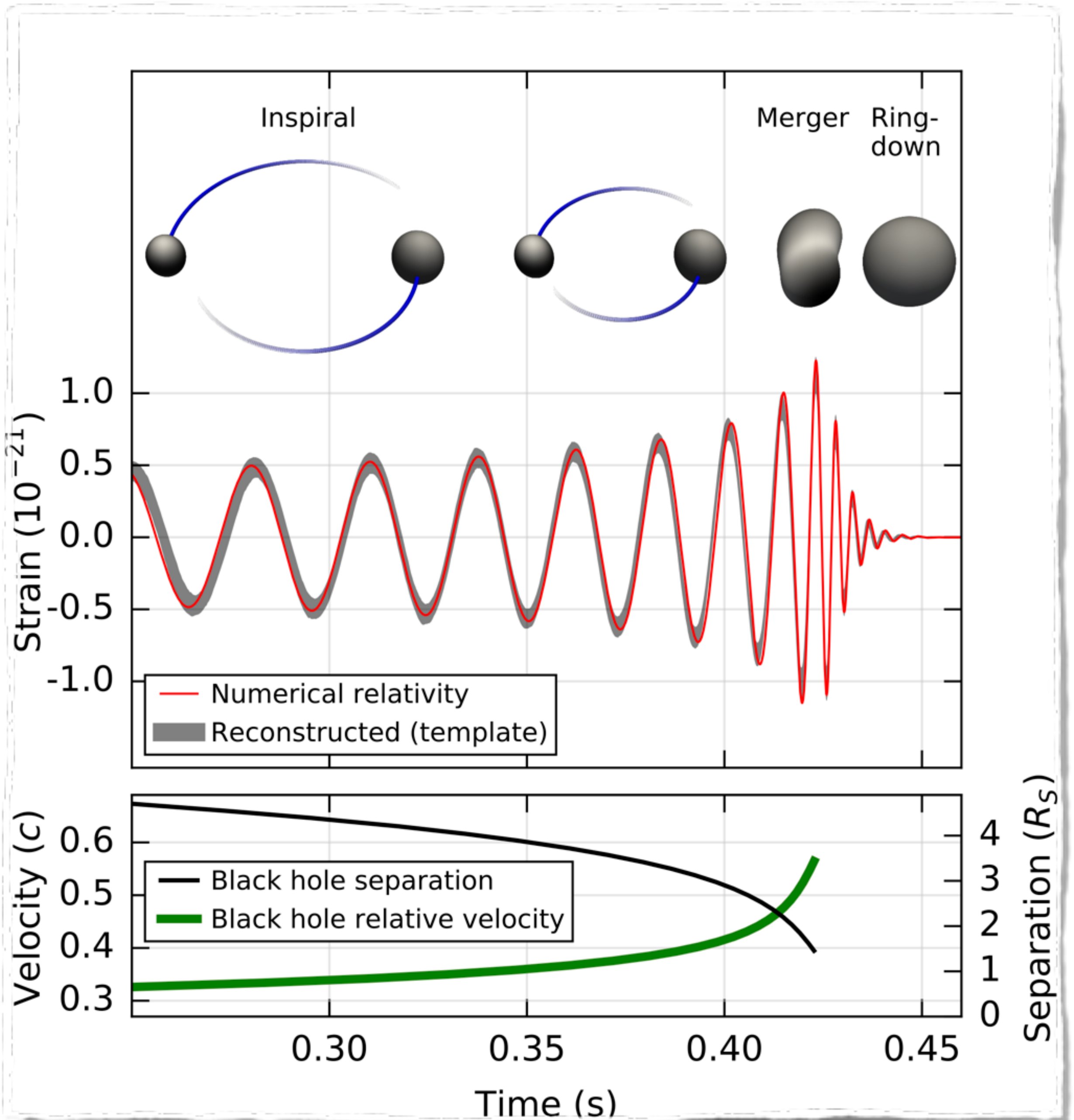


[LIGO & Virgo Scientific Collaborations (arXiv:1602.03841)]

Gravitational Waves (GW) detected !

[LIGO/VIRGO]

Let us celebrate …



[LIGO & Virgo Scientific Collaborations (arXiv:1602.03841)]

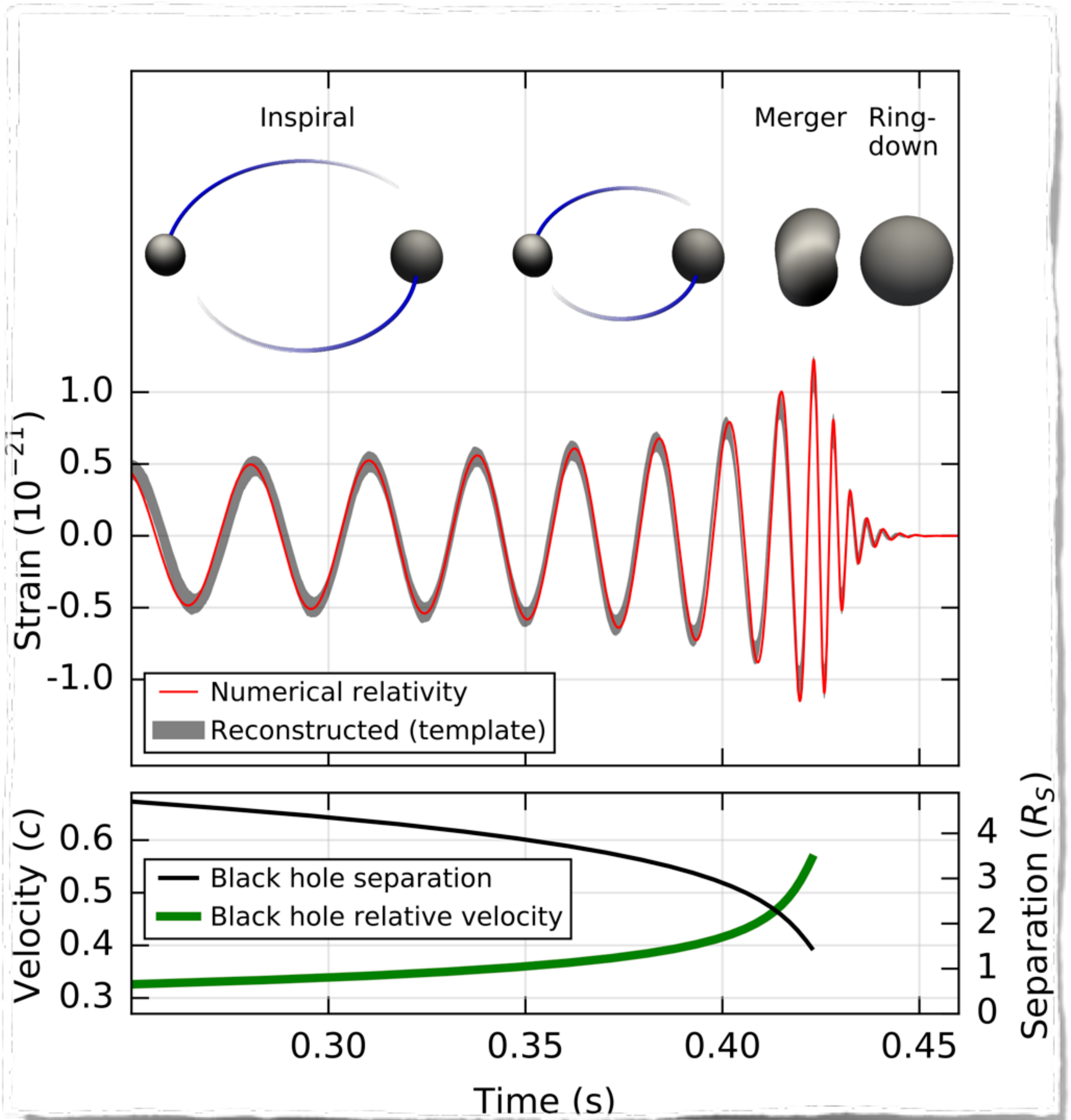
Gravit… al

)

milestone in physics

…/VIRGO]

Let us celebrate …



[LIGO & Virgo Scientific Collaborations (arXiv:1602.03841)]

Grav… …al

…)

…VIRGO]

milestone in physics

Einstein 1915 … LIGO/VIRGO 2015-2021

What have we learnt ?

milestone
in physics

What have we learnt ?

* O(10) Solar mass
Black Holes (BH) exist

Grav … al

)

[… /VIRGO]

milestone
in physics

What have we learnt ?

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm, and Neutron Star physics

milestone in physics

What have we learnt ?

*** O(10) Solar mass Black Holes (BH) exist**

*** We can test the BH's paradigm, and Neutron Star physics**

*** We can further test General Relativity (GR)**
[so far no deviation]

milestone in physics

What have we learnt ?

*** O(10) Solar mass Black Holes (BH) exist**

*** We can test the BH's paradigm, and Neutron Star physics**

*** We can further test General Relativity (GR)**
[so far no deviation]

*** We can observe the Universe through GWs**

*** ...**

Grav al)

milestone in physics

VIRGO]

How did we learn this?

*** O(10) Solar mass Black Holes (BH) exist**

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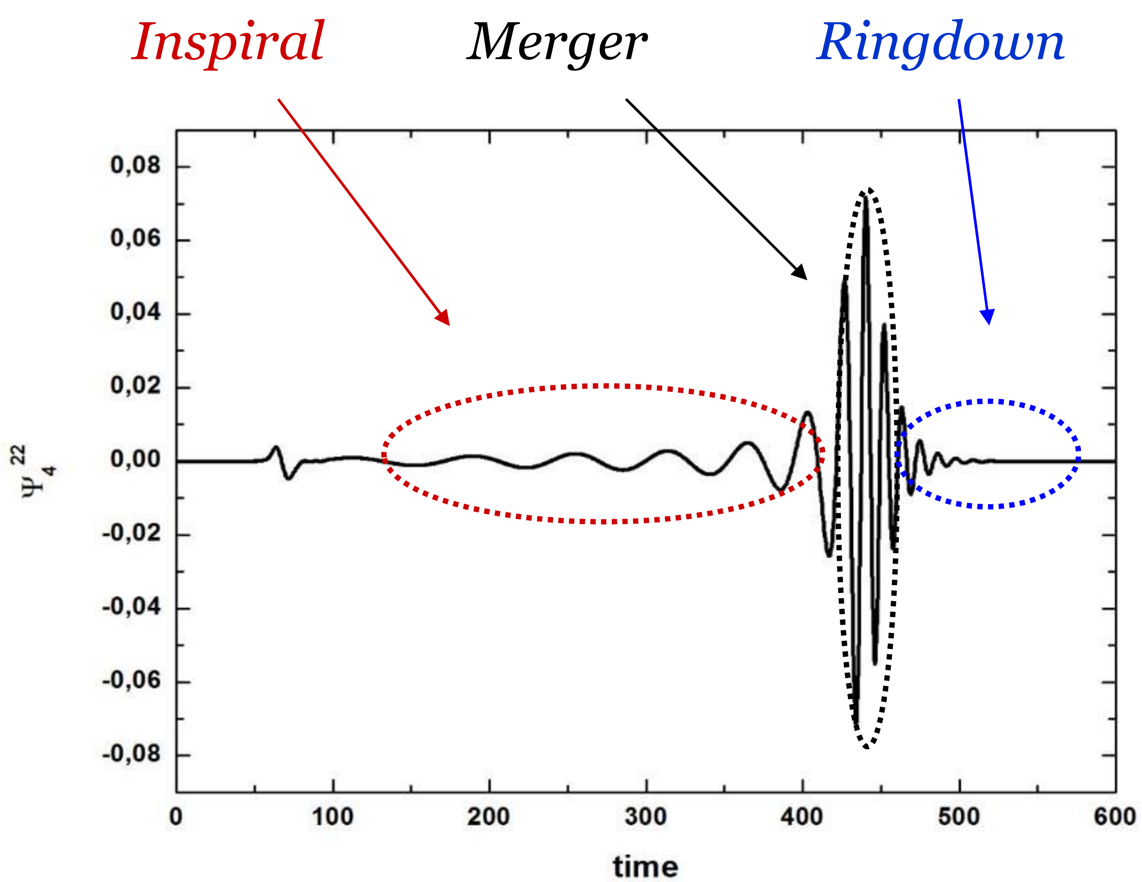
*** ...**

Binary wave functions



Let us celebrate !

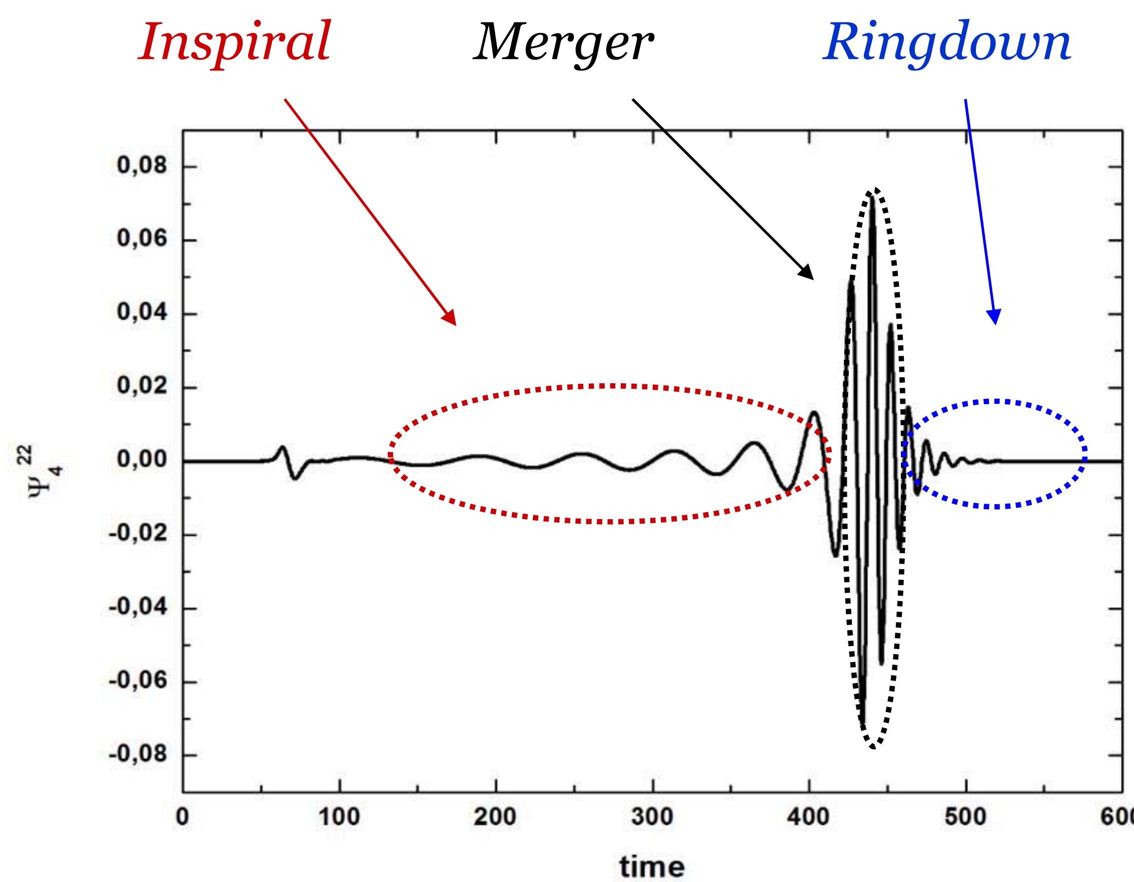
* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm, and Neutron Star physics

* We can further test General Relativity (GR)
[so far <u>no</u> deviation]

* We can observe the Universe through GWs

* ...



Let us celebrate !

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm, and Neutron Star physics

* We can further test General Relativity (GR) [so far <u>no</u> deviation]

(binaries)

Extremely interesting !

* We can observe the Universe through GWs

* ...

Let us celebrate !

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm, and Neutron Star physics

* We can further test General Relativity (GR)
[so far <u>no</u> deviation]

* We can observe the Universe through GWs

* ...

(binaries)

Extremely interesting !

BUT ...

Let us celebrate !

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm, and Neutron Star physics

* We can further test General Relativity (GR)
[so far no deviation]

(binaries)

* We can observe the Universe through GWs

* ...

Extremely interesting !

BUT ...

... We will focus on something else !

Let us celebrate !

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm, and Neutron Star physics

* We can further test General Relativity (GR)
[so far no deviation]

(binaries)

* We can observe the Universe through GWs

* ...

Extremely interesting !

BUT ...

... We will focus on something else !

Let us celebrate !

* O(10) Solar m...s
Black Ho... ...st

N... (binaries)

* W... ...test
Gen... ...elativity (GR)
[so far no deviation]

* We can observe the Universe through GWs

* ...

Stay tuned !

more fun guaranteed to come ...

Extremely interesting !

BUT ...

... We will focus on something else !

* We can observe the Universe through GWs

* We can observe the Universe through GWs

* We can observe the Universe through GWs

* We can observe the Universe through GWs

* Cosmology with GWs

* We can observe the Universe through GWs

* Cosmology with GWs

* Late Universe:

* Early Universe:

*** We can observe the Universe through GWs**

* Cosmology with GWs

*** Late Universe:** Standard sirens: distances in cosmology;
Measuring H0 and EoS dark energy;
cosmological parameters;
modify gravity, lensing, …

*** We can observe the Universe through GWs**

* Cosmology with GWs

*** Late Universe:**

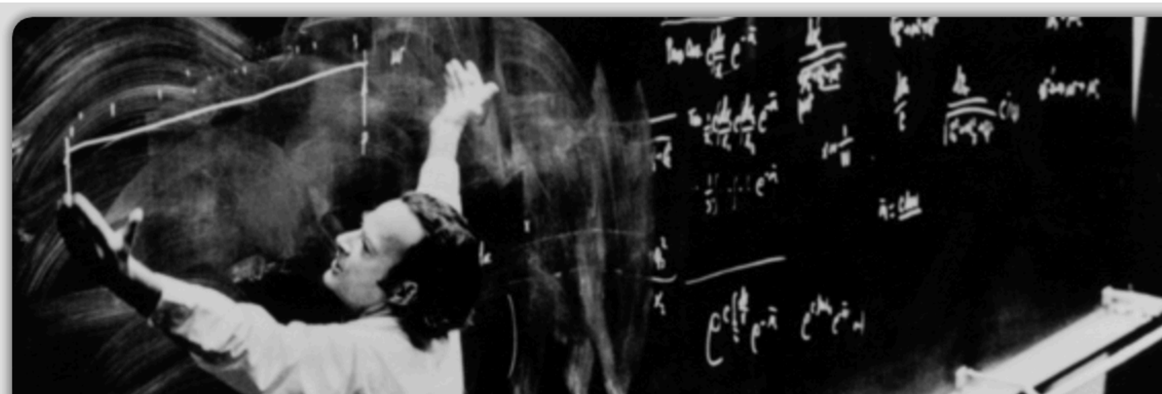
*** Early Universe:** High Energy Particle Physics

* We can observe the Universe through GWs

* Cosmology with GWs

* **Late Universe:**

* **Early Universe:** High Energy Particle Physics



GGI LECTURES ON THE THEORY OF FUNDAMENTAL INTERACTIONS

Galileo Galilei Institute for Theoretical Physics

Firenze

10-28 JANUARY 2022

* We can observe the Universe through GWs

* Cosmology with GWs

* **Late Universe:**

* **Early Universe:** High Energy Particle Physics

* We can observe the Universe through GWs

* Cosmology with GWs

* **Late Universe:** Are we going to forget about this ?

* **Early Universe:** High Energy Particle Physics

* **We can observe the Universe through GWs**

* Cosmology with GWs

* **Late Universe:** Nope, we simply postpone …

* **Early Universe:** High Energy Particle Physics

* We can observe the Universe through GWs

* Cosmology with GWs

* **Early Universe:** High Energy Particle Physics

*** We can observe the Universe through GWs**

*** Cosmology with GWs**

* **Early Universe:** High Energy Particle Physics

Can we really probe High Energy Physics using Gravitational Waves (GWs) ? How ?

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Answering these questions lies at the heart of what these lectures are about !

Can we really probe High Energy Physics using Gravitational Waves (GWs) ? How ?

Before answering …

… let us ask another question

GWs: probe of the early Universe

WHY ??

GWs: probe of the early Universe

WHY ??

ONE and ONLY ONE reason

GWs: probe of the early Universe

1. **WEAKNESS of GRAVITY**:

   ADVANTAGE: GW DECOUPLE upon Production
   DISADVANTAGE: DIFFICULT DETECTION

GWs: probe of the early Universe

1. **WEAKNESS of GRAVITY**:

   ADVANTAGE: GW DECOUPLE upon Production
   DISADVANTAGE: DIFFICULT DETECTION

2. **ADVANTAGE**: GW $\rightarrow$ Probe for Early Universe

$$\rightarrow \left\{ \begin{array}{l} \textbf{Decouple} \rightarrow \text{Spectral Form Retained} \\ \textbf{Specific HEP} \;\Leftrightarrow\; \text{Specific GW} \end{array} \right.$$

GWs: probe of the early Universe

1. **WEAKNESS of GRAVITY**:

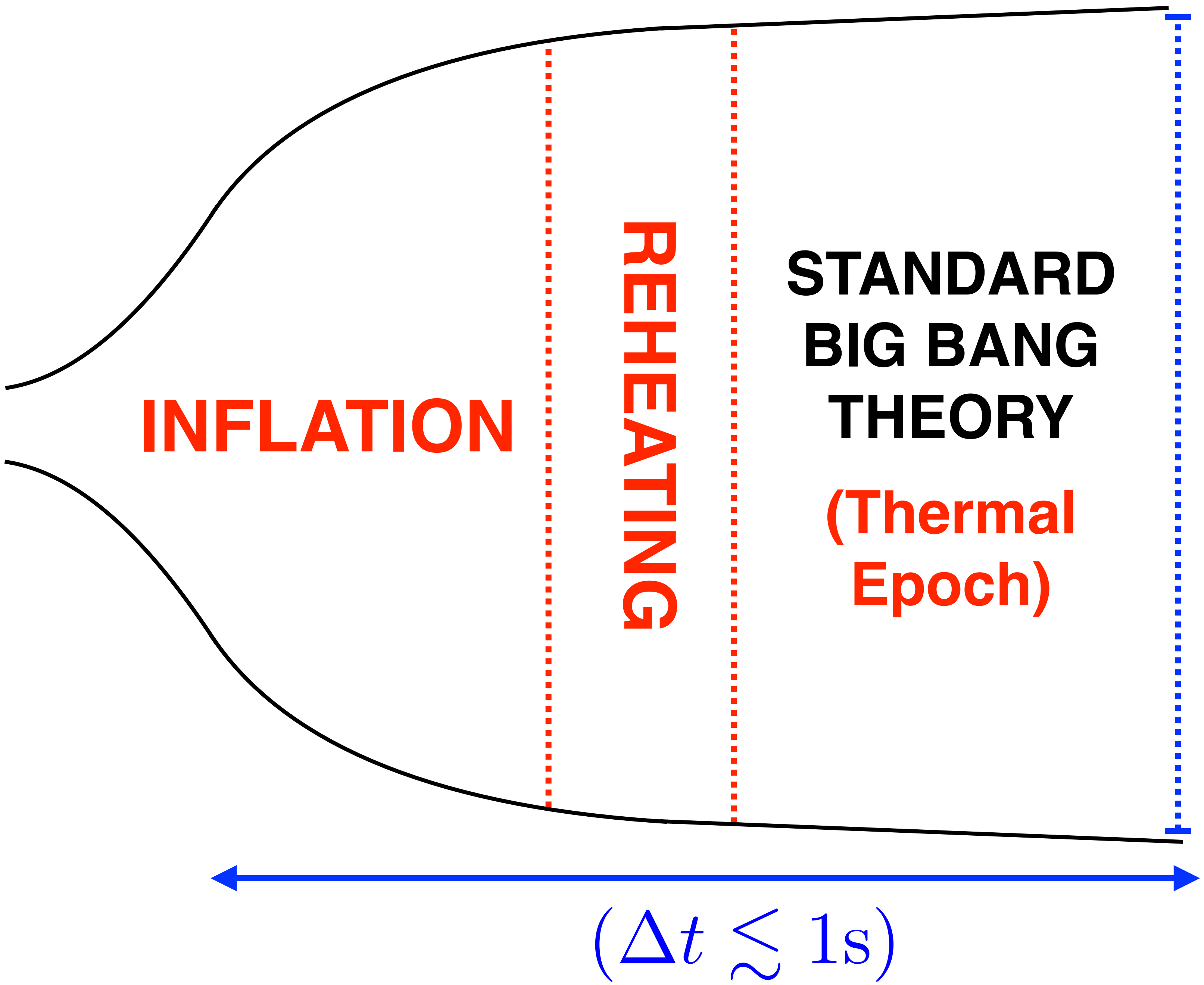
   ADVANTAGE: GW DECOUPLE upon Production
   DISADVANTAGE: DIFFICULT DETECTION

2. **ADVANTAGE**: GW $\longrightarrow$ Probe for $\boxed{\text{Early Universe}}$

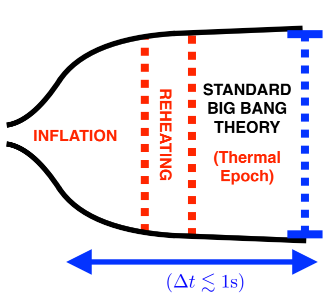
$$\longrightarrow \left\{ \begin{array}{l} \textbf{Decouple} \longrightarrow \text{Spectral Form Retained} \\ \textbf{Specific HEP} \;\Leftrightarrow\; \text{Specific GW} \end{array} \right.$$

What processes of the early Universe ?

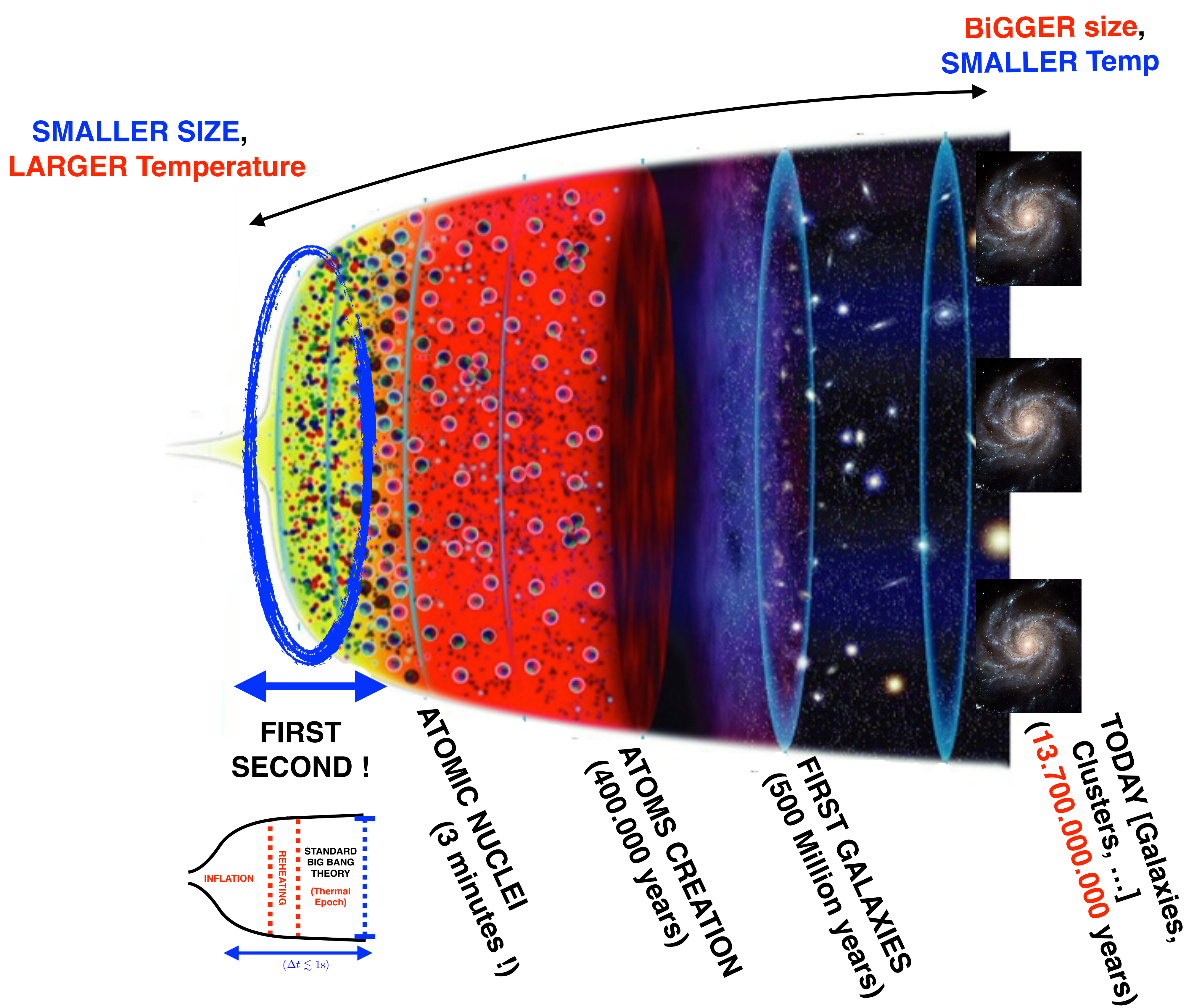
The Early Universe



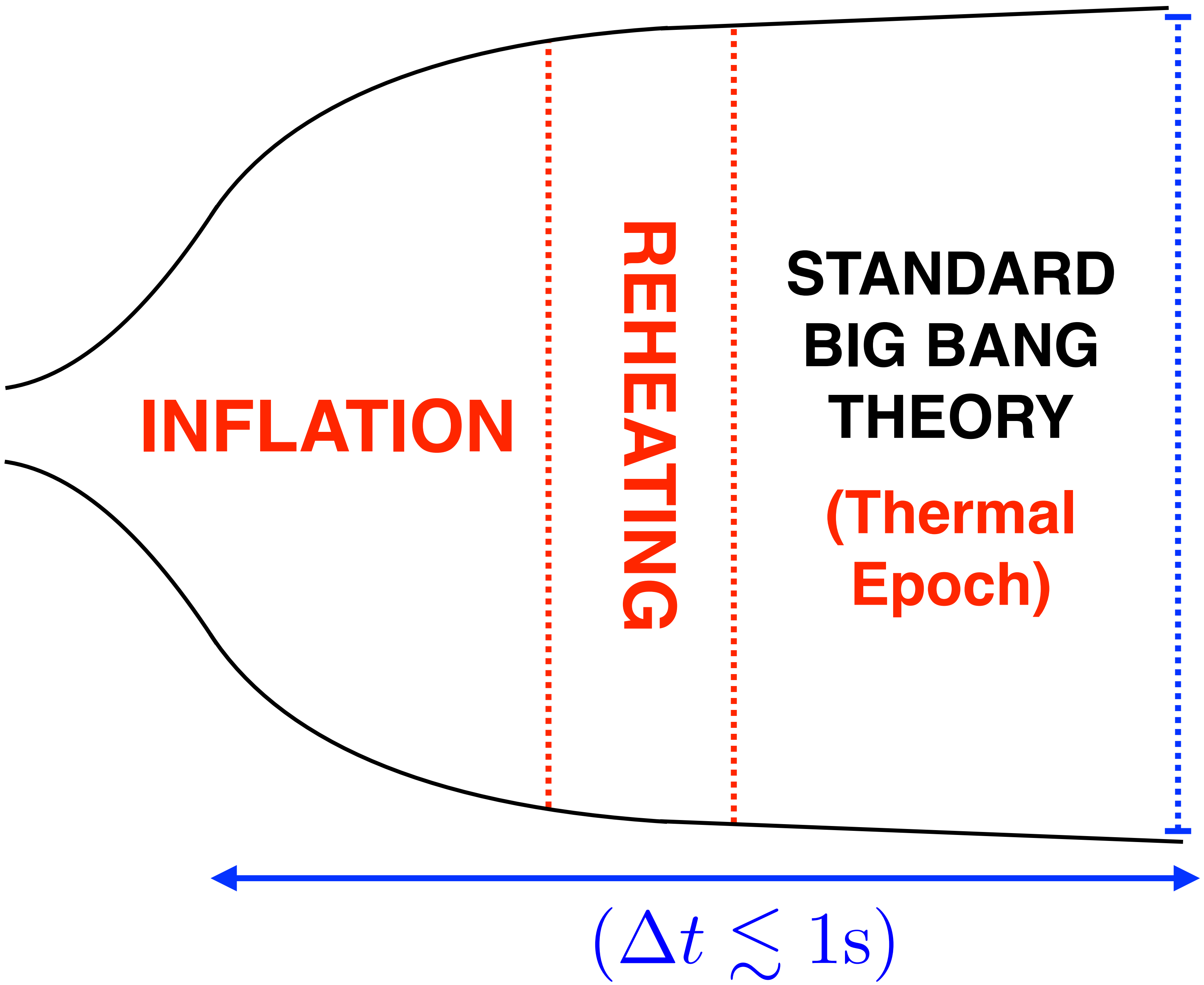
The Early Universe



The Early Universe

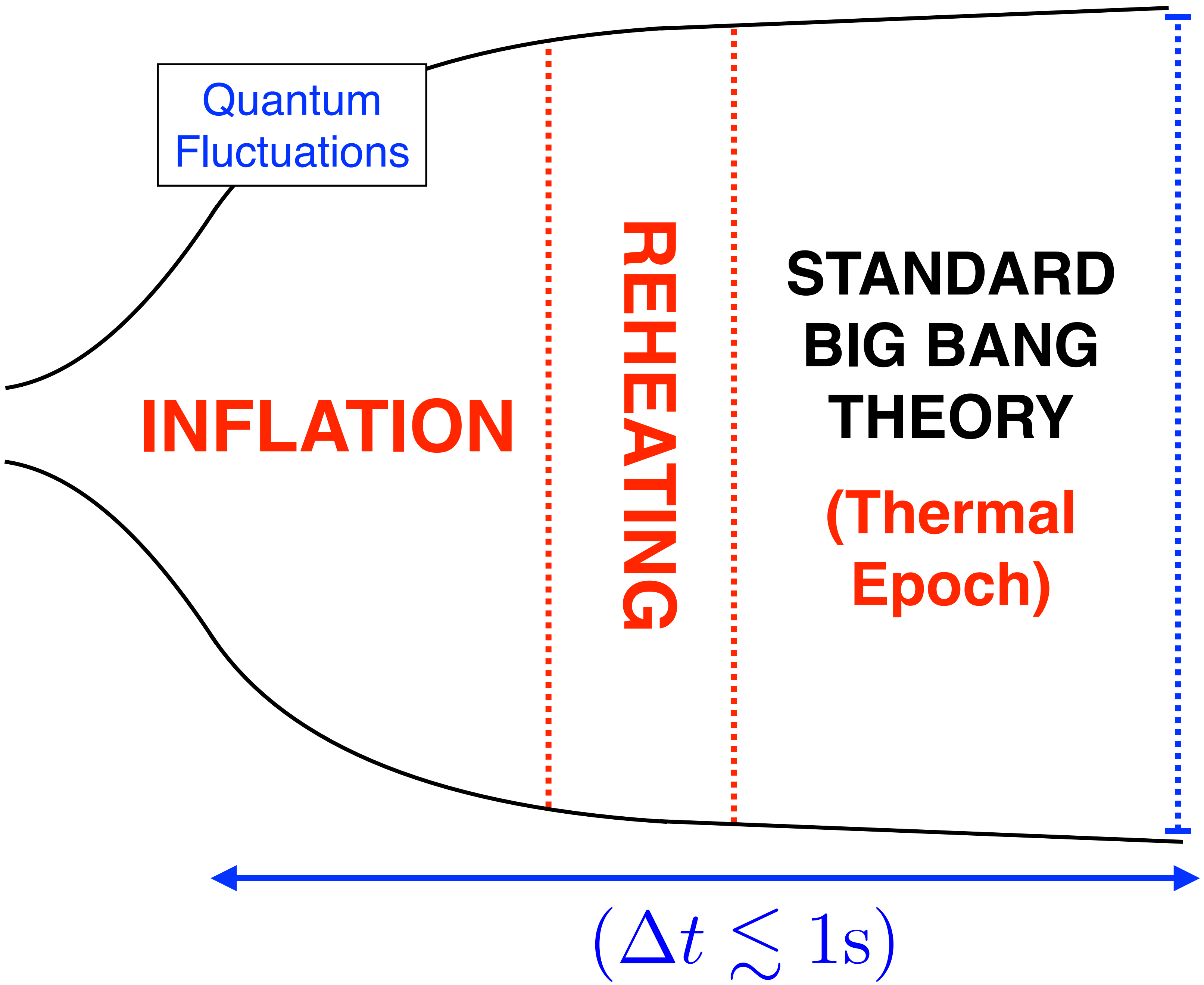


The Early Universe

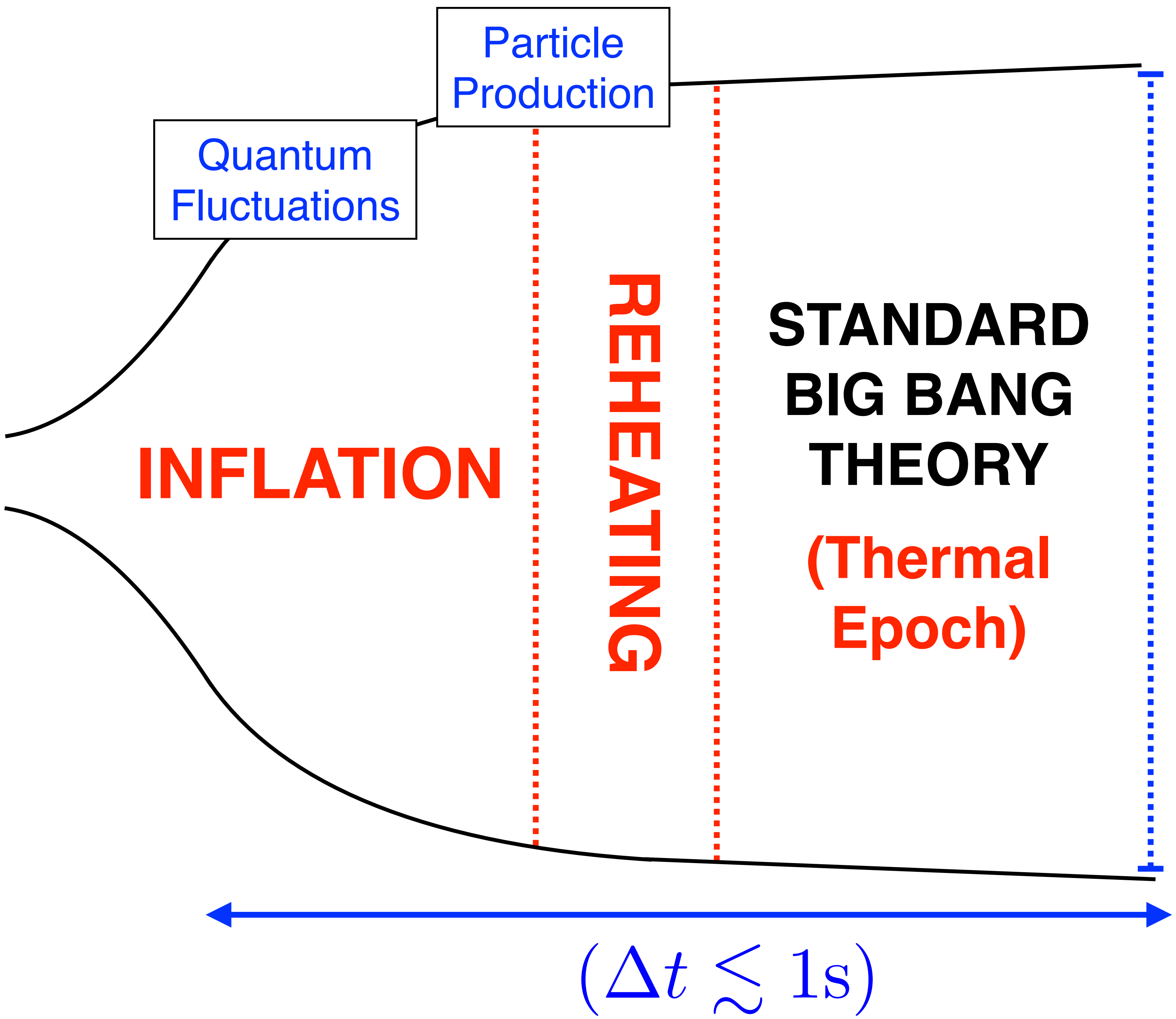


What phenomena are we interested in ?

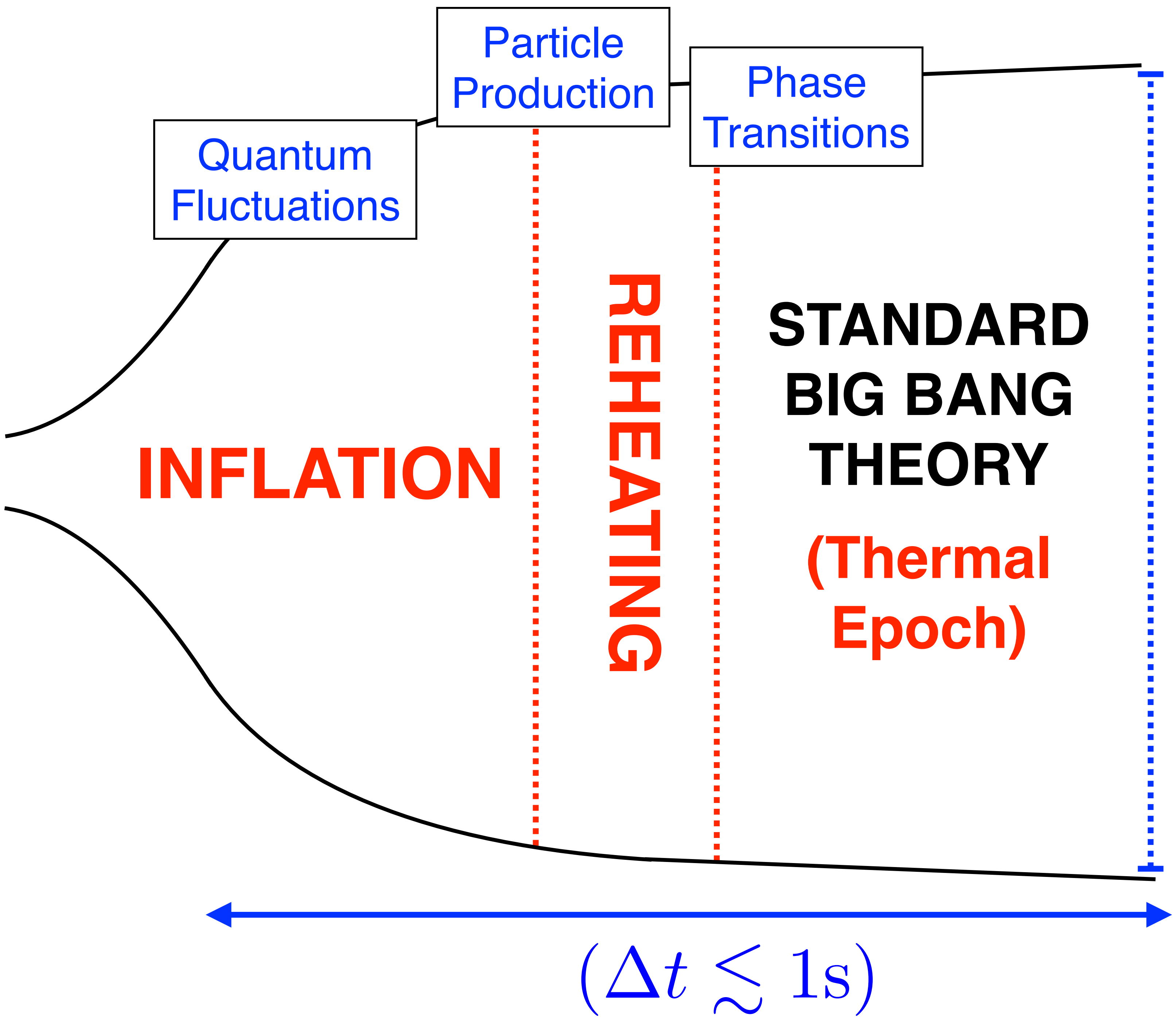
The Early Universe



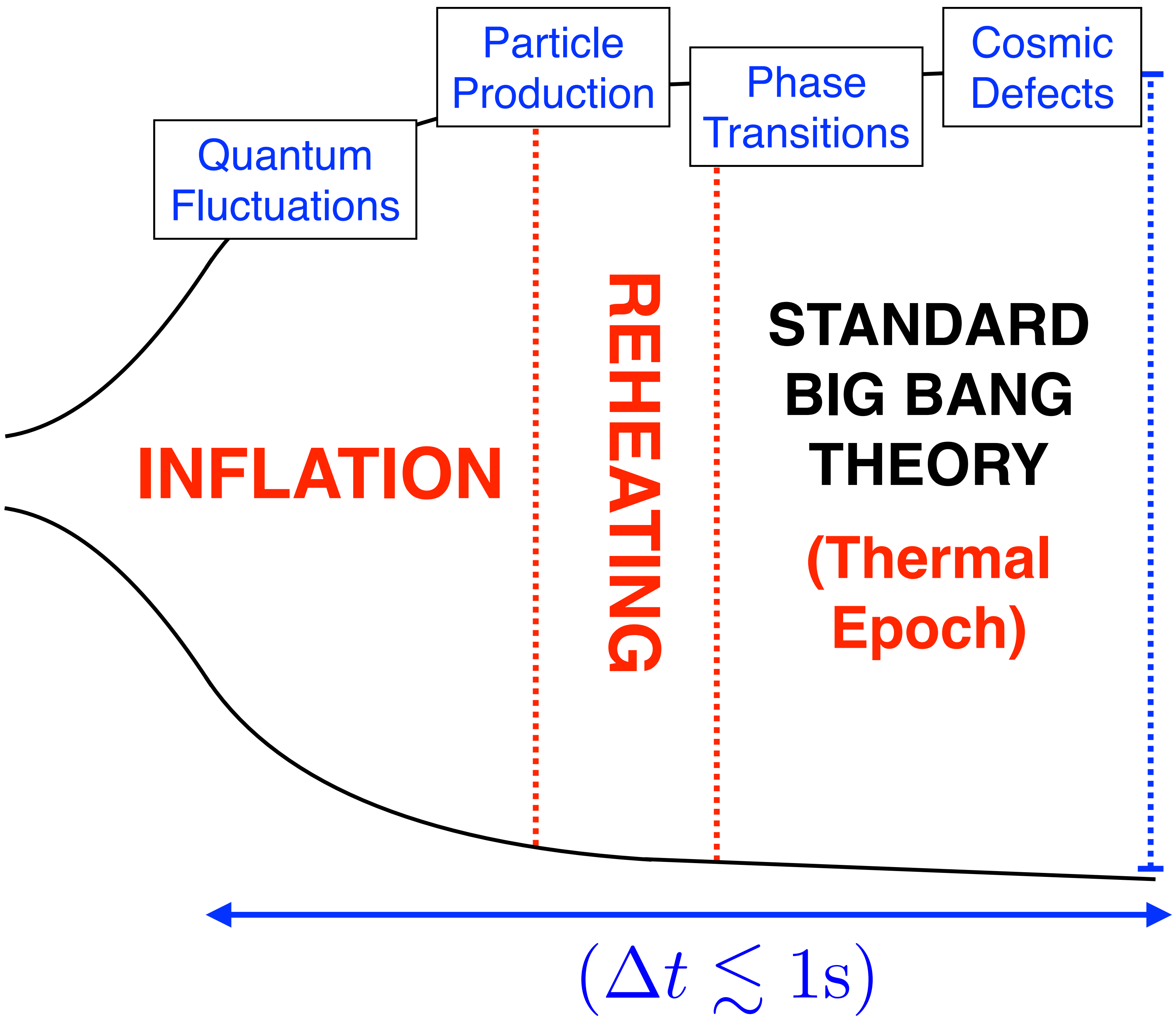
The Early Universe



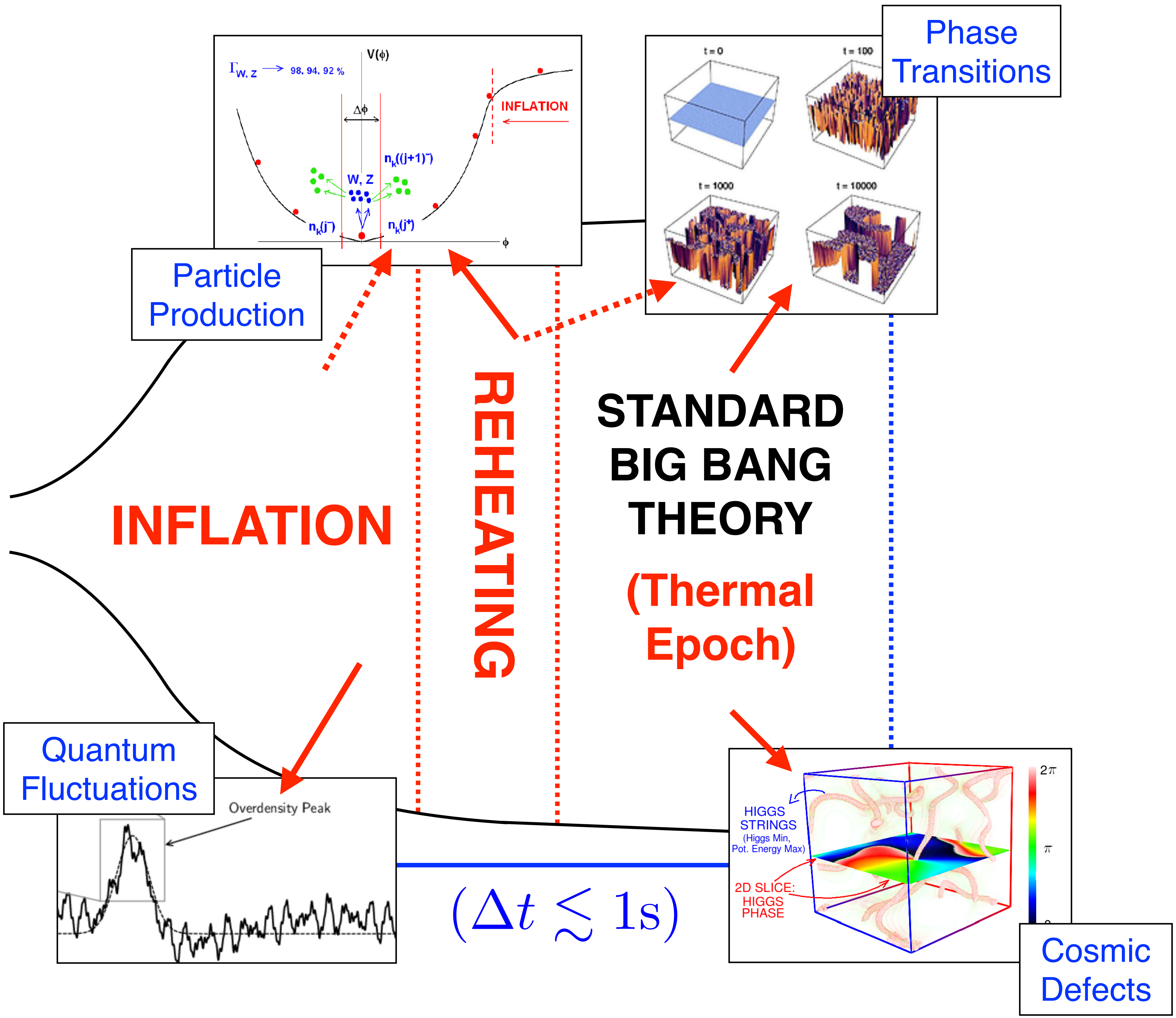
The Early Universe



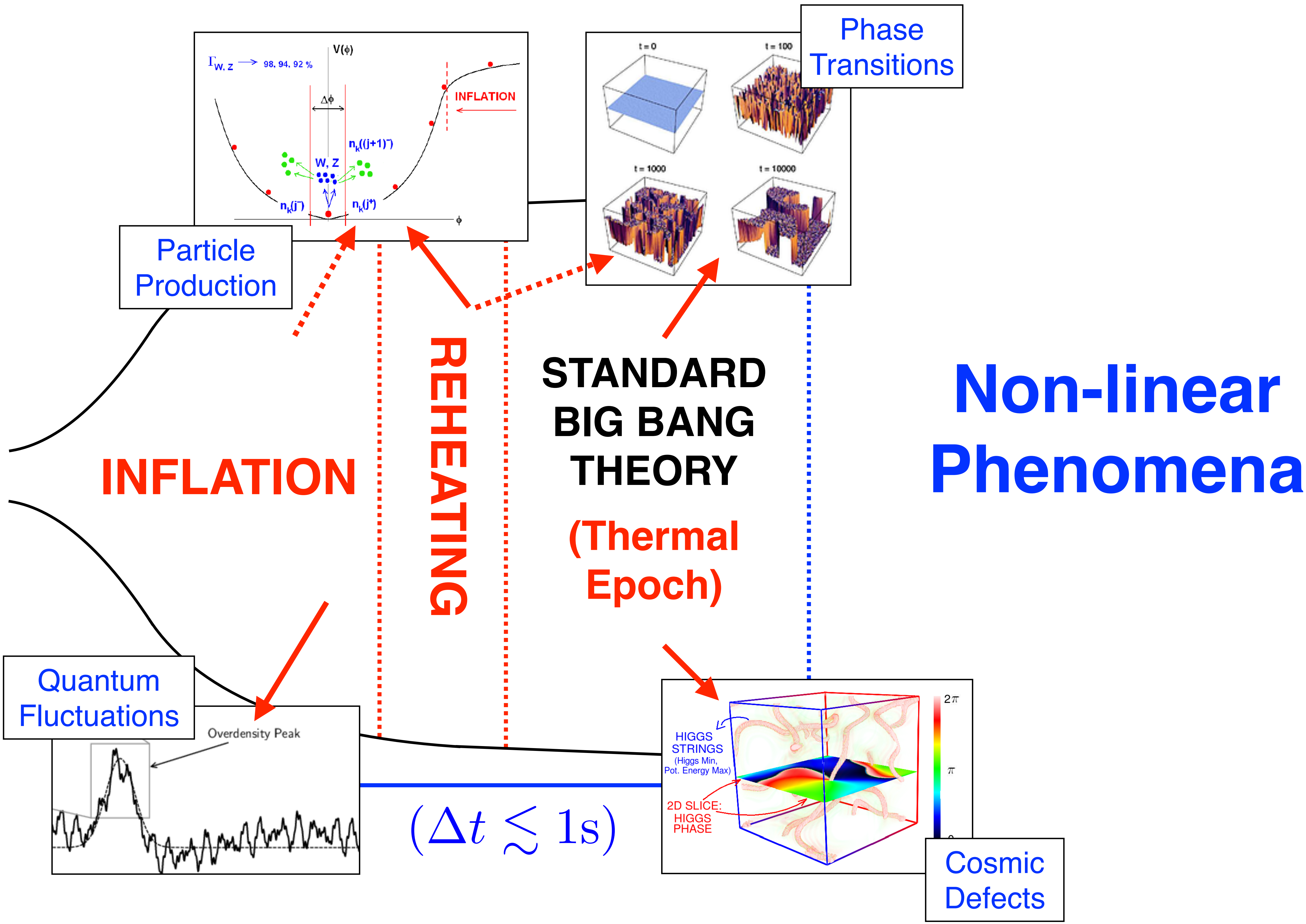
The Early Universe



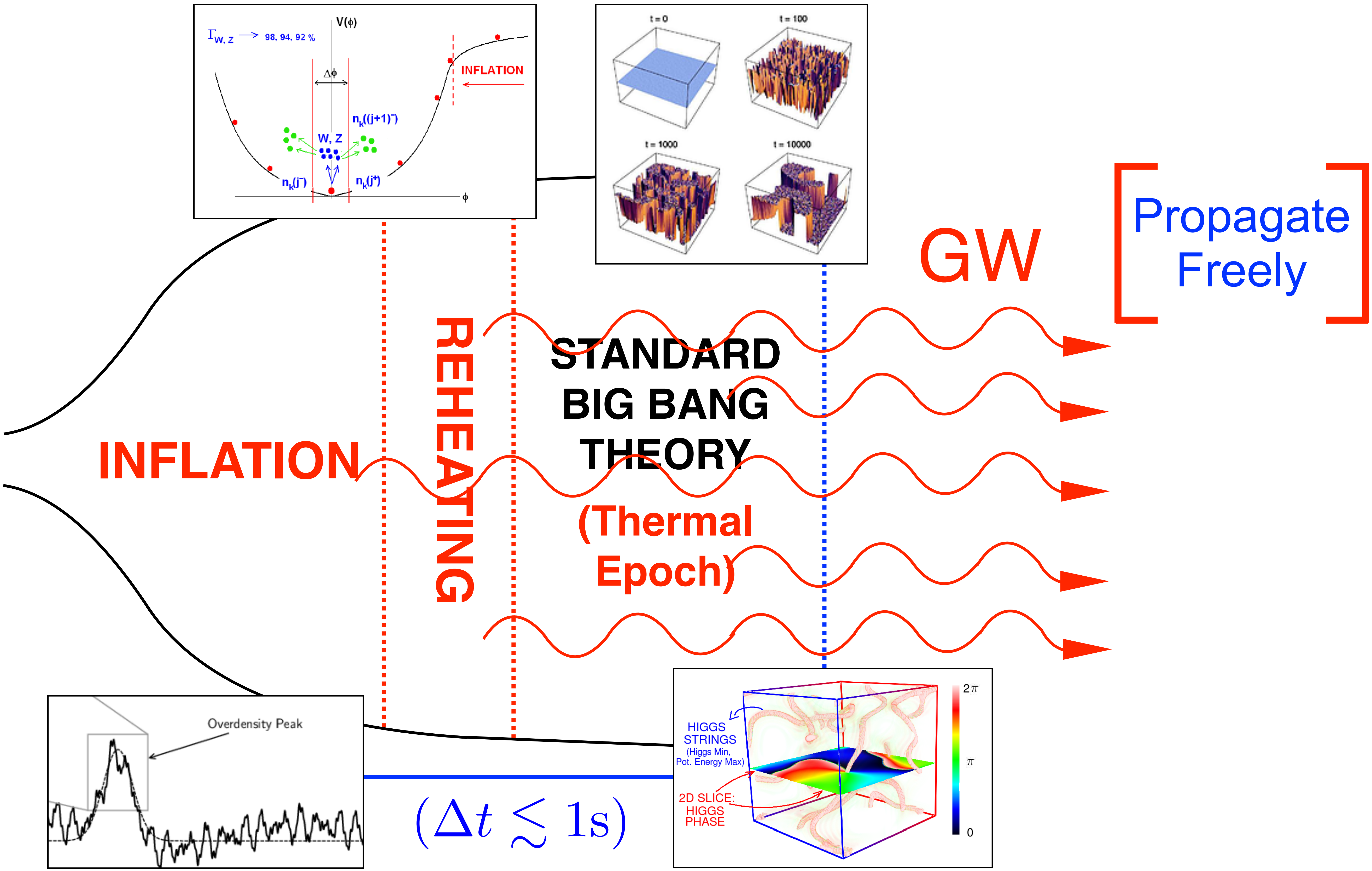
The Early Universe



The Early Universe



The Early Universe



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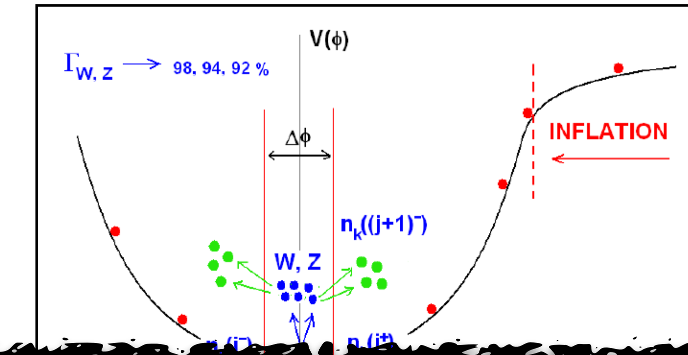
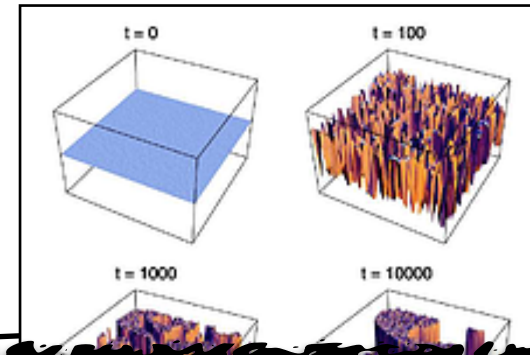
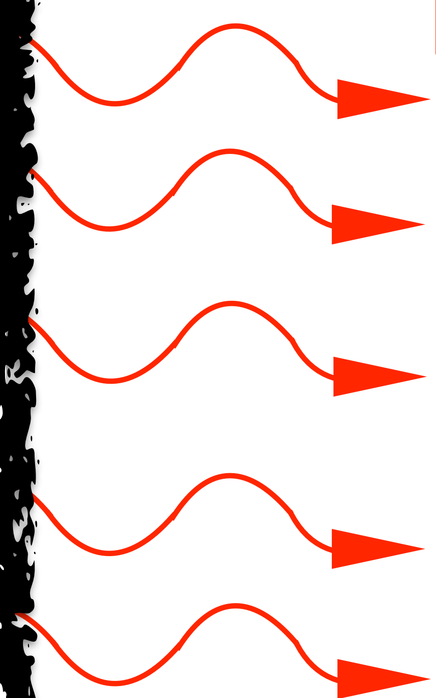


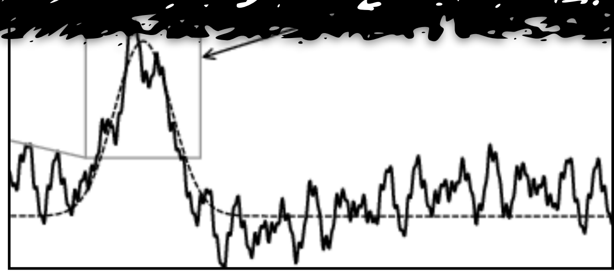


Grav. Wave
Cosmological
Backgrounds

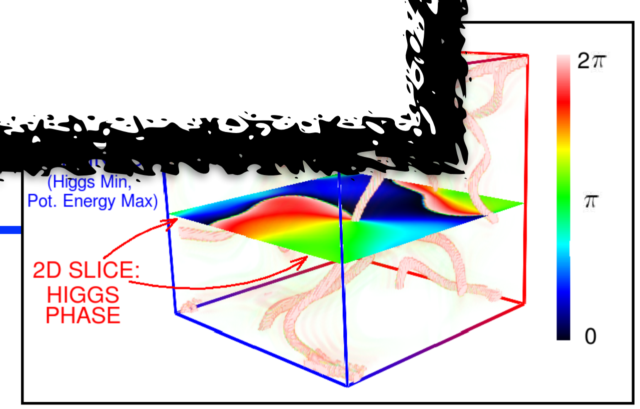
GW



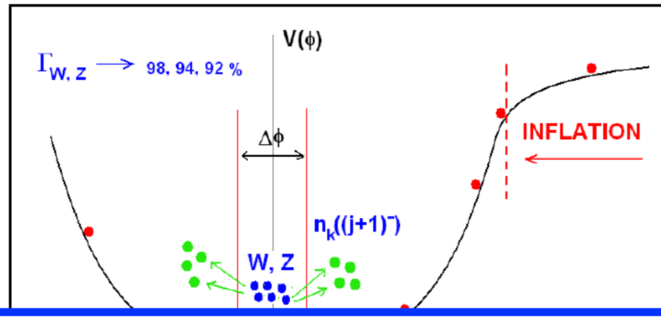
[Propagate Freely]

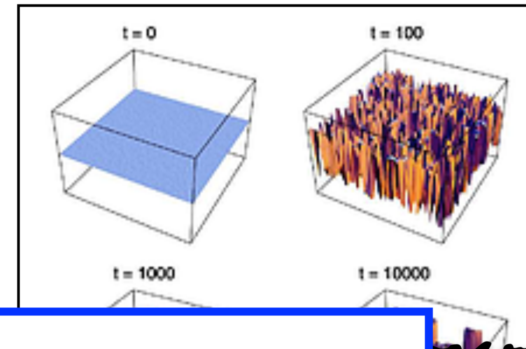


$(\Delta t \lesssim 1\mathrm{s})$



The Early Universe



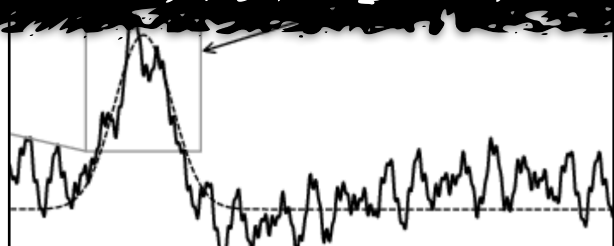


Unique cosmic messenger

Grav. Wave Cosmological Backgrounds

GW

[Propagate Freely]



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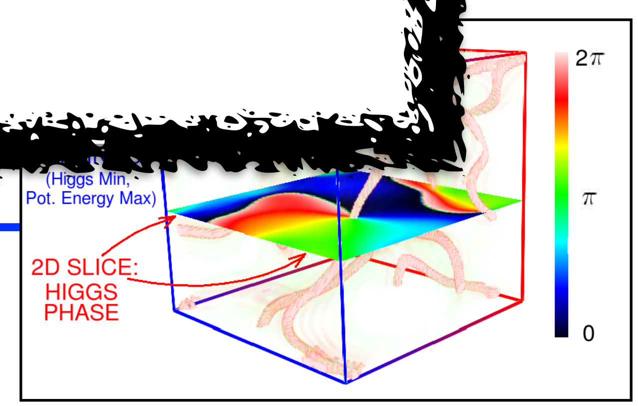
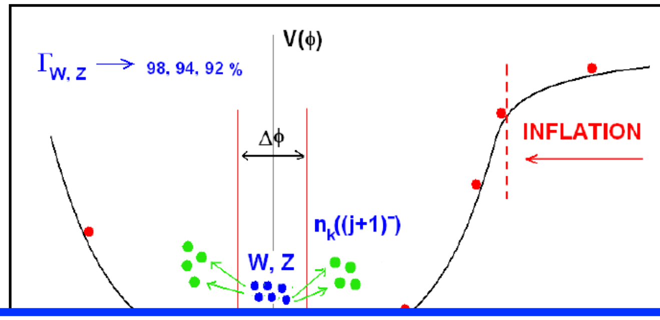
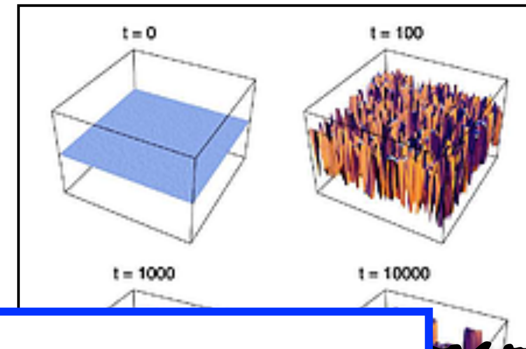


The Early Universe



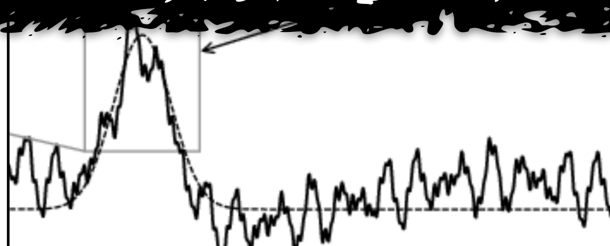


Unique cosmic messenger

Grav. Wave Cosmological Backgrounds

GW

[Probe of the early Universe]



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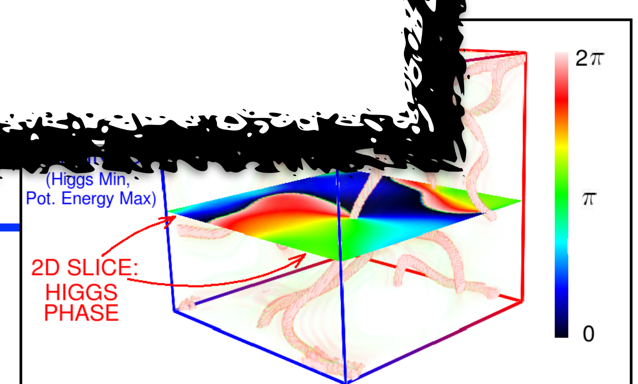
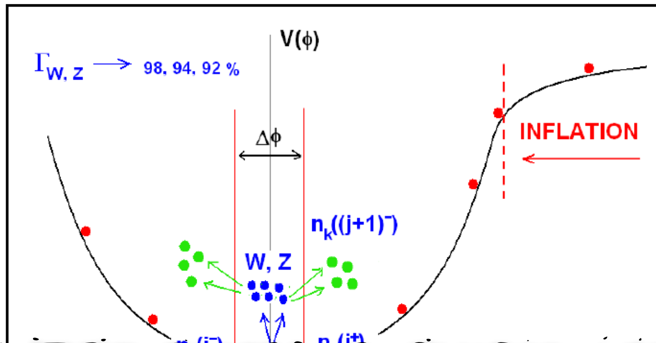
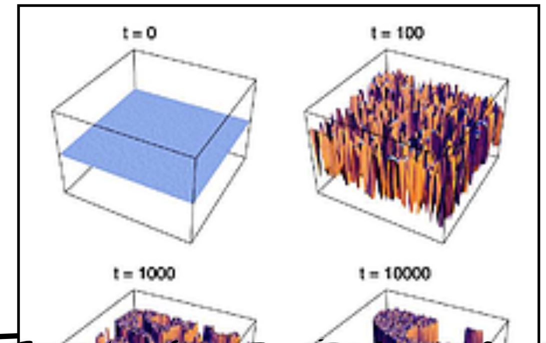


The Early Universe





* **Energies above terrestrial means**

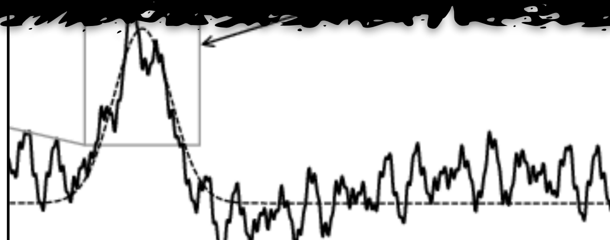
* **Fundamental Physics**

* **Beyond the Standard Model**

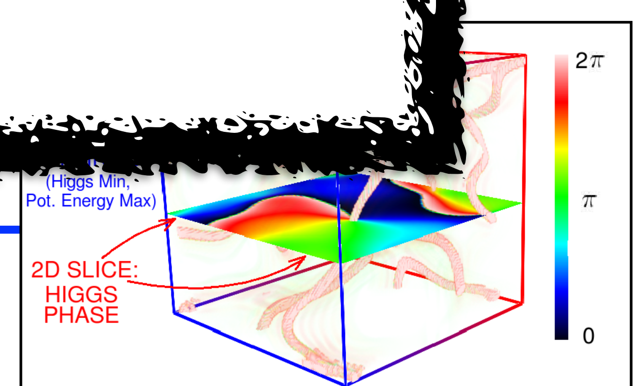
* **~ Origin of the Universe**

GW

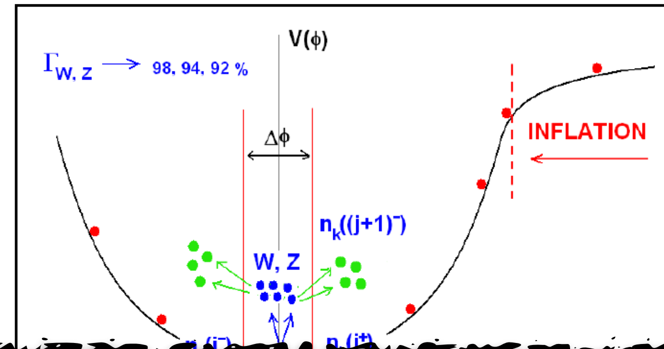
[Probe of the early Universe]

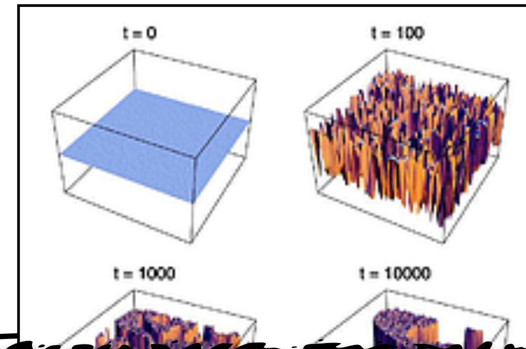


$(\Delta t \lesssim 1\mathrm{s})$



The Early Universe



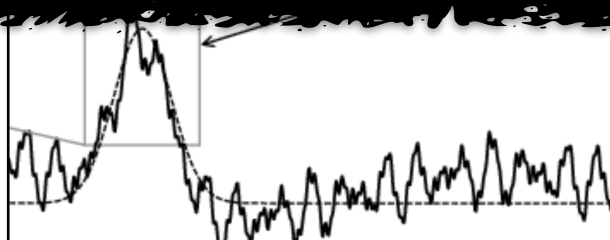


* Energies above terrestrial means
* Fundamental Physics
* Beyond the Standard Model
* ~ Origin of the Universe

Otherwise inaccessible !

GW

[Probe of the early Universe]



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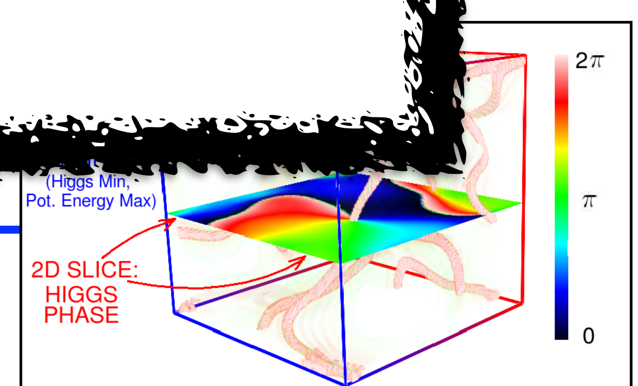
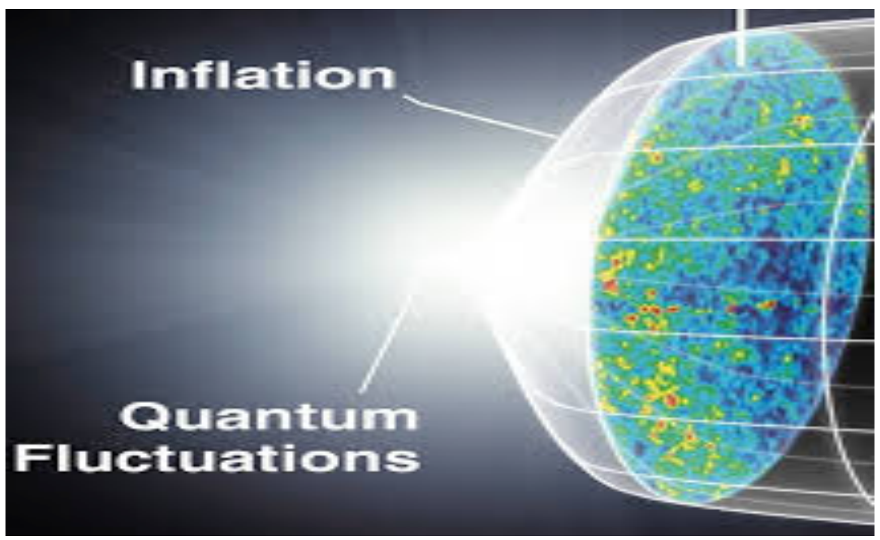


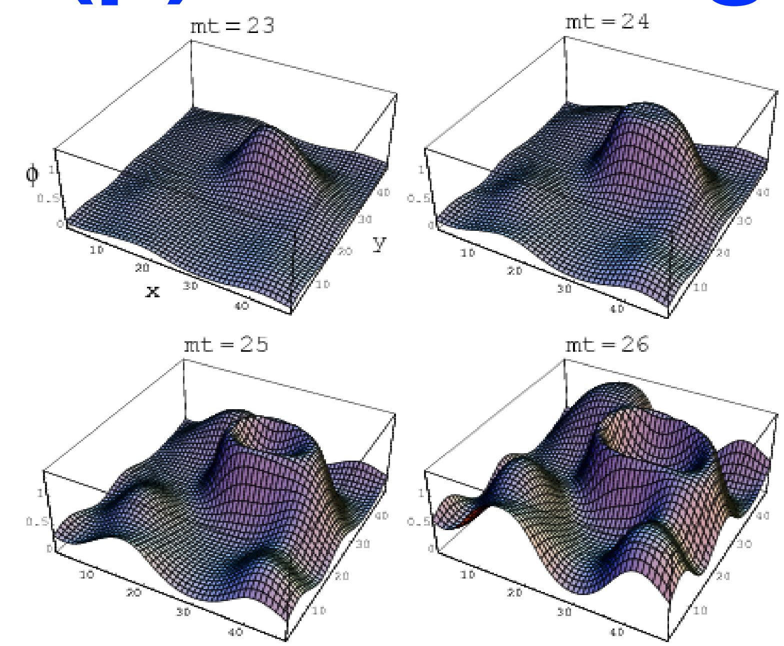
COSMOLOGICAL GW BACKGROUNDS

Inflationary Period



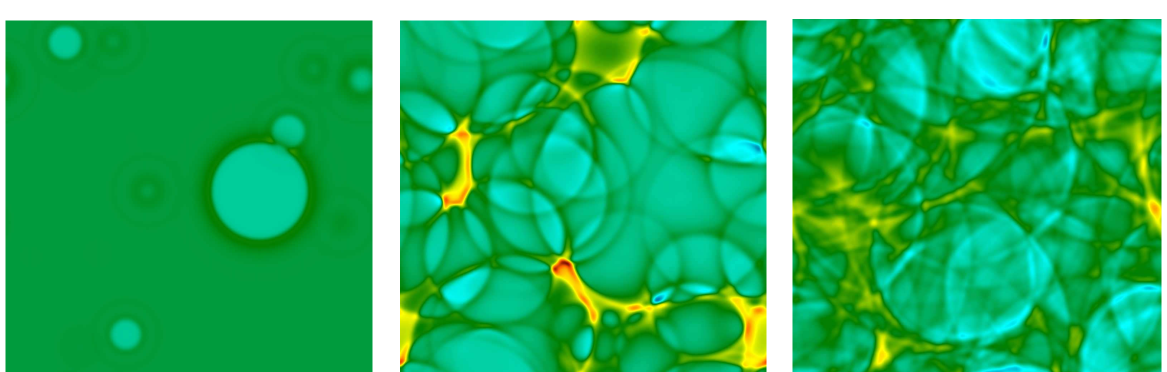
(Image: Google Search)

(p)Reheating



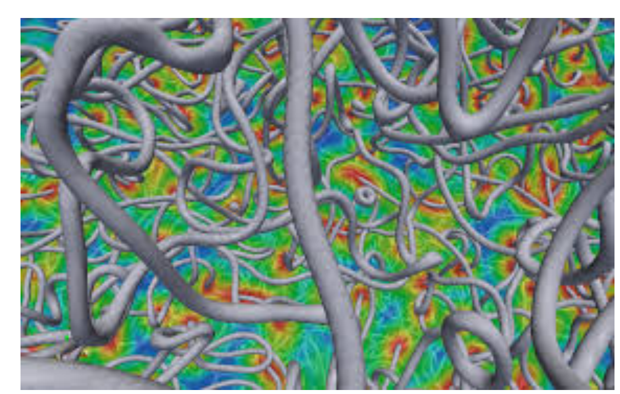
(Fig. credit: Phys.Rev. D67 103501)

Phase Transitions



(Image: PRL 112 (2014) 041301)

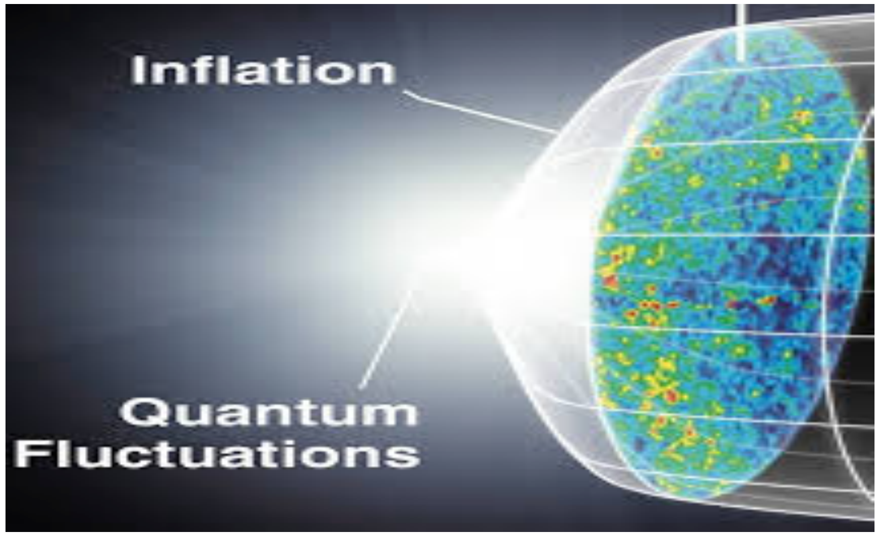
Cosmic Defects



(Image: Daverio et al, 2013)

COSMOLOGICAL GW BACKGROUNDS

Inflationary Period



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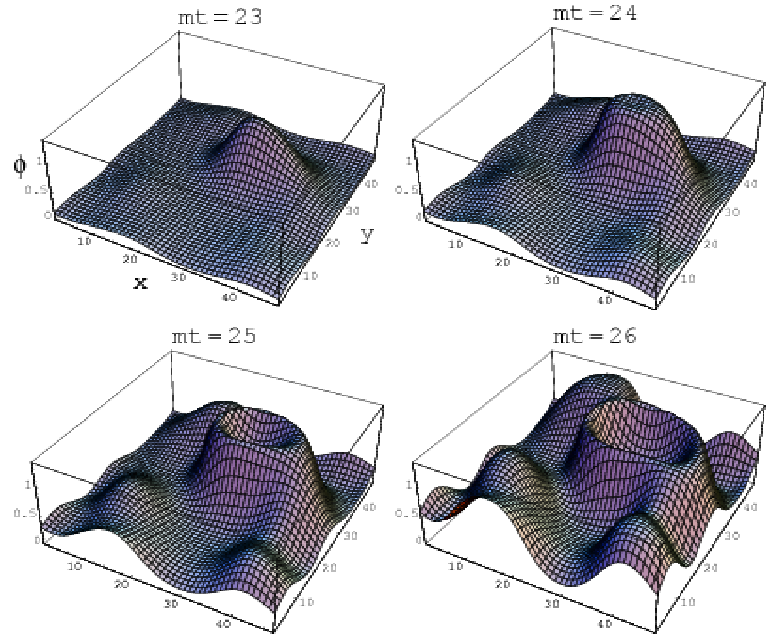
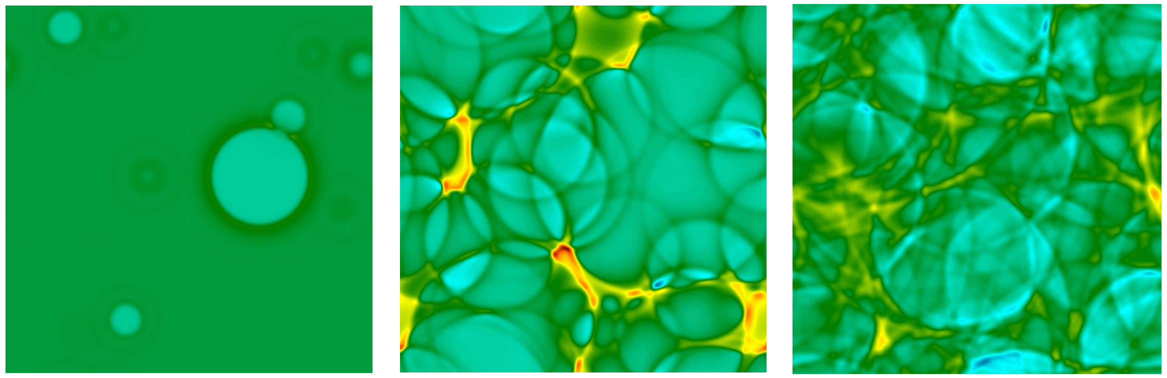
(p)Reheating



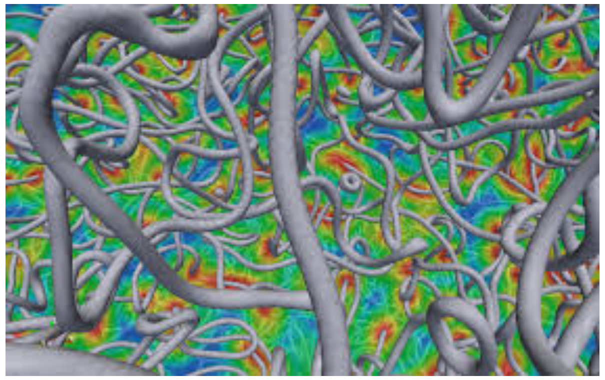
(Fig. credit: Phys.Rev. D67 103501)

Phase Transitions

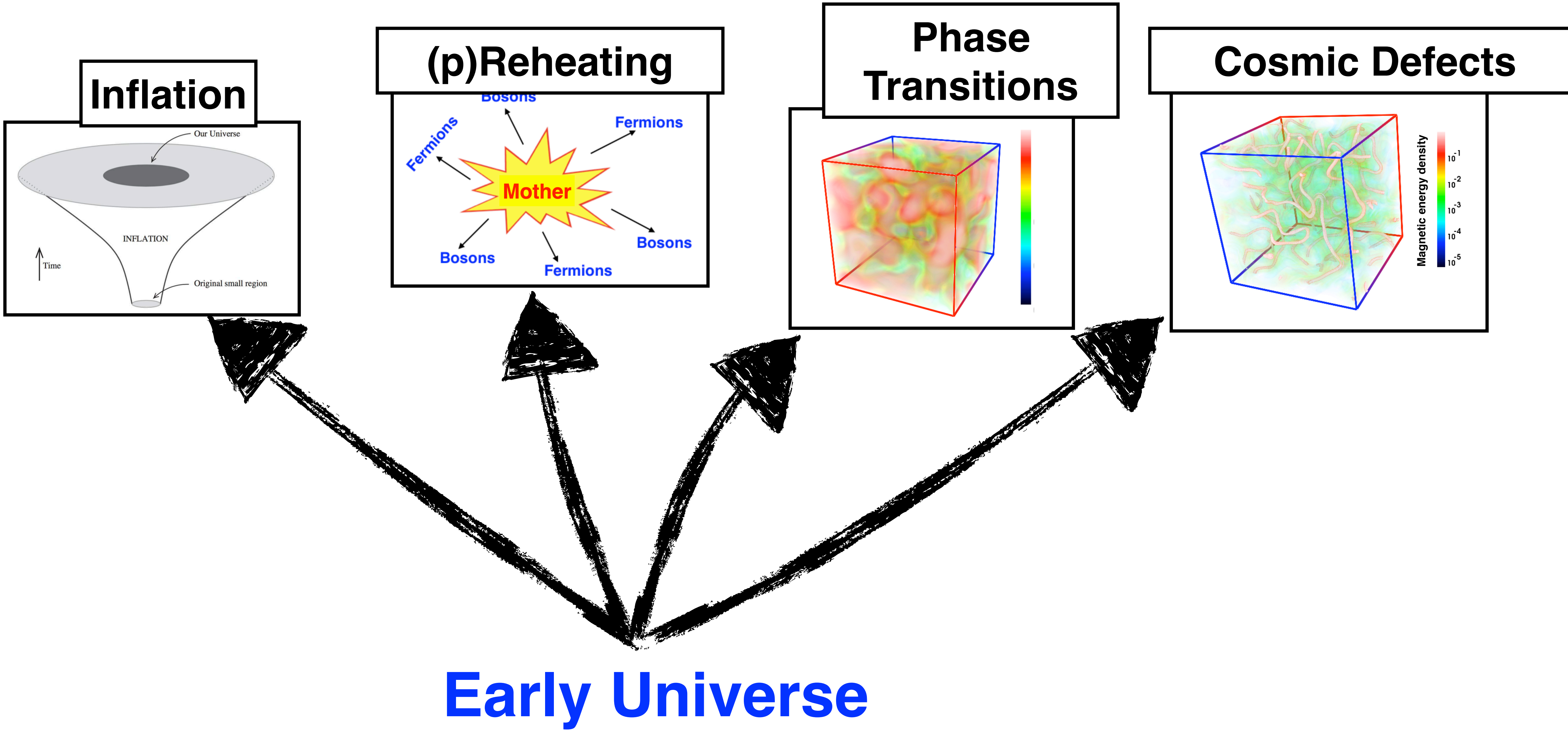


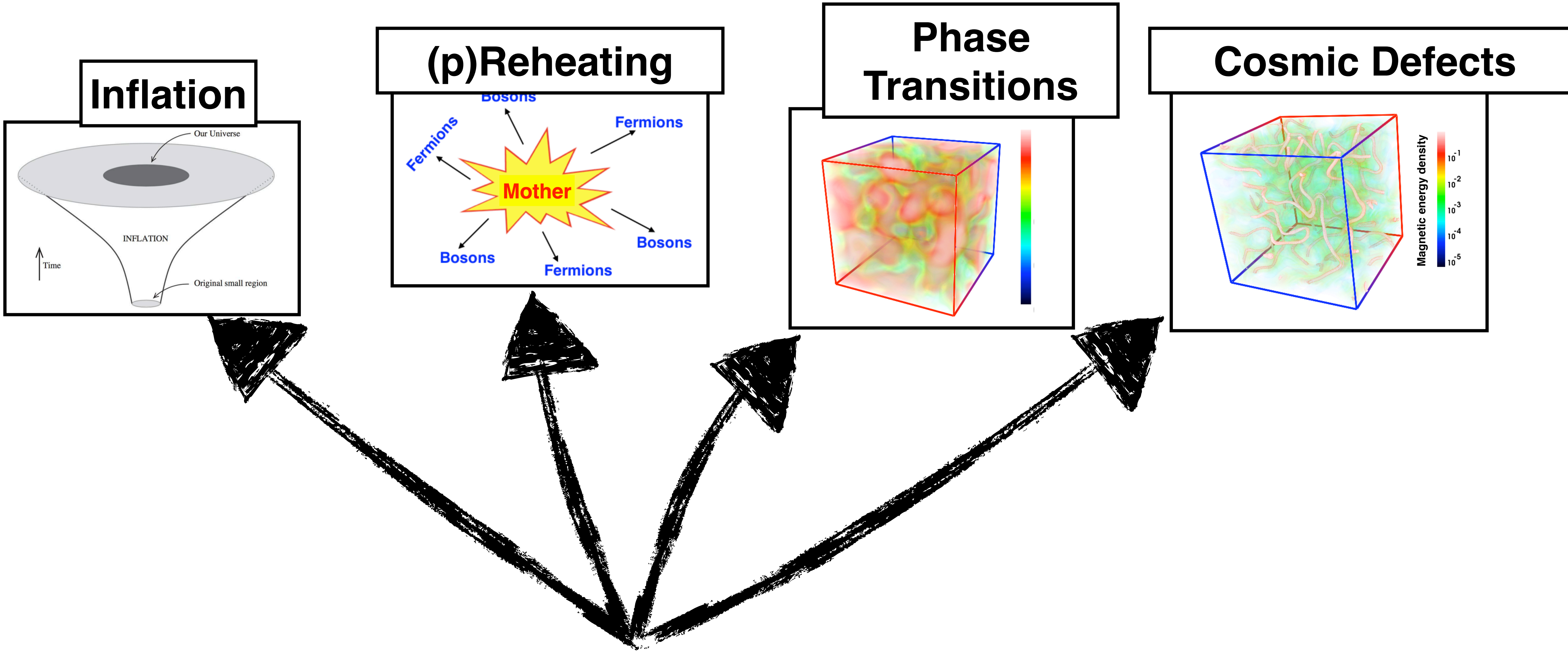
(Image: PRL 112 (2014) 041301)

Cosmic Defects



(Image: Daverio et al, 2013)

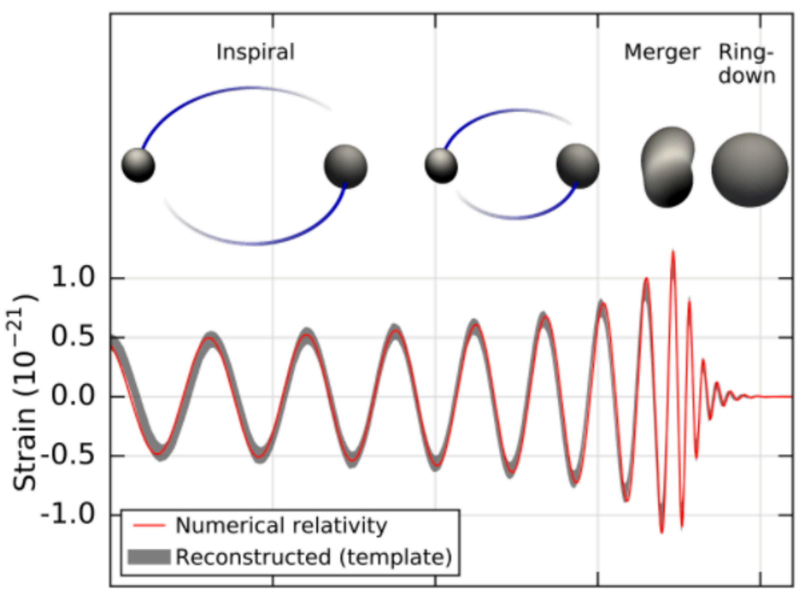

Inflation
Our Universe
INFLATION
Time
Original small region
(p)Reheating
Bosons
Fermions
Fermions
Mother
Bosons
Bosons
Fermions
Phase Transitions
Cosmic Defects
Magnetic energy density
Early Universe



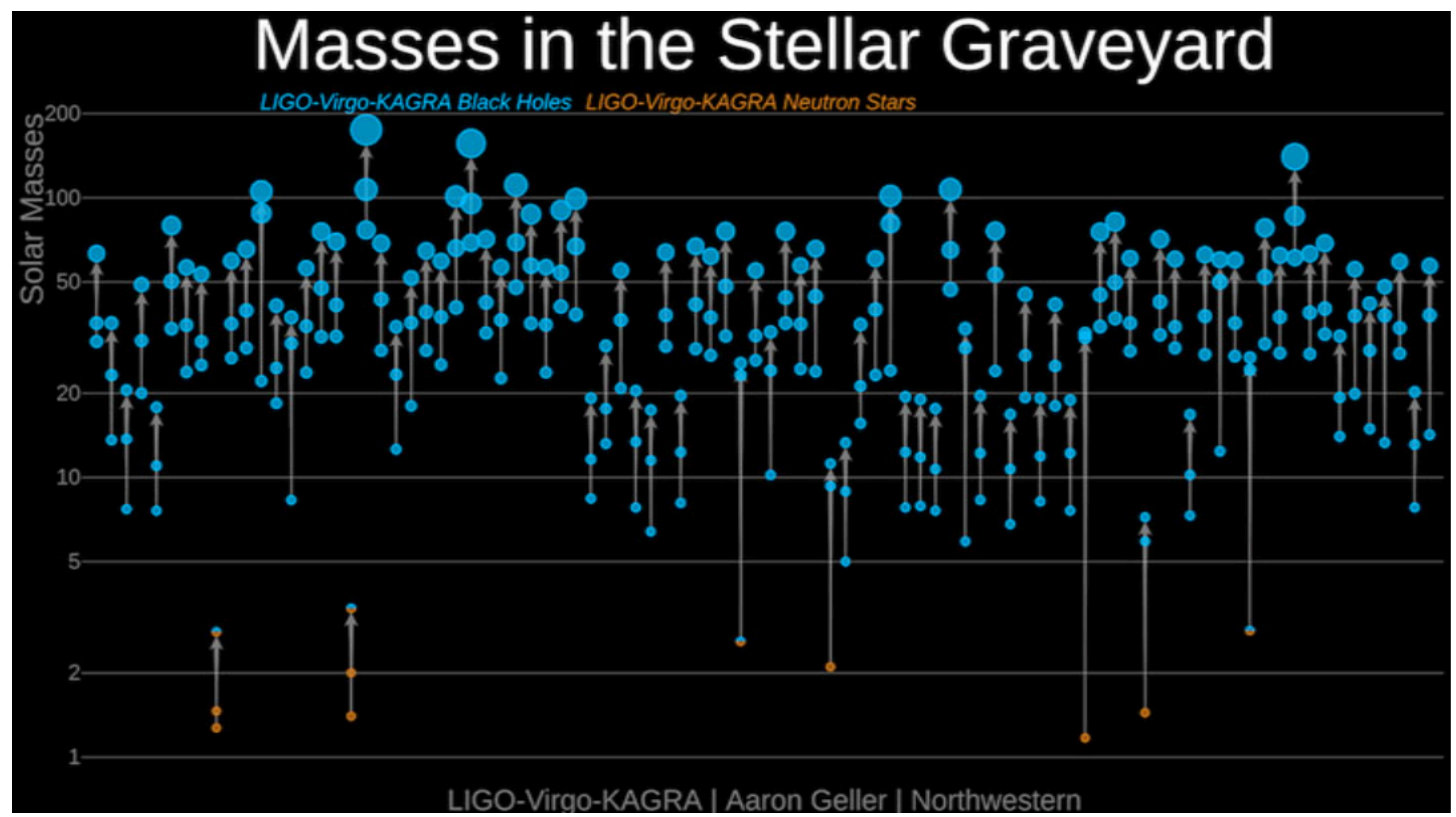
Early Universe

Late Universe ?

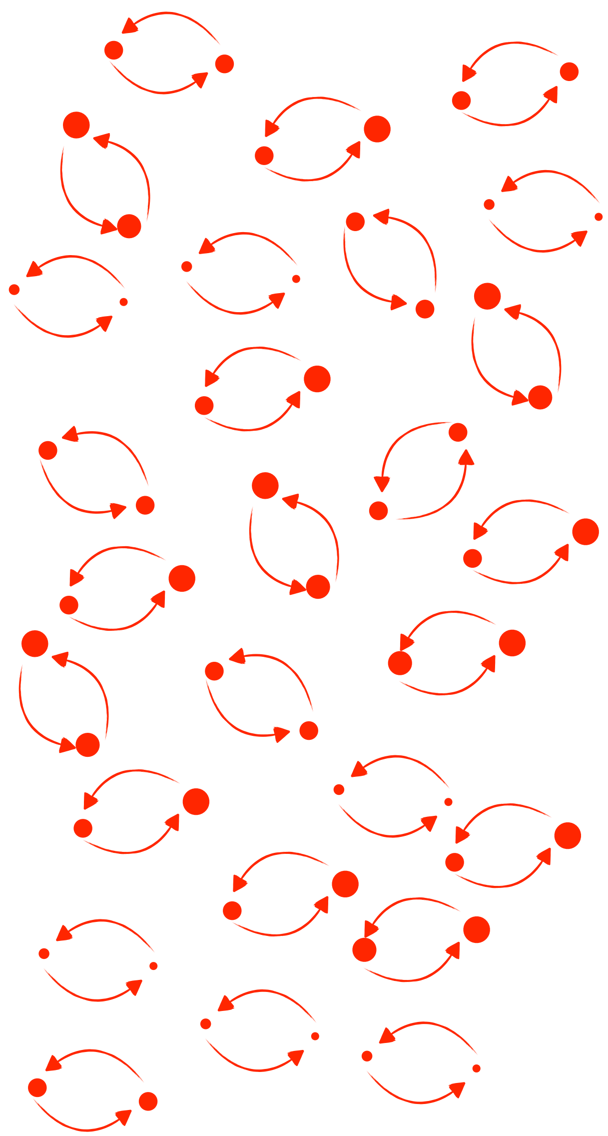
Late Universe $(0 \leq z \lesssim 10)$

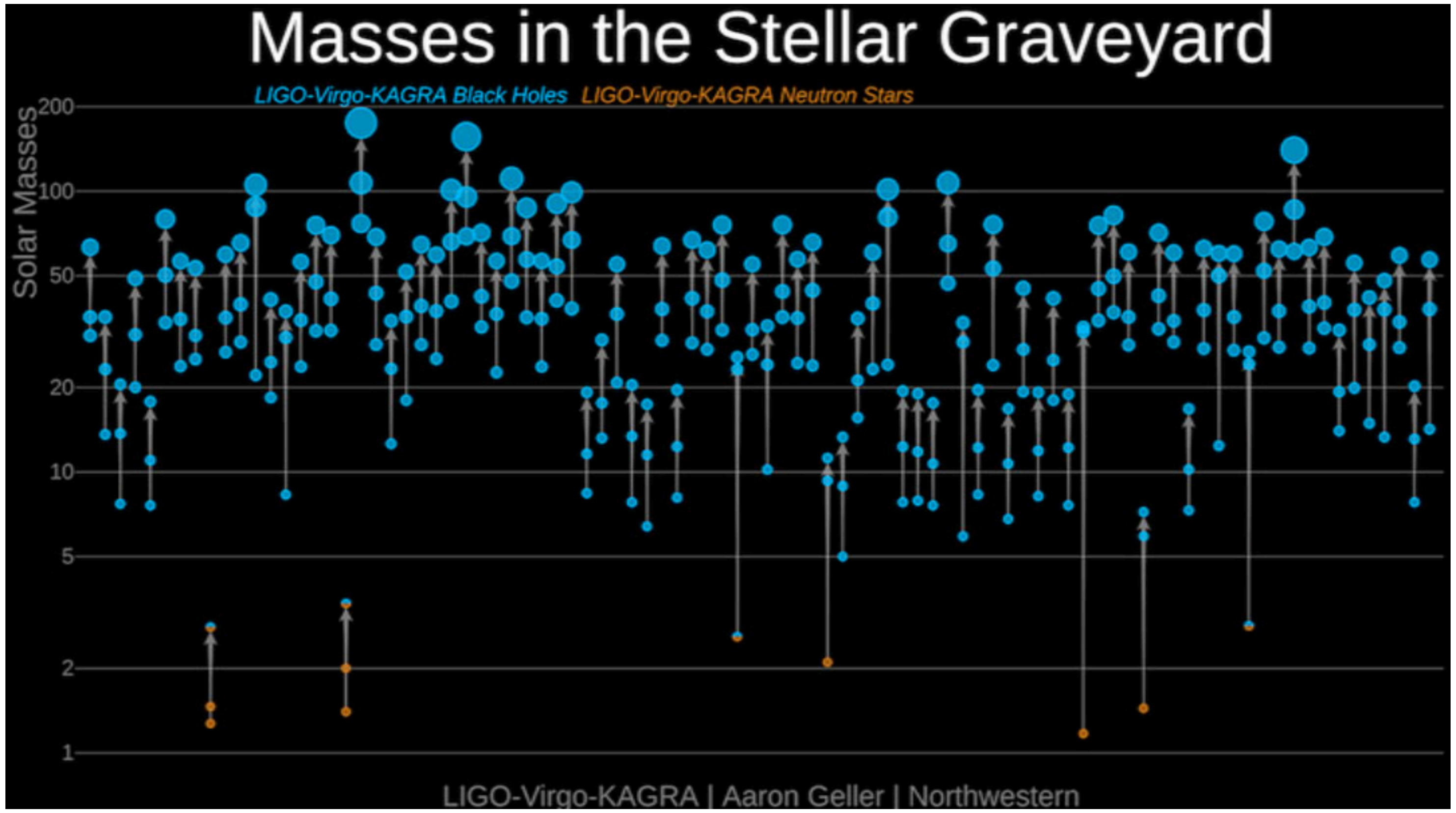


LIGO/VIRGO 2015-now

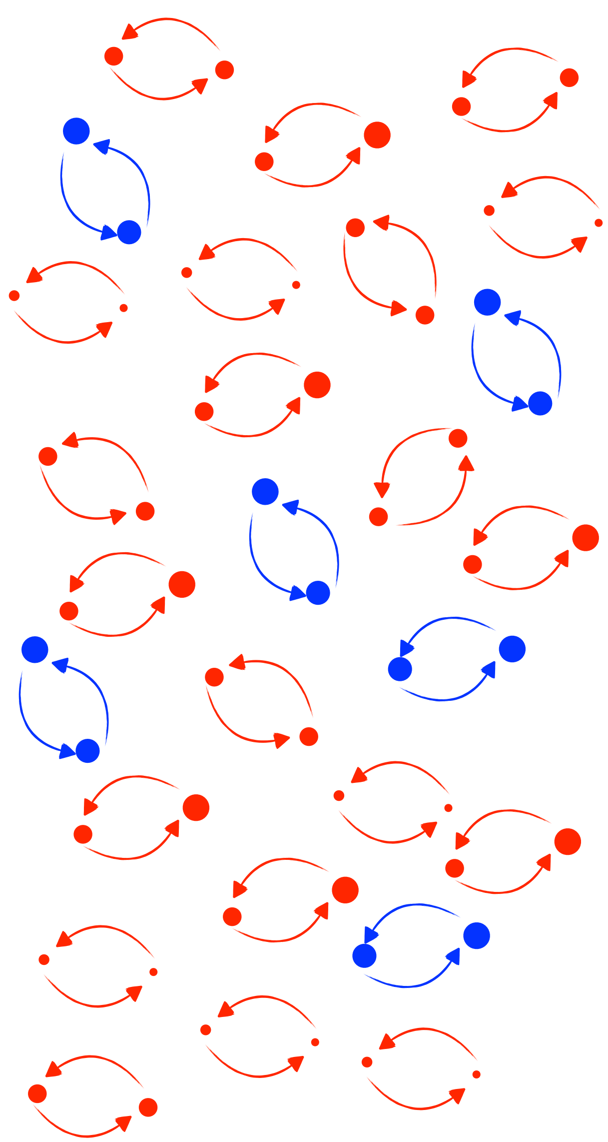


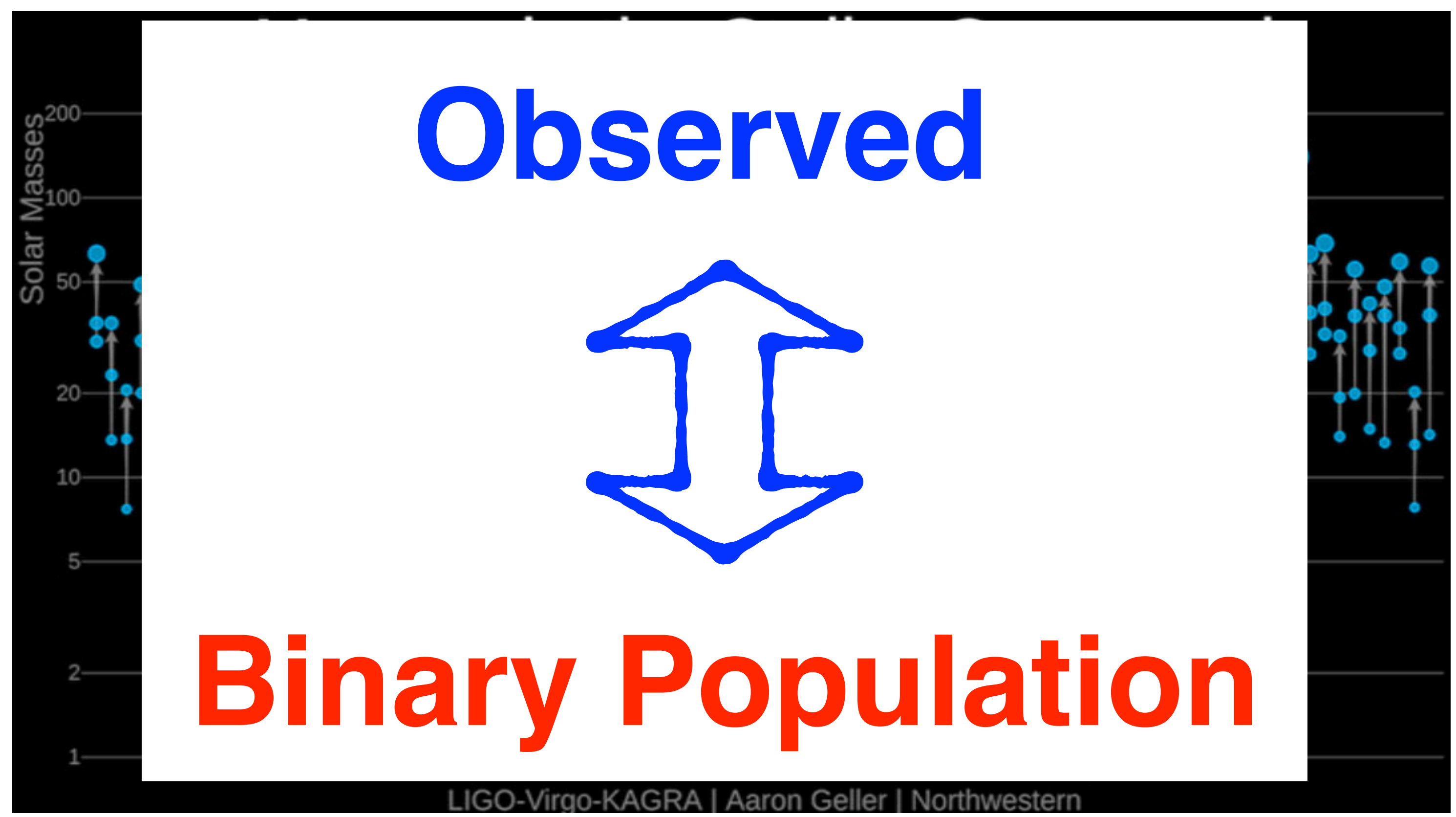
Late Universe $\quad (0 \leq z \lesssim 10)$



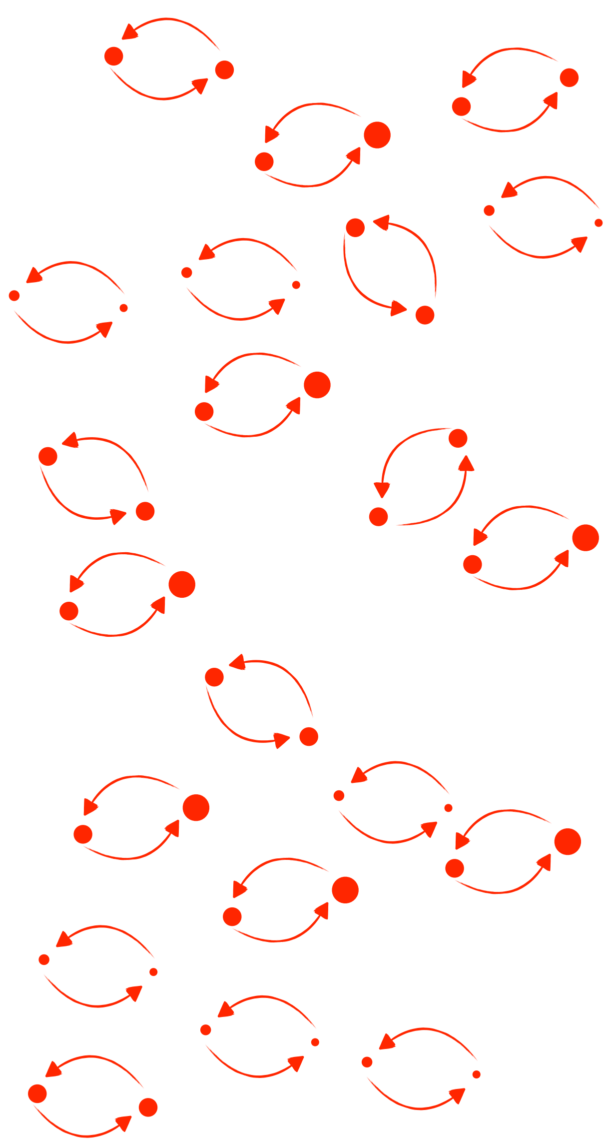


Late Universe $(0 \leq z \lesssim 10)$



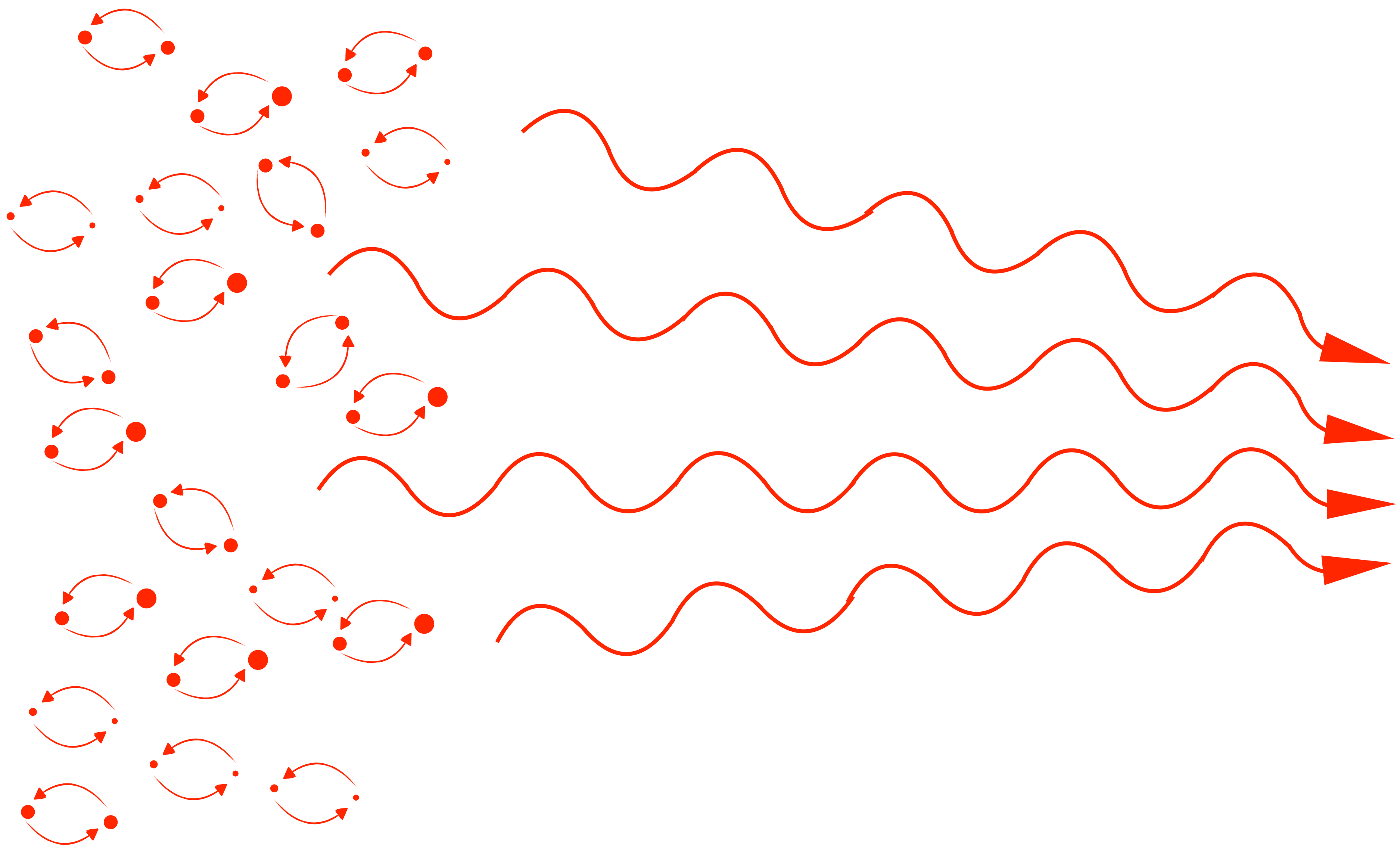


Late Universe $\quad (0 \leq z \lesssim 10)$

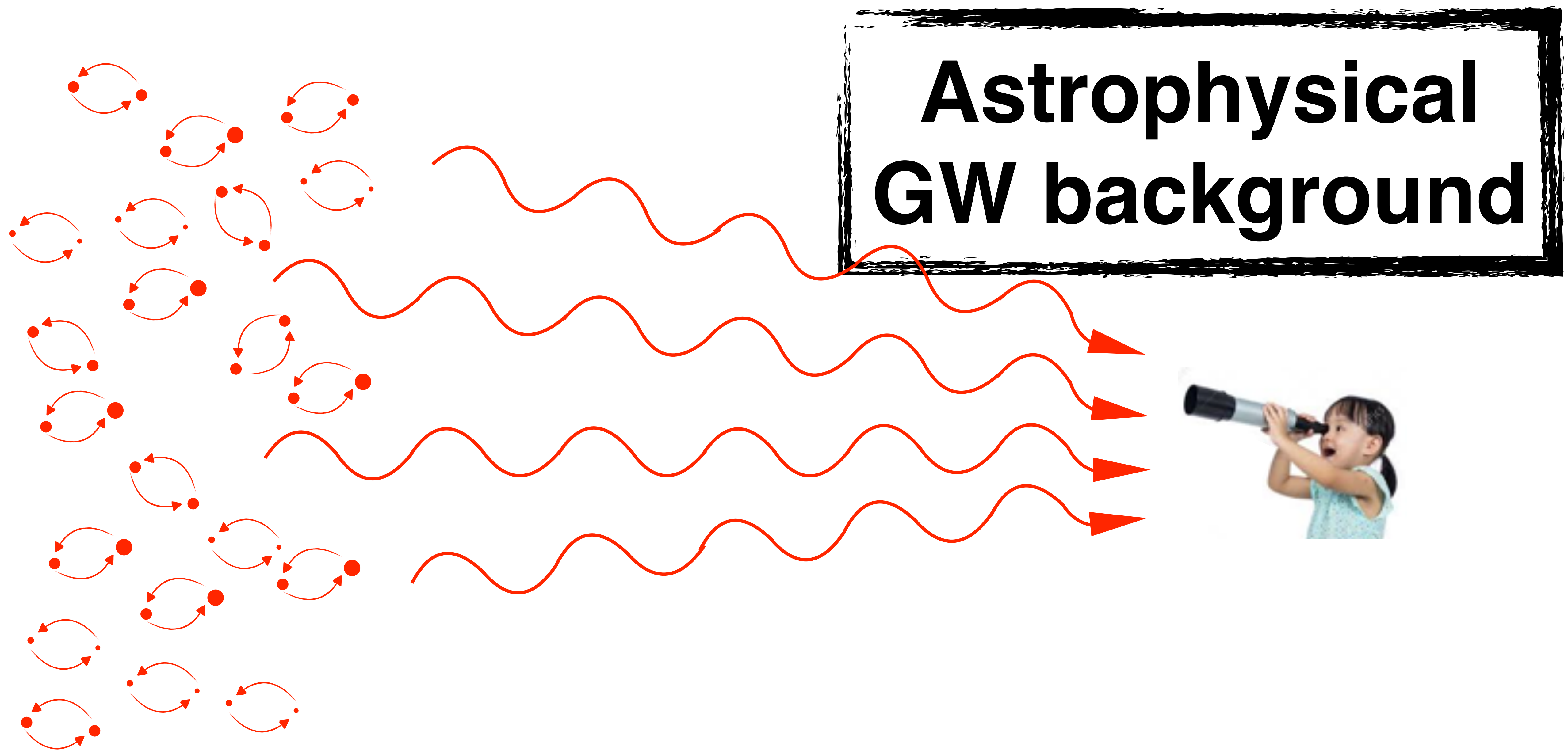


Mostly unresolved binaries !

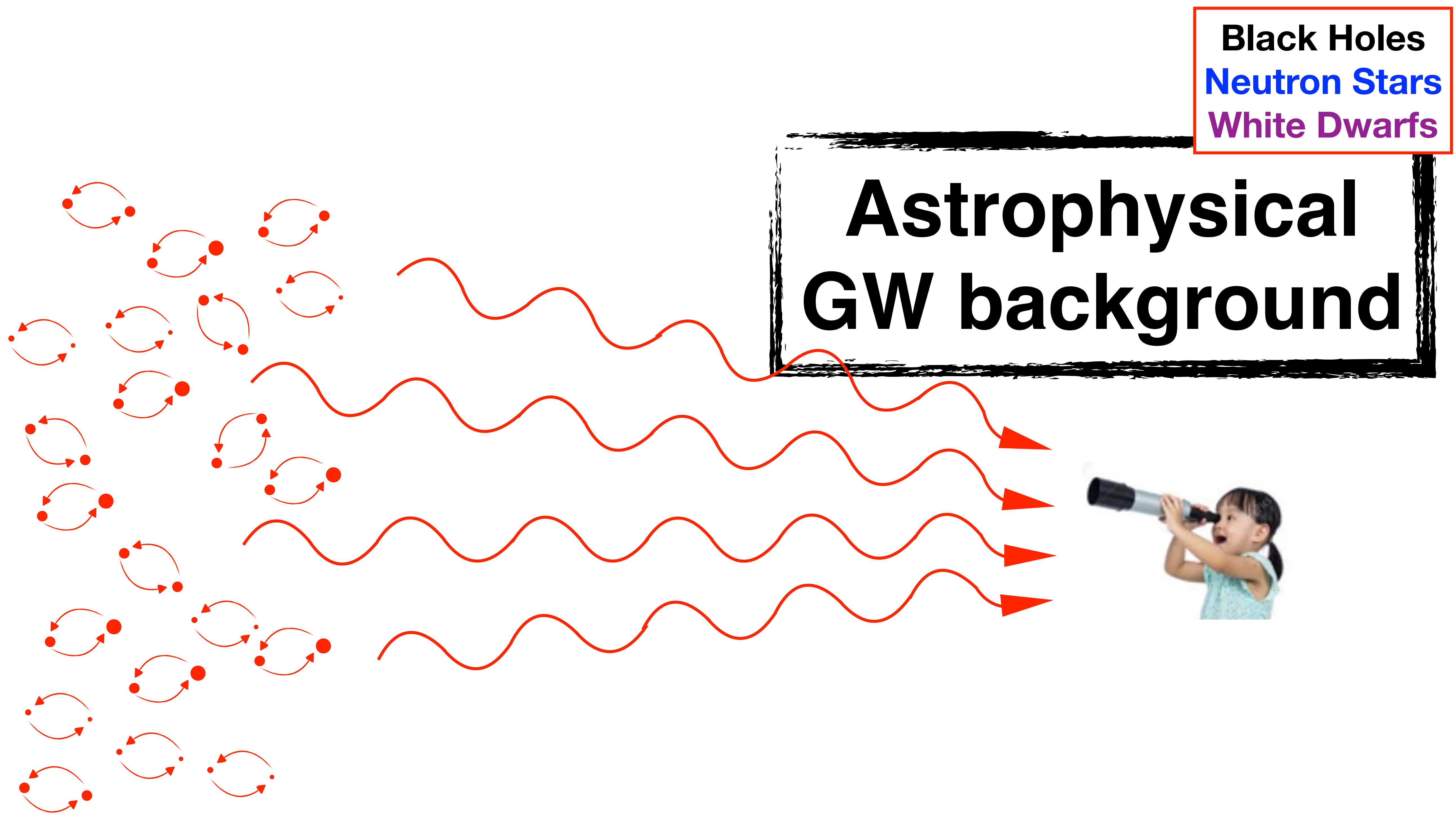
Late Universe $(0 \leq z \lesssim 10)$

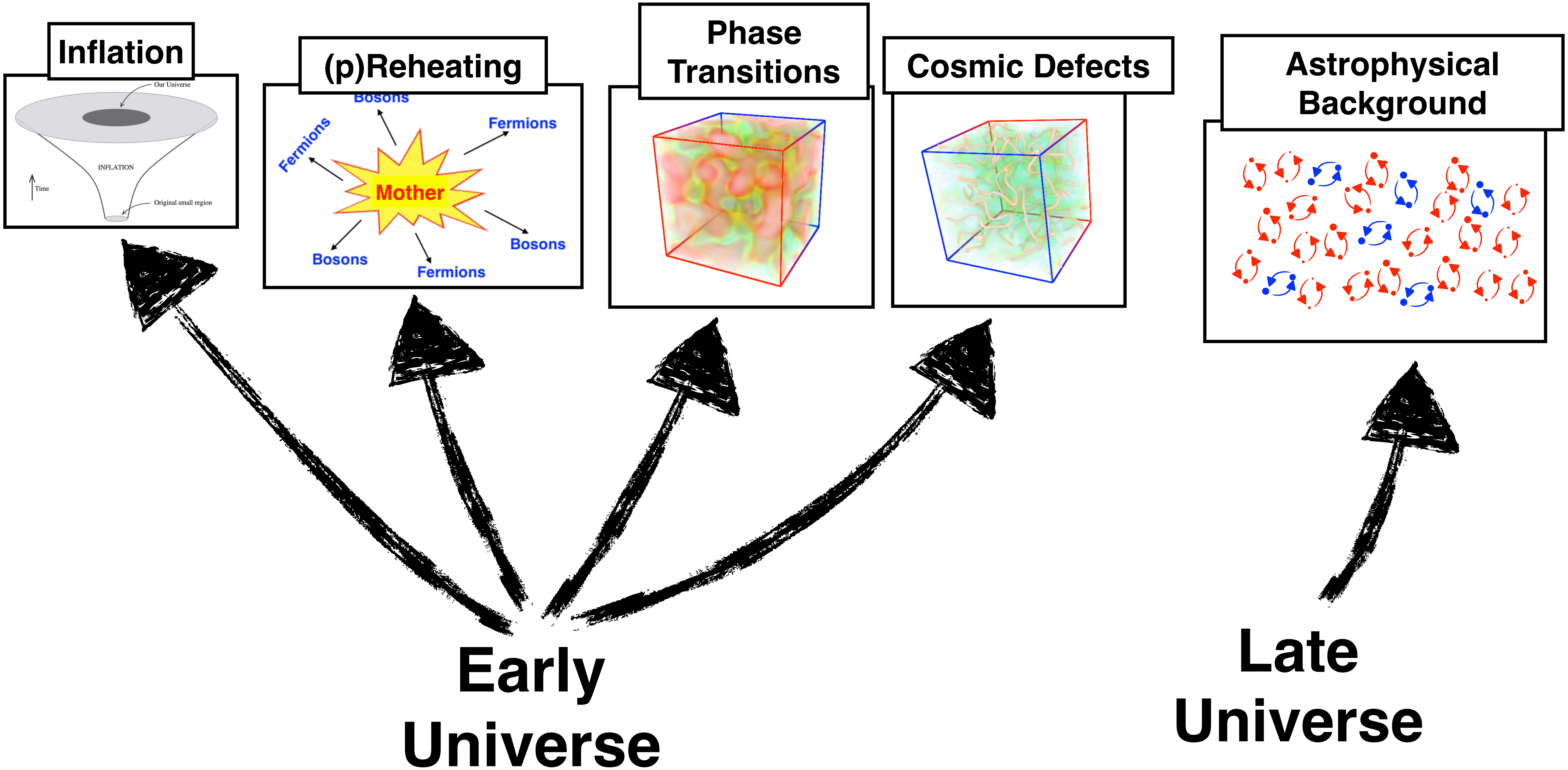


Late Universe $(0 \leq z \lesssim 10)$



Late Universe $(0 \leq z \lesssim 10)$





Gravitational Wave Backgrounds

Summary &
Perspective

Gravitational Wave Backgrounds

Cosmological

Early
Universe

Astrophysical

Late
Universe

Gravitational Wave Backgrounds

Probe of High Energy Physics

Cosmological

Early Universe

Astrophysical

Late Universe

Gravitational Wave Backgrounds

Probe Binary Population(s)

Cosmological

Early Universe

Astrophysical

Late Universe

Gravitational Wave Backgrounds

HOLY GRAIL

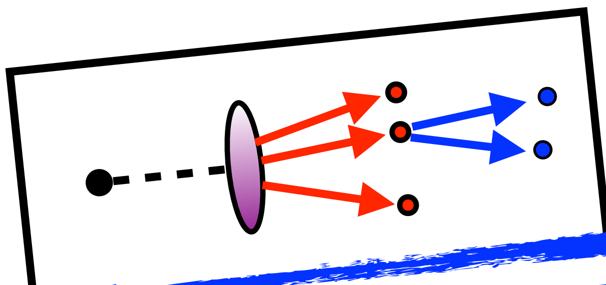
Cosmological

Early
Universe

Astrophysical

Late
Universe

Gravitational Wave Backgrounds



HOLY GRAIL

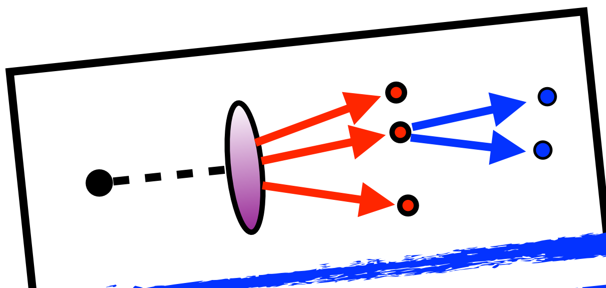
Cosmological

Early
Universe

Astrophysical

Late
Universe

Gravitational Wave Backgrounds



HOLY GRAIL

Cosmological

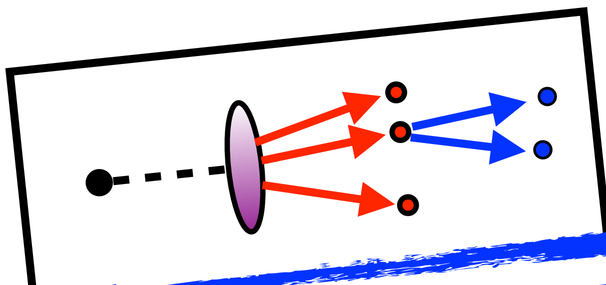
Early Universe

GUARANTEED

Astrophysical

Late Universe

Gravitational Wave Backgrounds



HOLY GRAIL

Cosmological

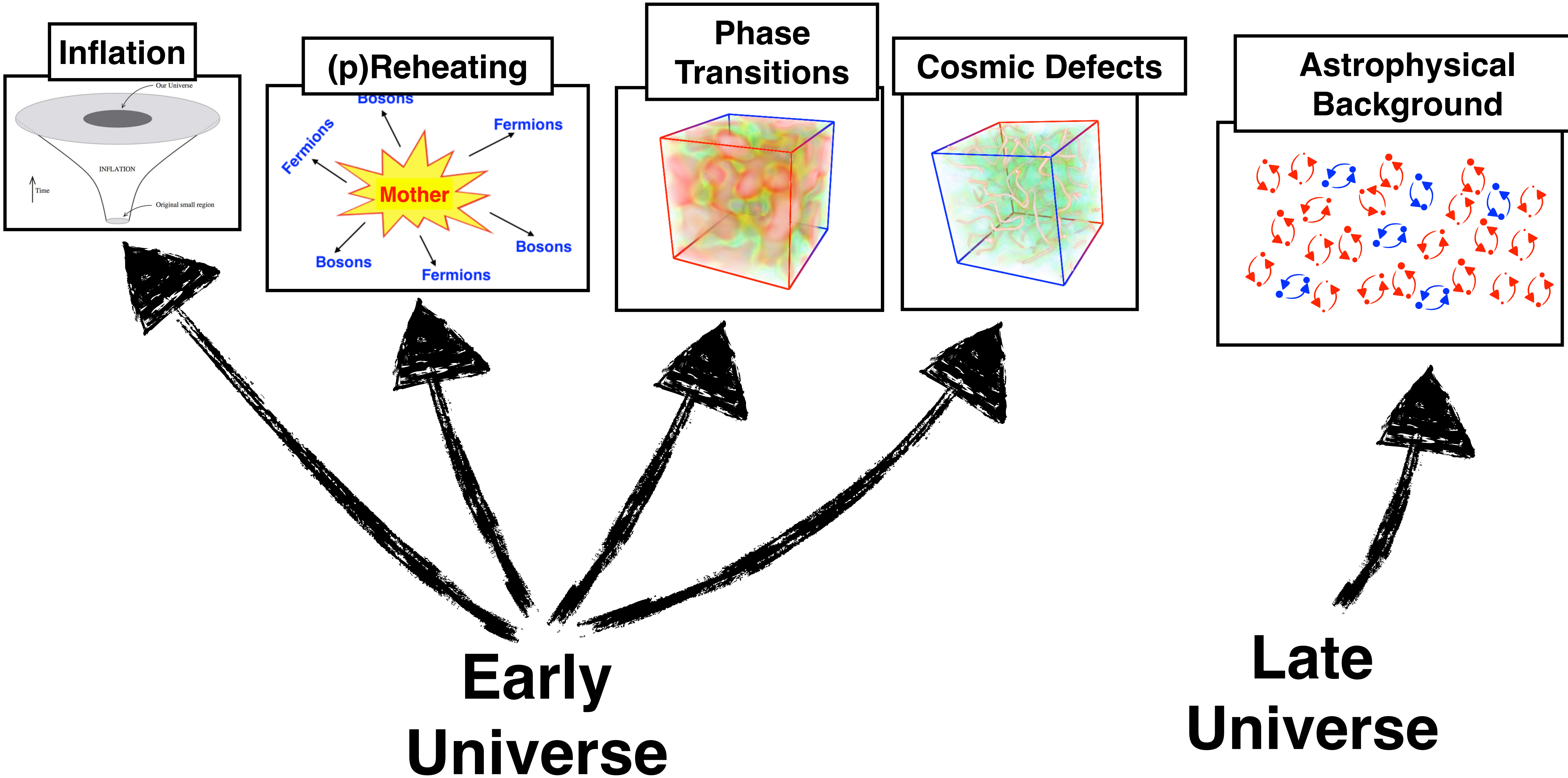
Early
Universe

Foreground

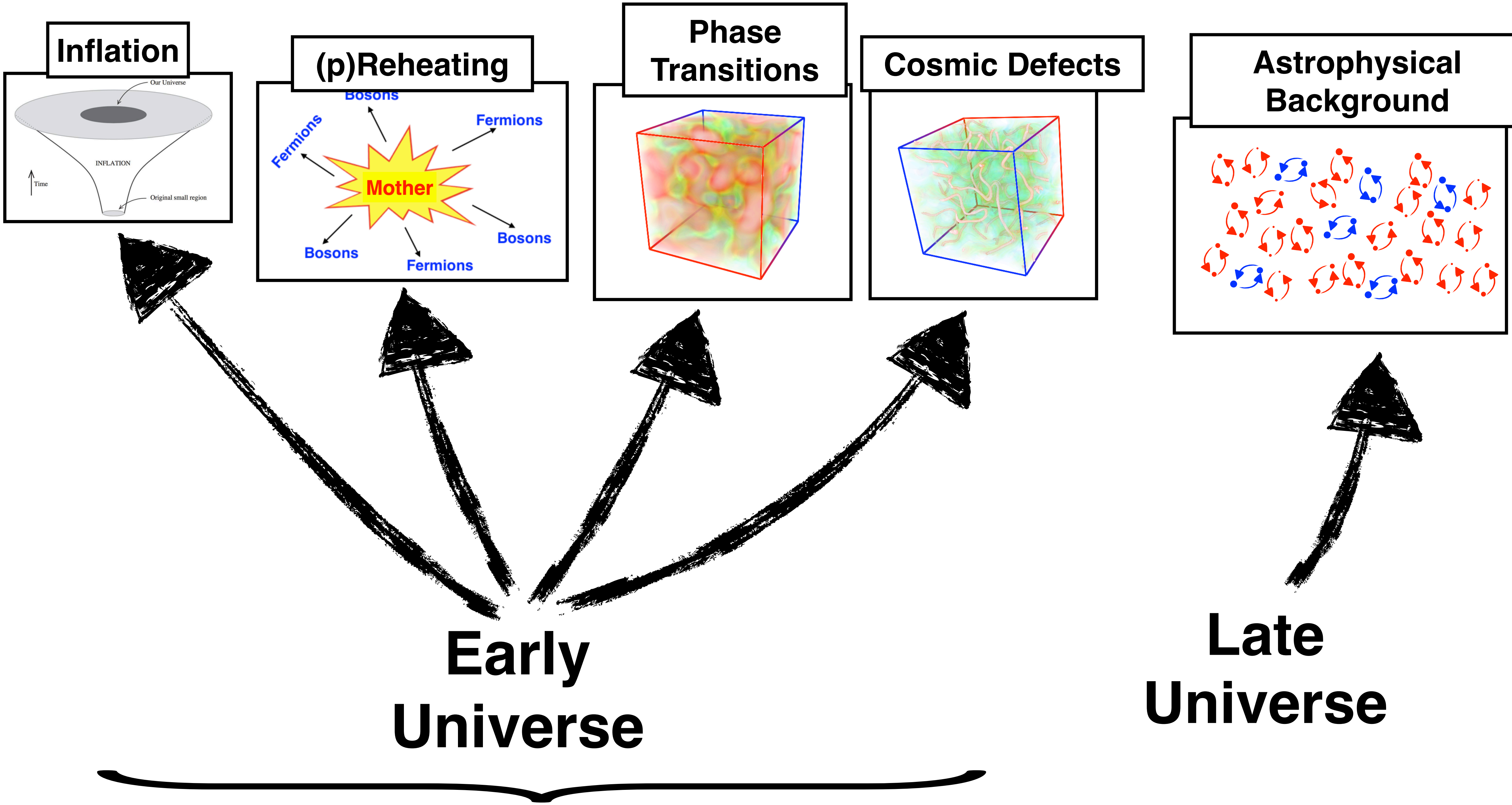
GUARANTEED

Astrophysical

Late
Universe

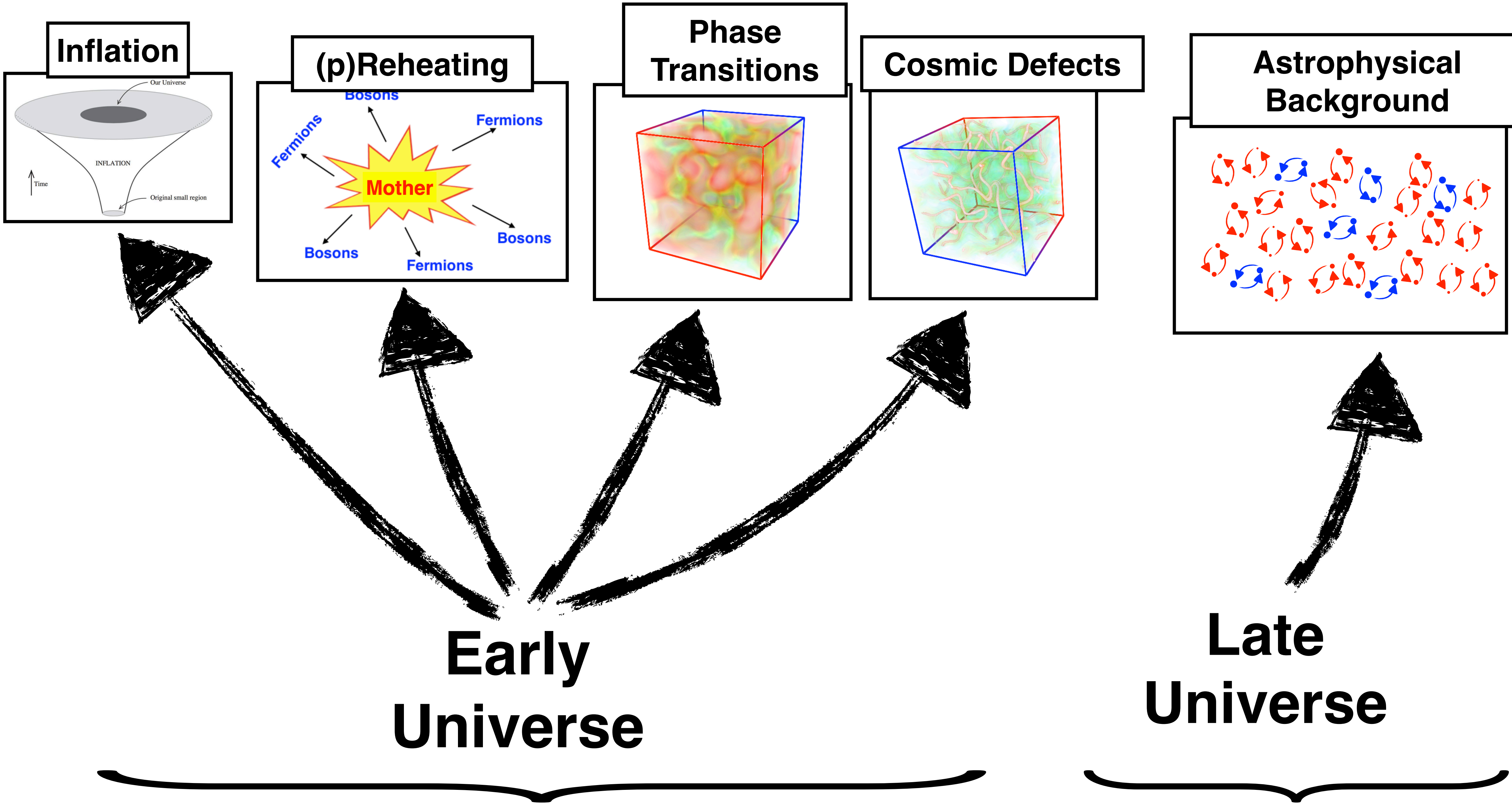


Gravitational Wave Backgrounds



Core of the lectures !

As these backgrounds probe Fundamental Physics (HEP)



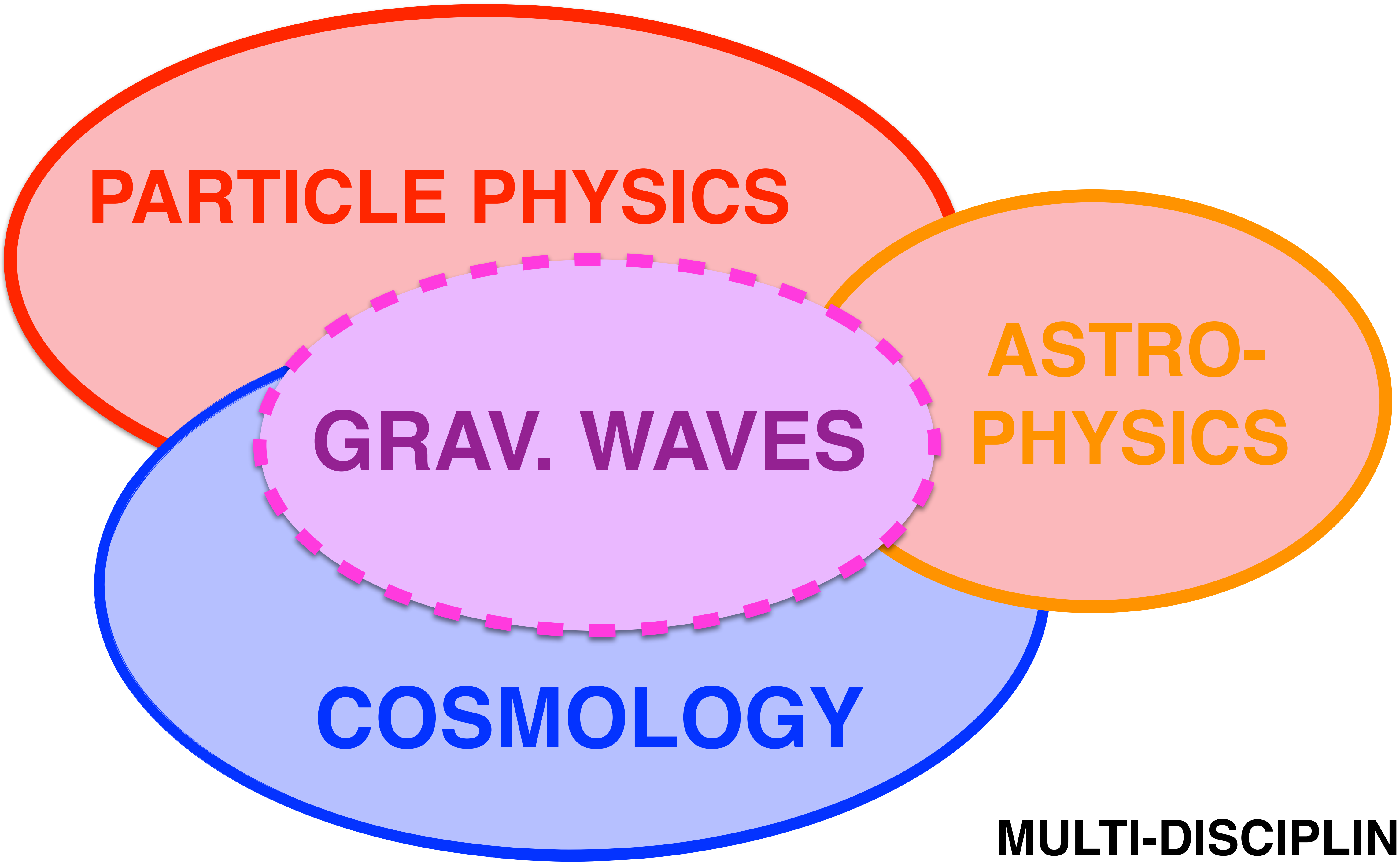
Core of the lectures !

As these backgrounds probe Fundamental Physics (HEP)

Peripheral Backgrounds act (for us) as a foreground !

Gravitational Wave Backgrounds

BROAD



MULTI-DISCIPLINARY

BACKGROUNDS OF GRAVITATIONAL WAVES

OUTLINE

Early Universe

1) GWs from Inflation

2) GWs from Preheating

3) GWs from Phase Transitions

4) GWs from Cosmic Defects

BACKGROUNDS OF GRAVITATIONAL WAVES

OUTLINE

0) Grav. Waves (GWs)

Early Universe

1) GWs from Inflation

2) GWs from Preheating

3) GWs from Phase Transitions

4) GWs from Cosmic Defects

BACKGROUNDS OF GRAVITATIONAL WAVES

OUTLINE

0) Cosmology/General Relativity & GW Def.

Early Universe

1) GWs from Inflation

2) GWs from Preheating

3) GWs from Phase Transitions

4) GWs from Cosmic Defects

BACKGROUNDS OF GRAVITATIONAL WAVES

OUTLINE

1) Cosmology/GR + GW definition

Early Universe

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

BACKGROUNDS OF GRAVITATIONAL WAVES

OUTLINE

1) Cosmology/GR + GW definition

Early Universe

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

Late Universe

6) Astrophysical Background(s) ⟵ IF there is time …

BACKGROUNDS OF GRAVITATIONAL WAVES

1st Bloc

1) Cosmology/GR + GWs

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

BACKGROUNDS OF GRAVITATIONAL WAVES

1st Bloc

1) Cosmology/GR + GWs

1.A) Cosmology

1.B) General Relativity

1.C) GW definition

3 Options:
- A) + B) + C)
- B) + C)
- C)

COSMOLOGY FRAMEWORK

BASICS of COSMOLOGY

hot Big Bang (hBB)
(evolution of the Universe)

$$\nearrow$$

Cosmology

$$\searrow$$

Inflation $= \begin{pmatrix} \text{initial} \\ \text{cond.} \end{pmatrix}$

BASICS of COSMOLOGY

hot Big Bang (hBB)
(evolution of the Universe)

Cosmology

Inflation $= \begin{pmatrix}\text{initial} \\ \text{cond.}\end{pmatrix}$

BASICS of COSMOLOGY

theoretical pillars

hot Big Bang (hBB)
(evolution of the Universe)

observational pillars

Cosmology

Inflation = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$

BASICS of COSMOLOGY

- Cosmology
  - hot Big Bang (hBB) (evolution of the Universe)
    - theoretical pillars
      - General Relativity
      - Cosmological Pple
    - observational pillars
      - Expansion
      - CMB
      - BBN
  - Inflation = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$

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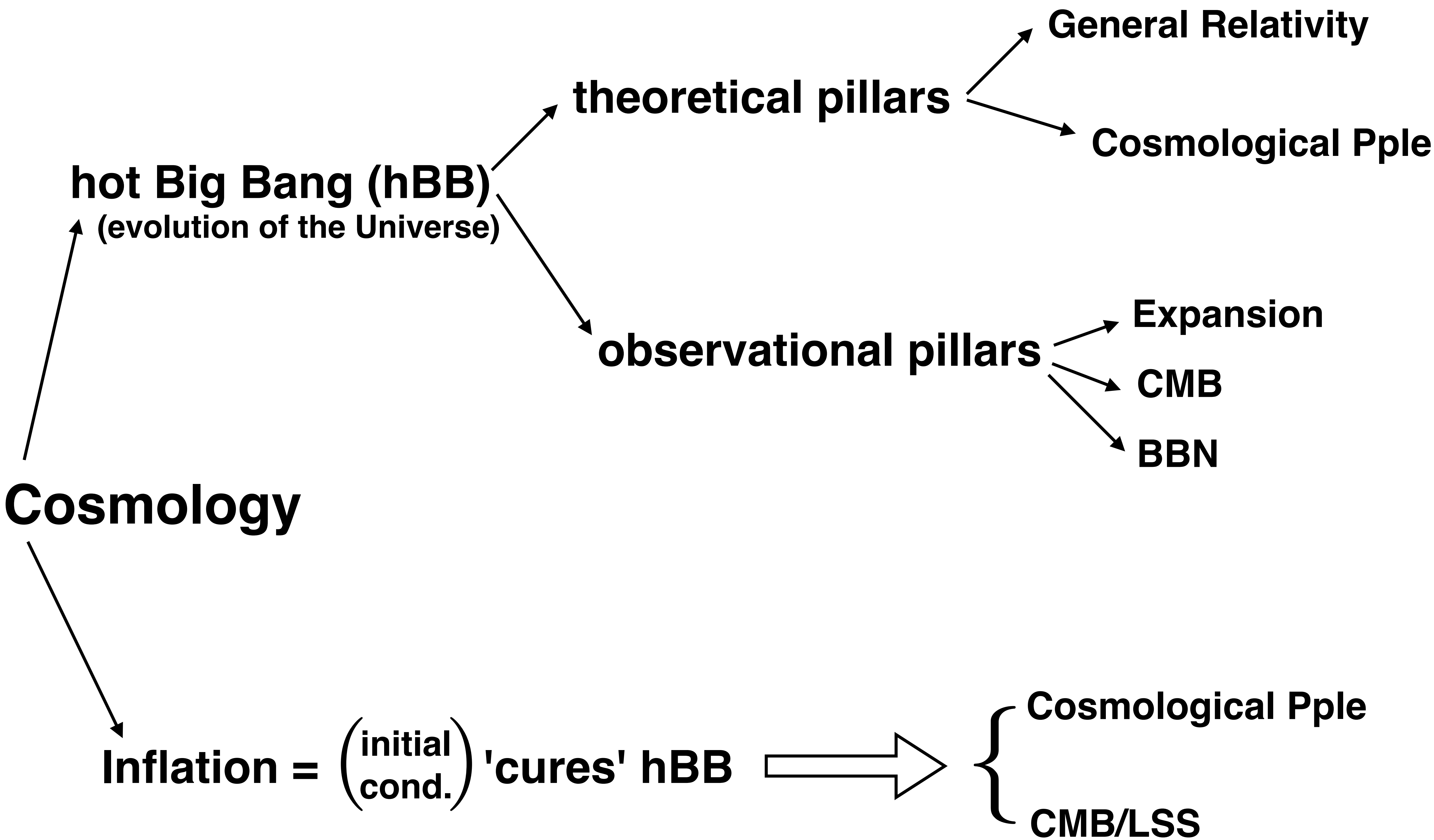
BASICS of COSMOLOGY

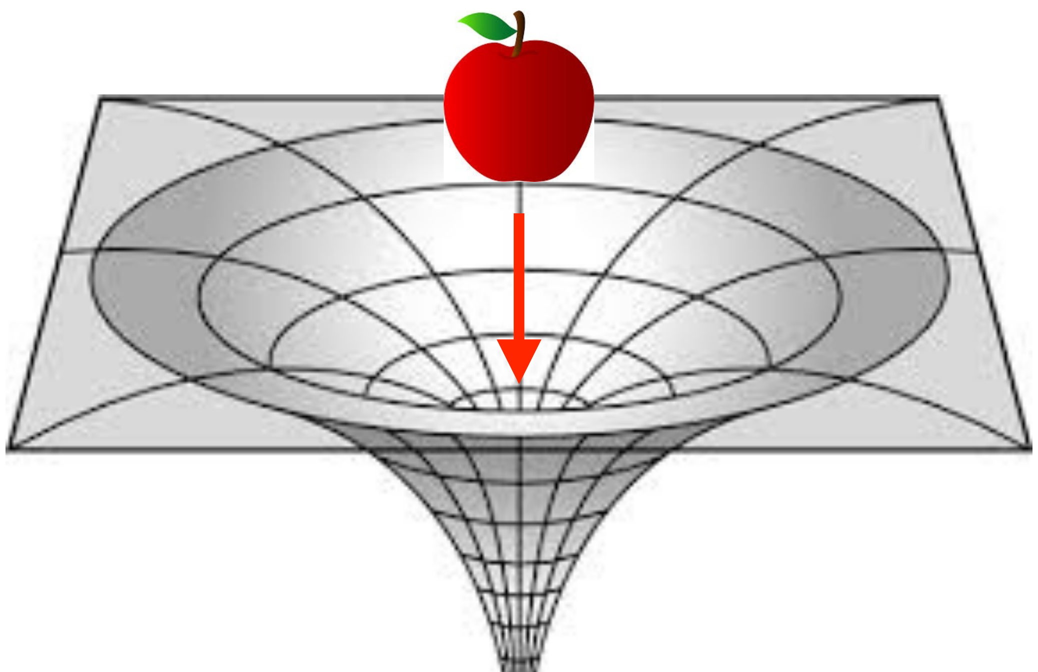


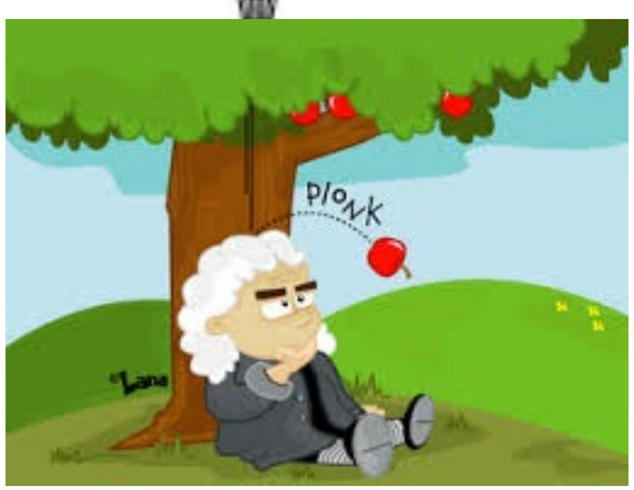
BASICS of COSMOLOGY

- **Cosmology**
  - **hot Big Bang (hBB)** (evolution of the Universe)
    - **theoretical pillars**
      - $\boxed{\text{General Relativity}}$
      - Cosmological Pple
    - **observational pillars**
      - Expansion
      - CMB
      - BBN
  - **Inflation** = $\left(\begin{array}{c}\text{initial}\\ \text{cond.}\end{array}\right)$ 'cures' hBB $\Longrightarrow$ $\left\{\begin{array}{l}\text{Cosmological Pple}\\ \\ \text{CMB/LSS}\end{array}\right.$

General Relativity (GR)

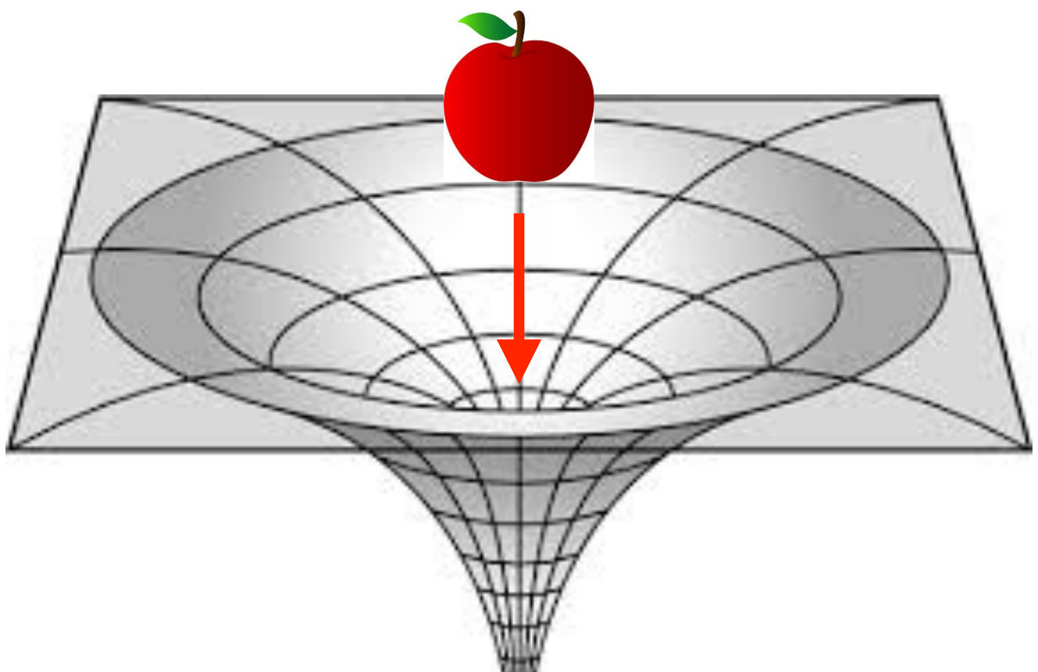
Gravitational Framework

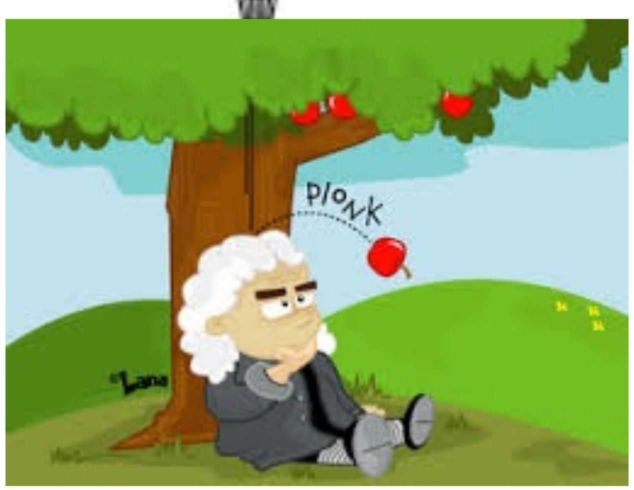




Gravitational Framework

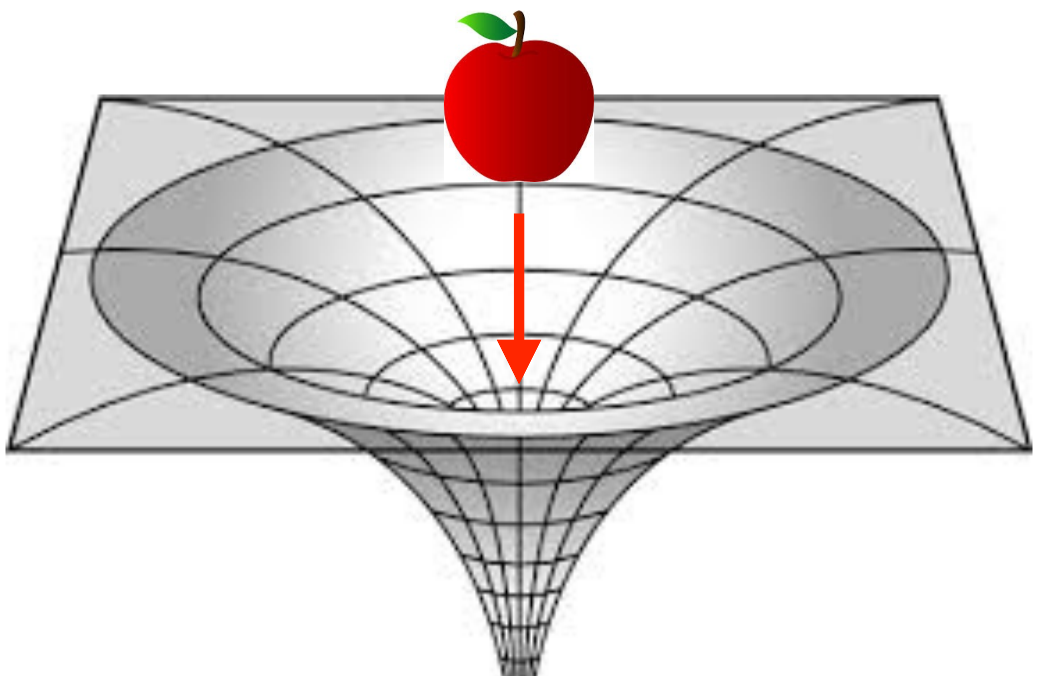
General Relativity (GR)

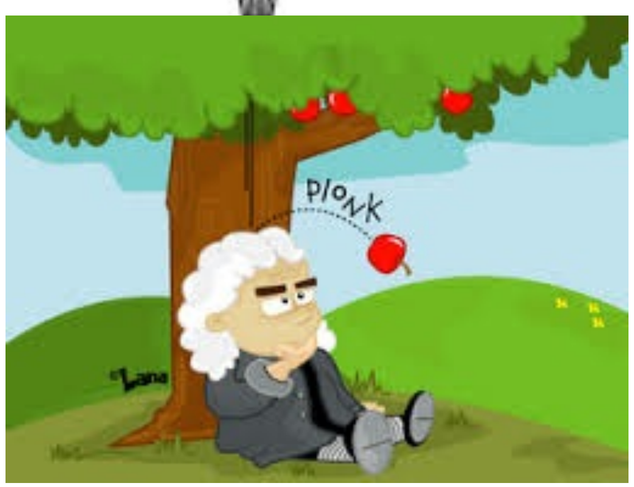




Gravitational Framework

General Relativity (GR)





$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry matter

$$\left[m_p = (8\pi G)^{-1/2} = 2.44 \cdot 10^{18}\,\mathrm{GeV} \right]$$

Reduced Planck mass

Gravitational Framework

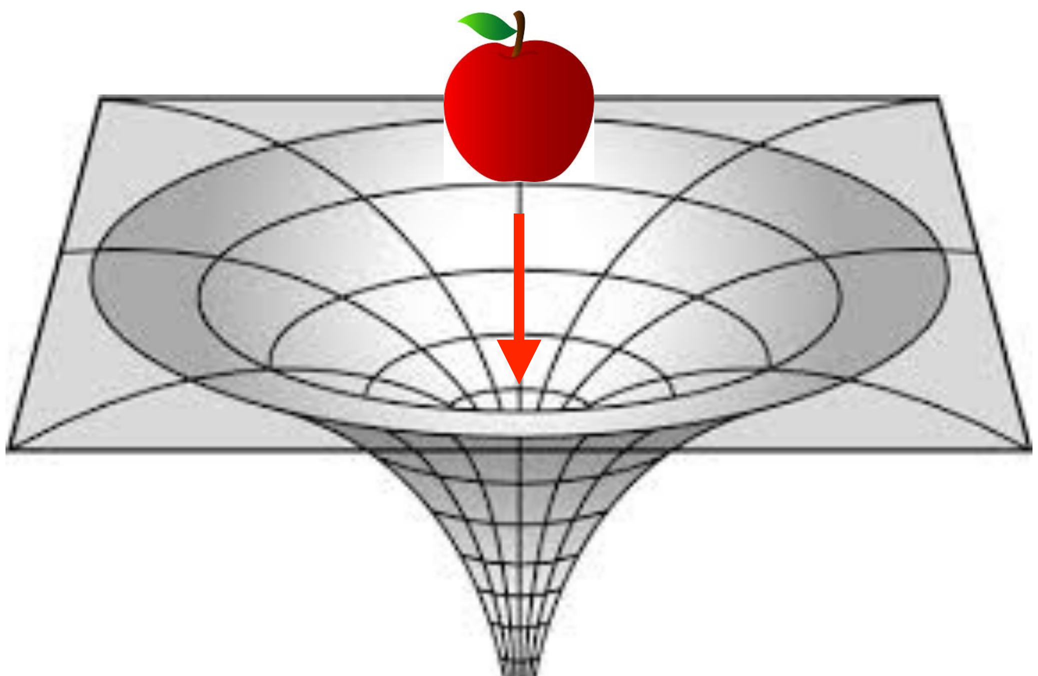
General Relativity (GR)

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

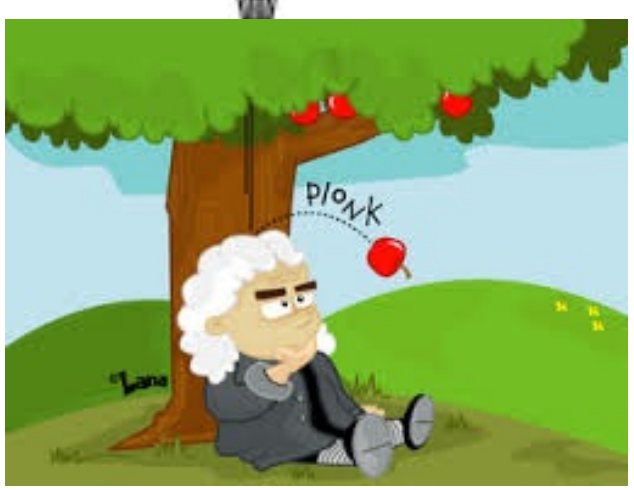
geometry matter

$$\left[m_p = (8\pi G)^{-1/2} = 2.44 \cdot 10^{18}\,\mathrm{GeV} \right]$$

Reduced Planck mass



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$



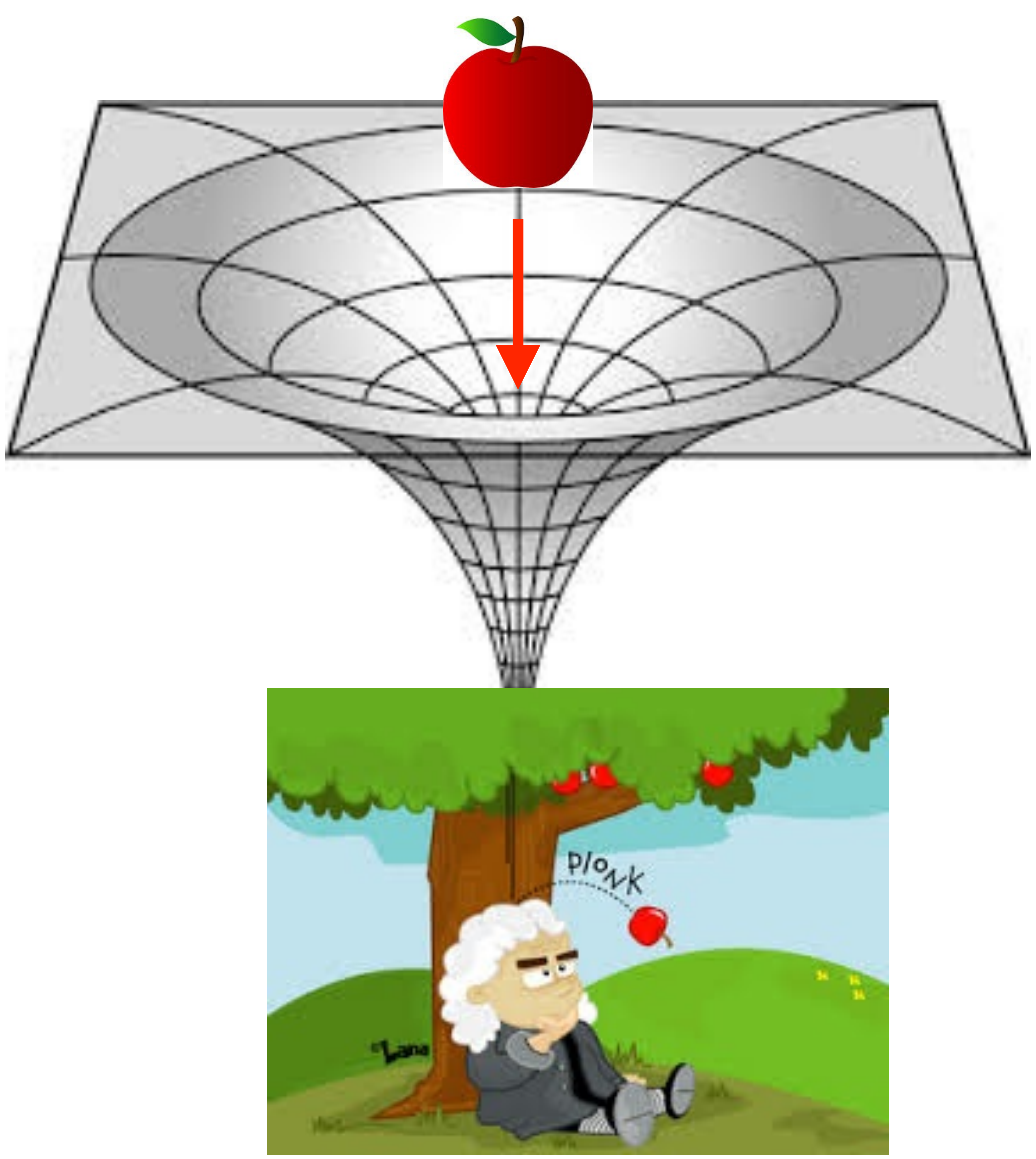
Gravitational Framework

General Relativity (GR)

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry matter

$$\left[m_p = (8\pi G)^{-1/2} = 2.44 \cdot 10^{18}\,\mathrm{GeV} \right]$$ Reduced Planck mass



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\mathrm{DIFF}: \quad x^\mu \to x'^\mu(x)$$

symmetry

GENERAL RELATIVITY

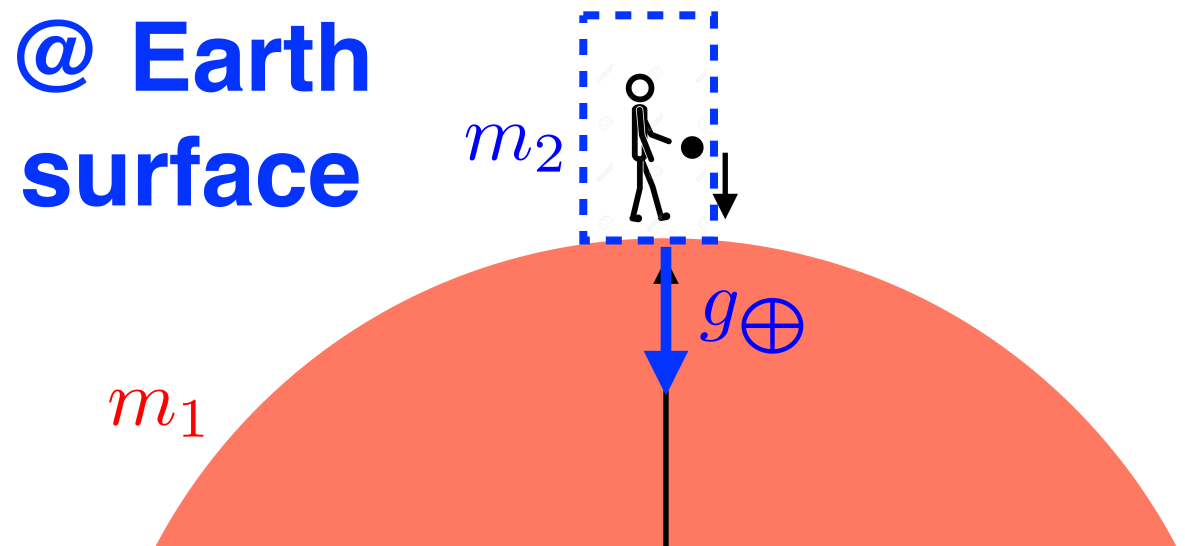
Primer (Introduction)

Skip (continue cosmology primer)

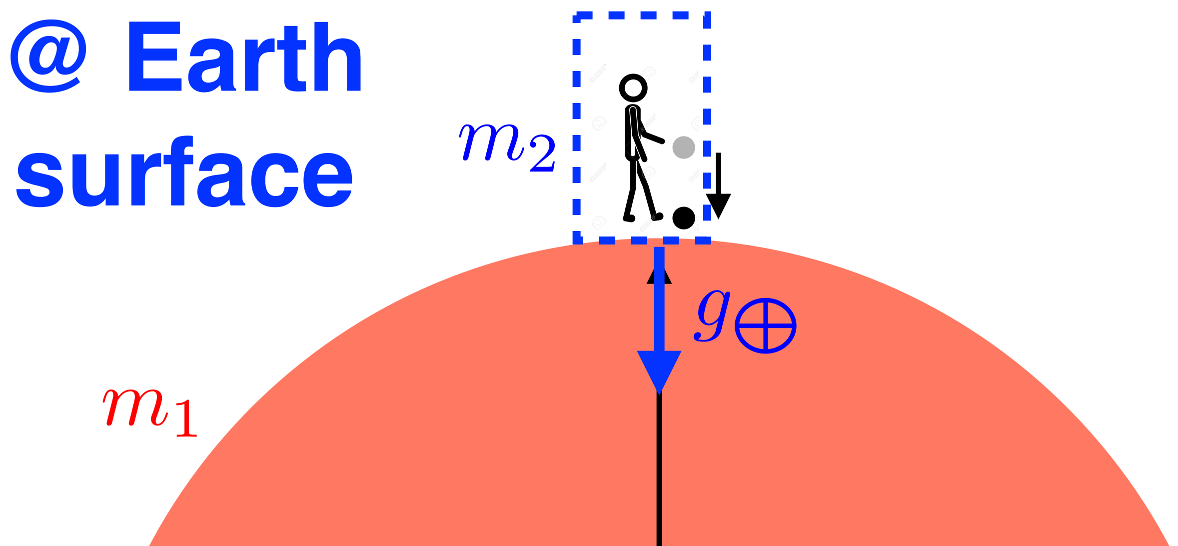
Skip (jump into GW definition)

A PRIMER ON GENERAL RELATIVITY

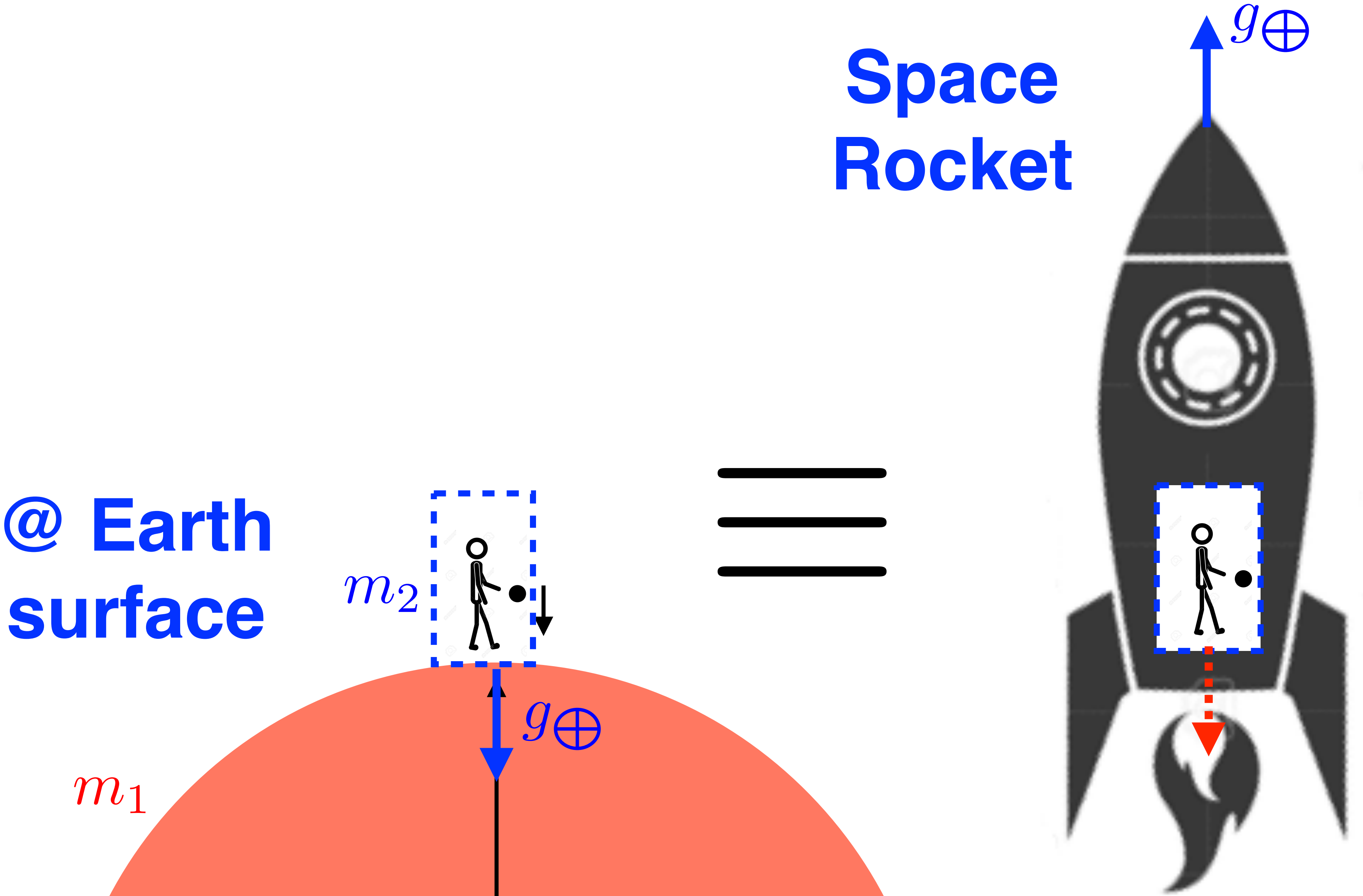
The Equivalence Principle



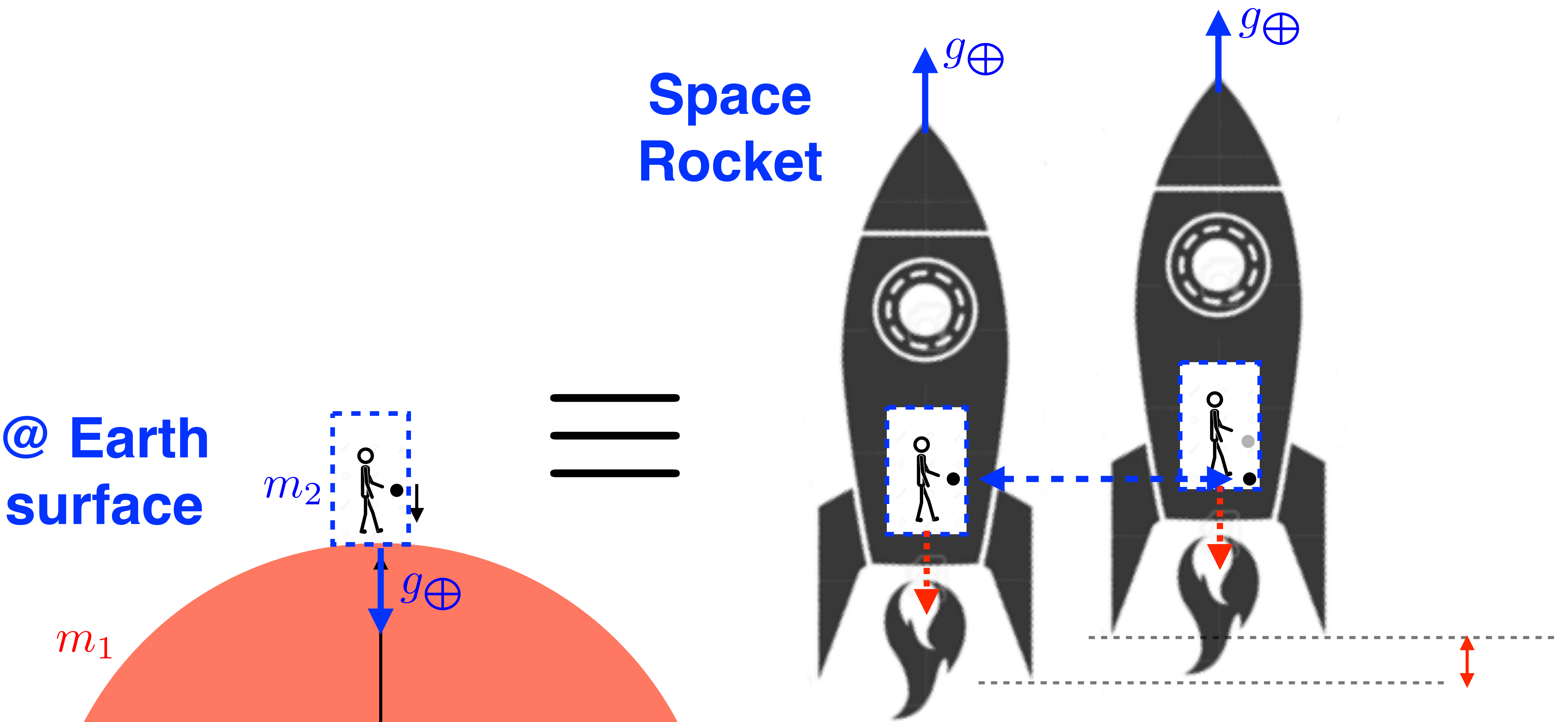
The Equivalence Principle



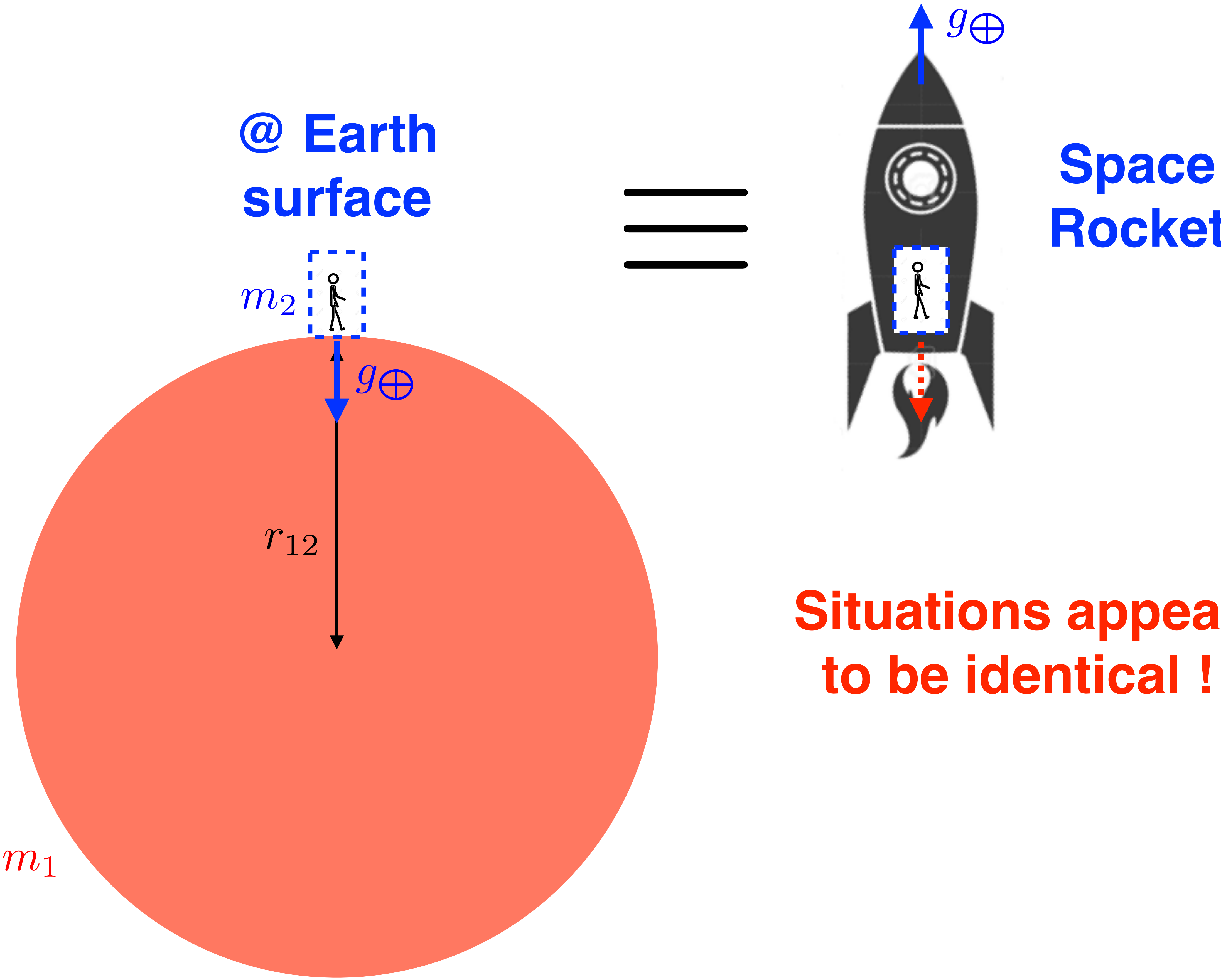
The Equivalence Principle



The Equivalence Principle

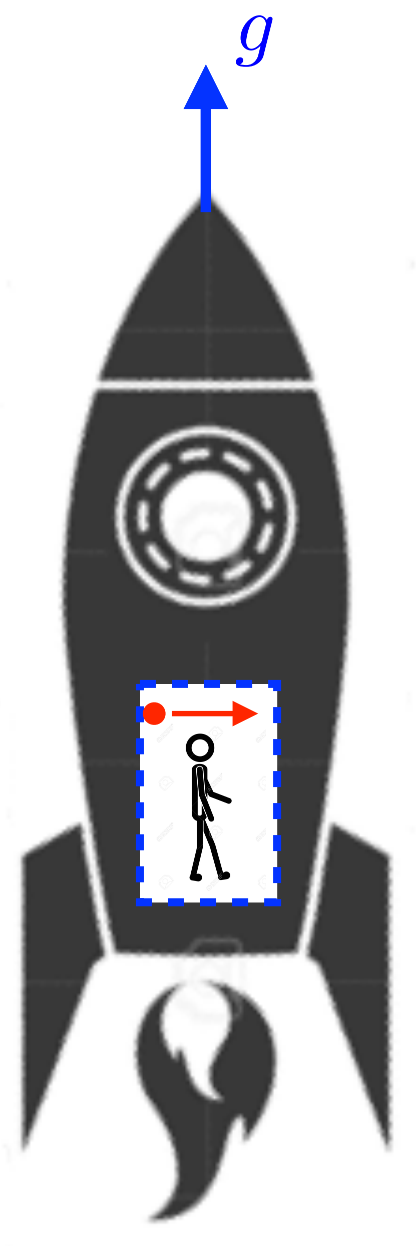


The Equivalence Principle



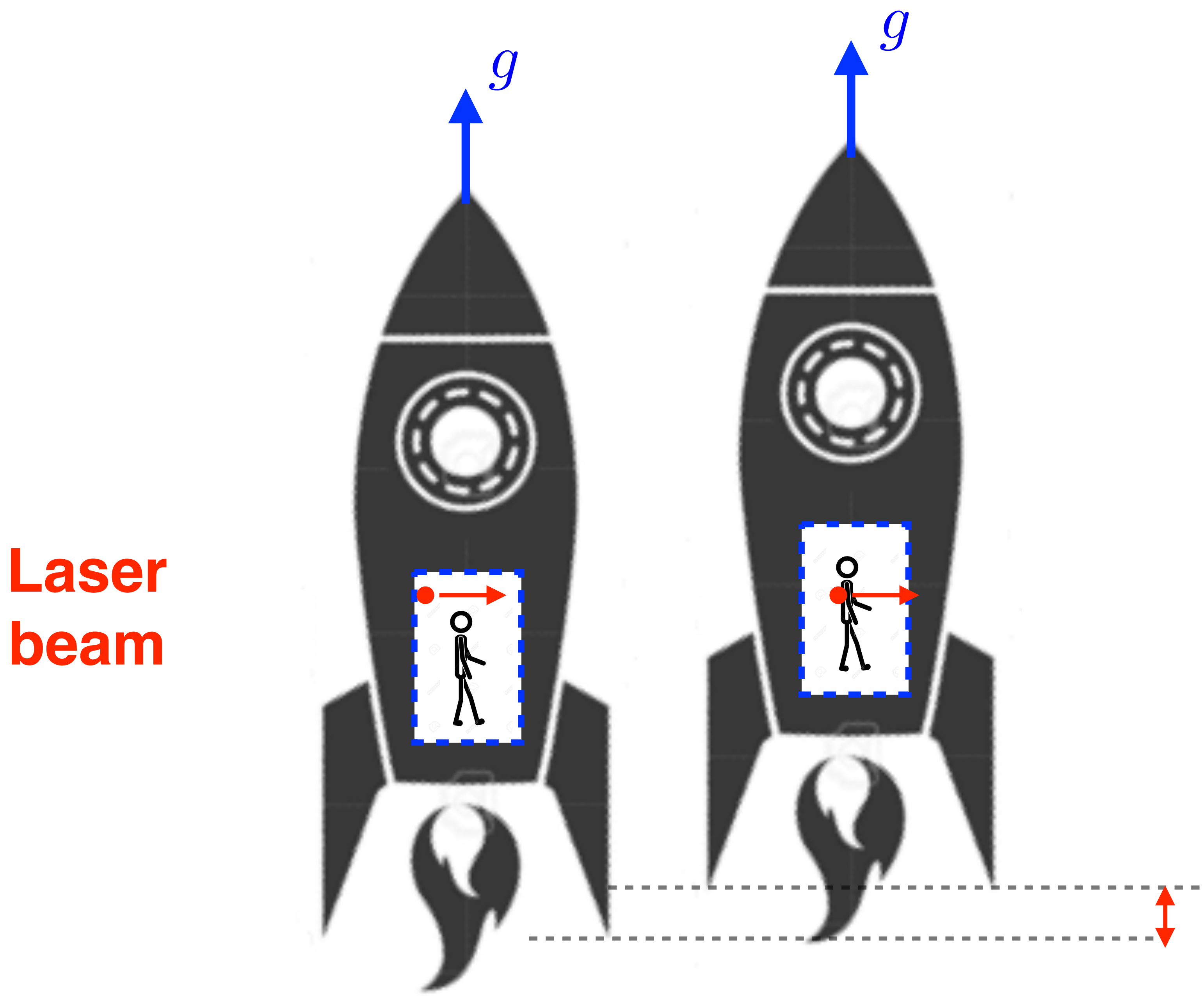
Situations appear to be identical !

The Equivalence Principle



Laser beam

The Equivalence Principle



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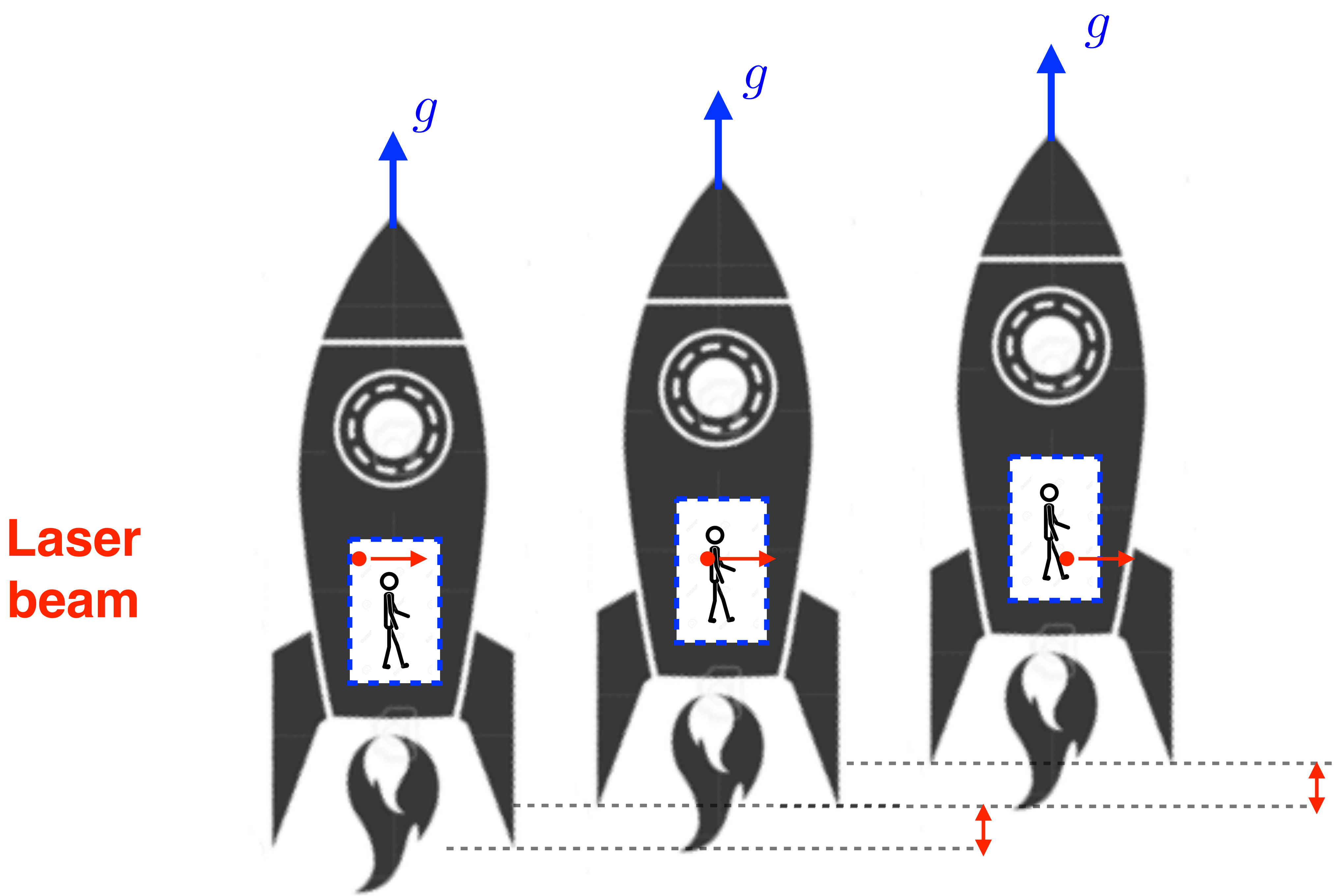
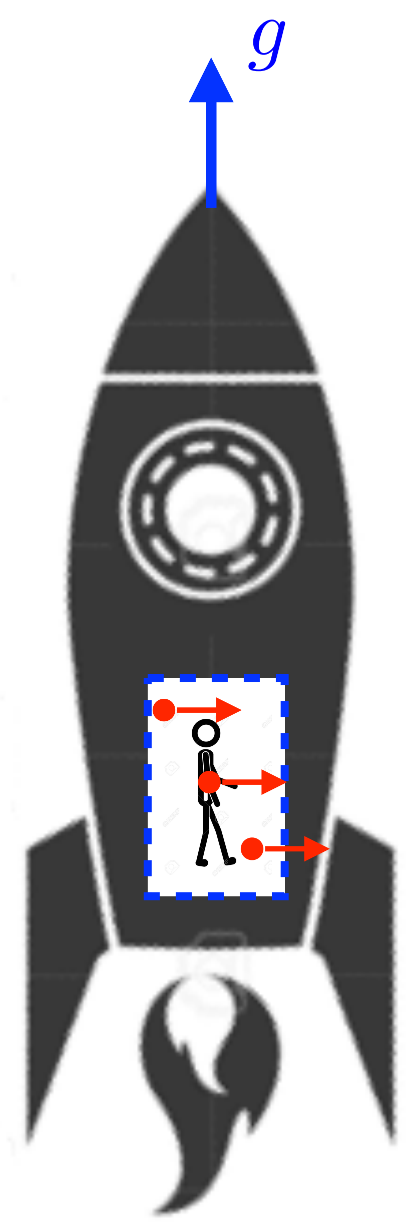
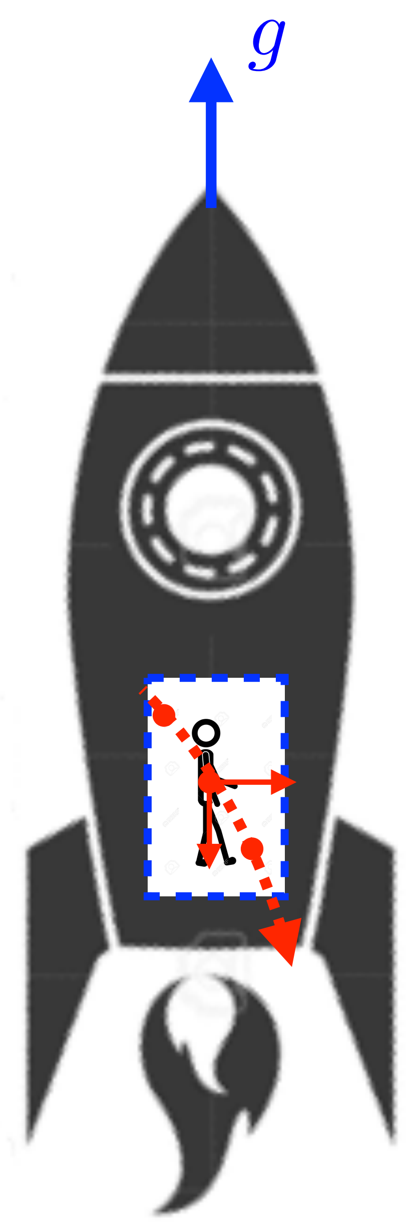


The Equivalence Principle



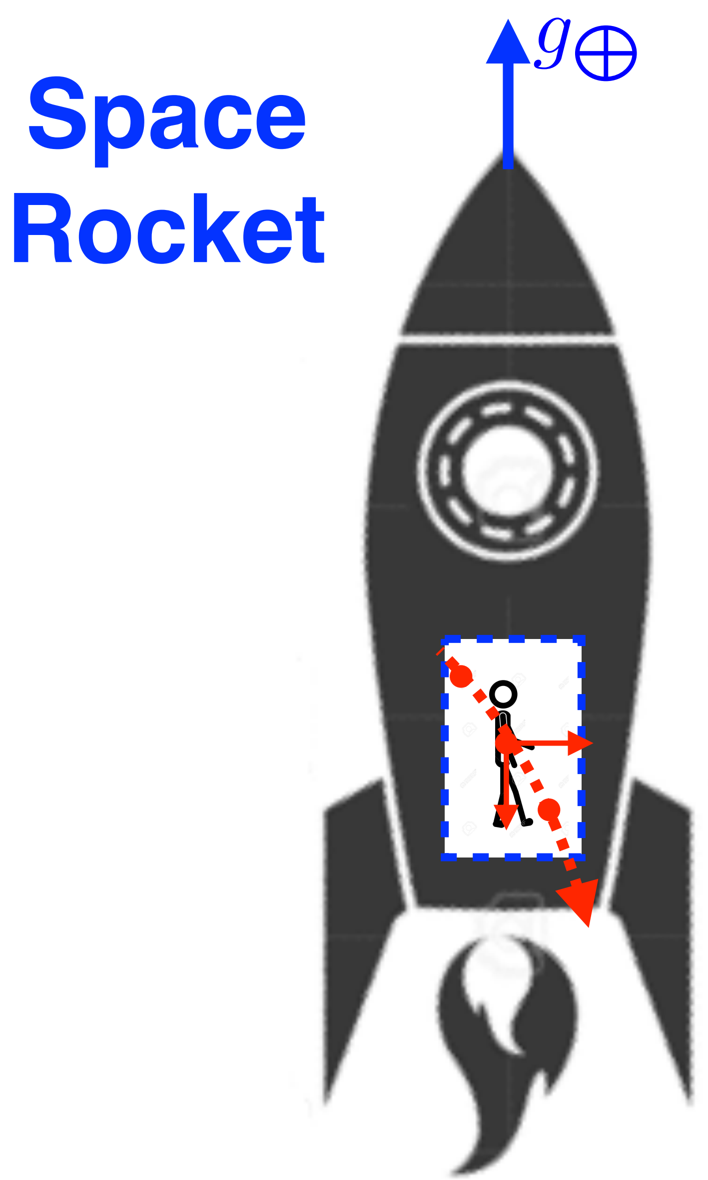
Laser beam

The Equivalence Principle

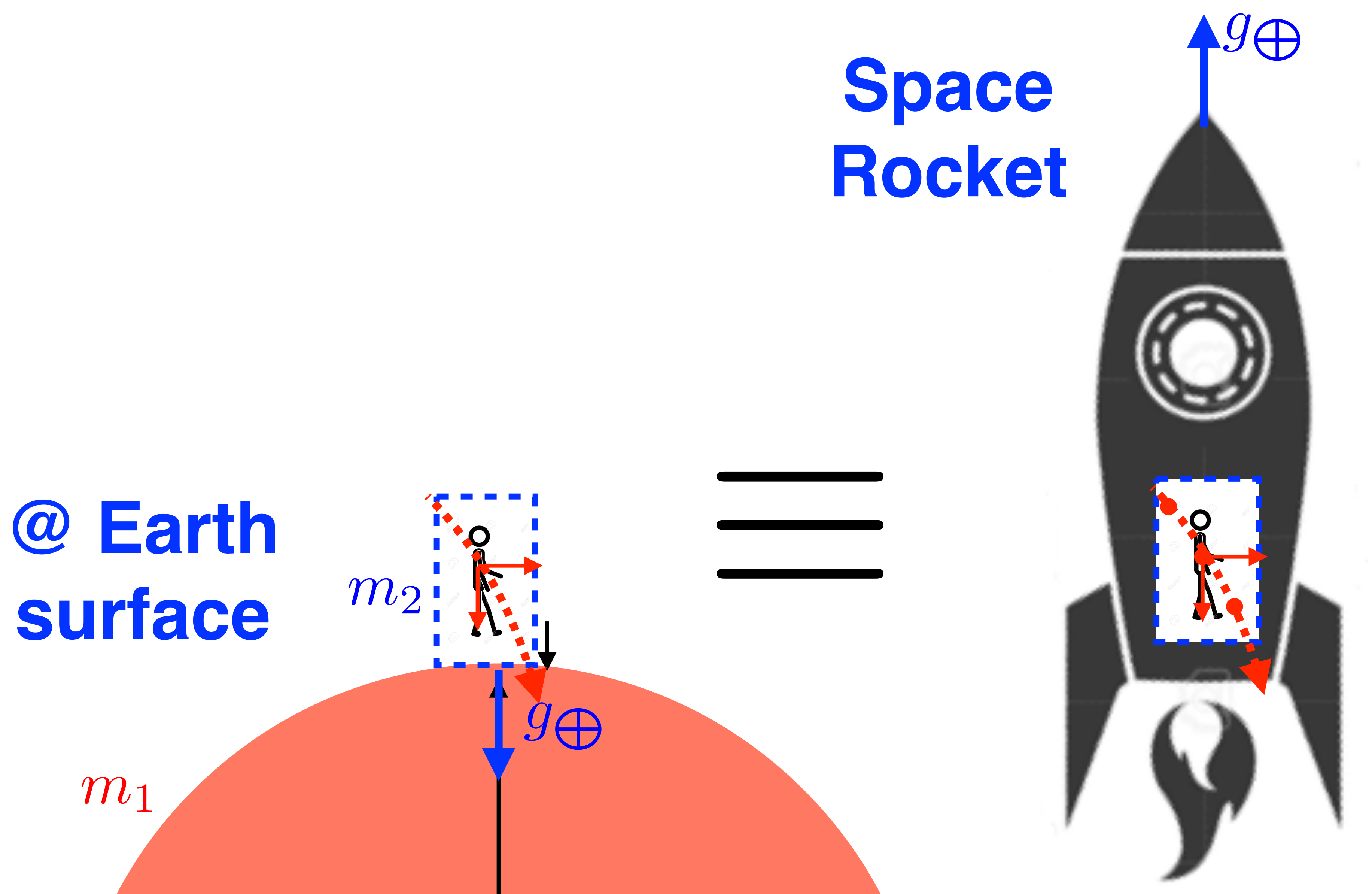


Laser beam

The Equivalence Principle



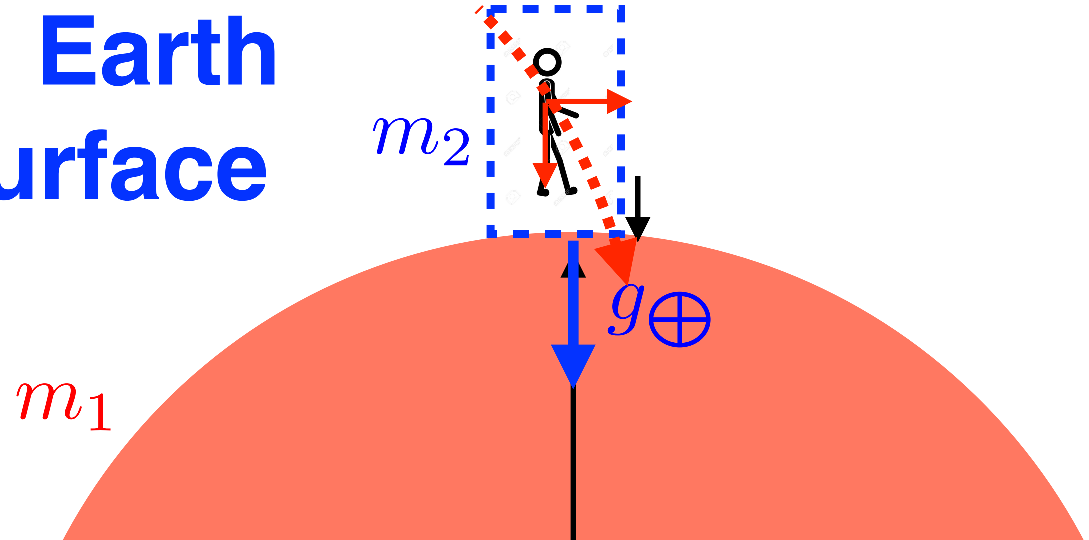
The Equivalence Principle



The Equivalence Principle

Gravitational field must bend light !

@ Earth surface

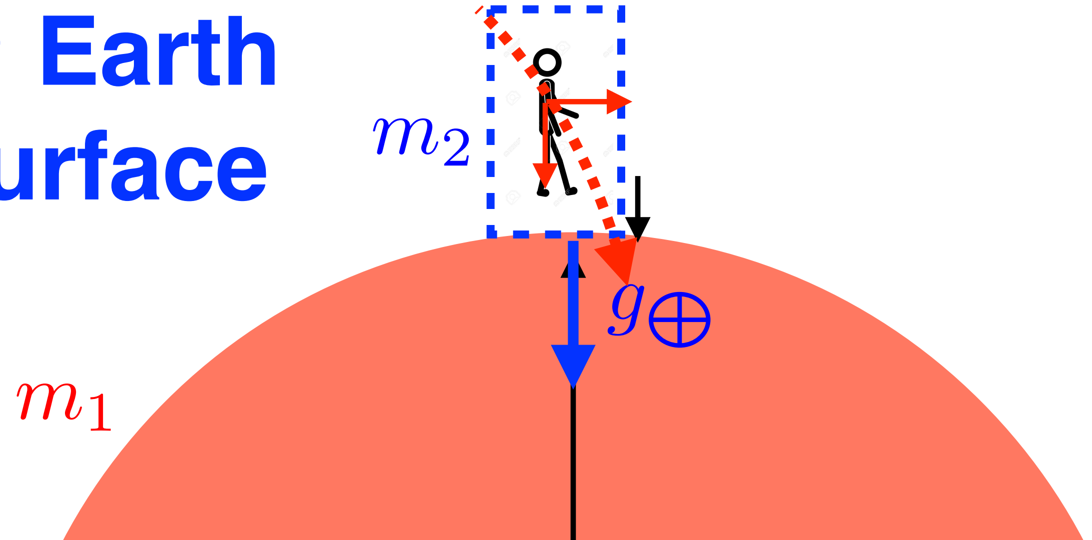


The Equivalence Principle

(Earth gravity too weak to observe the effect, but …)

Gravitational field must bend light !

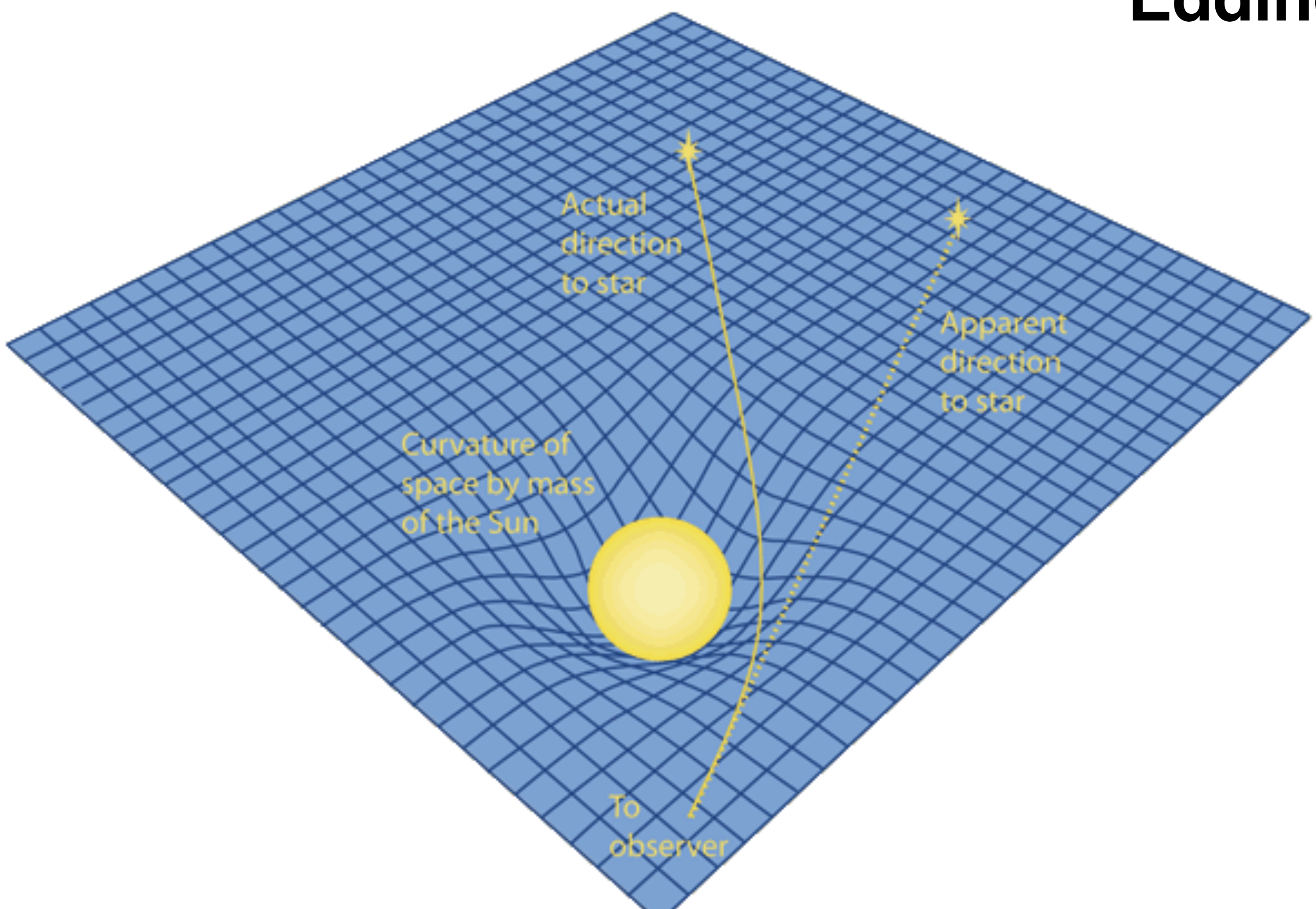
@ Earth surface



The Equivalence Principle

(Earth gravity too weak to observe the effect, but …) → **e.g. the sun does a better job !**

Eddington 1919



The Equivalence Principle

(Earth gravity too weak to observe the effect, but …) →

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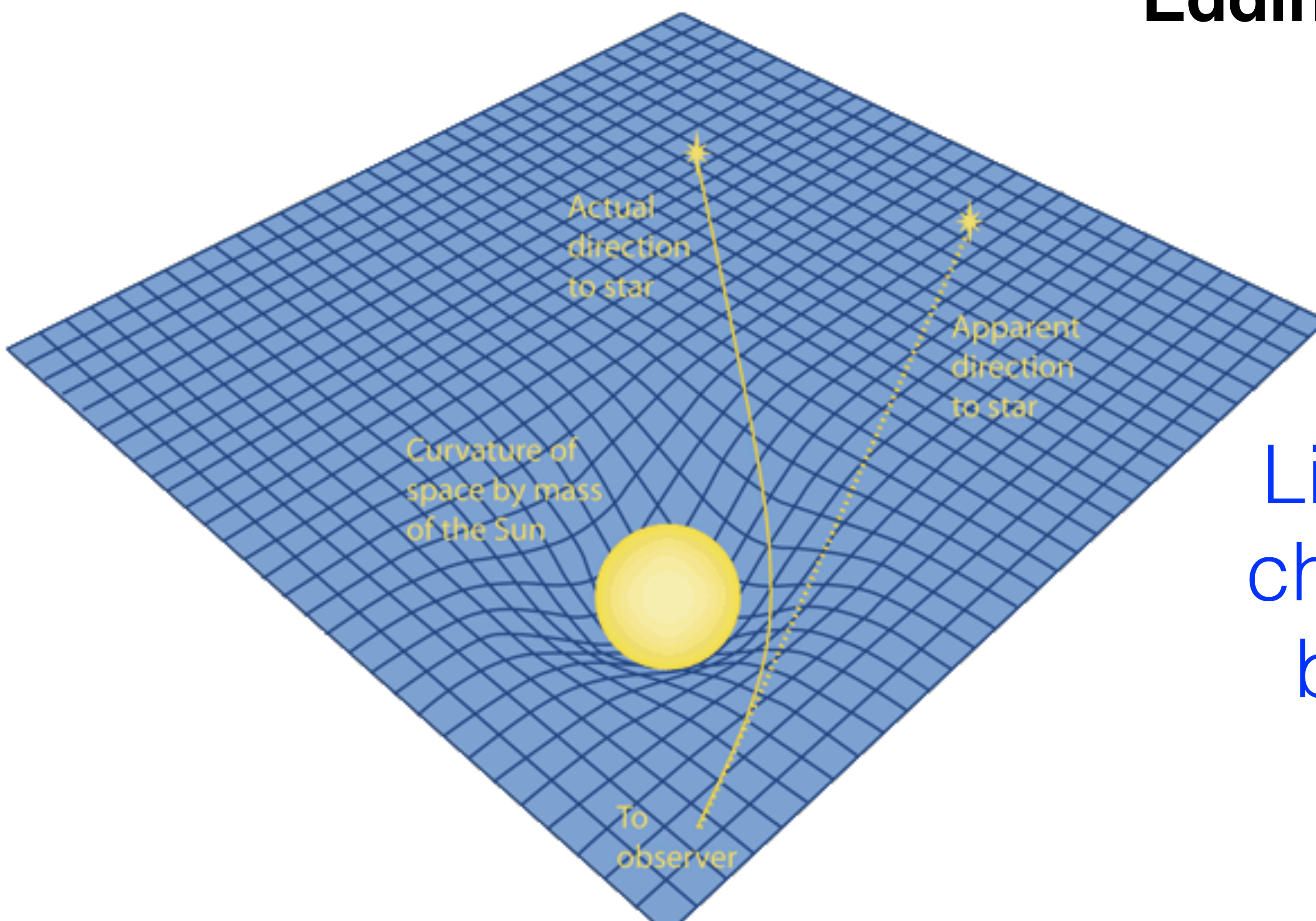
Eddington 1919



Light does not change speed, but direction

The Equivalence Principle

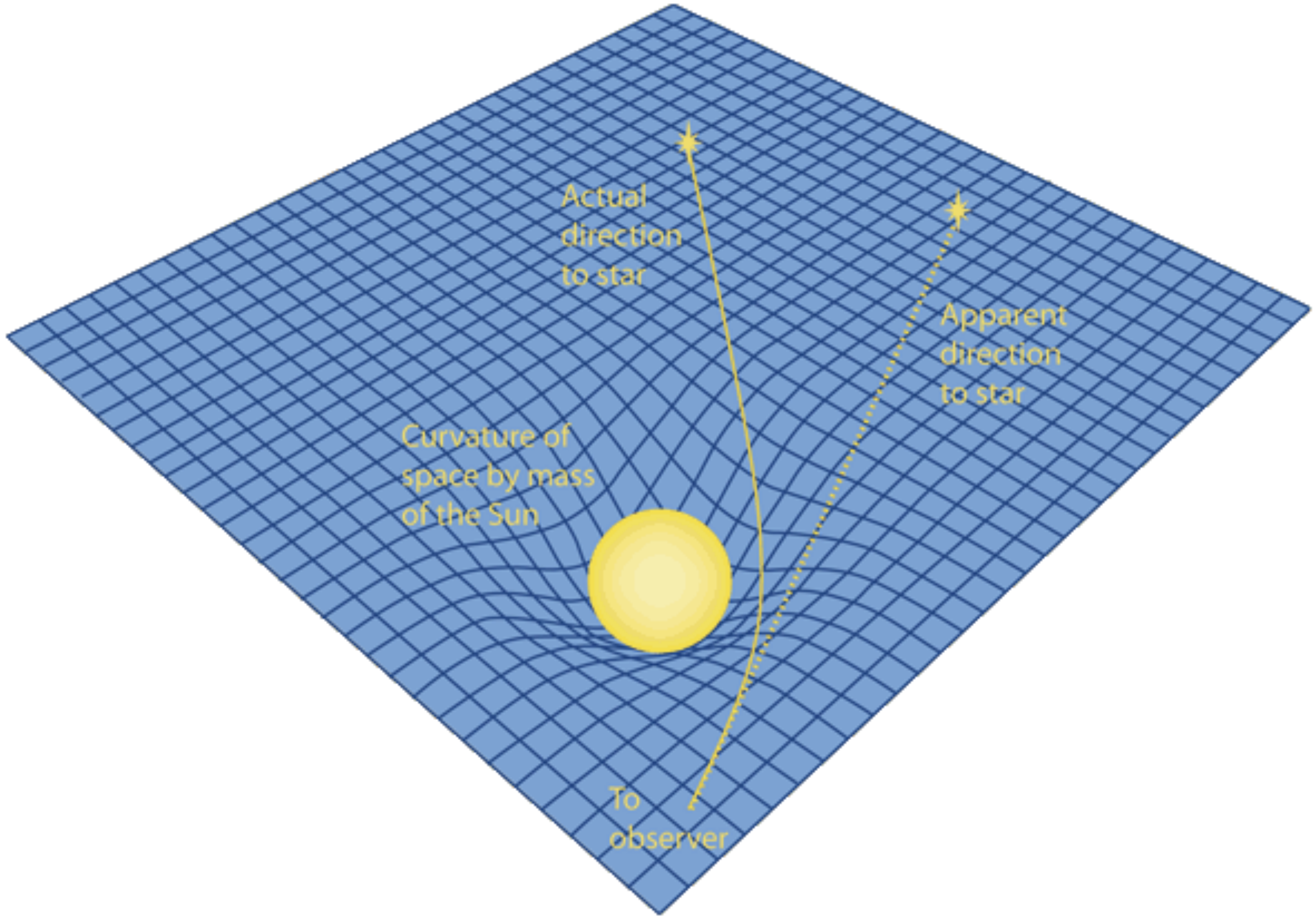
Einstein understood like this…

light bending,

light red/blue-shifting,

gravitational time dilation,

…

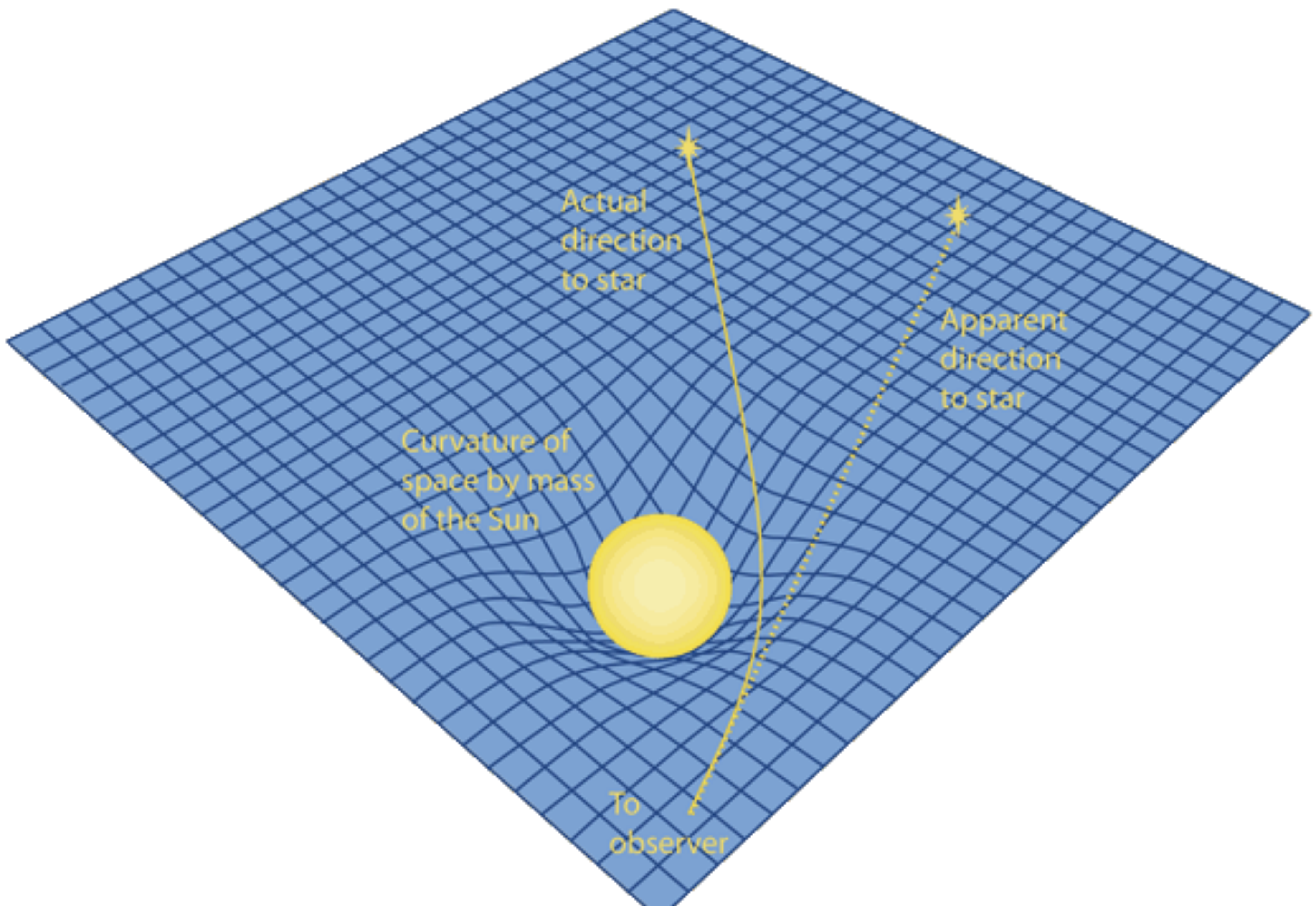


The Equivalence Principle

Einstein understood like this…

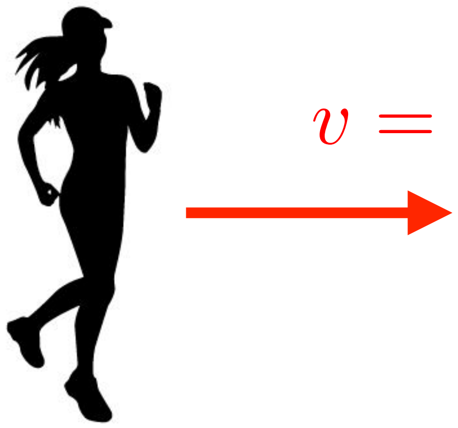
a mathematical formulation was needed !

…

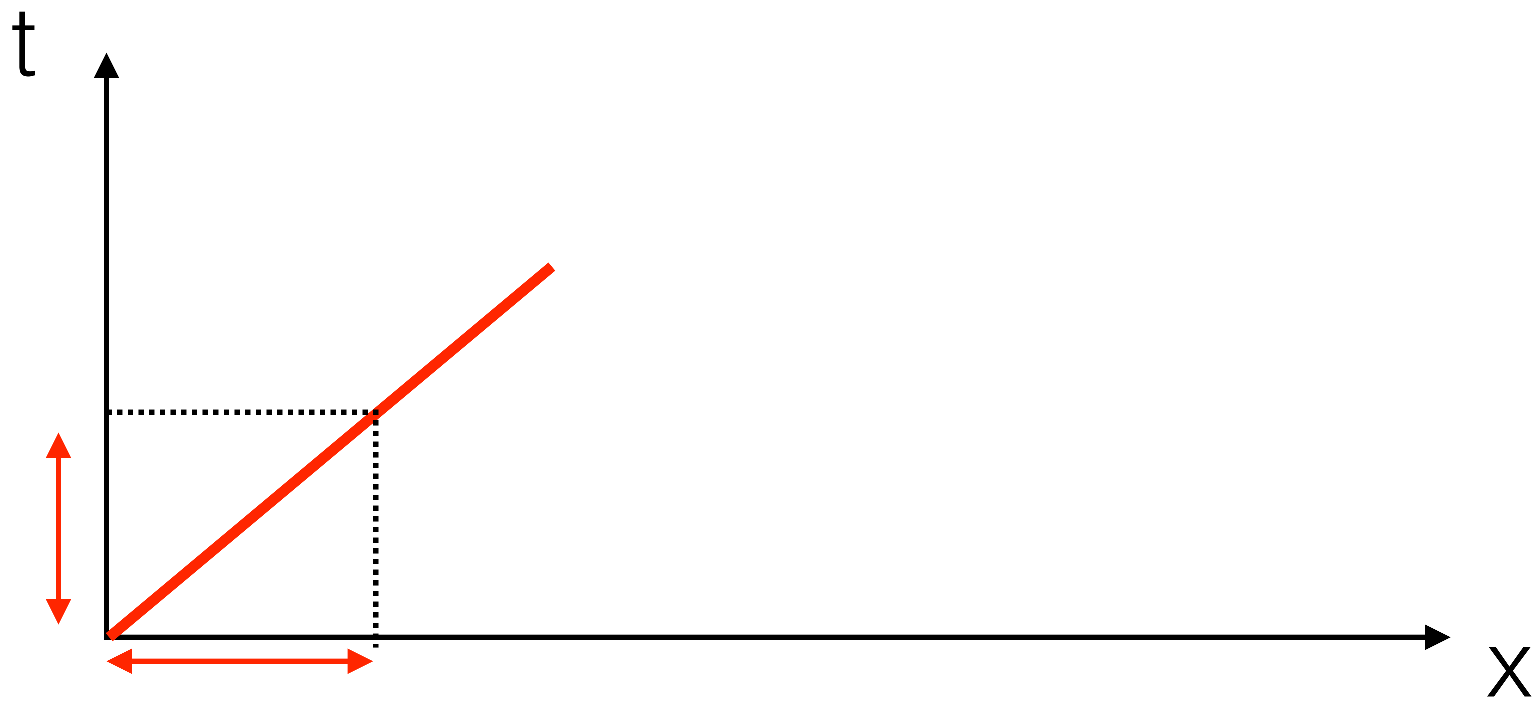


Mathematical formulation of General Relativity (GR)

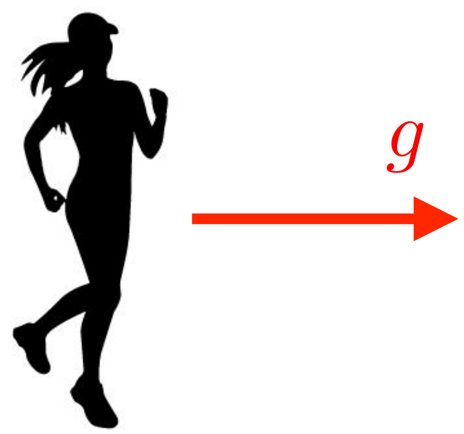
General Relativity Equations

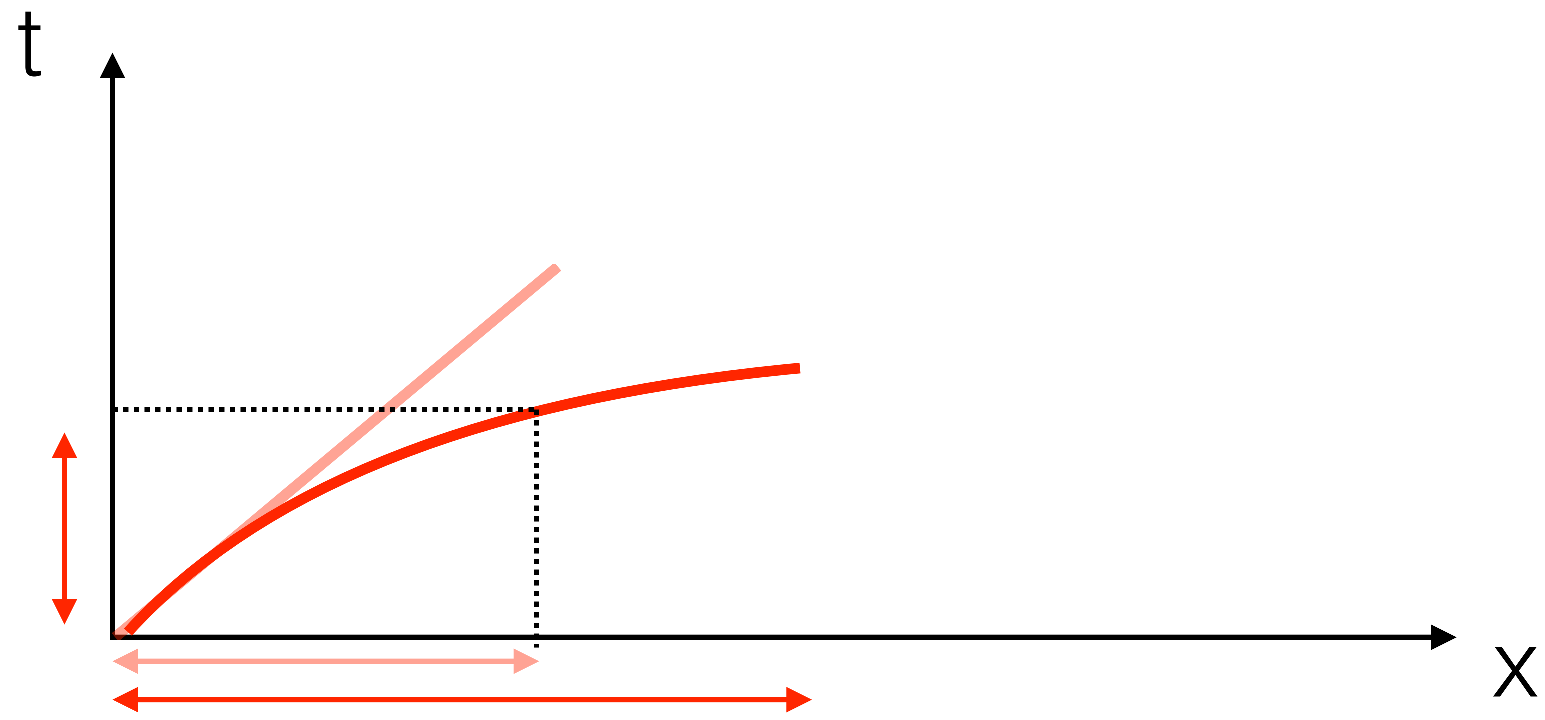


$v = const.$

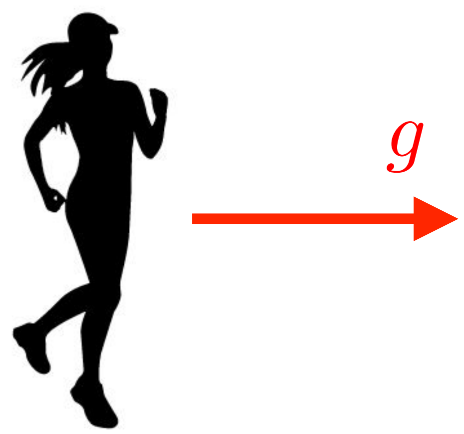


General Relativity Equations

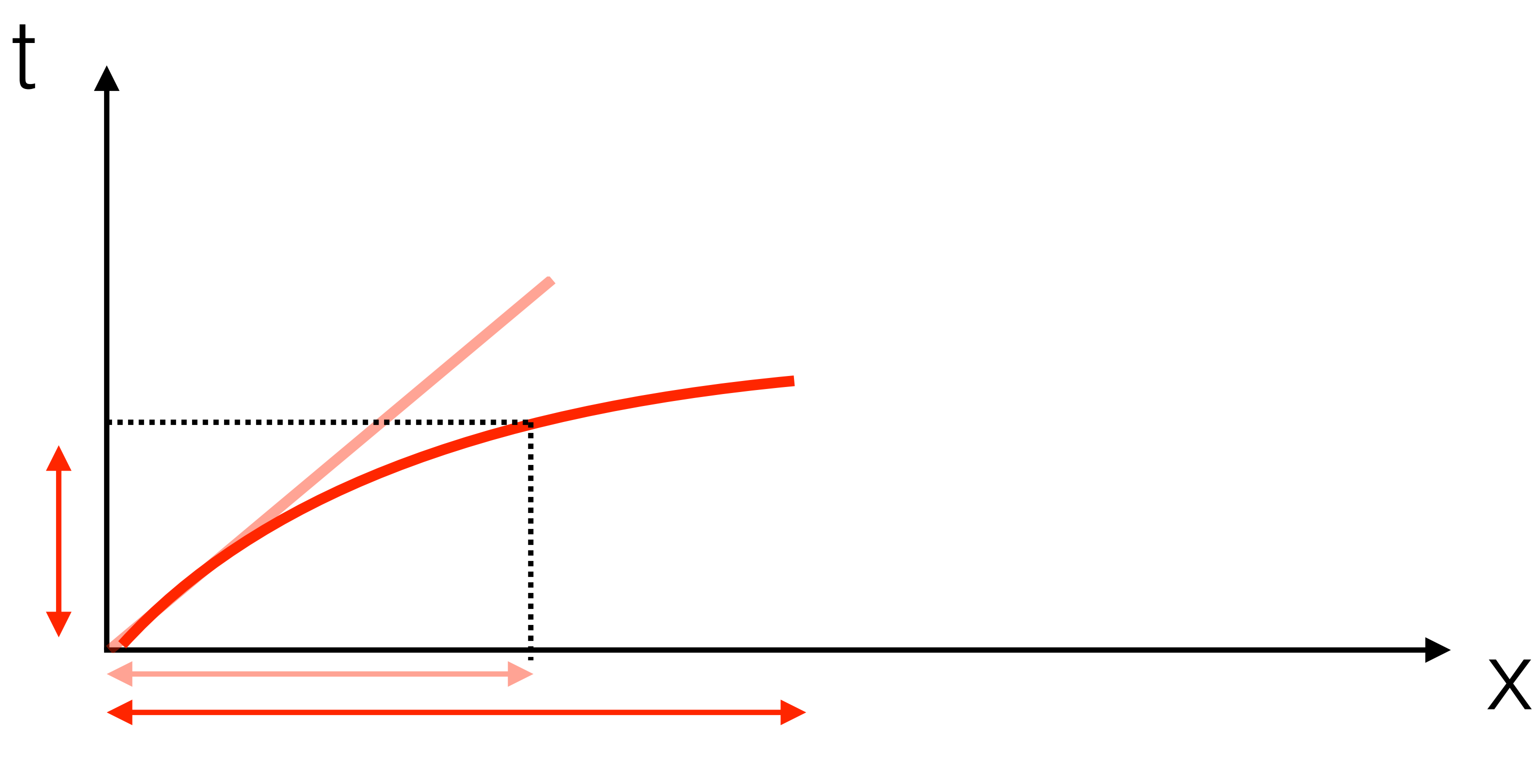




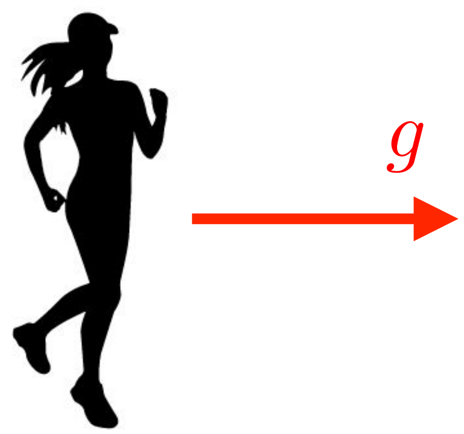
General Relativity Equations



Acceleration $\equiv$ Gravity



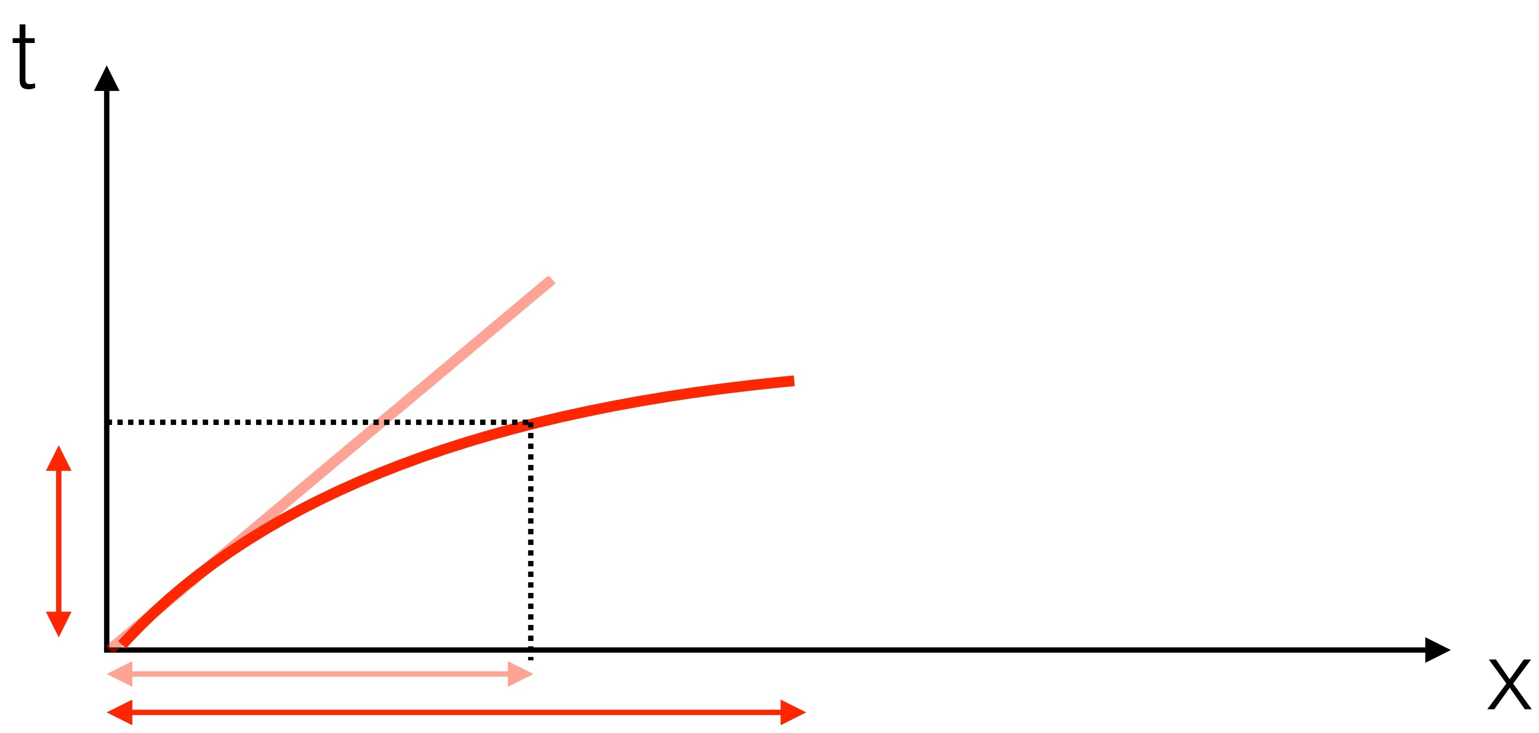
General Relativity Equations



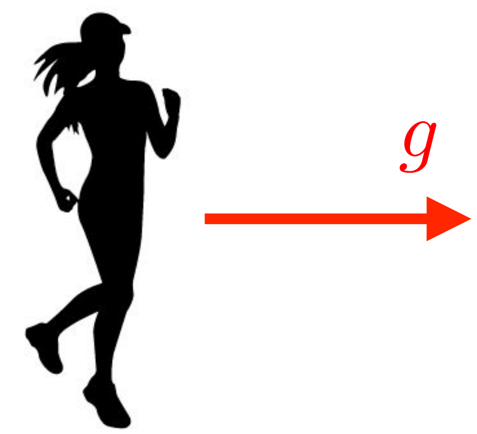
Acceleration $\equiv$ Gravity

$\equiv$

curved space-time trajectory



General Relativity Equations

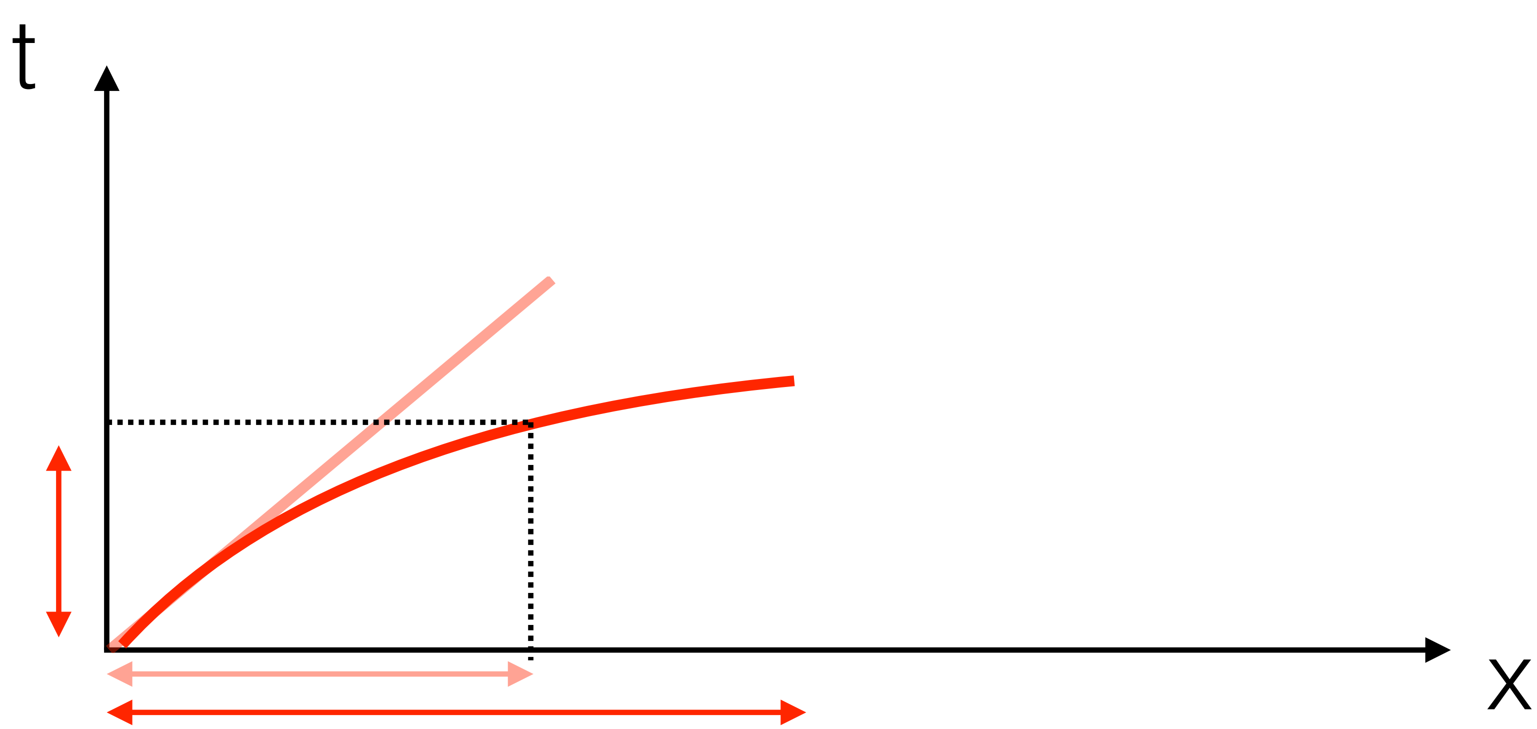


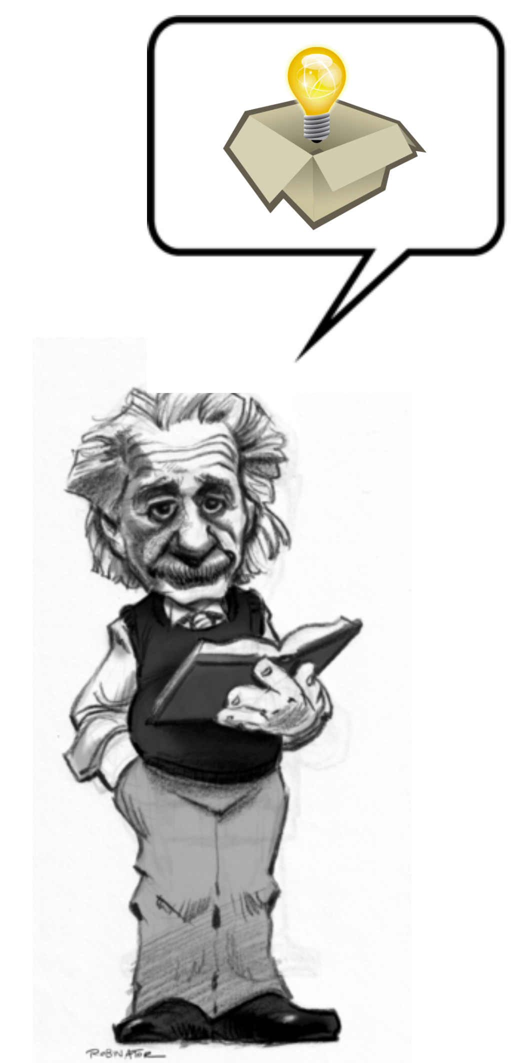
Acceleration $\equiv$ Gravity

$\equiv$

curved space-time

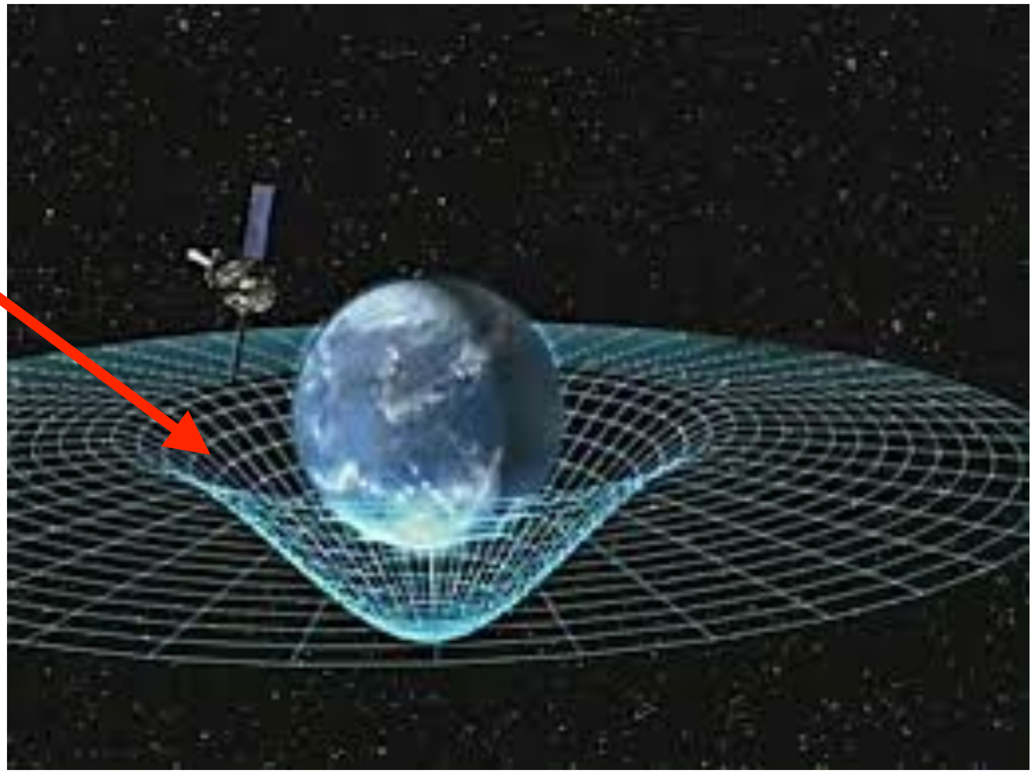
trajectory

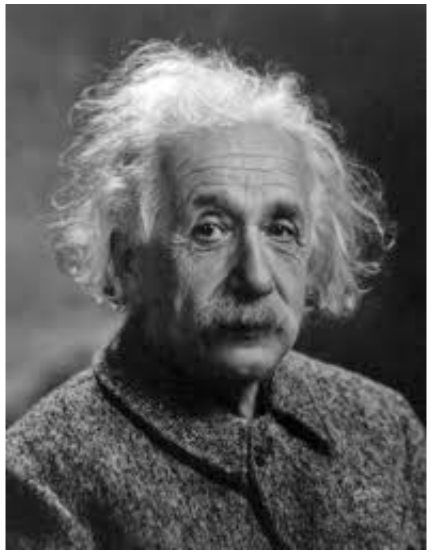




General Relativity Equations

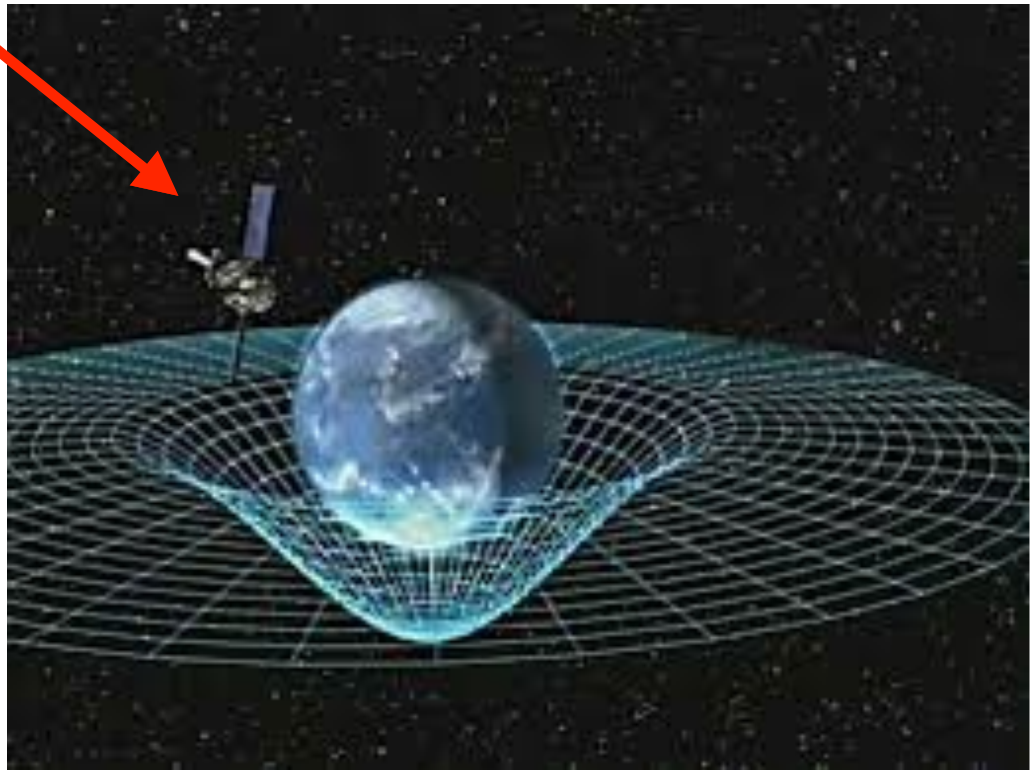
Presence of Matter (Energy/p)
dictates 'Space-Time' Geometry

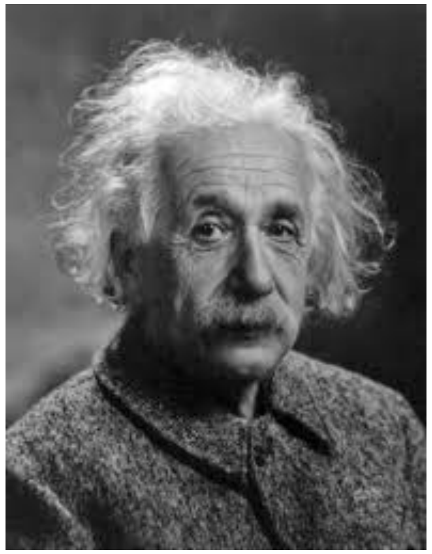




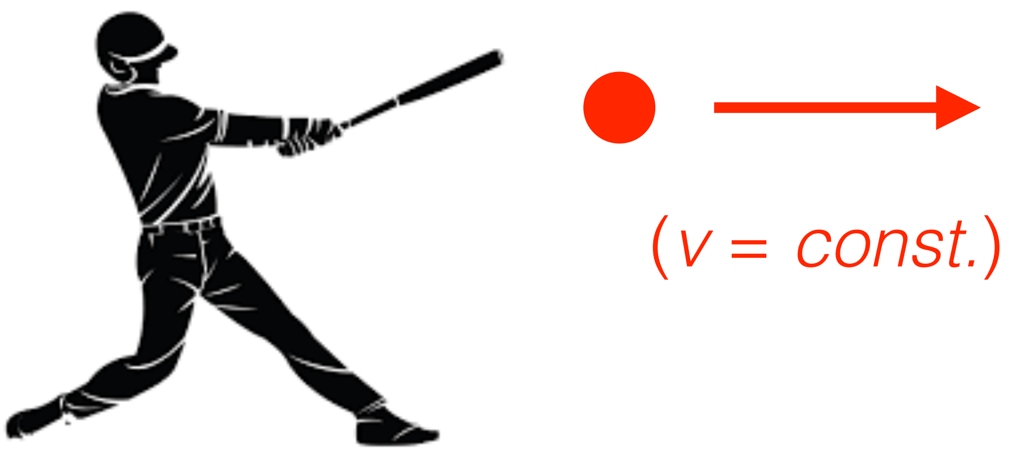
General Relativity Equations

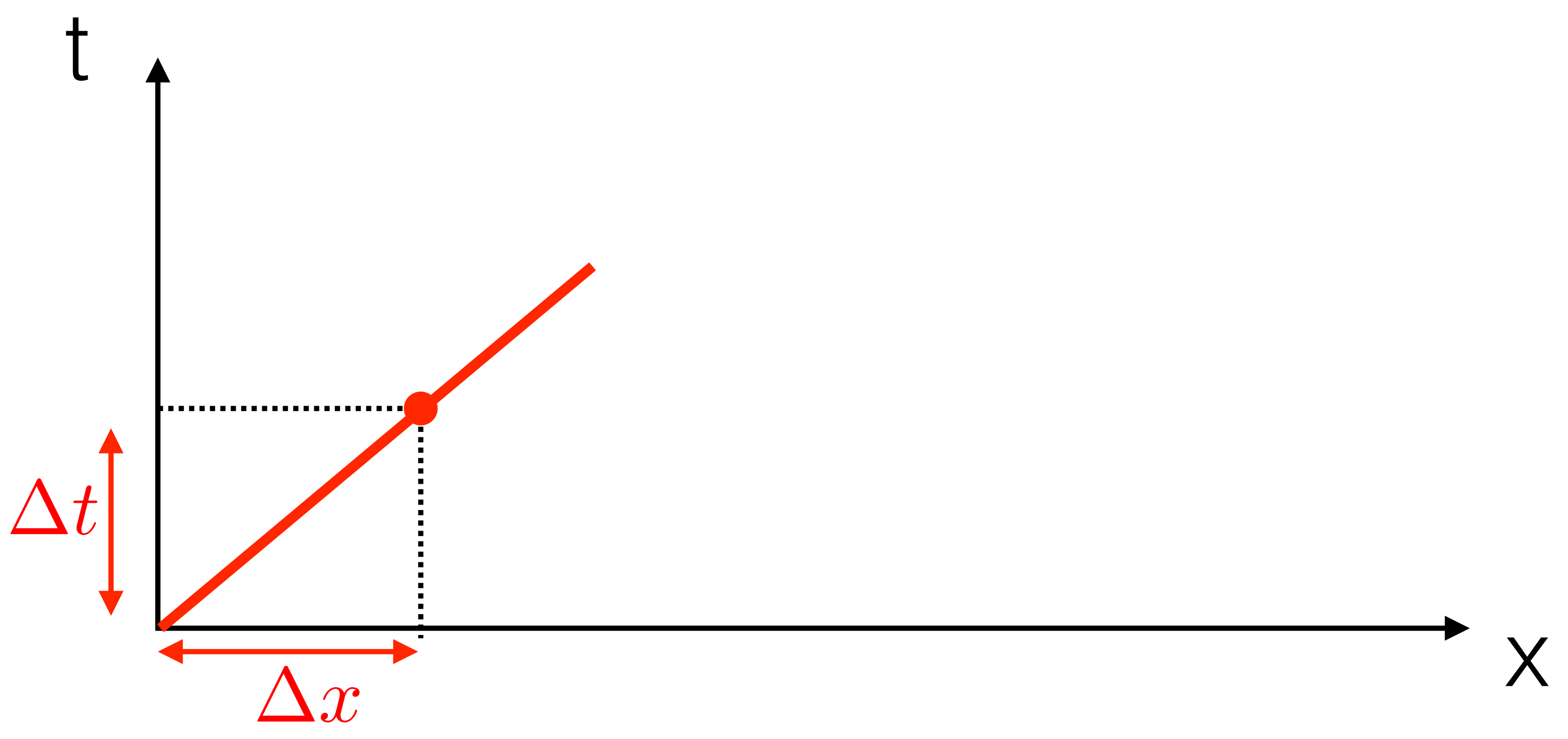
'Space-Time' Geometry
dictates **Movement of Matter**



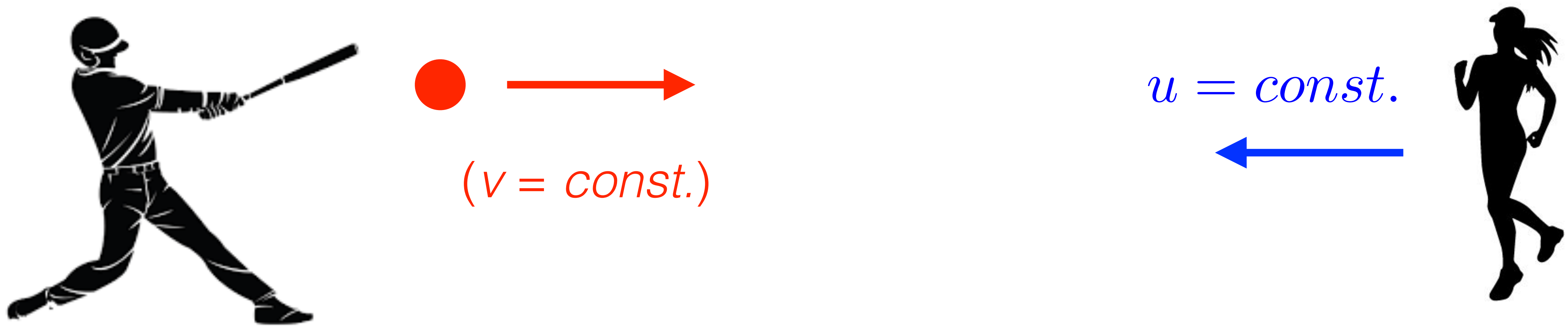


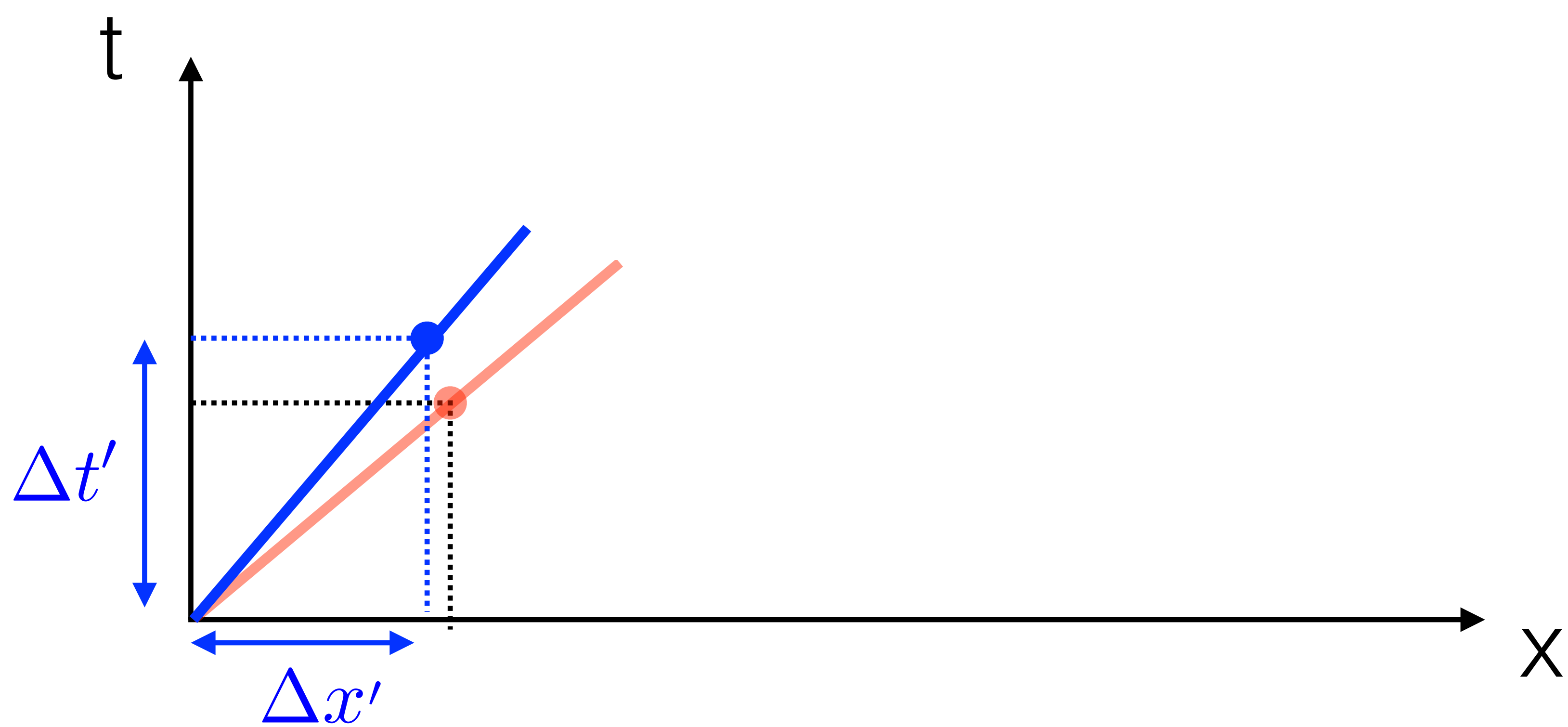
General Relativity Equations





General Relativity Equations



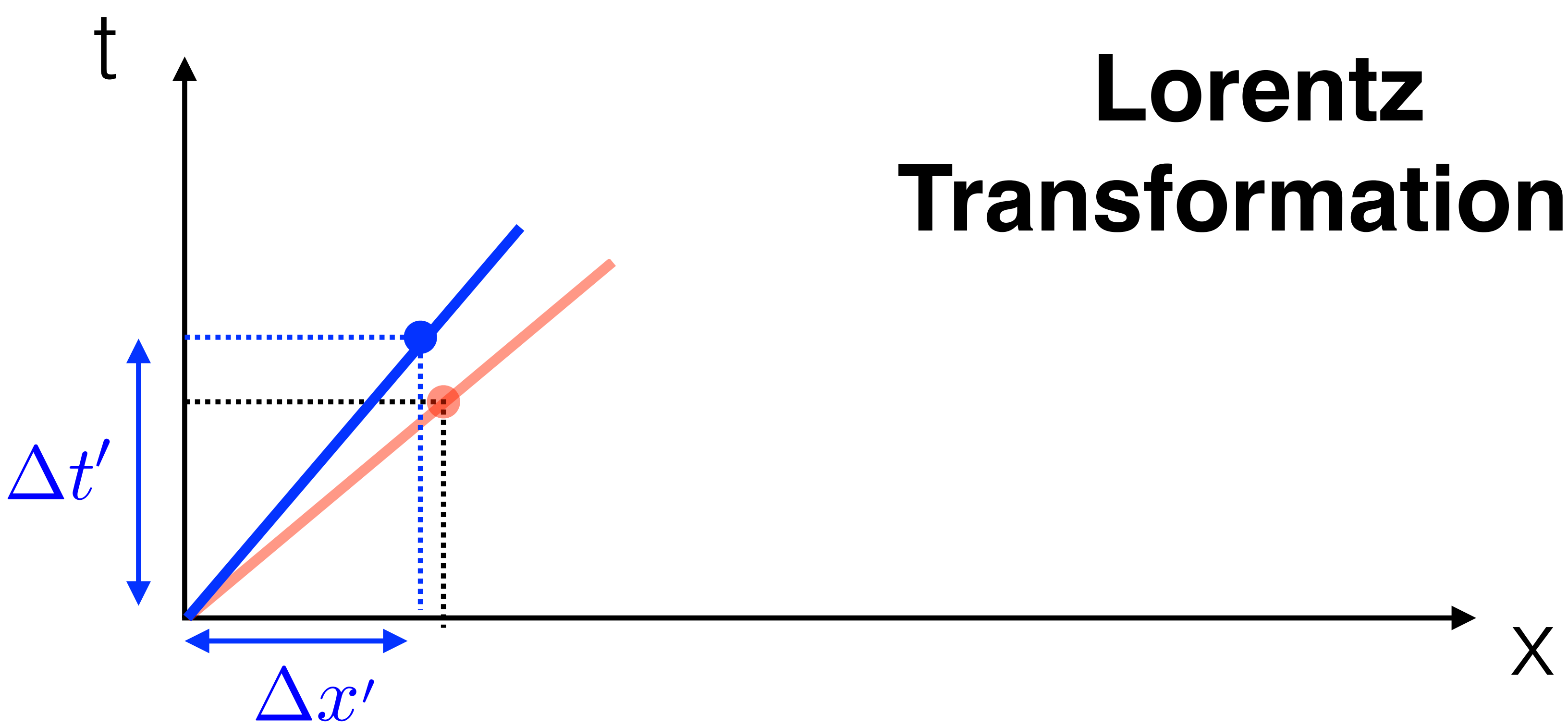


General Relativity Equations

$(v = const.)$

$u = const.$

$$(\Delta x, \Delta t) \neq (\Delta x', \Delta t')$$



Lorentz Transformations

General Relativity Equations

$u = const.$

$$(\Delta x, \Delta t) \neq (\Delta x', \Delta t')$$

$$s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 = s'^2 = (c\Delta t')^2 - (\Delta x')^2$$

General Relativity Equations

$u = const.$

$$(\Delta x, \Delta t) \neq (\Delta x', \Delta t')$$

$$s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 = s'^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$\boxed{ds^2 = c^2 dt^2 - \sum_j dx_j dx^j}$$

Special Relativity

Space-time interval invariant

General Relativity Equations

$$ds^2 = -c^2 dt^2 + \sum_j dx_j dx^j$$

Space-time invariant interval (**Special Relativity**)

General Relativity Equations

$$ds^2 = -c^2dt^2 + \sum_j dx_j dx^j \longrightarrow \boxed{ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu}$$

Einstein convention
Summation over repeated indices

Space-time invariant interval (**Special Relativity**)

$$\eta \equiv \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

General Relativity Equations

$$ds^2 = -c^2 dt^2 + \sum_j dx_j dx^j \longrightarrow \boxed{ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu}$$

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Space-time invariant
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Minkowski Metric

$$\eta \equiv \mathrm{diag}(-,+,+,+)$$

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$$g_{\mu\nu}(x) = \frac{\partial x'^\alpha}{dx^\mu}\frac{\partial x'^\beta}{dx^\nu}g'_{\alpha\beta}(x')$$

General Relativity Equations

General Relativity: Generalisation of Special Relativity

General Relativity Equations

General Relativity: Generalisation of Special Relativity

* **Equivalence Principle** $\longrightarrow$ Geodesic motion

$$\boxed{\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}[g_{**}]\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0}$$

$$\Gamma^\mu{}_{\alpha\beta} = \frac{1}{2}g^{\mu\lambda}\left(g_{\lambda\alpha,\beta} + g_{\lambda\beta,\alpha} - g_{\alpha\beta,\lambda}\right)$$

Christoffel Symbol

General Relativity Equations

General Relativity: Generalisation of Special Relativity

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* **Principle of Relativity** $\longrightarrow$ $x'^\mu = x'^\mu(\{x^\alpha\})$ Arbitrary change of coordinates

$$g_{\mu\nu}(x)dx^\mu dx^\nu = g'_{\alpha\beta}(x')dx'^\alpha dx'^\beta$$

;

$$g_{\mu\nu}(x) = \frac{\partial x'^\alpha}{dx^\mu}\frac{\partial x'^\beta}{dx^\nu}g'_{\alpha\beta}(x')$$

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$$;\quad G_{\mu\nu}[g_{**}] \equiv \boxed{\underbrace{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R}_{\text{space-time geometry}} = \underbrace{\frac{1}{m_p^2}T_{\mu\nu}}_{\text{matter (energy/p)}}}$$

General Relativity Equations

General Relativity: Generalisation of Special Relativity

$$x'^{\mu} = x'^{\mu}(\{x^{\alpha}\})$$

Arbitrary change of coordinates

;

$$g_{\mu\nu}(x)dx^{\mu}dx^{\nu} = g'_{\alpha\beta}(x')dx'^{\alpha}dx'^{\beta}$$

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General Relativity Equations

General Relativity: Generalisation of Special Relativity

$$x'^{\mu} = x'^{\mu}(\{x^{\alpha}\})$$ Arbitrary change of coordinates ;

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$$R_{\alpha\beta} = \Gamma^{\mu}{}_{\alpha\beta,\mu} - \Gamma^{\mu}{}_{\alpha\mu,\beta} + \Gamma^{\mu}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\alpha\beta} - \Gamma^{\mu}{}_{\lambda\beta}\Gamma^{\lambda}{}_{\alpha\mu}$$ Ricci tensor

$$\Gamma^{\mu}{}_{\alpha\beta} = \frac{1}{2}g^{\mu\lambda}\left(g_{\lambda\alpha,\beta} + g_{\lambda\beta,\alpha} - g_{\alpha\beta,\lambda}\right) \sim (metric)^2$$ Christoffel Symbol

General Relativity Equations

General Relativity: Generalisation of Special Relativity

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(Ricci scalar)

$$R_{\alpha\beta} = \Gamma^{\mu}{}_{\alpha\beta,\mu} - \Gamma^{\mu}{}_{\alpha\mu,\beta} + \Gamma^{\mu}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\alpha\beta} - \Gamma^{\mu}{}_{\lambda\beta}\Gamma^{\lambda}{}_{\alpha\mu} \quad ; \quad R \equiv g^{\mu\nu}R_{\mu\nu}$$

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General Relativity Equations

General Relativity (GR)

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry matter

metric

$$G_{\mu\nu} \equiv \mathcal{D}[g_{\alpha\beta}] \quad = \quad m_p^{-2} T_{\mu\nu}(\mathrm{Matt}, \mathrm{Rad}, \mathrm{Top.Defects}, \mathrm{DarkEnergy}, ...)$$

2nd order, non-Linear

source

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2nd order, non-Linear

source

Extremely difficult to solve !

END of digression on

GENERAL RELATIVITY

Gravitational Framework

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Extremely difficult to solve !

Jump to GW def ?

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2nd order, non-Linear

source

$$\begin{tikzcd} & T^{[U]}_{\mu\nu} \arrow[dd, bend left] \\ \textbf{Universe ?} \arrow[ur] \arrow[dr] & \\ & g^{[U]}_{\mu\nu} \end{tikzcd}$$

BASICS of COSMOLOGY

Cosmology

- **hot Big Bang (hBB)** (evolution of the Universe)
  - **theoretical pillars**
    - General Relativity ✓
    - Cosmological Pple
  - **observational pillars**
    - Expansion
    - CMB
    - BBN
- **Inflation** = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ 'cures' hBB $\Longrightarrow$ $\begin{cases}\text{Cosmological Pple}\\ \text{CMB/LSS}\end{cases}$

Cosmological Principle

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry of the Universe

matter within the Universe

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geometry of the Universe — matter within the Universe

→

Principle of Symmetry:

The Universe is Homogeneous & Isotropic

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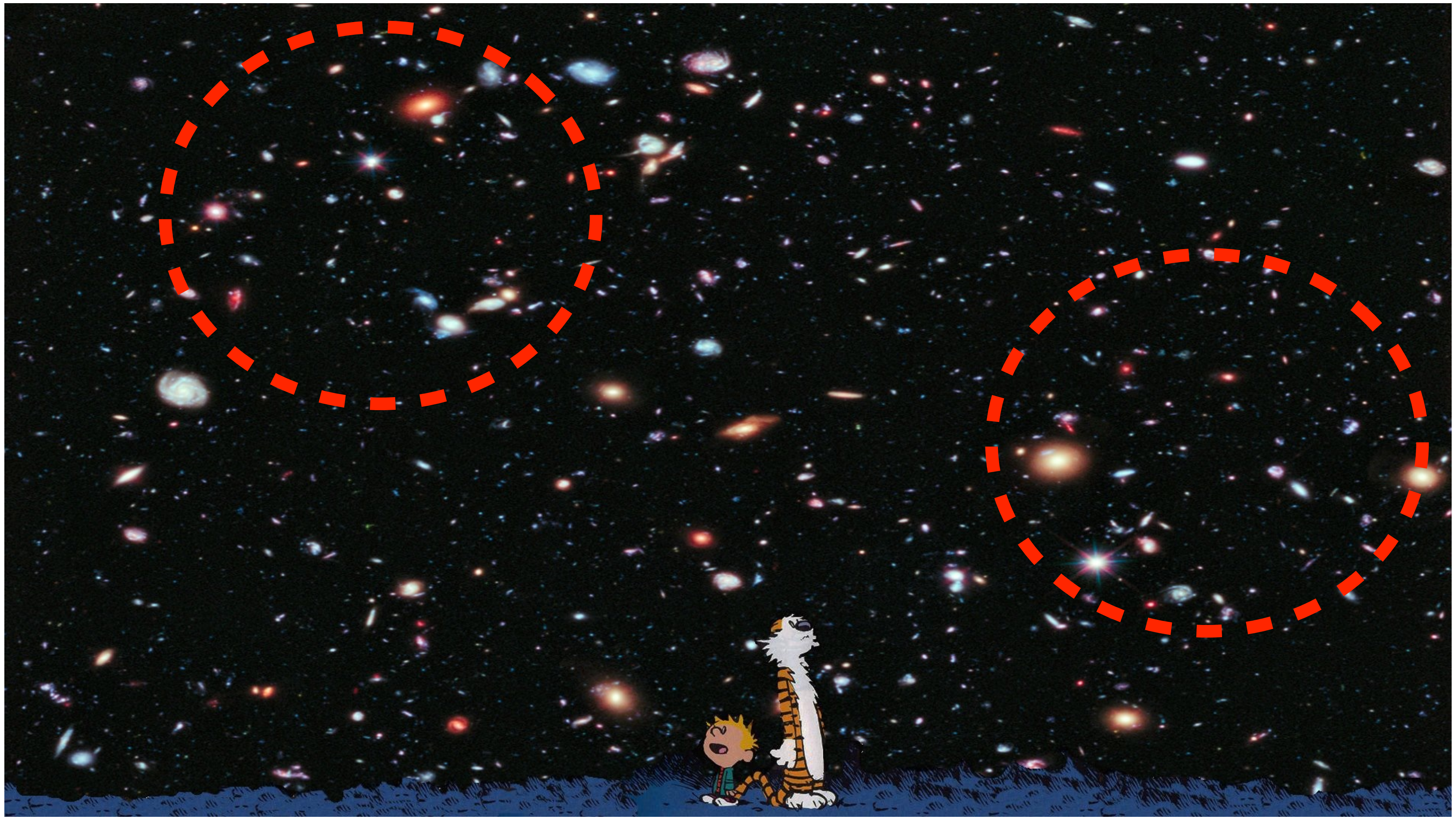

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geometry of the Universe — matter within the Universe

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$$g_{\mu\nu}^{[U]} \equiv \mathrm{diag}\left(-1, \frac{a^2(t)}{1-kr^2}, a^2(t)r^2, a^2(t)r^2\sin^2\theta\right)$$

FLRW

Friedmann-Lemaître -Robertson-Walker

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geometry of the Universe — matter within the Universe

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$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right\}$$

Scale Factor (dynamical)

Curvature (const.)

$$\begin{cases} k < 0, \text{Open} \\ k = 0, \text{Flat} \\ k > 0, \text{Close} \end{cases}$$

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geometry of the Universe — matter within the Universe

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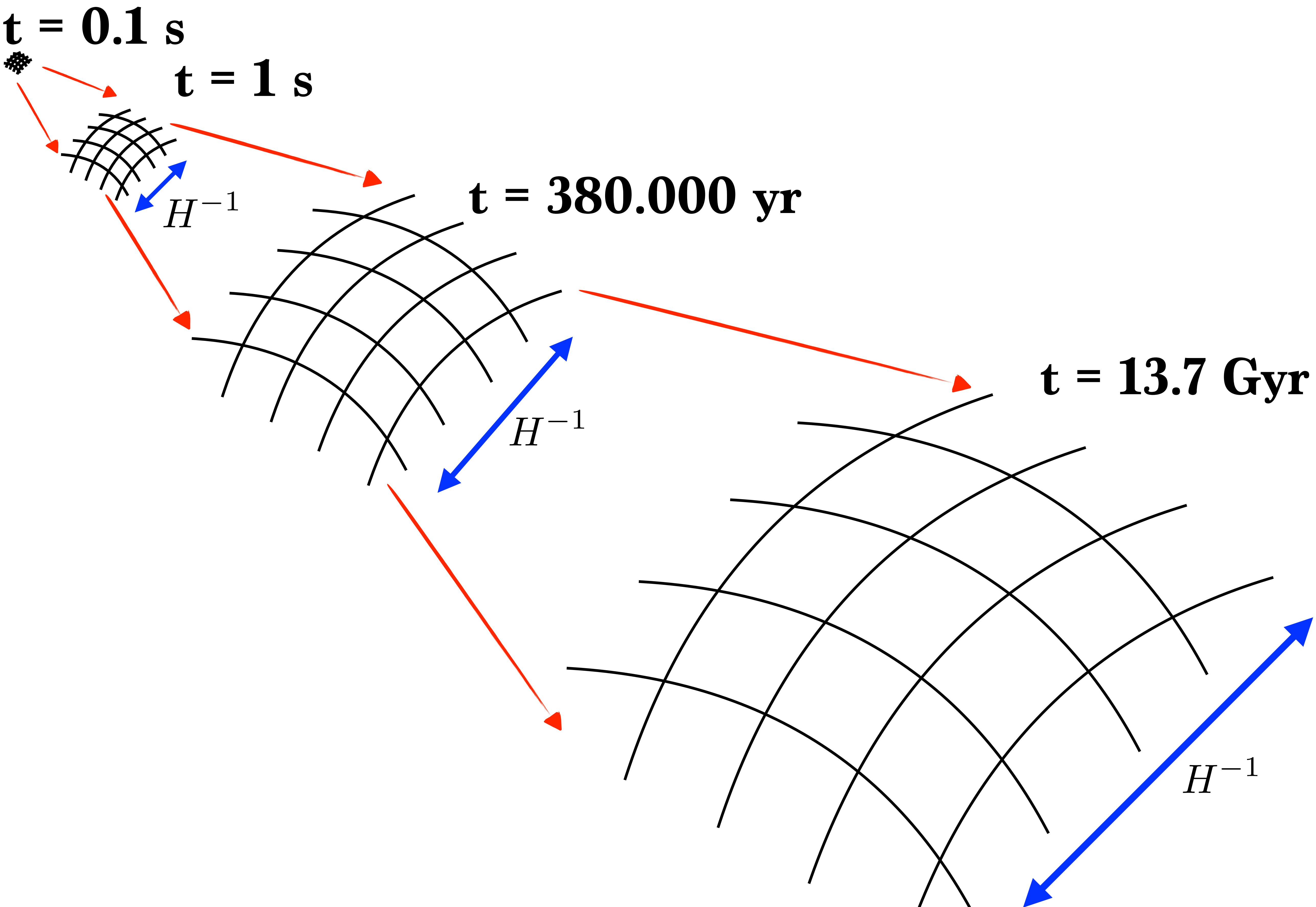
invariant: $(c = \mathrm{const.})$

$$\left\{\begin{array}{l} k \to k/c^2 \\ r \to c \cdot r \\ a \to a/c \end{array}\right. \Longrightarrow \left\{\begin{array}{ll} a, r, k & \text{unphysical} \\ \dfrac{k}{a^2},\ a\cdot r,\ kr^2 & \text{physical} \end{array}\right.$$

Expanding Universe



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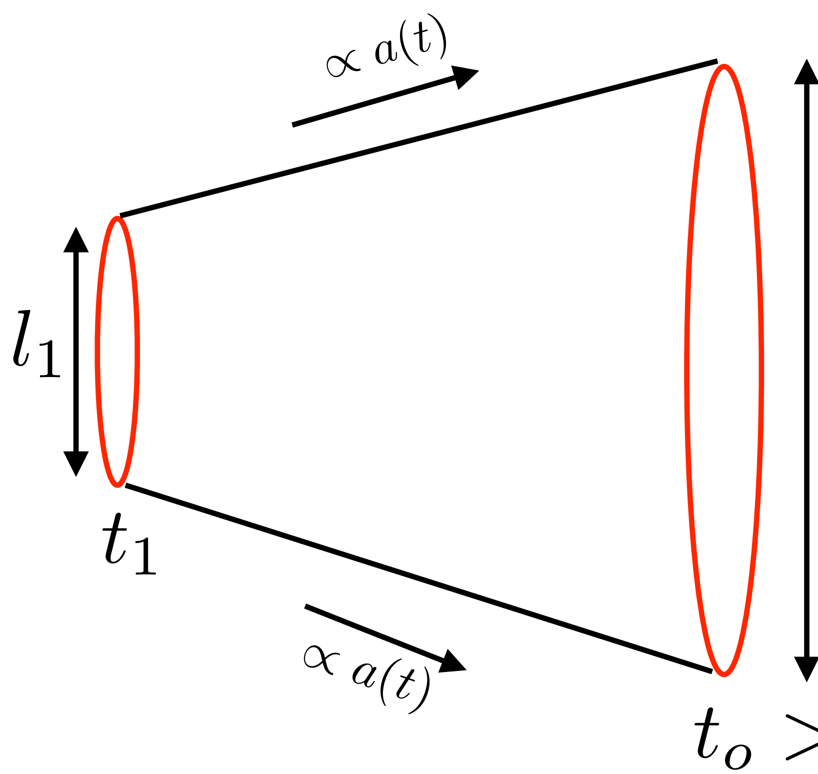

$$l_o \equiv \left(\frac{a(t_o)}{a(t_1)} \right) l_1$$

Redshift

$$z_1 \equiv \frac{a_o - a_1}{a_1}$$

$$\boxed{1 + z \equiv \frac{a(t_o)}{a(t)}}$$

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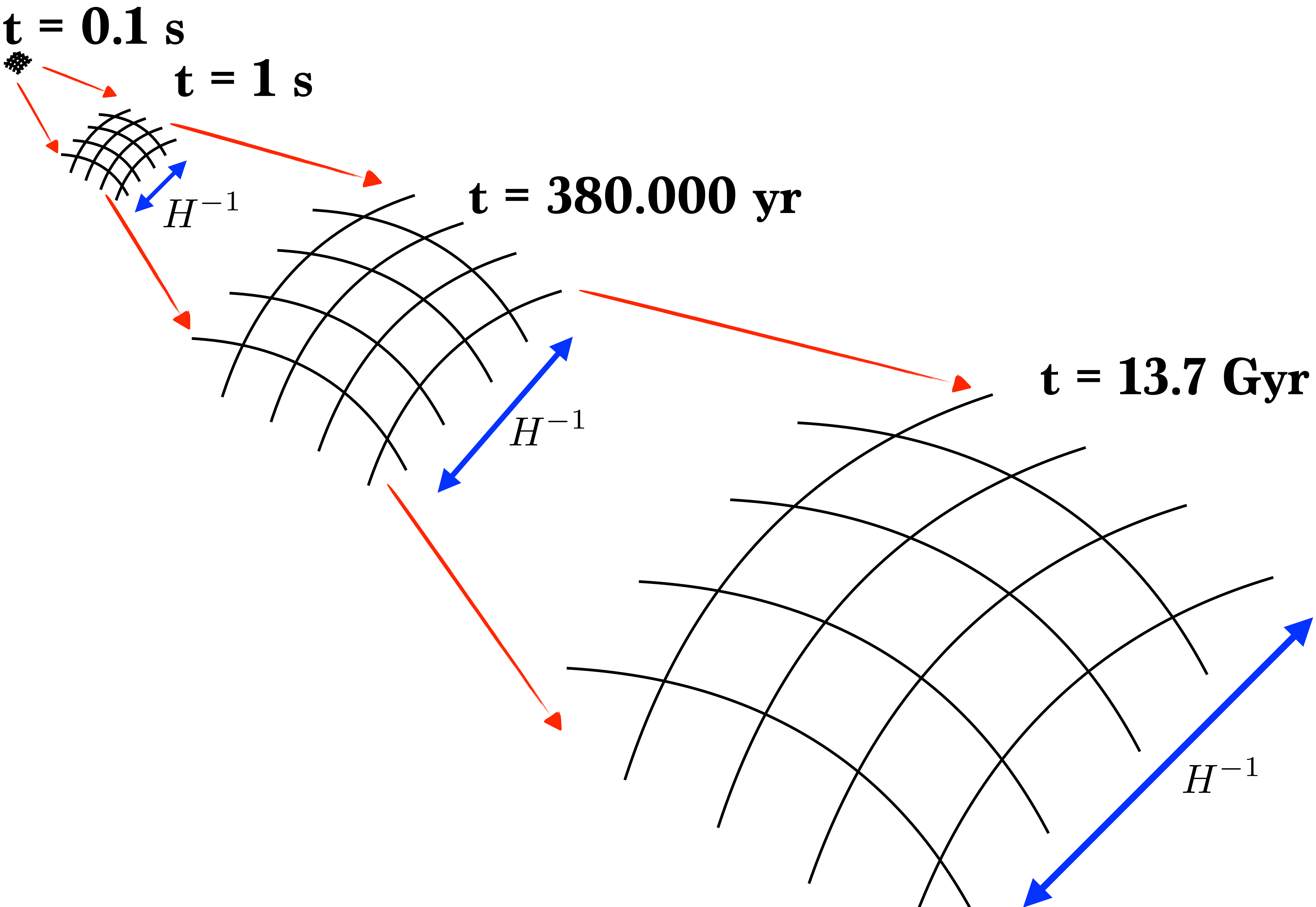

Expanding Universe

H & I

$$T^{\mu}_{\nu} \equiv \mathrm{diag}(-\rho, p, p, p)$$

$$m_p^2 G^{\mu}_{\nu}\left[g_{**}^{(FRW)}\right] = T^{\mu}_{\nu}$$

Expanding Universe

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Friedmann Equations

$$\boxed{\begin{aligned} \frac{1}{a}\frac{d^2a}{dt^2} &= -\frac{\rho}{6m_p^2}(1+3w) \\ H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 &= \frac{\rho}{3m_p^2} - \frac{K}{a^2} \end{aligned}}$$

(I)

(II)

$$\left(w \equiv \frac{p}{\rho}\right)$$ Equation of State (EoS)

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(I)

(II)

$$\left(w \equiv \frac{p}{\rho}\right)$$ Equation of State (EoS)

(I) + (II) $\Rightarrow$

$$\boxed{\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w)}$$ (III)

Expanding Universe

UNIVERSE:

1) GR

+

2) H & I

Friedmann Equations

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\rho}{6m_p^2}(1+3w) \qquad \text{(I)}$$

$$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \qquad \text{(II)}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w) \qquad \text{(III)}$$

$$\left(w \equiv \frac{p}{\rho}\right)$$

Equation of State (EoS)

Expanding Universe

(II) $H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2}$ $\longrightarrow$ $\boxed{\rho_c \equiv 3m_p^2 H^2}$

Critical density $(\rho = \rho_c \Leftrightarrow K = 0)$

Expanding Universe

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$$\rho = \sum_i \rho_i \;\; ; \quad \Omega_i \equiv \frac{\rho_i}{\rho_c} \;\Rightarrow\; \Omega \equiv \frac{\rho}{\rho_c} = \sum_i \Omega_i \;\Rightarrow\; \boxed{\Omega - 1 \equiv \frac{k}{a^2 H^2}}$$

Cosmic Sum

Expanding Universe

$$\text{(II)} \quad H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$$

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Cosmic Sum

$$\begin{cases} \Omega > 1 \Rightarrow \text{Close}(k > 0) \\ \Omega = 1 \Rightarrow \text{Flat}(k = 0) \\ \Omega < 1 \Rightarrow \text{Open}(k < 0) \end{cases}$$

one-to-one correlation

Expanding Universe

(II) $$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$$

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Cosmic Sum

(III)
$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w) \Rightarrow \rho \propto e^{-3\int \frac{da}{a}(1+w)} = \begin{cases} 1/a^3 & \text{, Mat.}(w=0) \\ 1/a^4 & \text{, Rad.}(w=1/3) \\ \text{const.} & \text{, C.C.}(w=-1) \end{cases}$$

Expanding Universe

(II) $H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$

Critical density $(\rho = \rho_c \Leftrightarrow K = 0)$

$$\rho = \sum_i \rho_i \; ; \quad \Omega_i \equiv \frac{\rho_i}{\rho_c} \Rightarrow \Omega \equiv \frac{\rho}{\rho_c} = \sum_i \Omega_i \Rightarrow \boxed{\Omega - 1 \equiv \frac{k}{a^2 H^2}}$$

Cosmic Sum

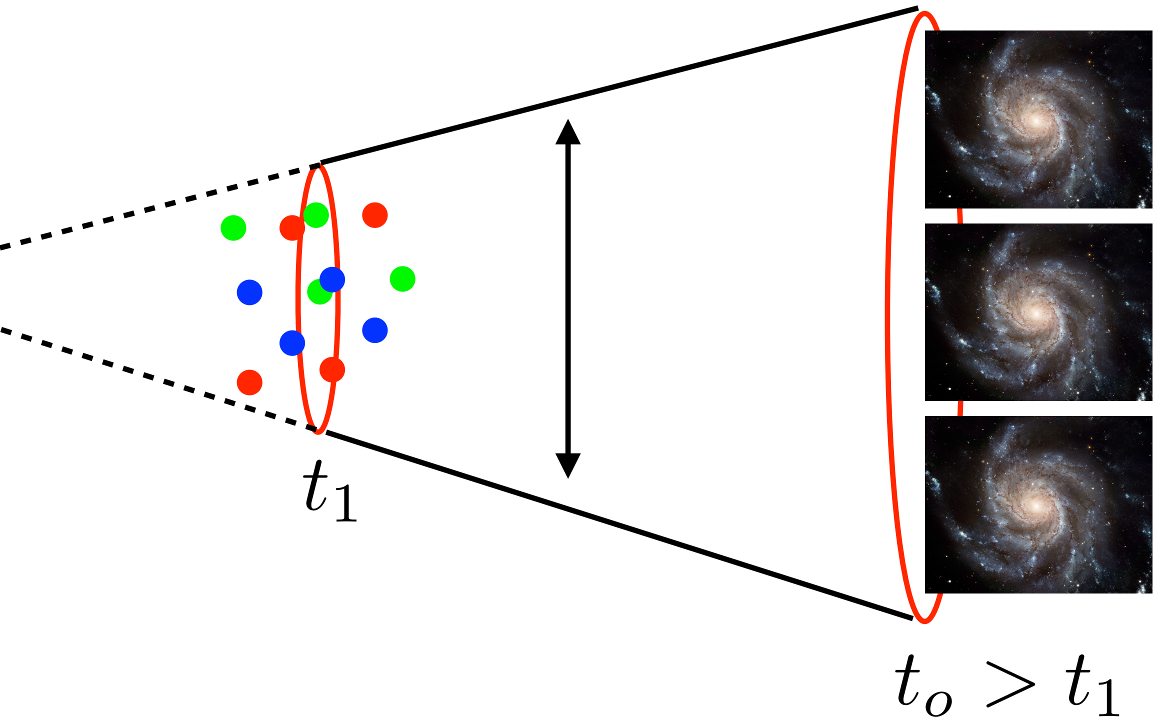
(III) + (II) :

$$H^2(a) = H_o^2 \left\{ \Omega_{\mathrm{R}}^{(o)} \left(\frac{a_o}{a}\right)^4 + \Omega_{\mathrm{M}}^{(o)} \left(\frac{a_o}{a}\right)^3 + \Omega_{\mathrm{k}}^{(o)} \left(\frac{a_o}{a}\right)^2 + \Omega_{\mathrm{DE}}^{(o)} e^{-3\int \frac{da}{a}(1+w)} \right\}$$

$$\equiv H_o^2 E^2(a)$$

$$E(a) \equiv \sqrt{\Omega_{\mathrm{R}}^{(o)} \left(\frac{a_o}{a}\right)^4 + \Omega_{\mathrm{M}}^{(o)} \left(\frac{a_o}{a}\right)^3 + \Omega_{\mathrm{k}}^{(o)} \left(\frac{a_o}{a}\right)^2 + \Omega_{\mathrm{DE}}^{(o)} e^{-3\int \frac{da}{a}(1+w)}} \qquad \Omega_{\mathrm{k}}^{(o)} \equiv -\frac{k}{a_o^2 H_o^2}$$

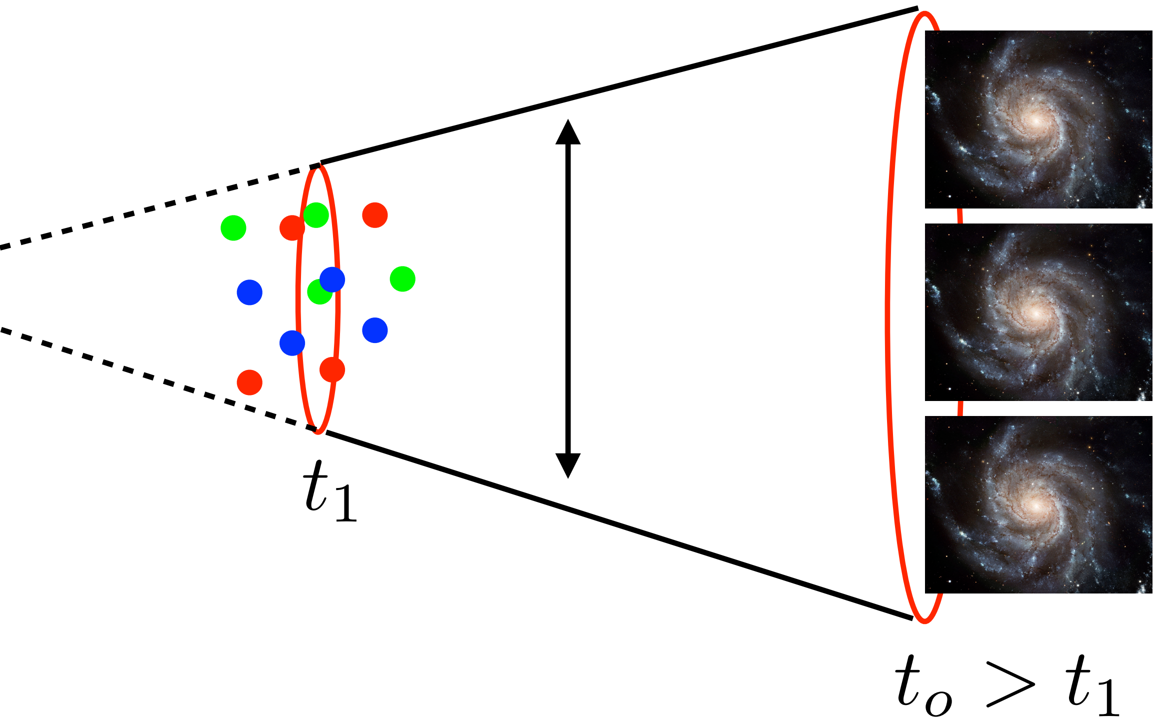
Expansion History



Past: particle ensemble

Statistical Mechanics

Expansion History

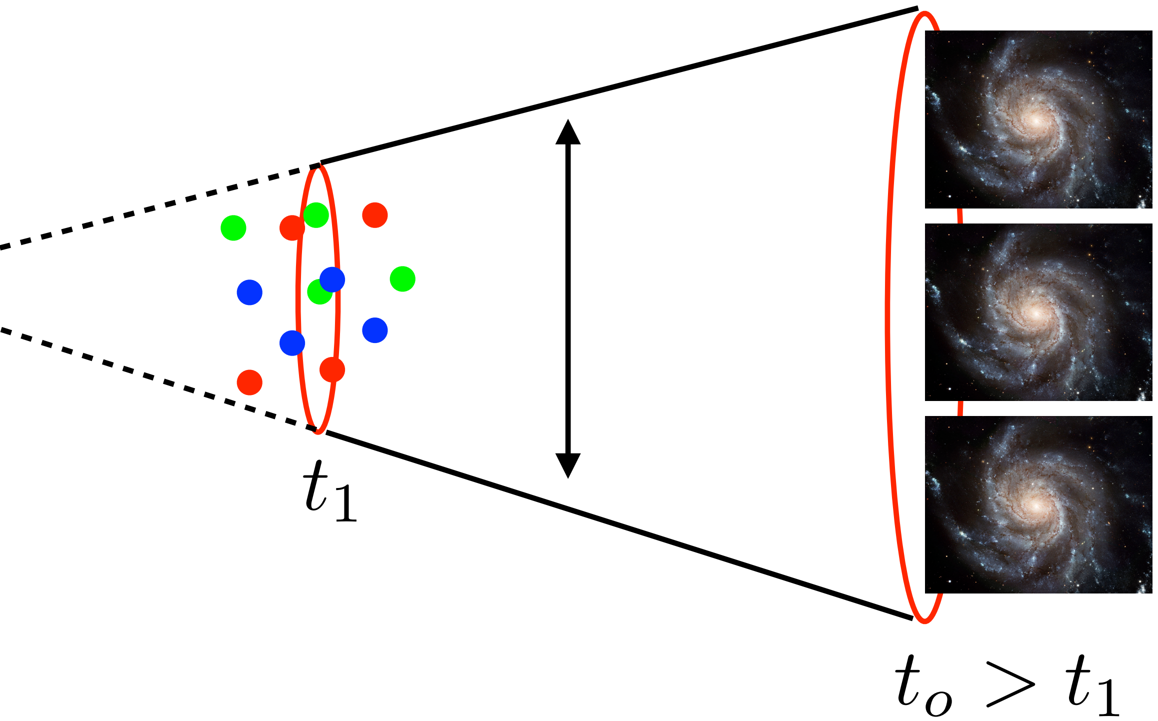


Past: particle ensemble

Statistical Mechanics

$$\text{(III)} \quad \frac{d\rho}{dt} + 3H(\rho + p) = 0 \longrightarrow \quad \frac{dU}{dt} + p\frac{dV}{dt} = 0\,, \qquad \begin{cases} U = a^3 \rho\,, \\ V = a^3 \end{cases}$$

Expansion History



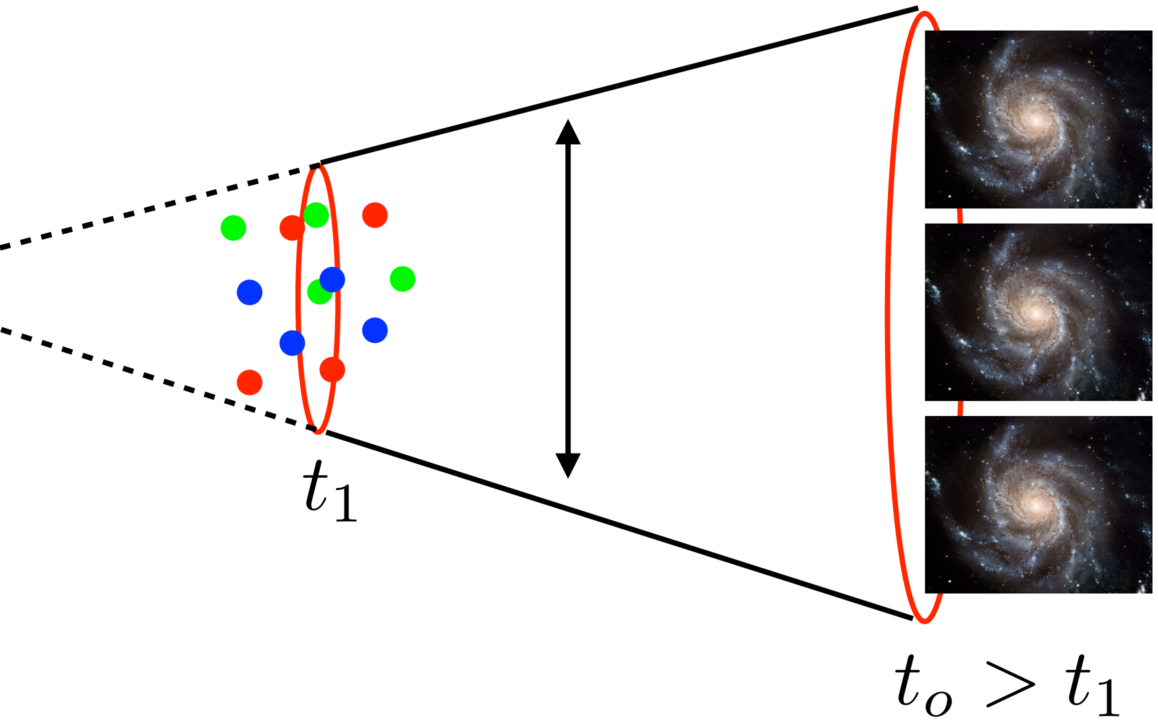
Past: particle ensemble

Statistical Mechanics

(III) $\frac{d\rho}{dt} + 3H(\rho + p) = 0 \longrightarrow \frac{dU}{dt} + p\frac{dV}{dt} = 0\,, \quad \begin{cases} U = a^3\rho\,, \\ V = a^3 \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} \frac{dU}{dt} + p\frac{dV}{dt} = T\frac{dS}{dt}\,, \longrightarrow \textbf{Thermal Eq.} \\ \frac{dS}{dt} = 0\,, \longrightarrow \textbf{Adiabatic Exp.} \end{cases}$$

Expansion History



Thermal Eq.

(densities)

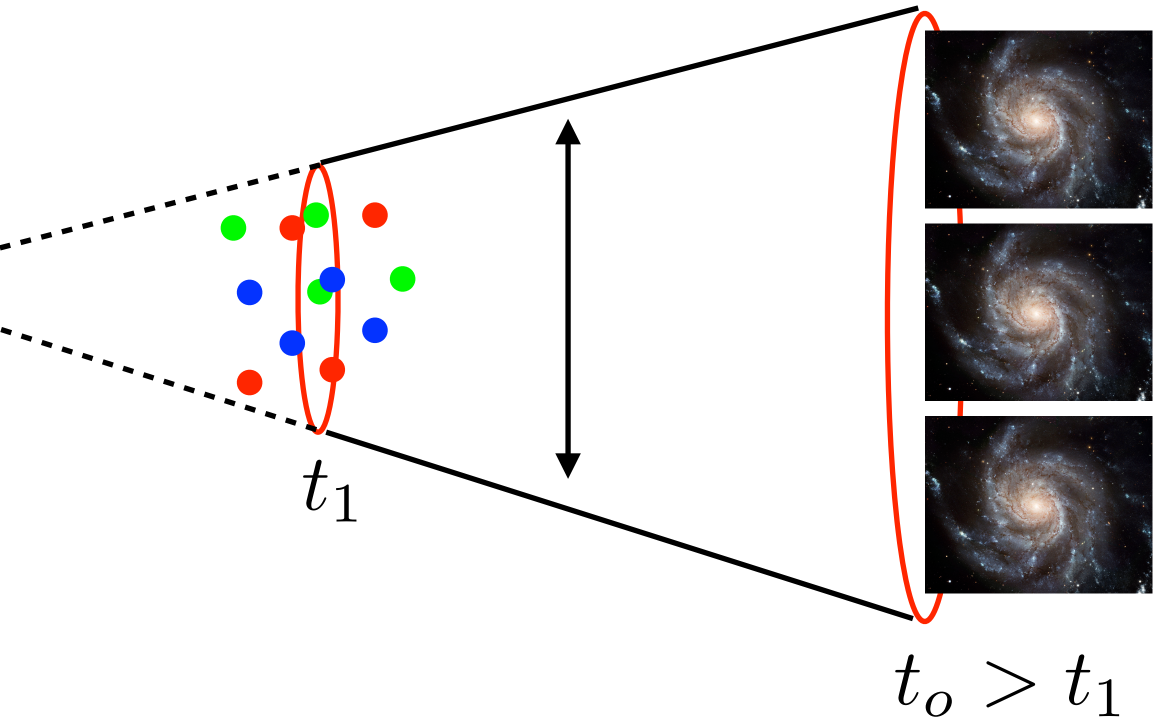
$$
\begin{cases}
n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\
\rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\
p = g_* \int d\vec{p}\, \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})\,, & \textbf{pressure}
\end{cases}
$$

dof

Dispersion relation

Statistical Distribution

Expansion History



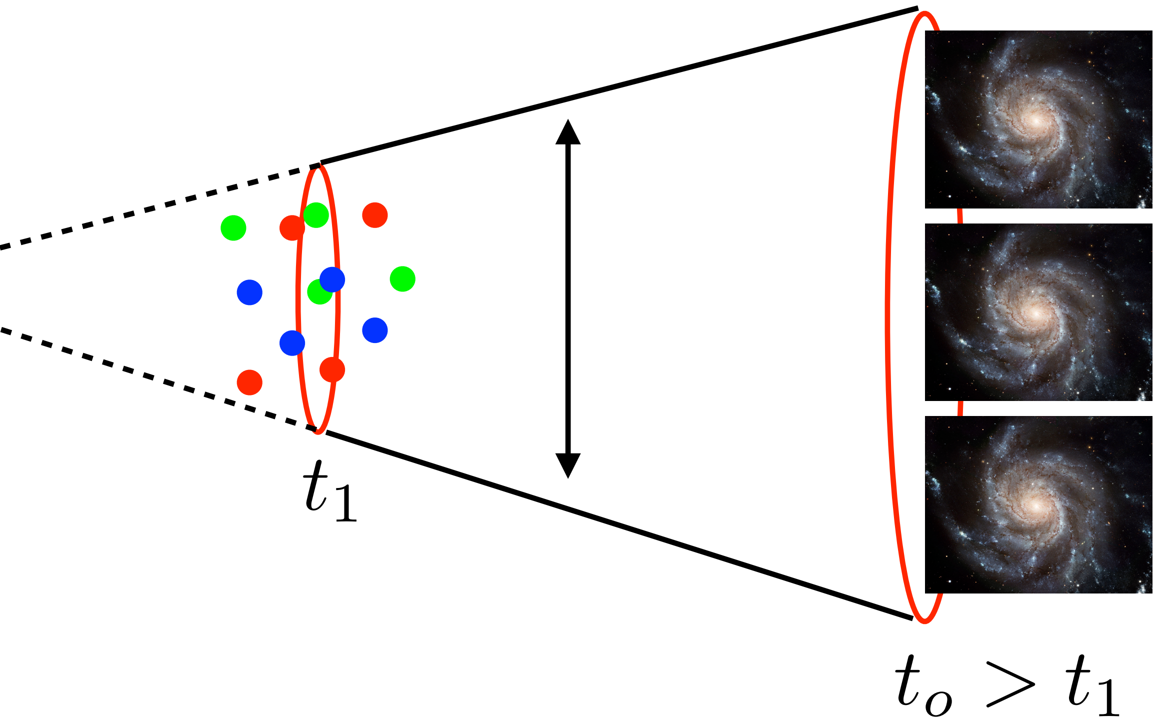
Thermal Eq. **(densities)**

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\ \rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\ p = g_* \int d\vec{p}\, \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})\,, & \textbf{pressure} \end{cases}$$

dof — *Dispersion relation* — *Statistical Distribution*

Bose-Einstein / Fermi-Dirac: $$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

Expansion History



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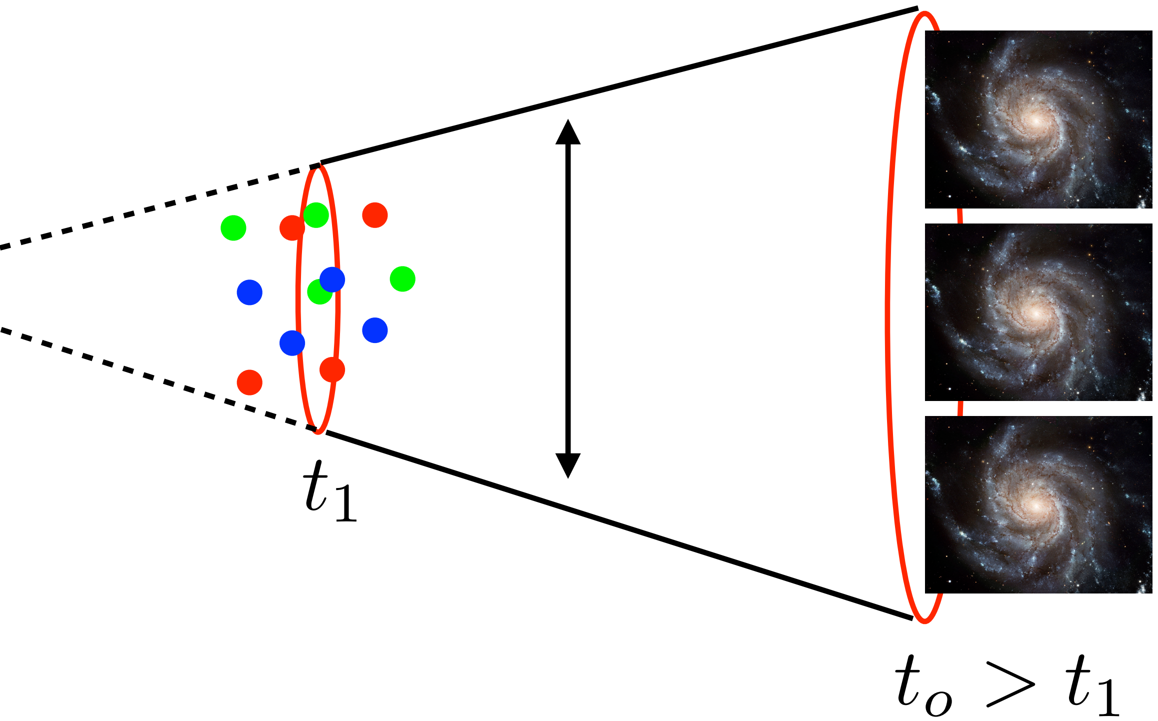
dof *Dispersion relation* *Statistical Distribution*

Bose-Einstein / Fermi-Dirac: $f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}\,, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$

$$\frac{\rho_R \propto 1/a^4}{\rho_M \propto 1/a^3} \propto 1/a\,, \quad \Rightarrow \quad z \geq z_{\rm EQ}\ (t \leq t_{\rm EQ})\,, \quad \rho_R > \rho_M$$

Expansion History



Thermal Eq. **(densities)**

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\ \rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\ p = \underbrace{g_*}_{\text{dof}} \int d\vec{p}\, \underbrace{\frac{|\vec{p}|^2}{3E(\vec{p})}}_{\text{Dispersion relation}} \underbrace{f(\vec{p})}_{\text{Statistical Distribution}}\,, & \textbf{pressure} \end{cases}$$

Bose-Einstein / Fermi-Dirac: $$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}\,, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

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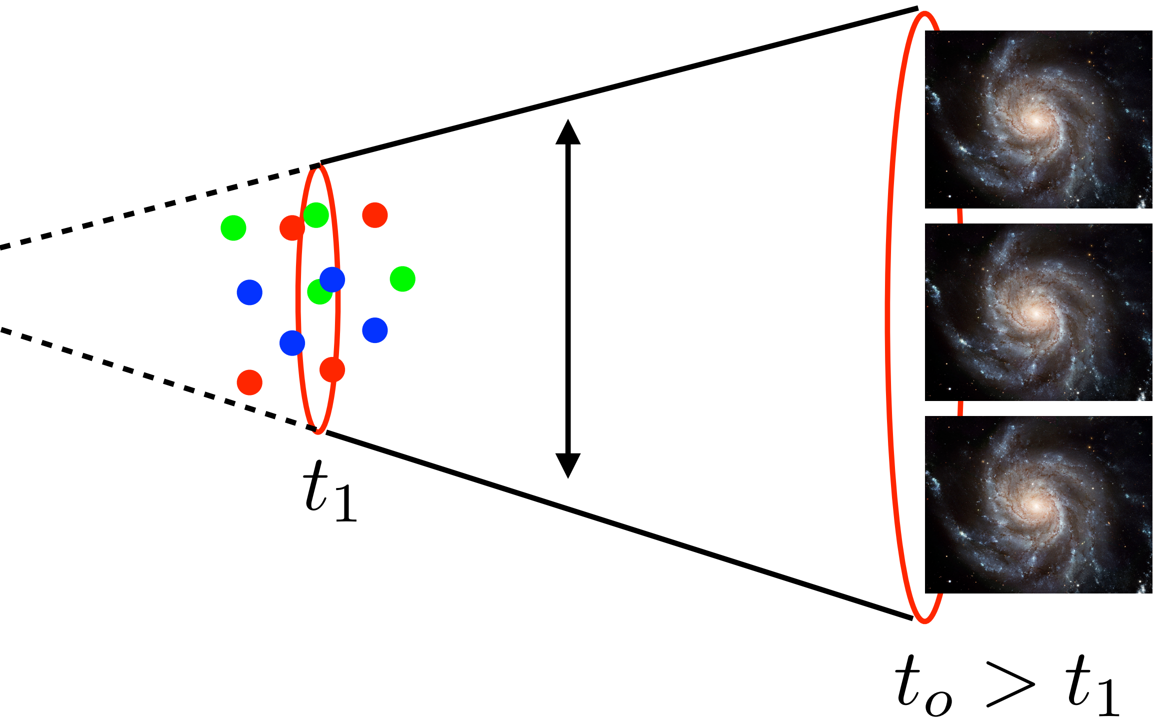
Past: Radiation Domination ! $\boxed{1 + z_{\rm EQ} = \Omega^{(o)}_{\rm M} / \Omega^{(o)}_{\rm Rad} \sim 3400}$

Expansion History



Thermal Eq. **(densities)**

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\ \rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\ p = g_* \int d\vec{p}\, \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})\,, & \textbf{pressure} \end{cases}$$

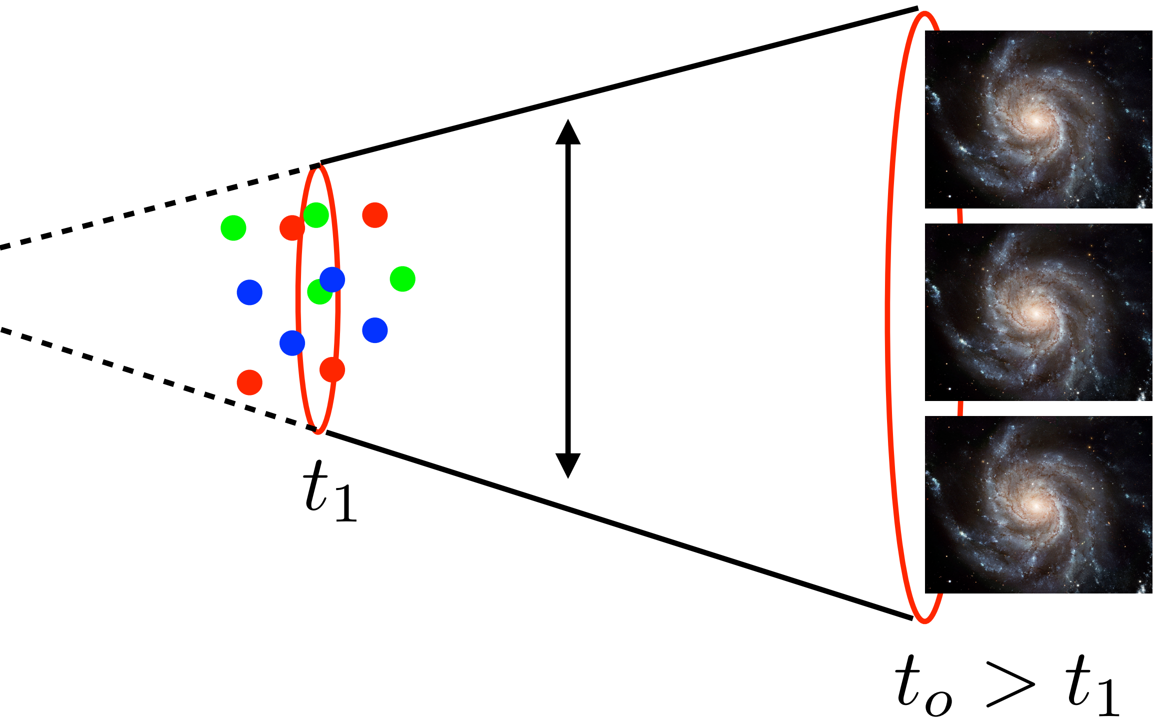
dof *Dispersion relation* *Statistical Distribution*

Bose-Einstein / Fermi-Dirac: $$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

Past: Radiation Domination (RD)

$$\rho_R^{(i)} = f_i g_*^{(i)} \frac{\pi^2}{30} T_i^4\,, \quad f_i = \begin{cases} 1, & \mathrm{B} \\ \frac{7}{8}, & \mathrm{F} \end{cases}$$

Expansion History



Thermal Eq. **(densities)**

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\ \rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\ p = g_* \int d\vec{p}\, \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})\,, & \textbf{pressure} \end{cases}$$

dof *Dispersion relation* *Statistical Distribution*

Bose-Einstein / Fermi-Dirac:
$$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}\,, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

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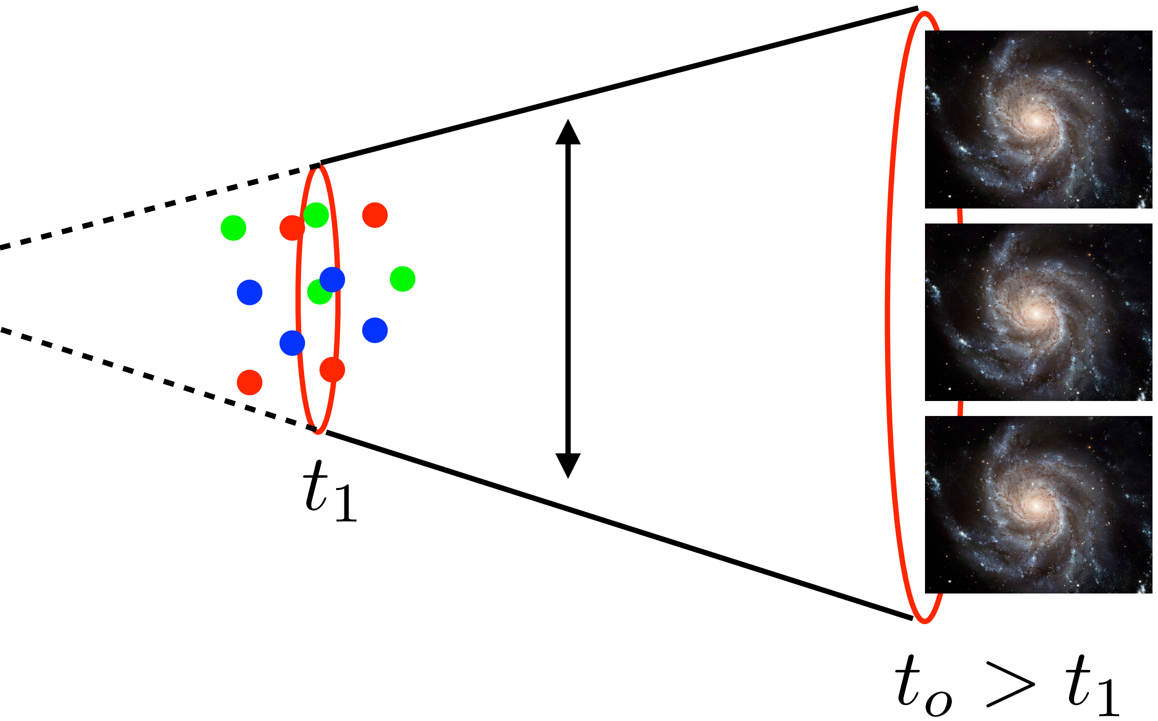
Expansion History



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dof *Dispersion relation* *Statistical Distribution*

Bose-Einstein / Fermi-Dirac: $$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}\,, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

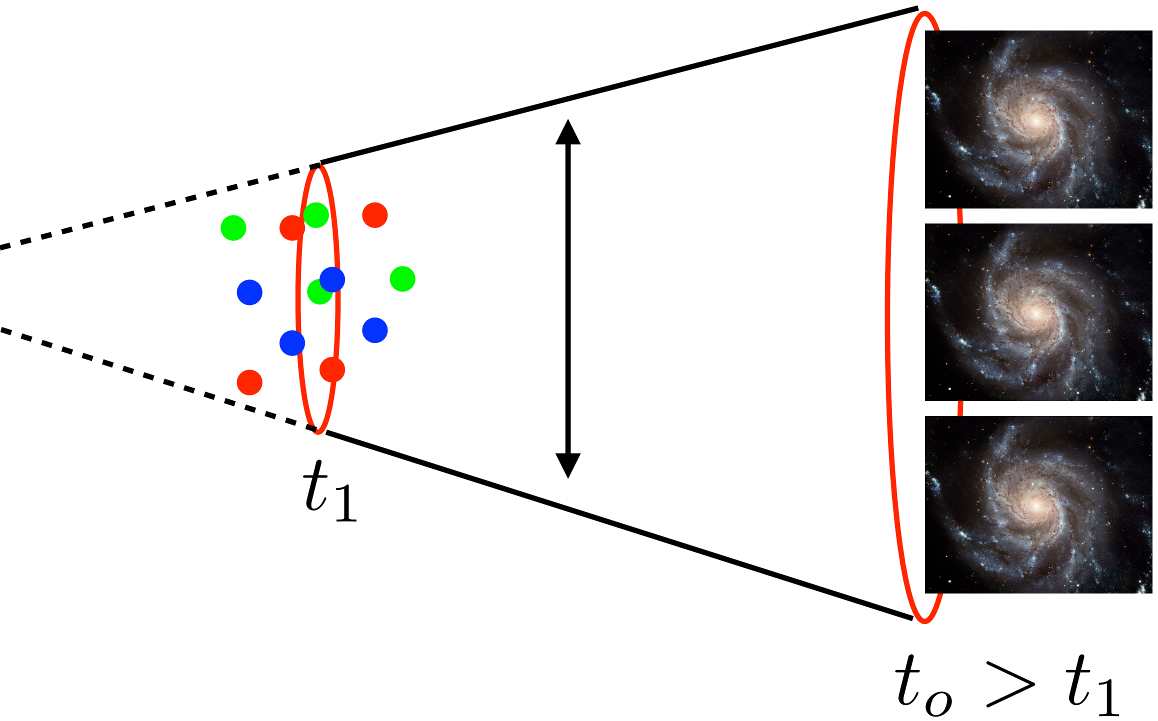
Past: Radiation Domination (RD)

$$\rho_R^{(i)} = f_i g_*^{(i)} \frac{\pi^2}{30} T_i^4\,, \quad f_i = \begin{cases} 1, & \mathrm{B} \\ \frac{7}{8}, & \mathrm{F} \end{cases}$$

$$\boxed{\rho_R = \sum_i \rho_R^{(i)} \equiv g_*(T) \frac{\pi^2}{30} T^4}$$

$$g_*(T) \equiv \sum_i g_{*,i}^{(B)} \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_i g_{*,i}^{(F)} \left(\frac{T_i}{T}\right)^4$$

Expansion History



Adiabatic Exp:

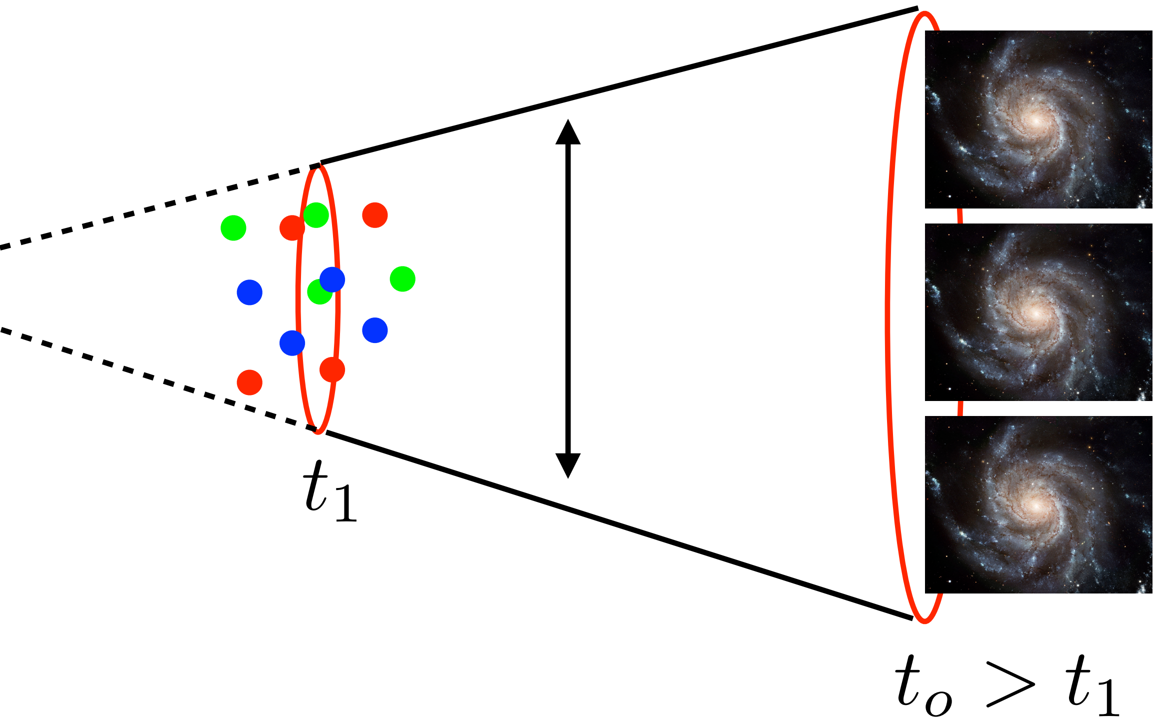
$$S = \frac{a^3(\rho + p)}{T} = const.$$

$$\Downarrow$$

$$\boxed{a^3 T^3 g_*^{(s)}(T) = const.}$$

$$g_*^{(s)}(T) \equiv \sum_i g_{*,i}^{(B)} \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_i g_{*,i}^{(F)} \left(\frac{T_i}{T}\right)^3$$

Expansion History



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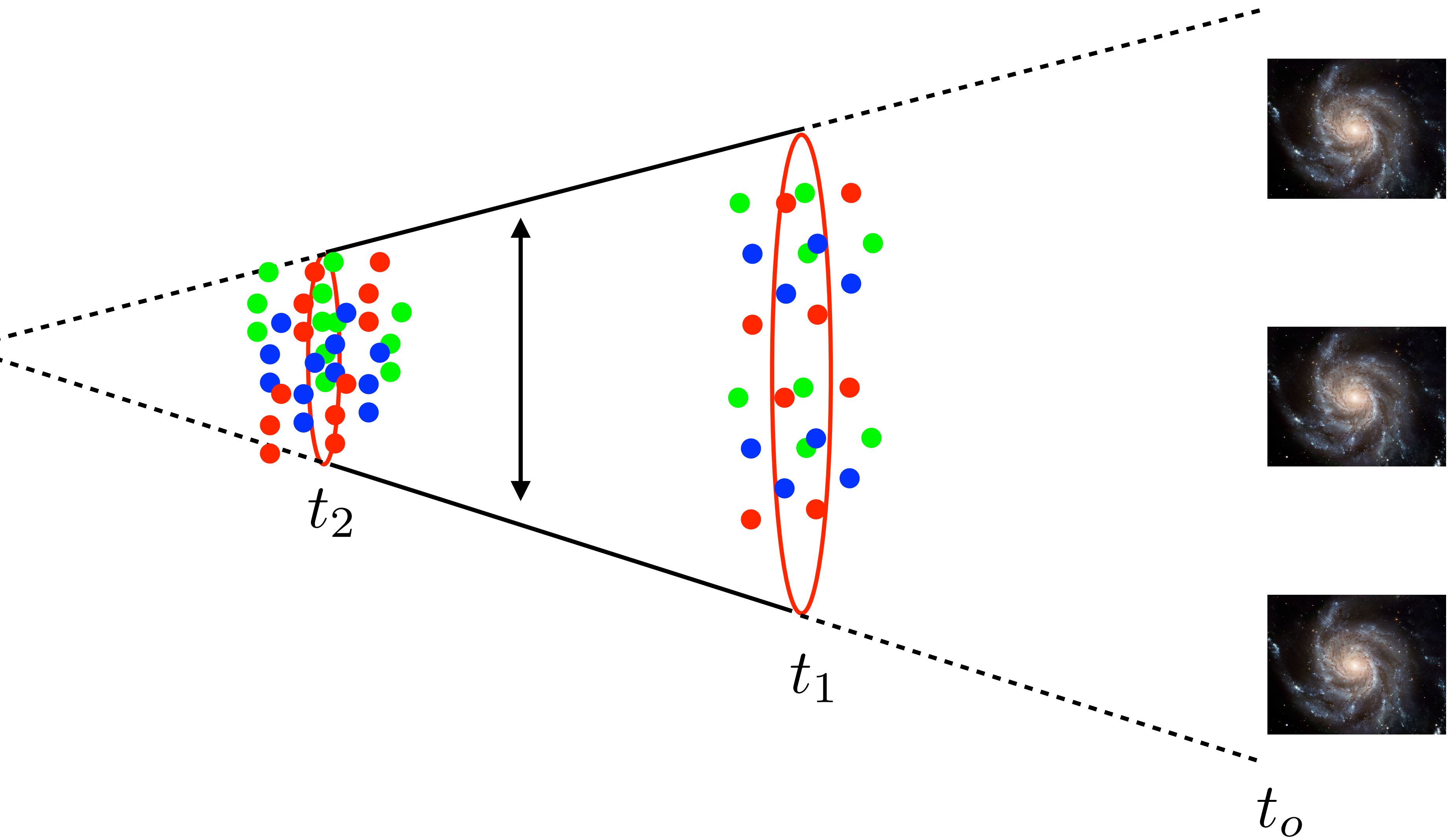
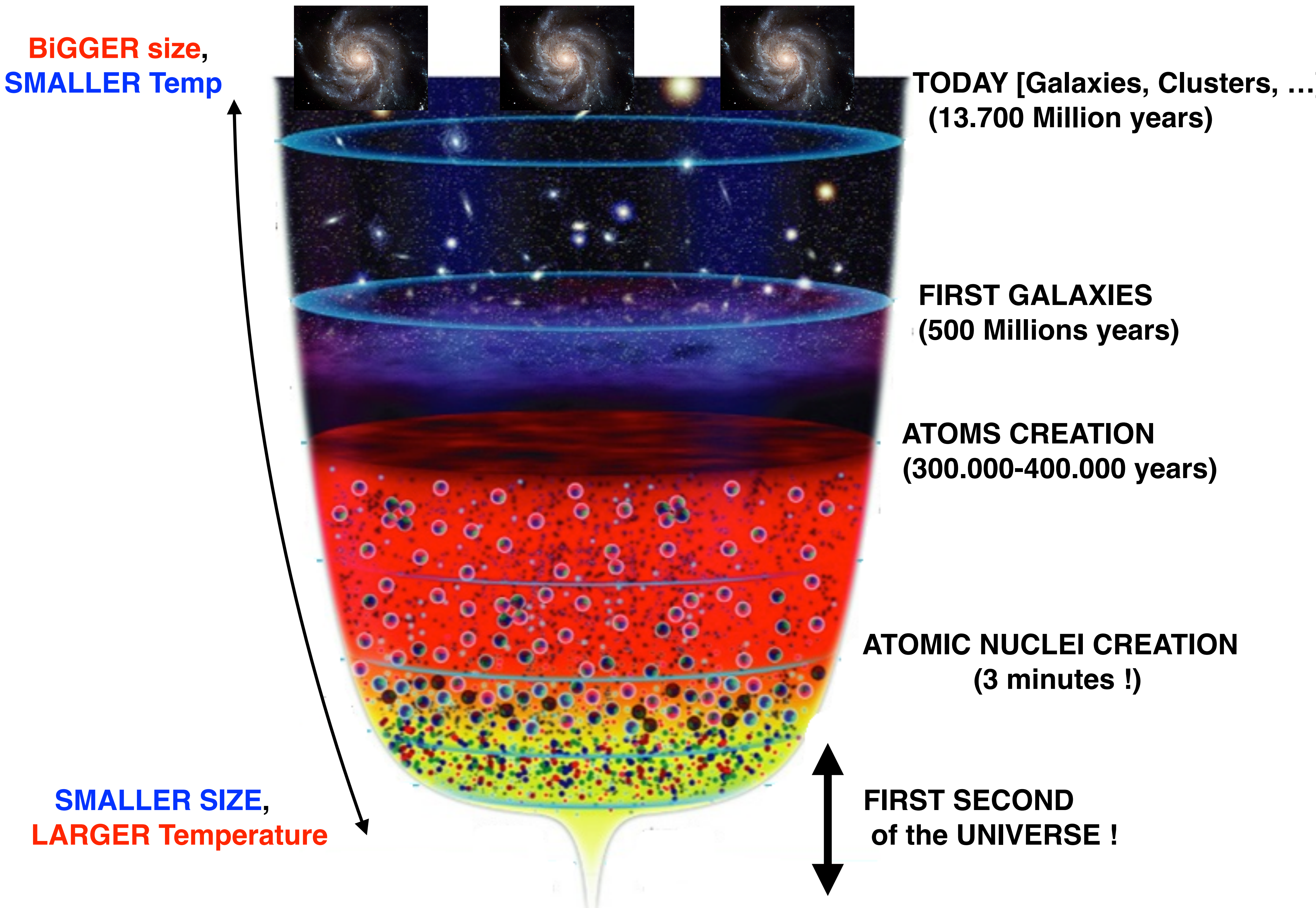
When do $g_*(T), g_*^{(s)}(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,,\ \ T \to T_i\,, \\ 2)\ \text{Mass threshold}\,, T < 2m_i\,, \end{cases}$$

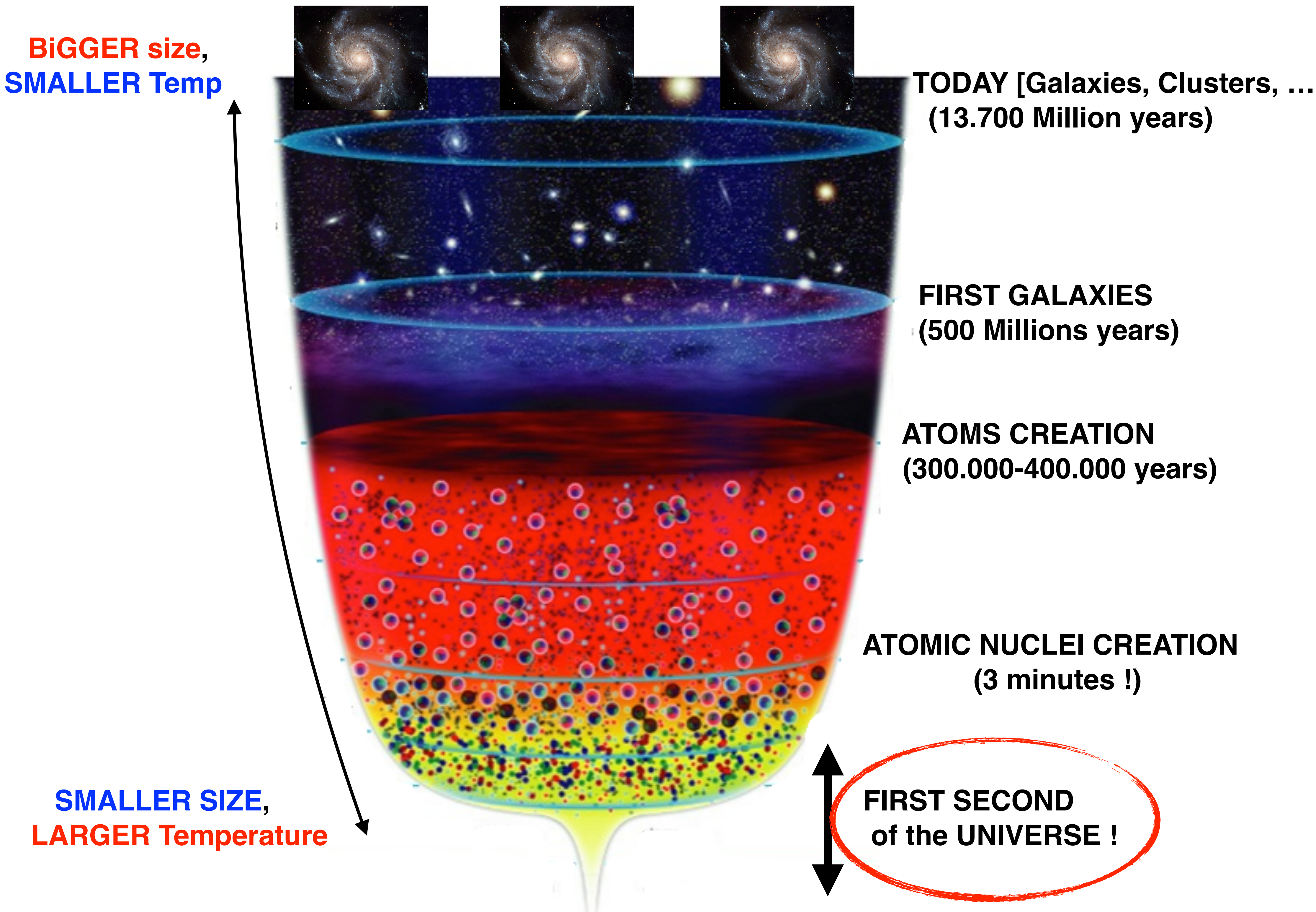
Expansion History



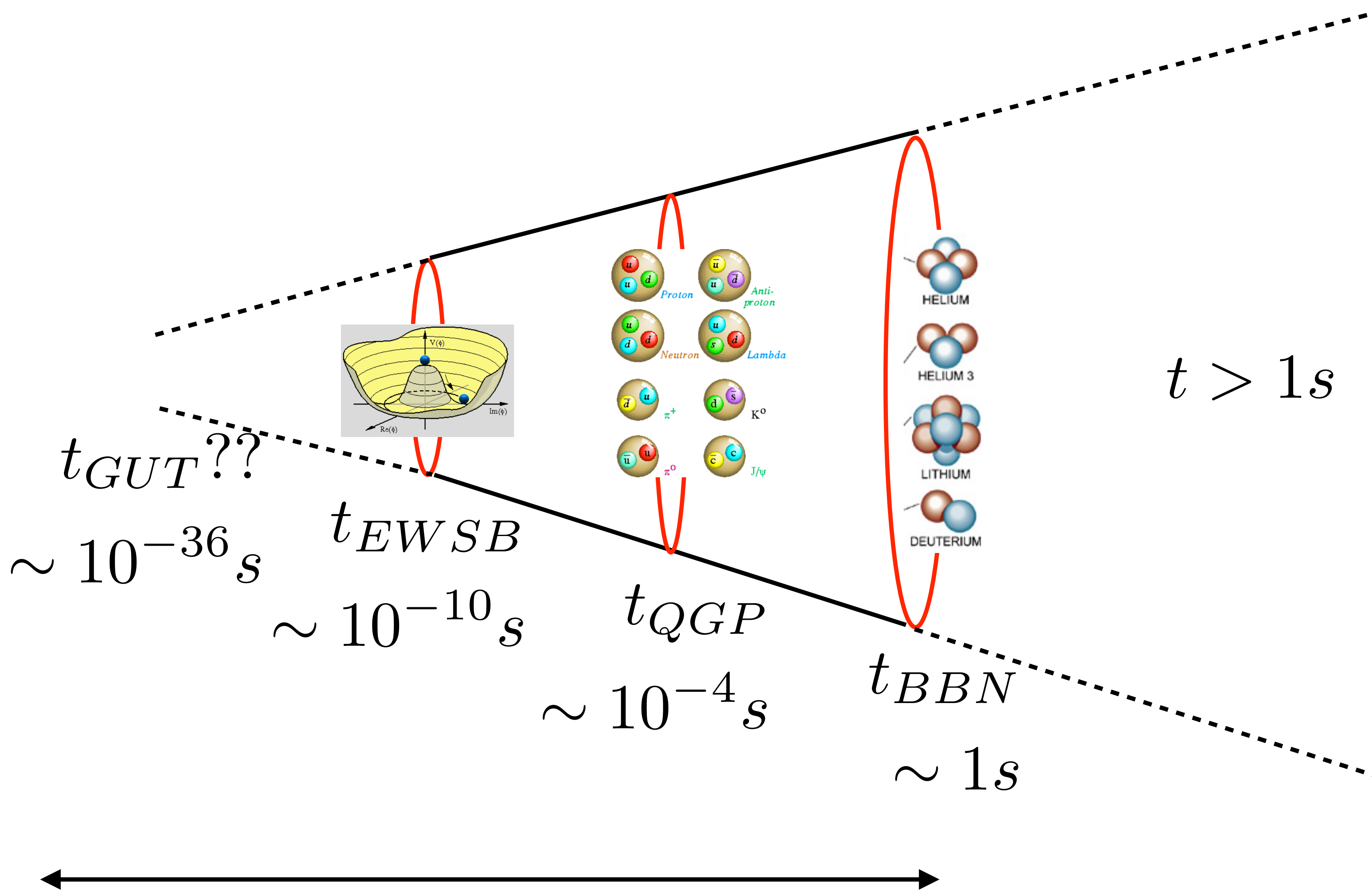
Expansion History



Expansion History



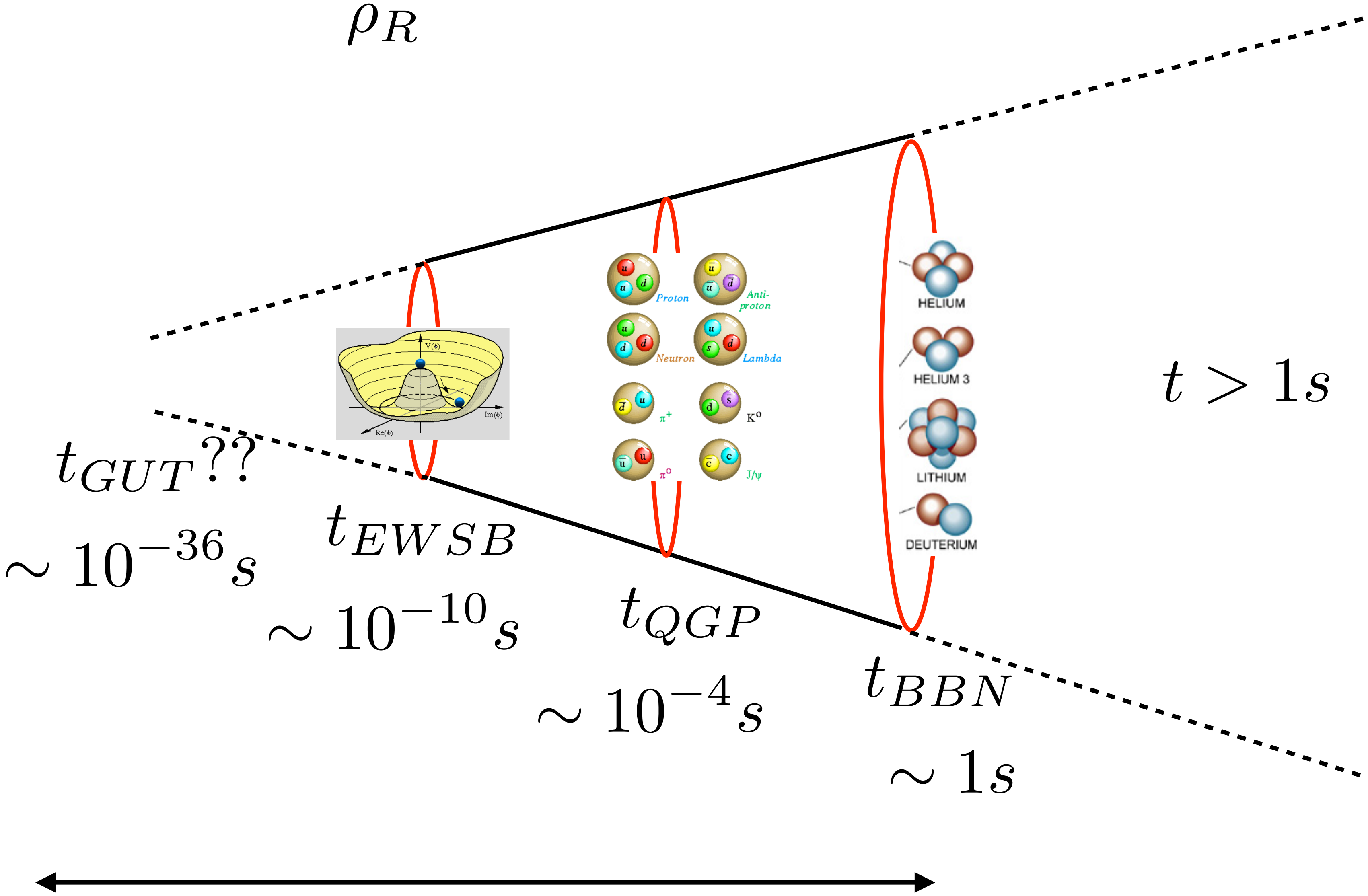
The Early Universe



hot early Universe $(\sim 1s)$

The Early Universe

$$H^2 = \frac{1}{(2t)^2} = \underbrace{\frac{1}{3m_p^2}\frac{\pi^2}{30}\overset{\#\ \text{relativistic } dof}{g_*(T)}T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \mathrm{s}$$



hot early Universe $(\sim 1s)$

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30}g_*(T)T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

T	$1\ \mathrm{MeV}$	$200\ \mathrm{MeV}$	$\mathcal{O}(10^2)\ \mathrm{GeV}$	$\mathcal{O}(10^{16})\ \mathrm{GeV}$	$\mathcal{O}(10^{19})\ \mathrm{GeV}$
t	$1\ \mathrm{s}$	$10^{-5}\ \mathrm{s}$	$10^{-10}\ \mathrm{s}$	$10^{-37}\ \mathrm{s}$	$10^{-43}\ \mathrm{s}$
	BBN	**QGP**	**EWSB**	**GUT**	**QG**

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30}\, g_*(T)\, T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

T	$1\ \mathrm{MeV}$	$200\ \mathrm{MeV}$	$\mathcal{O}(10^2)\ \mathrm{GeV}$	$\mathcal{O}(10^{16})\ \mathrm{GeV}$	$\mathcal{O}(10^{19})\ \mathrm{GeV}$
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	BBN	**QGP**	**EWSB**	**GUT**	**QG**

observational
evidence !

The Early Universe

\# relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30} g_*(T) T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

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	BBN	**QGP**	**EWSB**	**GUT**	**QG**
	↓	↓	↓		
	observational evidence !	theoretical input	theoretical input		

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30}\, g_*(T)\, T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2}\ \mathrm{s}$$

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	BBN	**QGP**	**EWSB**	**GUT**	**QG**
	↓	↓	↓	↓	↓
	observational evidence !	theoretical input	theoretical input	theoretical speculation	theoretical speculation

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2} \underbrace{\frac{\pi^2}{30} g_*(T) T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \ \text{s}$$

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	BBN	**QGP**	**EWSB**	**GUT**	**QG**
	observational evidence !	theoretical input	theoretical input	theoretical speculation	theoretical speculation

How can we probe the early Universe ?

The Early Universe

\# relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}g_*(T)T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \text{ s}$$

When do $g_*(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,,\ T \to T_i\,, \\ 2)\ \text{Mass threshold}\,, T < 2m_i\,, \end{cases}$$

The Early Universe

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}\underbrace{g_*(T)}_{\text{\# relativistic } dof}T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

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Decoupling:

$$\Gamma_{\rm int} = \underbrace{n}_{\substack{\text{number}\\\text{density}}} \times \langle \underbrace{\sigma}_{\substack{\text{cross}\\\text{section}}} \underbrace{v}_{\substack{\text{velocity}\\(c\,=\,1)}} \rangle \quad \longrightarrow \quad N_{\rm int} = \int_t^{t+\Delta t} dt' \Gamma_{\rm int}(t') \sim \frac{\Gamma_{\rm int}(t)}{H(t)} \begin{cases} \ll 1 \Rightarrow \text{decoupling} \\ \gg 1 \Rightarrow \text{Thermal Eq.} \end{cases}$$

The Early Universe

\# relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}g_*(T)T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \text{ s}$$

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cross section $\sim \begin{cases} \dfrac{(g^2/4\pi)^2}{T^2}, & \text{massless gauge} \\ T^2/M_*^4, & \text{massive gauge} \end{cases}$

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30} g_*(T) T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \text{ s}$$

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$$\Gamma_{\mathrm{int}} = n \times \langle \sigma v \rangle \quad \longrightarrow \quad N_{\mathrm{int}} = \int_t^{t+\Delta t} dt' \Gamma_{\mathrm{int}}(t') \sim \frac{\Gamma_{\mathrm{int}}(t)}{H(t)} \begin{cases} \ll 1 \Rightarrow \text{ decoupling} \\ \gg 1 \Rightarrow \text{ Thermal Eq.} \end{cases}$$

Neutrino Decoupling:

$$\Gamma_\nu = \underbrace{\langle \sigma_{\mathrm{EW}} \rangle}_{\sim G_F^2 T^2} \times \underbrace{n}_{\sim T^3} \sim T^5 / M_W^4 \lesssim H(t) \quad \Leftrightarrow \quad T \lesssim T_{\nu-\mathrm{dec}} = 0.8 \mathrm{MeV}$$

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30} g_*(T) T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

When do $g_*(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,,\ \ T \to T_i\,, \\ 2)\ \text{Mass threshold}\,, T < 2m_i\,, \end{cases}$$

Decoupling:

$$\Gamma_{\mathrm{int}} = n \times \langle \sigma v \rangle \quad \longrightarrow \quad N_{\mathrm{int}} = \int_t^{t+\Delta t} dt' \Gamma_{\mathrm{int}}(t') \sim \frac{\Gamma_{\mathrm{int}}(t)}{H(t)} \begin{cases} \ll 1 \Rightarrow \text{decoupling} \\ \gg 1 \Rightarrow \text{Thermal Eq.} \end{cases}$$

Graviton Decoupling:

$$\Gamma_g = \underbrace{\langle \sigma_g \rangle}_{\sim G^2 T^2} \times \underbrace{n}_{\sim T^3} \sim T^5 / M_p^4 \lesssim H(t)$$

The Early Universe

relativistic *dof*

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30} g_*(T) T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

When do $g_*(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,,\ T \to T_i\,, \\ 2)\ \text{Mass threshold}\,, T < 2m_i\,, \end{cases}$$

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$$\Gamma_{\rm int} = n \times \langle \sigma v \rangle \quad \longrightarrow \quad N_{\rm int} = \int_t^{t+\Delta t} dt' \Gamma_{\rm int}(t') \sim \frac{\Gamma_{\rm int}(t)}{H(t)} \begin{cases} \ll 1 \Rightarrow \text{decoupling} \\ \gg 1 \Rightarrow \text{Thermal Eq.} \end{cases}$$

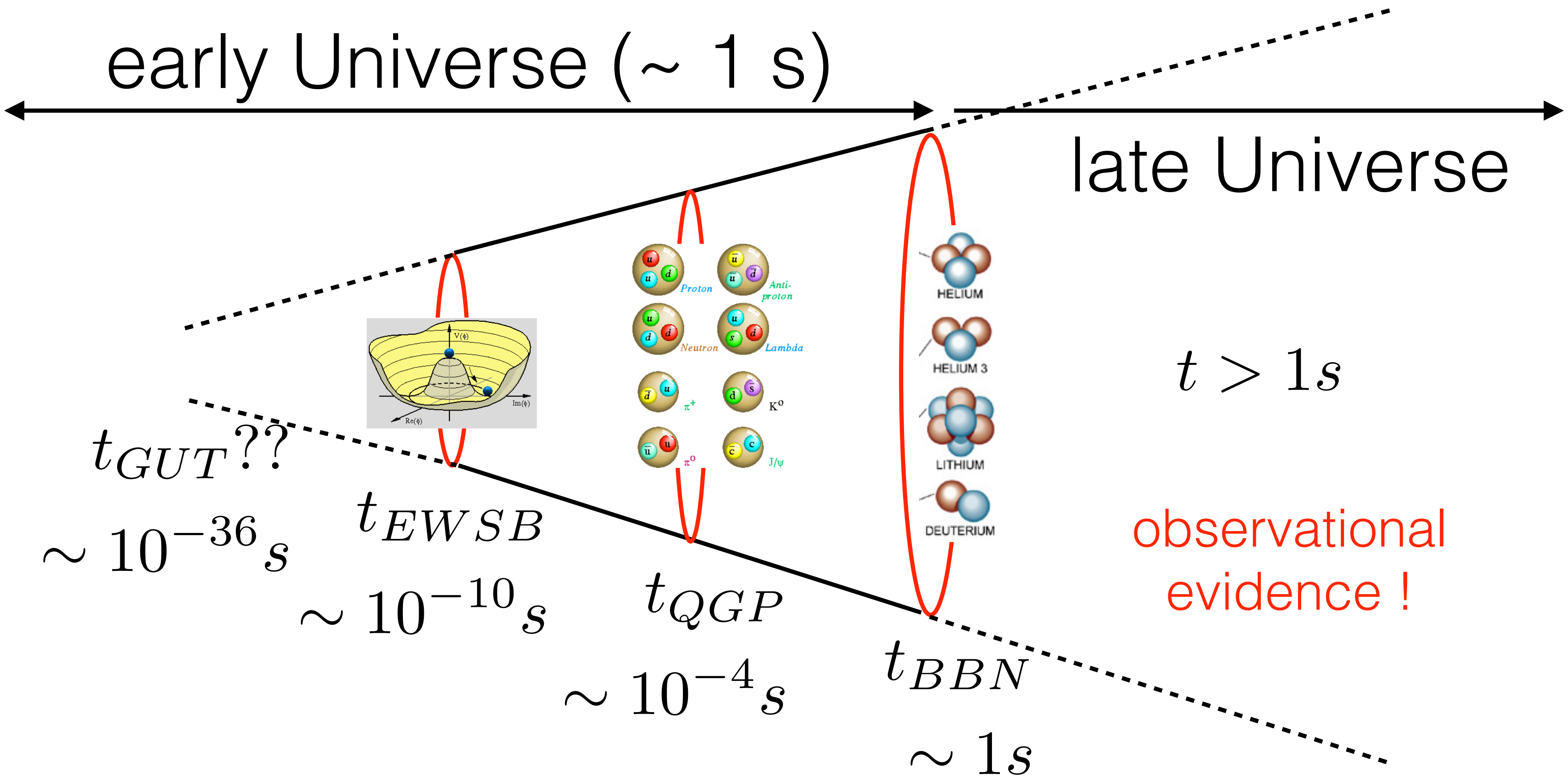
Graviton Decoupling:

$$\Gamma_g = \underbrace{\langle \sigma_g \rangle}_{\sim G^2 T^2} \times \underbrace{n}_{\sim T^3} \sim T^5/M_p^4 \lesssim H(t) \quad \Leftrightarrow \quad \boxed{T \lesssim T_g \simeq M_p}$$

Graviton(s) decouple below Planck Scale !

The Early Universe

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}g_*(T)T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \text{ s}$$



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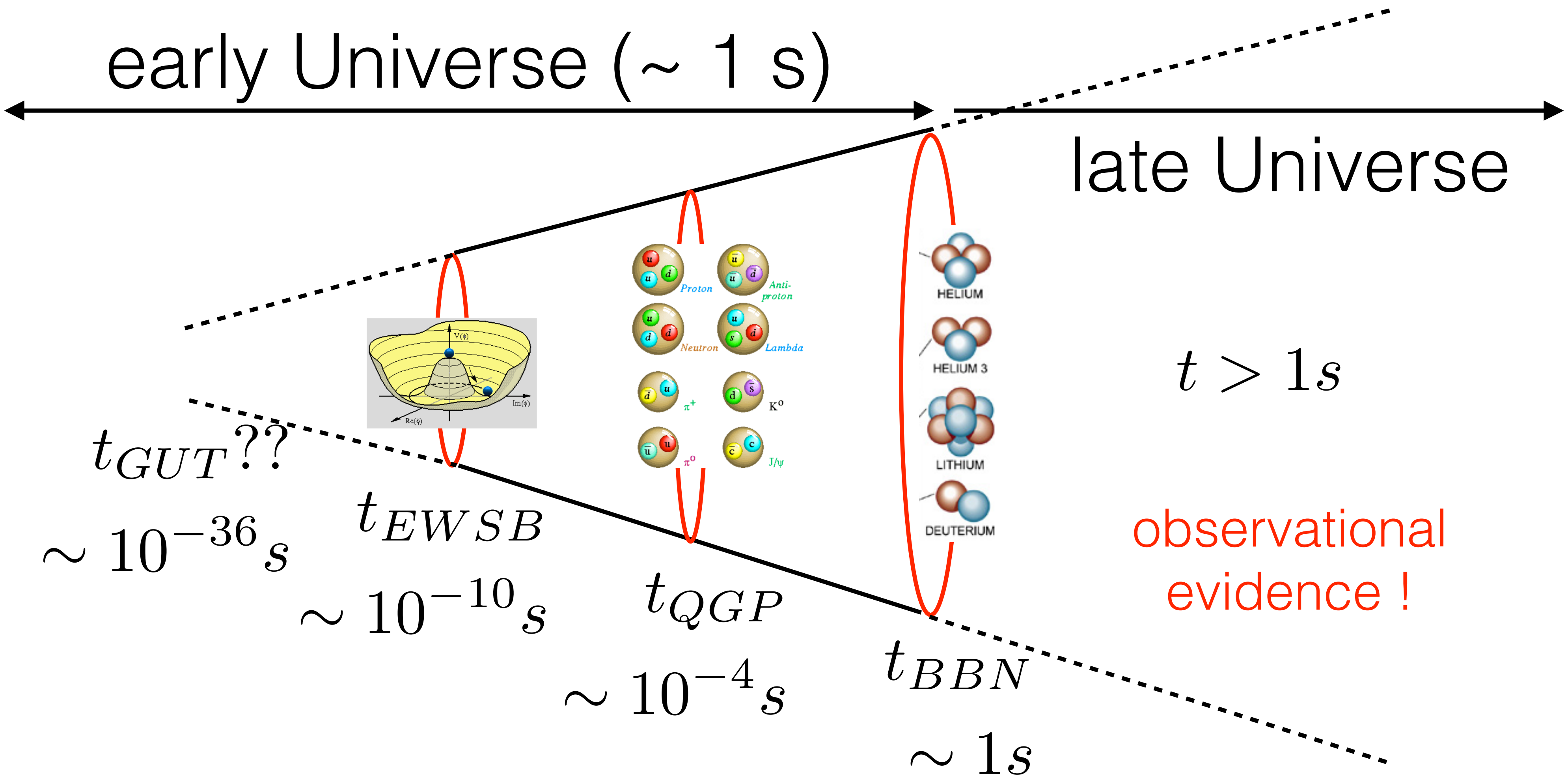

How can we probe the early Universe ? GWs !

BASICS of COSMOLOGY

Cosmology

- hot Big Bang (hBB)
  (evolution of the Universe)
  - theoretical pillars ✓
    - General Relativity
    - Cosmological Pple
  - observational pillars
    - Expansion ✓
    - CMB
    - BBN
- Inflation = (initial cond.) 'cures' hBB ⟹
  - Cosmological Pple
  - CMB/LSS

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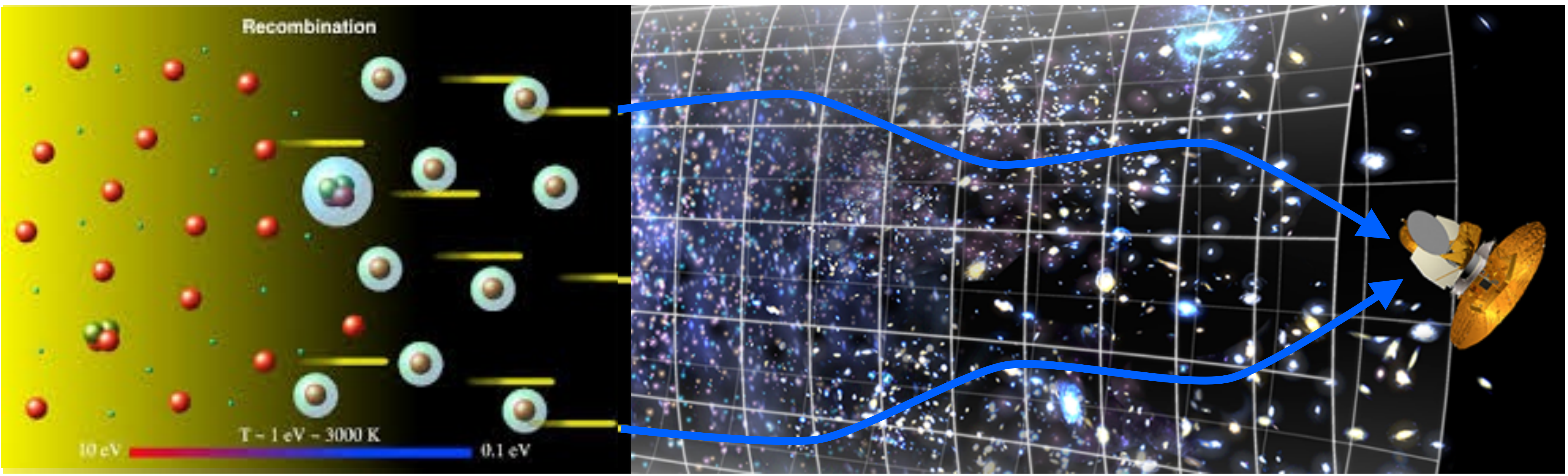
  - CMB/LSS

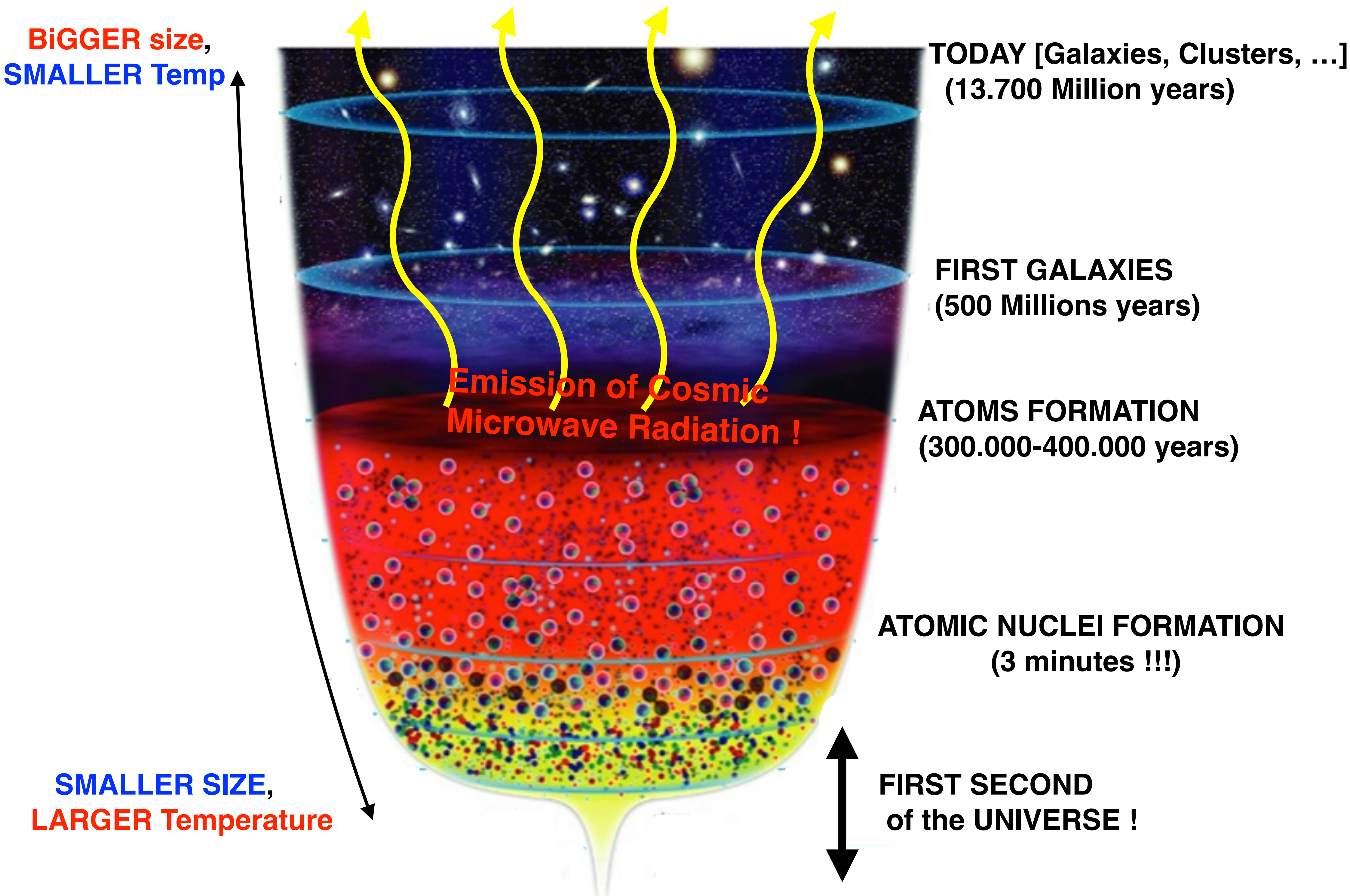
Recombination & release of Cosmic Microwave Background



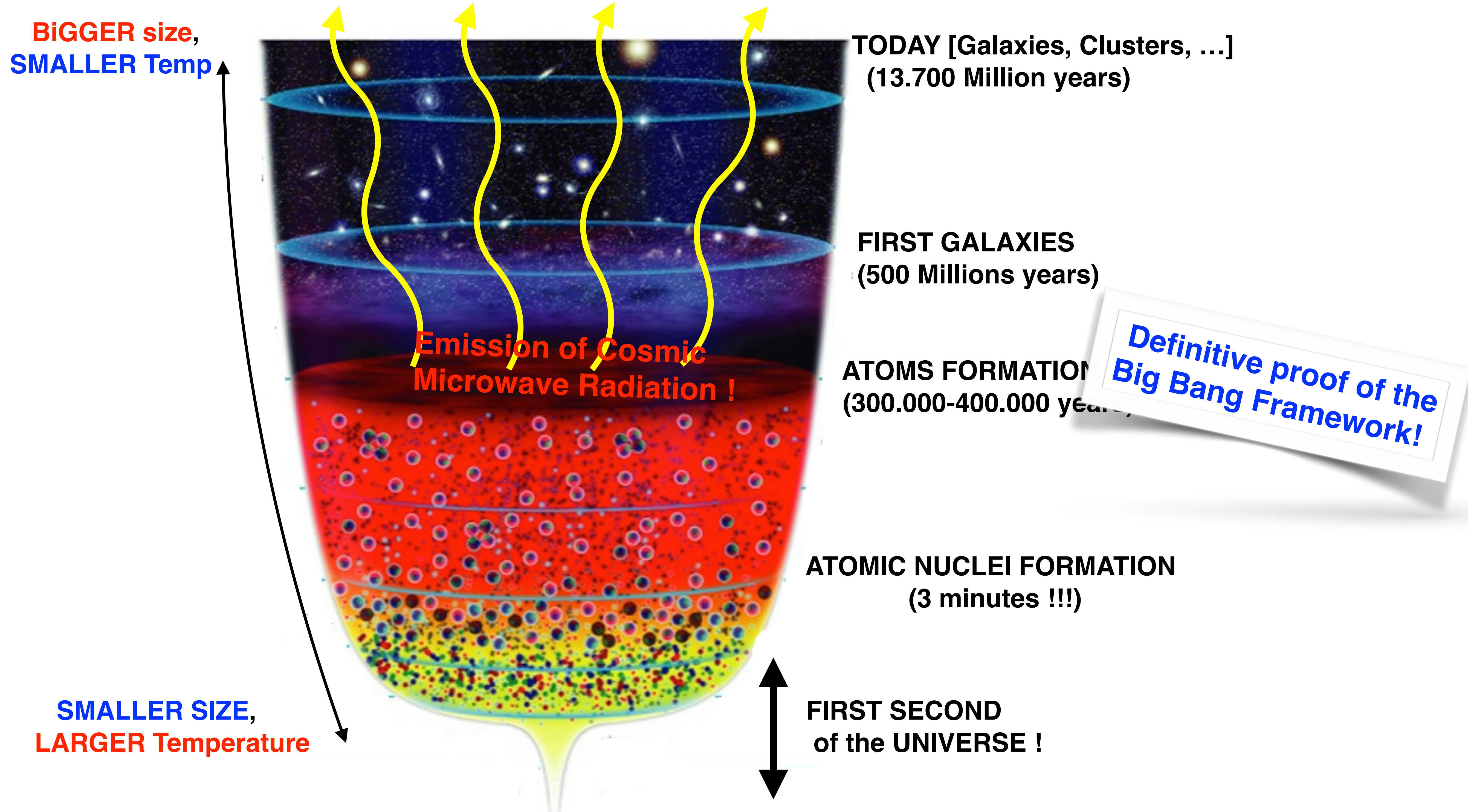
Atom Formation: **Free propagation of light !**

(Recombination)

Cosmic Microwave Background (CMB)



Cosmic Microwave Background (CMB)

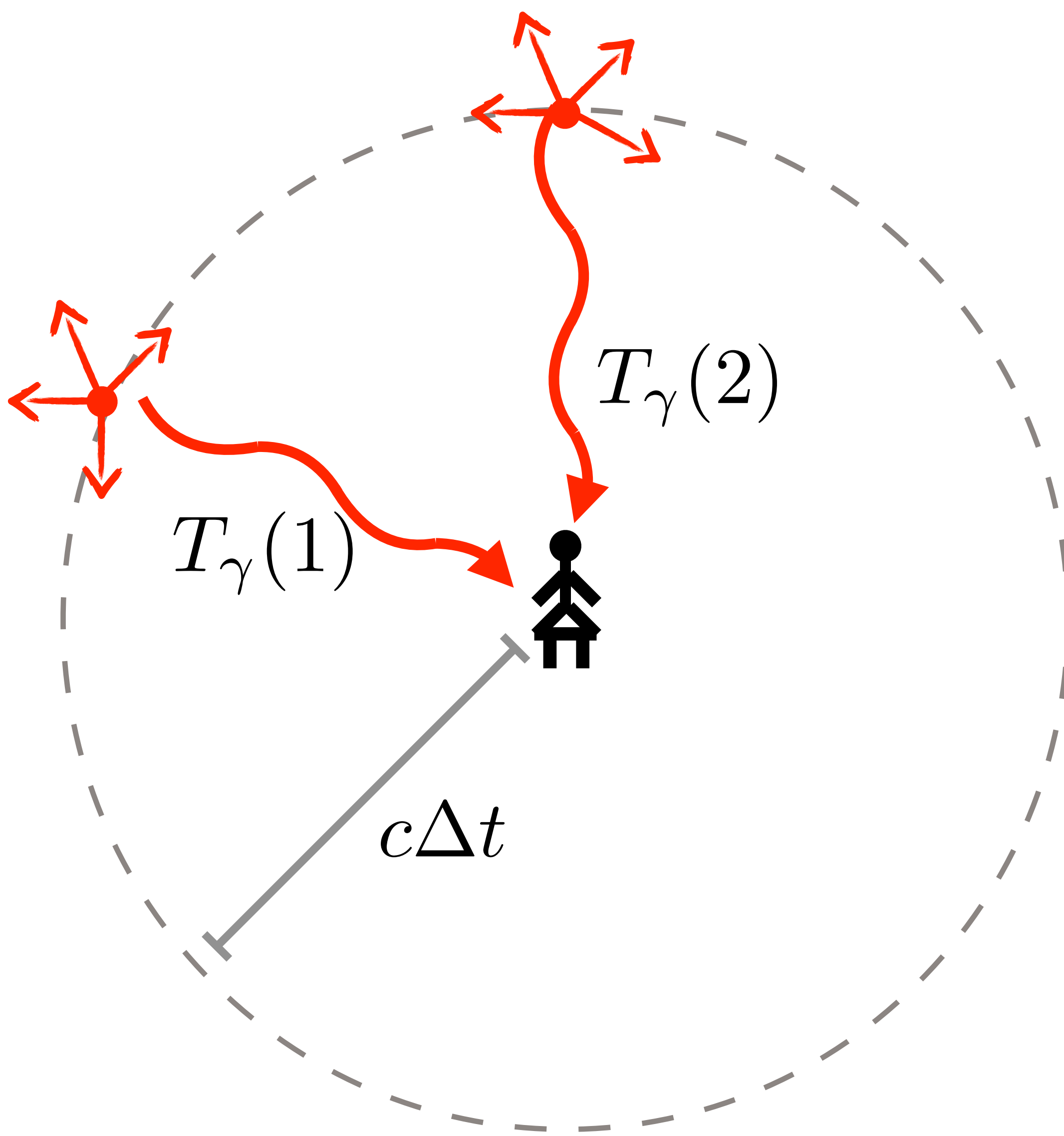


Cosmic Microwave Background (CMB)

¿ Where is that light?

Cosmic Microwave Background (CMB)

¿ Where is that light?

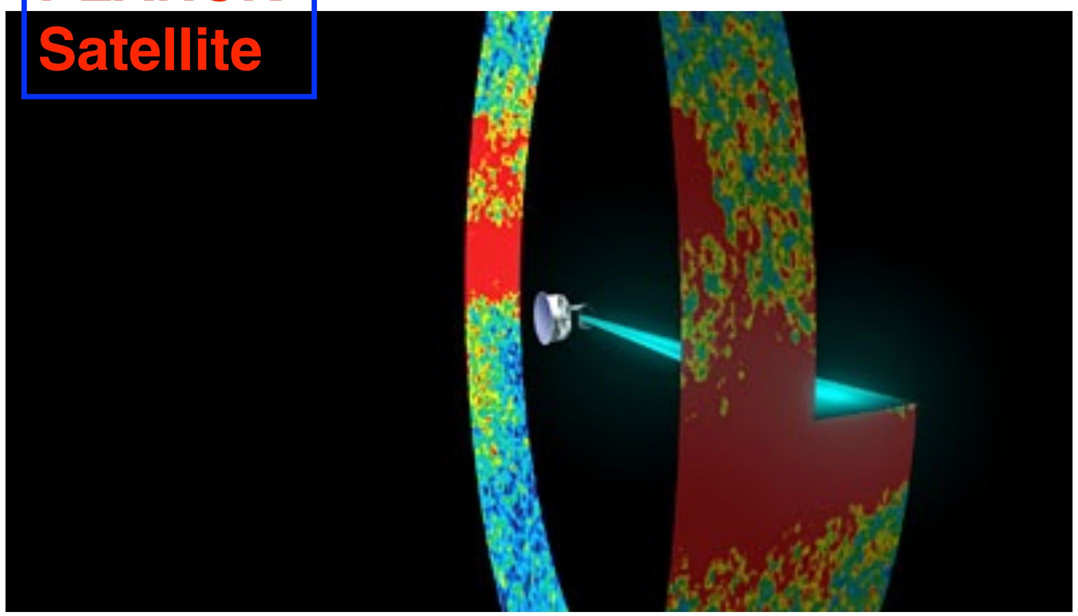


Cosmic Microwave Background (CMB)

Cosmic Microwave Background (CMB)

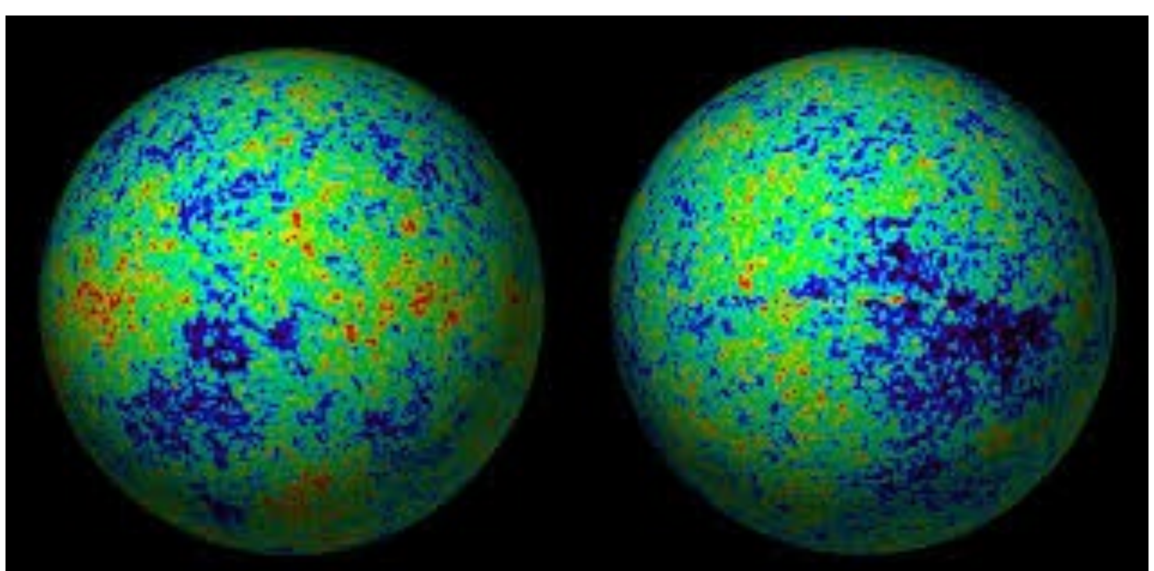
¿ Where is that light?

PLANCK
Satellite



Cosmic Microwave Background (CMB)

Definitive proof of the Big Bang Framework!

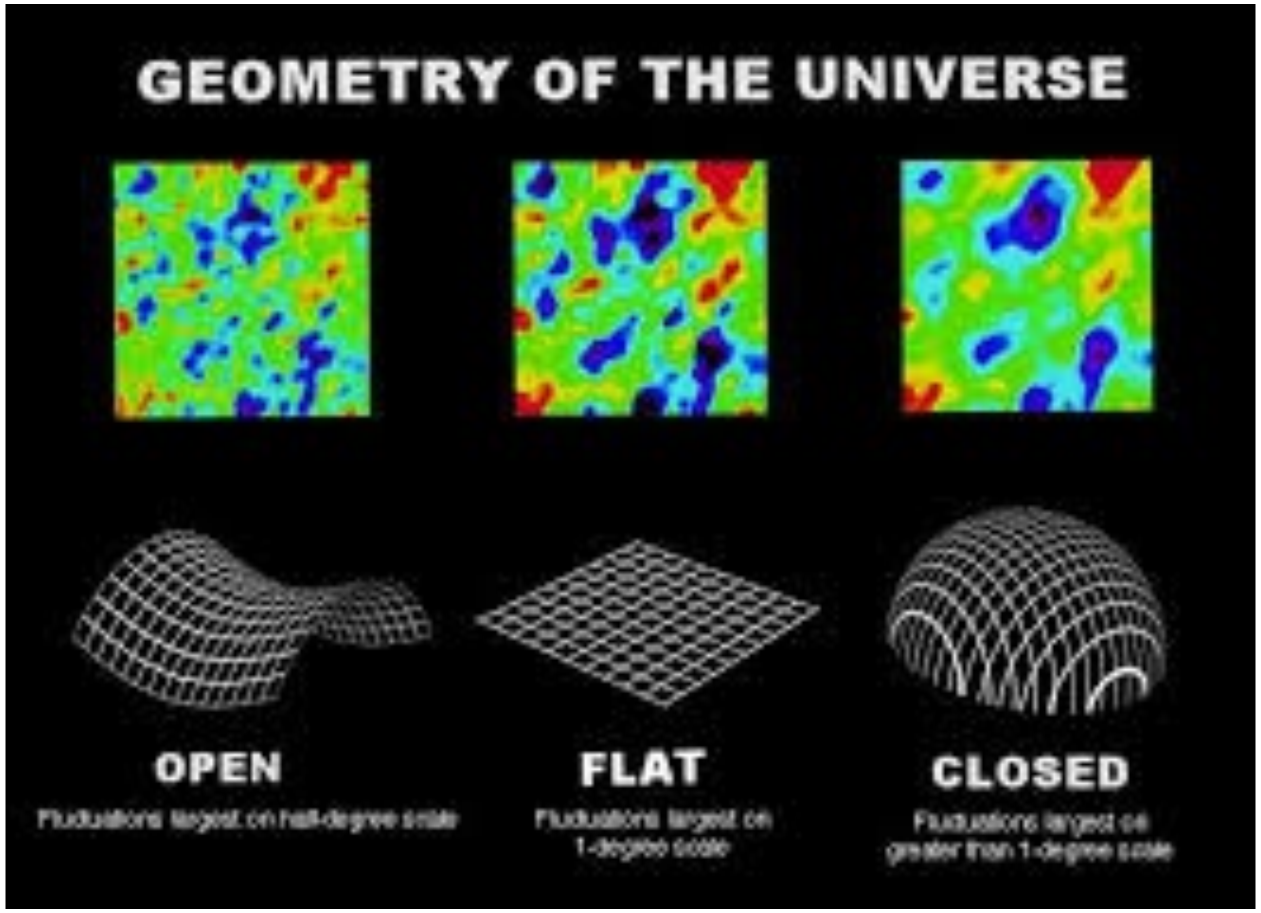


(almost-)ISOTROPIC

But ....

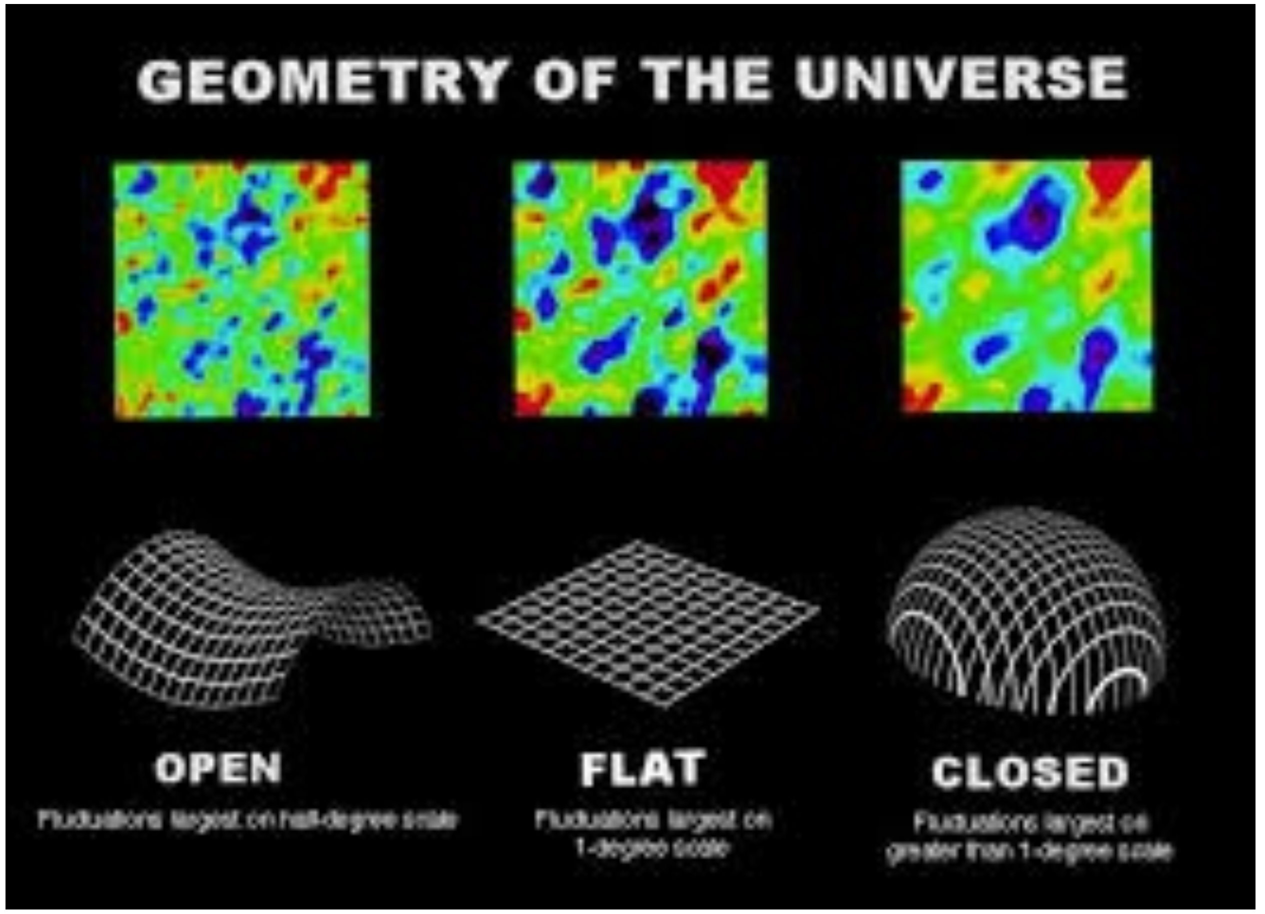
There are small 'Anisotropies' (variations of 1/100.000 only !)

Cosmic Microwave Background (CMB)



Properties of the Anisotropies **then .... Geometry of the Universe !**
(position of 1st acoustic peak)

Cosmic Microwave Background (CMB)



Properties of the Anisotropies **then .... Geometry of the Universe !**
(position of 1st acoustic peak)

The UNIVERSE has FLAT GEOMETRY (k = 0)

BASICS of COSMOLOGY

- **Cosmology**
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    - **theoretical pillars** ✓
      - General Relativity
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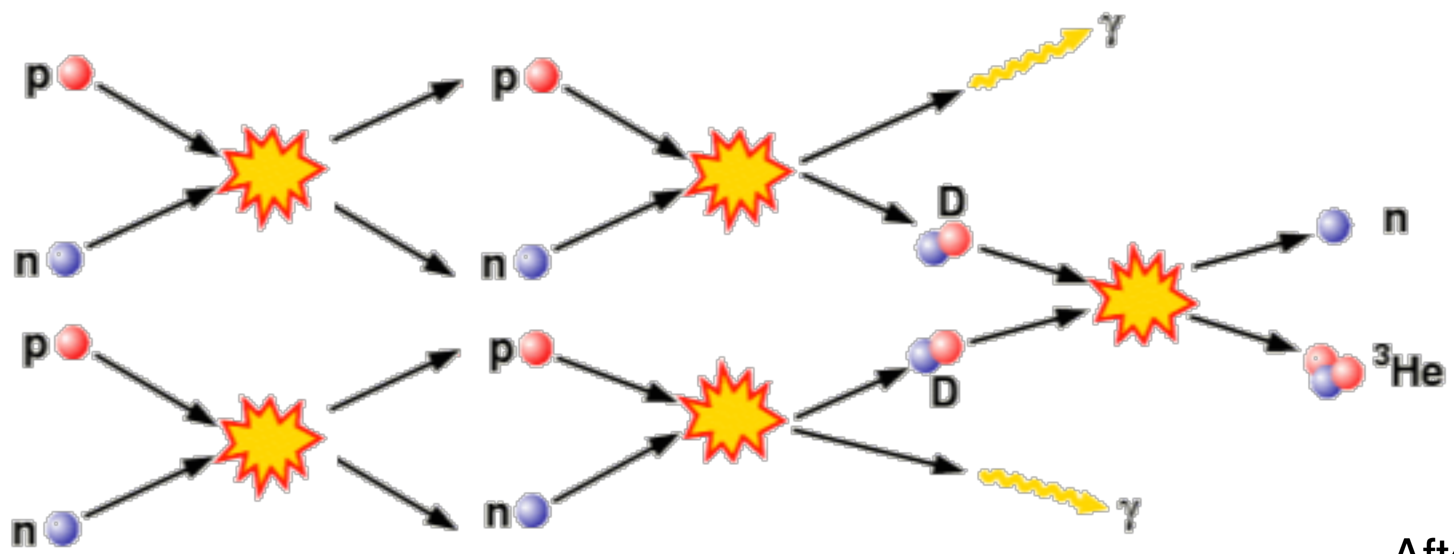
    - **observational pillars**
      - Expansion ✓
      - CMB ✓
      - $\boxed{\text{BBN}}$
  - **Inflation** = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ 'cures' hBB $\Longrightarrow$ $\left\{\begin{array}{l}\text{Cosmological Pple}\\ \text{CMB/LSS}\end{array}\right.$

Big Bang Nucleosynthesis

Formation of atomic nuclei (1s - 3 mins)



Past, hotter

(1 second)

Afterwards, colder

(3 minutes)

Protons, Neutrons

Interact strongly

Universe cools down…

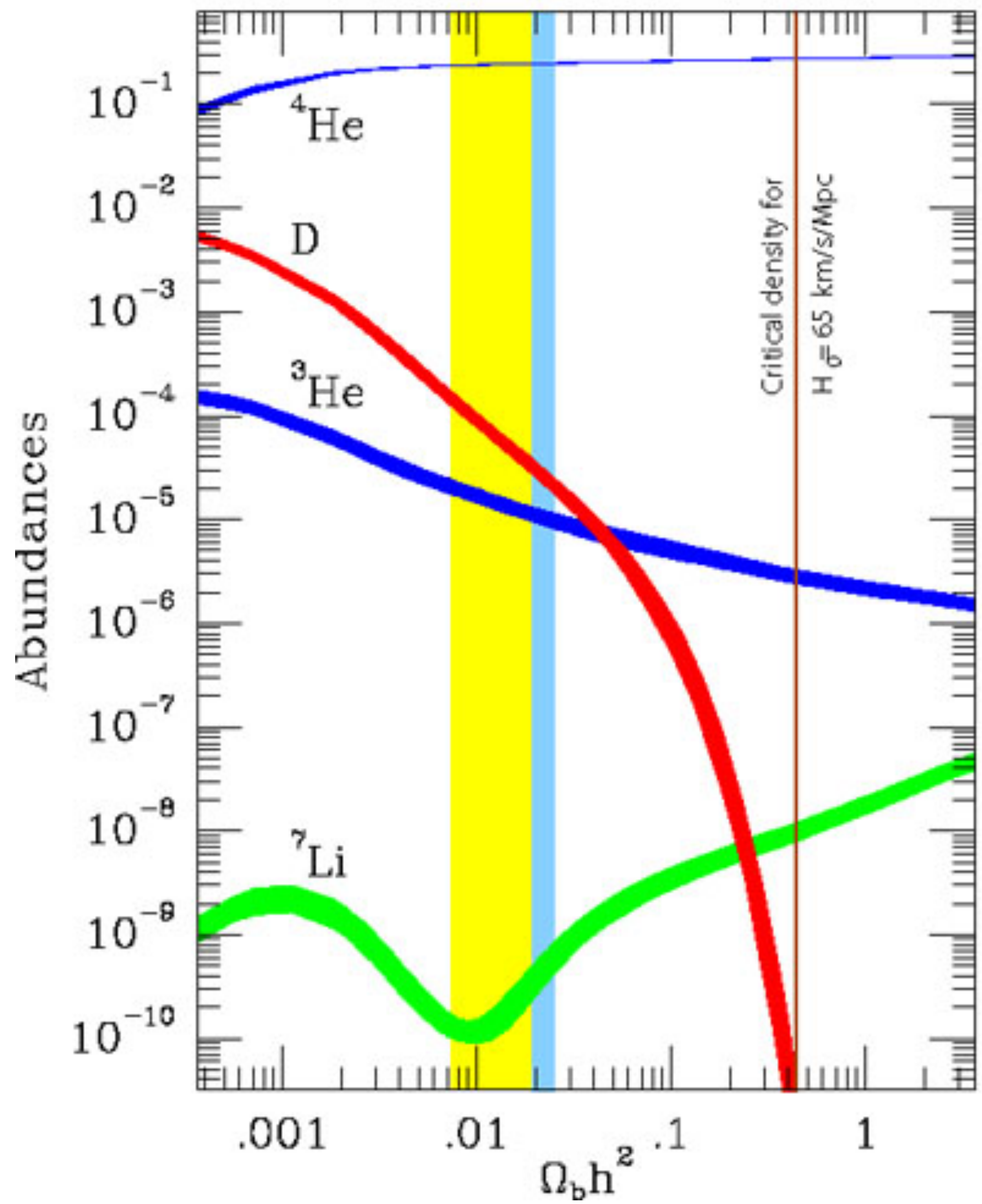
… protons and neutrons **don't have sufficient energy anymore**

Then **they join together forming atomic nuclei**: Nuclear Physics!

Atomic Nuclei created !

Big Bang Nucleosynthesis

Formation of atomic nuclei (1s - 3 mins)



NUCLEAR PHYSICS
(measured in the lab)

Predicts abundances of

$$H,\ ^4He,\ D,\ ^3He,\ ^7Li, \ldots$$

Another definitive proof of hot Big Bang framework !

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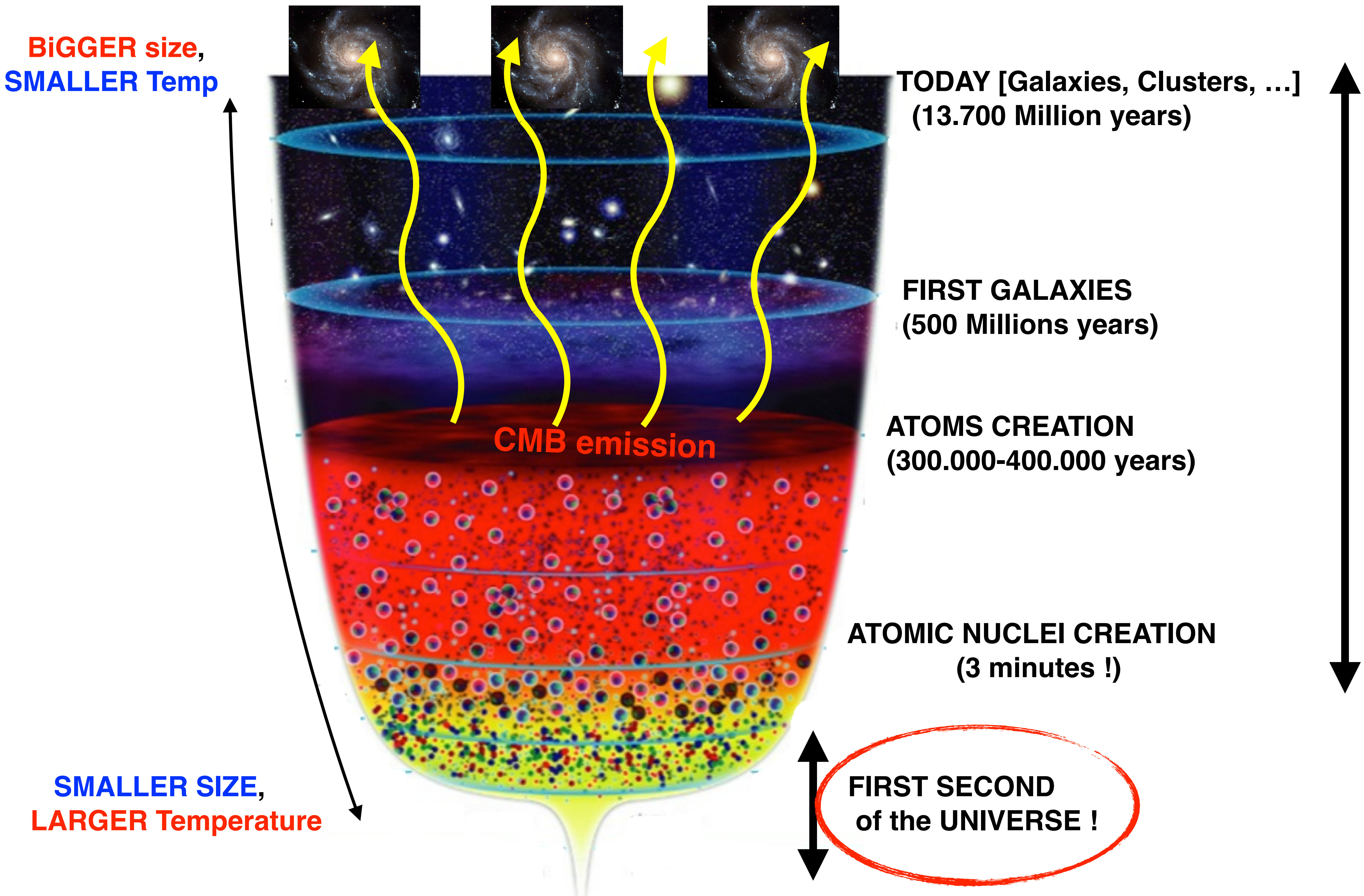
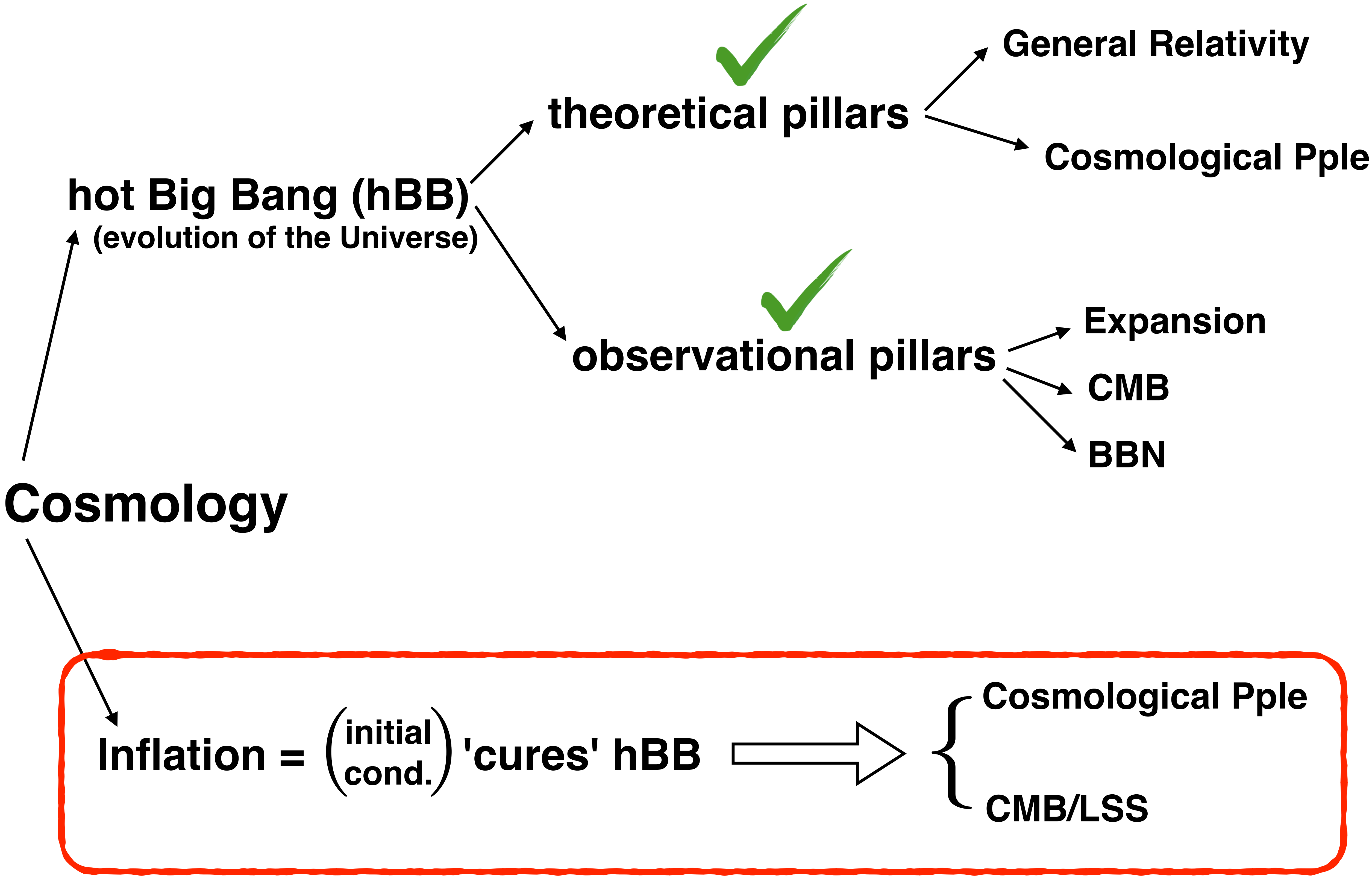
hot Big Bang (hBB)



BASICS of COSMOLOGY



BASICS of COSMOLOGY

theoretical pillars

General Relativity

Cosmological Pple

hot Big Bang (hBB)
(evolution of the Universe)

observational pillars

Expansion

CMB

BBN

Cosmology

Inflati

logical Pple

CMB/LSS

To Be Continued

BACK SLIDES

Shortcomings of the hBB framework

Electron/Positron Annihilation

Shortcomings of the hBB framework

hBB shortcomings

(motivation for inflation)

UNIVERSE → 1) GR + 2) H & I ⇒

Friedmann Equations

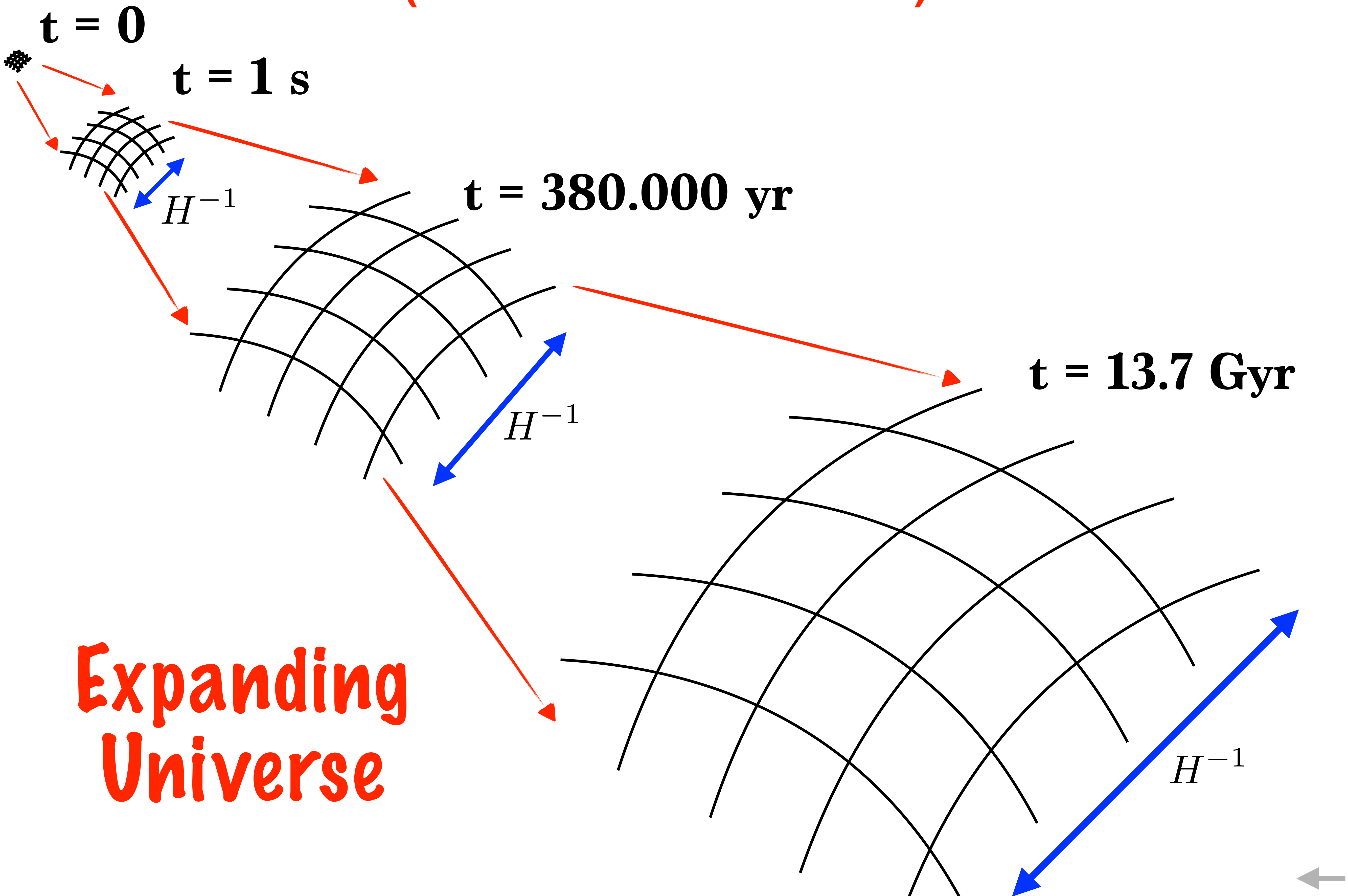
$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\rho}{6m_p^2}(1+3w)$$

$$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w)$$

$$\left(w \equiv \frac{p}{\rho}\right)$$

hBB shortcomings
(motivation for inflation)



hBB shortcomings
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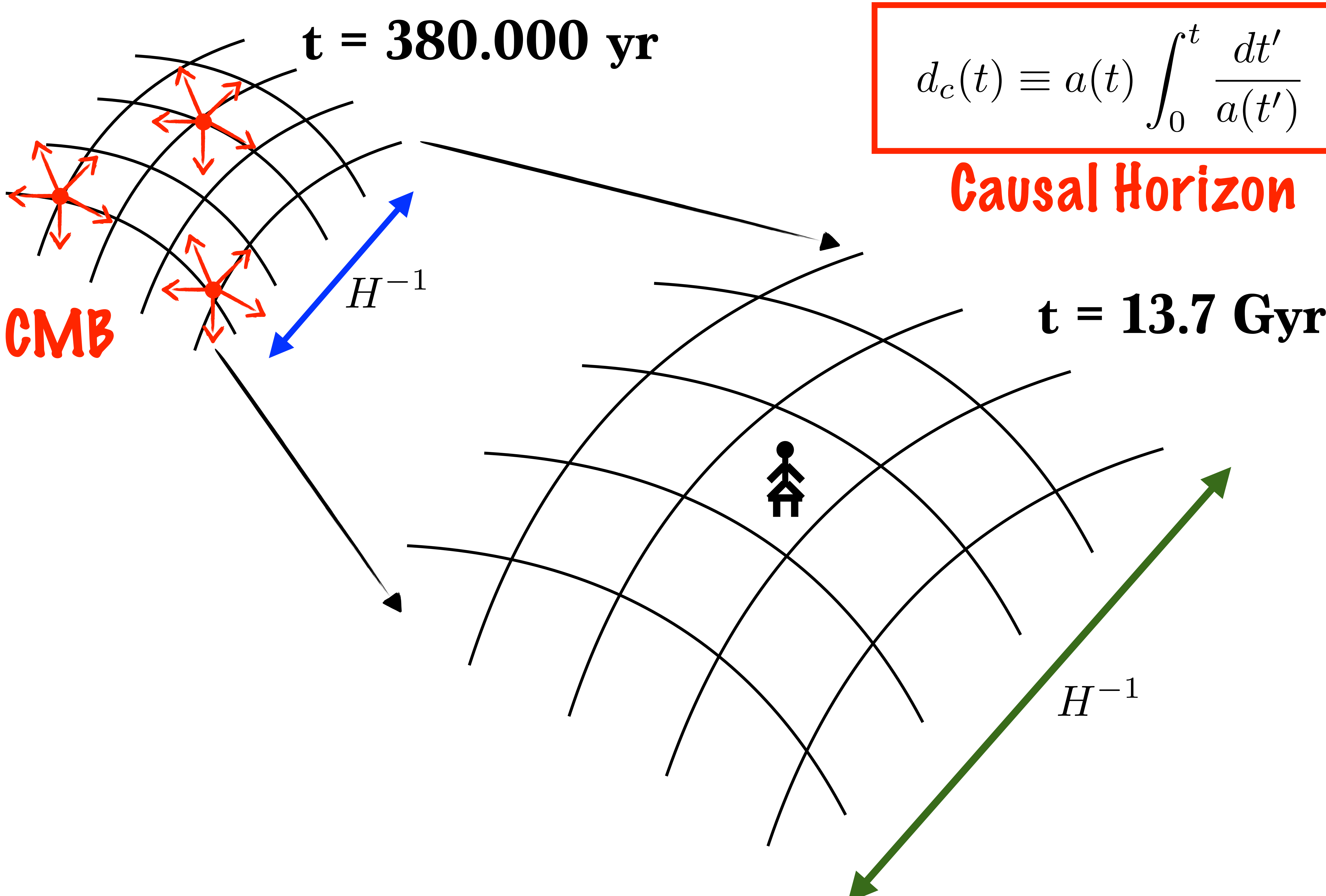
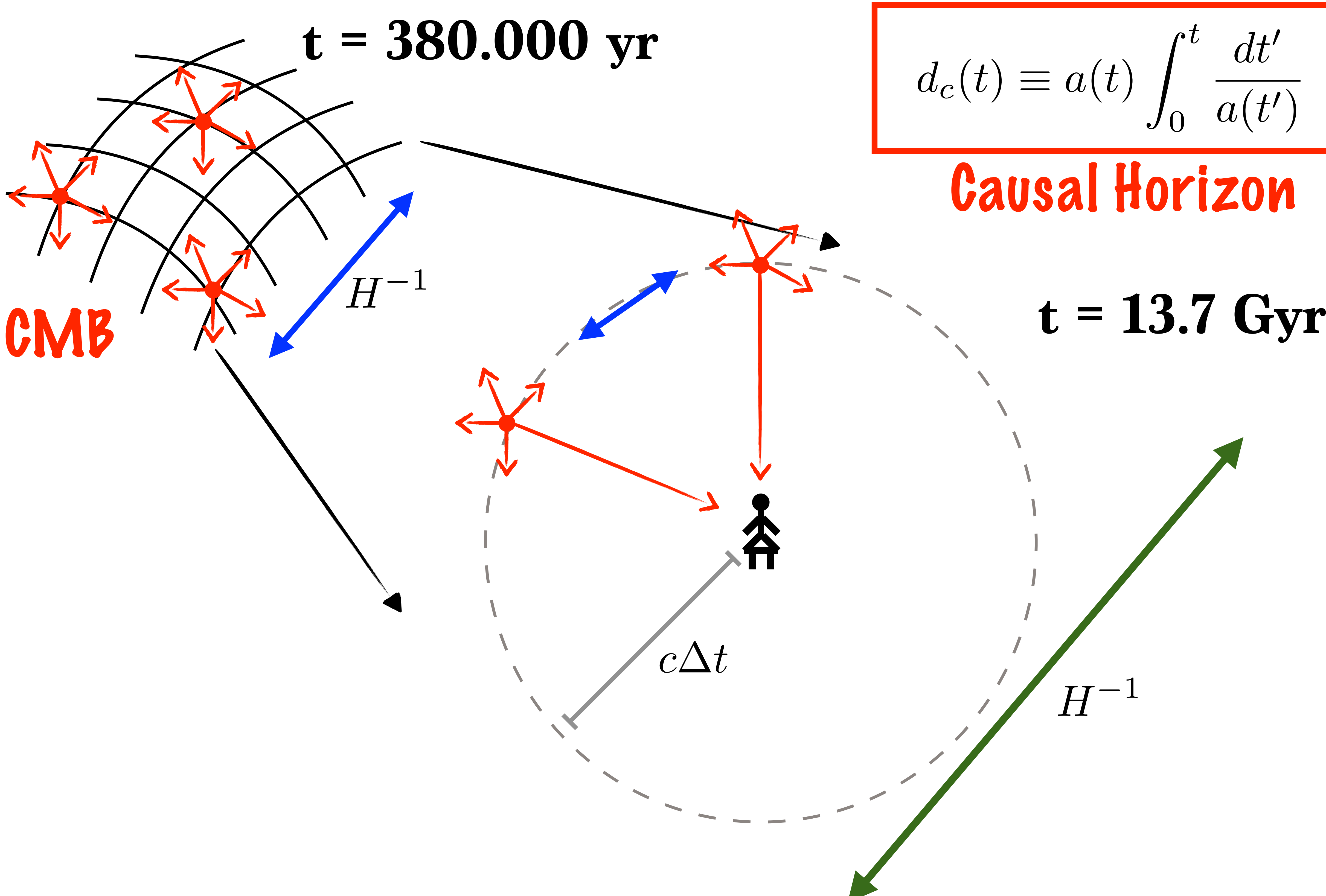


$$\boxed{d_c(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}}$$

Causal Horizon

hBB shortcomings

(motivation for inflation)



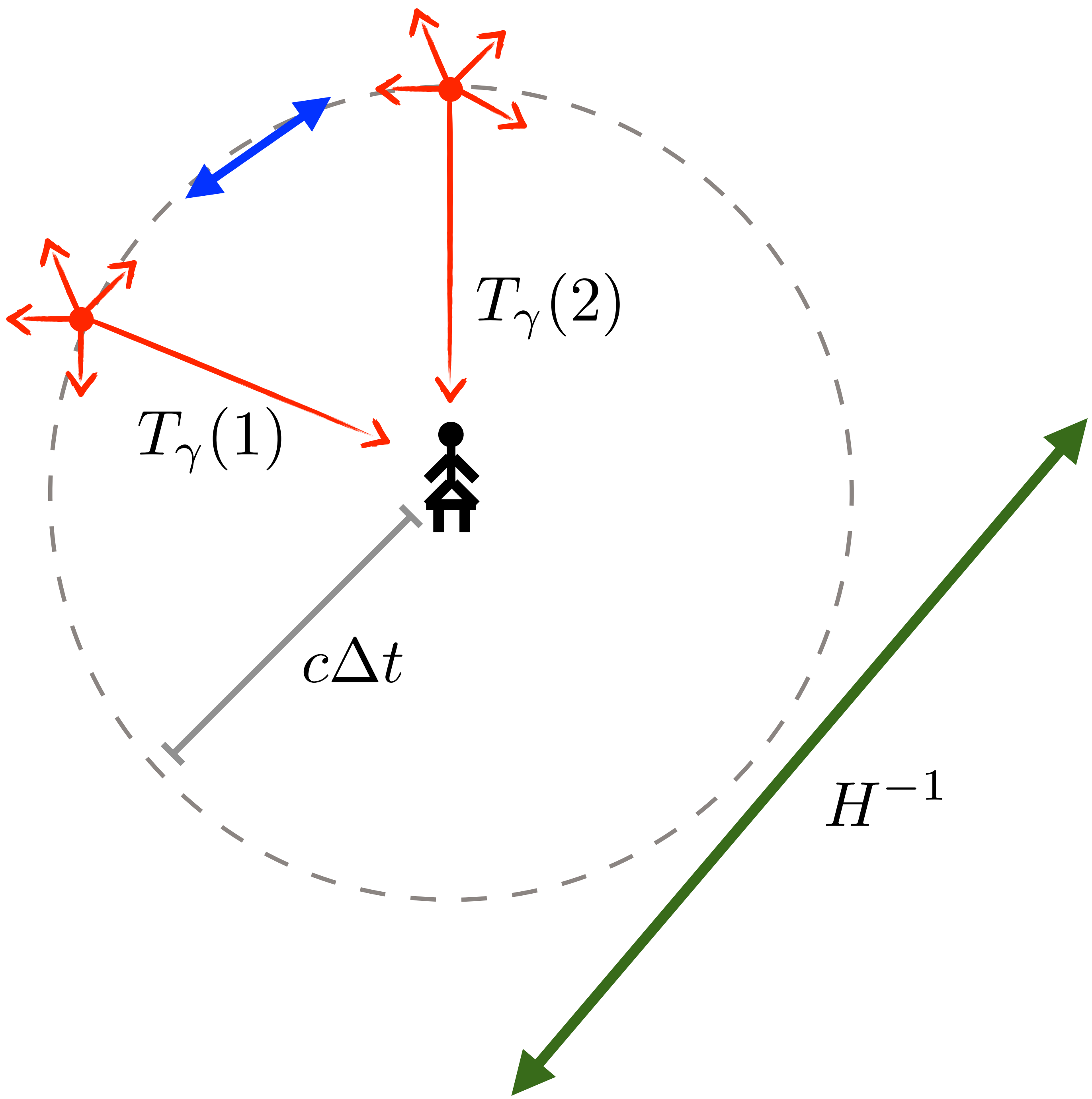
$$d_c(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}$$

Causal Horizon

hBB shortcomings

(motivation for inflation)

t = 13.7 Gyr



IF $\boxed{T_\gamma(1) = T_\gamma(2)}$

CAUSALITY VIOLATION !

hBB:

H&I @ Scales >> 1/H

iLL-defined!

hBB shortcomings
(motivation for inflation)

hBB $\nearrow$ 1) Horizon Problem $\longrightarrow$ Causality Violation !

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem $\longrightarrow$ Causality Violation !

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem
(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \mathrm{RD} \\ \propto a, & \mathrm{MD} \end{cases}$$

hBB shortcomings
(motivation for inflation)

hBB

1) **Horizon Problem** → Causality Violation !

2) **Curvature Problem**

(If curvature $\neq 0$, it grows unstable!)

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$$\text{Today}: \ |\Omega - 1|_o \lesssim 0.1$$

hBB shortcomings
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$$\text{Today}: \ |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \quad |\Omega - 1|_{\rm BBN} = \left(\frac{a_{\rm eq}}{a_o}\right)\left(\frac{a_{\rm BBN}}{a_{\rm eq}}\right)^2 |\Omega - 1|_o$$

$$\simeq \frac{1}{(1+z_{\rm eq})}\left(\frac{T_{\rm eq}}{T_{\rm BBN}}\right)^2 |\Omega - 1|_o$$

$$\simeq 10^{-3} 10^{-12} |\Omega - 1|_o \lesssim 10^{-18}$$

hBB shortcomings
(motivation for inflation)

hBB

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hBB shortcomings
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hBB shortcomings

(motivation for inflation)

hBB

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(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

$$\text{Today}: \ |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \quad |\Omega - 1|_{\rm BBN} \lesssim 10^{-18}$$

$$\boxed{|\Omega - 1|_{\rm GUT}} = \left(\frac{a_{\rm GUT}}{a_{\rm BBN}}\right)^2 |\Omega - 1|_{\rm BBN} \sim (T_{\rm BBN}/T_{\rm GUT})^2 |\Omega - 1|_{\rm BBN}$$

$$\simeq 10^{-38} |\Omega - 1|_{\rm BBN} \boxed{\lesssim 10^{-56}}$$

hBB shortcomings
(motivation for inflation)

hBB ↗ **1) Horizon Problem** ⟶ **Causality Violation !**

hBB ↘ **2) Curvature Problem**

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

$$\text{Today}: \quad |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \quad \boxed{|\Omega - 1|_{\rm BBN} \lesssim 10^{-18}} \quad \boxed{|\Omega - 1|_{\rm GUT} \lesssim 10^{-56}}$$

It might well be that k = 0 ...

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem $\longrightarrow$ Causality Violation !

2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

$\boxed{\text{Need extra ‘Ingredient’ ! } \longrightarrow \text{ INFLATION !}}$

Electron/Positron Annihilation

The Early Universe

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2} \underbrace{\frac{\pi^2}{30} g_*(T) T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

When do $g_*(T), g_*^{(s)}(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,, \ T \to T_i\,, \\ 2)\ \underline{\text{Mass threshold}}\,, T < 2m_i\,, \end{cases}$$

Annihilation (mass threshold):

$T < 2m_i \ \Rightarrow$ Can't produce it anymore

[Boltzman Supression $\sim e^{-m/T}$]

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Example e+/e- Annihilation

$$e^+ + e^- \leftrightarrow 2\gamma\,, \quad T > 511 \ \mathrm{keV}$$

$$\cancel{e^+ + e^-} \to 2\gamma\,, \quad T < 511 \ \mathrm{keV}$$

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$$T_\gamma(t > t_{e^\pm}) = \left(\frac{g_s^<}{g_s^>}\right)^{1/3} T_\nu = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

The Early Universe

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2} \underbrace{\frac{\pi^2}{30} g_*(T) T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \ \text{s}$$

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$$T_\gamma(t > t_{e^\pm}) = \left(\frac{g_s^<}{g_s^>}\right)^{1/3} T_\nu = (11/4)^{1/3} T_\nu$$

$\nu, e^\pm, \gamma$ — $\nu - \text{dec}$ ($T_\nu \sim 0.8$ MeV) — $e^\pm, \gamma$ (ν) — $\cancel{e^\pm}$ ($T = 0.5$ MeV) — $\gamma' s$

$$g_s^< = \frac{7}{8} \cdot 2 \cdot 2 + 2 = \frac{11}{2} \qquad g_s^> = 2$$