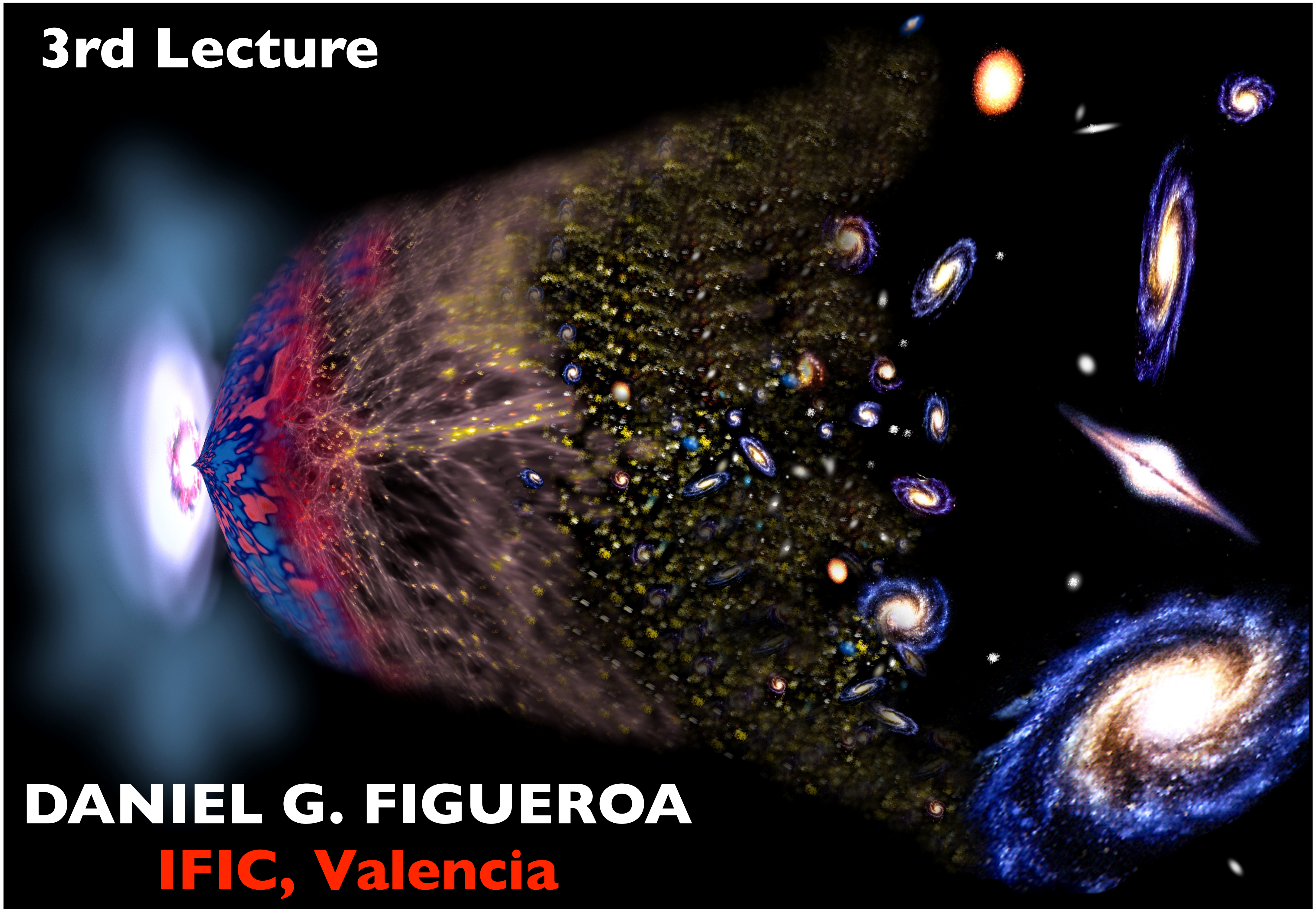
GRAVITATIONAL WAVE
— BACKGROUNDS —



GGI LECTURES ON THE THEORY OF FUNDAMENTAL INTERACTIONS — 2022 Program (3rd week)

Definition of GWs
4th approach

Gravitational Wave Definition

4th approach to GWs

(for a curved space-time)

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \delta g_{\mu\nu}(x)\,, \quad |\delta g_{\mu\nu}| \ll 1$$

(separation not well defined)

Gravitational Wave Definition

4th approach to GWs

(for a curved space-time)

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See e.g. Maggiore's 1st Book on GWs

More subtle problem! Solution: Separation of scales !

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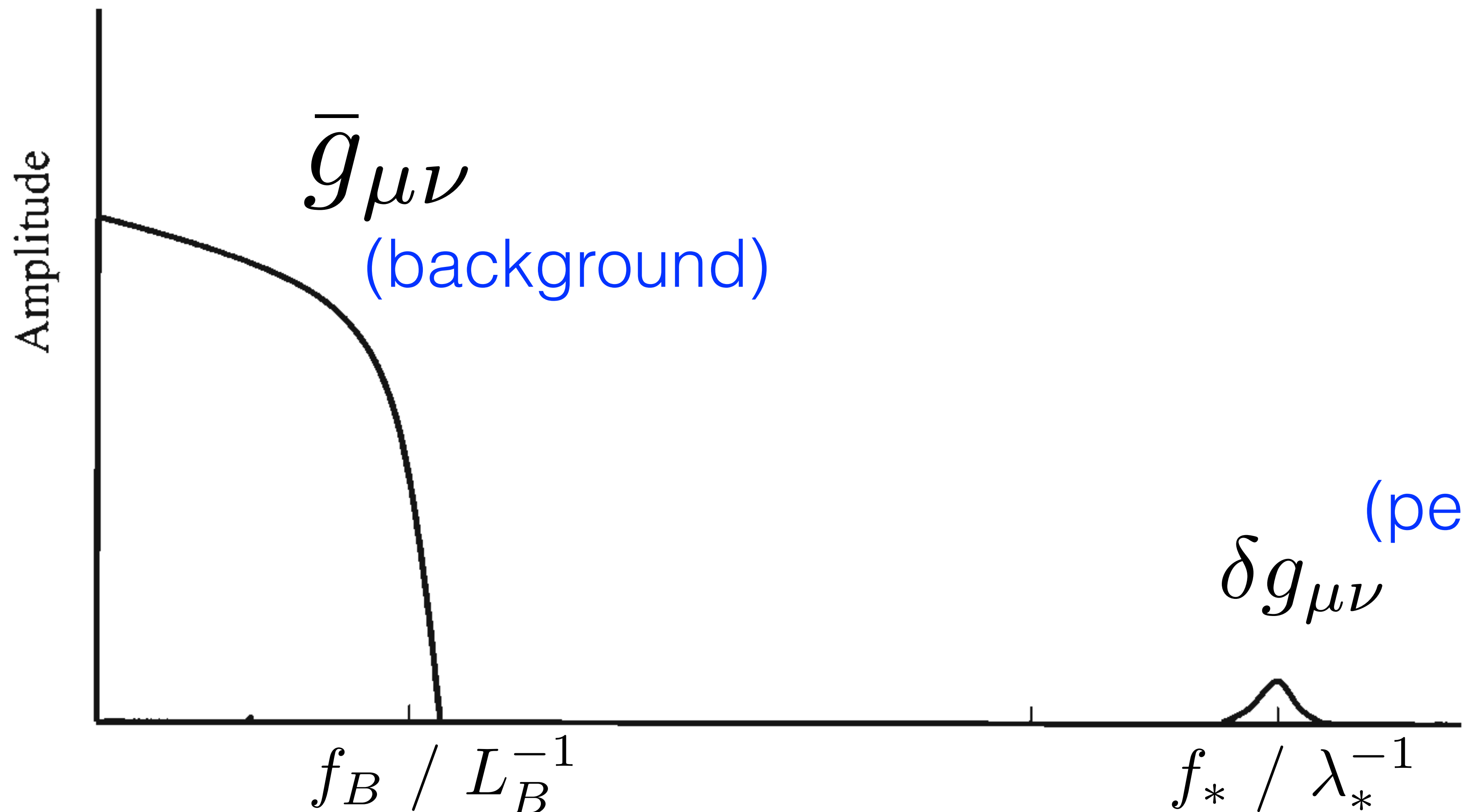

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$$R_{\mu\nu} = \frac{1}{m_p^2}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

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Gravitational Wave Definition

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Gravitational Wave Definition

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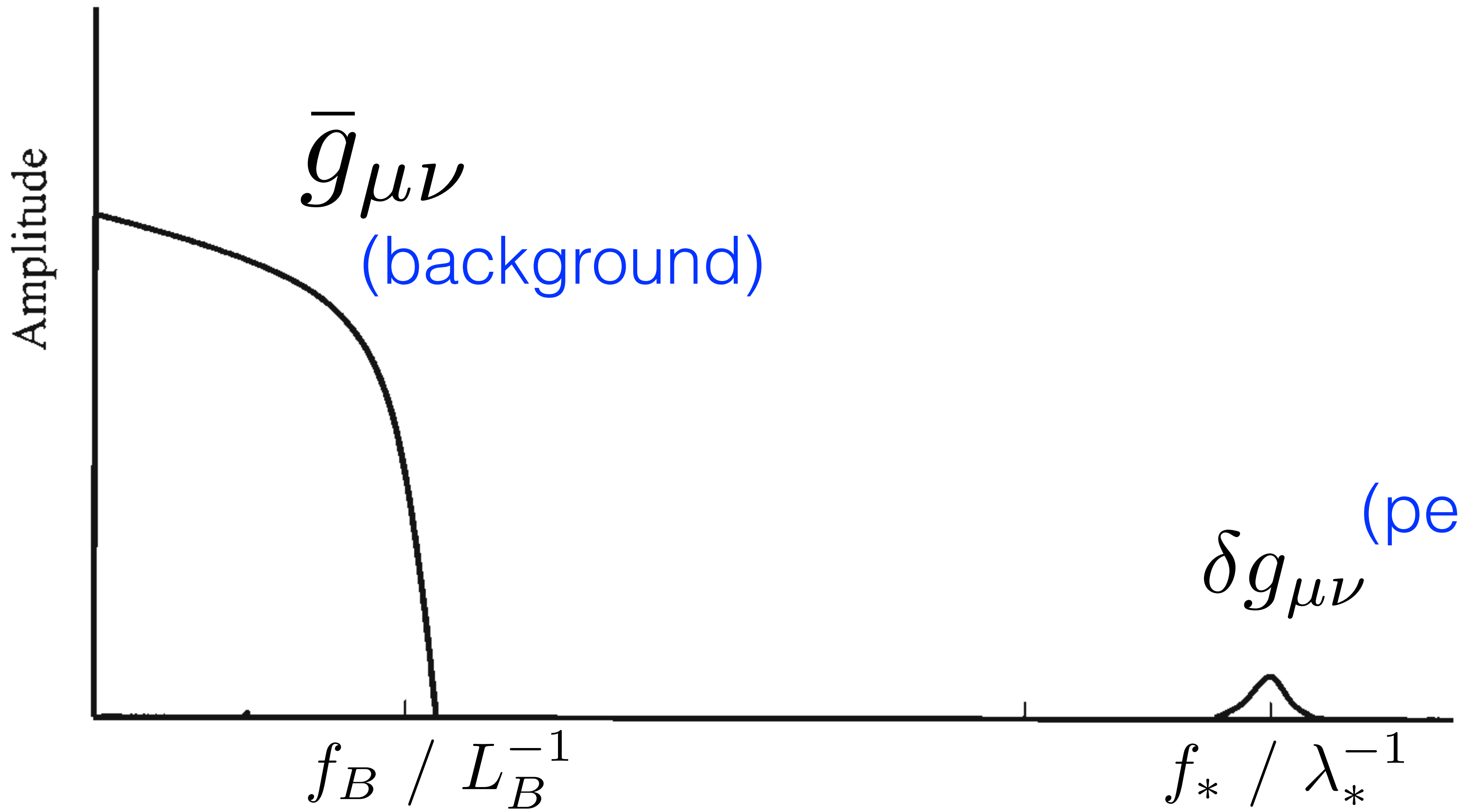
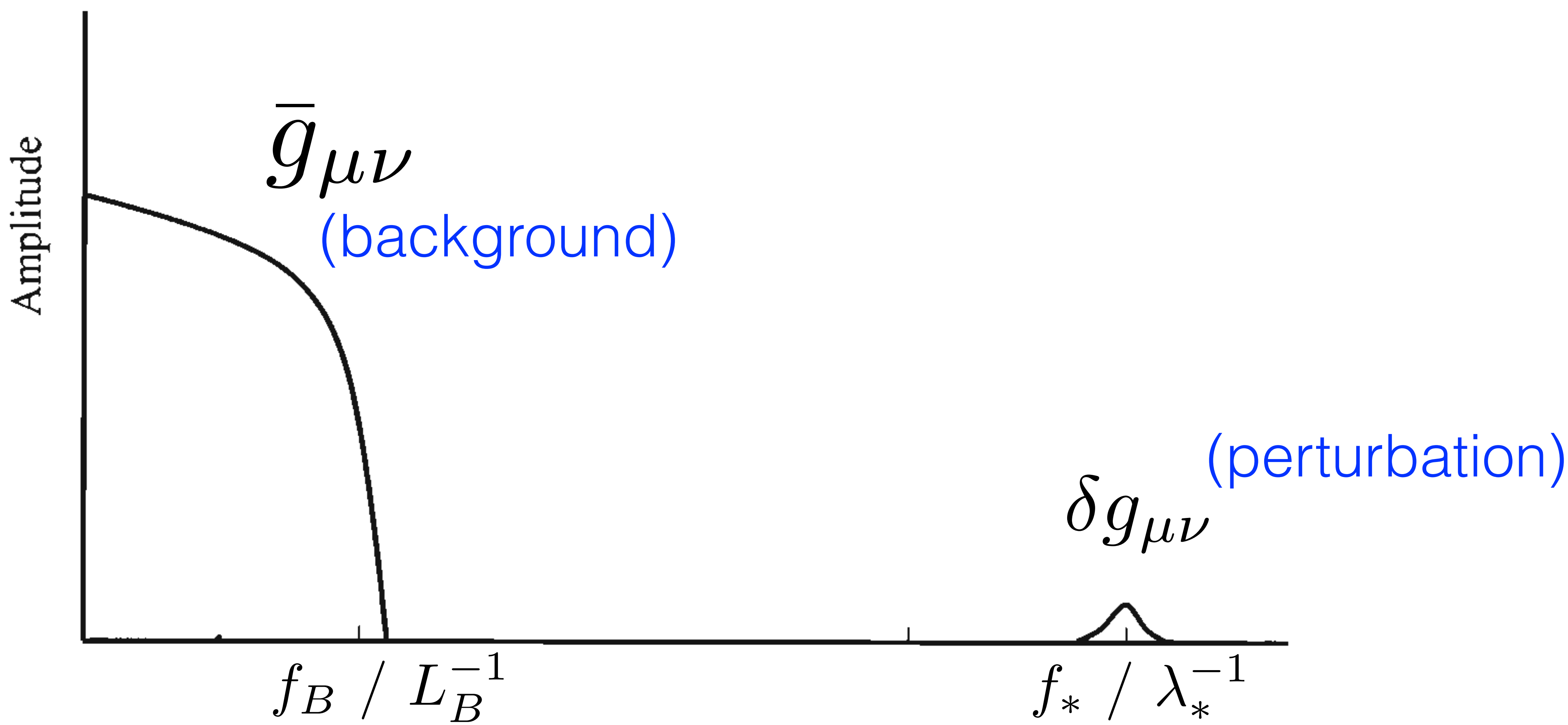
$\mathcal{O}(\delta g^2)$



Gravitational Wave Definition

(space/time average)

Low Freq. / Long Scale: $\bar{R}_{\mu\nu} = -\left\langle R^{(2)}_{\mu\nu} \right\rangle + \frac{1}{m_p^2}\left\langle T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right\rangle$



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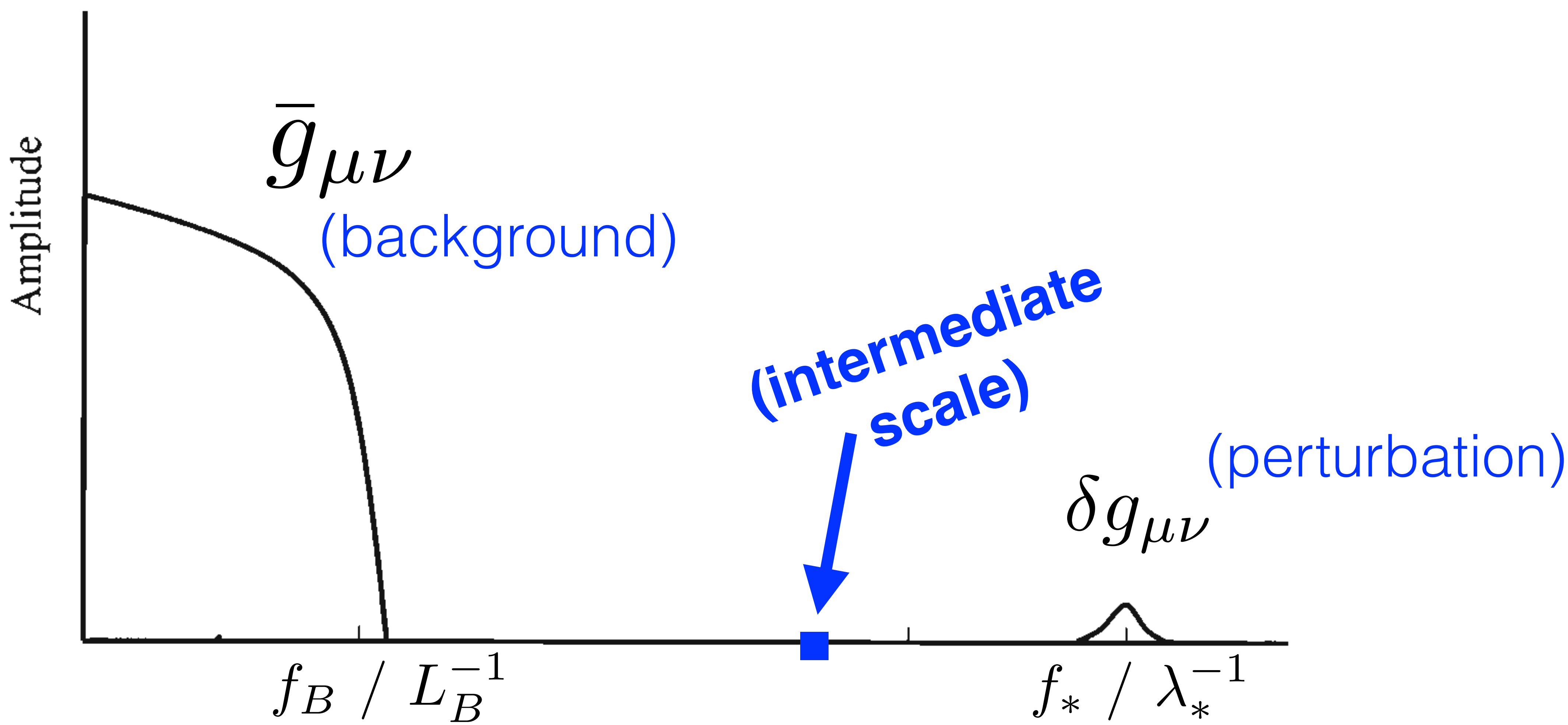

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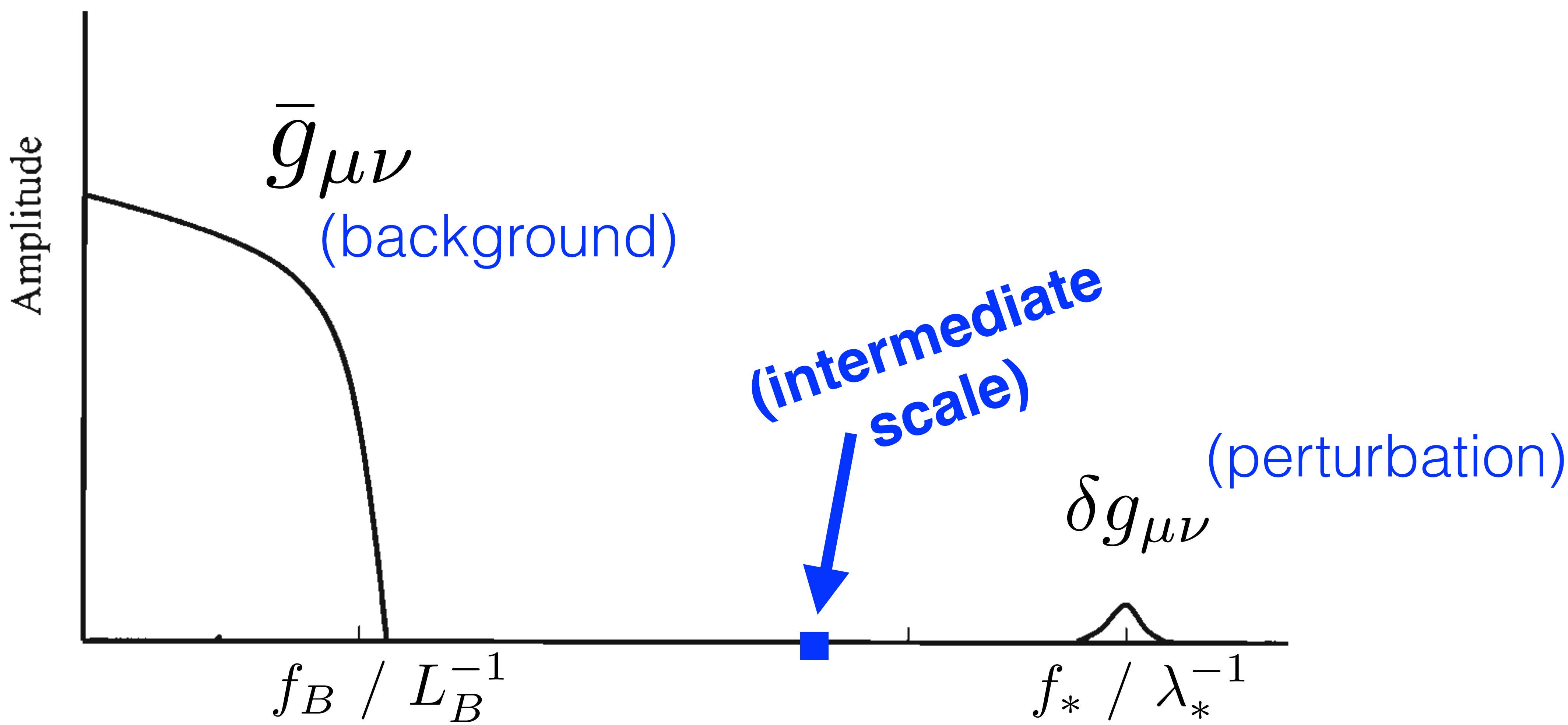

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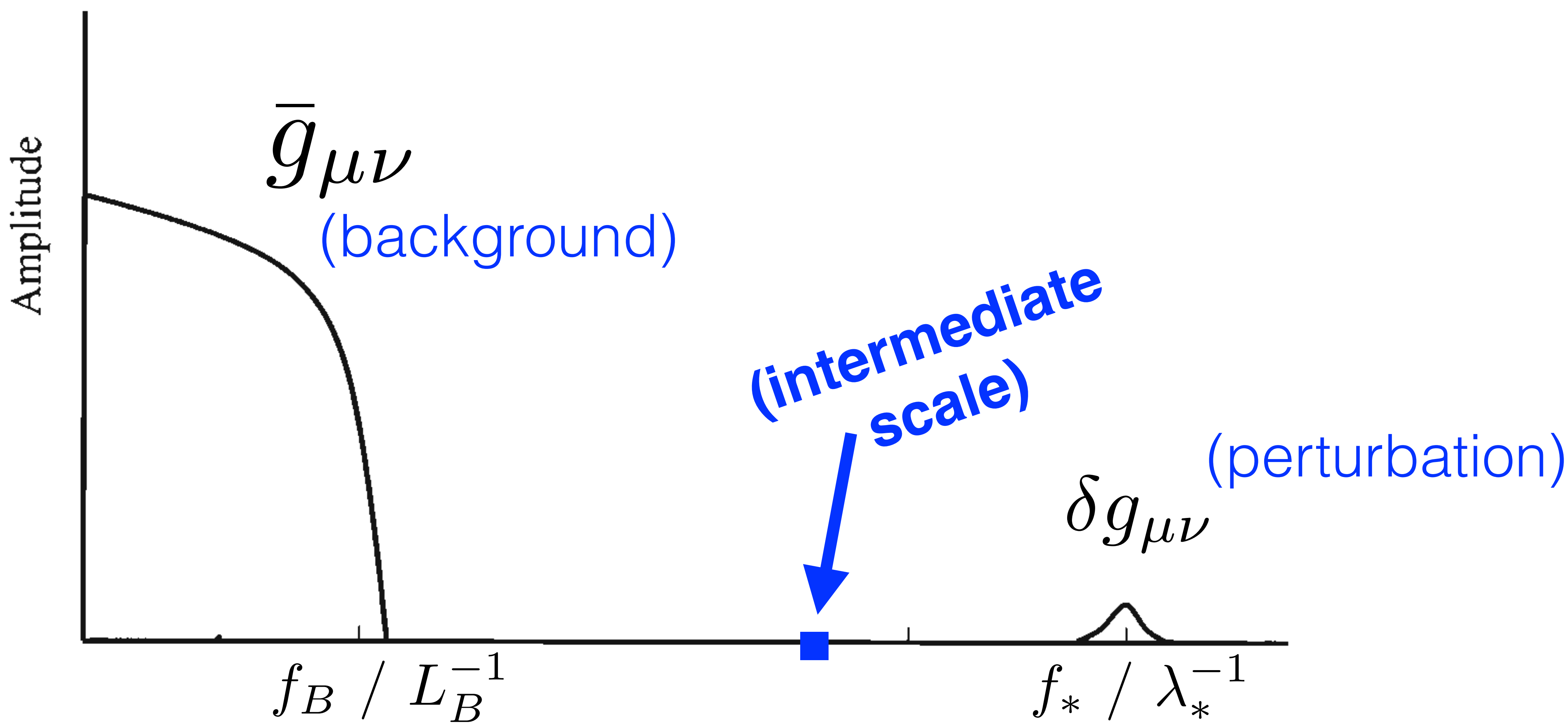

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It can be shown that only TT *dof* contribute to < … >

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$$\boxed{t_{\mu\nu} = \frac{m_p^2}{4}\,\langle\partial_\mu\delta g_{ij}^{\mathrm{TT}}\,\partial_\nu\delta g_{ij}^{\mathrm{TT}}\rangle}$$

GW energy-momentum tensor

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It can be shown that only TT *dof* contribute to < … >

$$\boxed{t_{\mu\nu} = \frac{m_p^2}{4}\langle\partial_\mu\delta g_{ij}^{\rm TT}\,\partial_\nu\delta g_{ij}^{\rm TT}\rangle} \quad \xrightarrow[(\delta g_{ij}\equiv h_{ij})]{} \quad \boxed{\rho_{\rm GW} \equiv \frac{m_p^2}{4}\langle\dot{h}_{ij}^{\rm TT}\,\dot{h}_{ij}^{\rm TT}\rangle}$$

GW energy-momentum tensor

GW energy density

Gravitational Wave Propagation

What about the
High Freq. / Short Scale?

$$R^{(1)}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\mathrm{High}} + \frac{1}{m_p^2}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\mathrm{High}}$$

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Lorentz gauge

Gravitational Wave Propagation

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vacuum

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Propagation of GWs
in curved space-time

Gravitational Wave Propagation

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$\longrightarrow$

vacuum

$$\boxed{D_\alpha D^\alpha \delta g^{\text{TT}}_{ij} = 0}$$

Propagation of GWs
in curved space-time
$\left(D_i \delta g^{\text{TT}}_{ij} = \bar{g}^{ij}\delta g^{\text{TT}}_{ij} = 0 \right)$

Gravitational Wave Propagation

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$\longrightarrow$

matter

$$\boxed{D_\alpha D^\alpha \overline{\delta g}_{\mu\nu} = \Pi_{\mu\nu}}$$

Creation of GWs
in curved space-time

Gravitational Wave Propagation

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Lorentz gauge

$$\boxed{D_\alpha D^\alpha \overline{\delta g}^{\rm TT}_{\mu\nu} = \Pi^{\rm TT}_{\mu\nu}}$$

matter

Creation of GWs
in curved space-time

TT dof = truly radiative !
[no gauge choice]

Definition of GWs

* 1st approach: Lin Grav in Minkowski ✓
* 2nd approach: SVT decomp. ✓
* 3rd approach: FLRW background ✓
* 4rd approach: General backgrounds ✓

Before we move into the 2nd Bloc…

Some perspective

GW Propagation/Creation in Cosmology

FLRW: $ds^2 = a^2(-\,dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \qquad \mathrm{TT}: \begin{cases} h_{ii} = 0 \\ h_{ij},_j = 0 \end{cases}$

(conformal time)

GW Propagation/Creation in Cosmology

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(conformal time)

Creation/Propagation GWs in FLRW

Eom: $\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\mathrm{TT}}_{ij},}$

Source: Anisotropic Stress

$\boxed{\Pi_{ij} = T_{ij} - \langle T_{ij} \rangle_{\mathrm{FLRW}}}$

GW Propagation/Creation in Cosmology

FLRW: $ds^2 = a^2(-dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j)$, $\quad \mathrm{TT}: \begin{cases} h_{ii} = 0 \\ h_{ij},_j = 0 \end{cases}$

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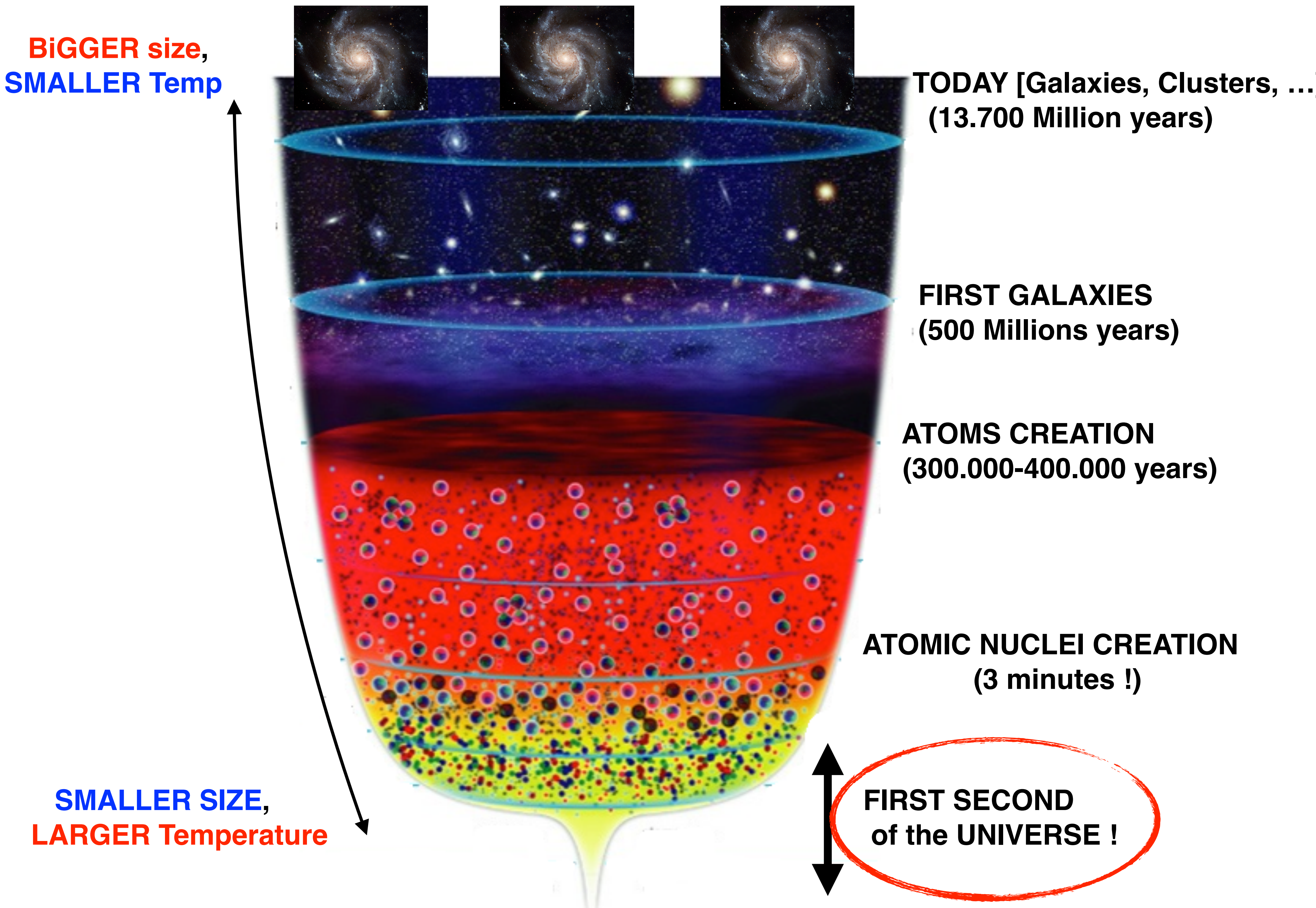
Source: Anisotropic Stress

$\boxed{\Pi_{ij} = T_{ij} - \langle T_{ij}\rangle_{\text{FLRW}}}$

GW Source(s): (SCALARS , VECTOR , FERMIONS)

$\Pi^{TT}_{ij} \propto \{\partial_i\chi^a\partial_j\chi^a\}^{TT}, \{E_iE_j + B_iB_j\}^{TT}, \{\bar\psi\gamma_iD_j\psi\}^{TT}$

Cosmic History



OUTLINE

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

A primer on Inflation

INFLATIONARY COSMOLOGY

'cures' hBB $\Longrightarrow$ Cosmological Pple

Inflation

INFLATIONARY COSMOLOGY

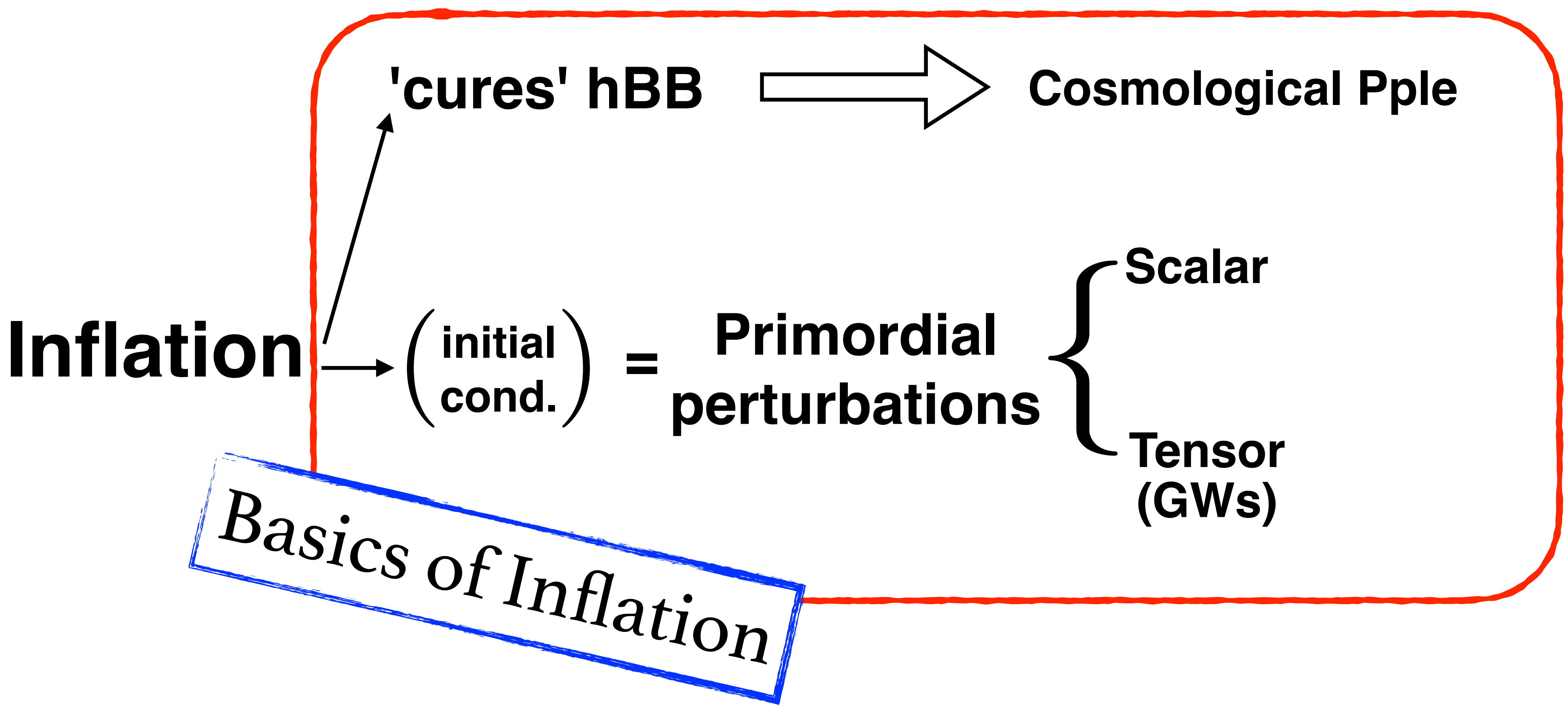
'cures' hBB $\Rightarrow$ Cosmological Pple

Inflation $\rightarrow$ (initial cond.) = Primordial perturbations

Scalar

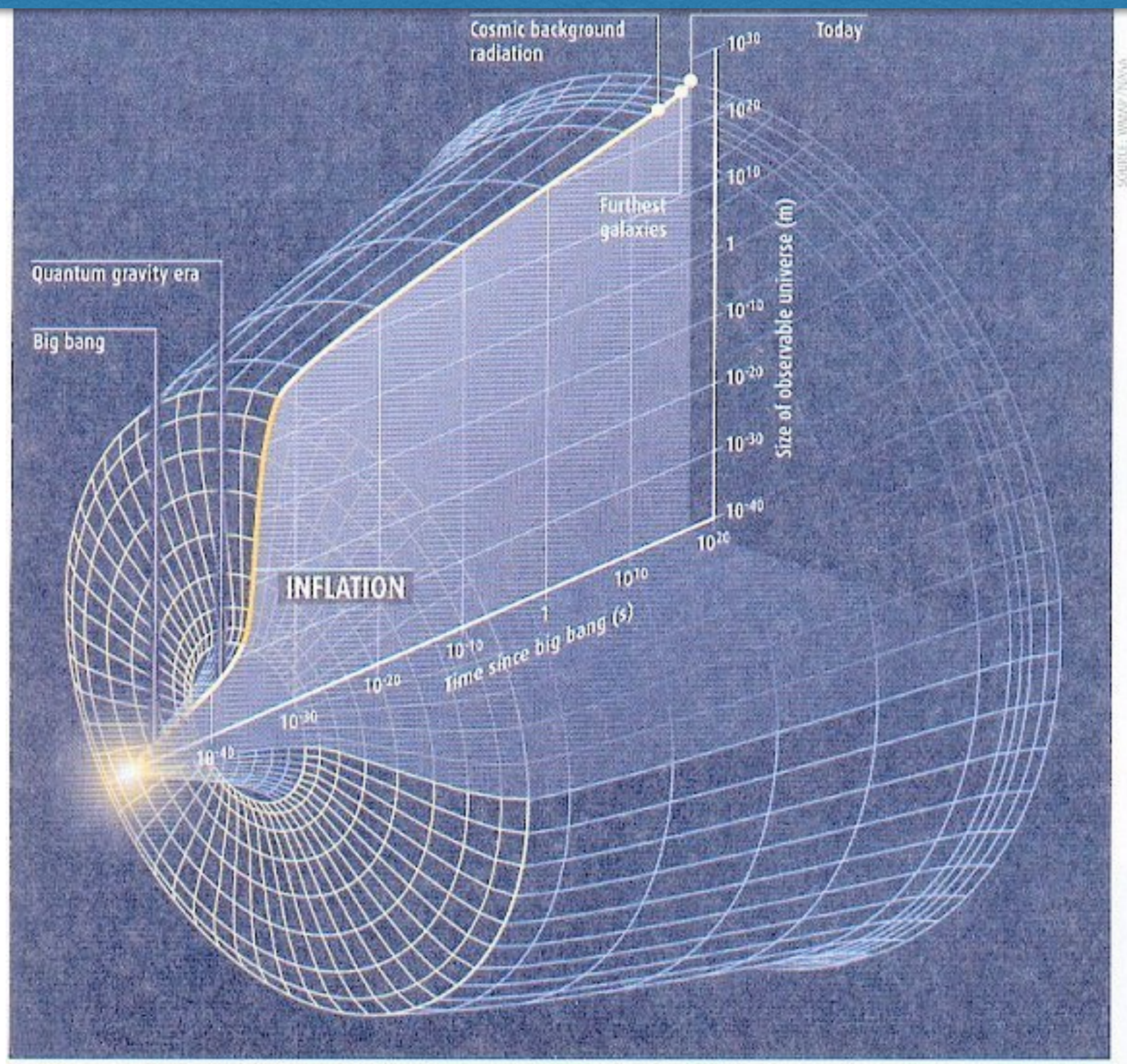
Tensor (GWs)

INFLATIONARY COSMOLOGY



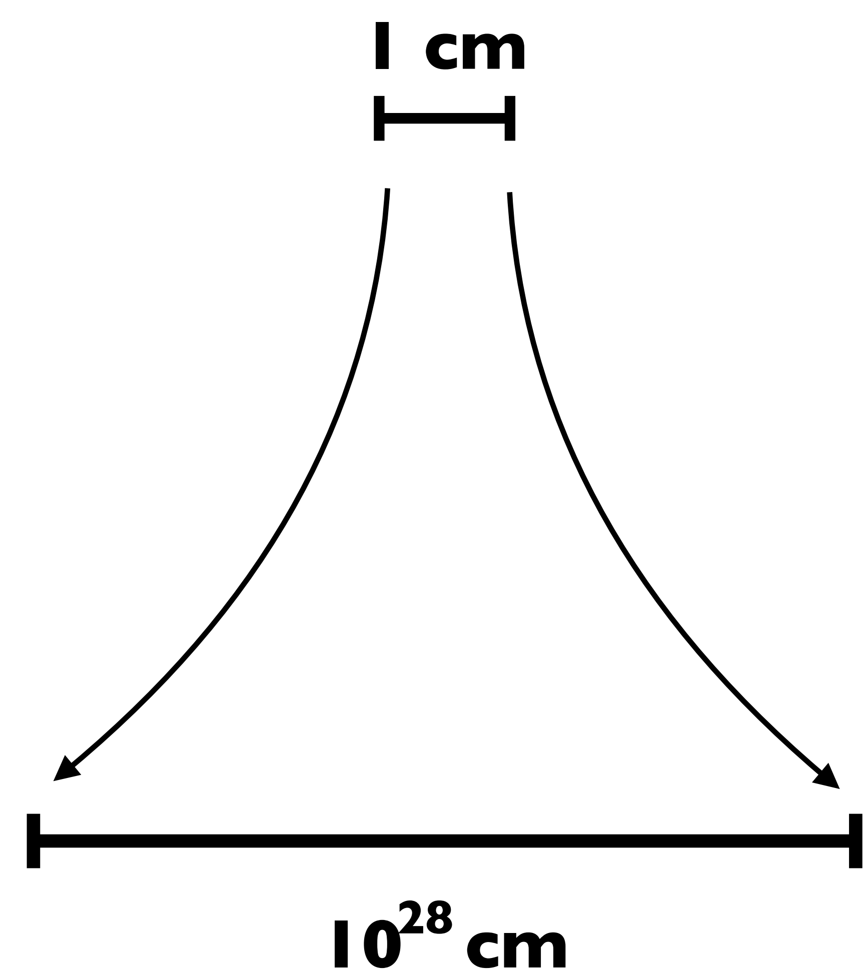
Inflation (basics)

COSMIC INFLATION



Needed for Consistency of the Big Bang theory

$$\boxed{a \sim e^{H_* t} \gtrsim e^{60}}$$



Inflation: Definition + Implementation

INF

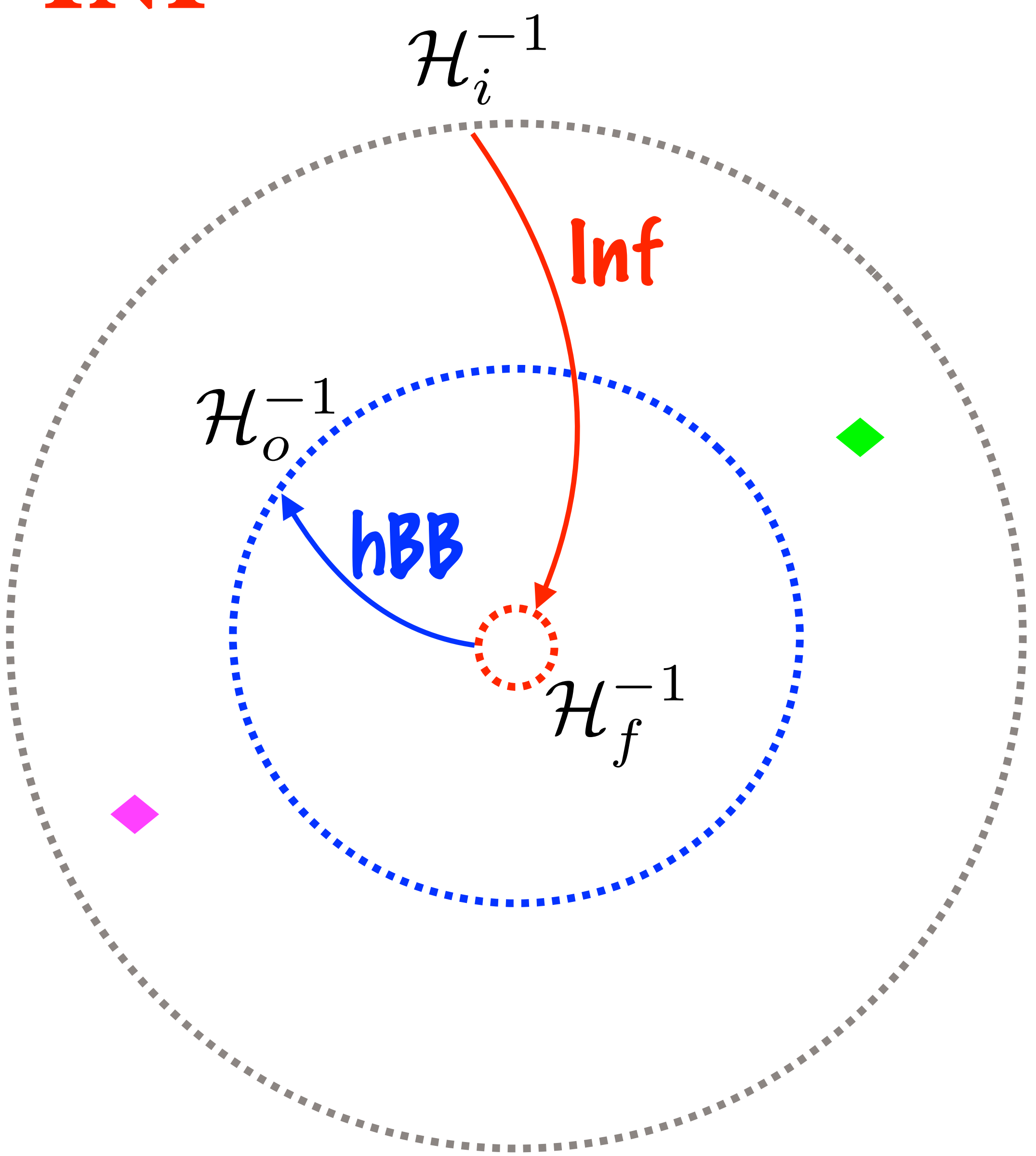
* Definition: $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

Inflation: Definition + Implementation

INF

* Definition:

$$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$



$$\frac{a_f}{a_i} \equiv e^N \quad (\#\text{ e-folds})$$

$$\begin{aligned} N &\geq \log(\mathcal{H}_f/\mathcal{H}_o) = \log(E_f/E_o) \\ &\gtrsim 60 + \log(E_f[GeV]/10^{16}) \end{aligned}$$

Inflation: Definition + Implementation

INF

* Definition: $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

* Consequences: If N $\gtrsim$ 60 $\longrightarrow$ Horizon Problem Solved !

  $\searrow$ Bonus: Null Curvature

$$\boxed{\left(\left|\frac{K}{\mathcal{H}^2}\right| \sim |K/H_i^2|e^{-2N} = |K/H_i^2|e^{-120} \ll 1\ \right)}$$

Inflation: Definition + Implementation

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* **Implementation:** $$\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$$ (ϕ Inflaton)

IF $\boxed{V(\phi) \gg \frac{1}{2}\dot{\phi}^2, \frac{1}{2}(\nabla\phi)^2}$

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\cancel{\frac{1}{2}\dot{\phi}^2} - \cancel{\frac{1}{6a^2}(\nabla\phi)^2} - V(\phi)}{\cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} + V(\phi)} \simeq \frac{-V(\phi)}{V(\phi)} \simeq -1$$

Inflation: Definition + Implementation

* **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

Slow Roll (SR) Regime: $$\boxed{\epsilon = \frac{\dot{\phi}^2}{2m_p^2 H^2} \ll 1 \ ; \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1}$$

Inflation: Definition + Implementation

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$$\boxed{\begin{aligned} \epsilon_V &\equiv \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2 \\ \eta_V &\equiv m_p^2\left(\frac{V''}{V}\right) \end{aligned}}$$

$$(\epsilon \simeq \epsilon_V \, , \quad \eta \simeq \eta_V - \epsilon_V)$$

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$(\epsilon \simeq \epsilon_V \, , \quad \eta \simeq \eta_V - \epsilon_V)$

$\longrightarrow$

$$\boxed{\text{If } \epsilon_V , \eta_V \ll 1 \;\; \Rightarrow \;\; \epsilon, \eta \ll 1}$$

SR $\Longrightarrow$ $\underbrace{\textbf{quasi dS}}_{\equiv}$ **for** $\Delta N = 60$

$$\boxed{a(t) \simeq a_i e^{\int_t H(\phi)dt'}}$$

Inflation: Definition + Implementation

* **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ $\quad (\phi \text{ Inflaton})$

Case of Study: $\boxed{V(\phi) = \dfrac{1}{2} m_\phi^2 \phi^2}$

$$\epsilon_V(\phi) = \eta_V(\phi) = 2(m_p/\phi)^2$$

$$N(\phi) = (\phi/2m_p)^2 - 1/2$$

Inflation: Definition + Implementation

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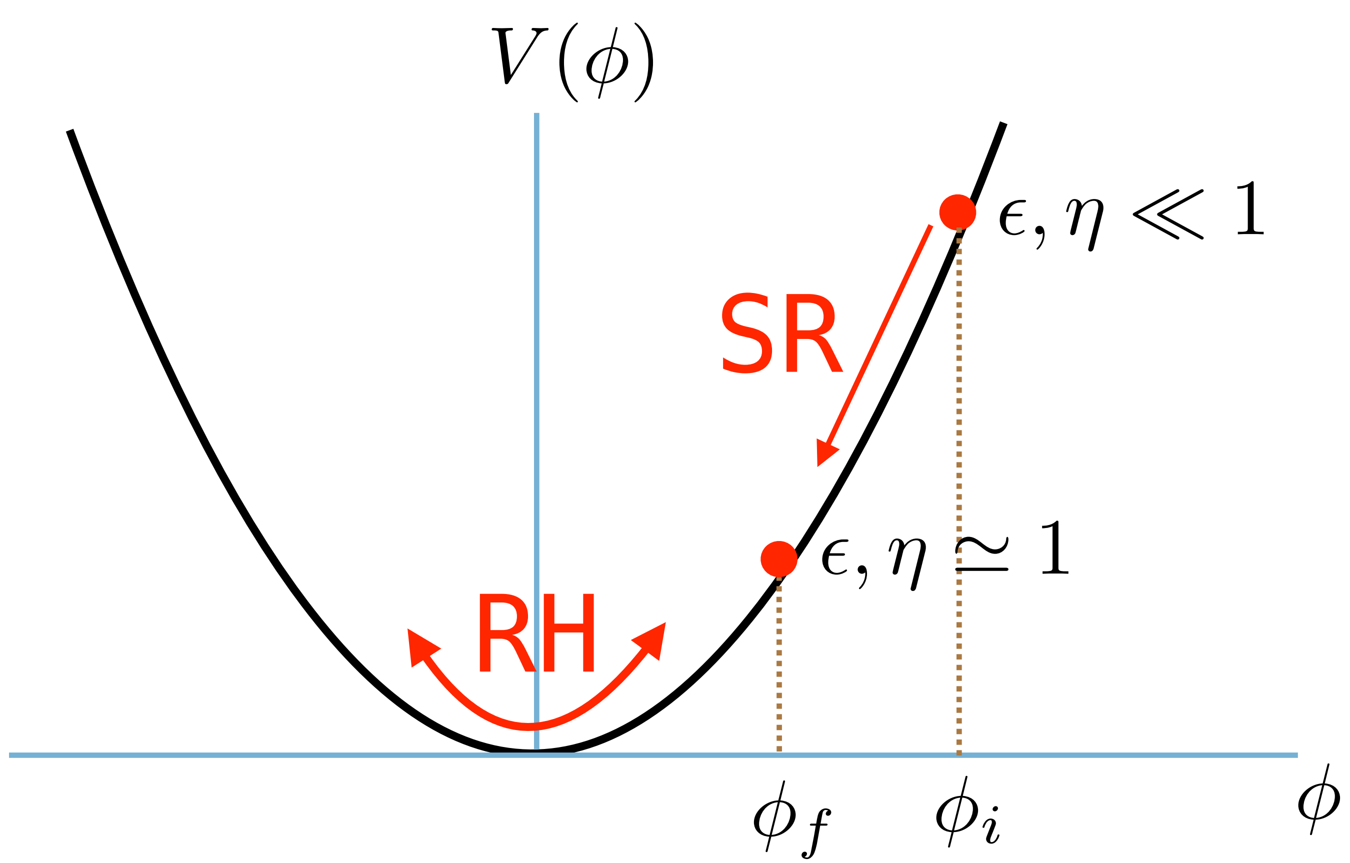

'Inflating' is easy with any potential of the type $V(\phi) \propto \phi^p$

Inflation & Primordial Perturbations

INF → **SR:** $\underset{\textbf{(Start)}}{\epsilon, \eta \ll 1} \rightarrow \underset{\textbf{(End)}}{\epsilon, \eta \simeq 1}$

→ $\boxed{a \sim e^{\int H dt'} \gtrsim e^{60}}$ (qdS)

No Hor. Problem !

Flat Universe !

Inflation & Primordial Perturbations

INF → **SR:** $\epsilon, \eta \ll 1 \rightarrow \epsilon, \eta \simeq 1$

(Start) **(End)**

$a \sim e^{\int H dt'} \gtrsim e^{60}$ (qdS)

No Hor. Problem !

Flat Universe !

*** Is that ALL?** **NO!**

$\phi(t)$

$g_{\mu\nu}(t)$

(Background)

Inflation & Primordial Perturbations

INF → **SR:** $\epsilon, \eta \ll 1 \rightarrow \epsilon, \eta \simeq 1$

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*** Is that ALL? NO!**

(Background)	(Fluctuations)
$\phi(t)$ →	$\phi(t) + \delta\phi(\vec{x}, t)$
$g_{\mu\nu}(t)$ →	$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$

INF

↓

Primordial fluctuations !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$\phi(t)$	$\longrightarrow$	$\phi(t) + \delta\phi(\vec{x}, t)$
$g_{\mu\nu}(t)$	$\longrightarrow$	$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$
(Background)		**(Fluctuations)**

but WHY fluctuations ?

Quantum Mechanics !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\begin{matrix}\phi(t) \\ \\ g_{\mu\nu}(t)\end{matrix}} \begin{matrix}\longrightarrow \\ \\ \longrightarrow\end{matrix} \boxed{\begin{matrix}\phi(t)+\delta\phi(\vec{x},t) \\ \\ g_{\mu\nu}(t)+\delta g_{\mu\nu}(\vec{x},t)\end{matrix}}$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

Quantum Mechanics !

VeV

Vacuum Quam. Fluct.

QM: $\left\{ \hat{\phi}(\vec{x},t) \;\rightarrow\; \langle\hat{\phi}(\vec{x},t)\rangle = \phi(t) \quad \Rightarrow \boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t)} \right.$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\begin{matrix} \phi(t) \\ \\ g_{\mu\nu}(t) \end{matrix}} \Longrightarrow \boxed{\begin{matrix} \phi(t) + \delta\phi(\vec{x},t) \\ \\ g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t) \end{matrix}}$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

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Vacuum Quam. Fluct.

$$\text{QM:}\begin{cases} \hat{\phi}(\vec{x},t) \;\rightarrow\; \langle \hat{\phi}(\vec{x},t) \rangle = \phi(t) \quad \Rightarrow \boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t)} \\ \\ \langle \hat{\delta\phi}(\vec{x},t) \rangle = 0 \quad \text{but...} \quad \Big\langle\, [\hat{\delta\phi}(\vec{x},t)]^2 \,\Big\rangle \neq 0 \end{cases}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$\phi(t)$	$\rightarrow$	$\phi(t) + \delta\phi(\vec{x}, t)$
$g_{\mu\nu}(t)$	$\rightarrow$	$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$
(Background)		**(Fluctuations)**

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QM:

$$\hat{\phi}(\vec{x}, t) \;\rightarrow\; \langle \hat{\phi}(\vec{x}, t) \rangle = \phi(t) \quad \Rightarrow \boxed{\hat{\phi}(\vec{x}, t) = \phi(t) + \hat{\delta\phi}(\vec{x}, t)}$$

$$\langle \hat{\delta\phi}(\vec{x}, t) \rangle = 0$$ **but...** $$\left\langle\, [\hat{\delta\phi}(\vec{x}, t)]^2 \,\right\rangle \neq 0$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t) \quad \rightarrow \quad \langle \hat{\delta\phi}^2(\vec{x},t)\rangle \neq 0}$$

but ... ~~Minkowski~~ → Curved Space: (quasi)dS

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

$$\phi(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\phi(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$\begin{aligned} ds^2 &= g^{\rm tot}_{\mu\nu}dx^\mu dx^\nu = (g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t))dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j \end{aligned}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

Inflation & Primordial Perturbations

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$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$B_i = \partial_i B - S_i$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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Expanding U. $\longrightarrow$ Vector Perturbations $S_i, F_i \propto \frac{1}{a}$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\partial_i h_{ij} = h_{ii} = 0$$

(tensors = GWs)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\phi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$

$$\text{dof} \xrightarrow{\text{scalars}} \delta\phi + \{\Phi, B, \Psi, E\}$$

$$\text{dof} \xrightarrow{\text{tensors}} h_{ij}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\} \xrightarrow{\text{Diff.}} \delta\phi' + \{\Phi', B', \Psi', E'\}$ ~~Gauge Inv.~~

tensors: $h_{ij} \xrightarrow{\text{Diff.}} h_{ij}$ Gauge Inv. !

Diff.: $\boxed{x^\mu \;\to\; x^\mu + \xi^\mu}$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

dof

scalars

$\delta\phi + \{\Phi, B, \Psi, E\}$

tensors

h_{ij}

$$\zeta \equiv -[\Psi + (H/\dot\rho)\delta\rho_\phi] \xrightarrow{\text{Diff.}} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot\phi)\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot\phi/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

All Gauge Inv. !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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dof

scalars $\delta\phi + \{\Phi, B, \Psi, E\}$

tensors h_{ij}

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$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

All Gauge Inv. !

Curvature Pert.

Tensor Pert. (GW)

Fixing Gauge: e.g. $E, \delta\phi = 0 \Rightarrow g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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Inflation & Primordial Perturbations

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Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\boxed{g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]} \quad S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\} \Rightarrow$$

$$\Rightarrow \boxed{S = S_{(0)} + S^{(s)}_{(2)} + S^{(t)}_{(2)}}$$

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\ a^3\frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]}$$

$$\boxed{S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dtdx^3a^3\left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]}$$

Background
Inflationary dynamics

(UV limit: deep inside Hubble radius)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S_{(2)}^{(s)} = \frac{1}{2}\int d^4x \; a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$d\tau \equiv dt/a(t)$ (Conformal time)

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \tfrac{1}{2}\int d^4x \; a^3 \tfrac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]}$$

$$=$$

$$\boxed{\frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]}$$

$\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot\phi}{H}\right]$ (Mukhanov variable)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$S_{(2)}^{(s)} = \frac{1}{2}\int d^4x \; a^3 \frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right] = \frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right] \Rightarrow$$

(F.T.: $v(\mathbf{x},t) = \int d\mathbf{k}\, e^{-i\mathbf{k}\mathbf{x}}\, v_{\mathbf{k}}(t)$ **)**

$$\Rightarrow \quad v_{\vec{k}}'' + (k^2 - z''/z)v_{\vec{k}} = 0$$

with $$\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right), \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z) v_{\vec{k}} = 0}$$

with

$$\boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right) \; , \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Quantization: $\boxed{v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}} \;\; , \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')}$ $\Rightarrow$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S_{(2)}^{(s)} = \tfrac{1}{2}\int d^4x \; a^3 \frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]} \Rightarrow$$

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➡ 2 linearly independent solutions (Hankel functions)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\; a^3 \frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]} \Rightarrow$$

$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0} \quad \text{with} \quad \boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right) \;, \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

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$$\Rightarrow \boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H^{(1)}_\nu(-k\tau)}$$

(we keep only one, $\hat{H}v_k = +kv_k$, $\langle v_k, v_k\rangle > 0$)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\; a^3\frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3\left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]} \Rightarrow$$

$$\Rightarrow \boxed{v''_{\vec k} + (k^2 - z''/z)v_{\vec k} = 0} \quad \text{with} \quad \boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\,, \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Quantization: $\boxed{v_{\vec k}(t) \to v_k(t)\hat a_{\vec k} + v^*_k(t)\hat a^\dagger_{-\vec k}\;, \quad [a_{\vec k}, a^\dagger_{\vec k'}] = (2\pi)^3\delta(\vec k - \vec k')}$ $\rightarrow$

$$\rightarrow \boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2}e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2}H^{(1)}_\nu(-k\tau)} \xrightarrow[\text{(sub-Hubble)}]{-k\tau \gg 1} \frac{1}{\sqrt{2k}}e^{-ik\tau}$$

(we keep only one, $\hat H v_k = +kv_k$, $\langle v_k, v_k\rangle > 0$)

Positive define freq

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\ a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]} \Rightarrow$$

$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0}$$ **with** $$\boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right), \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Quantization: $$\boxed{v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}}\ , \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')} \rightarrow$$

$$\rightarrow \boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H^{(1)}_\nu(-k\tau)}$$

(Bunch-Davies) Vacuum Fluct.

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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(Bunch-Davies) Vacuum Fluct.

$$\boxed{\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]}$$

$$\Rightarrow \quad \left\langle \hat{\mathcal{R}}_{\vec{k}} \hat{\mathcal{R}}_{\vec{k}'} \right\rangle \equiv \frac{1}{z^2} \left\langle \hat{v}_{\vec{k}} \hat{v}_{\vec{k}'} \right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2} \left| v_k(\eta) \right|^2} \delta(\vec{k} + \vec{k}')$$

Inflation & Primordial Perturbations

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$$\boxed{\equiv P_{\mathcal{R}}(k,\eta)}$$

Scalar Power Spectrum

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluct:

$$\boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H_\nu^{(1)}(-k\tau)}$$

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$$\boxed{\equiv P_{\mathcal{R}}(k,\tau)}$$

Scalar Power Spectrum

$$\boxed{\Delta^2_{\mathcal{R}}(k,\tau) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k,\tau)}$$

Dimensionless Scalar PS

$$\xrightarrow{(\, k << aH \,)}$$

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2} \left(\frac{k}{aH}\right)^{2\eta - 4\epsilon}}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$d\tau \equiv dt/a(t)$$ (Conformal time)

$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right] = \sum_s \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3\mathbf{k}\left[({v^s_{\mathbf{k}}}')^2 - \left(k^2 - \frac{a''}{a}\right)(v^s_{\mathbf{k}})^2\right]$$

$$h_{ij}(\vec{k},\tau) = \epsilon^{(s)}_{ij} h^{(s)}_{\vec{k}} \longrightarrow v^{(s)} \equiv \frac{a}{2} m_p h^{(s)}_{\vec{k}}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right] = \sum_s \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3\mathbf{k}\left[(v_{\mathbf{k}}^{s\prime})^2 - \left(k^2 - \frac{a''}{a}\right)(v_{\mathbf{k}}^s)^2\right] \Rightarrow$$

$\Rightarrow$ [Same Procedure as with Scalar Pert.

Quantize → Bunch-Davies → Power Spectrum]

Quantization of Gravity dof !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$\Rightarrow$ [Same Procedure as with Scalar Pert.

Quantize → Bunch-Davies → Power Spectrum]

Quantization of Gravity dof !

$$\Delta_h^2(k,\tau) \equiv \frac{k^3}{2\pi^2}P_h(k,\tau) \xrightarrow{(\,k \ll aH\,)} \boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}}$$

Inflation & Primordial Perturbations

H & I

INFLATION

$$\boxed{g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$$

Inflation & Primordial Perturbations

H & I

INFLATION

$$\boxed{g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$$

$$\langle \mathcal{R}\mathcal{R}^* \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta_{\mathcal{R}}^2(k)$$

$$\langle h_{ij} h_{ij}^* \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta_h^2(k)$$

Quantum fluctuations !

Inflation & Primordial Perturbations

H & I

INFLATION

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}}$$

$$\boxed{n_s - 1 \equiv 2(\eta - 2\epsilon)}$$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

Inflation & Primordial Perturbations

INFLATION →

H & I

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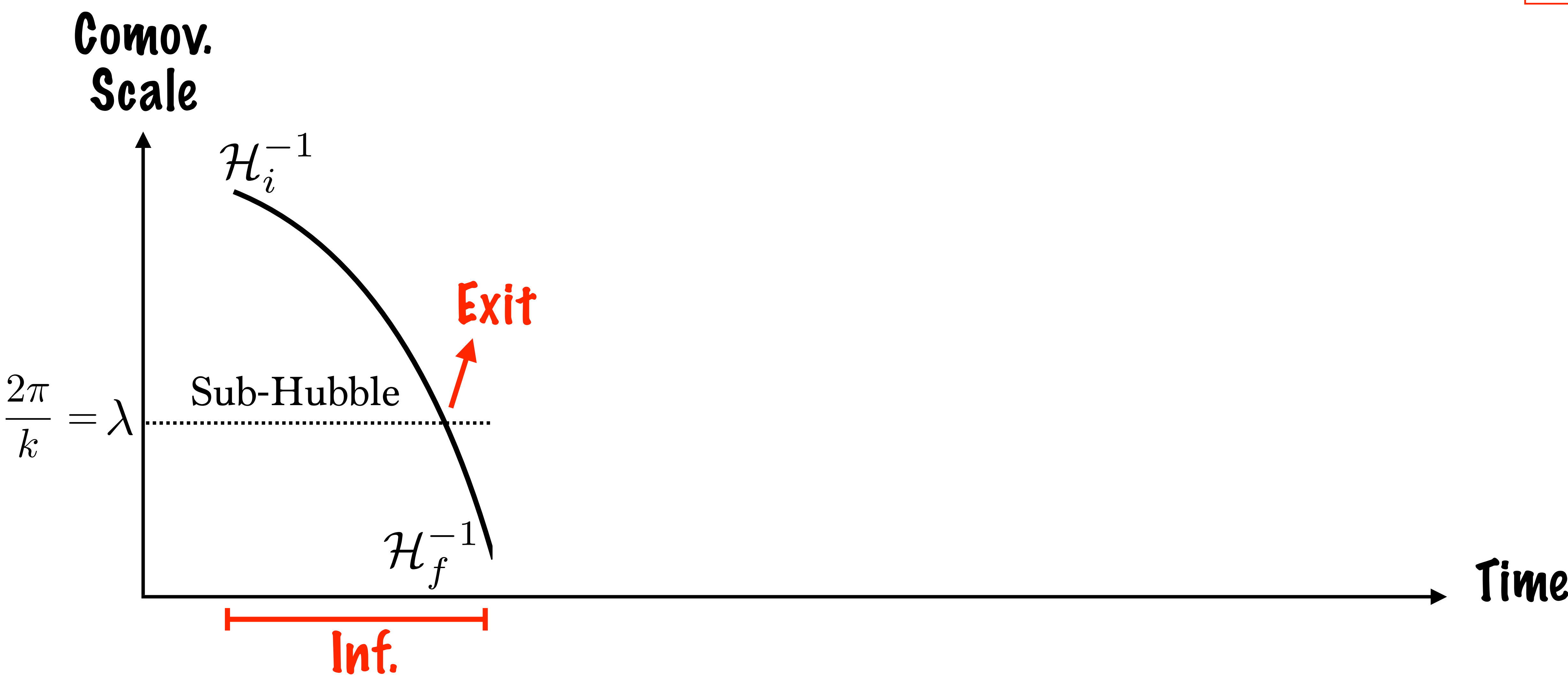

Inflation & Primordial Perturbations

H & I

INFLATION $\rightarrow$ $g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}}$$

$\boxed{n_s - 1 \equiv 2(\eta - 2\epsilon)}$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

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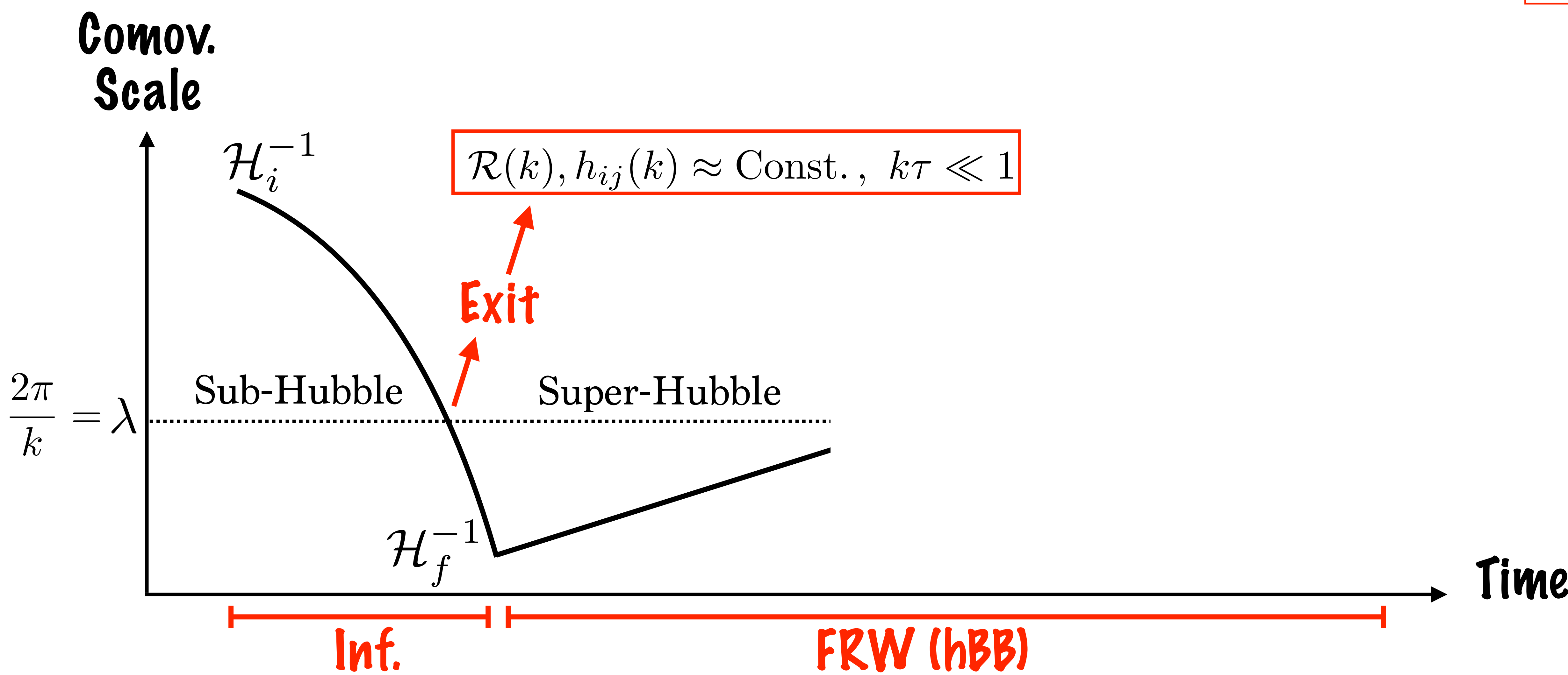


Inflation & Primordial Perturbations

H & I

INFLATION

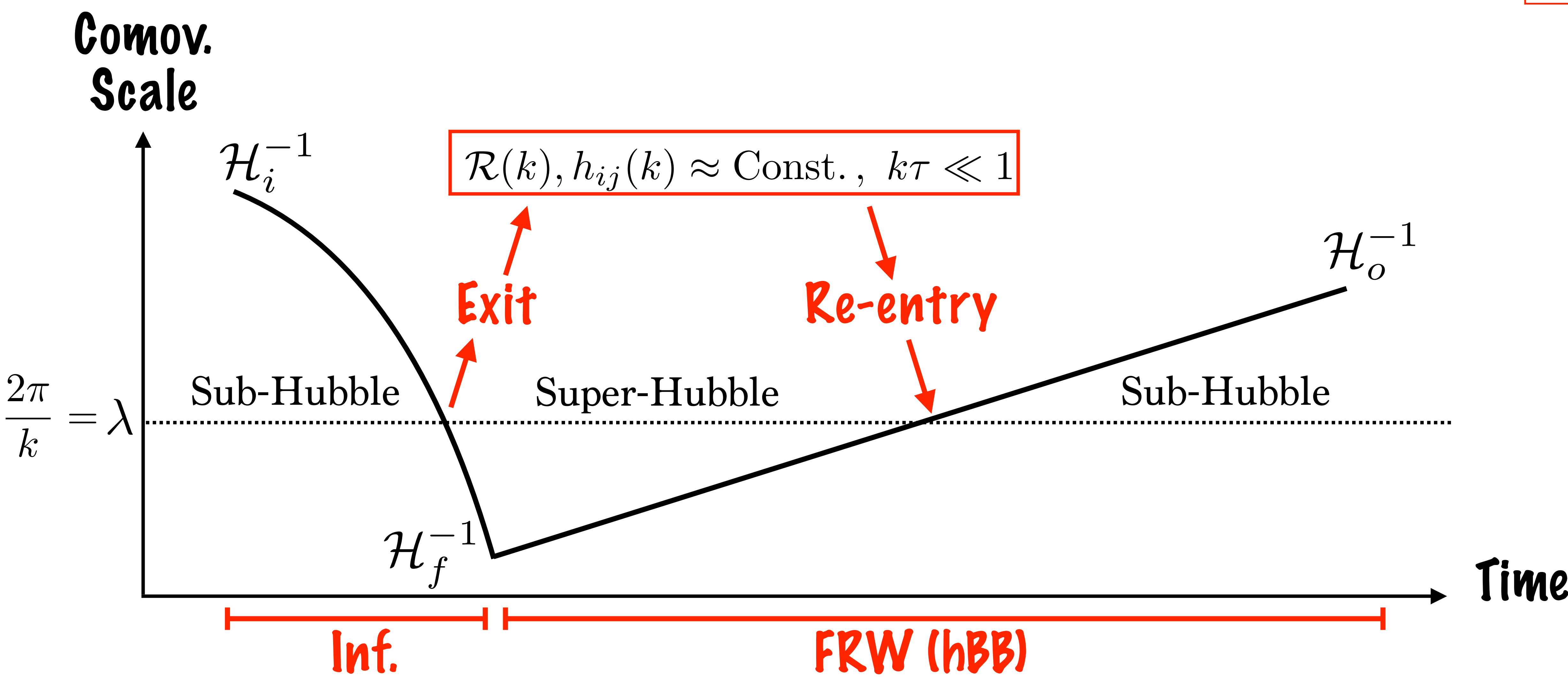
$$g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$n_s - 1 \equiv 2(\eta - 2\epsilon)$

$$\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}$$

$n_t \equiv -2\epsilon$



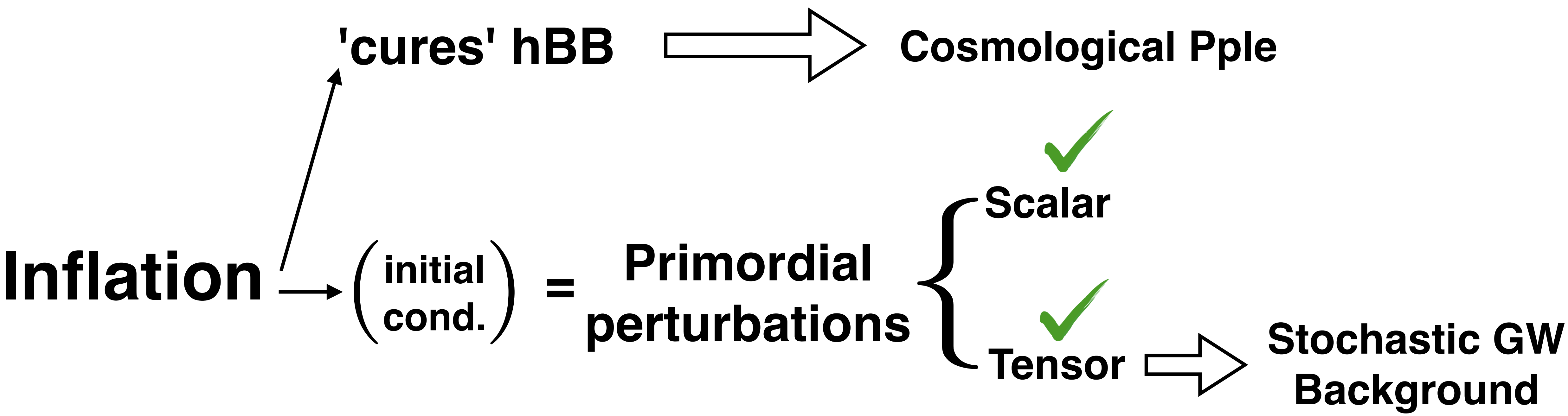
End primer on Inflation

INFLATIONARY COSMOLOGY

'cures' hBB $\Longrightarrow$ Cosmological Pple

Inflation $\rightarrow$ $\left(\begin{array}{c}\text{initial}\\ \text{cond.}\end{array}\right)$ = Primordial perturbations $\begin{cases}\text{Scalar}\\ \text{Tensor}\end{cases}$

INFLATIONARY COSMOLOGY



Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x}, t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r}^{+}\right)\, e_{ij}^r(\hat{\mathbf{k}})}$$

conformal time

quantum fields

Polarizations: +, x

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^{+}_{\mathbf{k}r}\right)\, e^{r}_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V$$

$$\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned}
\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \\
&\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \\
&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k}',t) \\
&\qquad\qquad \times \frac{1}{V}\int_V d\mathbf{x}\; e^{-i\mathbf{x}(\mathbf{k}-\mathbf{k}')}\,,
\end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned}
\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V \\
&\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \\
&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \cancel{\frac{d\mathbf{k}'}{(2\pi)^3}}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k}',t) \\
&\qquad \times \frac{1}{V} \boxed{(2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{k}')}
\end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times}\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\left(h_k(t)\,e^{i\mathbf{k}\mathbf{x}}\,\hat{a}_{\mathbf{k}r} + h_k^*(t)e^{-i\mathbf{k}\mathbf{x}}\,\hat{a}^+_{\mathbf{k}r}\right)\,e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\boxed{\rho_{\rm GW}(t)} = \frac{1}{32\pi G a^2(t)}\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V$$

$$\equiv \frac{1}{32\pi G a^2(t)}\frac{1}{V}\int_V d\mathbf{x}\,\dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)$$

$$= \boxed{\frac{1}{32\pi G a^2(t) V}\int \frac{d\mathbf{k}}{(2\pi)^3}\;\dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k},t)}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r}^{+}\right)\, e_{ij}^r(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V$$

$\longrightarrow$ Volume/Time Average

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r}^{+} \right)\, e_{ij}^{r}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\, e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}_{ij}^*\,(\mathbf{k}',t) \right\rangle$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t) \dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\, e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}(\mathbf{k},t)\, \dot{h}^*_{ij}(\mathbf{k}',t) \right\rangle$$

$$\boxed{\left\langle \dot{h}_{ij}(\mathbf{k},t)\, \dot{h}^*_{ij}(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t) \delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times}\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\left(h_k(t)\, e^{i\mathbf{kx}}\,\hat{a}_{\mathbf{k}r} + h_k^*(t)e^{-i\mathbf{kx}}\,\hat{a}^+_{\mathbf{k}r}\right)\; e^{r}_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)}\int\frac{dk}{k}\;k^3\,\mathcal{P}_{\dot{h}}(k,t) \;\; = \int\frac{d\rho_{\rm GW}}{d\log k}\,d\log k$$

$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\,\dot{h}^*_{ij}\,(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\,\mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r}^{+}\right)\, e_{ij}^{r}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)} \int \frac{dk}{k}\, k^3\, \mathcal{P}_{\dot{h}}(k,t) \;= \int \frac{d\rho_{\rm GW}}{d\log k}\, d\log k$$

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot{h}}(k,t)}$$

$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}_{ij}^*\,(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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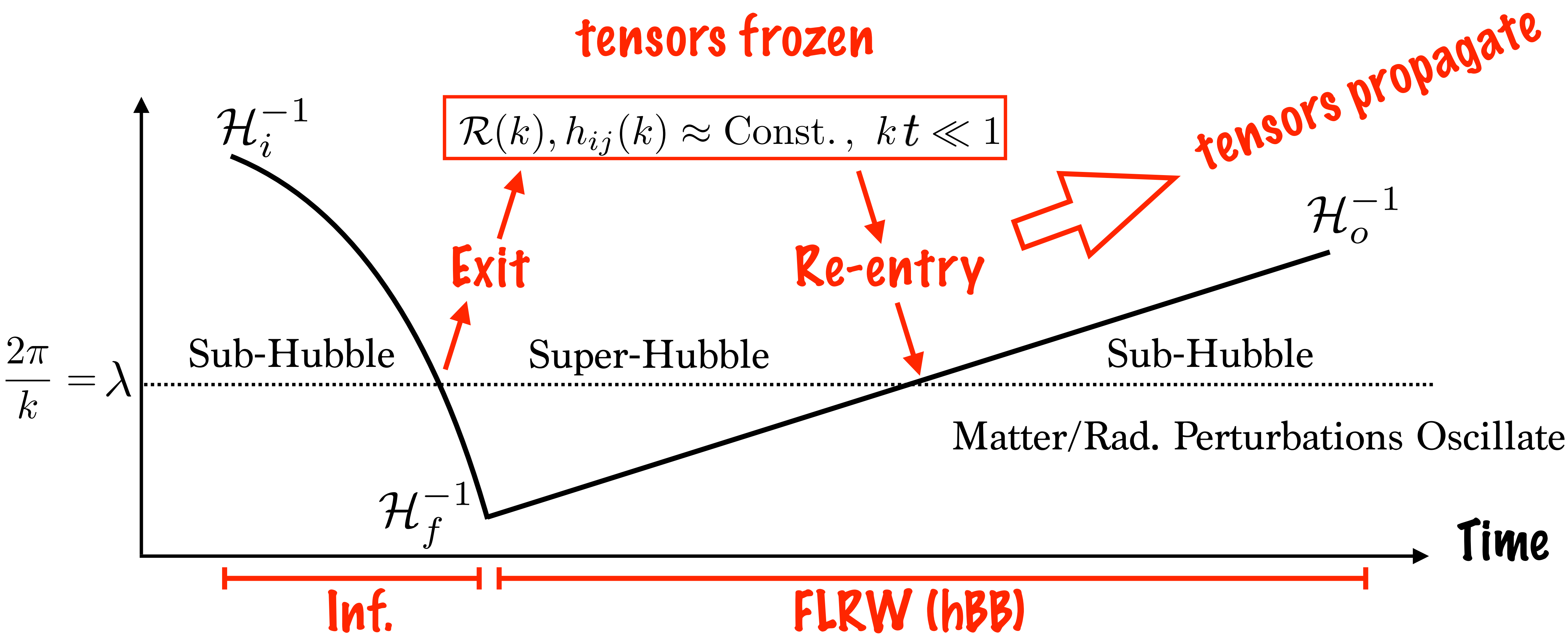

Irreducible GW background from Inflation

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$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\,\dot{h}^*_{ij}\,(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Horizon Re-entry $\Rightarrow$ tensors propagate

Rad Dom: $h_r(\mathbf{k},t) = \dfrac{A_r(\mathbf{k})}{a(\eta)}\, e^{ikt} + \dfrac{B_r(\mathbf{k})}{a(\eta)}\, e^{-ikt}$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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$$\left\langle \dot h \dot h\right\rangle = k^2\langle hh\rangle = \left(\frac{a_*}{a}\right)^2\frac{k^2}{2}\langle|h_*|^2\rangle = \left(\frac{a_o}{a}\right)^2\frac{k^2}{2(1+z_*)^2}\frac{2\pi^2}{k^3}\Delta^2_{h_*}$$

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Redshift

Inflationary Tensor Spectrum !

Irreducible GW background from Inflation

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GW normalized energy density spectrum (today)

Inflationary tensor spectrum

Irreducible GW background from Inflation

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GW normalized energy density spectrum (today)

Transfer Funct

$$T(k)\begin{cases}\propto k^0(\mathrm{RD})\\ \propto k^{-2}(\mathrm{MD})\end{cases}$$

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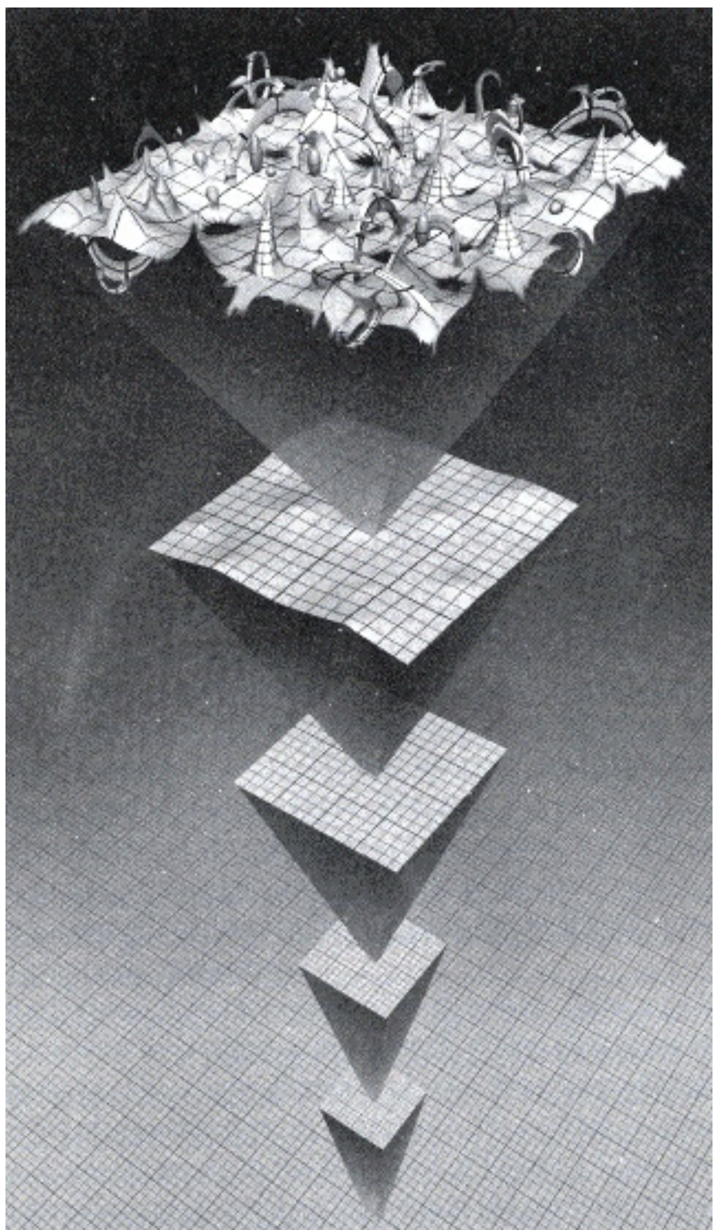
Inflationary Hubble Rate

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$\boxed{n_t \equiv -2\epsilon}$ Small red-tilt, i.e. (almost-) scale-invariant

Irreducible GW background from Inflation

$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} + \delta g_{\mu\nu} \quad ; \quad [\delta g_{\mu\nu}]^{\mathrm{TT}} = h_{ij} \; , \; \begin{cases} h_{ii} = 0 \\ \partial_i h_{ij} = 0 \end{cases}$$



Quantum Fluctuations

$$\left\langle h_{ij}(\vec{k},t) \right\rangle = 0$$

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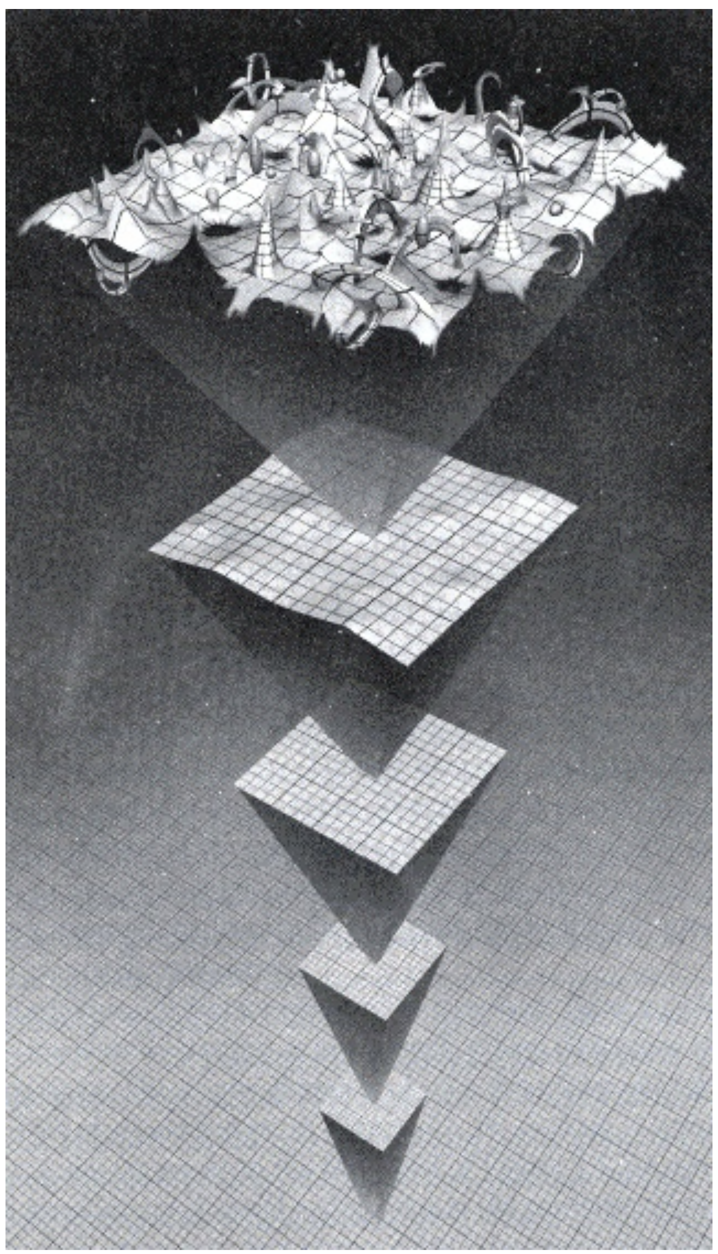

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$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{n_t}}$$

$n_t \equiv -2\epsilon$

energy scale

Irreducible GW background from Inflation

$$\boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{n_t}}$$

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$$\boxed{\Omega_{\rm GW}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \underbrace{\frac{\Omega_{\rm Rad}^{(o)}}{24}}\Delta_{h_*}^2(k)}$$

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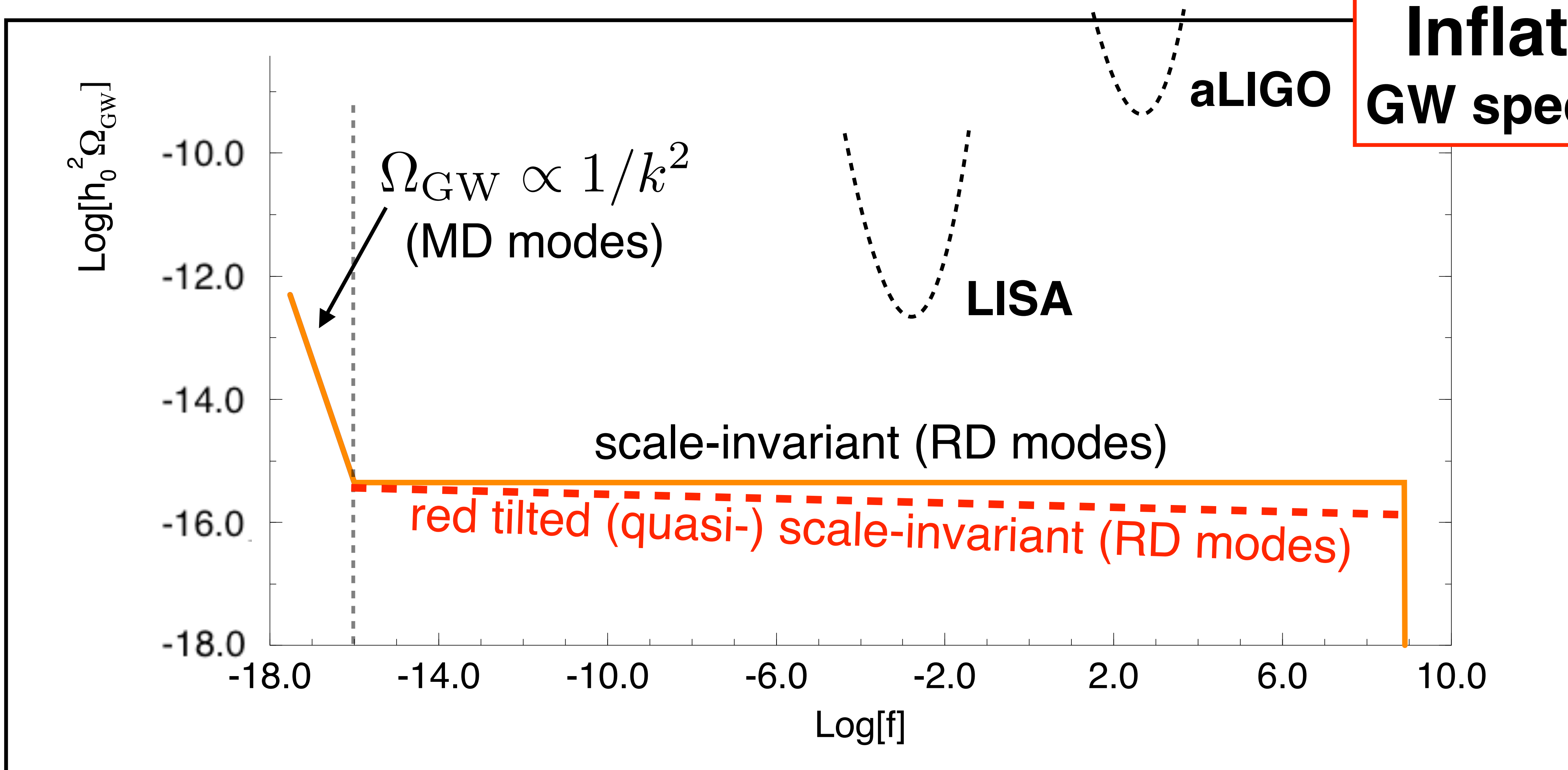
Inflation GW spectrum



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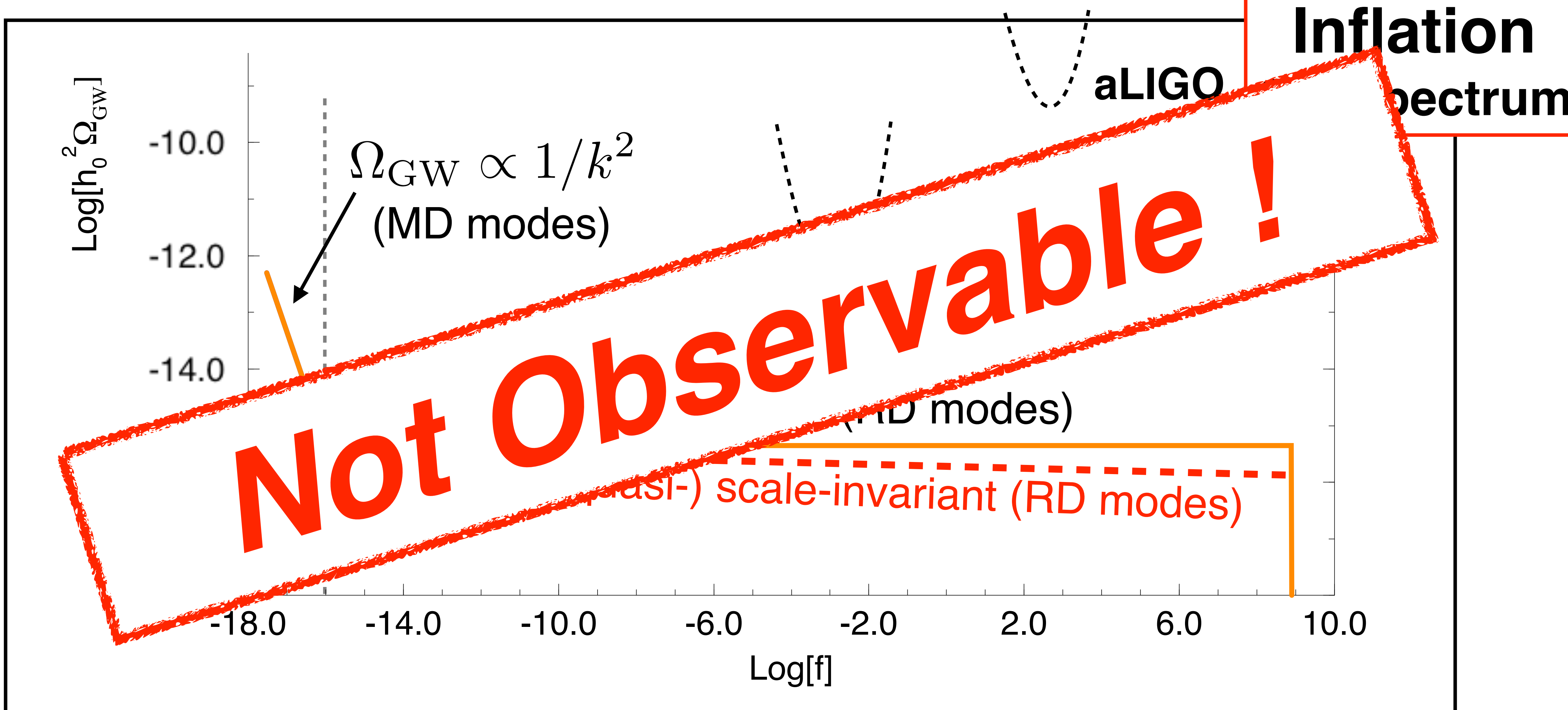

Not Observable !

Irreducible GW background from Inflation

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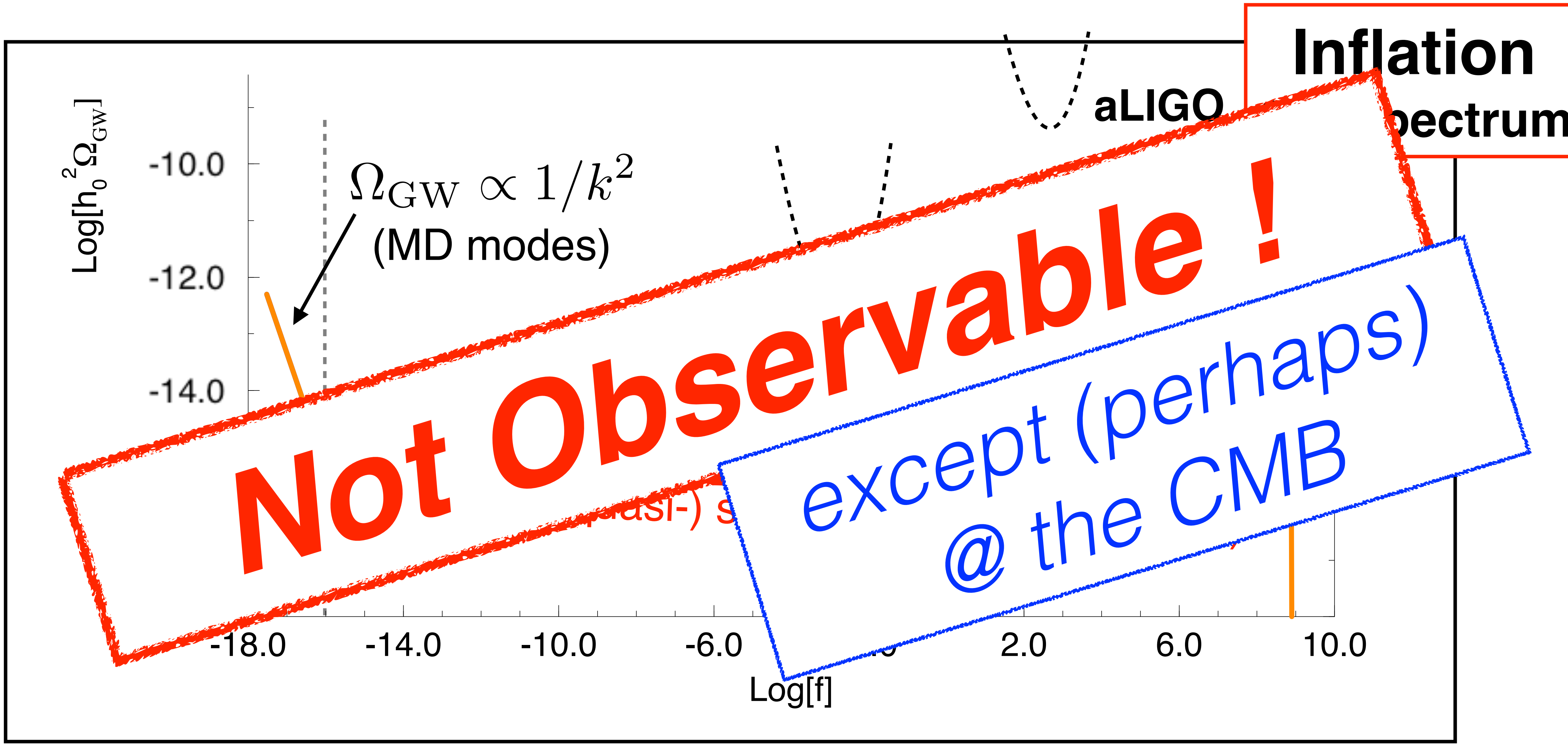

Not Observable !

except (perhaps) @ the CMB

Irreducible GW background from Inflation

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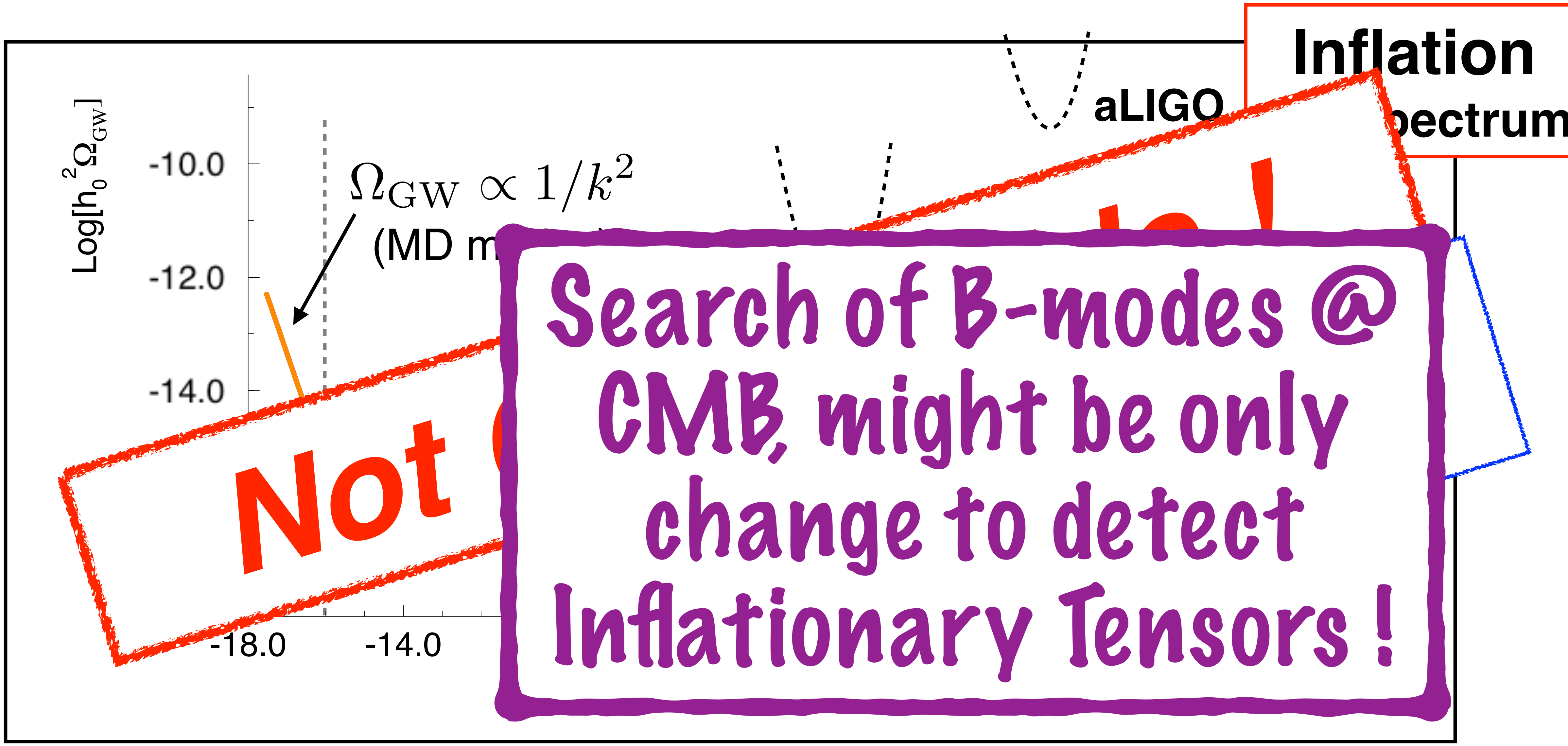

Search of B-modes @ CMB, might be only change to detect Inflationary Tensors !

Irreducible GW background from Inflation

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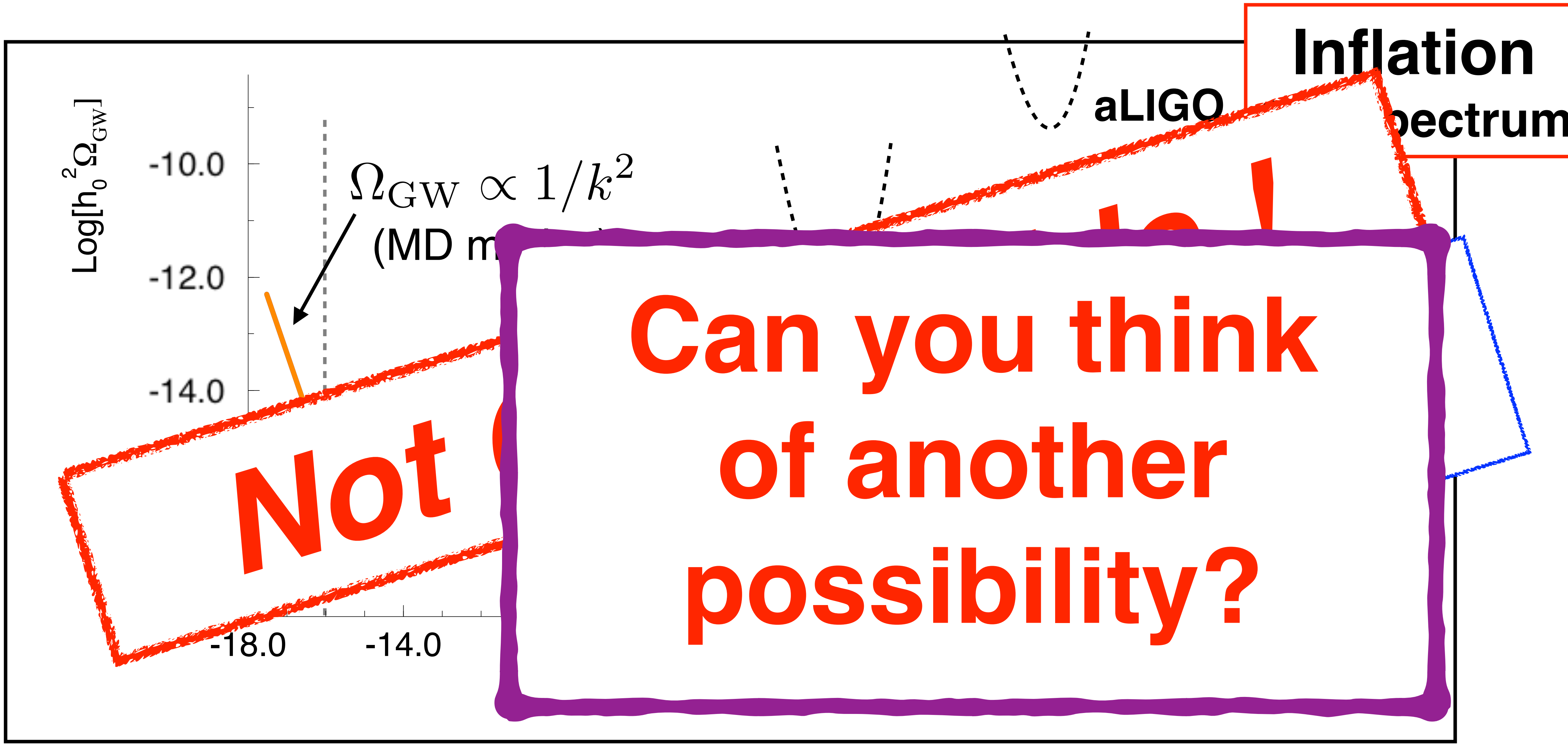

Can you think of another possibility?

Irreducible GW background from Inflation

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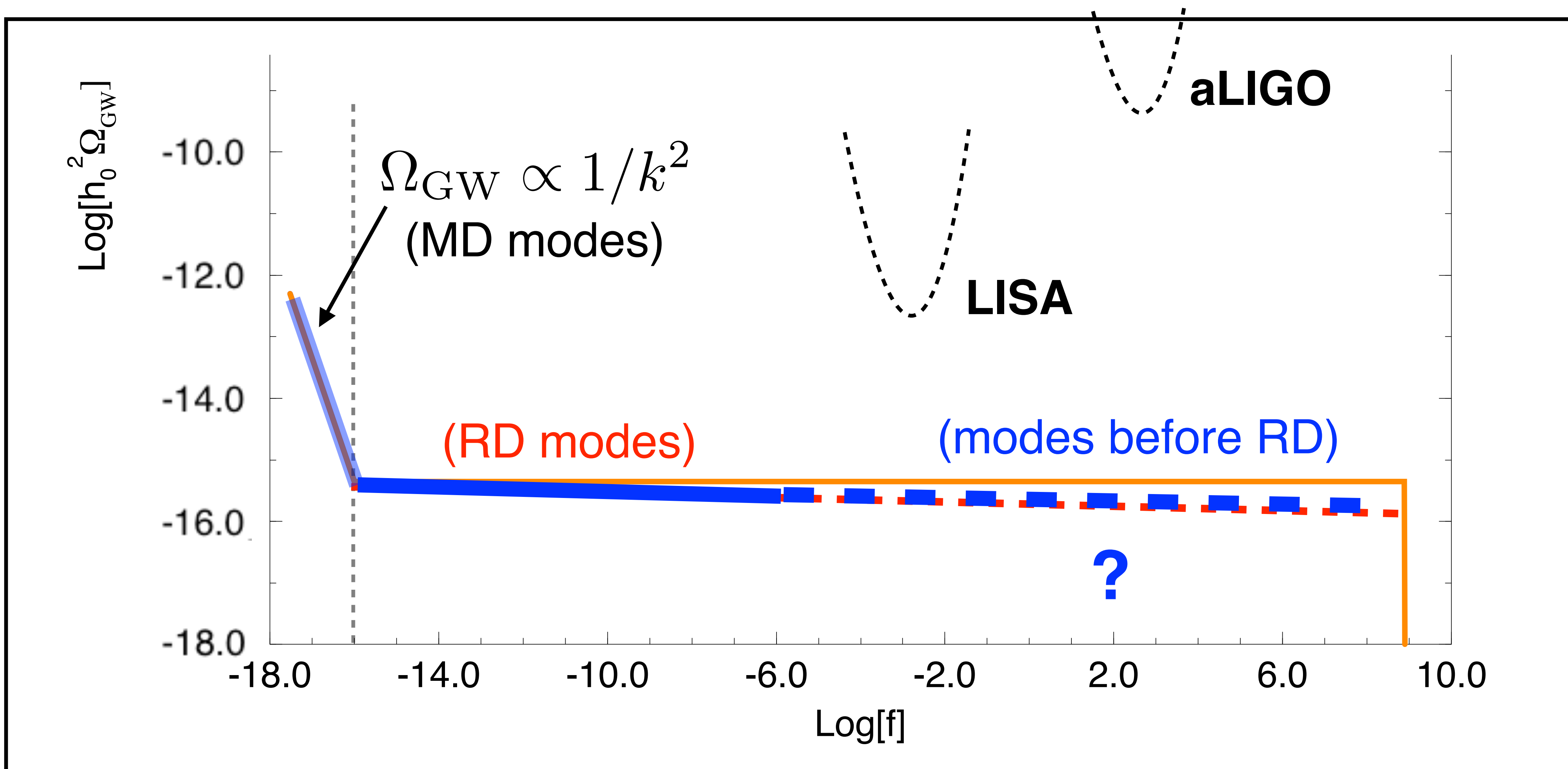
energy scale

Transfer Funct.: $T(k) \propto k^0(\mathrm{RD})$

Period before RD: $T(k) \propto k^{2\frac{(w_s-1/3)}{(w_s+1/3)}}$



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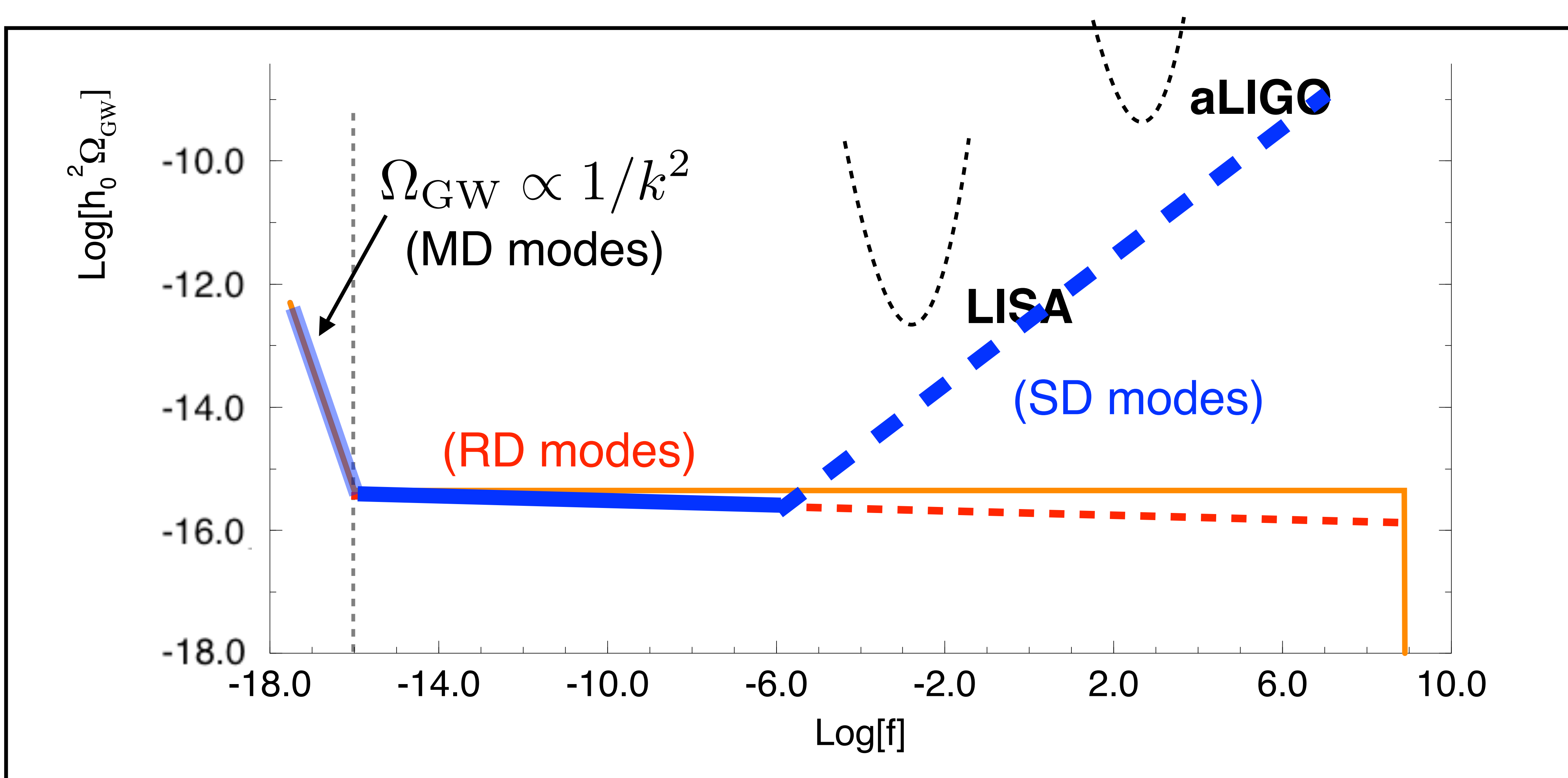
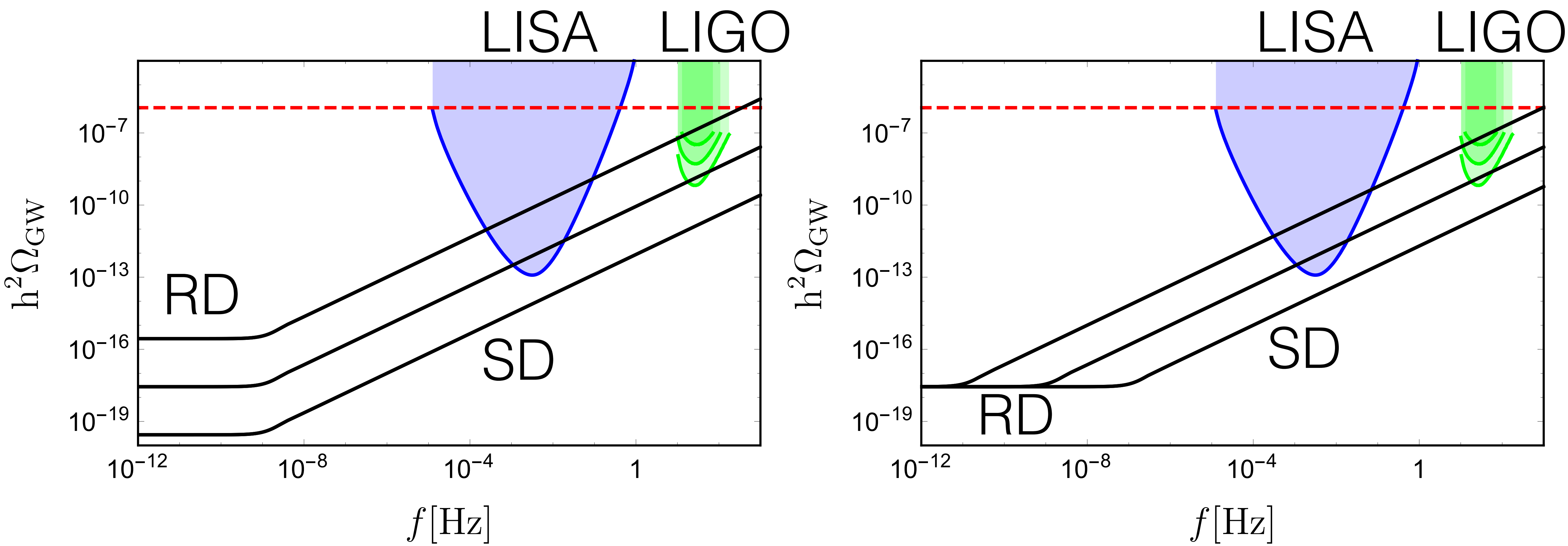
energy scale

Transfer Funct.: $T(k) \propto k^0(\mathrm{RD})$

Stiff Period: $T(k) \propto k^{2\frac{(w_s-1/3)}{(w_s+1/3)}} \quad (1/3 < \omega_s < 1)$



STIFF EQ of STATE $(1/3 < \omega_s < 1)$



$$\Omega_{\rm GW}(f) \propto H_{\rm inf}^2 \left(\frac{f}{f_{\rm RD}}\right)^{\frac{2(w-1/3)}{(w+1/3)}}$$

Not Scale Invariant !

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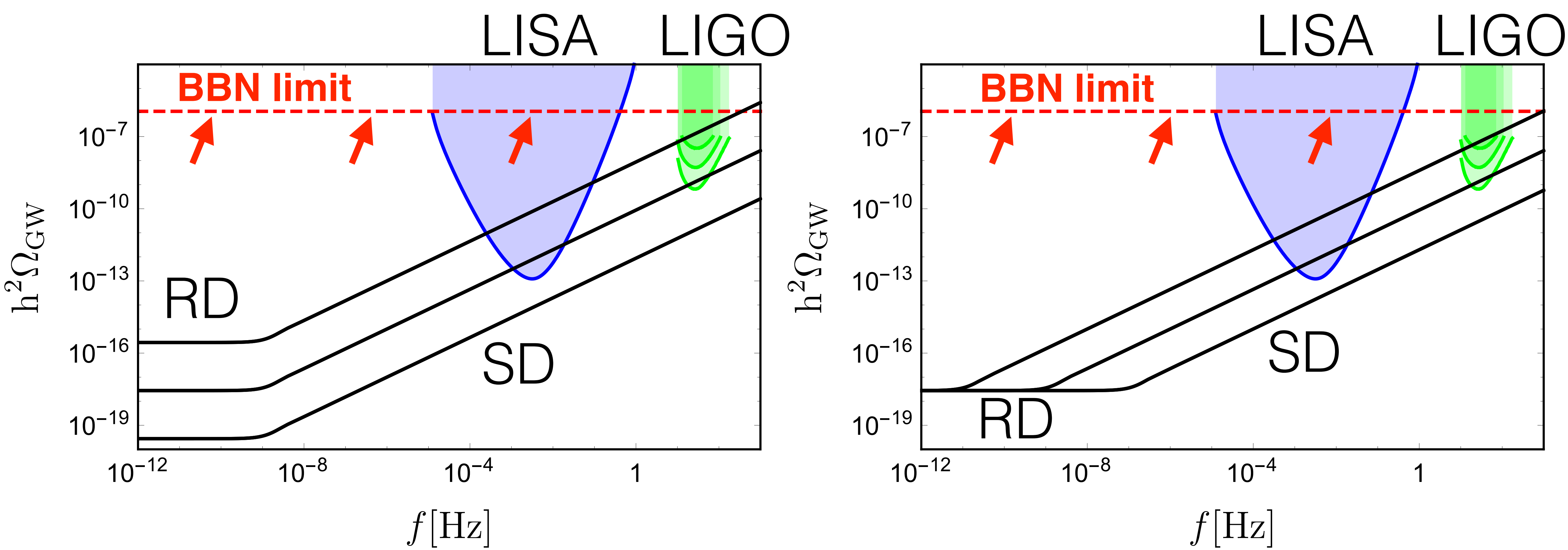

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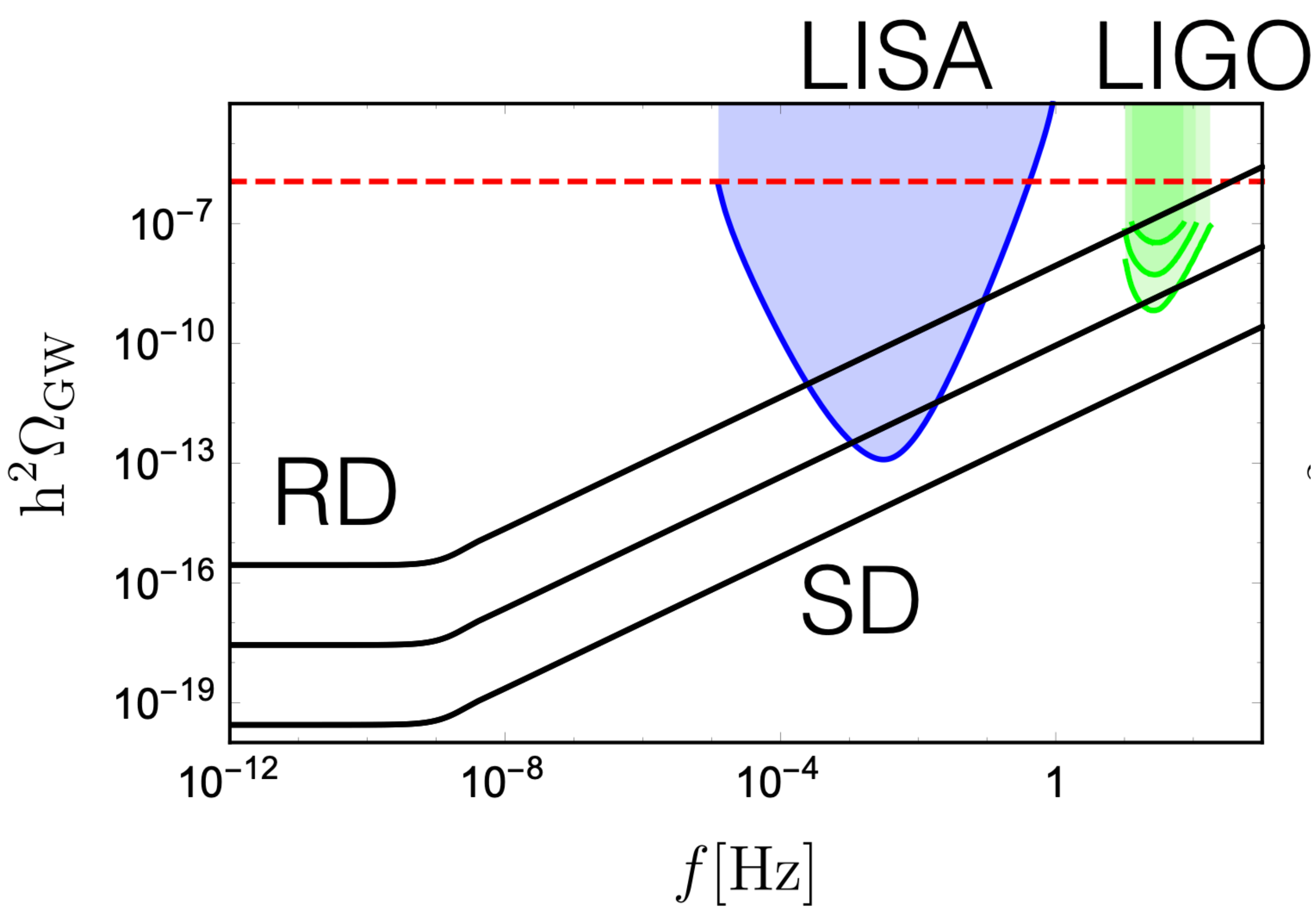

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DGF, Tanin '19, 1905.11960

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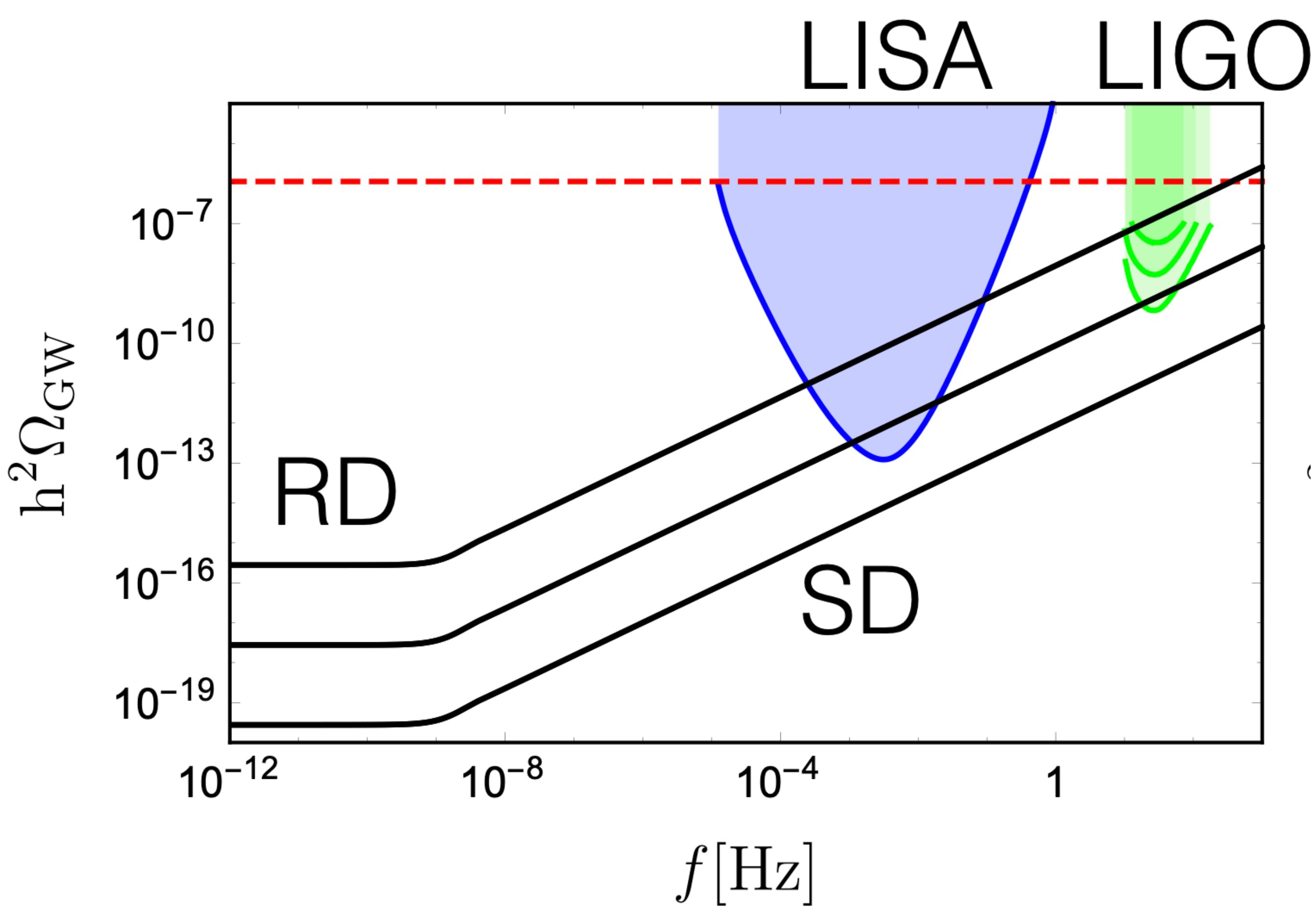

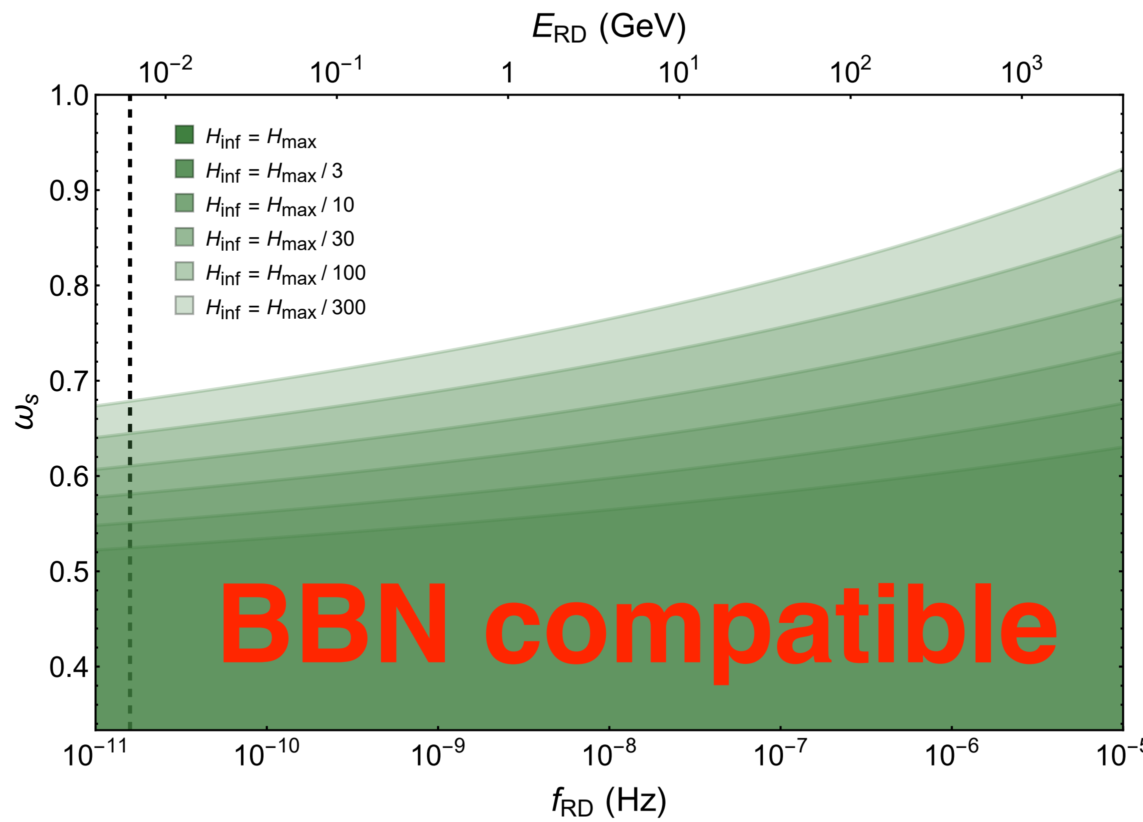

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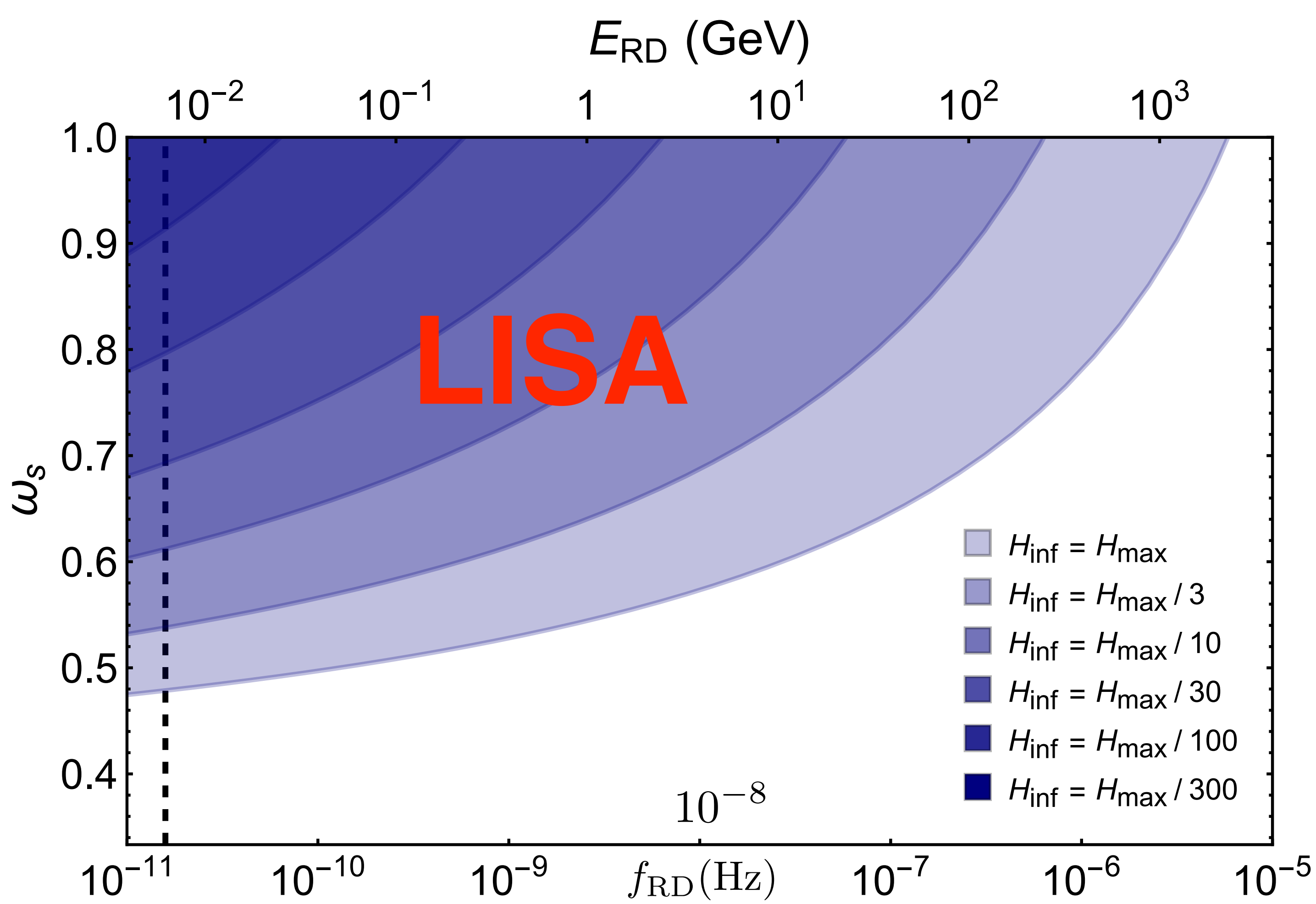

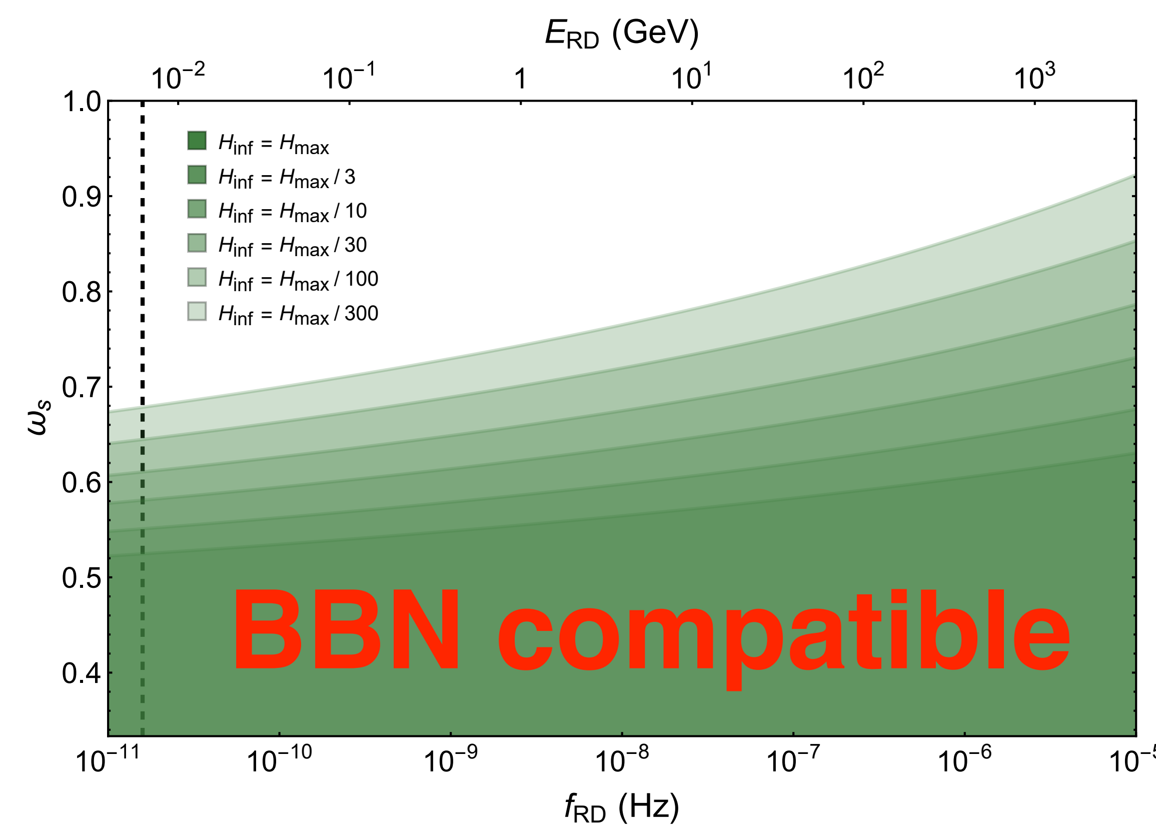

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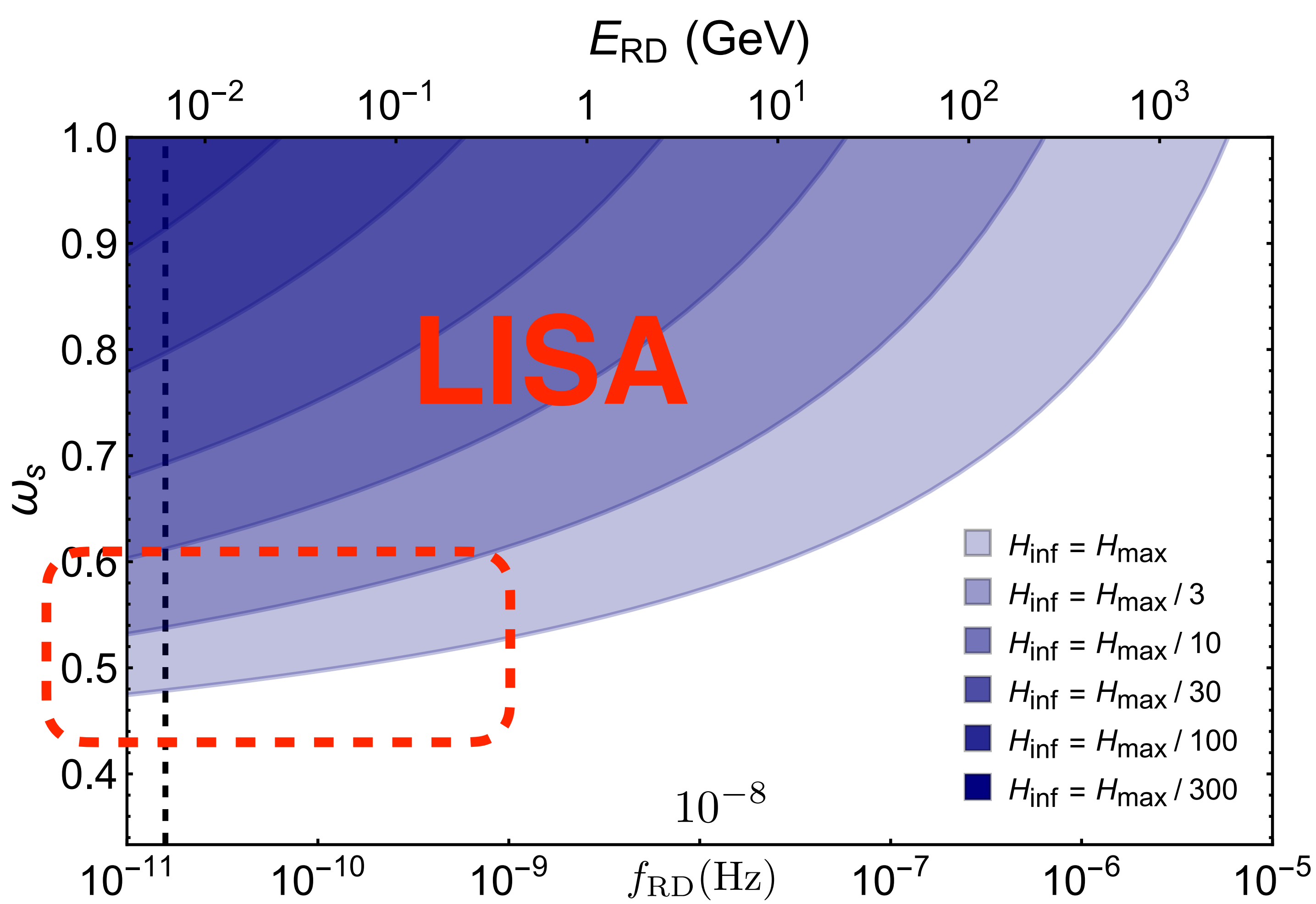

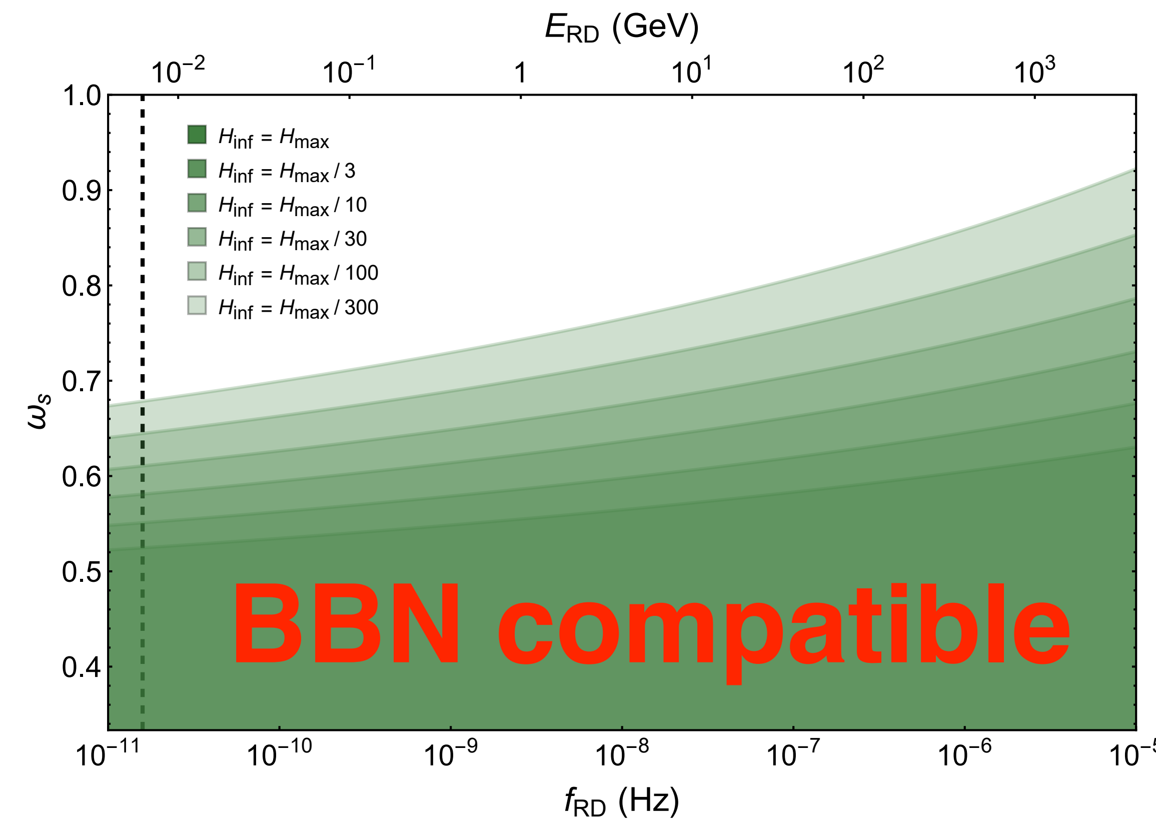

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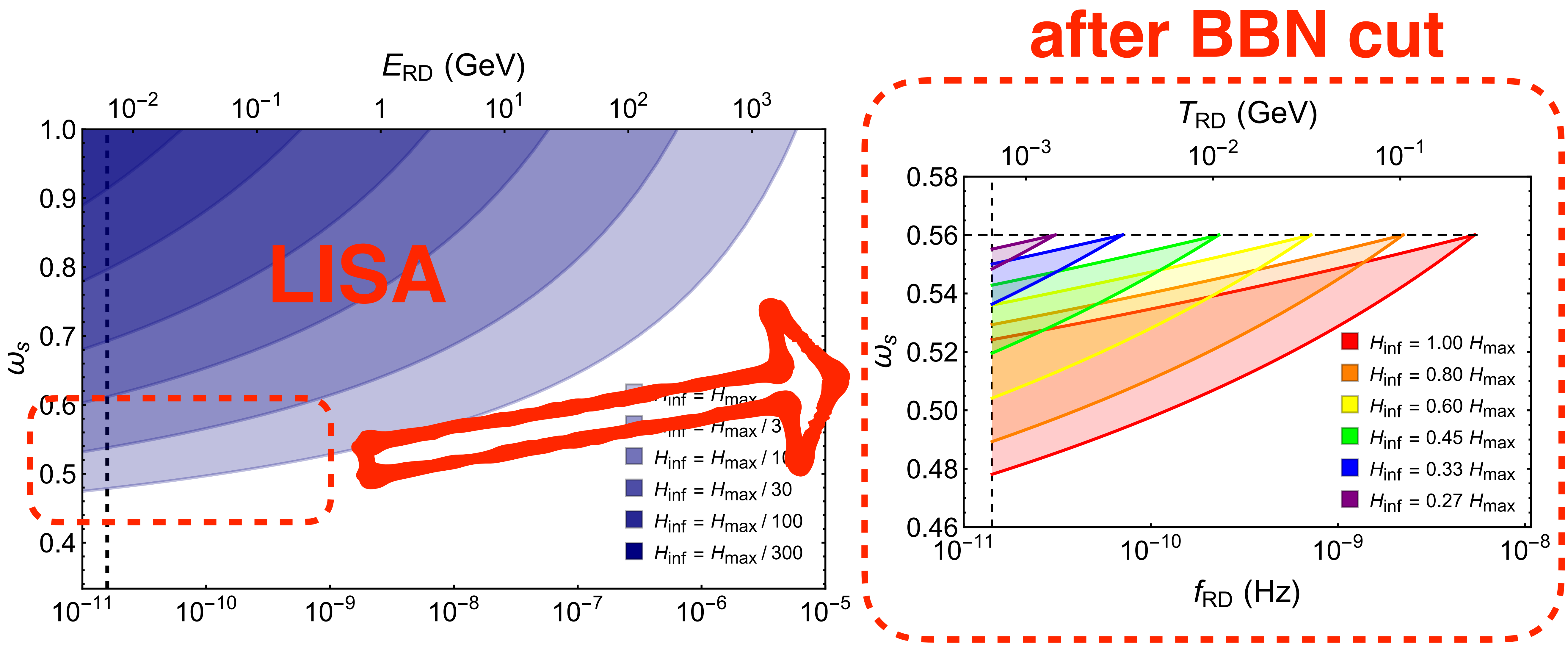

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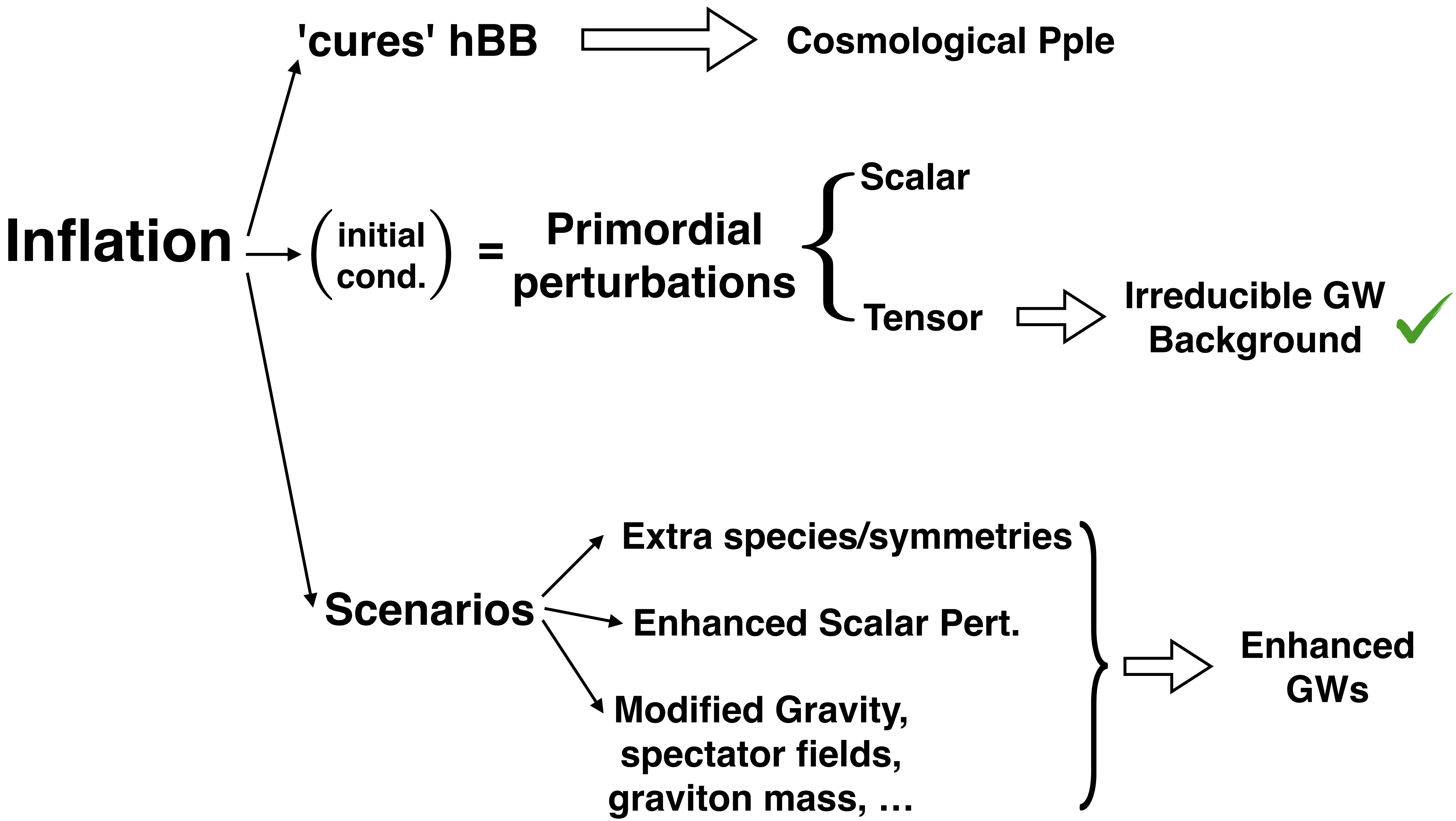
INFLATIONARY COSMOLOGY



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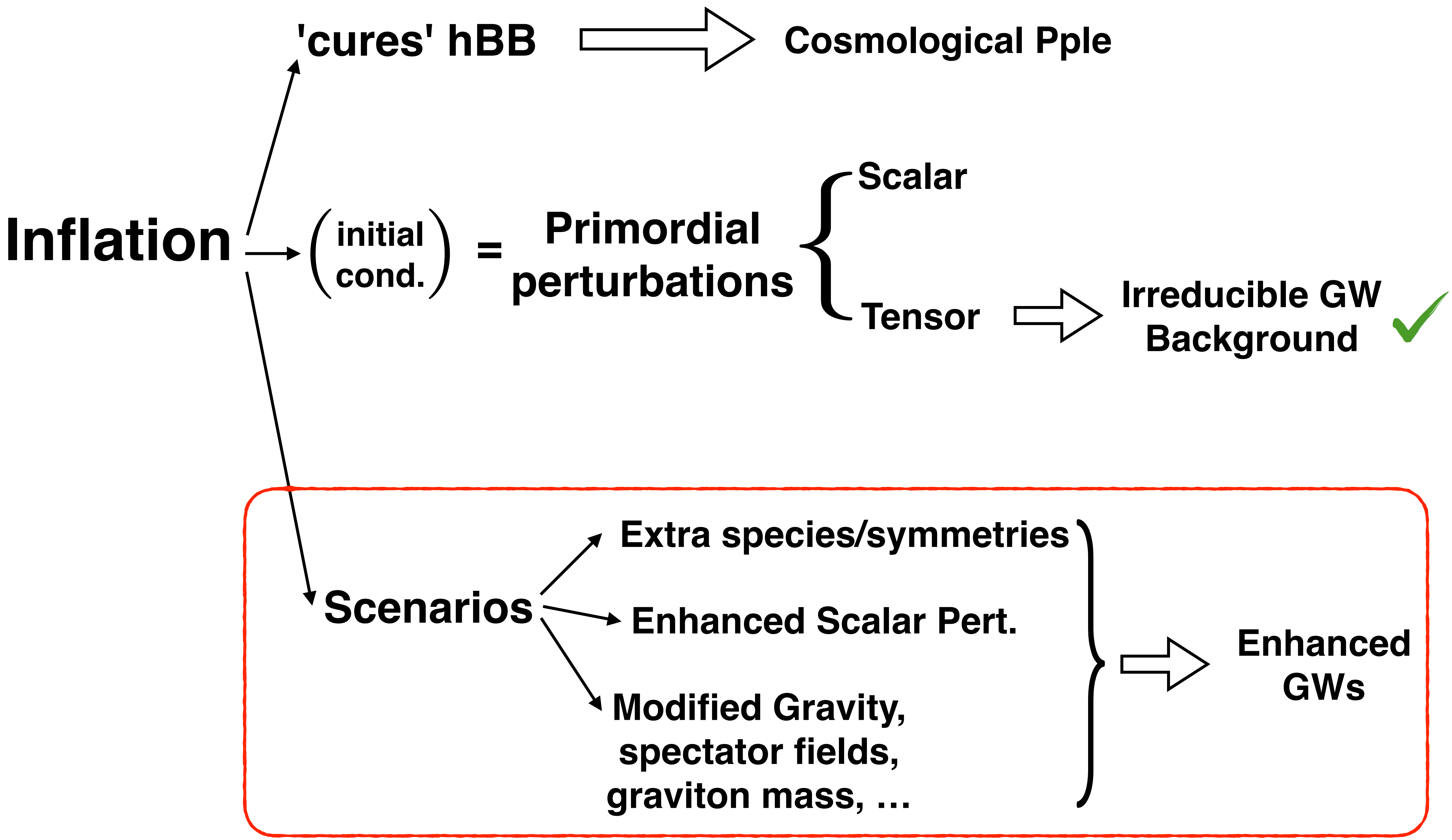

INFLATIONARY COSMOLOGY

'cures' hBB ⇒ Cosmological Pple

Inflation → (initial cond.) = Primordial perturbations

- Scalar
- Tensor ⇒ Irreducible GW Background ✓

Scenarios

- Extra species/symmetries
- Enhanced Scalar Pert.
- Modified Gravity, spectator fields, graviton mass, …

⇒ Enhanced GWs

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + C$ on couplings to other fields

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Shift symmetry $\phi \to \phi + C$ on couplings to other fields

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + V_{\cancel{\text{shift}}}(\phi) + \frac{c_\psi}{f}\,\partial_\mu \phi\, \bar{\psi}\,\gamma^\mu\,\gamma_5\,\psi + \frac{\alpha}{f}\,\phi\, F_{\mu\nu}\,\tilde{F}^{\mu\nu}}$$

derivative couplings to: fermions gauge fields

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto ’90; . . .

Shift symmetry $\phi \to \phi + C$ on couplings to other fields

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + V_{\cancel{\text{shift}}}(\phi) + \frac{c_\psi}{f}\,\partial_\mu \phi\, \bar{\psi}\, \gamma^\mu \gamma_5\, \psi + \frac{\alpha}{f}\, \phi\, F_{\mu\nu}\, \tilde{F}^{\mu\nu}}$$

derivative couplings to: fermions gauge fields

Not the QCD axion;

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fermions gauge fields

(derivative couplings)

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(derivative couplings)

$= \partial_\mu K^\mu$

topological term

$[\phi \partial_\mu K^\mu = K^\mu \partial_\mu \phi]$

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breaks shift-symm

fermions

gauge fields

(derivative couplings)

$= \partial_\mu K^\mu$ topological term

$[\phi\partial_\mu K^\mu = K^\mu \partial_\mu \phi]$

$\boxed{\text{With shift symmetry, } \Delta V \propto V_{\cancel{\text{shift}}}}$ ⇨ Protected against radiative corrections !

INFLATIONARY MODELS

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Shift symmetry $\phi \to \phi + const.$

$$V(\varphi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

inflaton ϕ = pseudo-scalar axion

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Photon: 2 helicities

$$\vec{A}(\tau, \mathbf{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[\vec{\epsilon}_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}) A_\lambda(\tau, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]$$

$$\left[a_\lambda(\mathbf{k}), a^\dagger_{\lambda'}(\mathbf{k}') \right] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

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$$\vec{A}'' - \nabla^2 \vec{A} - \frac{1}{f} \phi' \, \vec{\nabla} \times \vec{A} = 0 \quad \Rightarrow \quad \left[\frac{\partial^2}{\partial \tau^2} + k^2 \pm \frac{2k\xi}{\tau} \right] A_\pm(\tau, k) = 0,$$

$$\boxed{\xi \equiv \frac{\dot{\phi}}{2fH}}$$

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Chiral instability

$$\boxed{A_+(\tau, k) \cong \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi a H} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}}$$

$$\boxed{\xi \equiv \frac{\dot{\phi}}{2fH}}$$

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$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

A_+ exponentially amplified,

A_- has no amplification

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Gauge field excitation creates chiral GWs !

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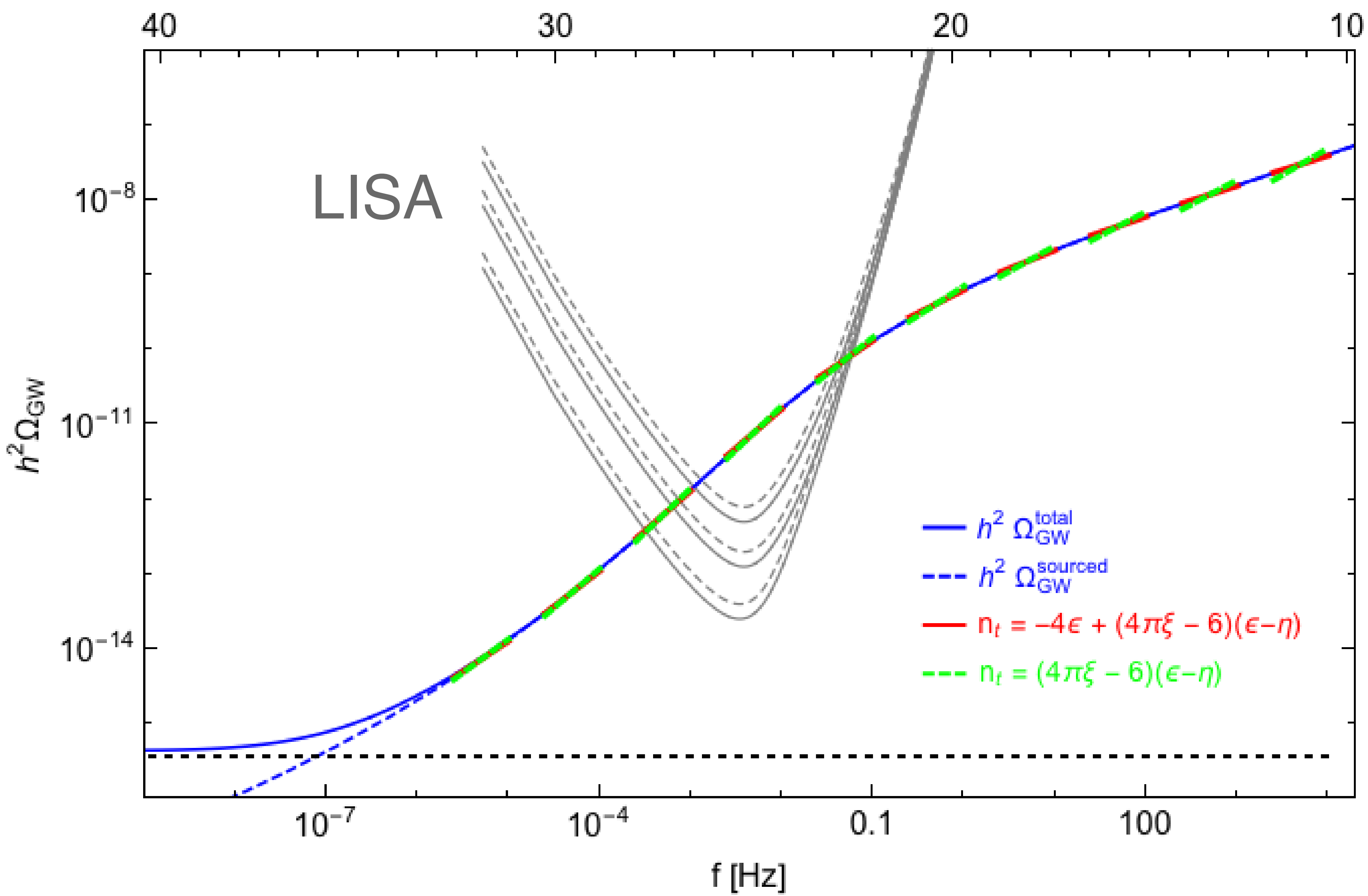
GW one-chirality only

A_μ **Chiral**

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



Gauge fields source a Blue-Tilted + Chiral + Non-G GW background

INFLATIONARY MODELS

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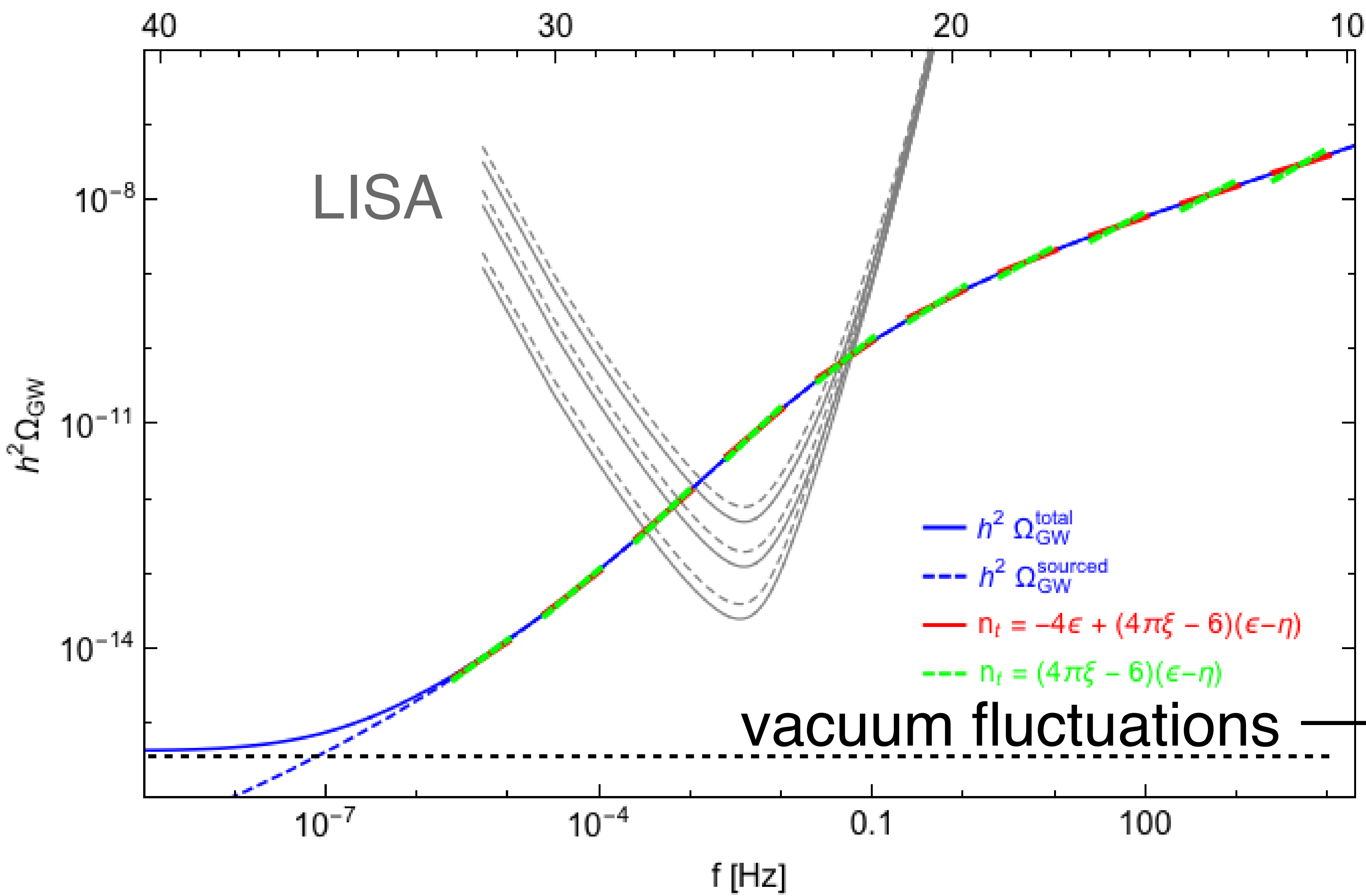

$(\Omega_{\rm Rad}/24)\Delta_h^2$

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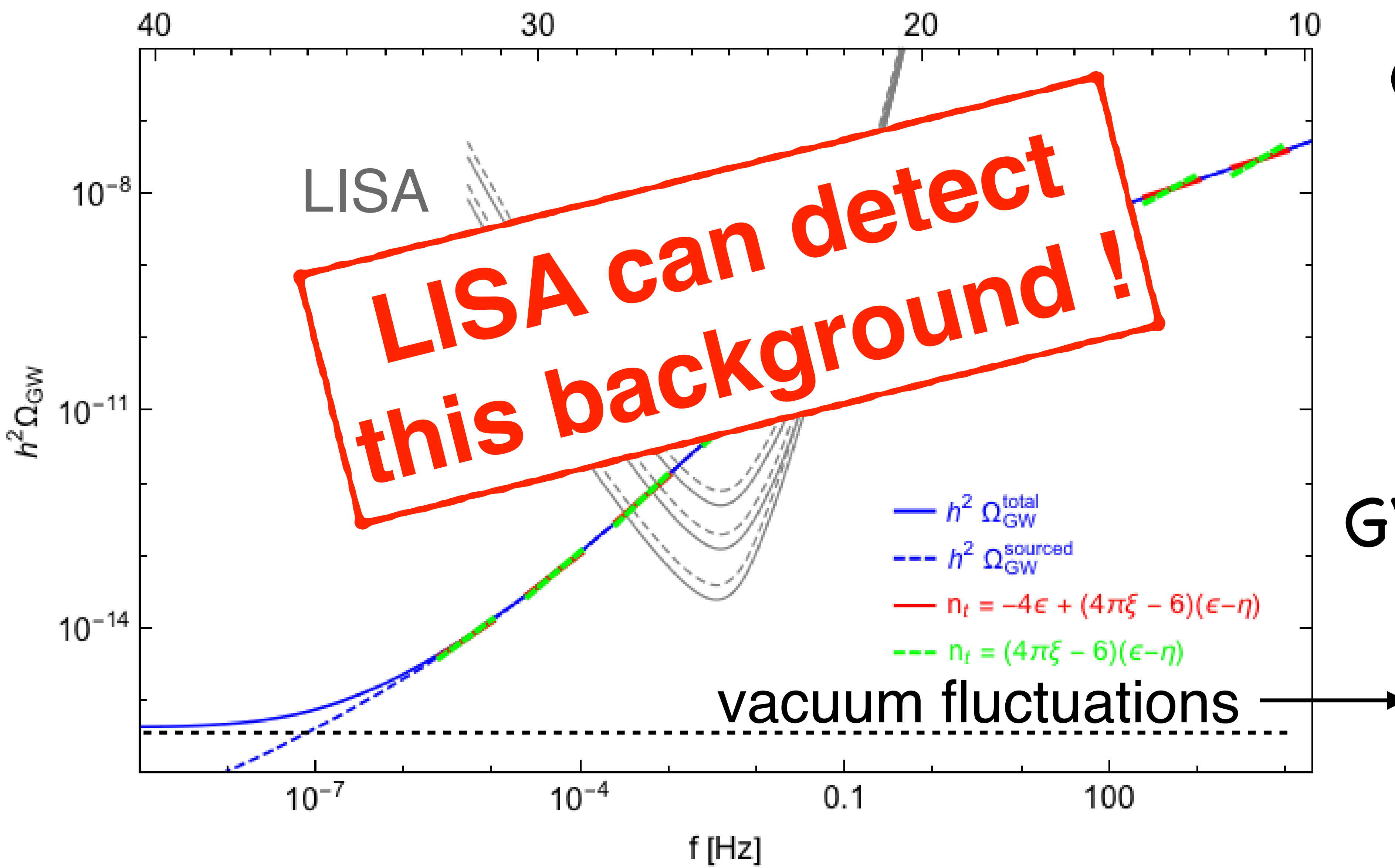

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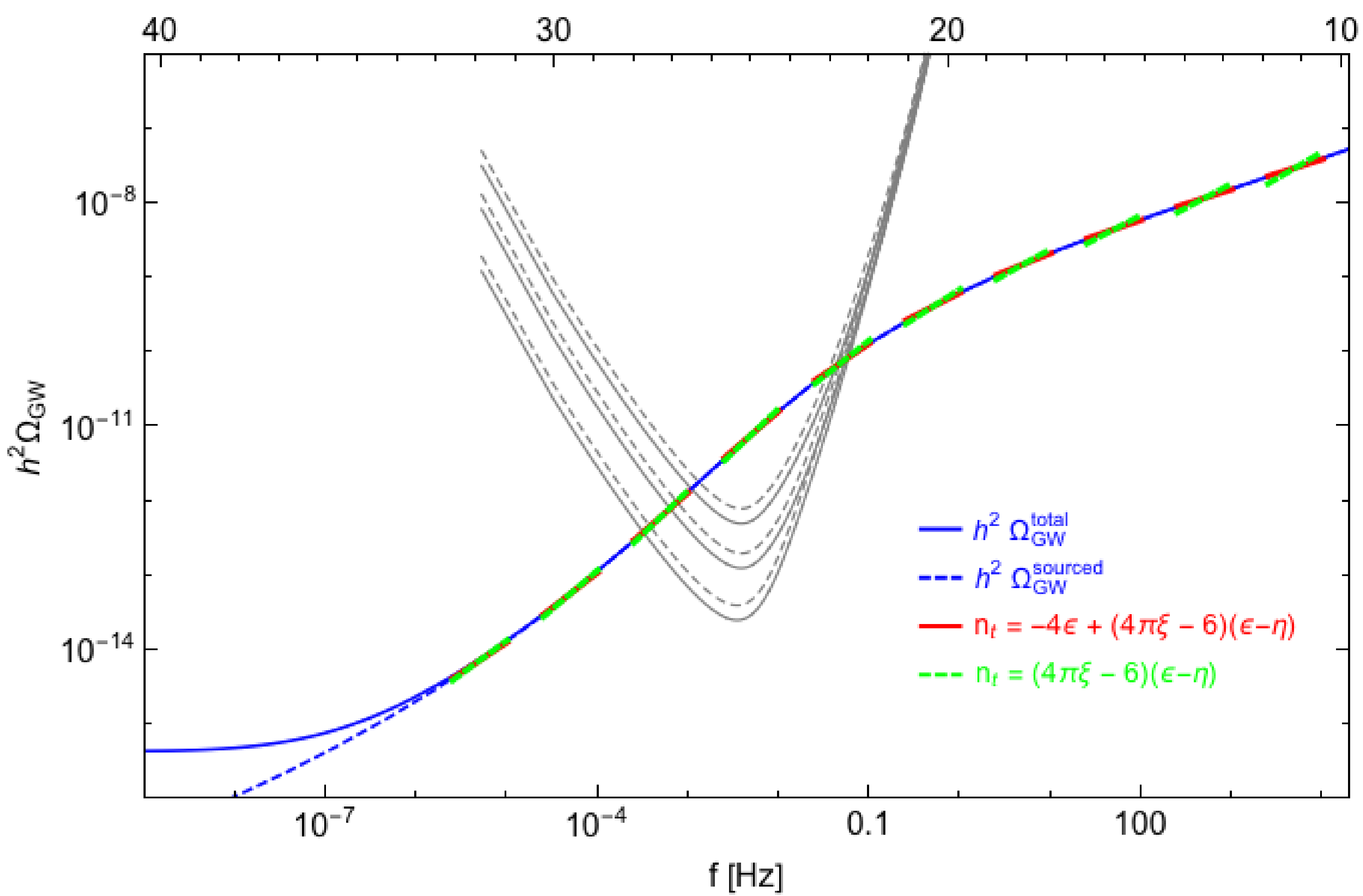
INFLATIONARY MODELS

Axion-Inflation

Bartolo et al ‘16

$$h^2\Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_T}$$



INFLATIONARY MODELS

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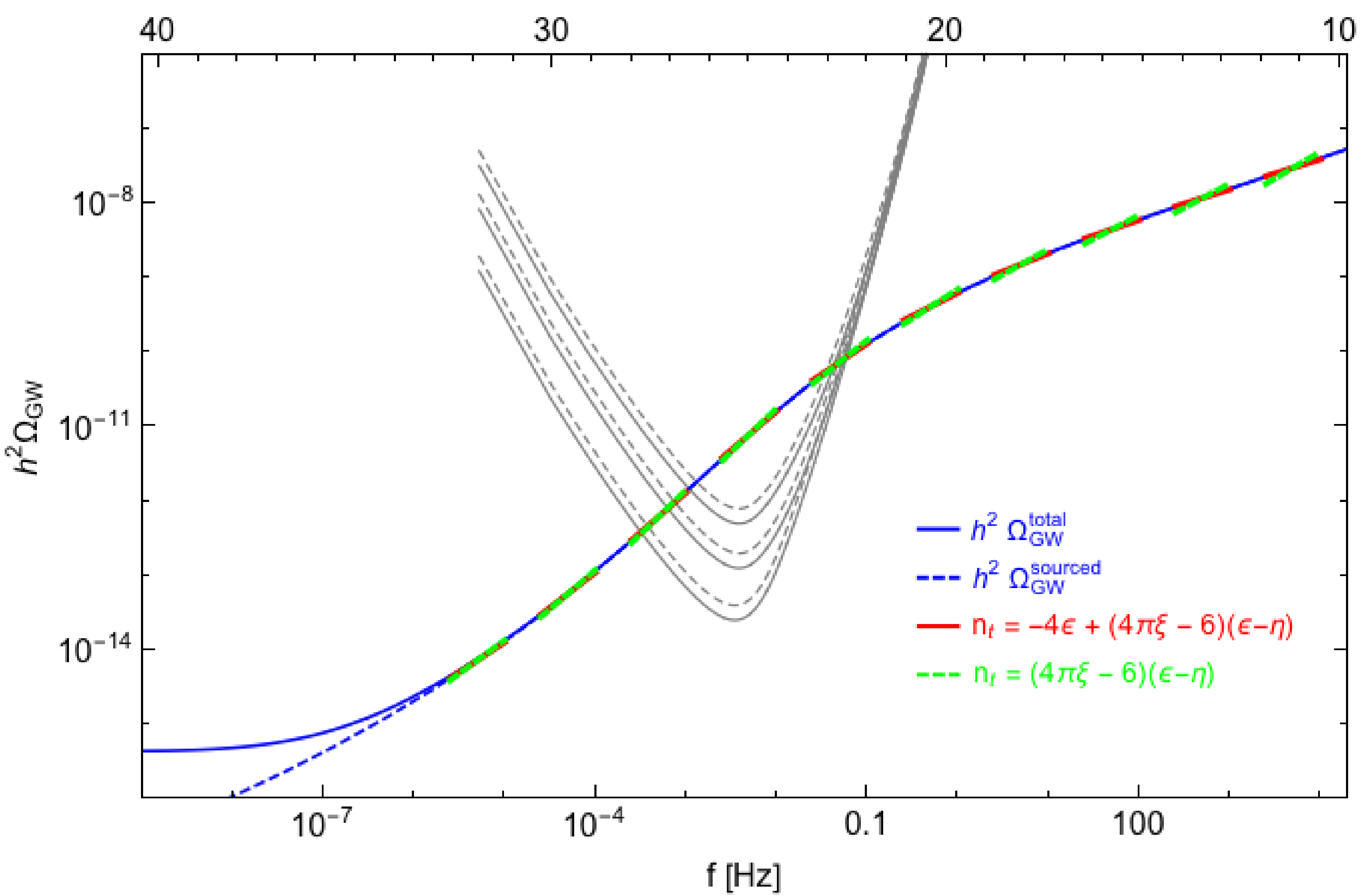
$$h^2\Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_T} \rightarrow \boxed{n_T \simeq (4\pi\xi - 6)(\epsilon_H - \eta)}$$

$$\boxed{\xi \equiv \frac{\alpha\dot{\phi}}{2fH}}$$

$$A_* = \boxed{1.5 \cdot 10^{-13} \frac{H^4}{M_{Pl}^4} \frac{e^{4\pi\xi}}{\xi^6}},$$

$\boxed{H,\ \xi,\ \epsilon_H - \eta}$ **(3 parameters!)**



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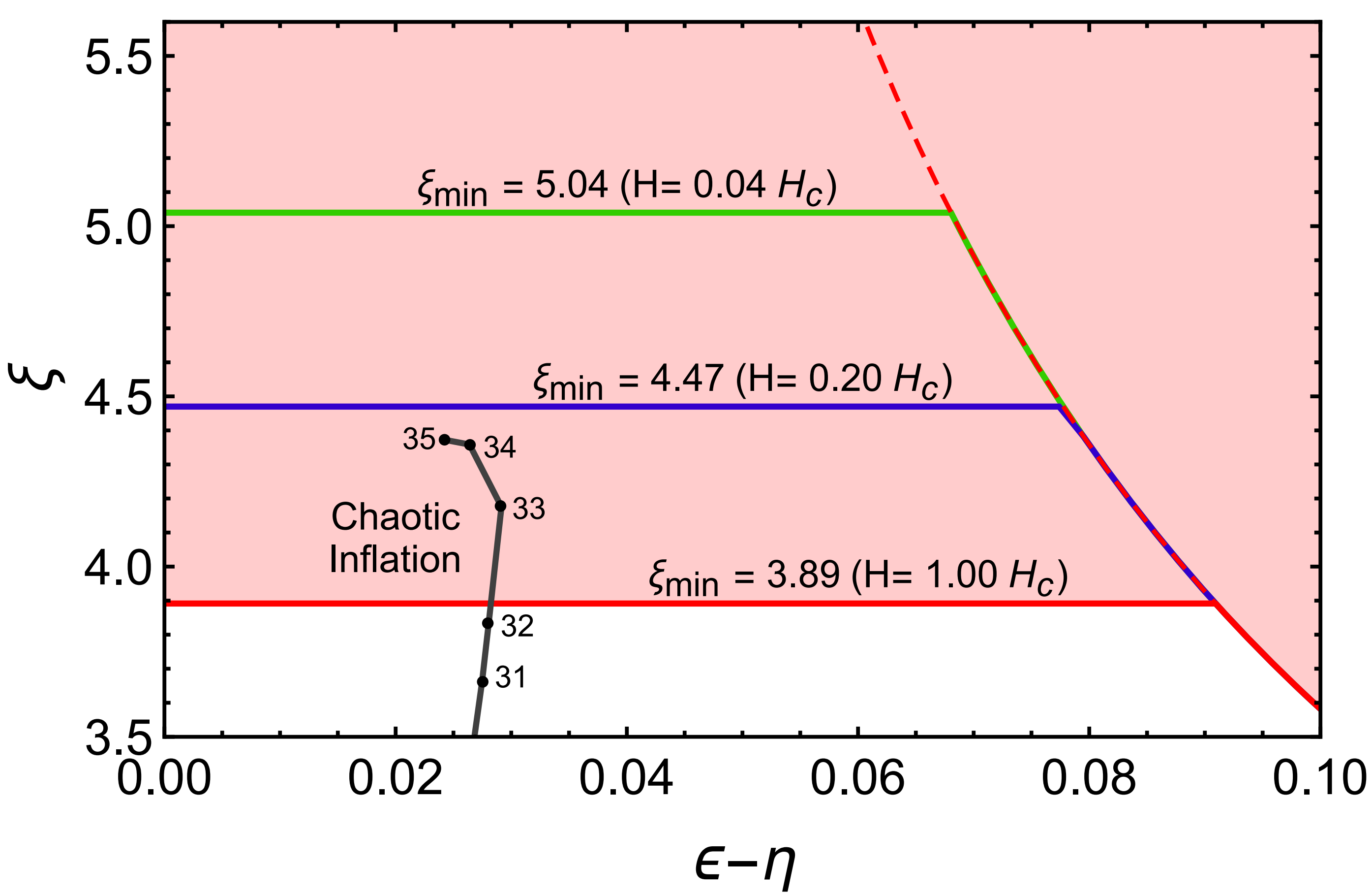
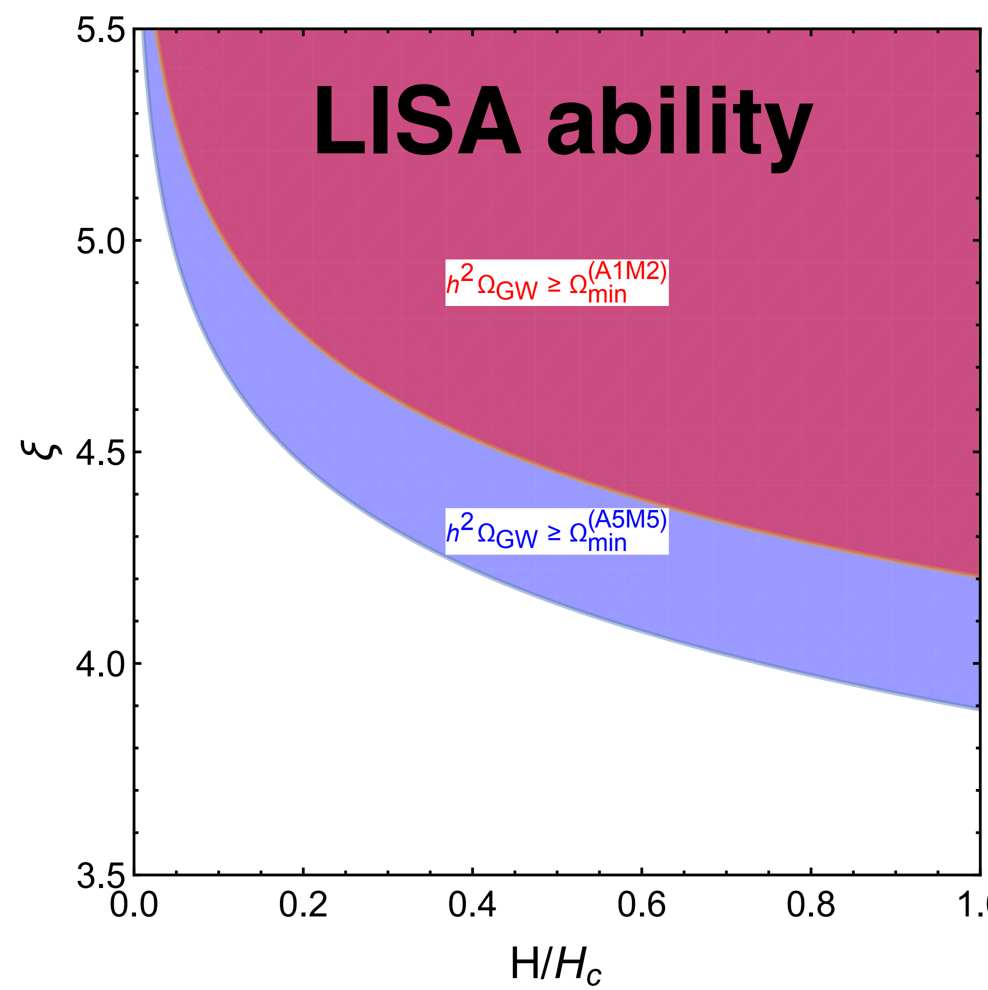
LISA ability





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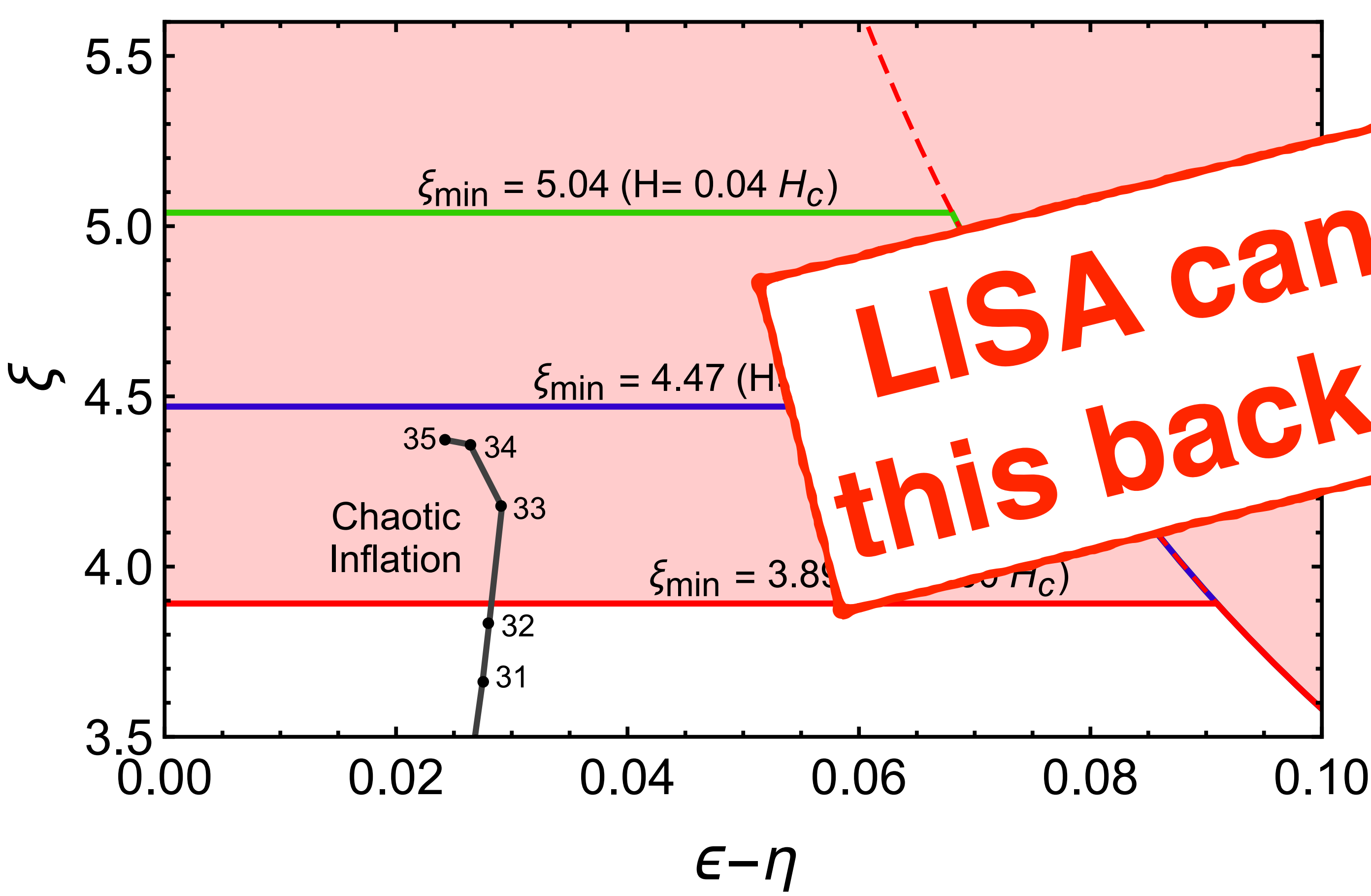

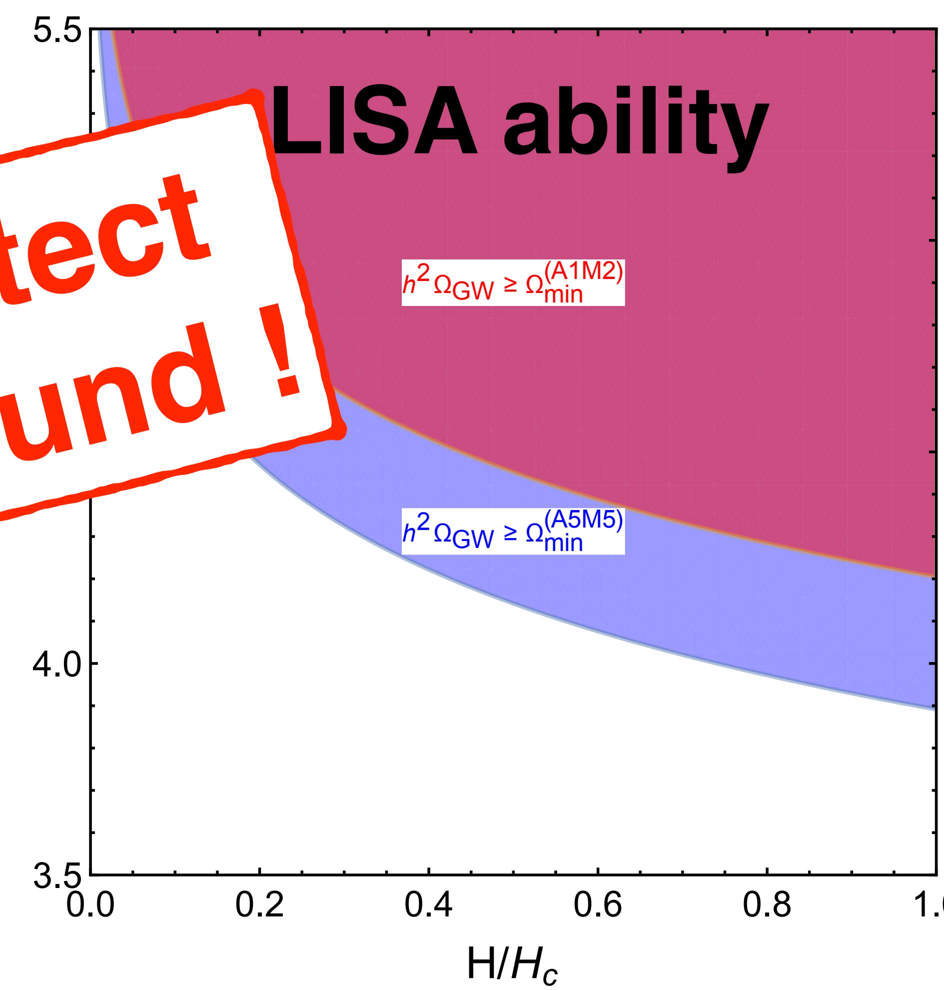

LISA can detect this background !

INFLATIONARY MODELS

Axion-Inflation: ***Shift*** **symmetry** $\longrightarrow$ **Natural (chiral) coupling to** A_μ

huge excitation of fields ! (photons)

INFLATIONARY MODELS

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What if there are arbitrary fields coupled to the inflaton ?

(i.e. no need of extra symmetry)

INFLATIONARY MODELS

Axion-Inflation: ***Shift*** **symmetry** → Natural (chiral) coupling to A_μ

huge excitation of fields ! (photons)

What if there are arbitrary fields coupled to the inflaton ? → **large excitation of these fields !?**

(i.e. no need of extra symmetry)

will they create GWs?

INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ?

(i.e. no need of extra symmetry)

GW generation !?

INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ?

(i.e. no need of extra symmetry) GW generation !?

$$-\mathcal{L}_\chi = (\partial\chi)^2/2 + g^2(\phi-\phi_0)^2\chi^2/2$$ **Scalar Fld**

$$-\mathcal{L}_\psi = \bar{\psi}\gamma^\mu\partial_\mu\psi + g(\phi-\phi_0)\bar{\psi}\psi$$ **Fermion Fld**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - gA_\mu)\Phi)|^2 - V(\Phi^\dagger\Phi)$$ **Gauge Fld** ($\Phi = \phi e^{i\theta}$)

INFLATIONARY MODELS

fields coupled to the inflaton ? → **large excitation ?**

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All 3 cases:

$$m = g(\phi(t) - \phi_0) \Rightarrow \overset{\text{non-adiabatic}}{\dot{m} \gg m^2} \quad \text{during} \quad \Delta t_{\rm na} \sim 1/\mu\,, \qquad \boxed{\mu^2 \equiv g\dot{\phi}_0}$$

$$\boxed{n_k = \mathrm{Exp}\{-\pi(k/\mu)^2\}}$$ **Non-adiabatic field excitation (particle creation)**

INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ✓

(i.e. no need of extra symmetry) GW generation !?

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Non-adiabatic field excitation (particle creation !)
(spin-independent)

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Non-adiabatic field excitation (particle creation !)
(spin-independent)

In all three cases (scalars, fermions, and vectors)

GWs generated by anisotropic distribution of the created species

(Only $k \ll \mu$ long-wave modes excited)

INFLATIONARY MODELS

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In all three cases (scalars, fermions, and vectors)

GWs power spectrum: $\mathcal{P}_h^{(\mathrm{tot})}(k) = \mathcal{P}_h^{(\mathrm{vac})}(k) + \mathcal{P}_h^{(\mathrm{pp})}(k)$

from particle production

GW Source(s): (SCALARS , VECTOR , FERMIONS)

$$\Pi_{ij}^{TT} \propto \{\partial_i\chi^a\partial_j\chi^a\}^{TT}, \quad \{E_iE_j + B_iB_j\}^{TT}, \quad \{\bar\psi\gamma_i D_j\psi\}^{TT}$$

INFLATIONARY MODELS

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$$\boxed{\frac{\Delta\mathcal{P}_h}{\mathcal{P}_h} \equiv \frac{\mathcal{P}_h^{(\mathrm{tot})} - \mathcal{P}_h^{(\mathrm{vac})}}{\mathcal{P}_h^{(\mathrm{vac})}} \equiv \frac{\mathcal{P}_h^{(\mathrm{pp})}}{\mathcal{P}_h^{(\mathrm{vac})}} \sim few \times \mathcal{O}(10^{-4})\frac{H^2}{m_{\mathrm{pl}}^2}W(k\tau_0)\left(\frac{\mu}{H}\right)^3 \ln^2(\mu/H)}$$

$\boxed{(W \lesssim 0.5)}$

N. Barnaby *et al.*, Phys. Rev. **D86**, 103508 (2012), [1206.6117].
J. L. Cook and L. Sorbo, Phys. Rev. **D85**, 023534 (2012), [1109.0022].

INFLATIONARY MODELS

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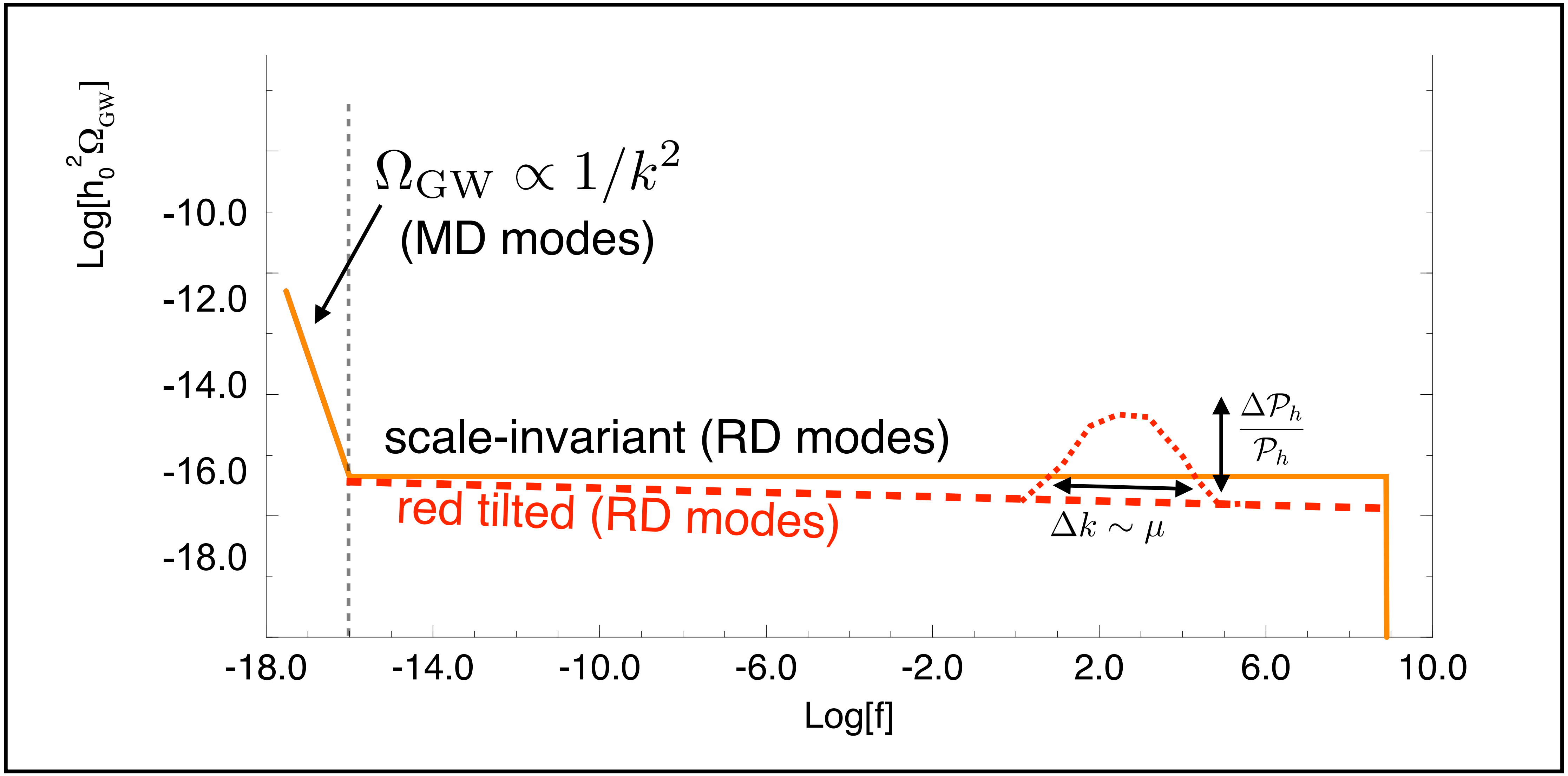
$$\mu^2 \equiv g\dot{\phi}_0$$

(Sorbo et al 2011, Peloso et al 2012-2013, Caprini & DGF 2018)



INFLATIONARY MODELS

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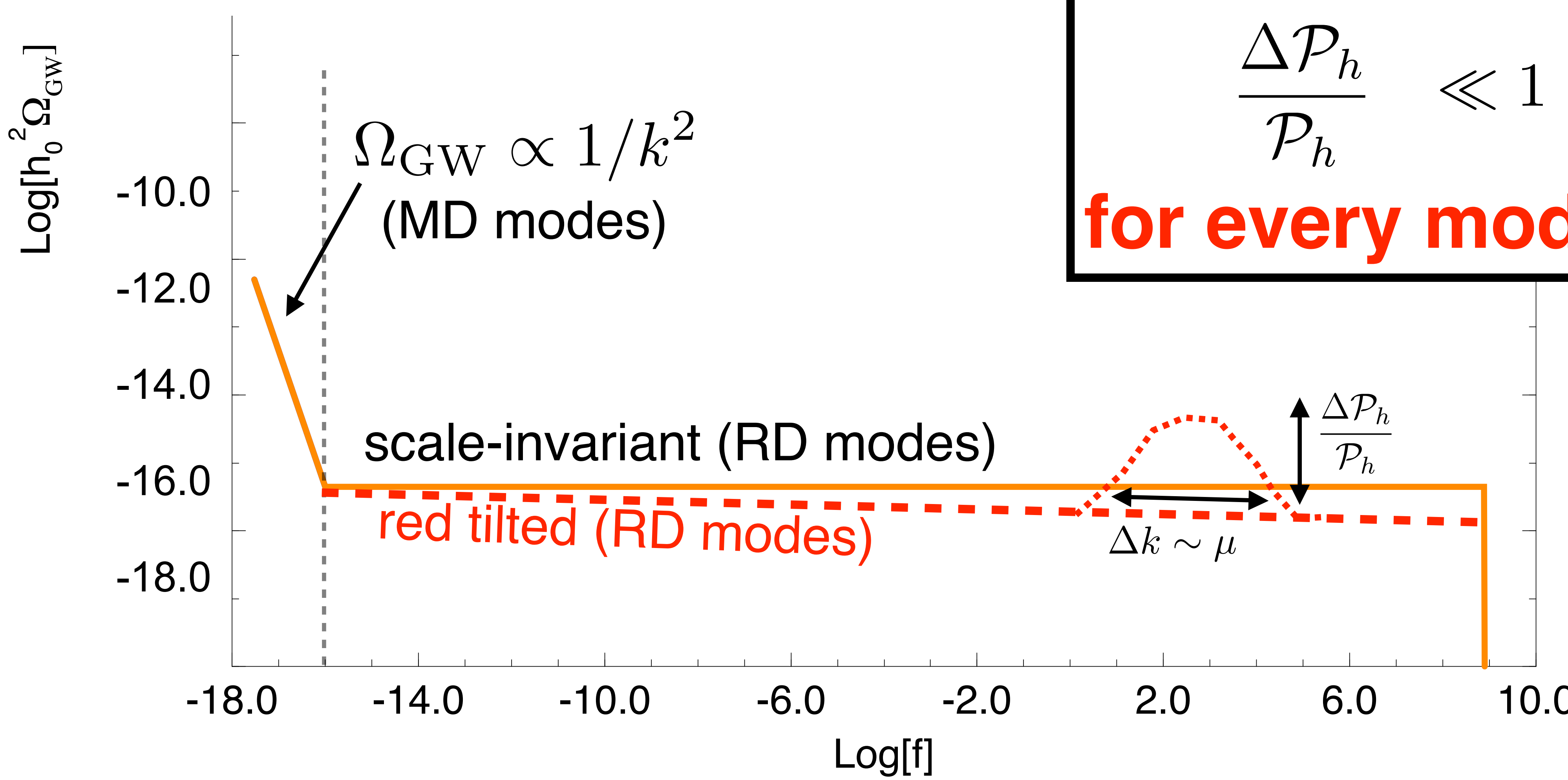
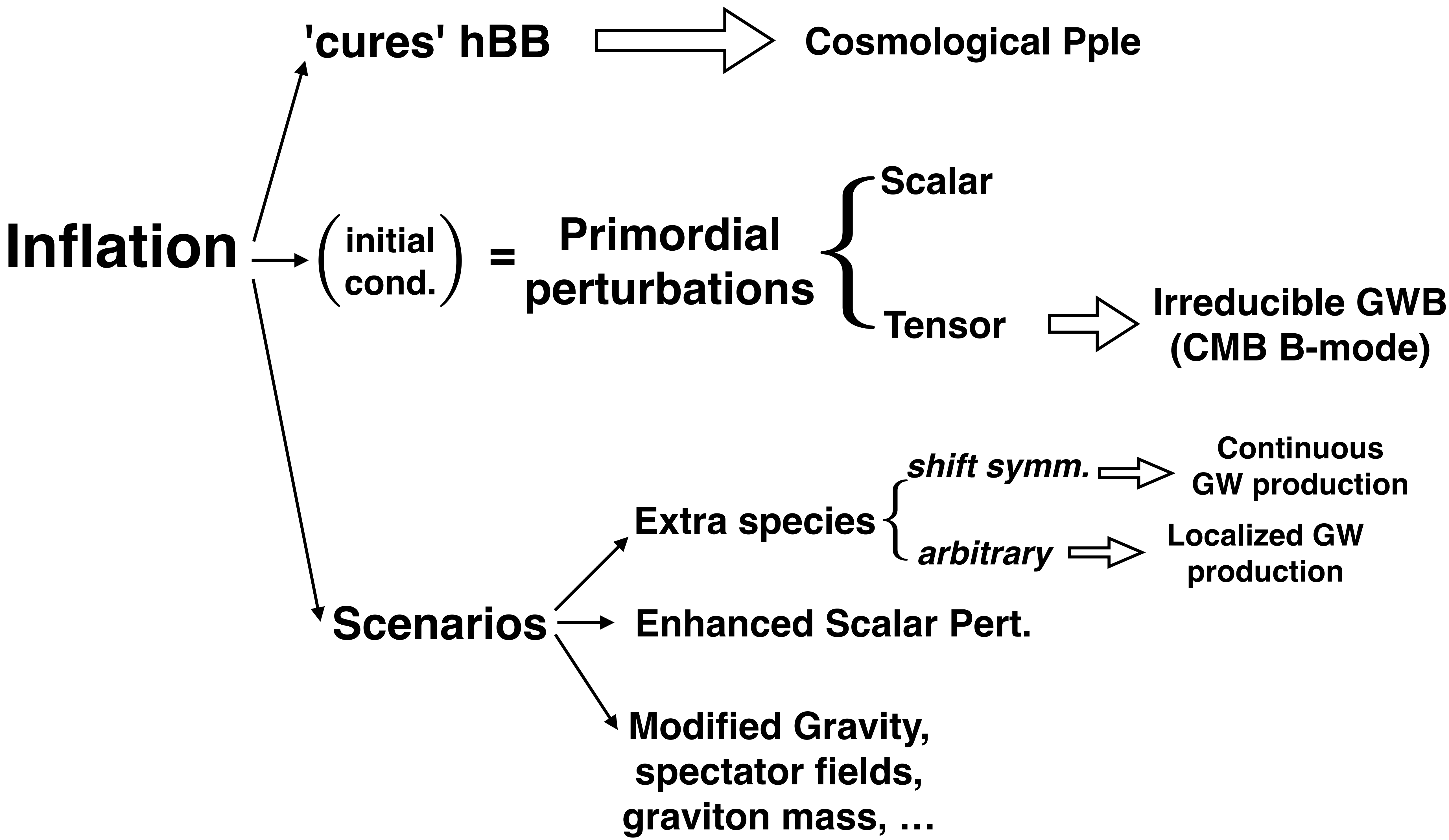
Unfortunately …

$$\frac{\Delta\mathcal{P}_h}{\mathcal{P}_h} \ll 1$$

for every model !



INFLATIONARY COSMOLOGY



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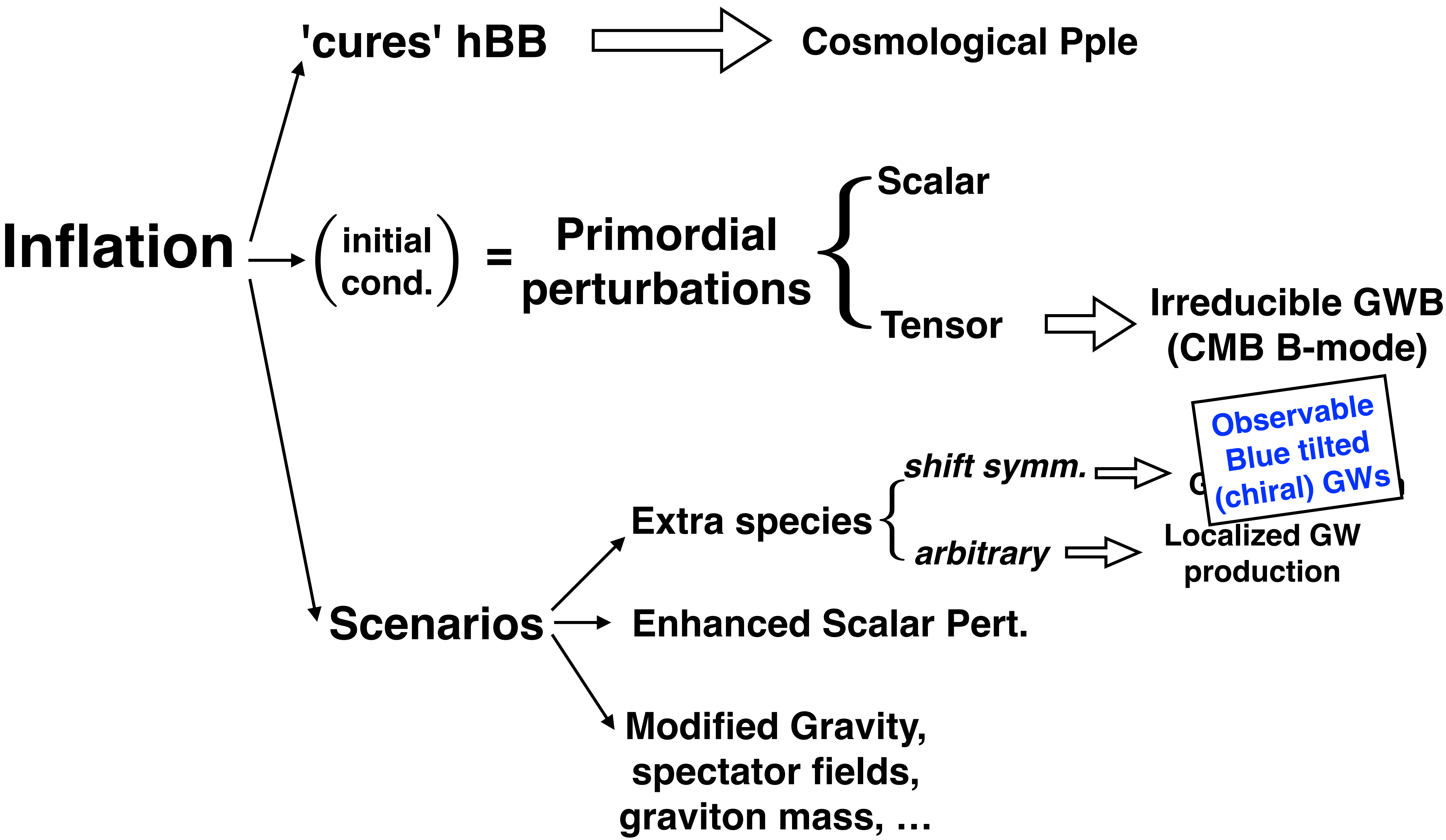

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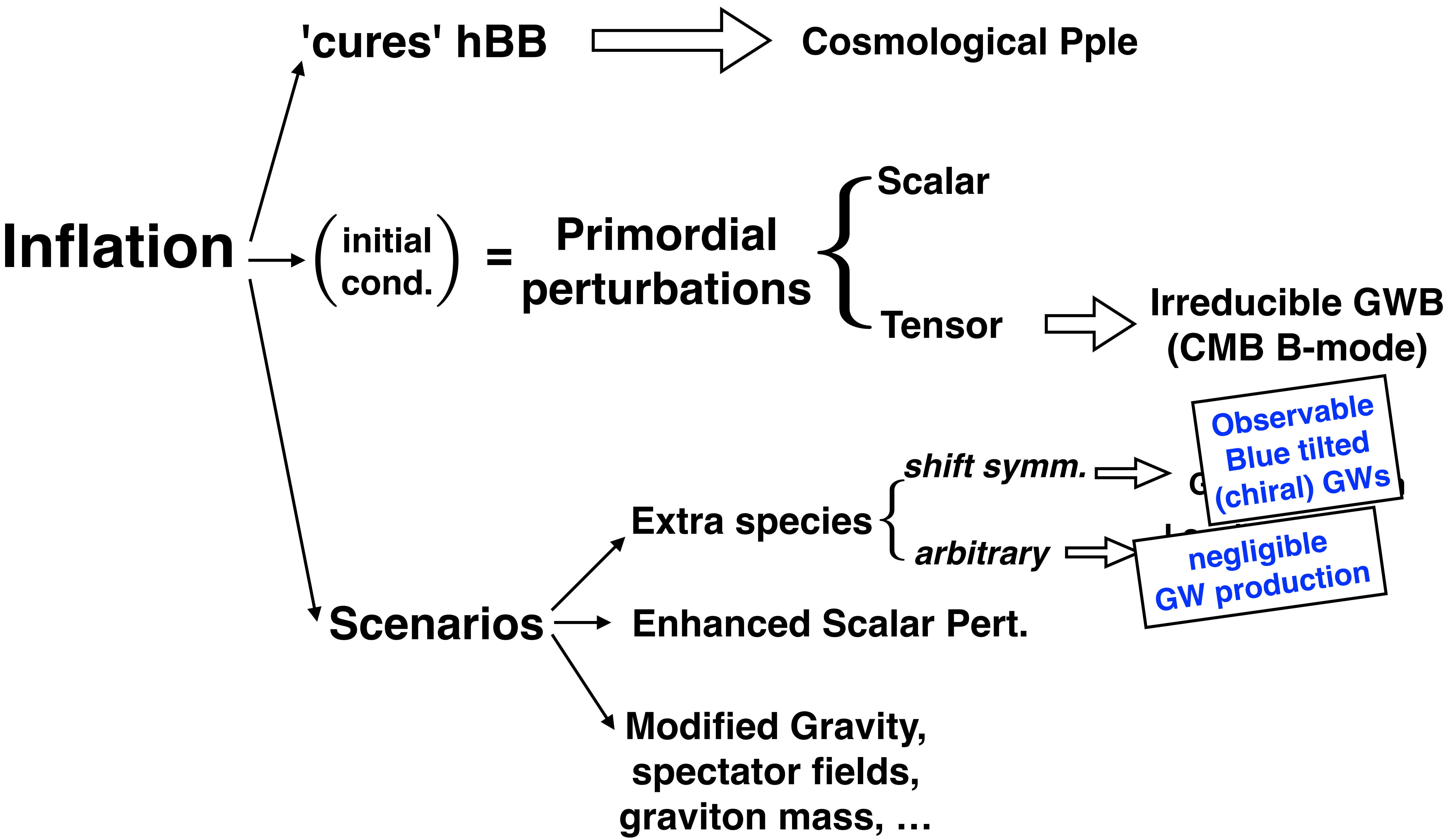

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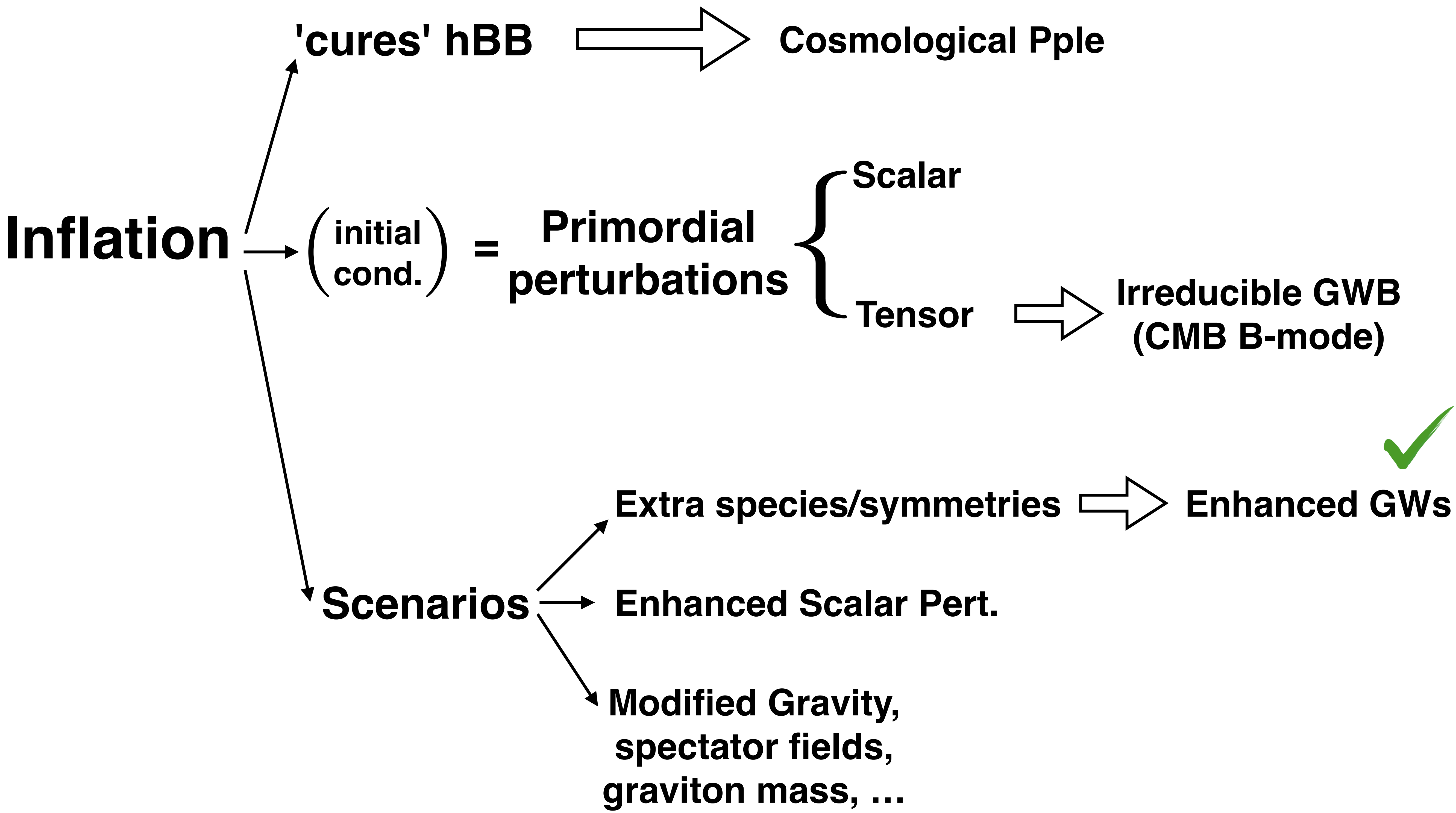

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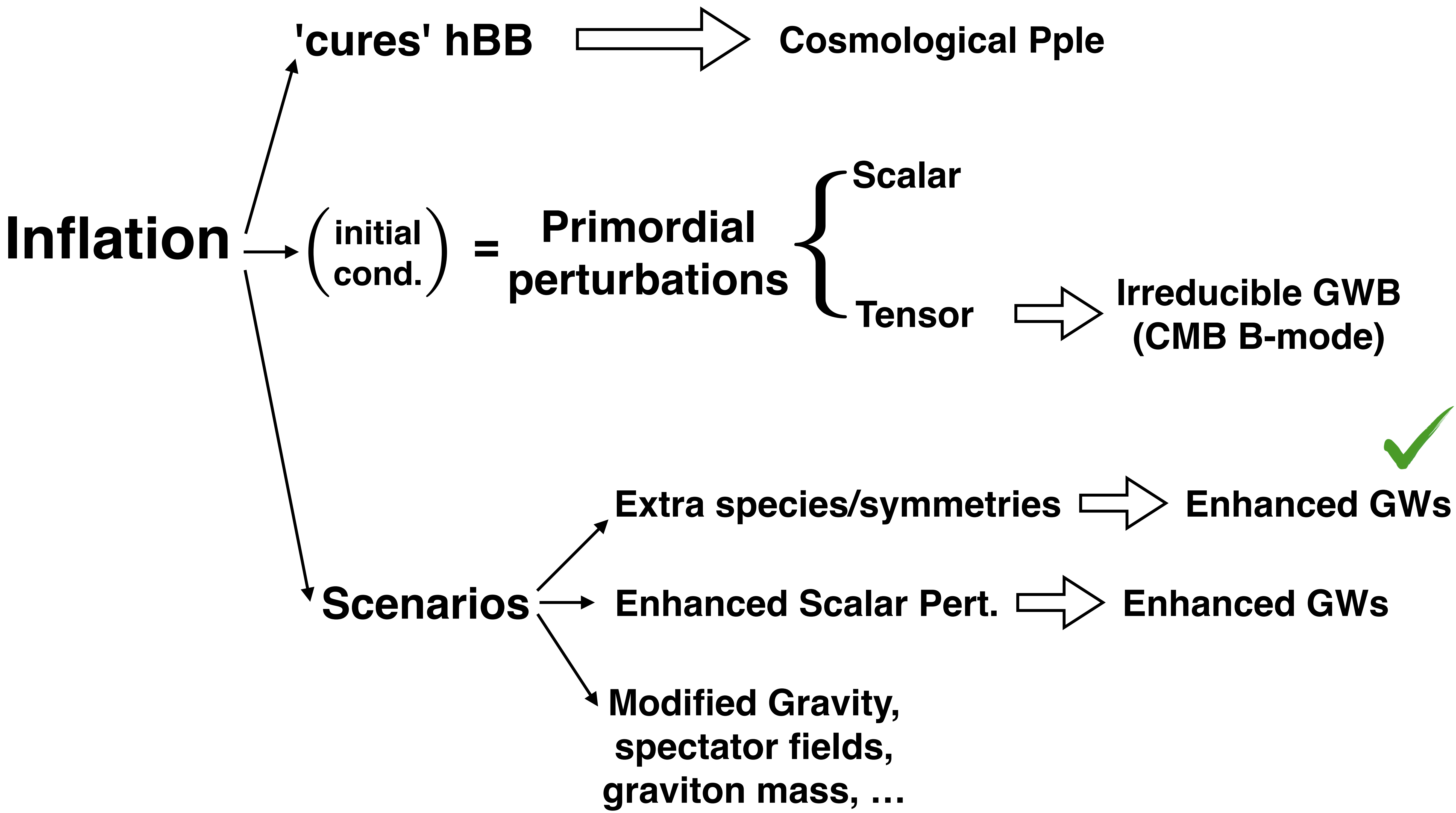

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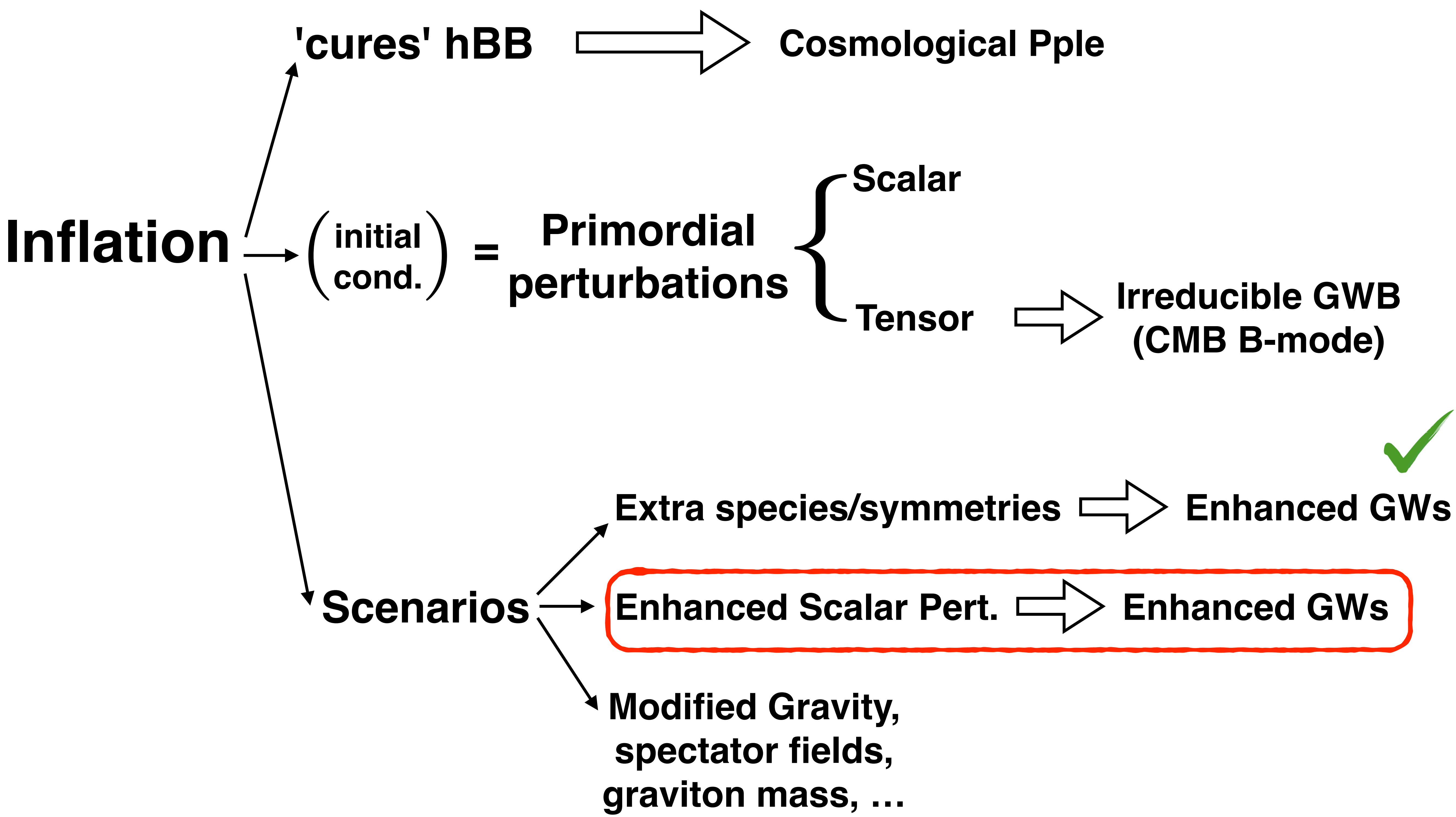


INFLATIONARY MODELS

H & I

INFLATION $\Rightarrow$ $g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}}$$

$$\boxed{n_s - 1 \equiv 2(\eta - 2\epsilon)}$$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

(quasi-)scale invariance $\longleftrightarrow$ Slow roll monotonic potentials

INFLATIONARY MODELS

H & I

INFLATION

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$\Delta^2_{\mathcal{R}}(k)$ H^4 1

$- 2\epsilon)$

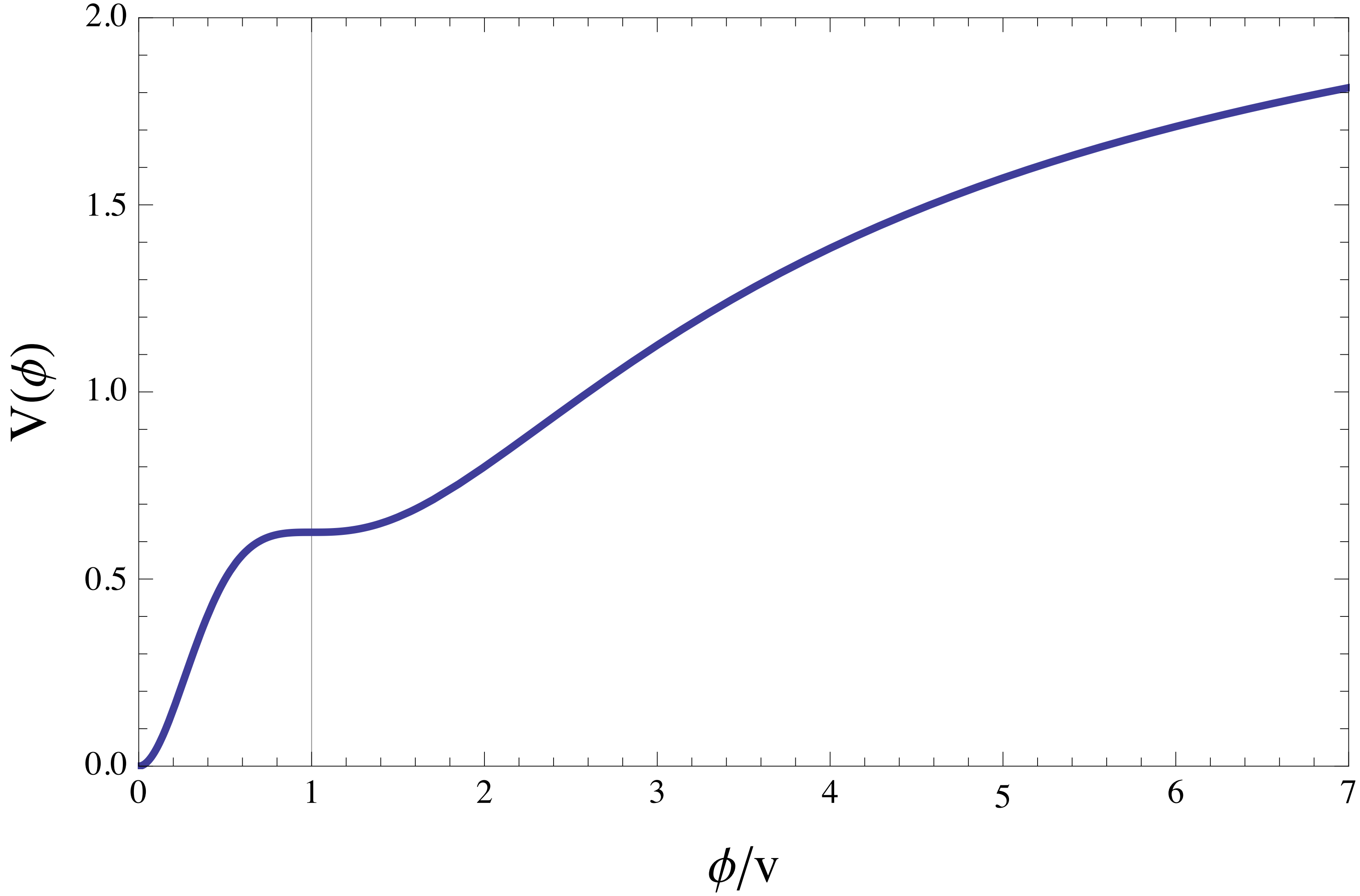
$\Delta^2_h(k$ $\left(\frac{\ }{aH}\right)$

$n_t \equiv -2\epsilon$

Monotonic Single Field Slow Roll

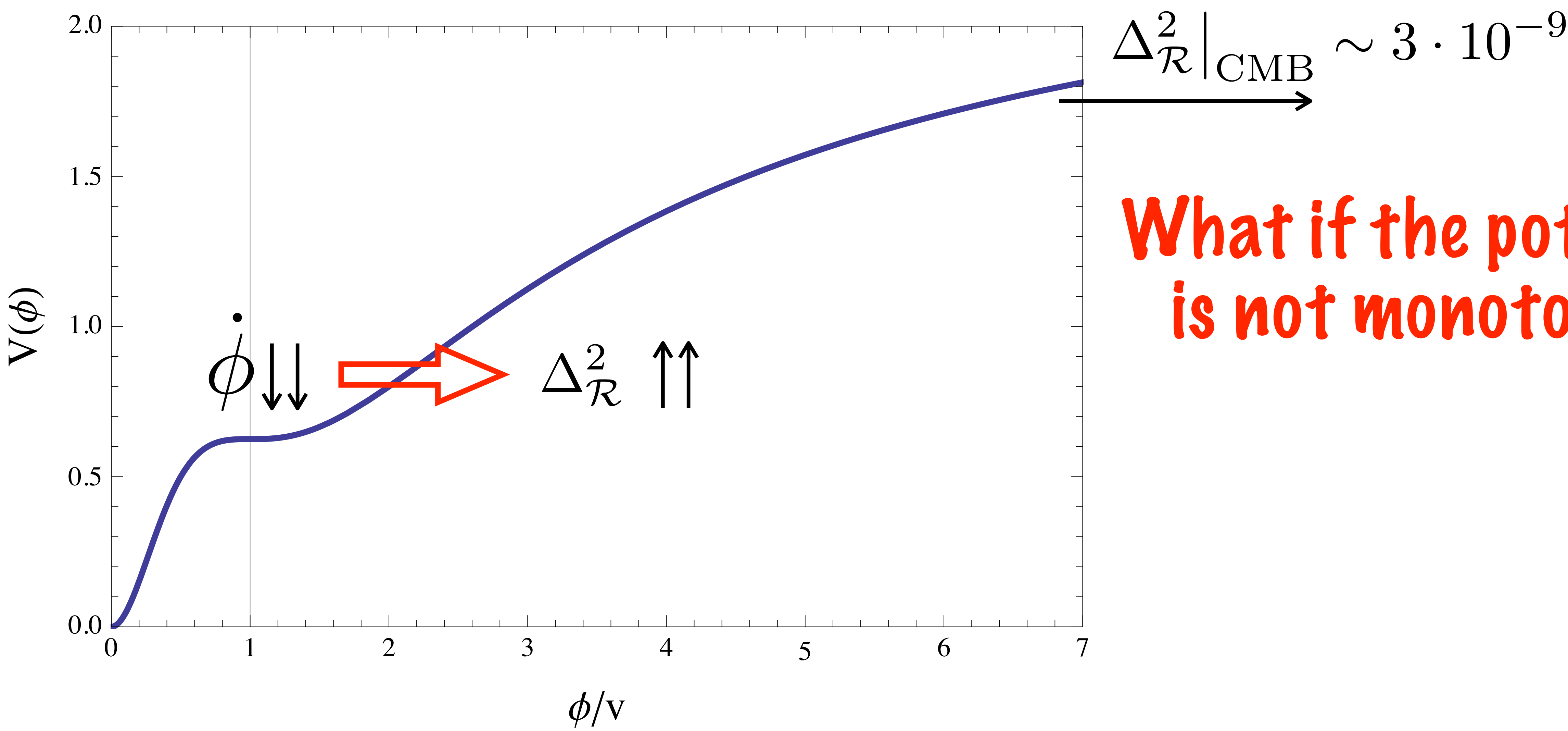
(quasi-)scale invariance $\longleftrightarrow$ Slow roll monotonic potentials

INFLATIONARY MODELS



What if the potential is not monotonic ?

INFLATIONARY MODELS



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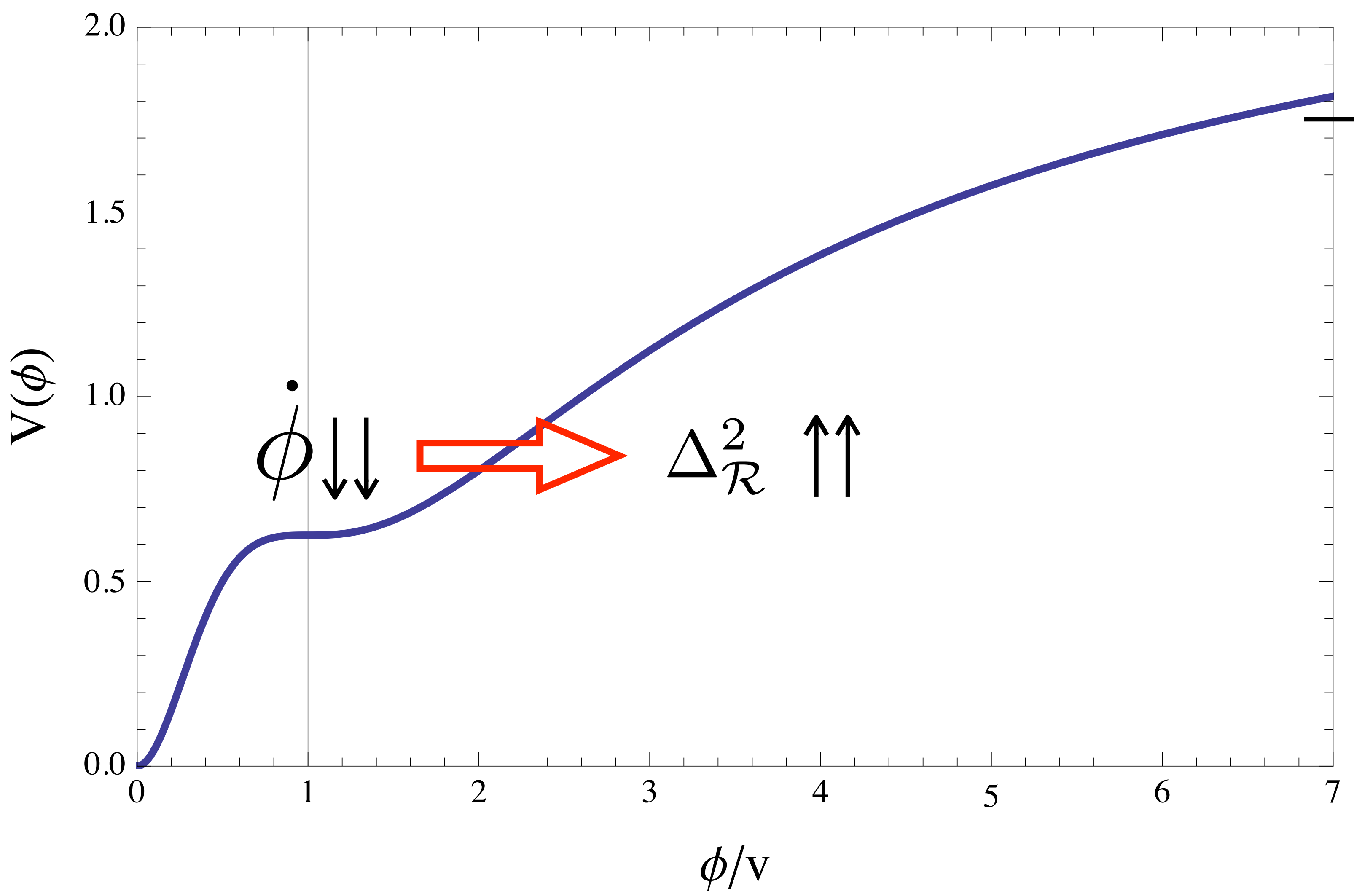
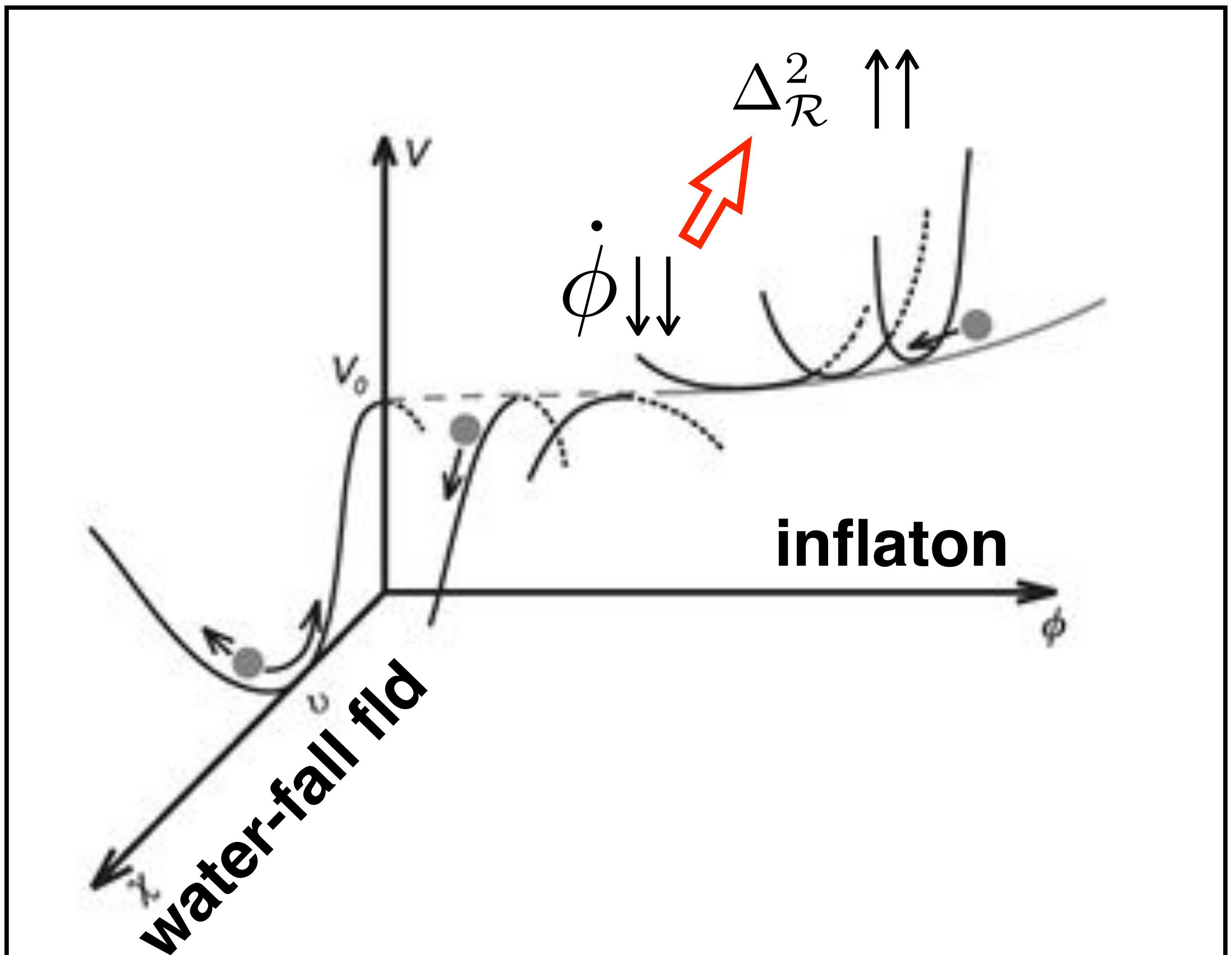


$\Delta^2_{\mathcal{R}}\big|_{\mathrm{CMB}} \sim 3 \cdot 10^{-9}$

What if the potential is not monotonic ?

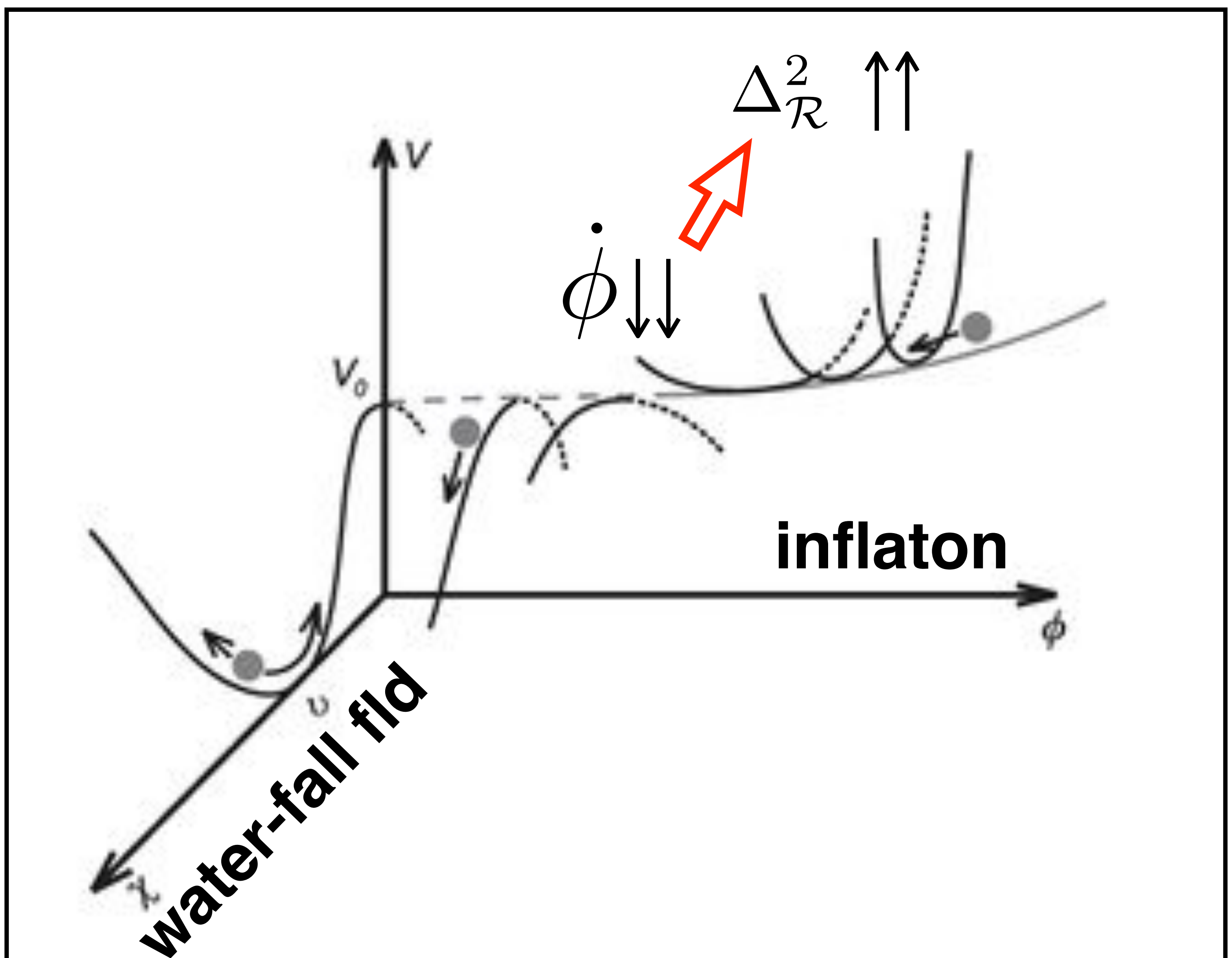
$\Delta^2_{\mathcal{R}}$ **greatly enhanced!**
(at small scales)

INFLATIONARY MODELS



What if it is
multi-field inflation ?

INFLATIONARY MODELS



What if it is multi-field inflation ?

also possible to greatly enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

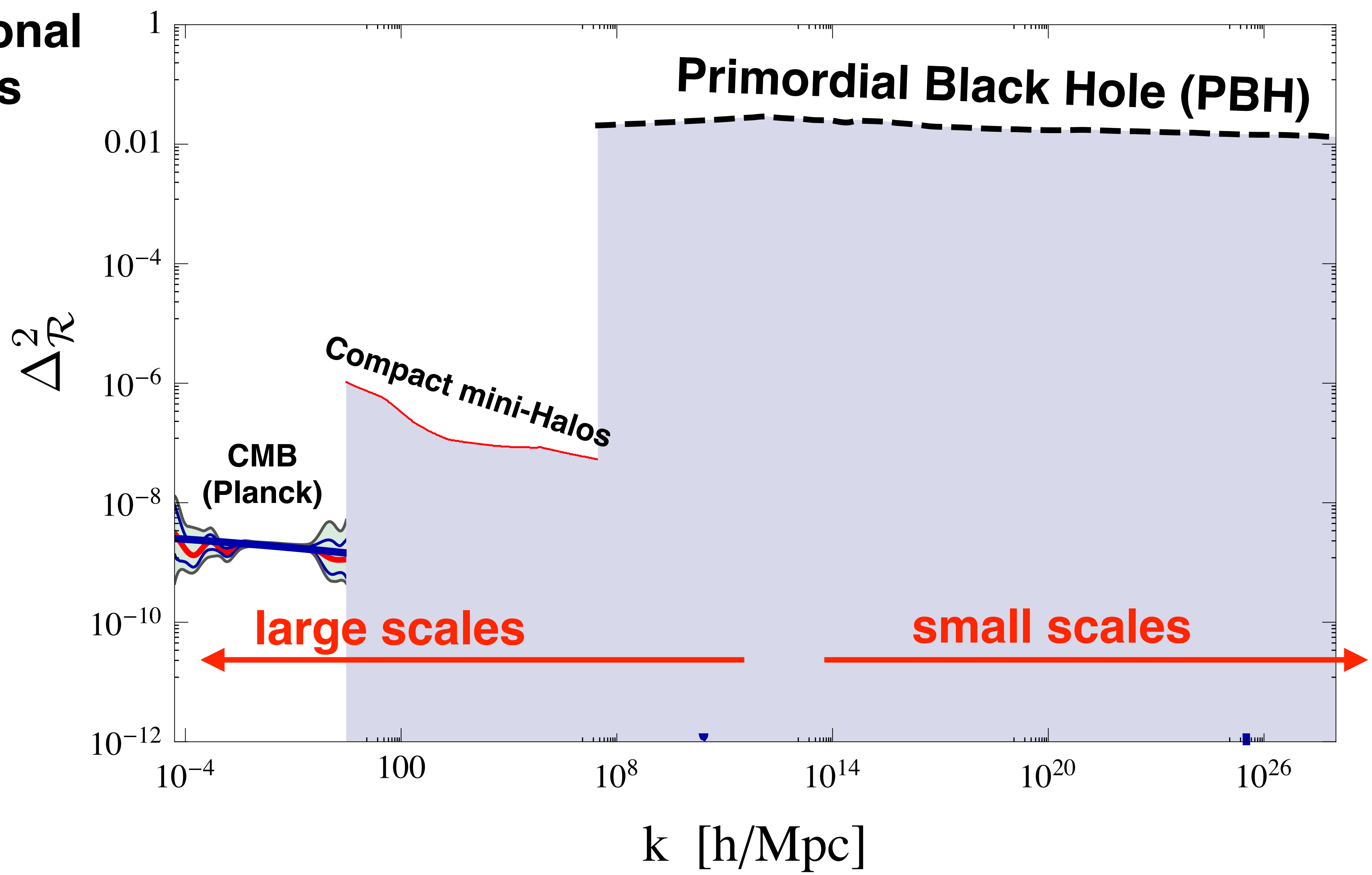
INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{\begin{array}{l}\text{non-monotonic} \\ \text{multi-field}\end{array}\right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

INFLATIONARY MODELS

INFLATION ➡ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ ⇨ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

Observational constraints



INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

Let us suppose $\boxed{\Delta^2_{\mathcal{R}} \gg \Delta^2_{\mathcal{R}}\big|_{\rm CMB} \sim 3 \cdot 10^{-9}}$, @ small scales

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + [(1-2\Psi)\delta_{ij} + 2F_{(i,j)} + h_{ij}]dx^i dx^j]$$

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$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}} \sim \Phi * \Phi \quad \textbf{(2nd Order Pert.)}$$

$$S_{ij} = 2\Phi\partial_i\partial_j\Phi - 2\Psi\partial_i\partial_j\Phi + 4\Psi\partial_i\partial_j\Psi + \partial_i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi$$
$$-\frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi)$$
$$-\frac{2c_s^2}{3w\mathcal{H}}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)$$

D. Wands et al, 2006-2010
Baumann et al, 2007
Peloso et al, 2018

INFLATIONARY MODELS

INFLATION ➡ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ ⇨ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

BBN	$\Omega_{gw,0} < 1.5 \times 10^{-5}$	$\longrightarrow$	$\triangle^2_{\mathcal{R}} < 0.1$
LIGO	$\Omega_{gw,0} < 6.9 \times 10^{-6}$	$\longrightarrow$	$\triangle^2_{\mathcal{R}} < 0.07$
PTA	$\Omega_{gw,0} < 4 \times 10^{-8}$	$\longrightarrow$	$\triangle^2_{\mathcal{R}} < 5 \times 10^{-3}$
LISA	$\Omega_{gw,0} < 10^{-13}$	$\longrightarrow$	$\triangle^2_{\mathcal{R}} < 1 \times 10^{-5}$
BBO	$\Omega_{gw,0} < 10^{-17}$	$\longrightarrow$	$\triangle^2_{\mathcal{R}} < 3 \times 10^{-7}$

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INFLATIONARY MODELS

INFLATION → IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ ⇒ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

IF $\Delta^2_{\mathcal{R}}$ very enhanced → Primordial Black Holes (PBH) may be produced!

INFLATIONARY MODELS

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PBH candidate for DM ? Yes !, for $\sim 10^{-15}$–$10^{-11} M_{\odot}$

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* If PBH are the DM, what is the GW from 2nd O(Φ)? Bartolo et al, '18

* If GW from from 2nd O(Φ) PBH, then Non-Gaussianity? Bartolo et al, '19

* If GW from from 2nd O(Φ) PBH, then Anisotropies? Bartolo et al, '19

INFLATIONARY MODELS

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Has LIGO detected PBH's ?

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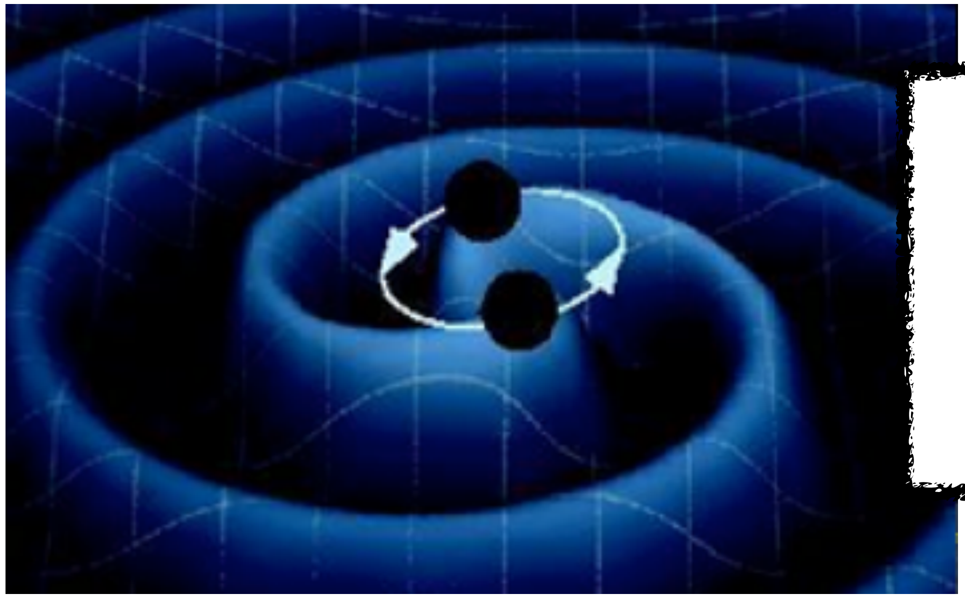
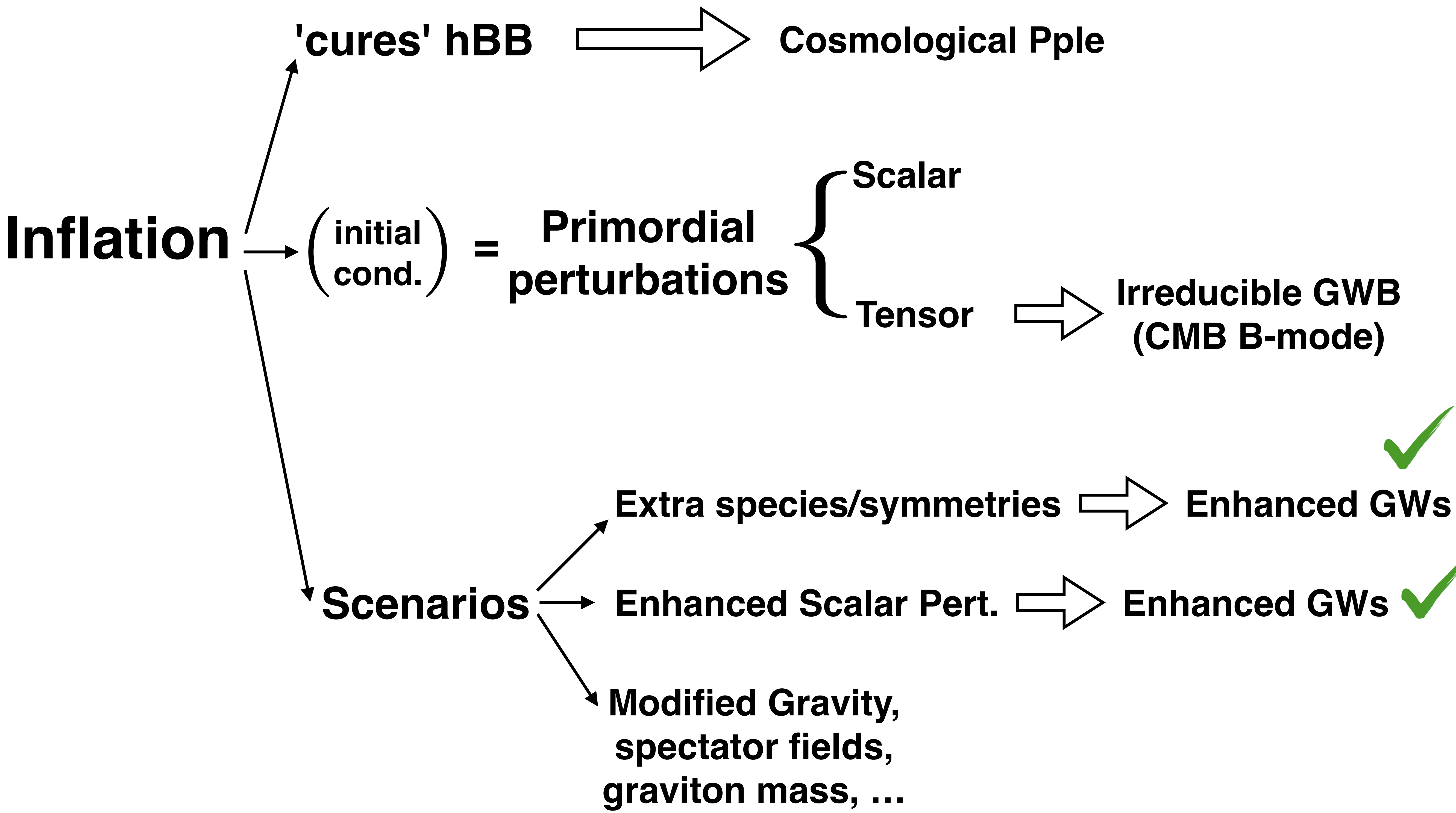


'We will know determining the mass/spin distribution'

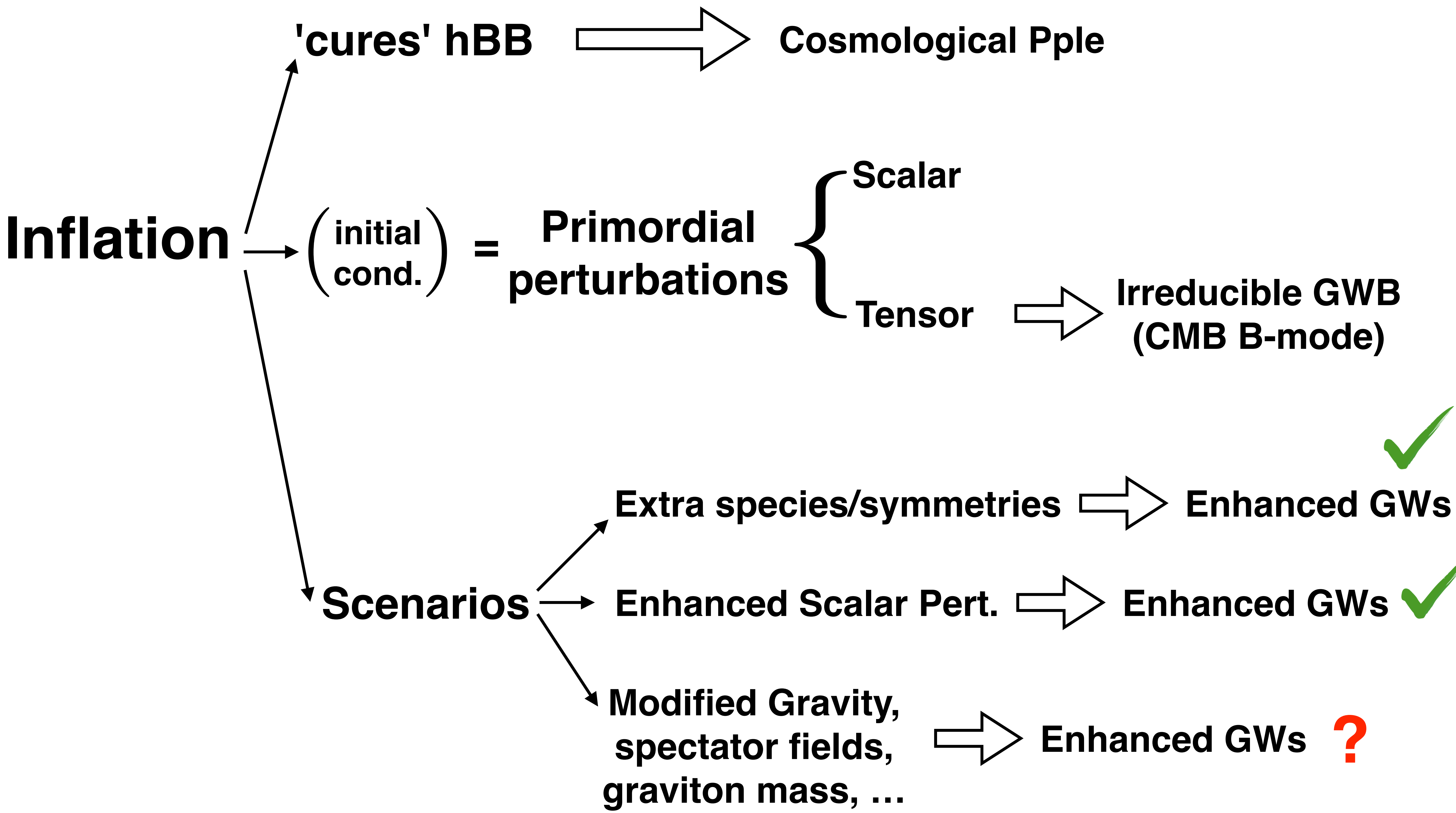
(M. Fishbach (LIGO), Moriond'19)

e.g. 2102.03809, 2105.03349, De Luca *et al*

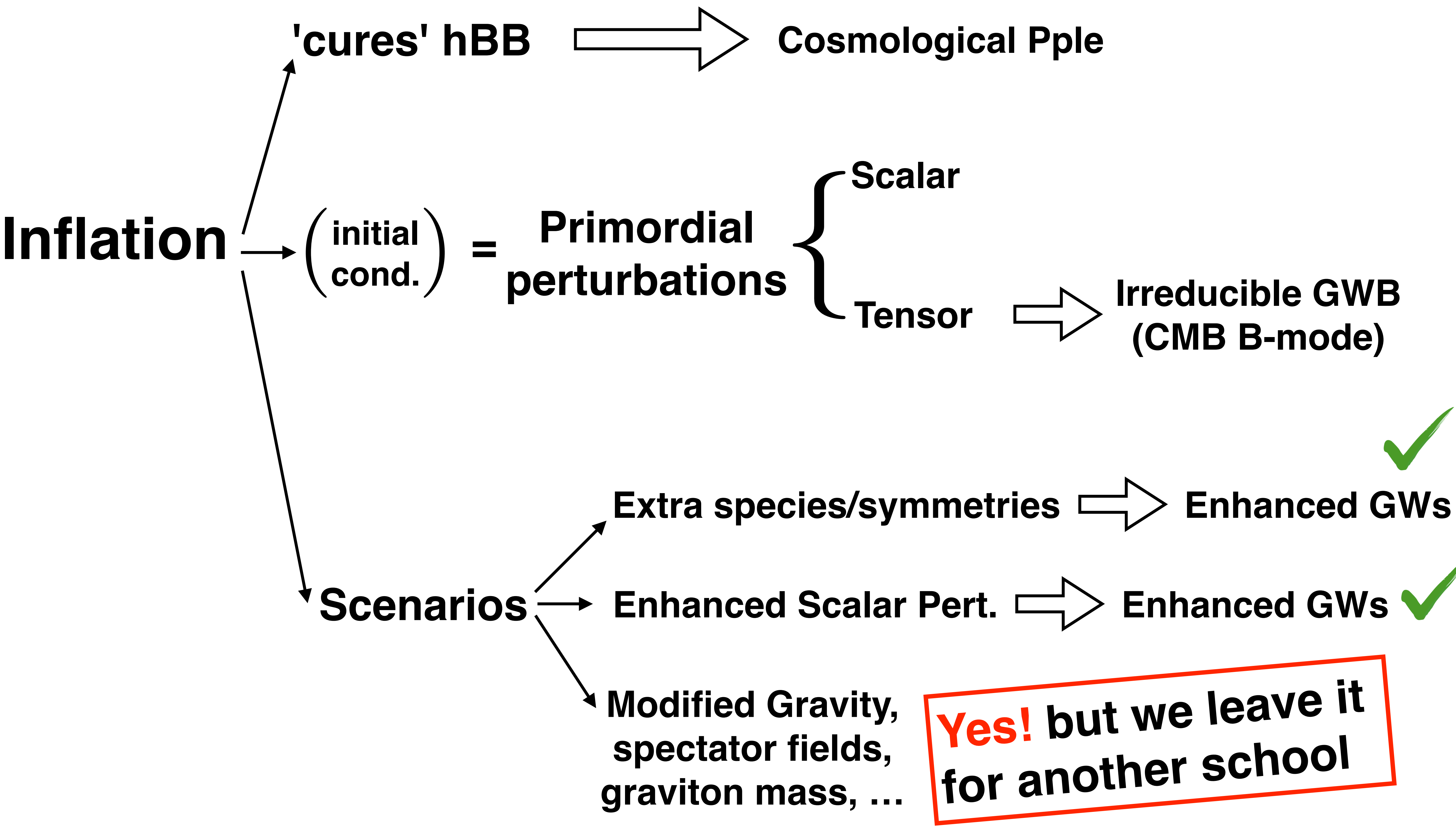
INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY

Inflation

'cures' hBB → Cosmological Pple

(initial cond.) = Primordial perturbations

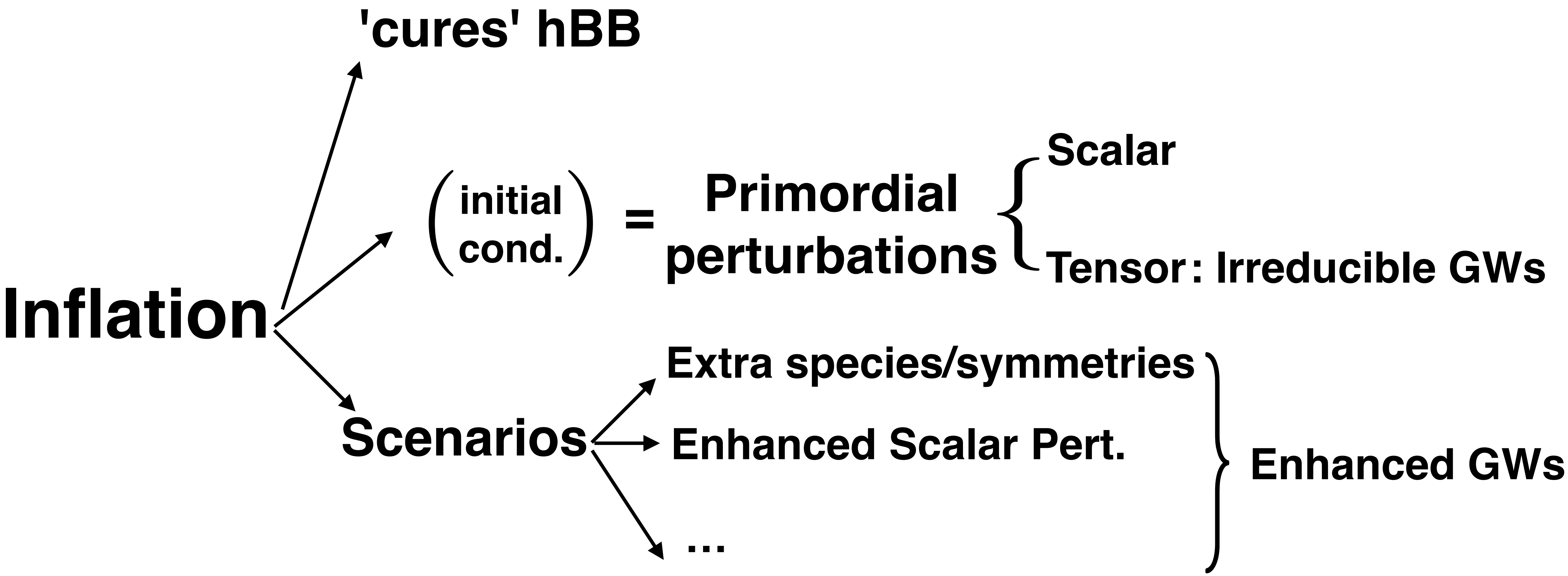
- Scalar
- Tensor → Irreducible GWB (CMB B-mode)

Scenarios

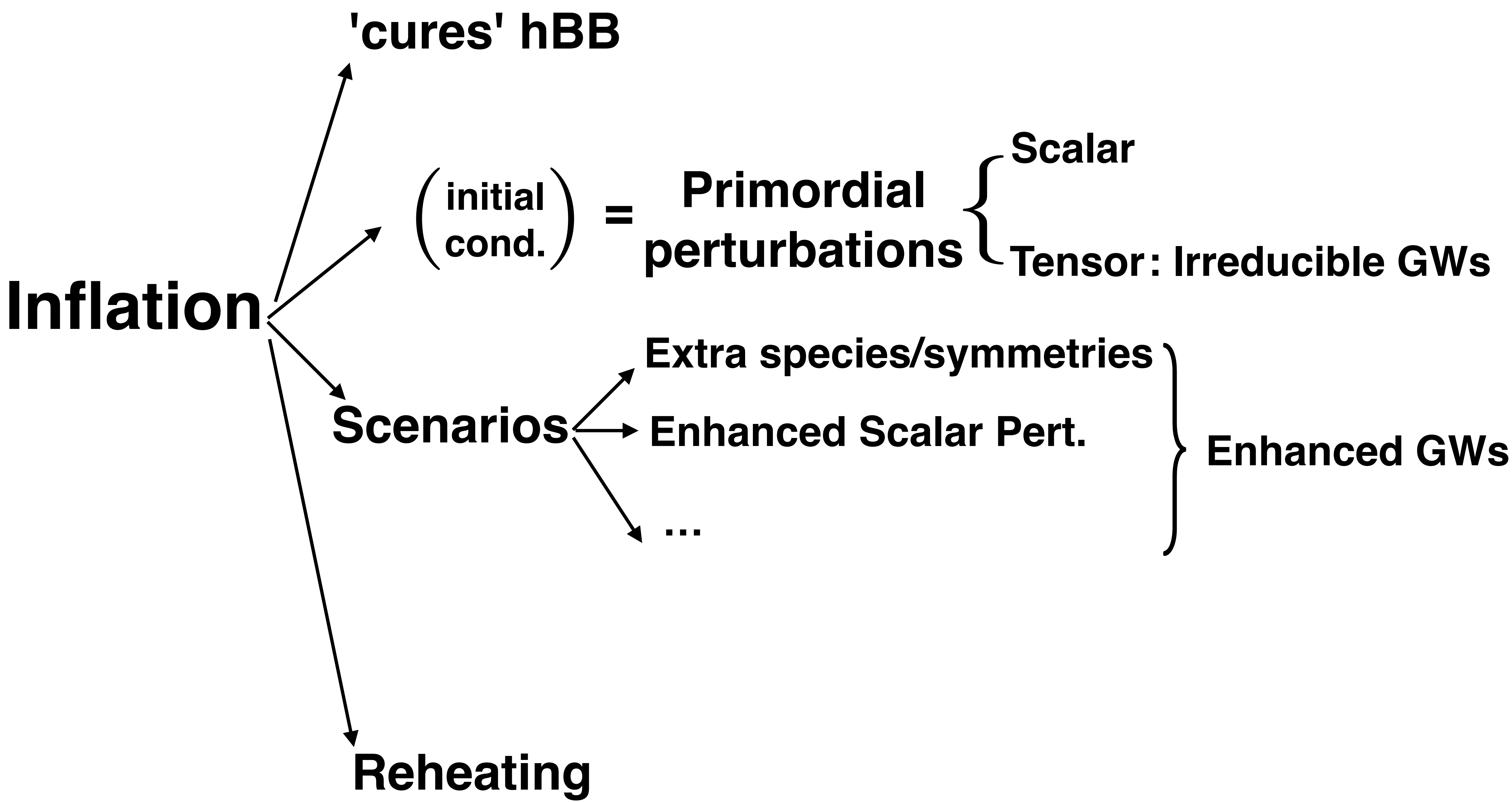
- Extra species/symmetries → Enhanced GWs ✓
- Enhanced Scalar Pert. → Enhanced GWs ✓
- Modified Gravity,
  spectat
  graviton

see GW review: 1801.04268

INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY

Inflation

- 'cures' hBB
- $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations $\begin{cases}\text{Scalar}\\ \text{Tensor: Irreducible GWs}\end{cases}$
- Scenarios
  - Extra species/symmetries
  - Enhanced Scalar Pert.
  - ...

  $\Big\}$ Enhanced GWs
- Reheating