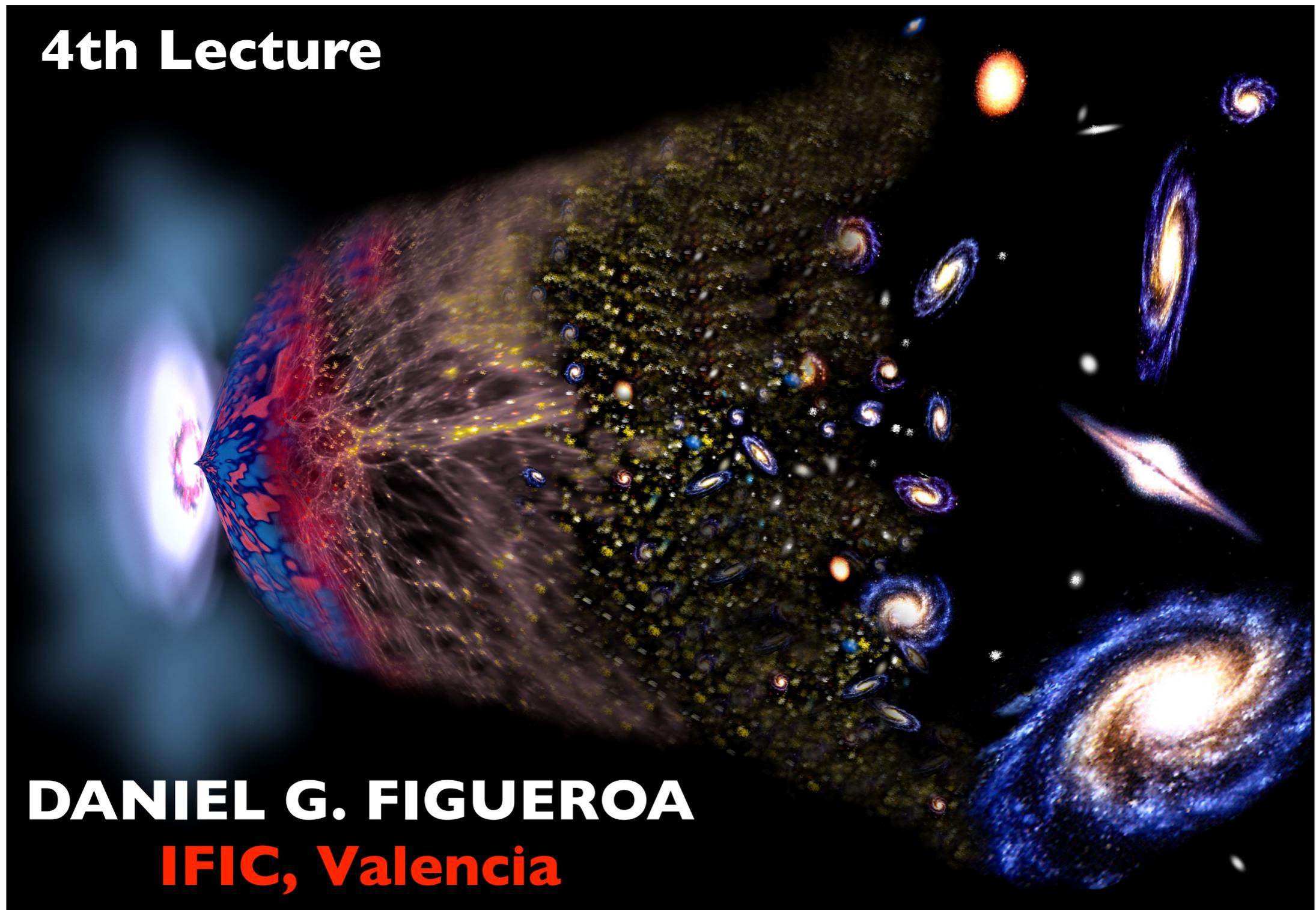


GRAVITATIONAL WAVE — BACKGROUNDS —

4th Lecture



DANIEL G. FIGUEROA
IFIC, Valencia

OUTLINE

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

2) GWs from Inflation ✓

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

OUTLINE

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

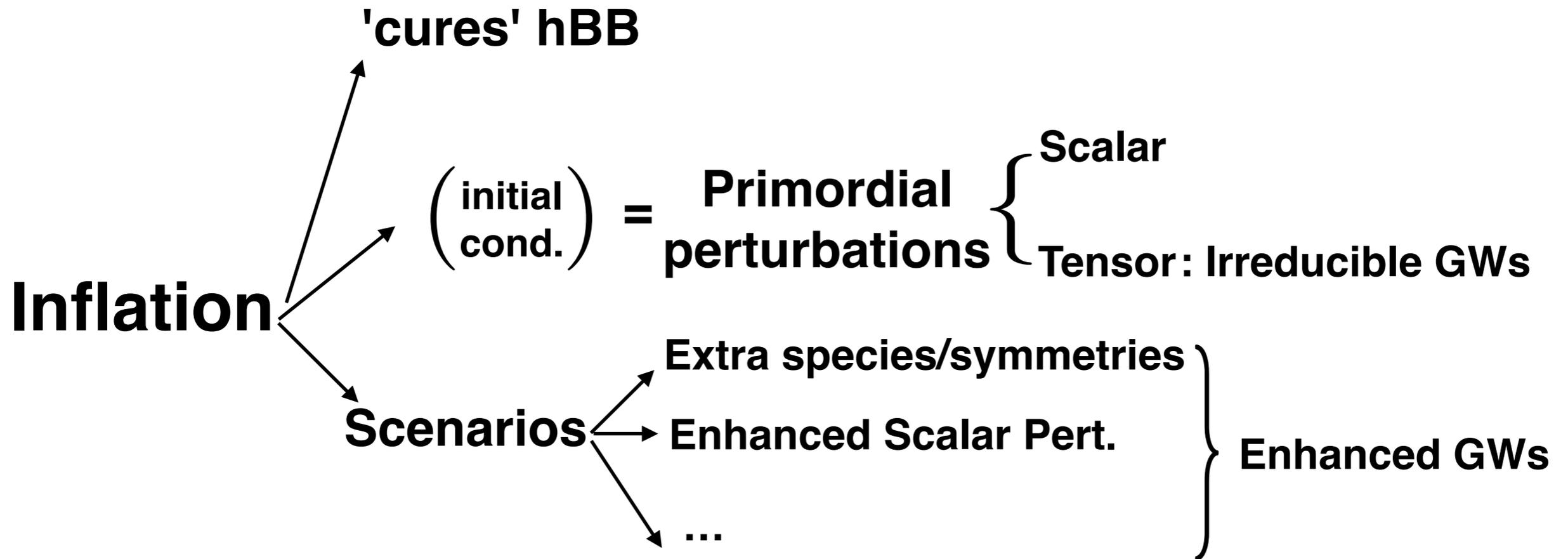
2) GWs from Inflation ✓

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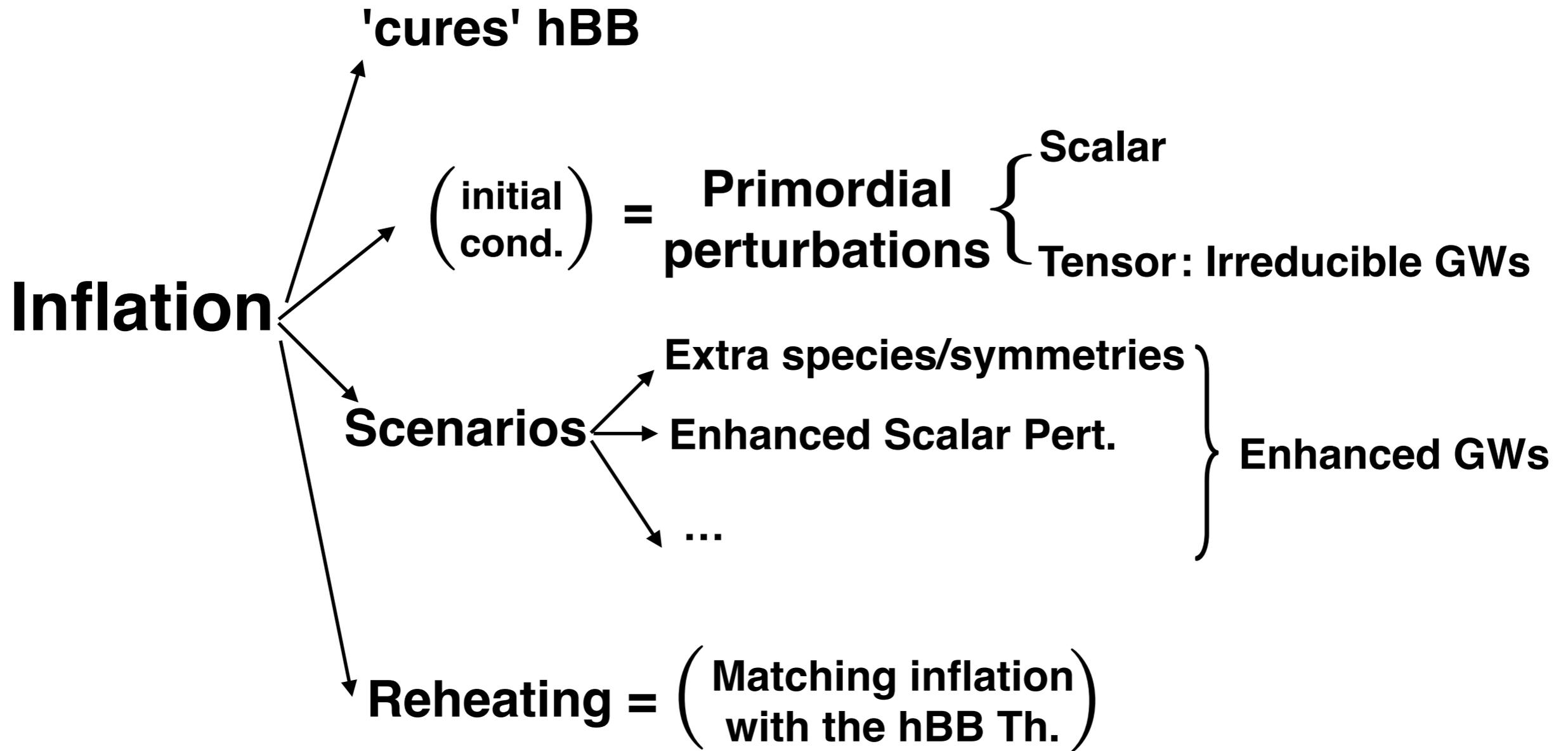
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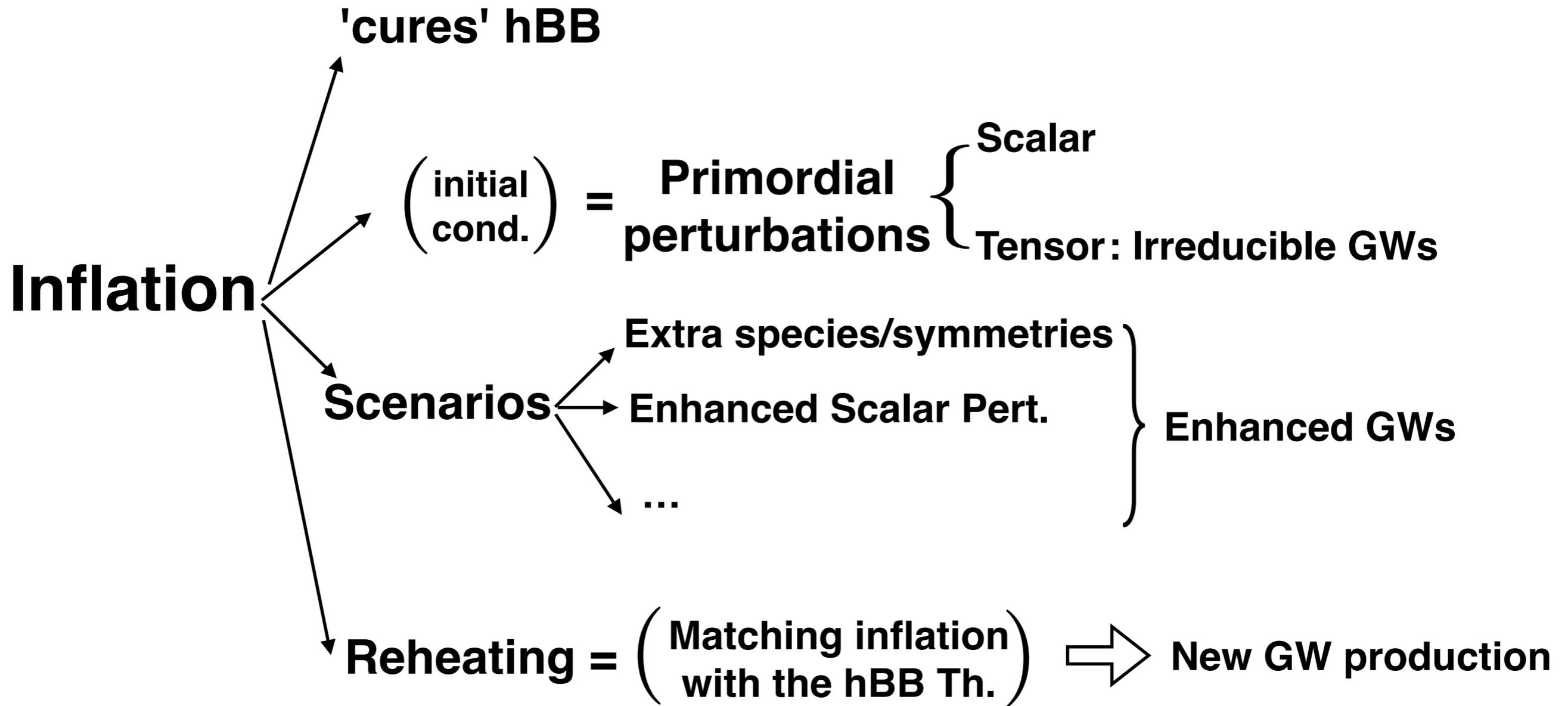
INFLATIONARY COSMOLOGY



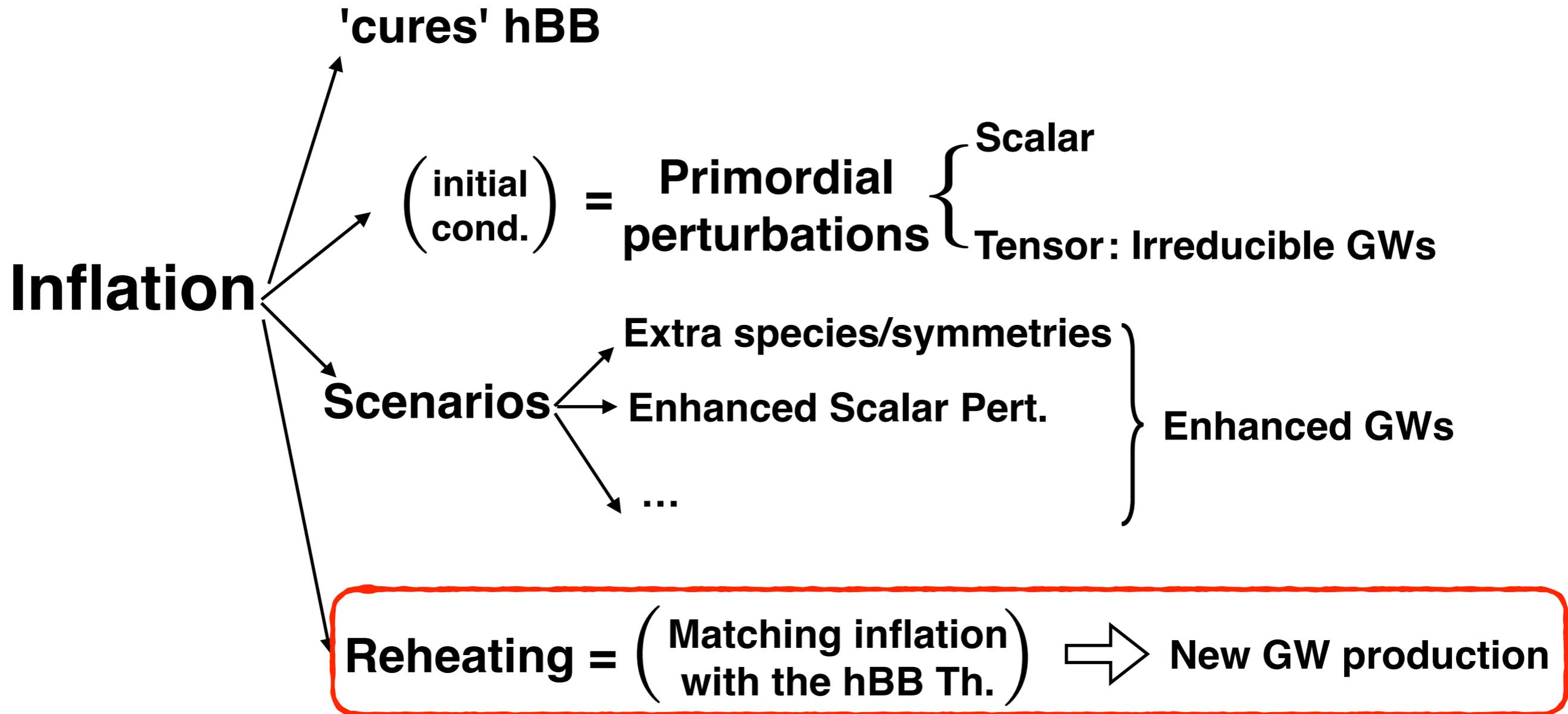
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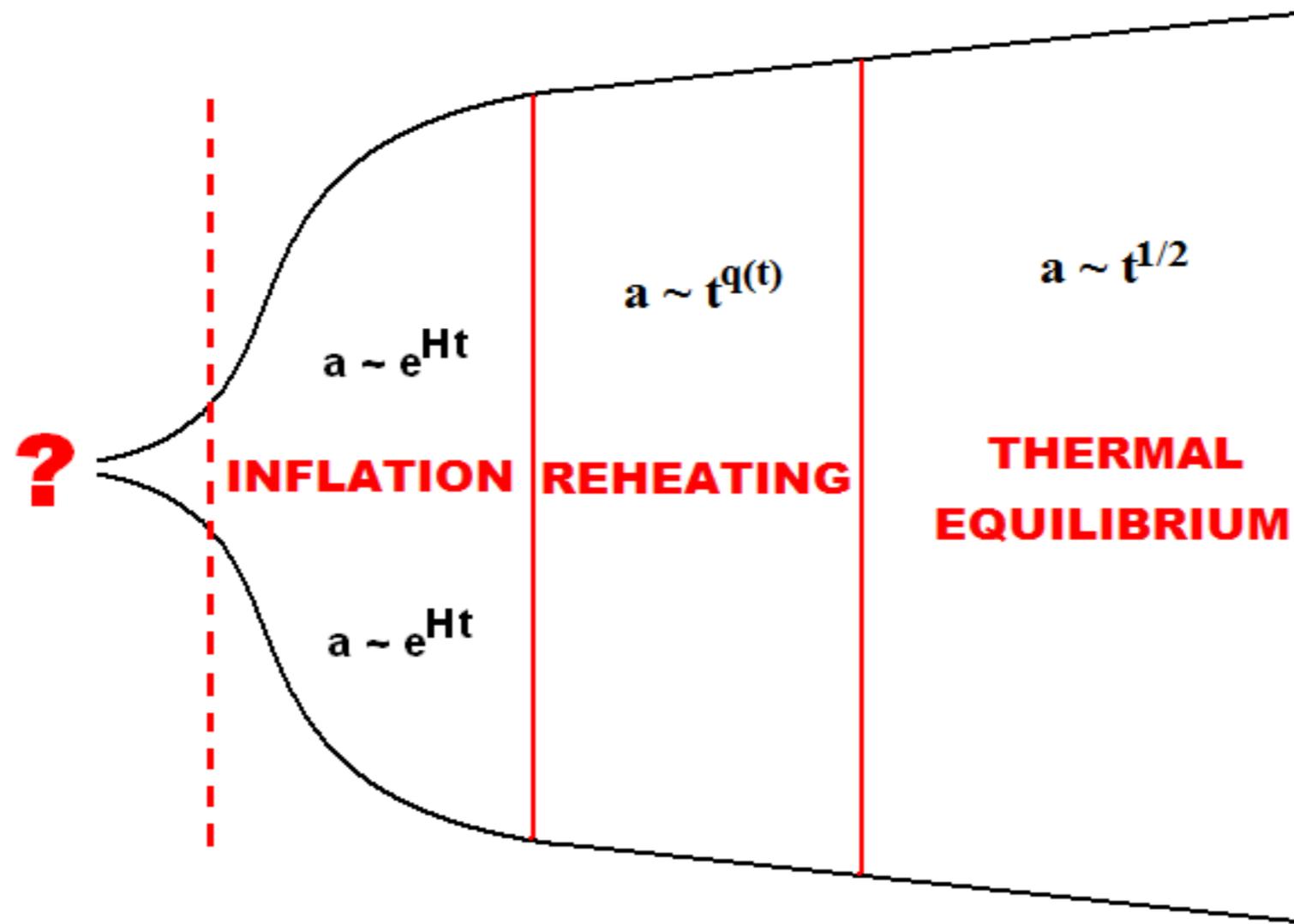


INFLATIONARY COSMOLOGY



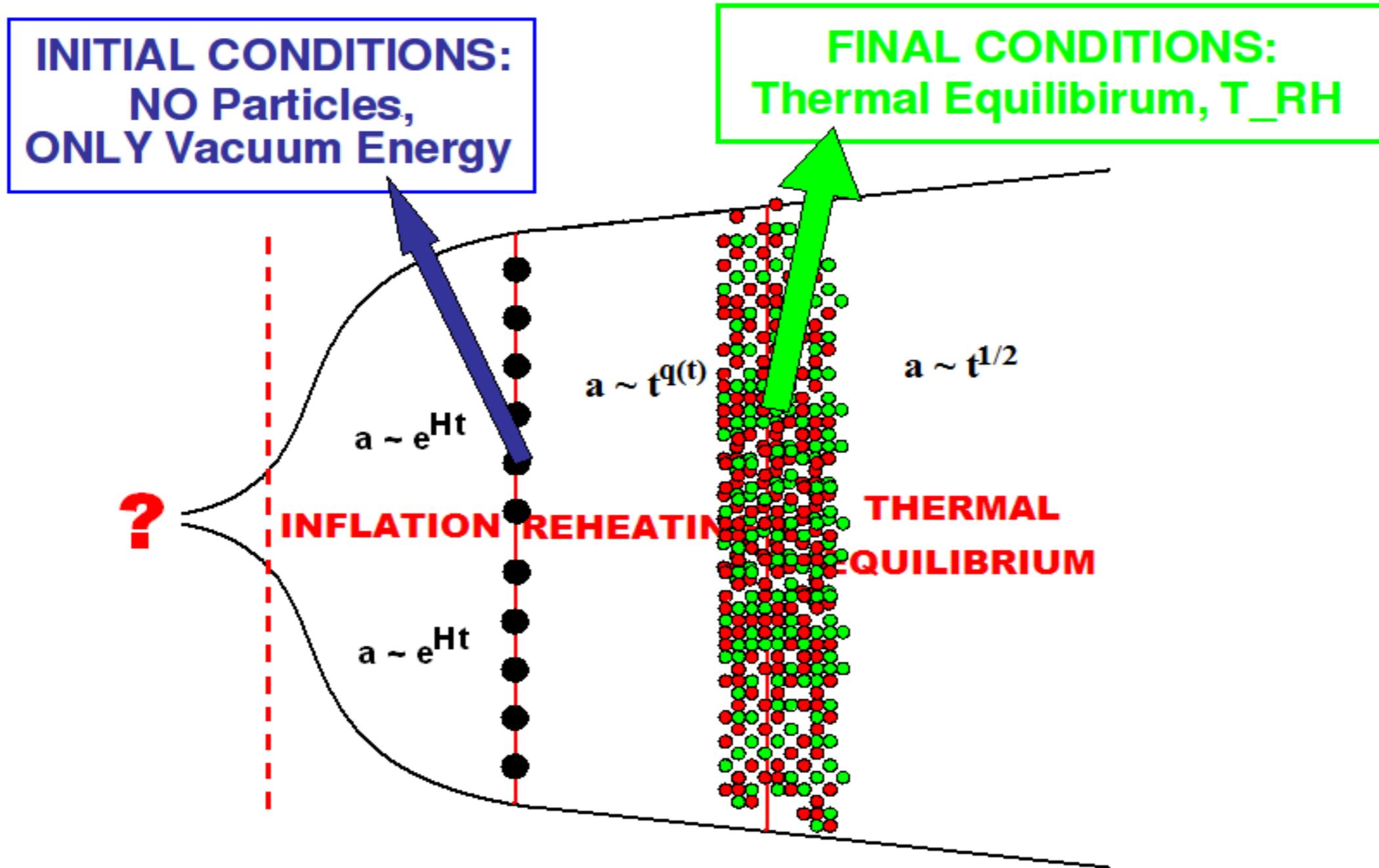
INFLATIONARY REHEATING

INFLATION → REHEATING → BIG BANG THEORY



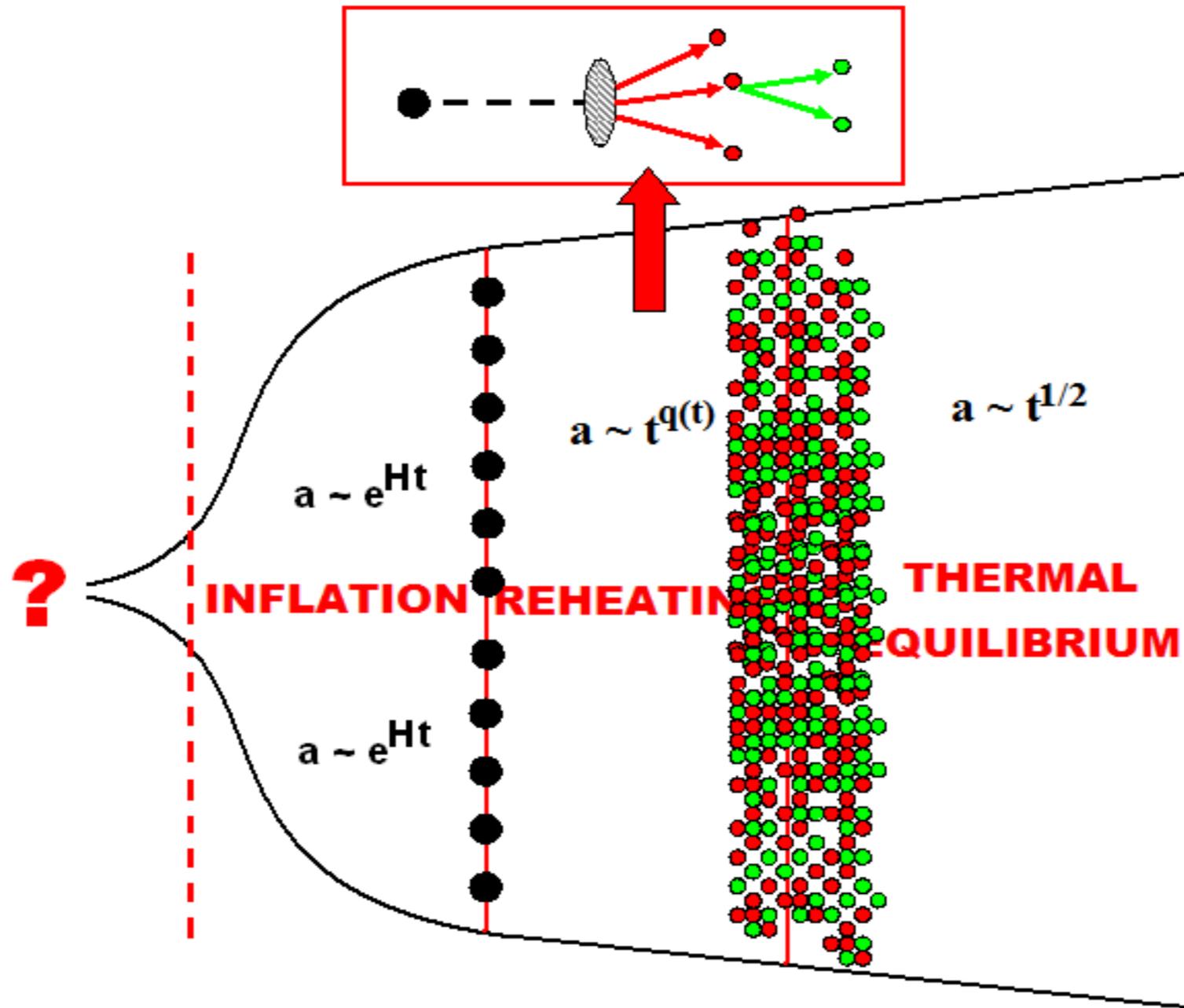
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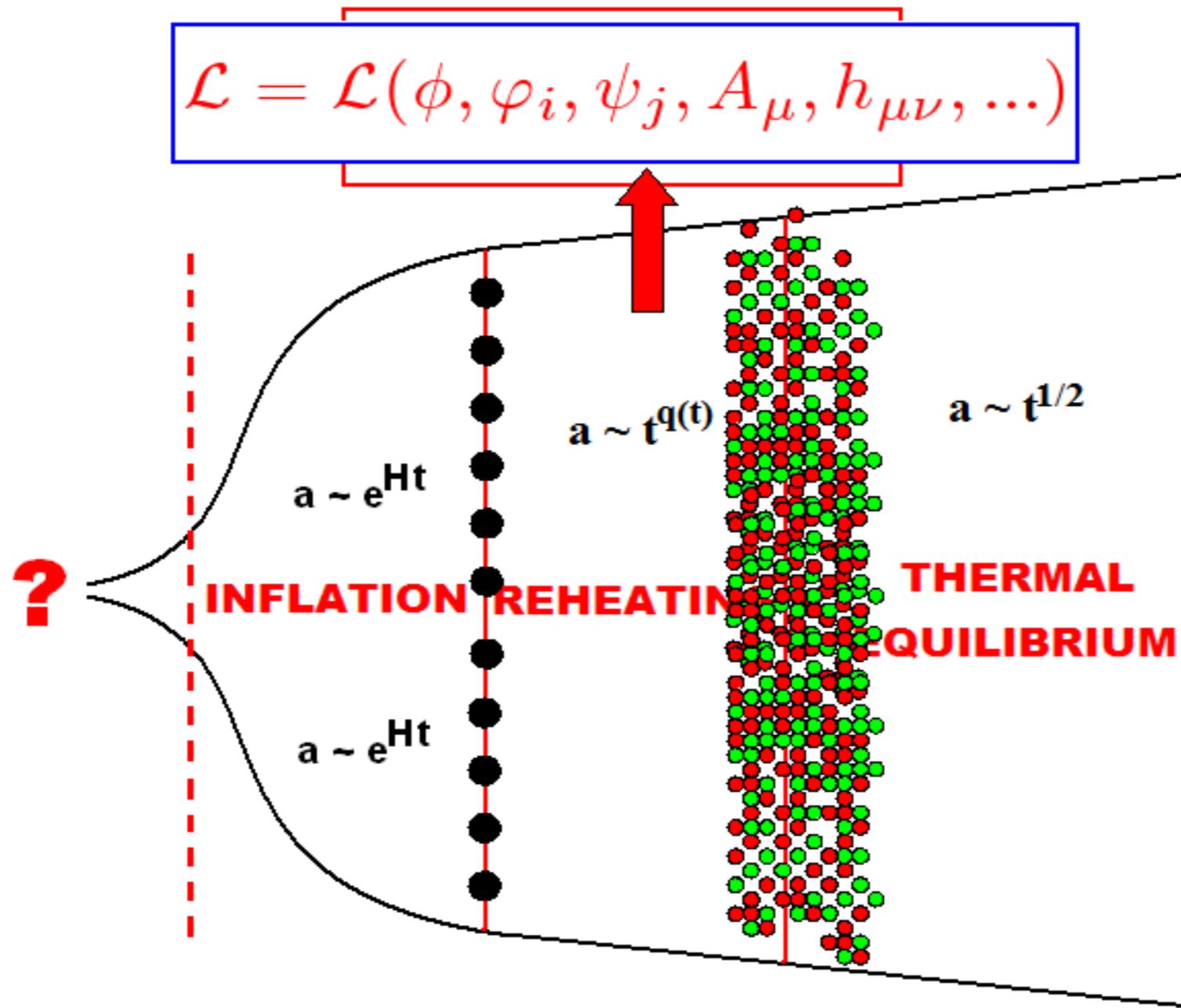
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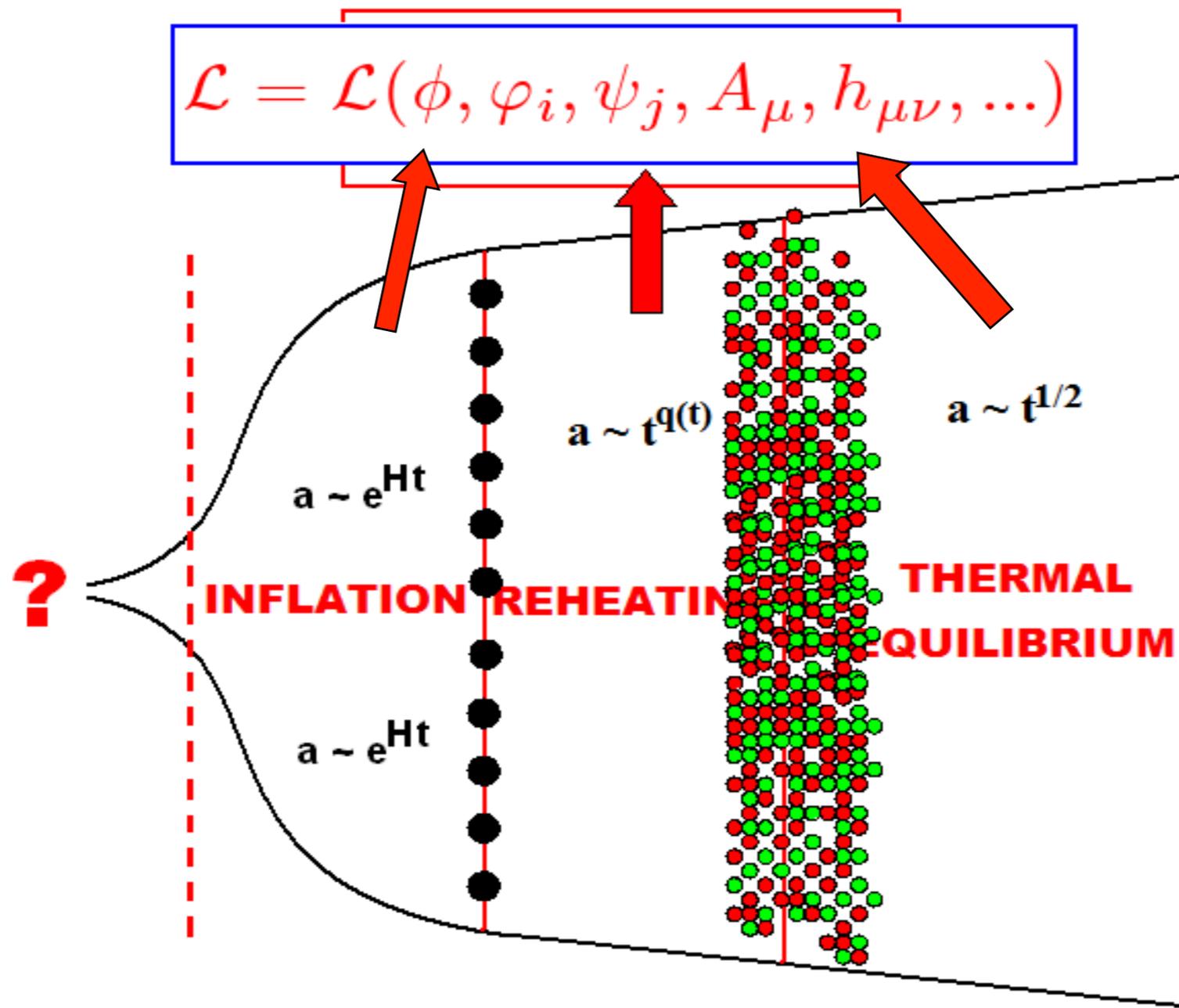
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INFLATIONARY REHEATING

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SCALAR REHEATING

1) $V(\phi, \chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2$ (Chaotic)

2) $V(\phi, \chi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\chi^2$ (Hybrid)

INFLATON **MATTER** **COUPLING**

SCALAR REHEATING

$$\begin{aligned} 1) \quad V(\phi, \chi) &= \frac{1}{4}\lambda\phi^4 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2 \quad (\text{Chaotic}) \\ 2) \quad V(\phi, \chi) &= \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\chi^2 \quad (\text{Hybrid}) \end{aligned}$$

INFLATON **MATTER** **COUPLING**

↑
(Ruled out for inflation,)
Not for reheating !

SCALAR REHEATING

$$\begin{aligned} 1) \quad V(\phi, \chi) &= \frac{1}{4}\lambda\phi^4 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2 \quad (\text{Chaotic}) \\ 2) \quad V(\phi, \chi) &= \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\chi^2 \quad (\text{Hybrid}) \end{aligned}$$

INFLATON **MATTER** **COUPLING**

$$\left\{ \begin{aligned} \ddot{\phi}(t) + 3H\dot{\phi} + V'(\phi) &= 0 \quad (\text{Inflaton Zero-Mode : Damped Oscillator}) \\ \square\phi_k + F(\int dq\phi_q\chi_{|k-q|})\phi_k + \dots &= 0 \quad (\text{Inflaton Fluctuations}) \\ \square\chi_k + F(\int dq\chi_q, \phi_{|k-q|})\chi_k + \dots &= 0 \quad (\text{Matter Fluctuations}) \end{aligned} \right.$$

SCALAR REHEATING

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INFLATON
MATTER
COUPLING

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DYNAMICS:

Non-Linear, Non-Perturbative & Far-From-Equilibrium

$$\mathbf{k}_i \pm \Delta\mathbf{k}_i \rightarrow \varphi_k(t), n_k(t) \sim \exp\{\mu_k t\}$$

SCALAR (P)REHEATING

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DYNAMICS:

Non-Linear, Non-Perturbative & Far-From-Equilibrium

$\mathbf{k}_i \pm \Delta\mathbf{k}_i \rightarrow \varphi_k(t), n_k(t) \sim \exp\{\mu_k t\} \rightarrow$ PREHEATING

SCALAR (P)REHEATING

$$1) \quad V(\phi, \chi) = \frac{\lambda}{n} \phi^n + \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} g^2 \phi^2 \chi^2 \quad (\text{monomial})$$

SCALAR (P)REHEATING

$$1) \quad V(\phi, \chi) = \frac{\lambda}{n} \phi^n + \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} g^2 \phi^2 \chi^2 \quad (\text{Chaotic})$$

SCALAR (P)REHEATING

- 1) Chaotic Scenarios: PARAMETRIC RESONANCE

SCALAR (P)REHEATING

1) Chaotic Scenarios: PARAMETRIC RESONANCE

MATTER FIELD FLUCTUATIONS

$$\text{Massless : } X_k'' + (\kappa^2 + q \operatorname{cn}^2(z)) X_k = 0 \quad (\text{Lamé Eq.}) \quad q \equiv \frac{g^2}{\lambda}; \quad \kappa \equiv \frac{k}{\omega_*}; \quad z \equiv \omega_* t$$

(n = 4)

$$[X = a^{3/2} \chi]$$

SCALAR (P)REHEATING

1) Chaotic Scenarios: PARAMETRIC RESONANCE

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$$\text{Massive : } X_k'' + (A_k - 2q \cos(2z)) X_k = 0 \quad (\text{Mathieu Eq.}) \quad q \equiv \frac{g^2 \phi_*^2}{4\omega_*^2}; \quad \kappa \equiv \frac{k}{\omega_*}$$

(n = 2)

$z \equiv \omega_* t; \quad \omega_* \equiv m_\phi$

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SCALAR (P)REHEATING

1) Chaotic Scenarios: PARAMETRIC RESONANCE

MATTER FIELD FLUCTUATIONS

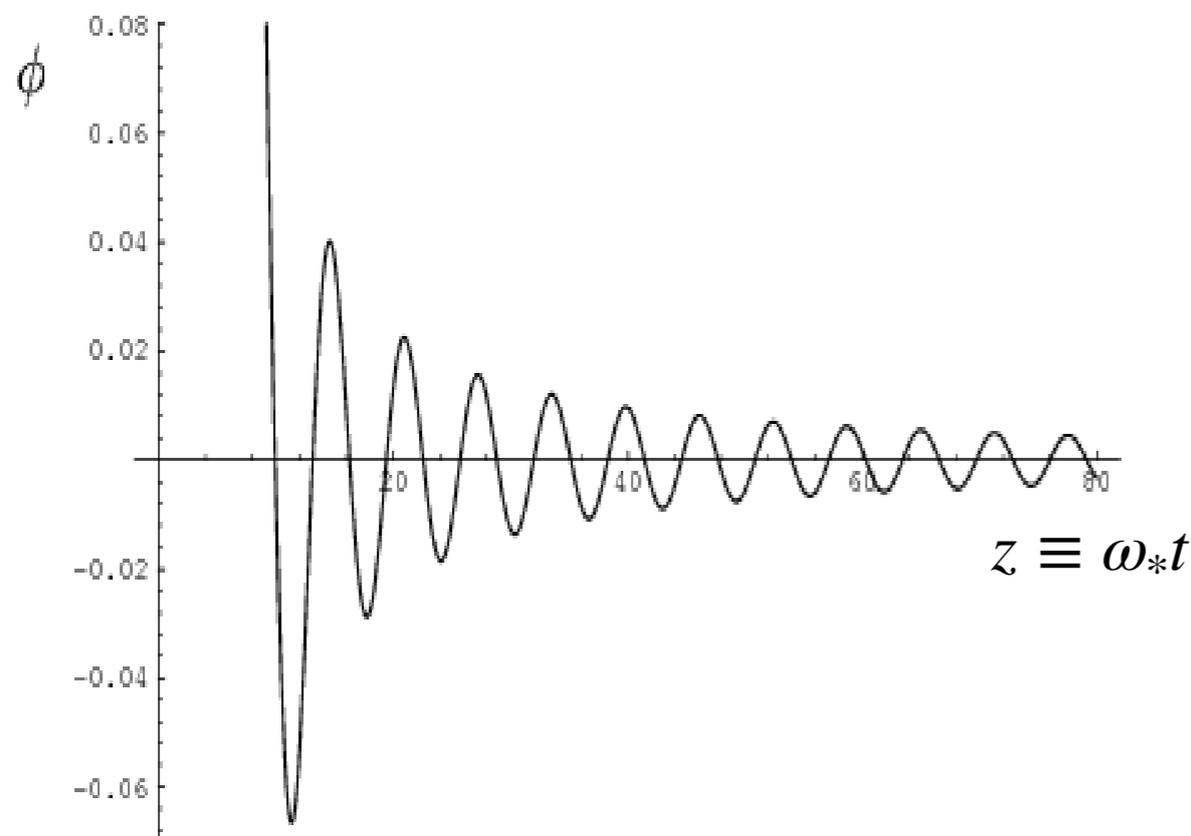
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SCALAR (P)REHEATING

1) Chaotic Scenarios: PARAMETRIC RESONANCE

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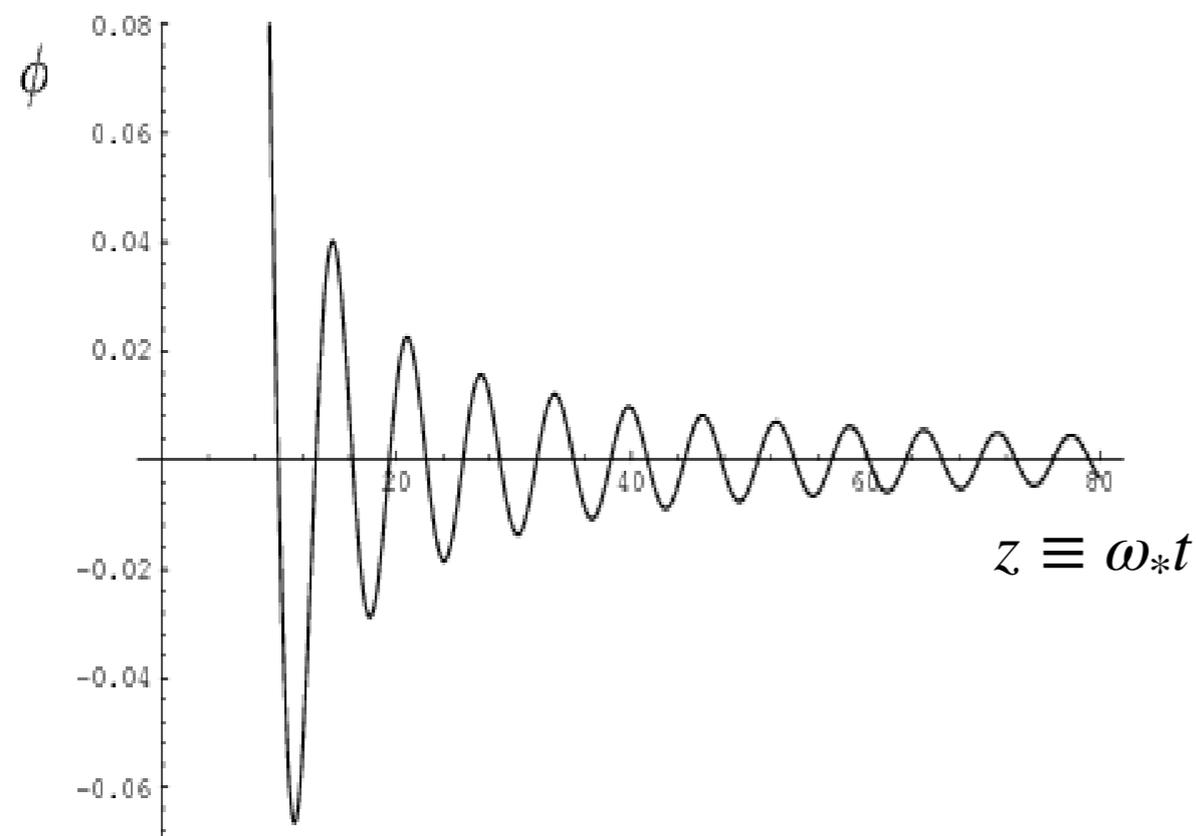
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(n = 2)

$$\left. \begin{array}{l} X_k \sim e^{\mu_k t} \\ n_k \sim e^{\mu_k t} \end{array} \right\}$$



SCALAR (P)REHEATING

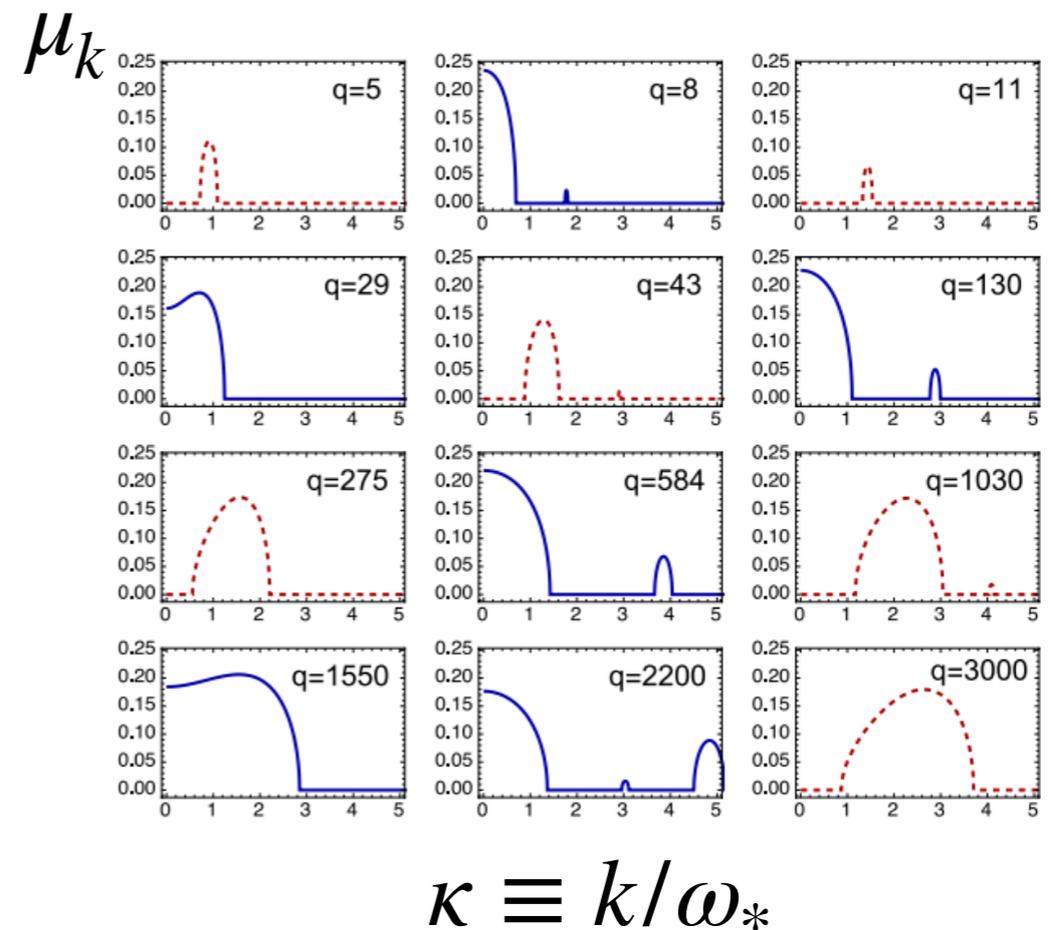
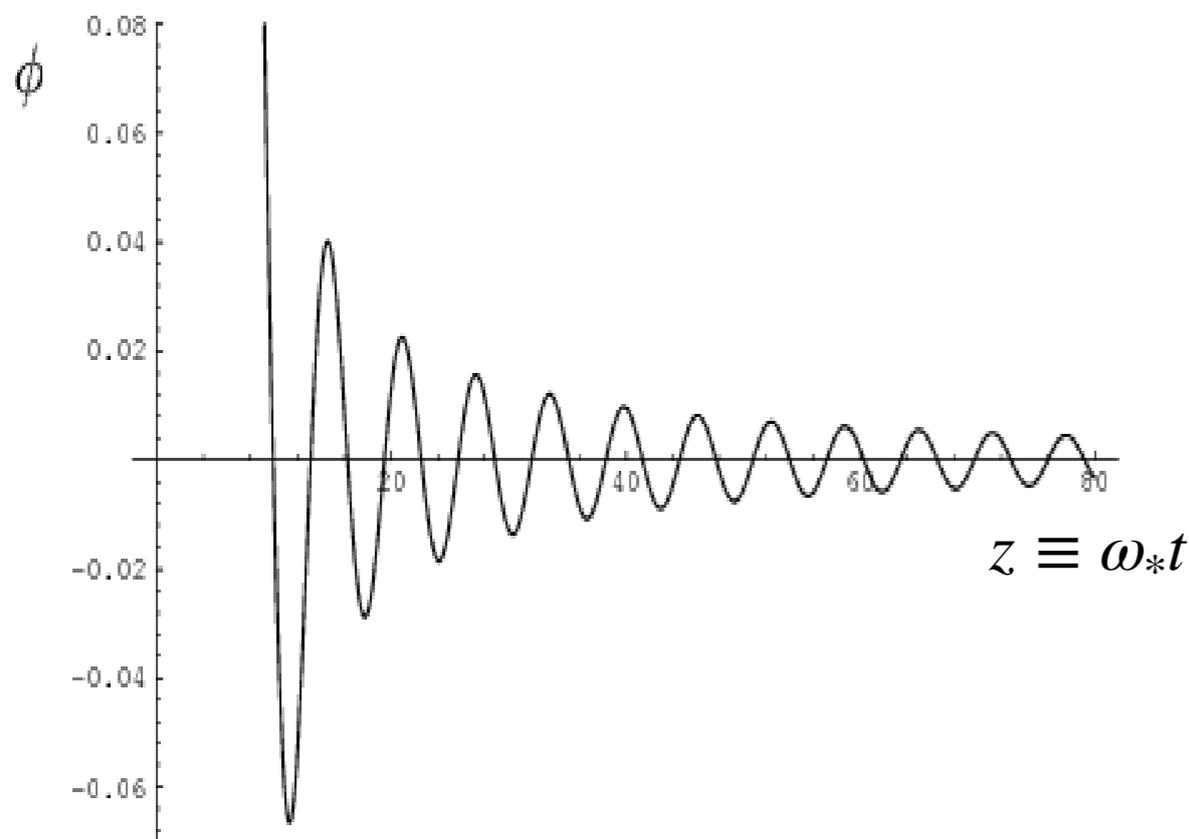
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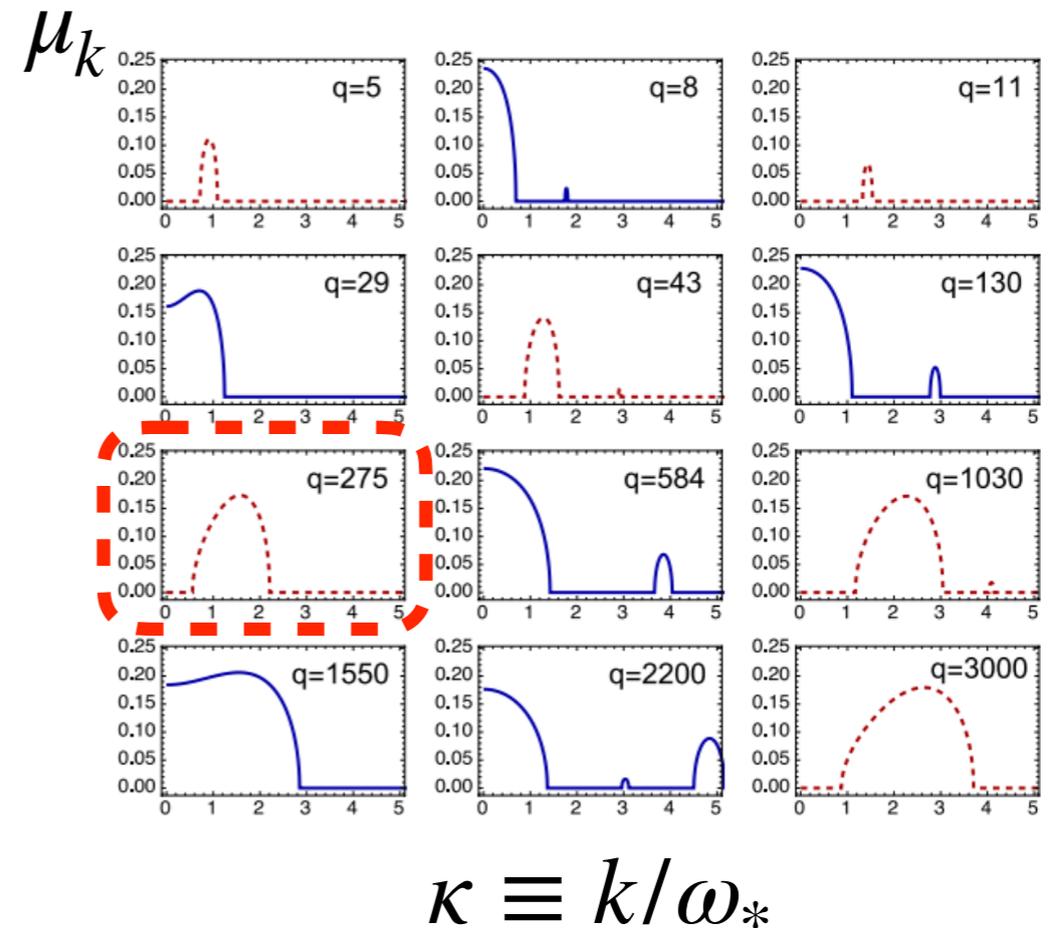
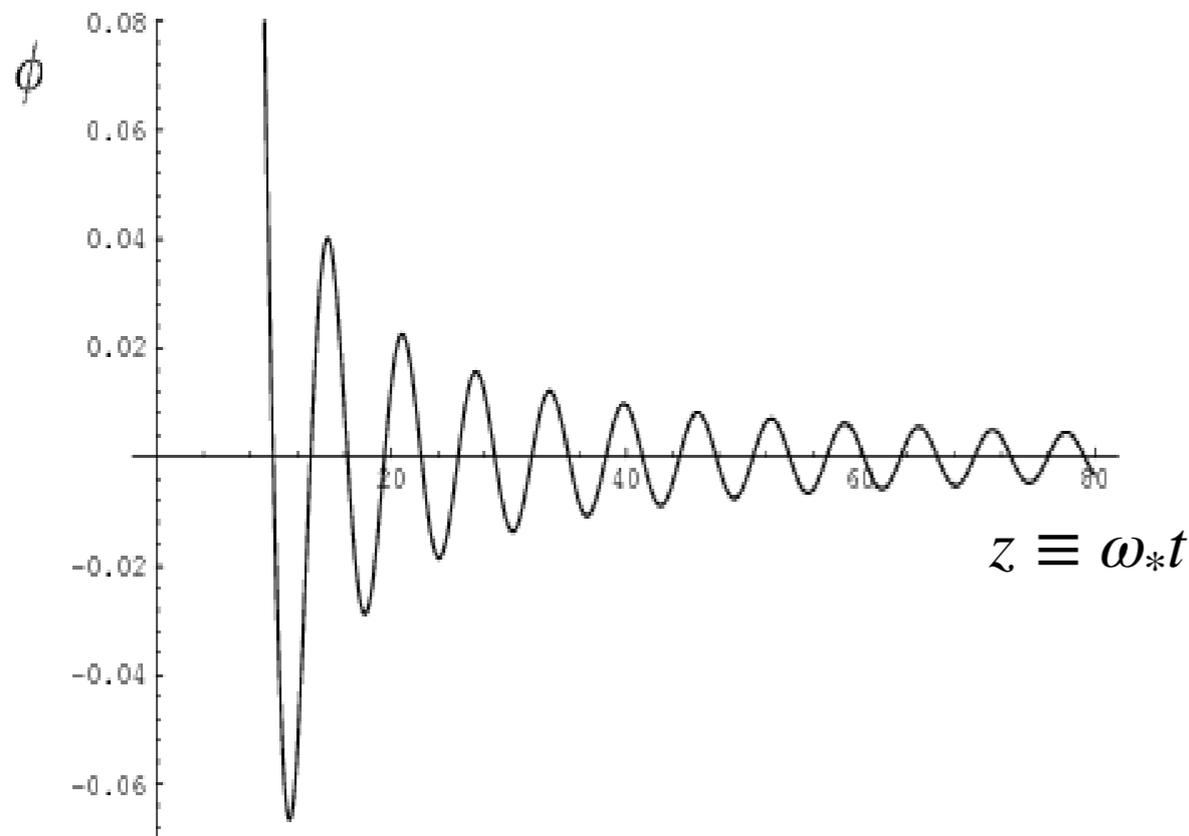
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SCALAR (P)REHEATING

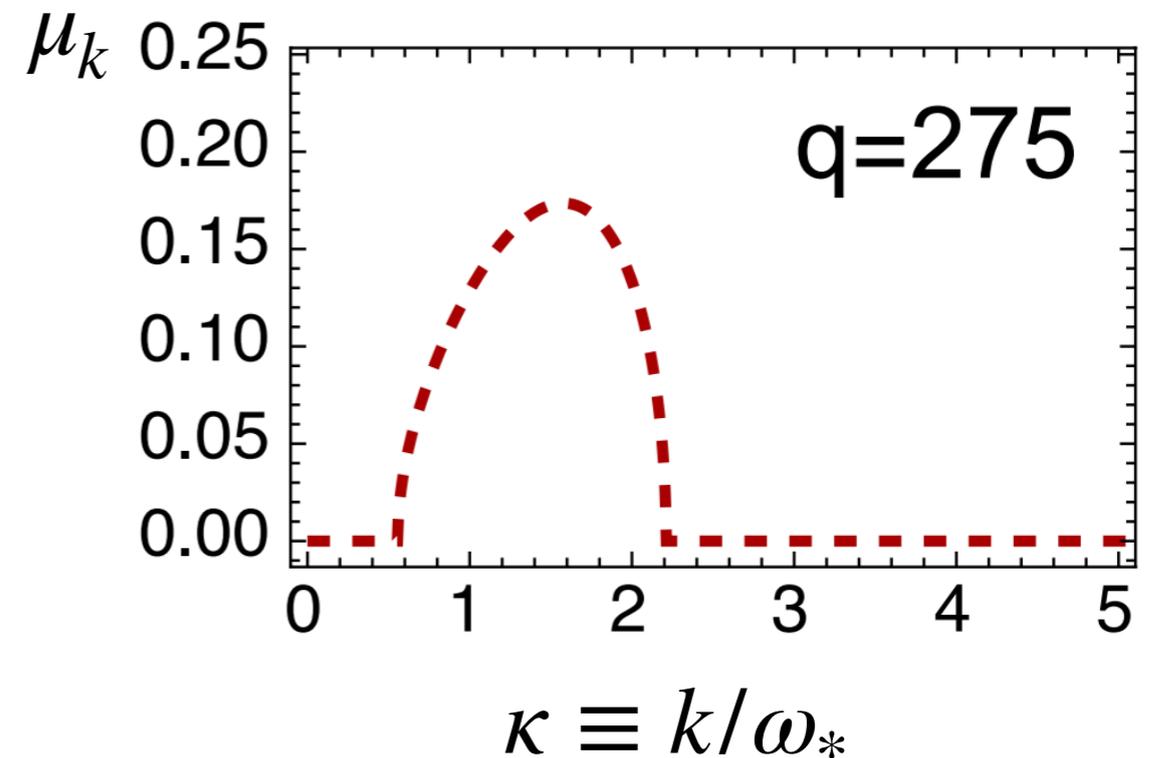
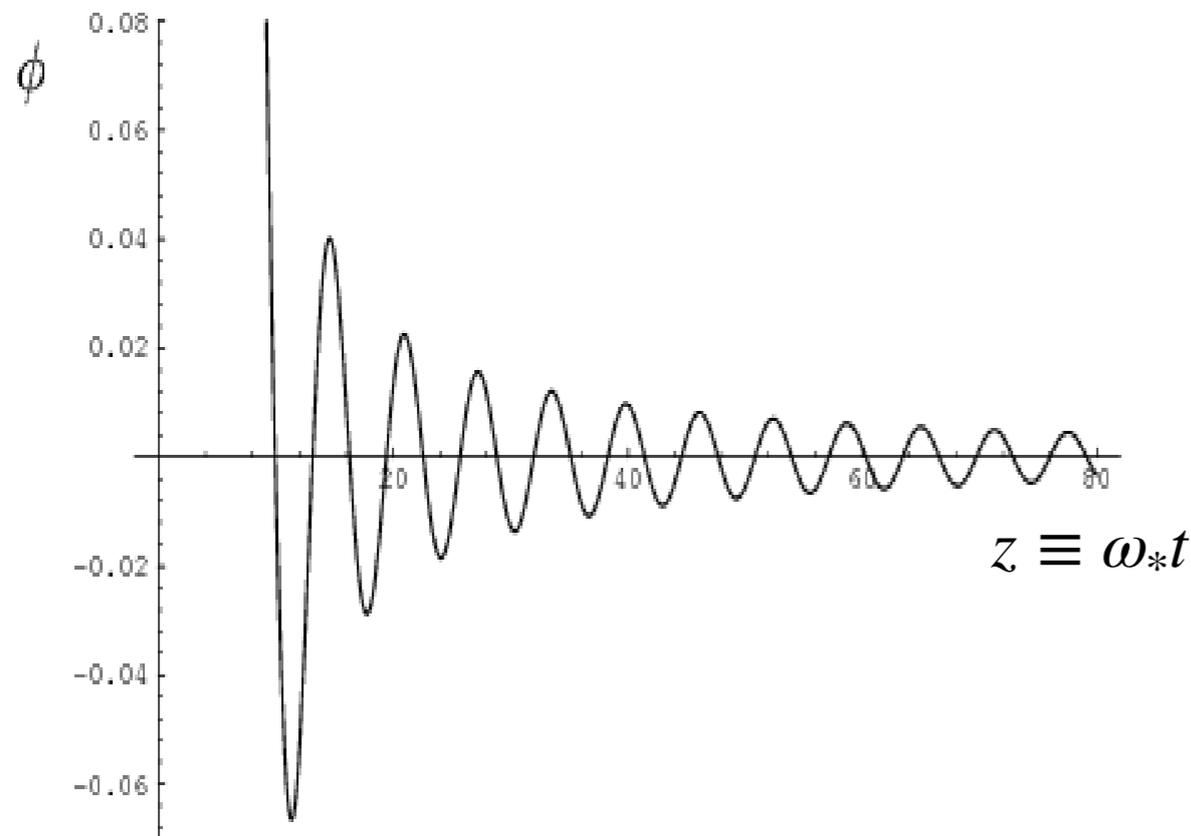
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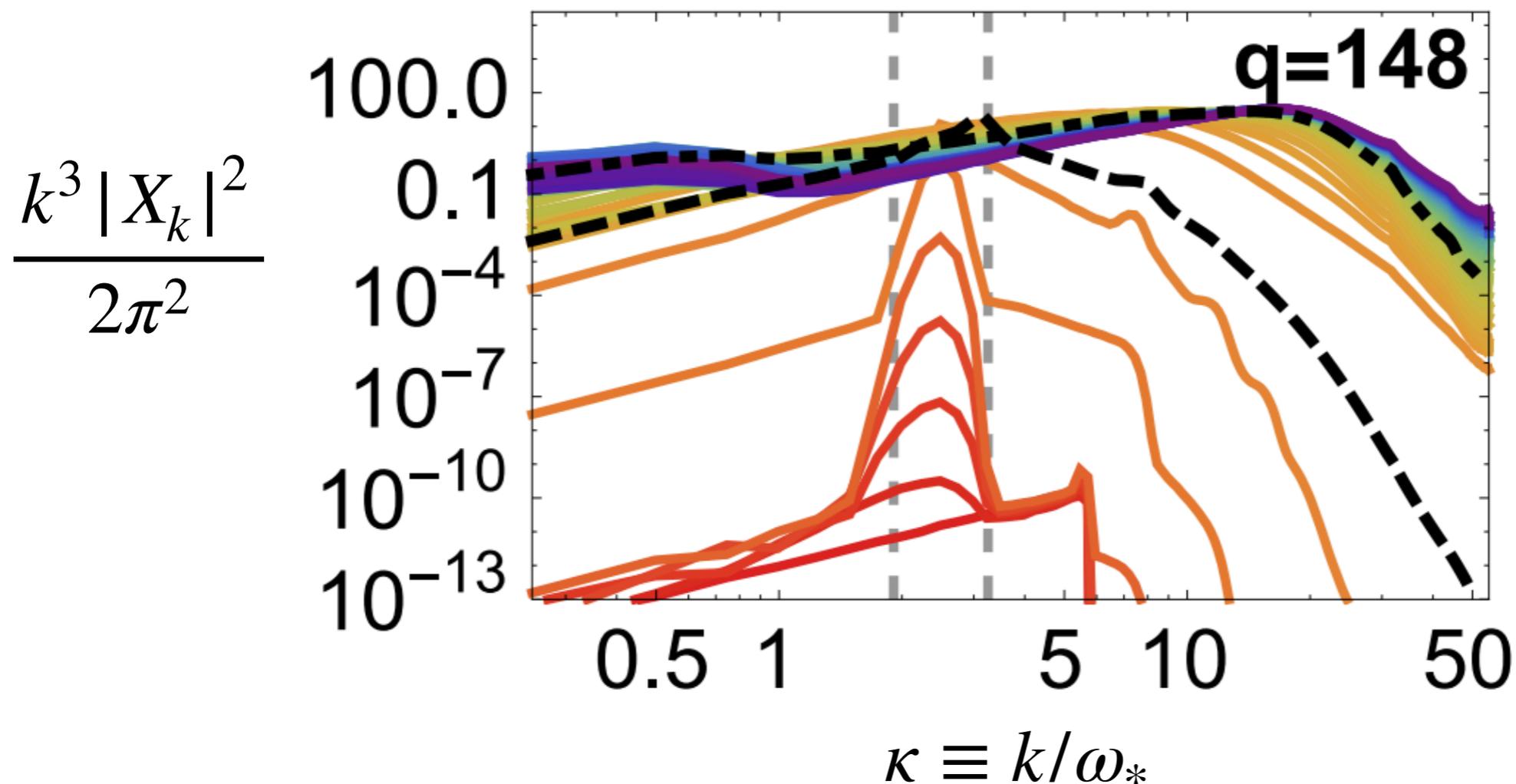
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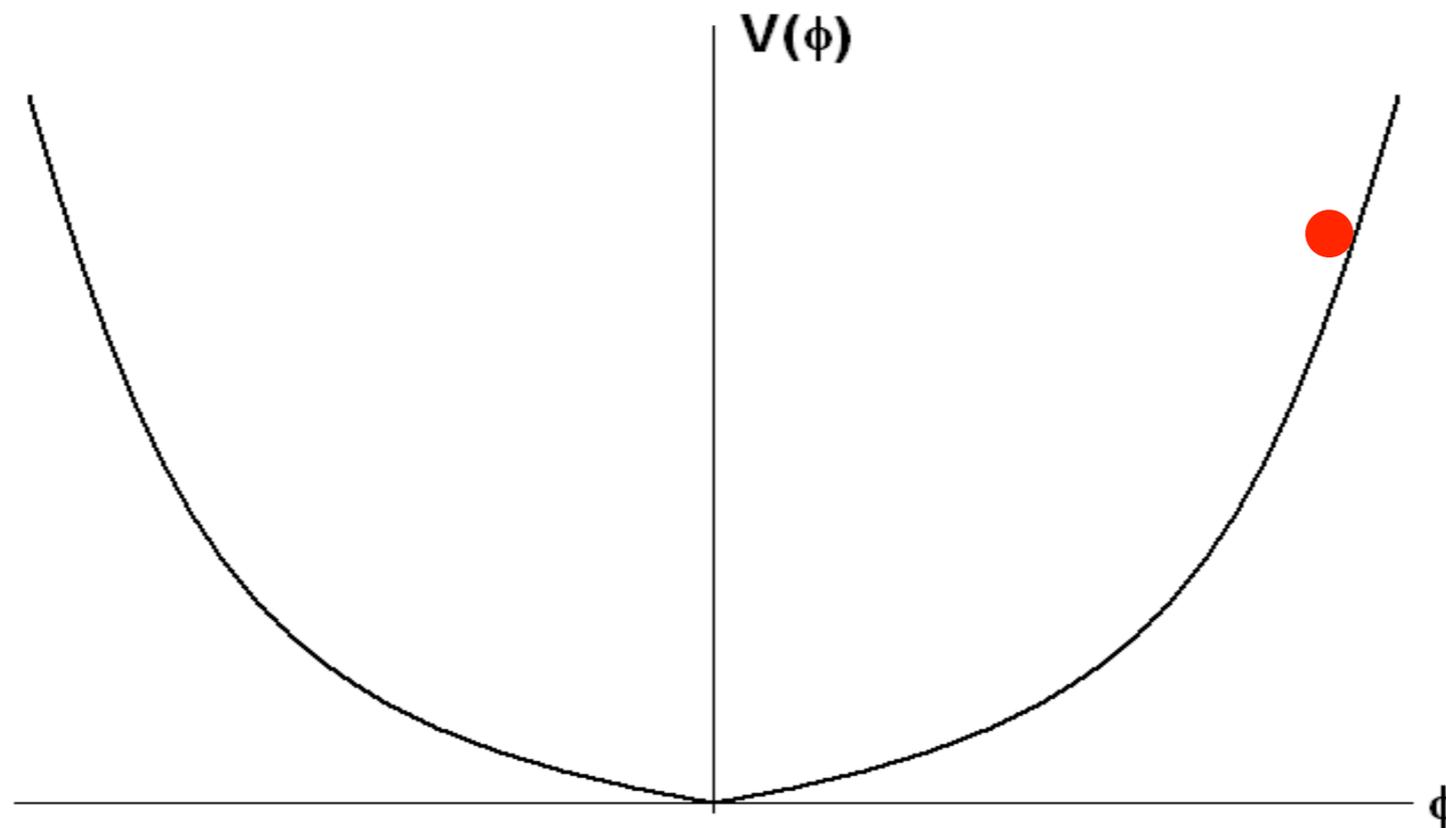


SCALAR (P)REHEATING

1) Chaotic Scenarios: PARAMETRIC RESONANCE

$$V(\phi, \chi) = V(\phi) + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2 \quad (\text{Chaotic Models})$$

$$X_k'' + [\kappa^2 + m^2(\phi)]X_k = 0 \quad (\text{Fluctuations of Matter})$$

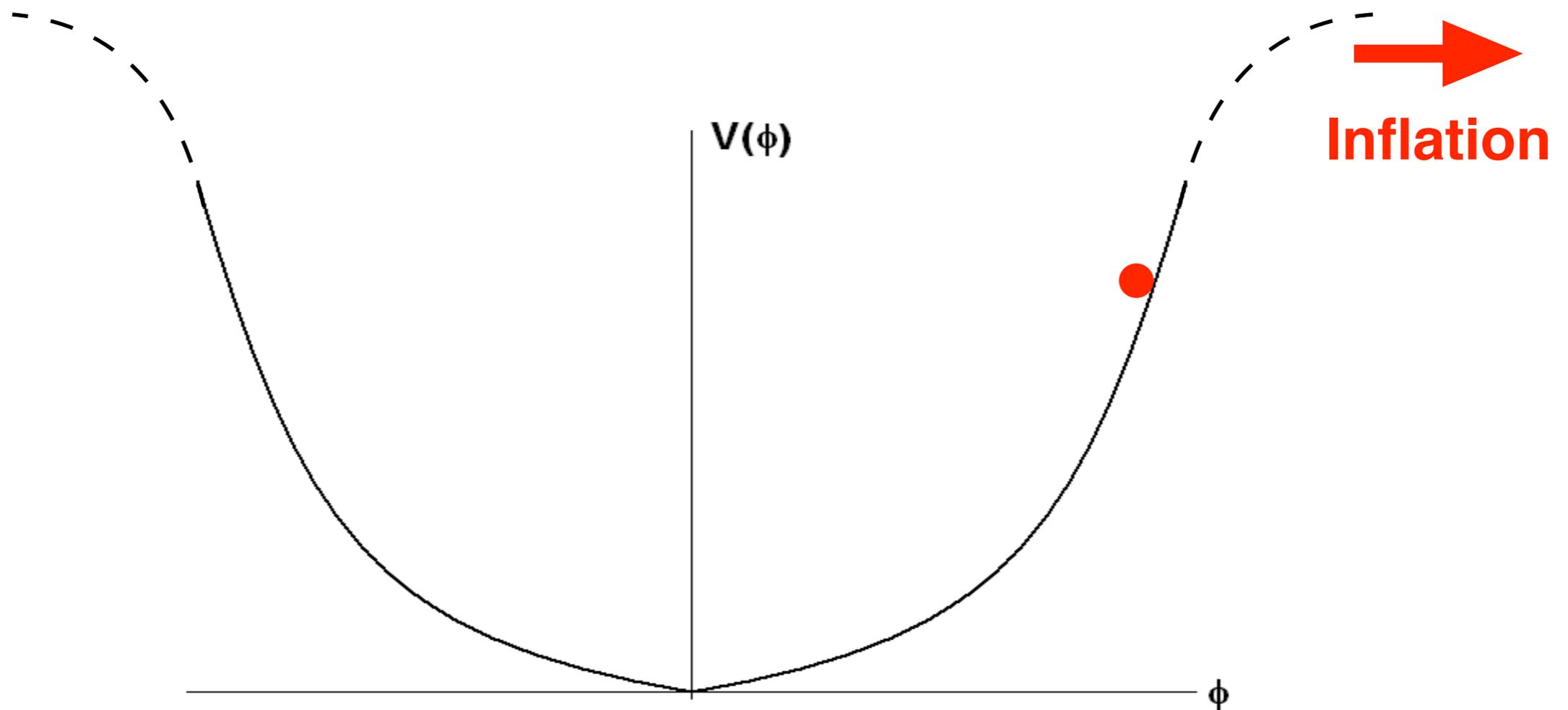


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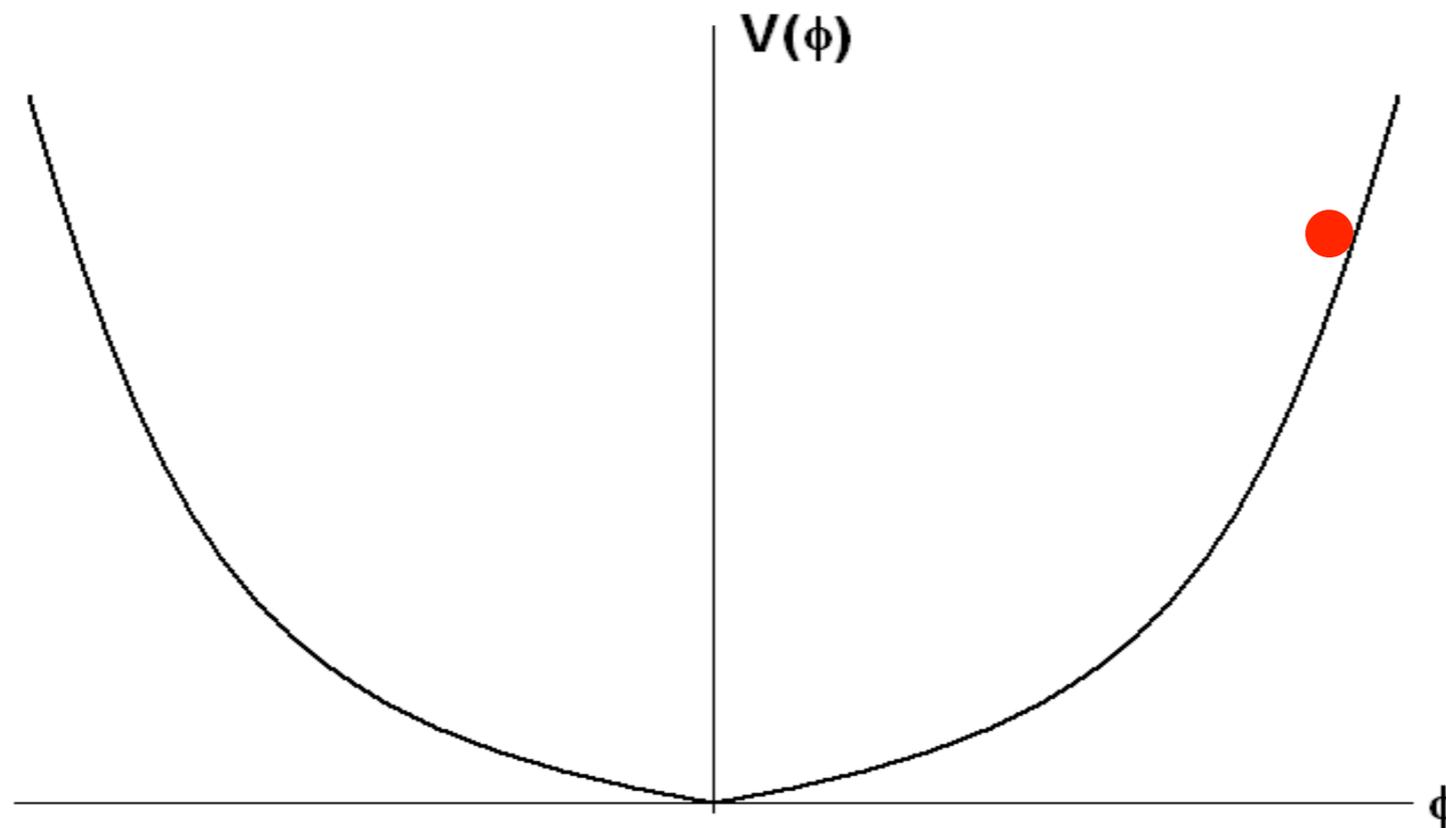


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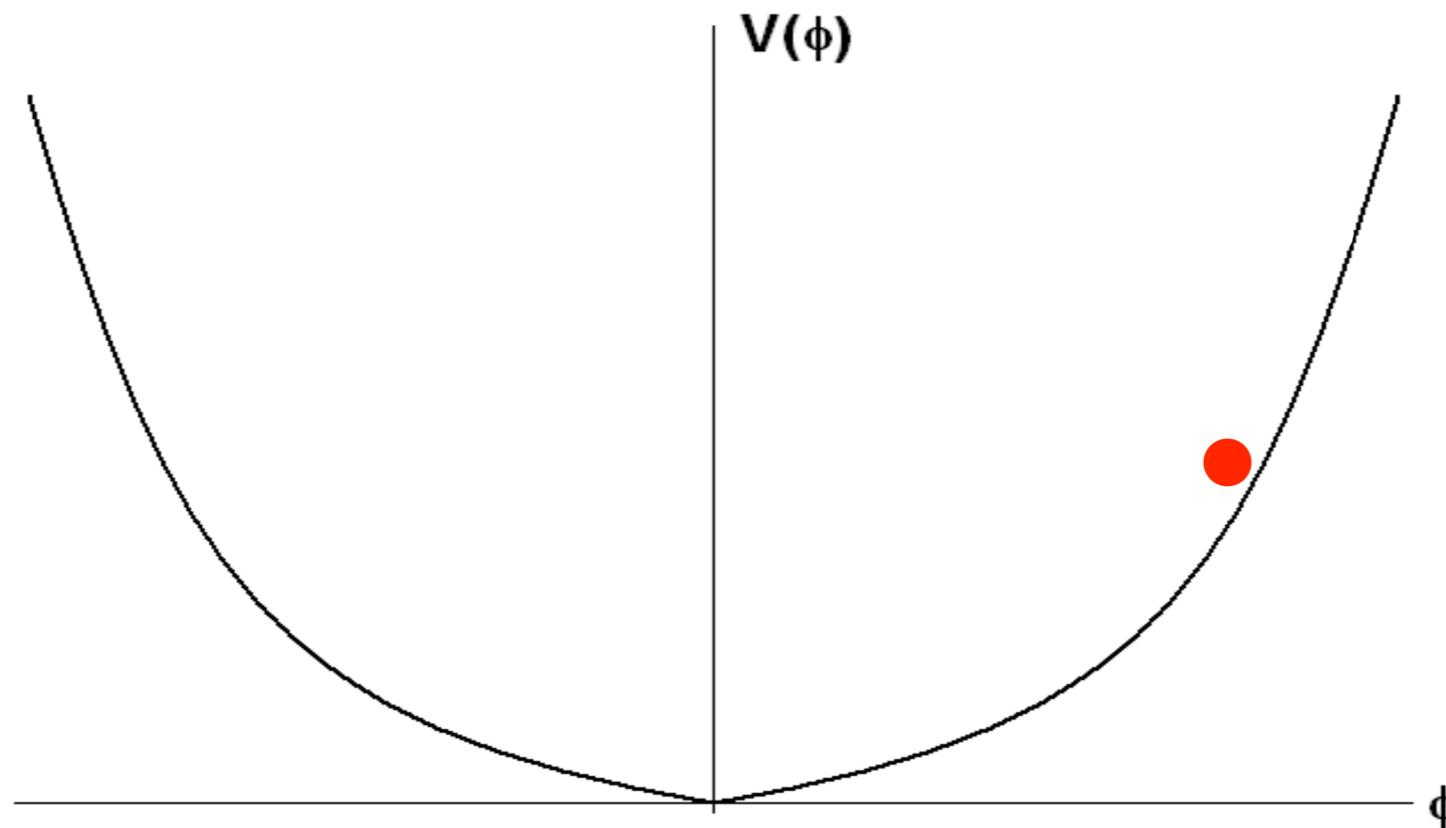


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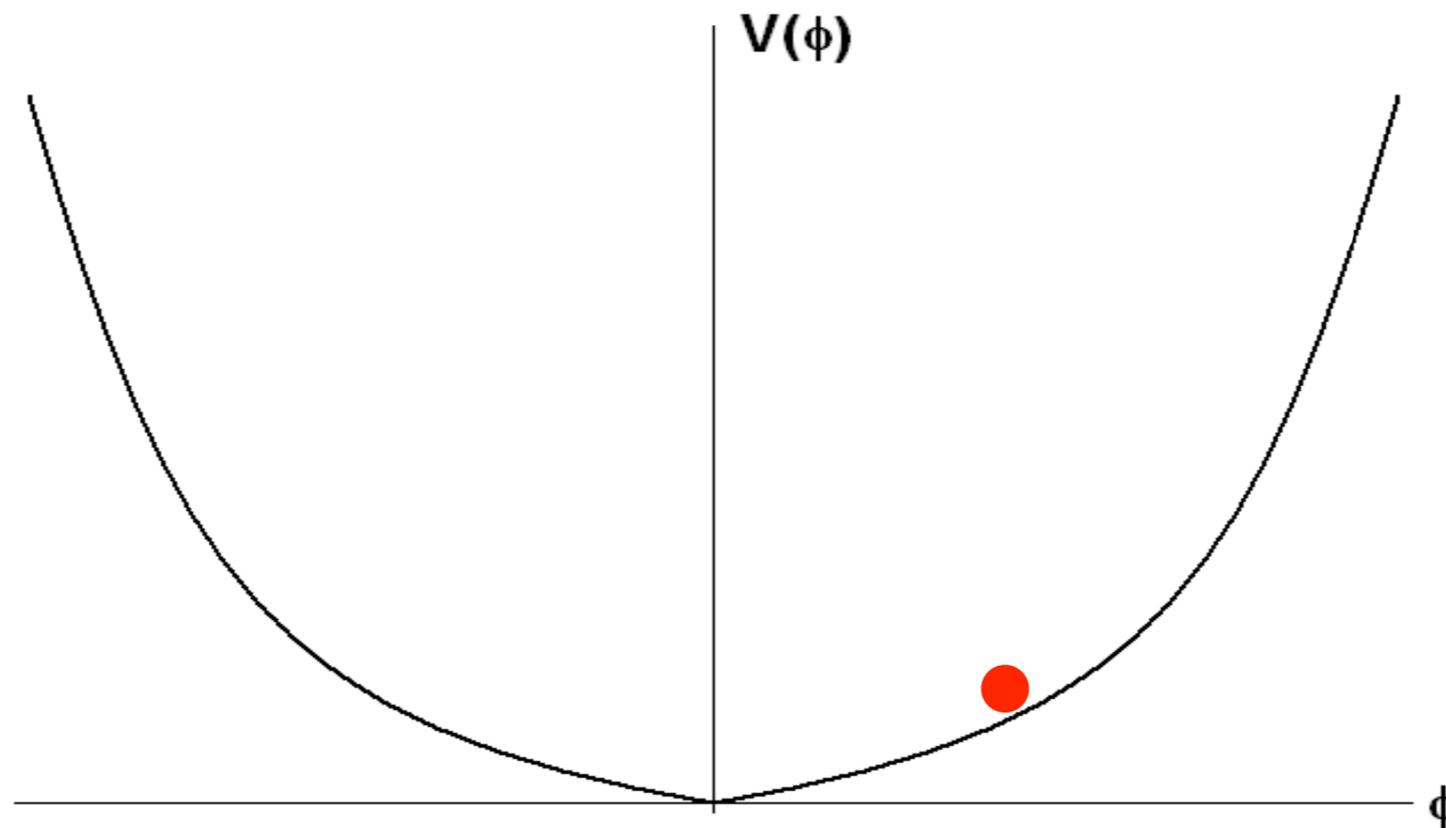


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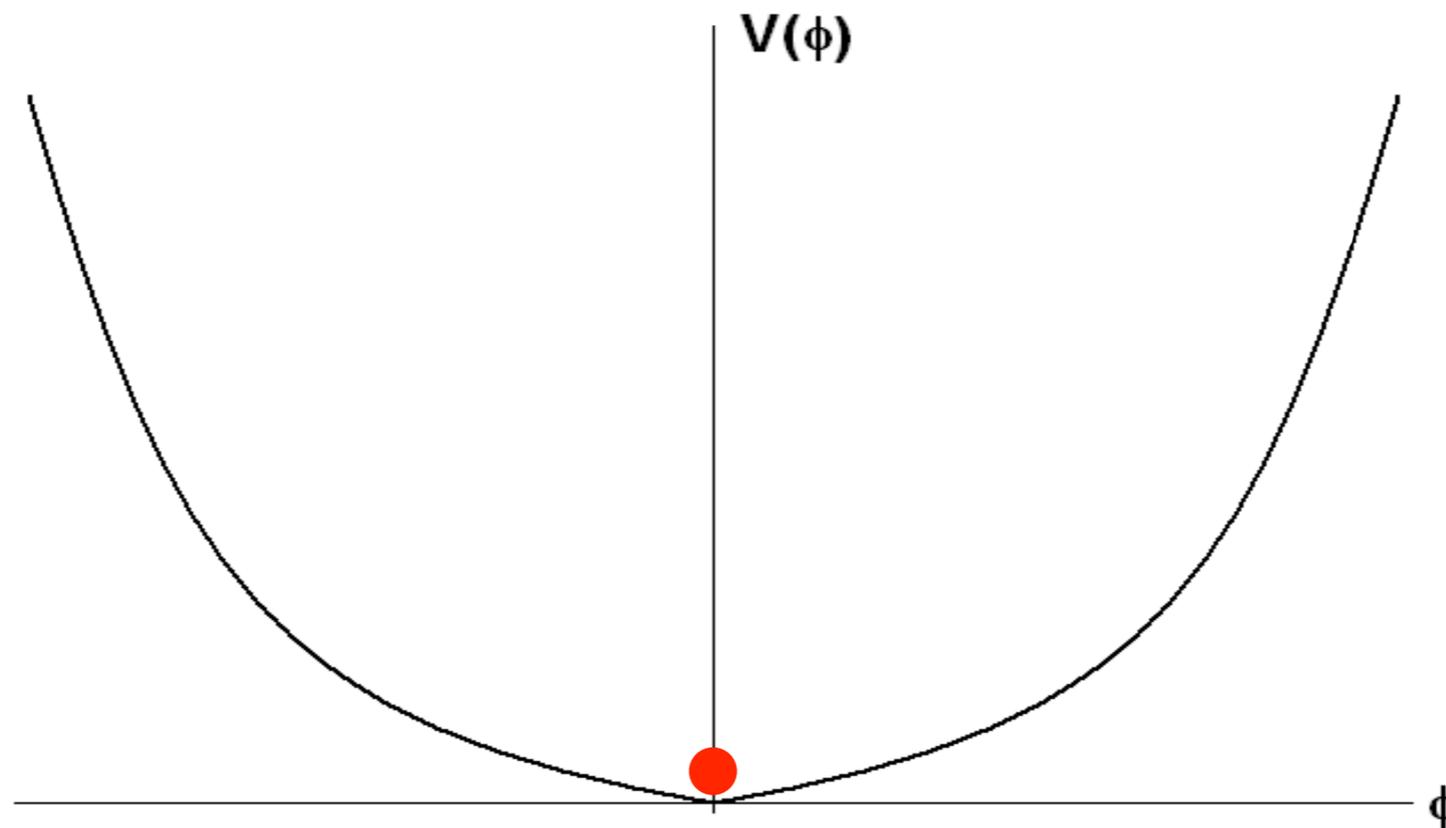


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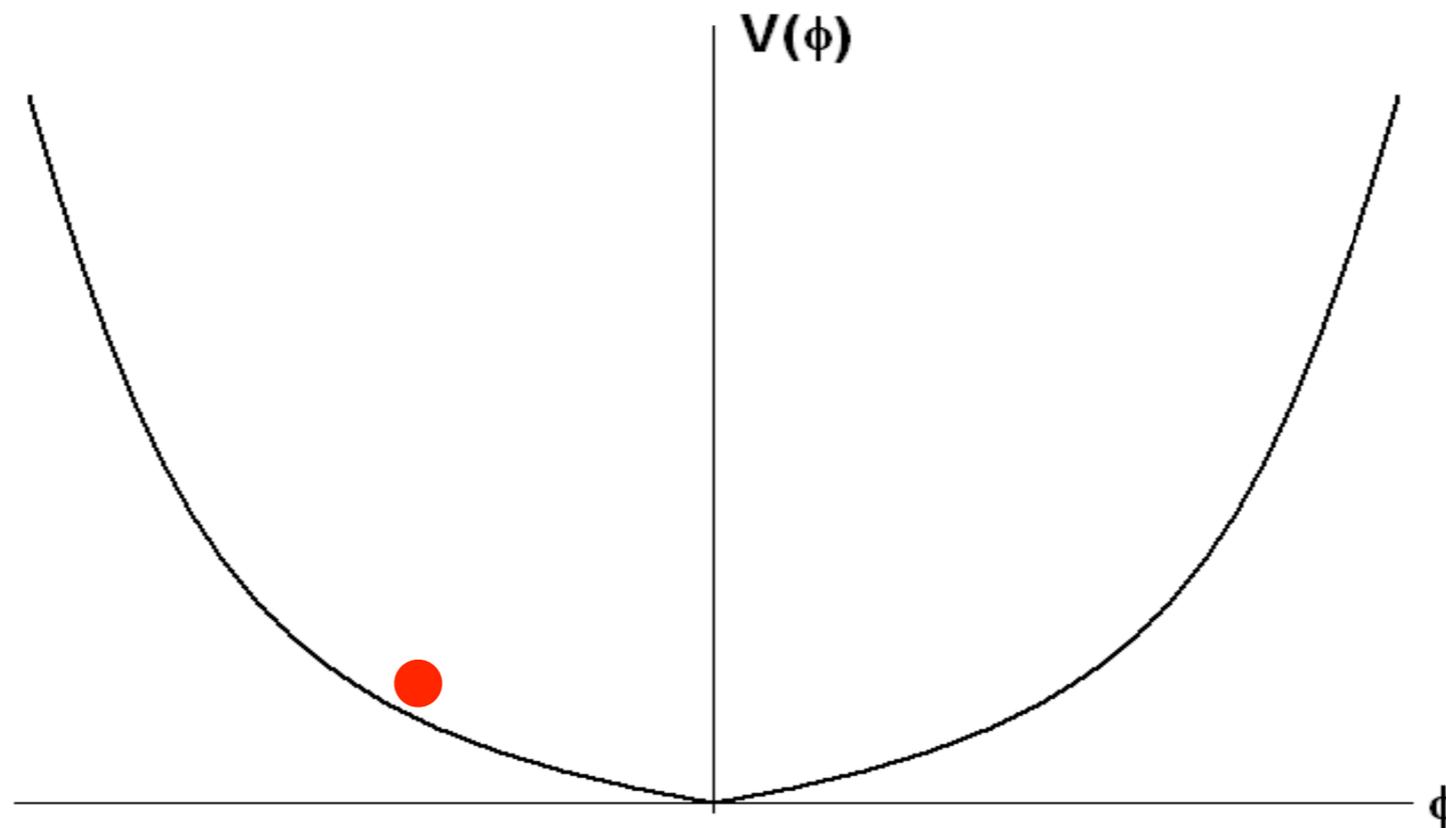


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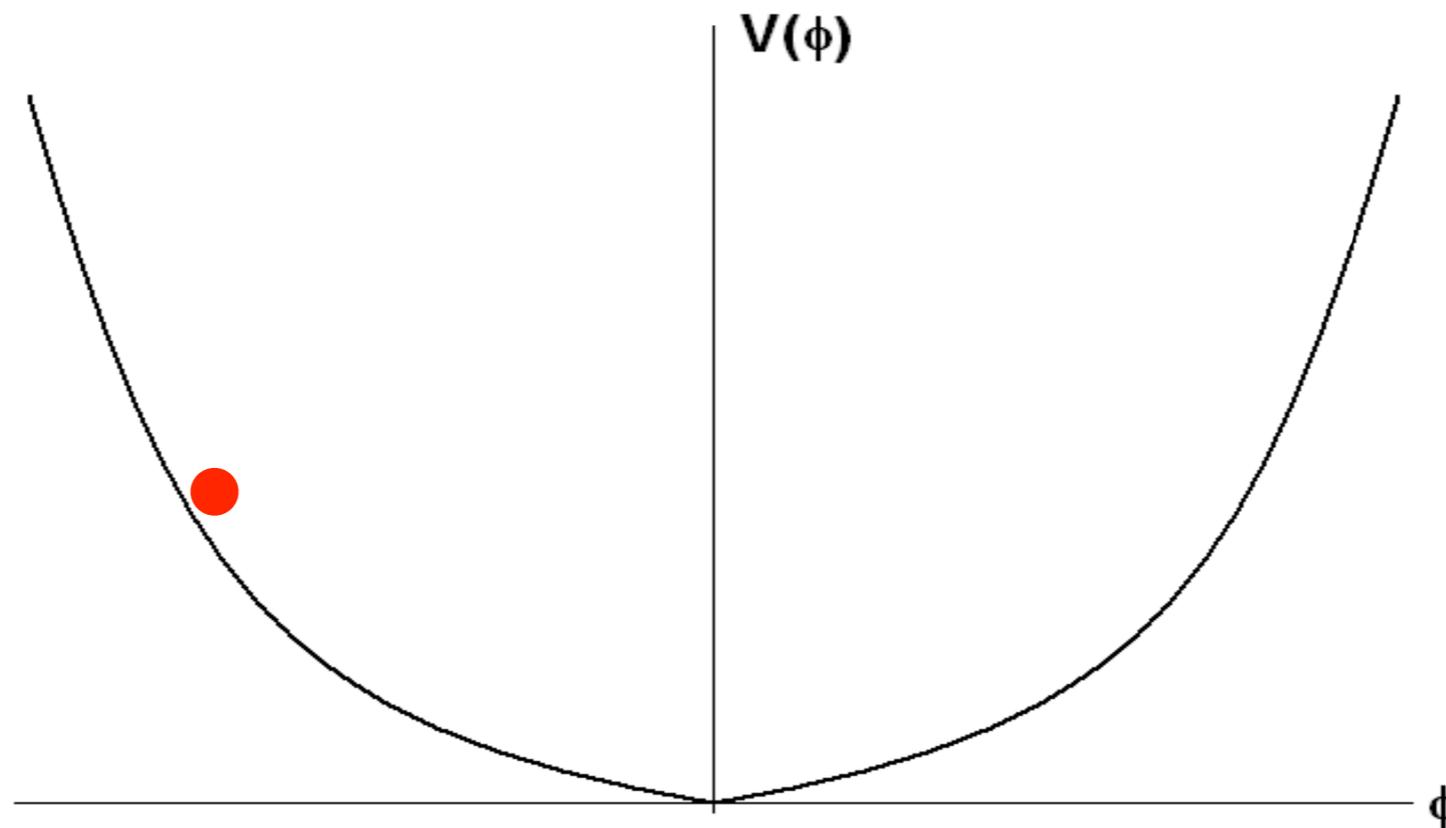


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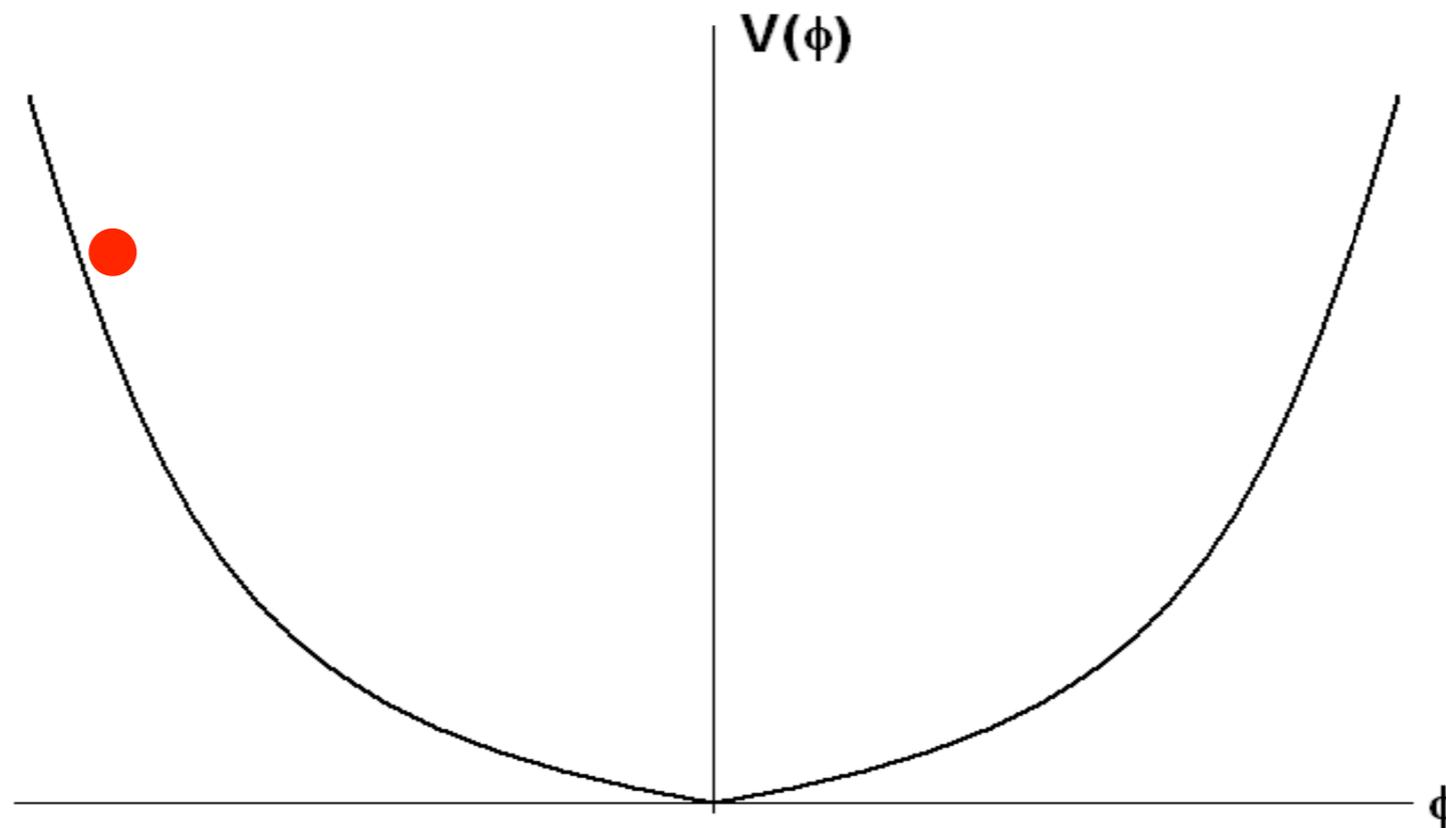


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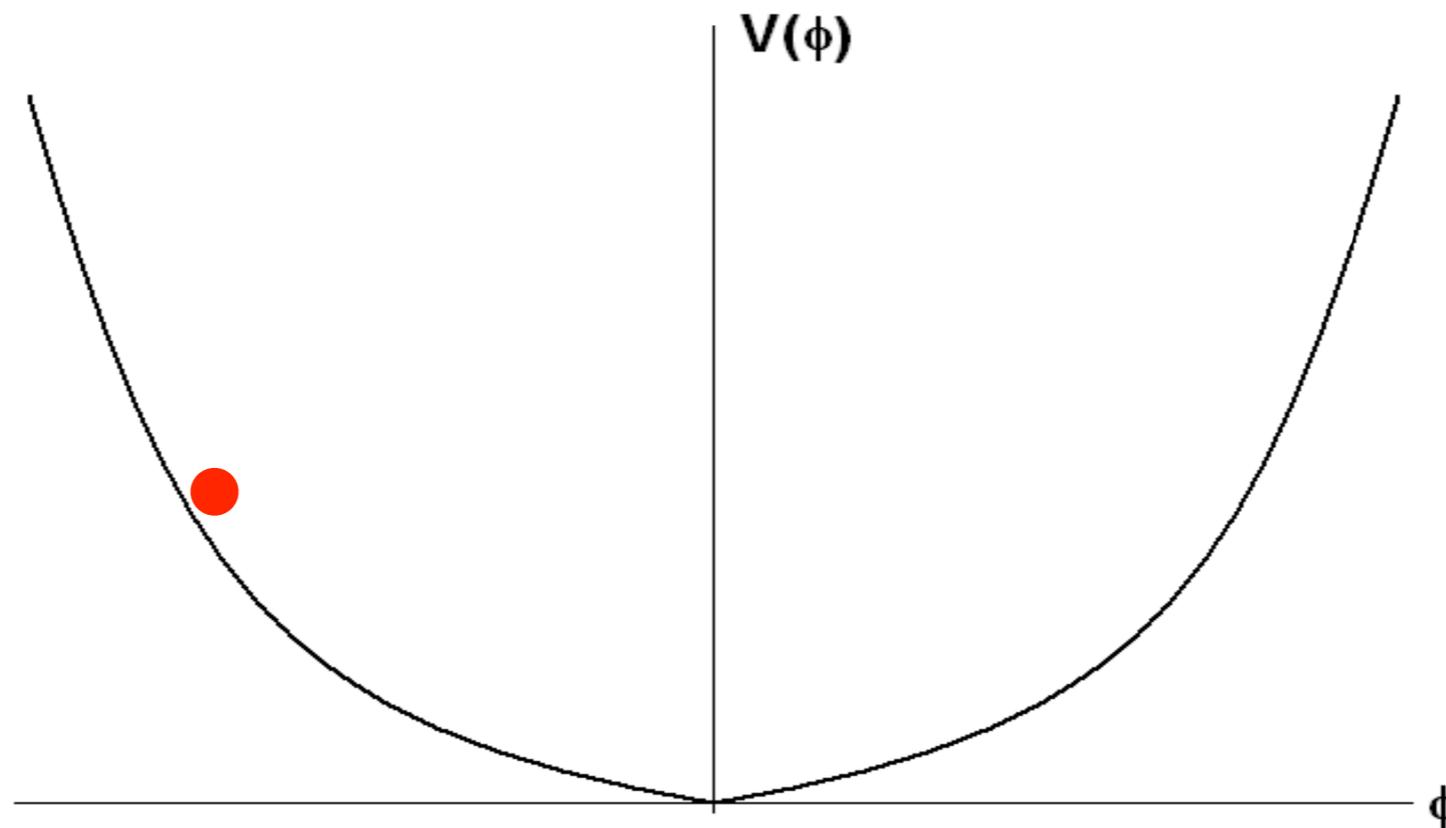


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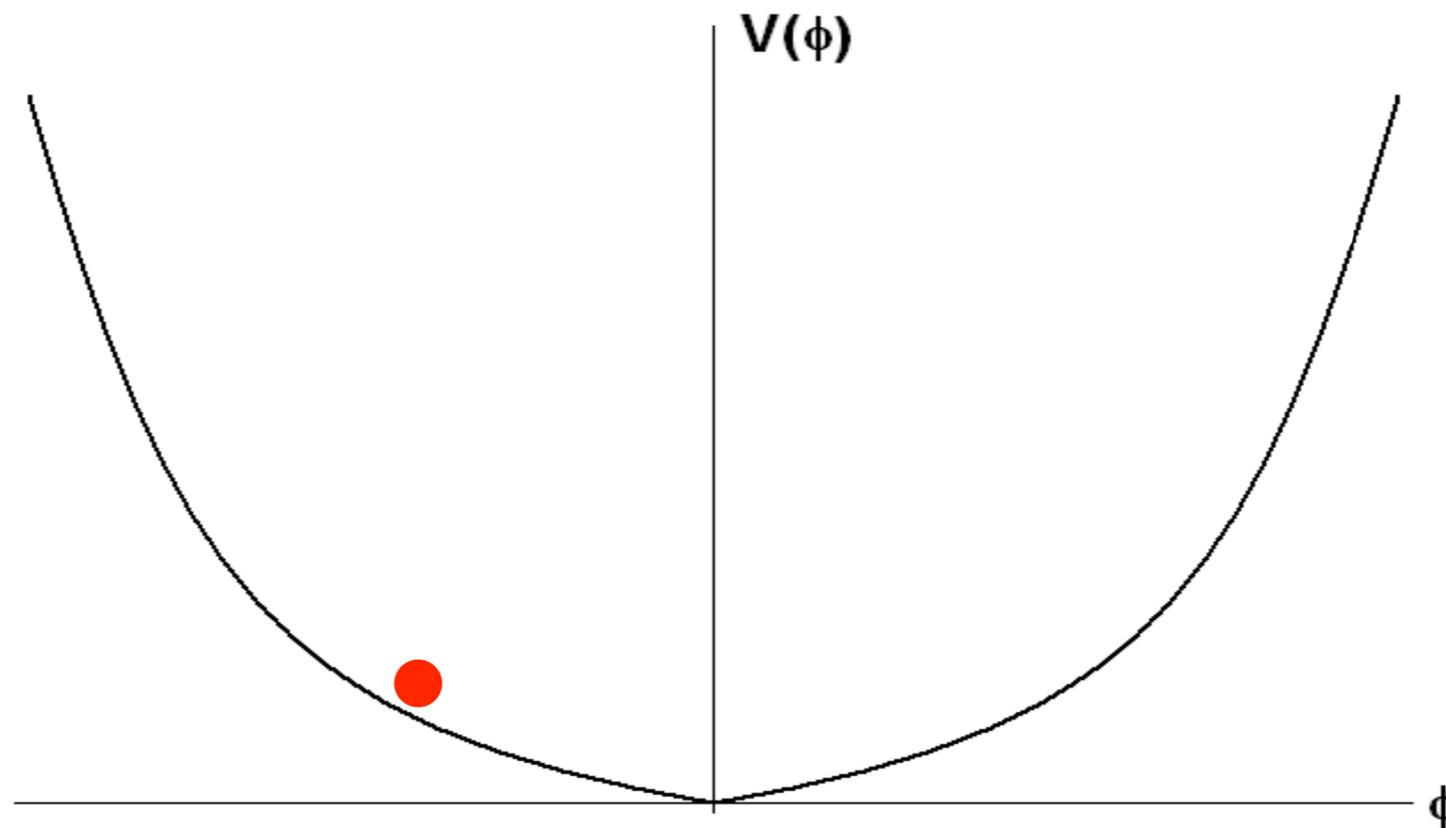


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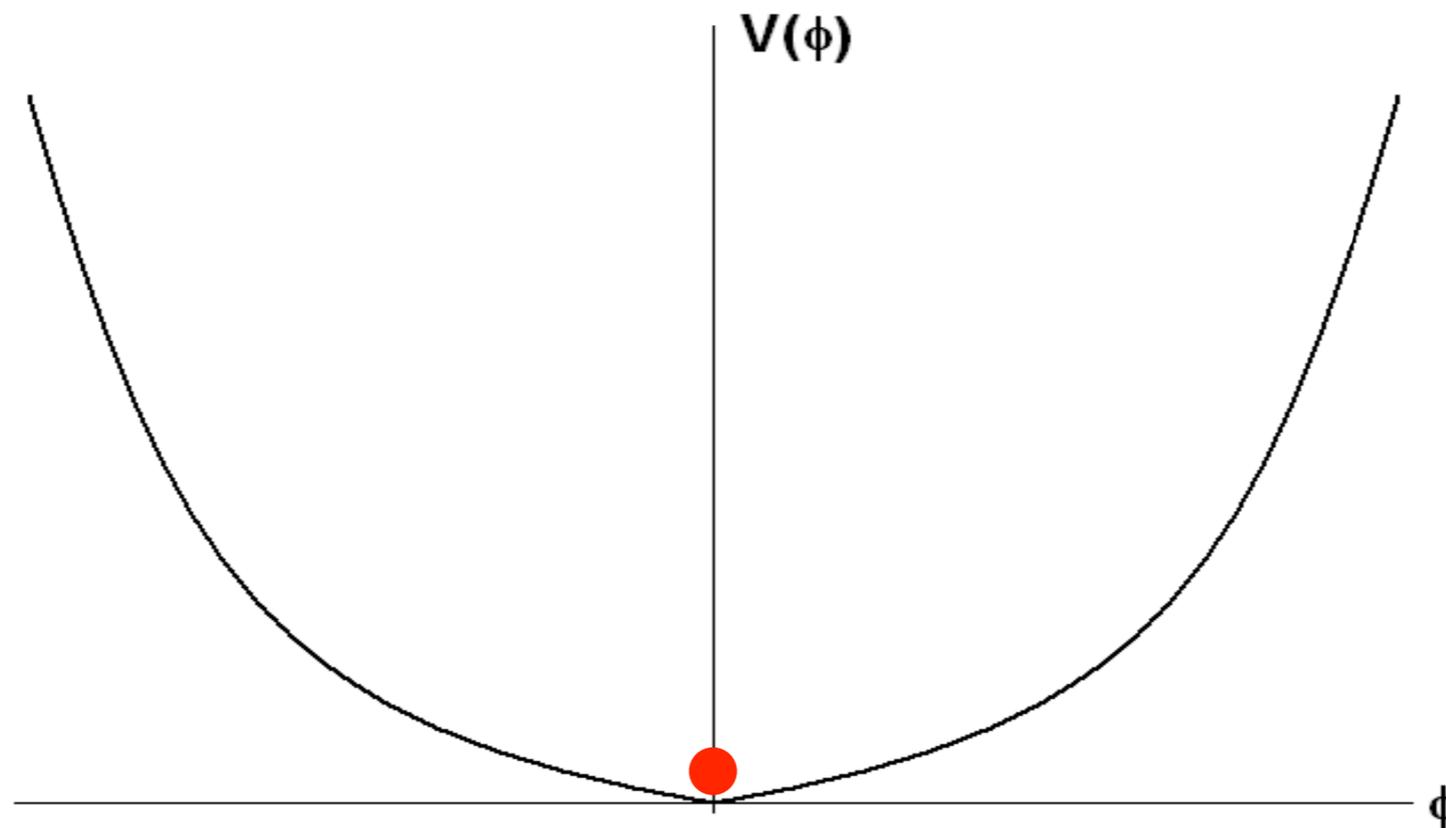


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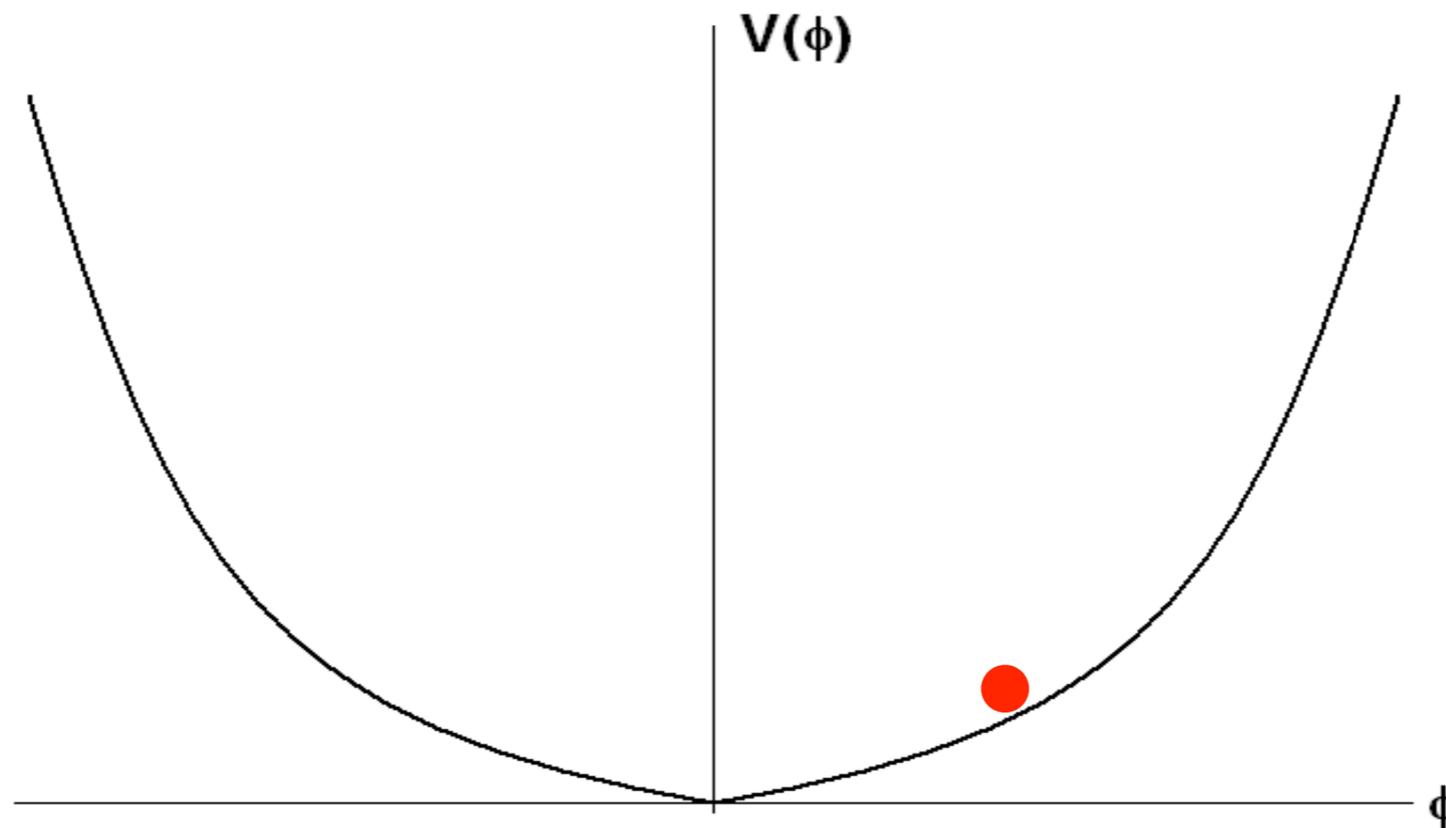


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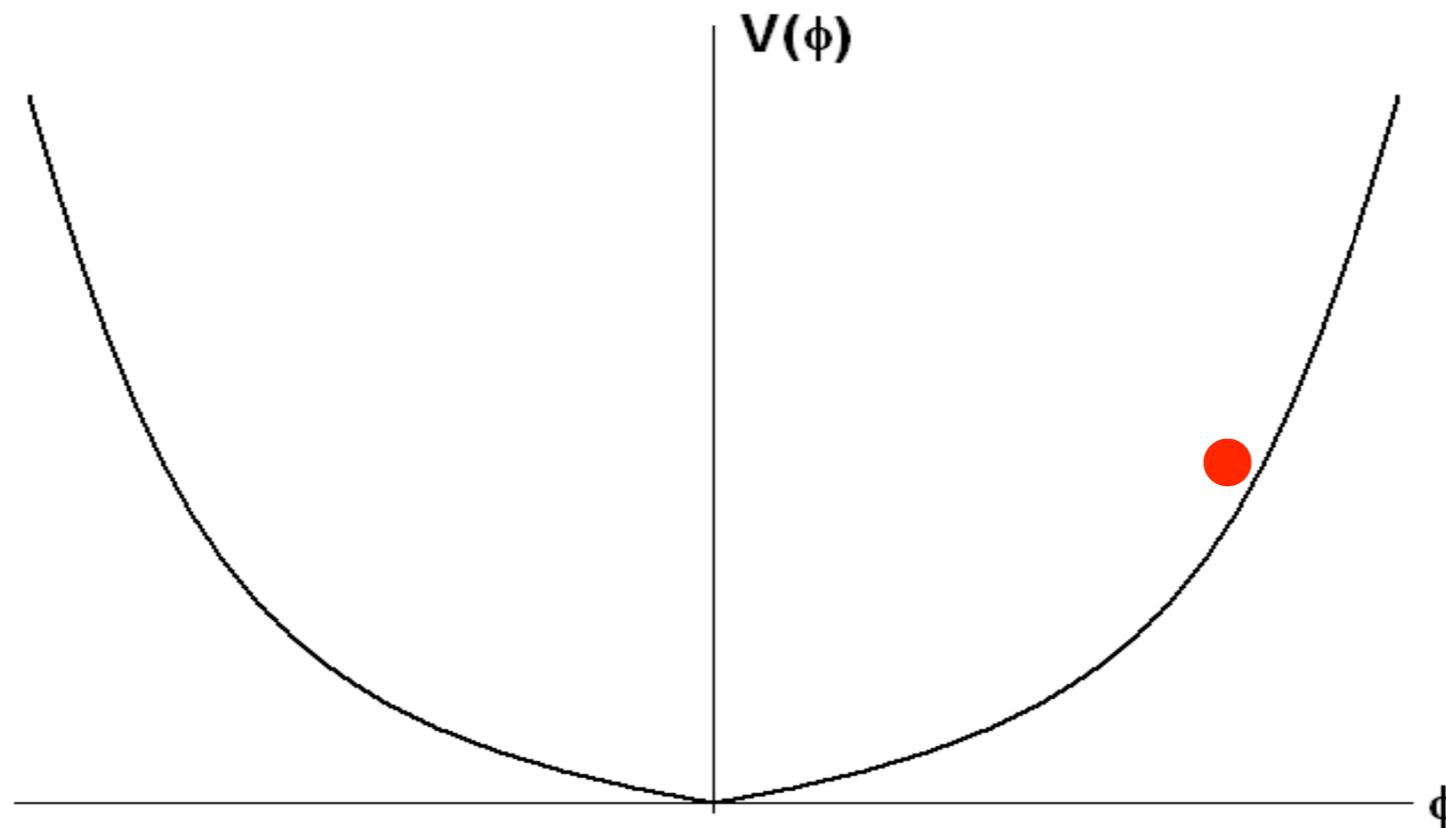


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$$X_k'' + [\kappa^2 + m^2(\phi)]X_k = 0 \quad (\text{Fluctuations of Matter})$$

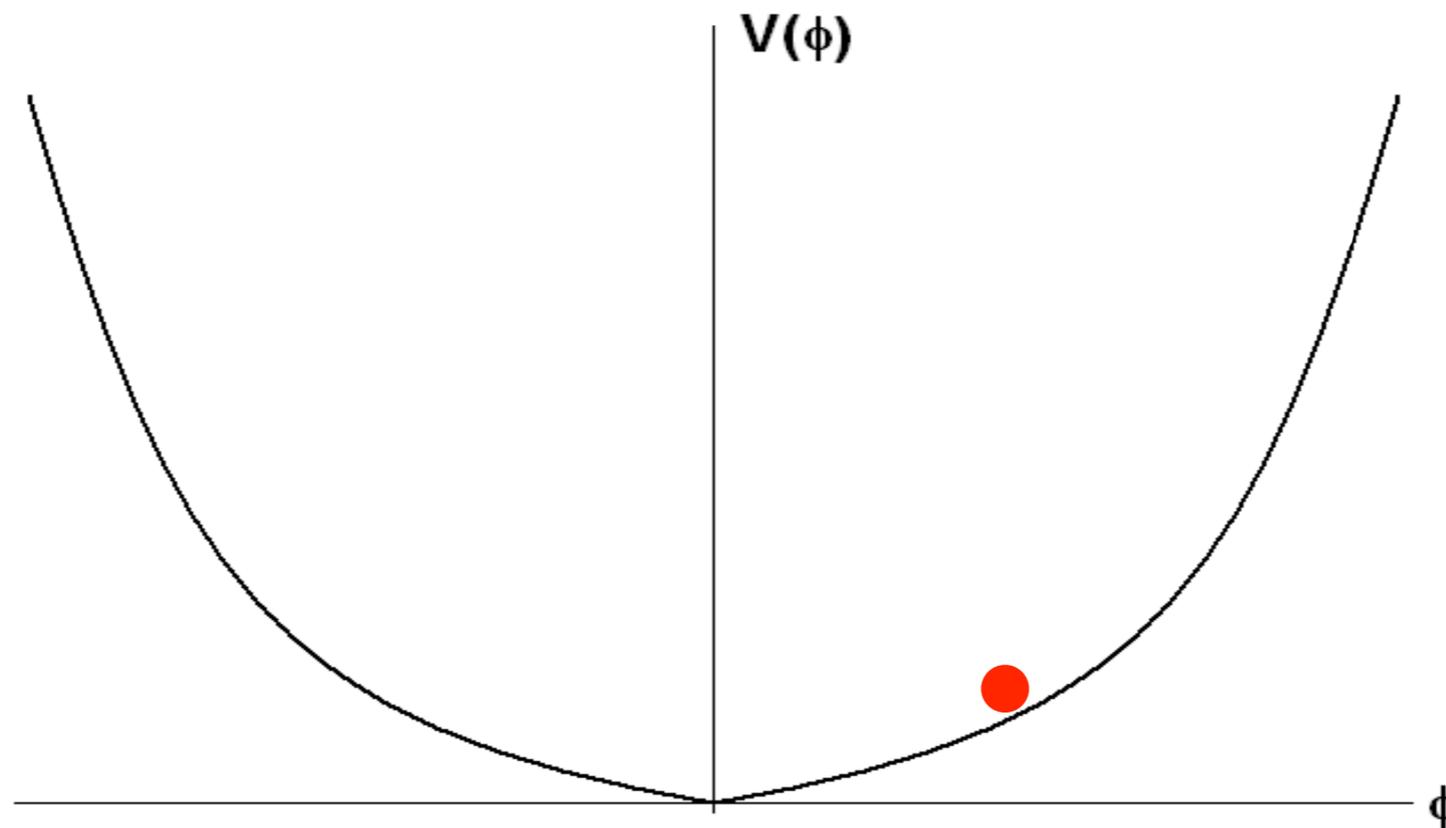


SCALAR (P)REHEATING

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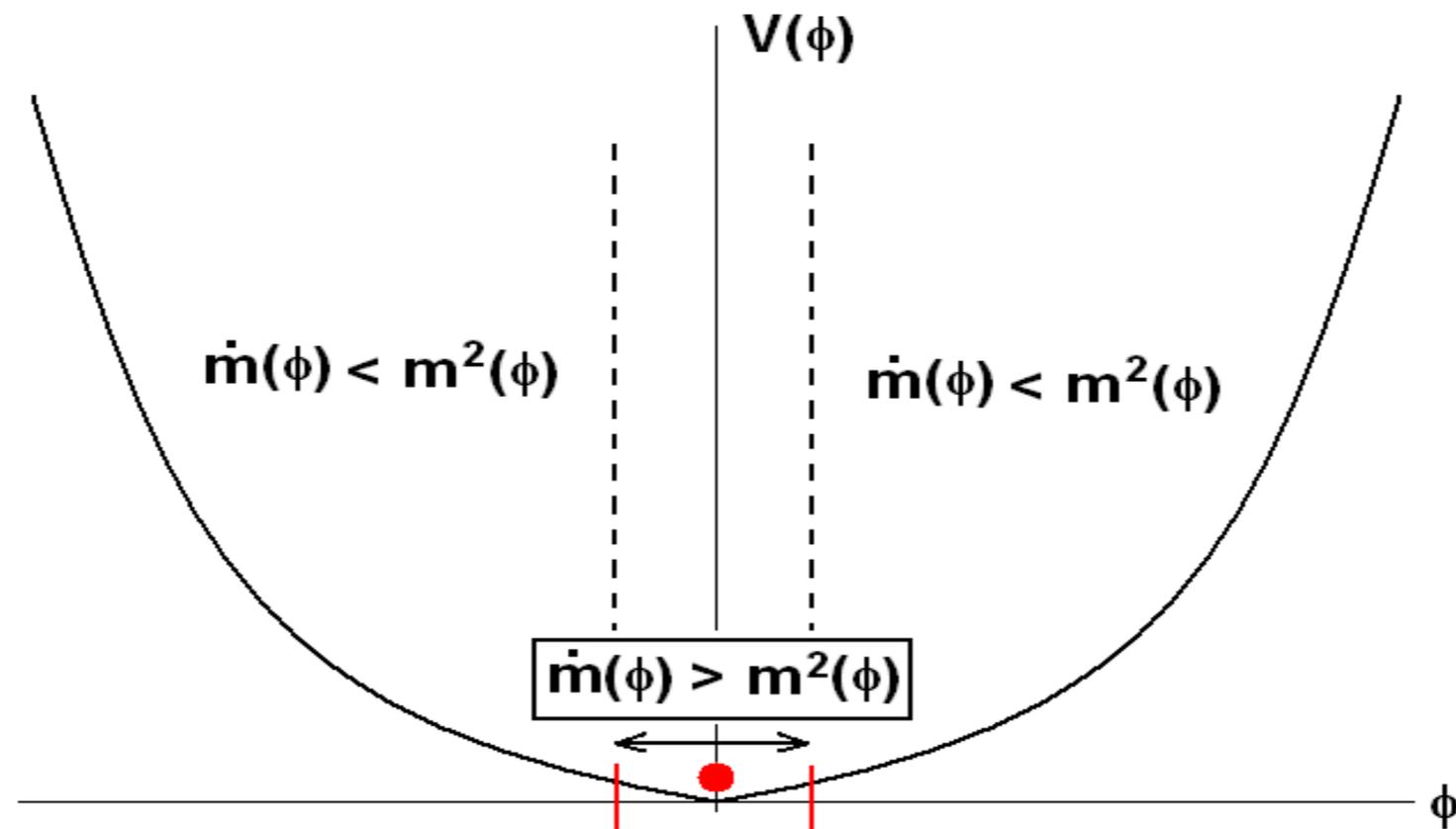


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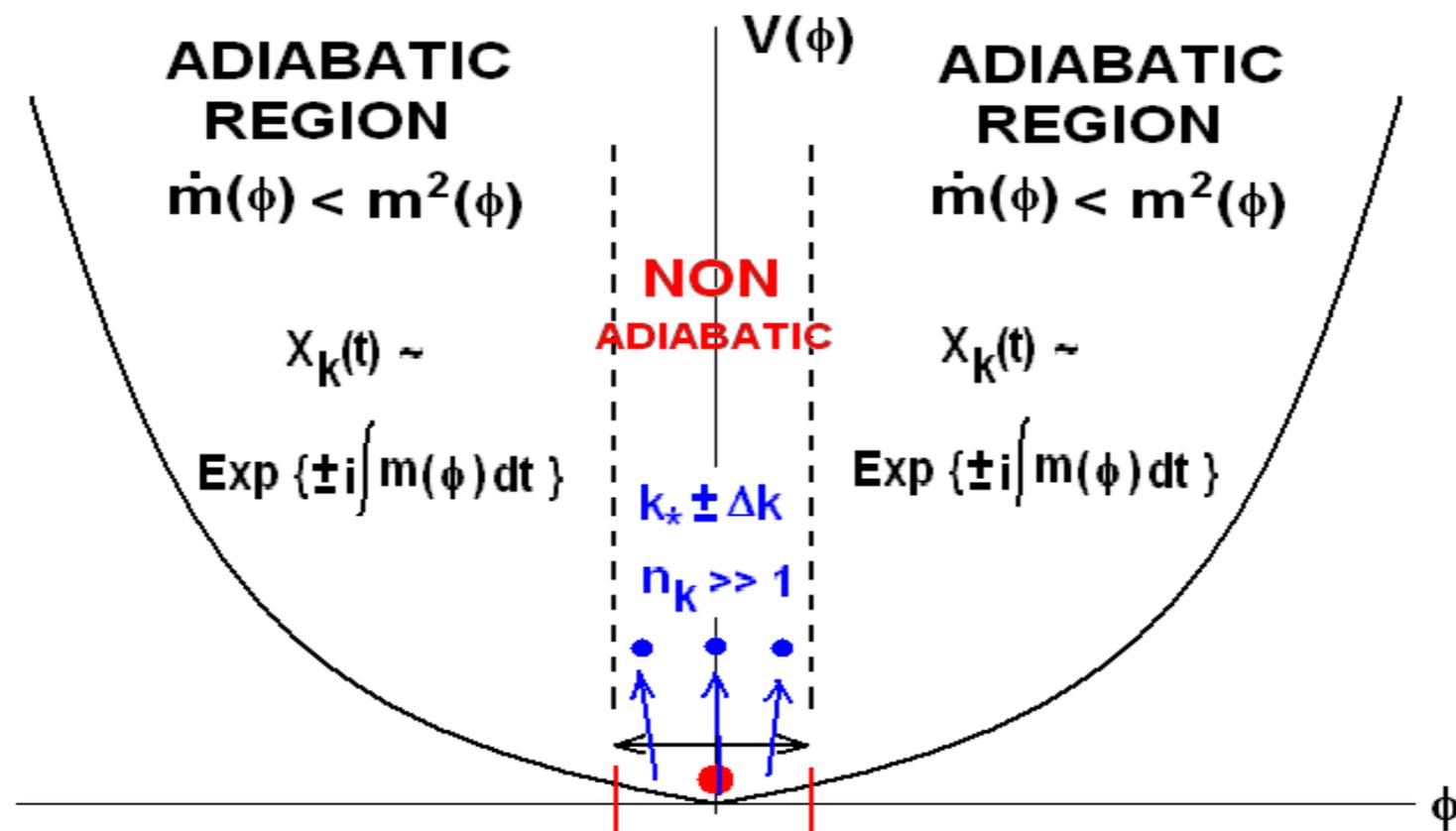


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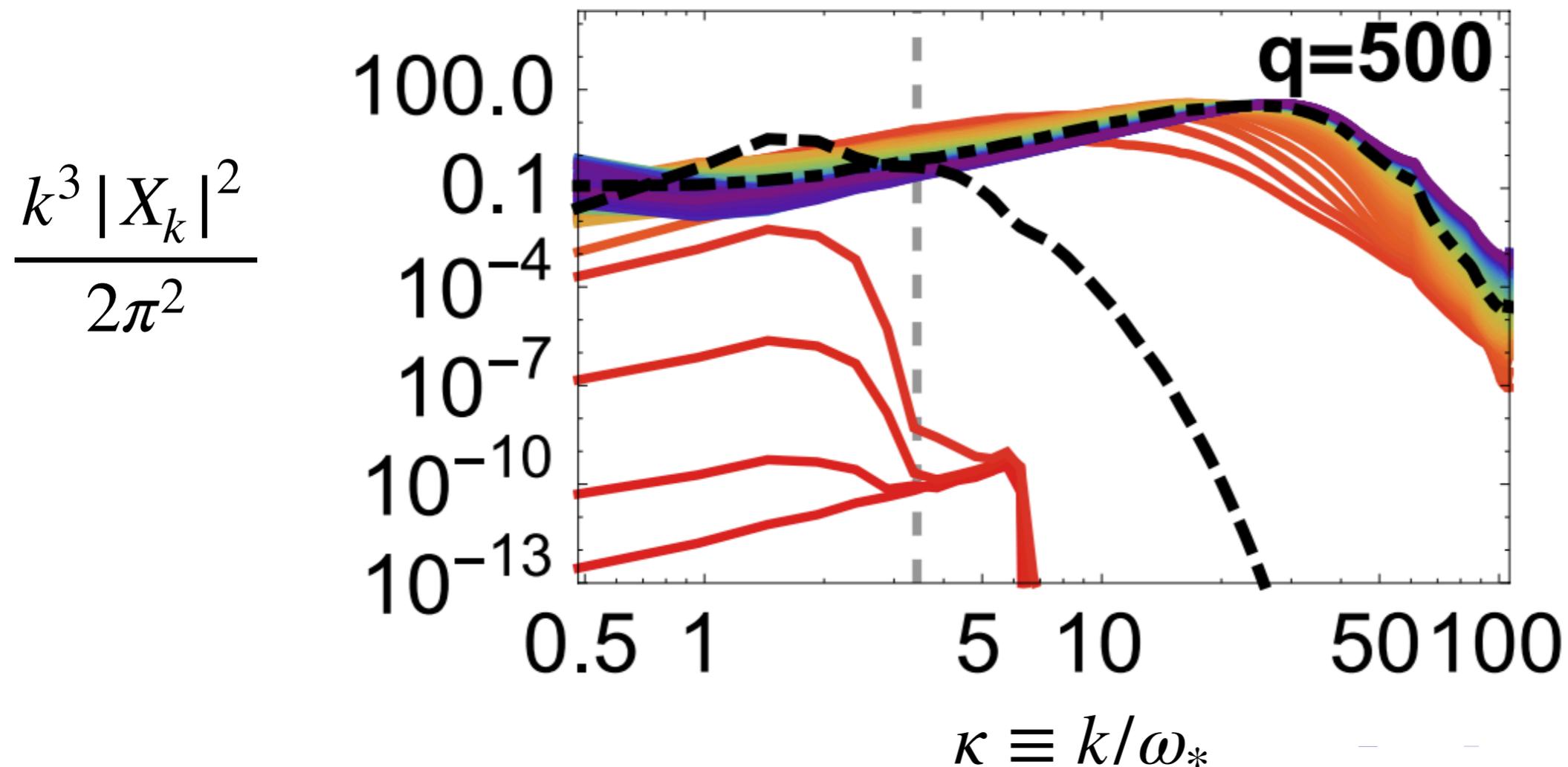


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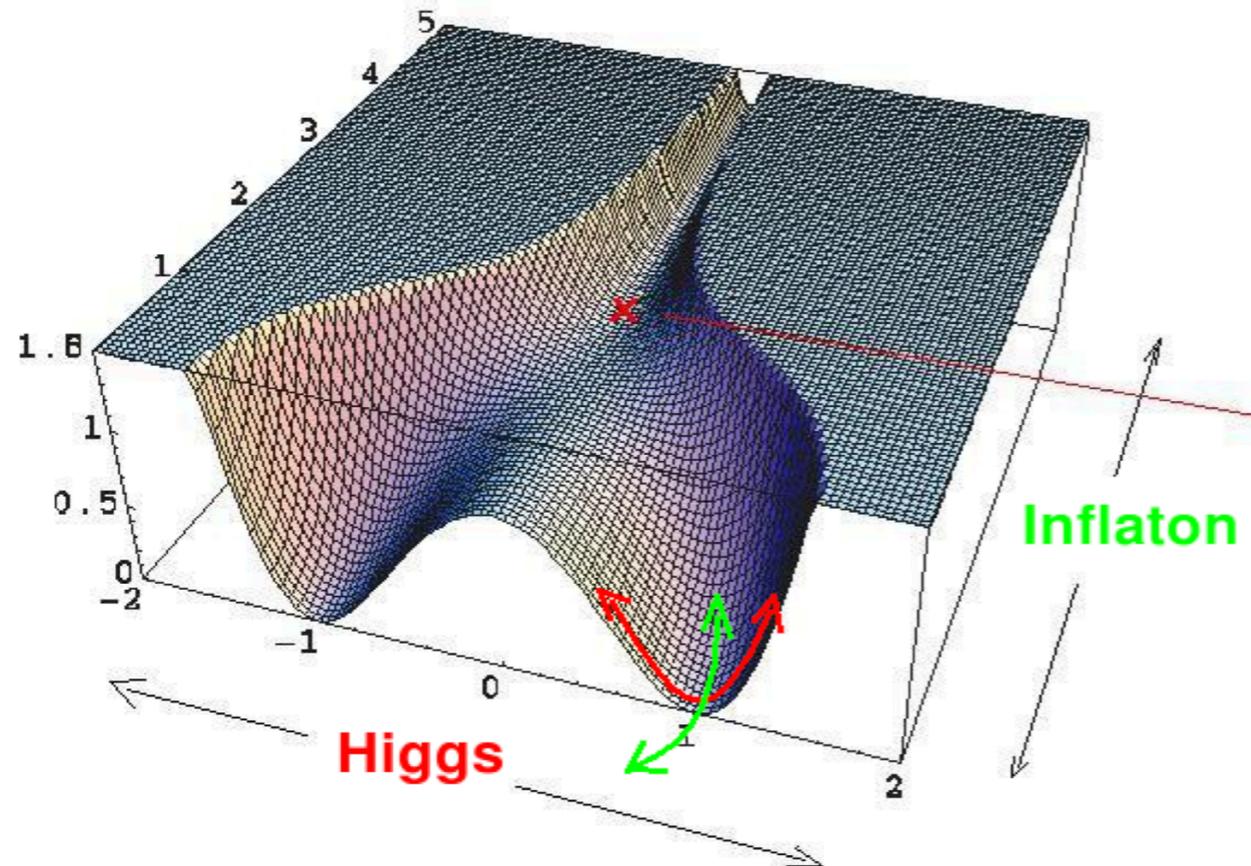


SCALAR (P)REHEATING

2) Hybrid Scenarios : SPINODAL INSTABILITY

$$\left. \begin{aligned} \ddot{\phi}(t) + (\mu^2 + g^2|\chi|^2)\phi(t) &= 0 \\ \ddot{\chi}_k + \left(k^2 + m^2 \left(\frac{\phi^2}{\phi_c^2} - 1\right) + \lambda|\chi|^2\right)\chi_k &= 0 \end{aligned} \right\}$$

Hybrid Preheating



SCALAR (P)REHEATING

2) Hybrid Scenarios : SPINODAL INSTABILITY

inflaton mass \nearrow coupling \nearrow

$$\ddot{\phi}(t) + (\mu^2 + g^2|\chi|^2)\phi(t) = 0$$

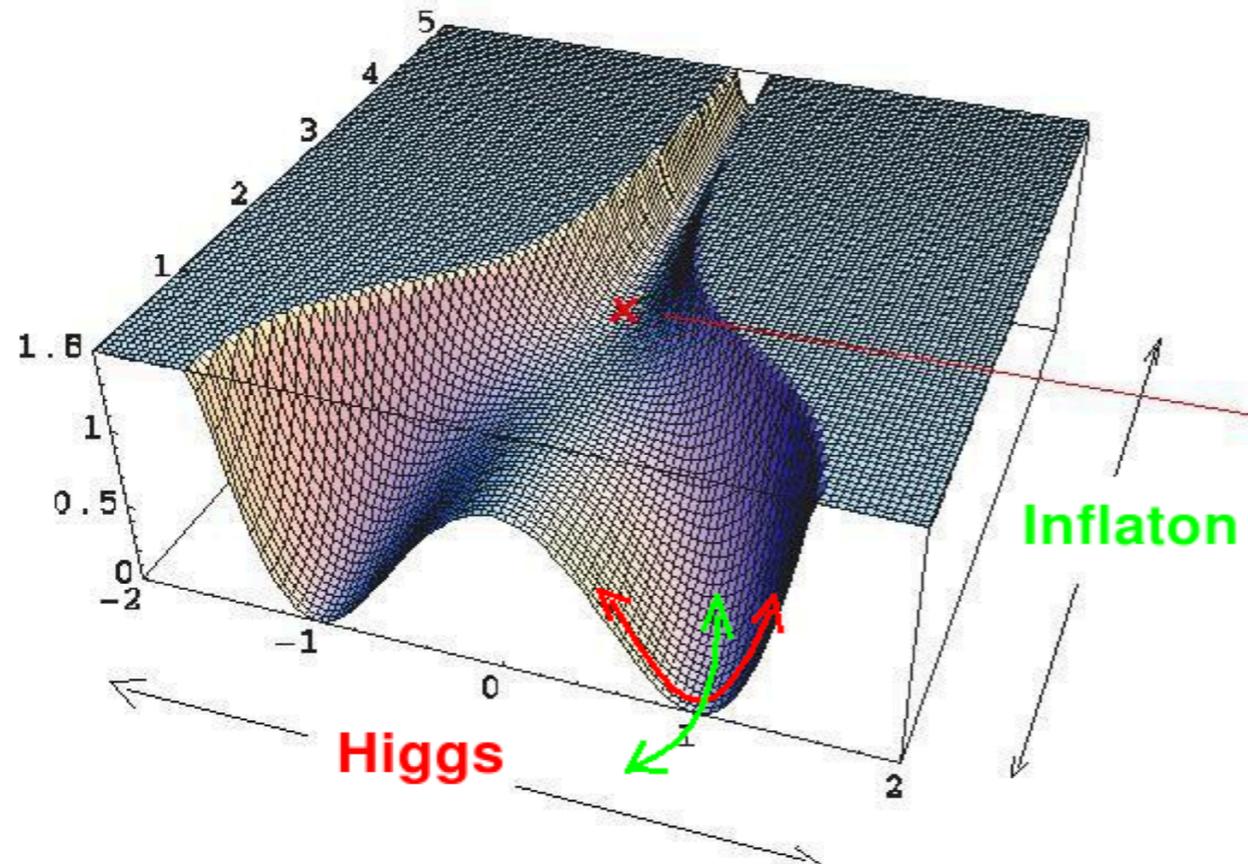
$$\ddot{\chi}_k + \left(k^2 + \underbrace{m^2 \left(\frac{\phi^2}{\phi_c^2} - 1 \right)}_{(g^2\phi^2 - m^2)} + \lambda|\chi|^2 \right) \chi_k = 0$$

Self-coupling \nwarrow V.E.V. \nwarrow

$$m = \sqrt{\lambda}v$$

$$\phi_c \equiv m/g \quad \text{Critical value}$$

Hybrid Preheating



SCALAR (P)REHEATING

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inflaton mass μ^2 coupling $g^2|\chi|^2$

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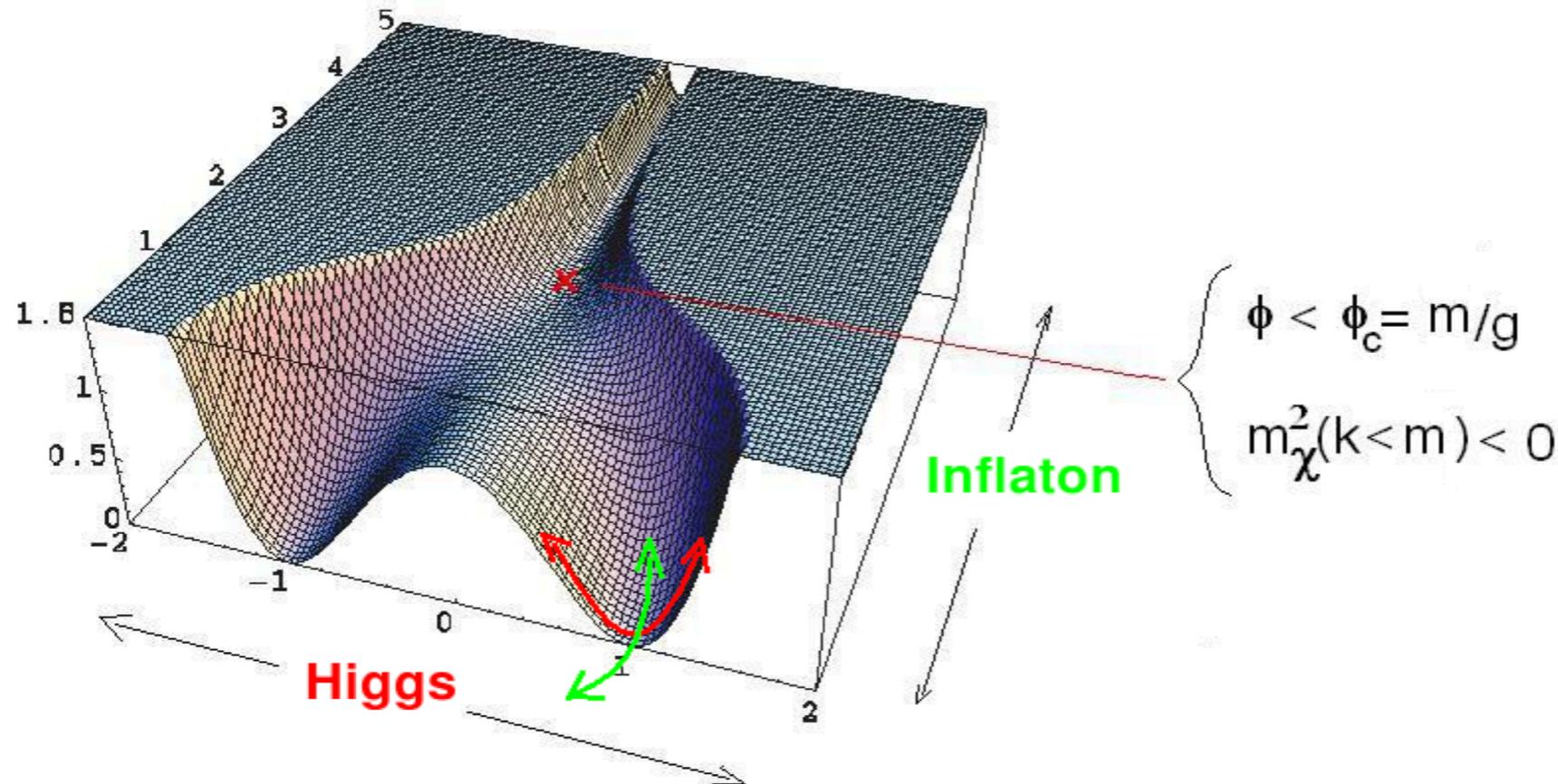
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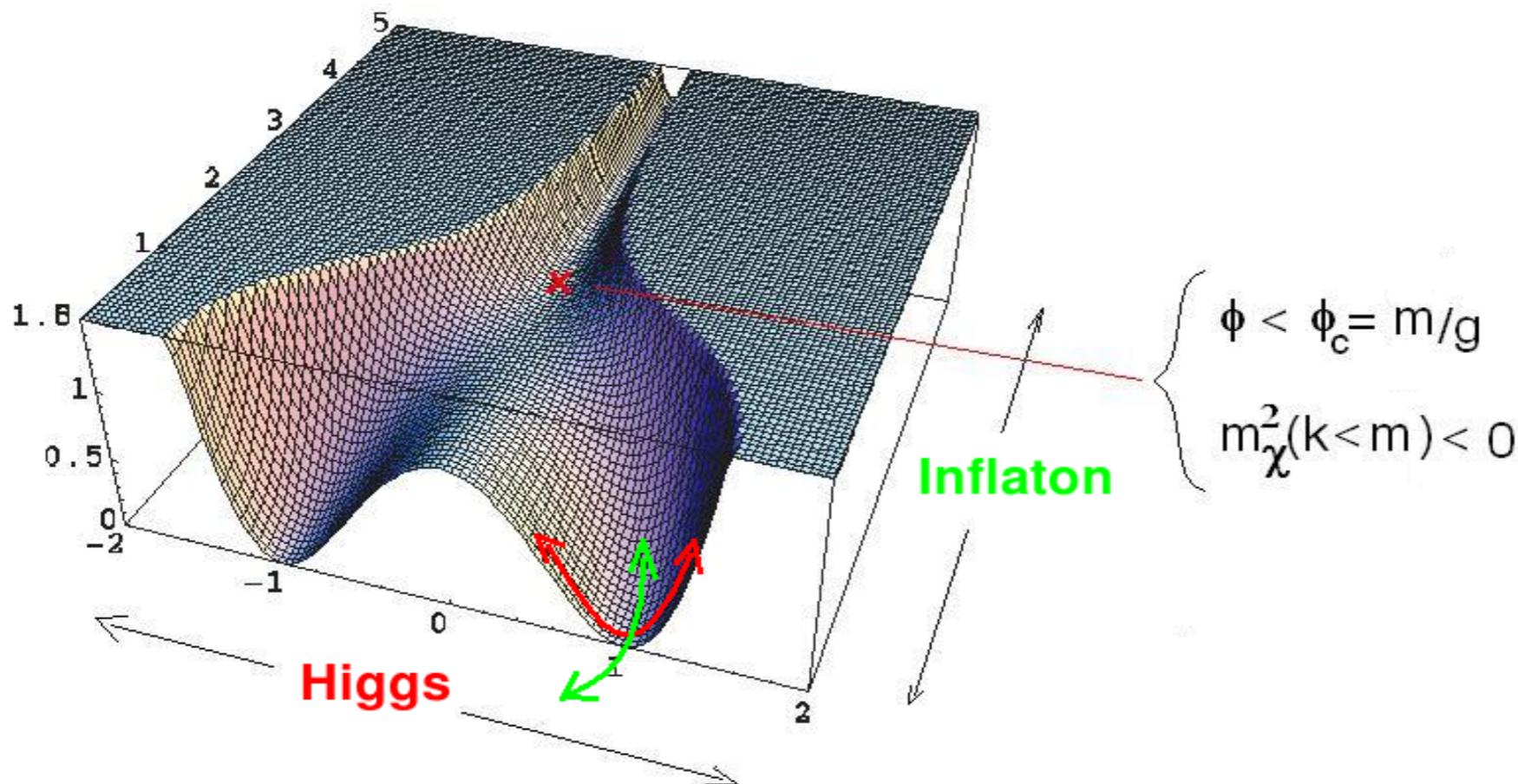
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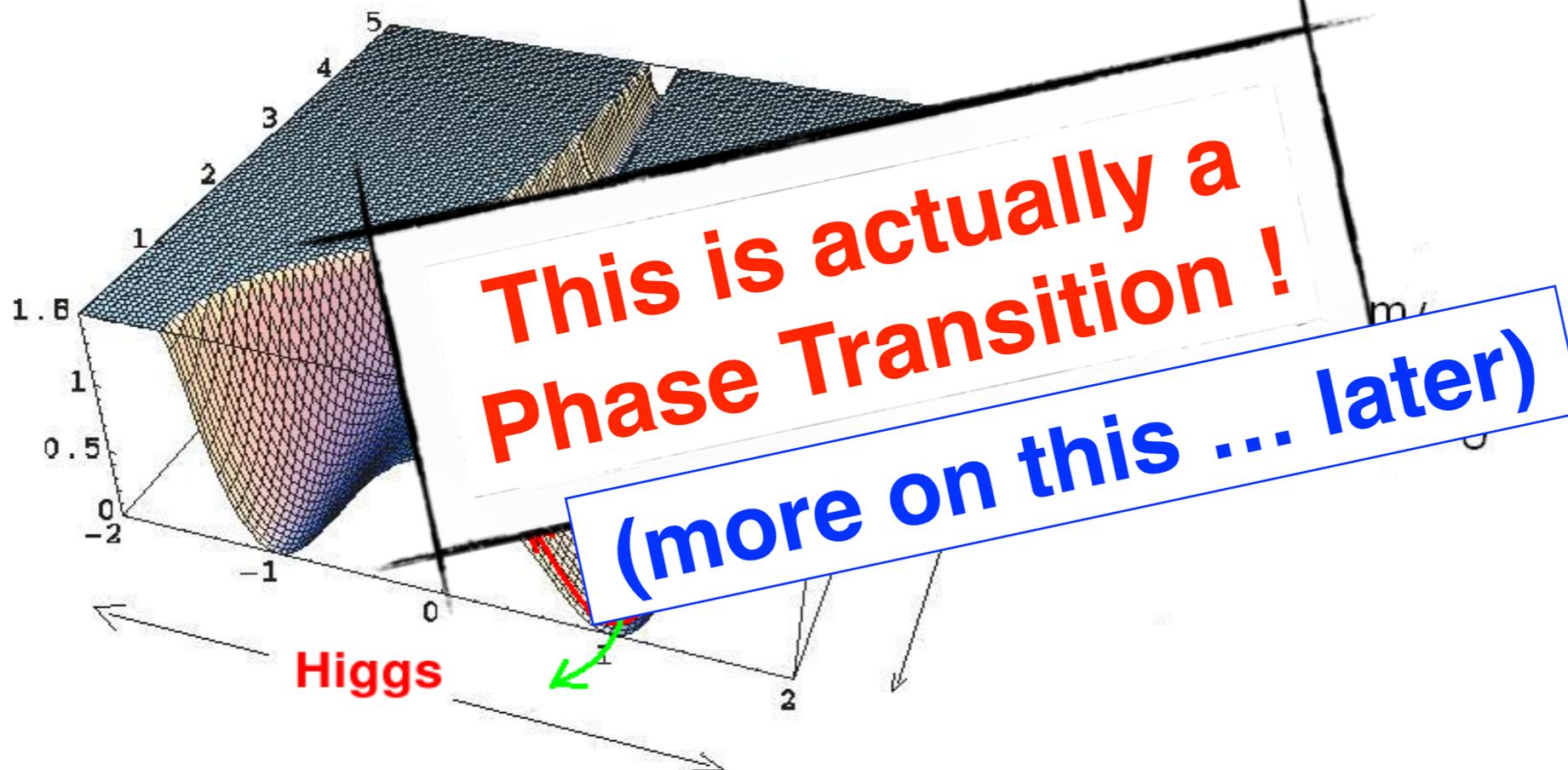
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INFLATIONARY PREHEATING

Physics of (p)REHEATING: $\ddot{\varphi}_k + \omega^2(k, t)\varphi_k = 0$

$$\left\{ \begin{array}{ll} \text{Hybrid Preheating : } \omega^2 = k^2 + m^2(1 - V t) < 0 & \text{(Tachyonic)} \\ \text{Chaotic Preheating : } \omega^2 = k^2 + \Phi^2(t) \sin^2 \mu t & \text{(Periodic)} \end{array} \right.$$

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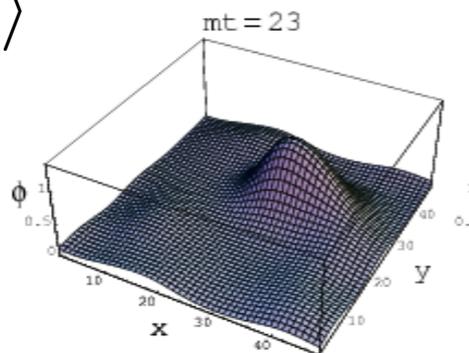
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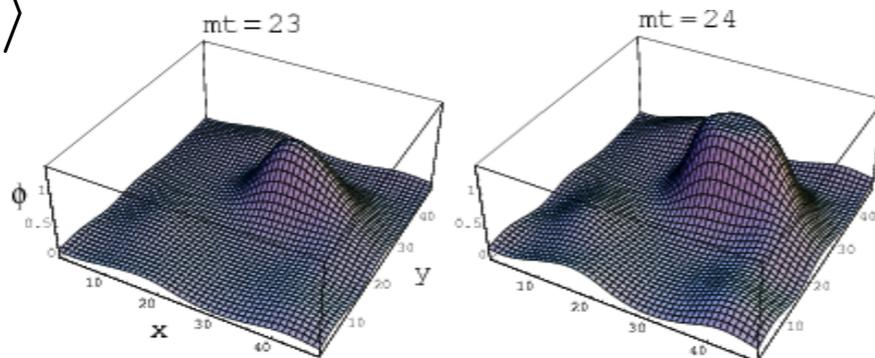
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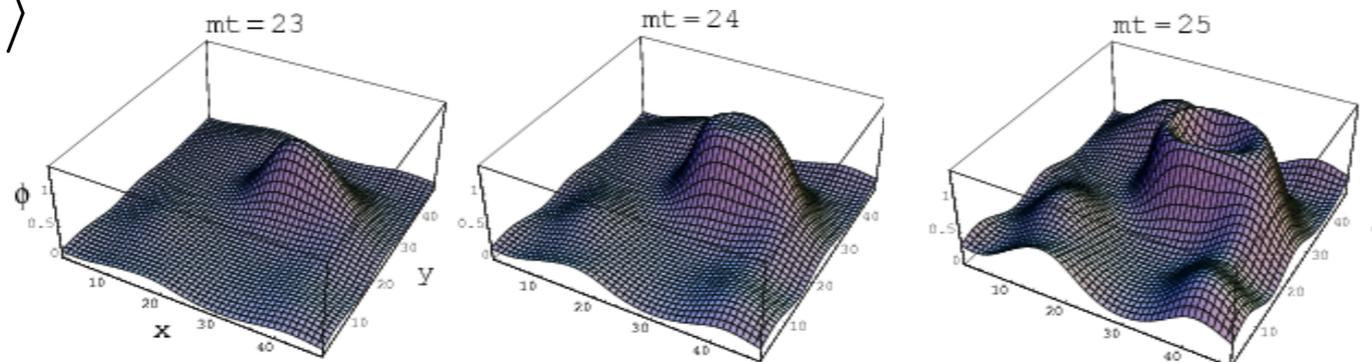
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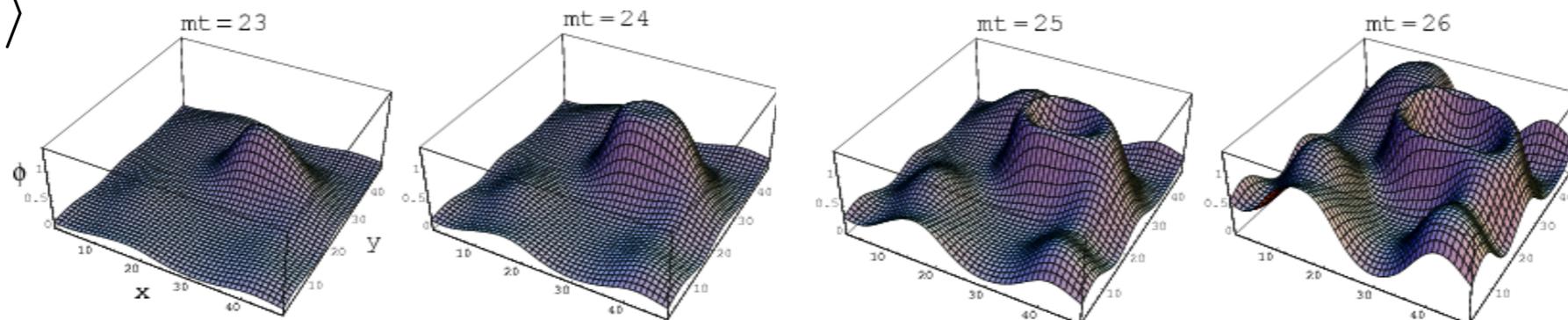
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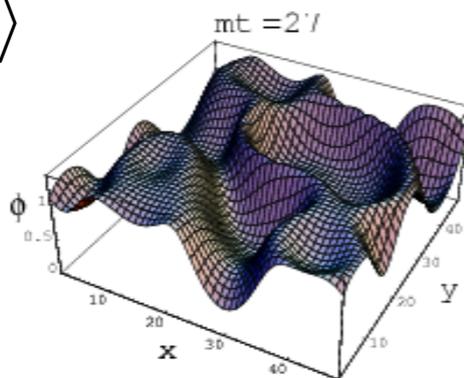
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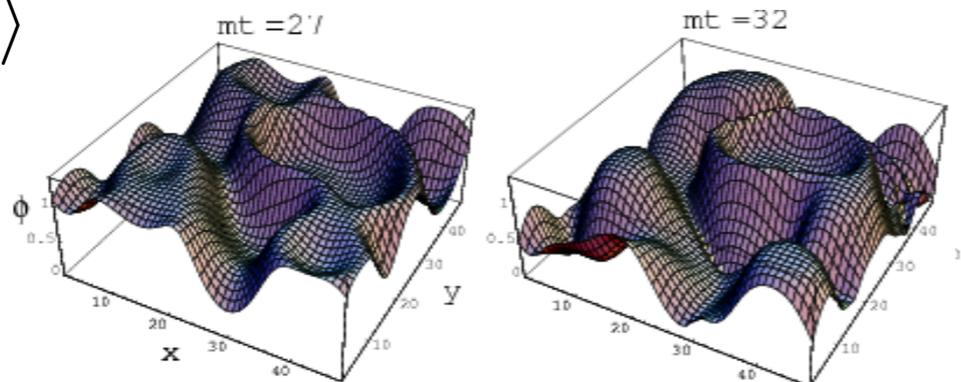
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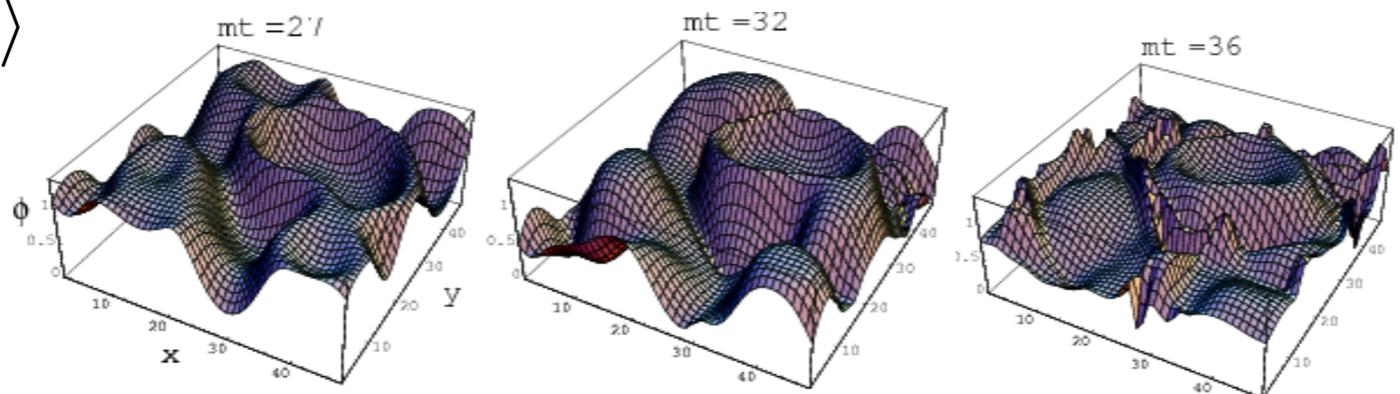
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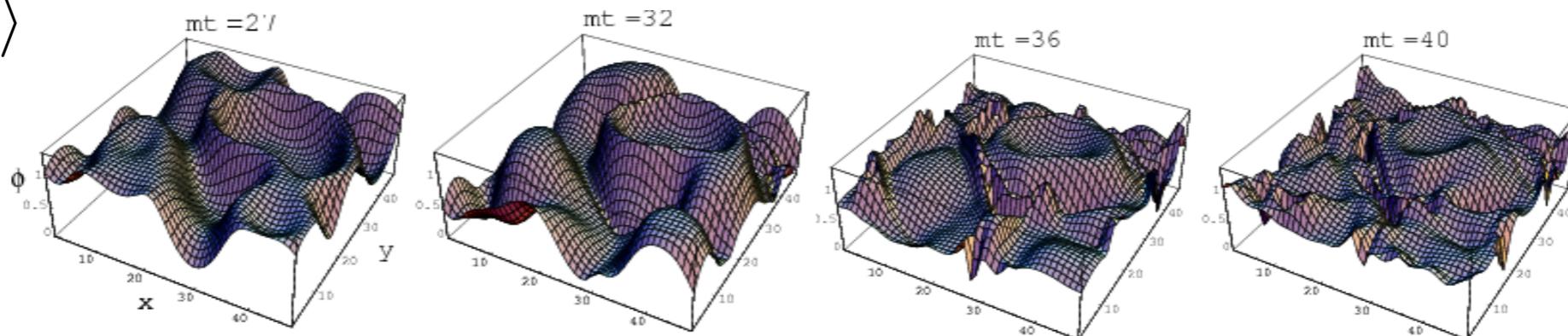
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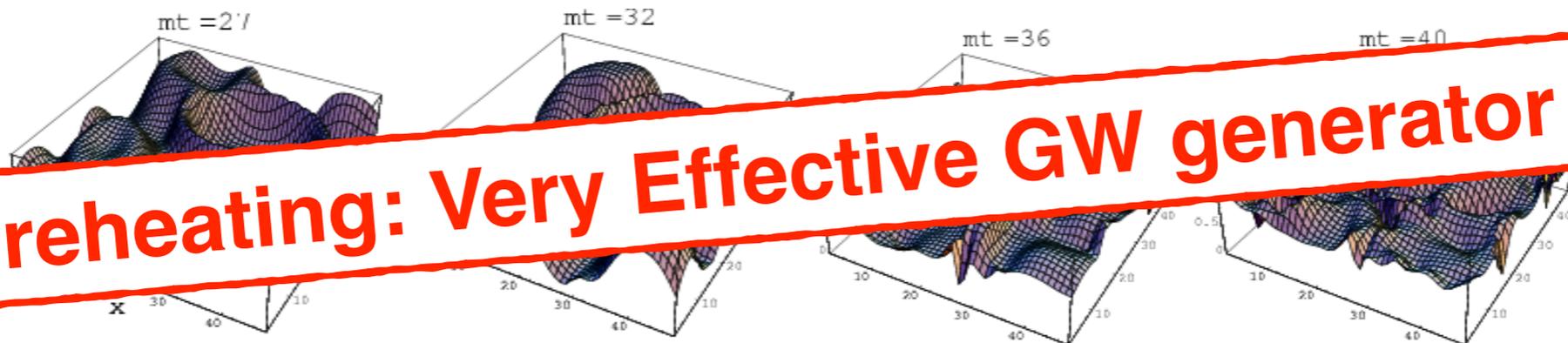
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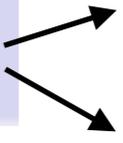
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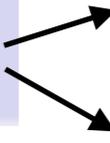


Preheating: Very Effective GW generator !

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

- Scalars ($n_k \gg 1$): $\square\phi + V_{,\phi} = 0$, $\square\chi_a + V_{,\chi_a} = 0$

Semi-classical regime $\pi_k \approx \kappa\phi_k + \dots$ (**Squeezed States**)

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$$ds^2 = a^2(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \quad \text{TT} : \begin{cases} h_{ii} = 0 \\ h_{ij,j} = 0 \end{cases}$$

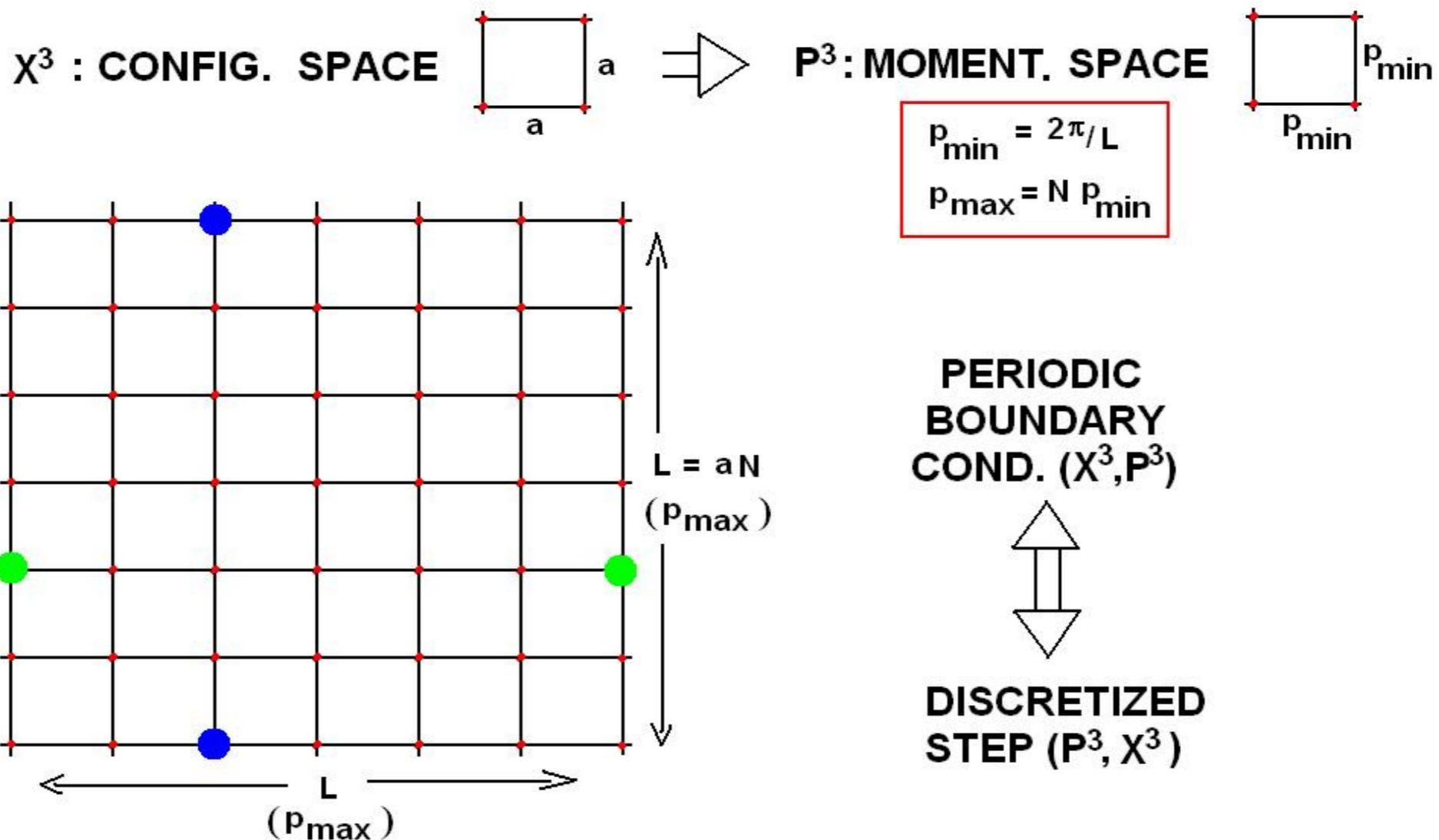
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Lattice Simulations: Dynamics

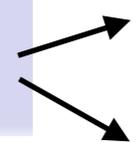
non-linear
out-Eq

$$\partial_\mu O(x) \rightarrow (O(x + \mu) - O(x - \mu))/2a_\mu$$

$$\partial_\mu \partial_\mu O(x) \rightarrow (O(x + 2\mu) + O(x - 2\mu) - 2O(x))/4a_\mu^2$$



INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

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TT: Non-local operation !

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TT: Non-local operation !

$$\Pi_{ij}(\mathbf{k}, t) \equiv \int d^3\mathbf{x} e^{+i\mathbf{k}\mathbf{x}}(\hat{k}) \Pi_{ij}(\mathbf{x}, t) \quad (\text{Fourier Transform})$$

$$\Pi_{ij}^{(TT)}(\mathbf{k}, t) \equiv \Lambda_{ij,lm}(\hat{k}) \Pi_{ij}(\mathbf{k}, t) \quad (\text{TT-Projection})$$

$$\Pi_{ij}^{(TT)}(\mathbf{x}, t) \equiv \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\mathbf{x}} \Lambda_{ij,lm}(\hat{k}) \Pi_{lm}(\mathbf{k}, t) \quad (\text{Fourier back})$$

INFLATIONARY PREHEATING

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 \rightarrow **out-Eq**

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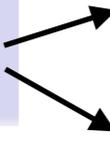
$\Pi_{ij}(\mathbf{k}, t) \equiv \int d^3 \mathbf{x} e^{+i\mathbf{k}\mathbf{x}} \hat{\Pi}_{ij}(\mathbf{x}, t)$ (Fourier Transform)

Numerically Prohibited !

(TT-Projection)

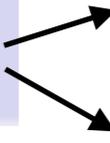
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INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

Building the Solution:
$$\left\{ \begin{array}{l} h_{ij}(\mathbf{k}, t) = \Lambda_{ij,lm}(\hat{\mathbf{k}}) u_{lm}(\mathbf{k}, t) \\ u_{lm}(\mathbf{k}, t) = \int_{t_0}^t dt' G(t - t') \Pi_{lm}^{\text{eff}}(\mathbf{k}, t') \end{array} \right.$$

INFLATIONARY PREHEATING

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1) Non-Physical eq.:

$$\ddot{u}_{ij}(\mathbf{x}, t) + 3H\dot{u}_{ij}(\mathbf{x}, t) - \frac{\nabla^2}{a^2} u_{ij}(\mathbf{x}, t) = \frac{2}{m_p^2} \{ \phi^a_{,i} \phi^a_{,j} \}(\mathbf{x}, t)$$

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

Building the Solution:
$$\begin{cases} h_{ij}(\mathbf{k}, t) = \Lambda_{ij,lm}(\hat{\mathbf{k}}) u_{lm}(\mathbf{k}, t) \\ u_{lm}(\mathbf{k}, t) = \int_{t_0}^t dt' G(t - t') \Pi_{lm}^{\text{eff}}(\mathbf{k}, t') \end{cases}$$

1) Non-Physical eq.:

$$\ddot{u}_{ij}(\mathbf{x}, t) + 3H\dot{u}_{ij}(\mathbf{x}, t) - \frac{\nabla^2}{a^2} u_{ij}(\mathbf{x}, t) = \frac{2}{m_p^2} \{ \phi^a_{,i} \phi^a_{,j} \}(\mathbf{x}, t)$$

2) Fourier transform: $u_{ij}(\mathbf{x}, t) \rightarrow u_{ij}(\mathbf{k}, t)$

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

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INFLATIONARY PREHEATING

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Only when needed !

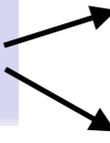
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INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
out-Eq

Outputs: $\rho_{GW} = \frac{1}{32\pi G} \frac{1}{L^3} \int d^3\mathbf{x} \dot{h}_{ij} \dot{h}_{ij} = \frac{1}{32\pi G} \frac{1}{L^3} \int d^3\mathbf{k} |\dot{h}_{ij}(t, \mathbf{k})|^2$

INFLATIONARY PREHEATING

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$$\rho_{GW} = \frac{1}{32\pi G L^3} \times \int k^2 dk \int d\Omega \Lambda_{ij,lm}(\hat{\mathbf{k}}) \dot{u}_{ij}(t, \mathbf{k}) \dot{u}_{lm}^*(t, \mathbf{k})$$

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics

non-linear
out-Eq

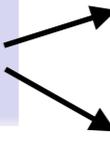
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INFLATIONARY PREHEATING

Lattice Simulations: Dynamics  **non-linear**
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INFLATIONARY PREHEATING

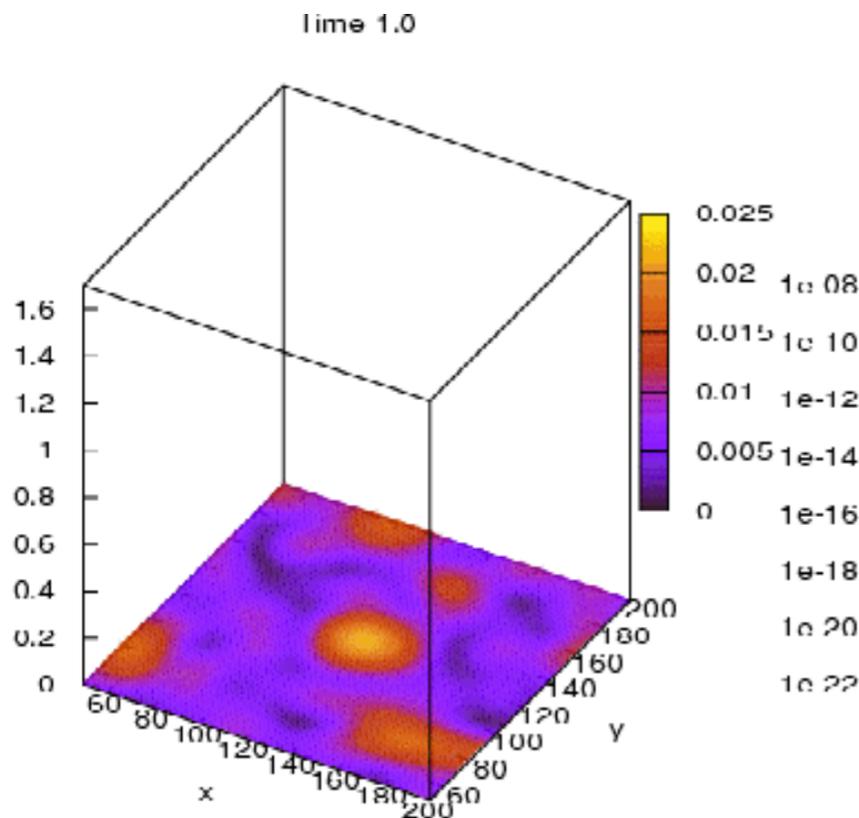
Lattice Simulations: Dynamics

non-linear
out-Eq

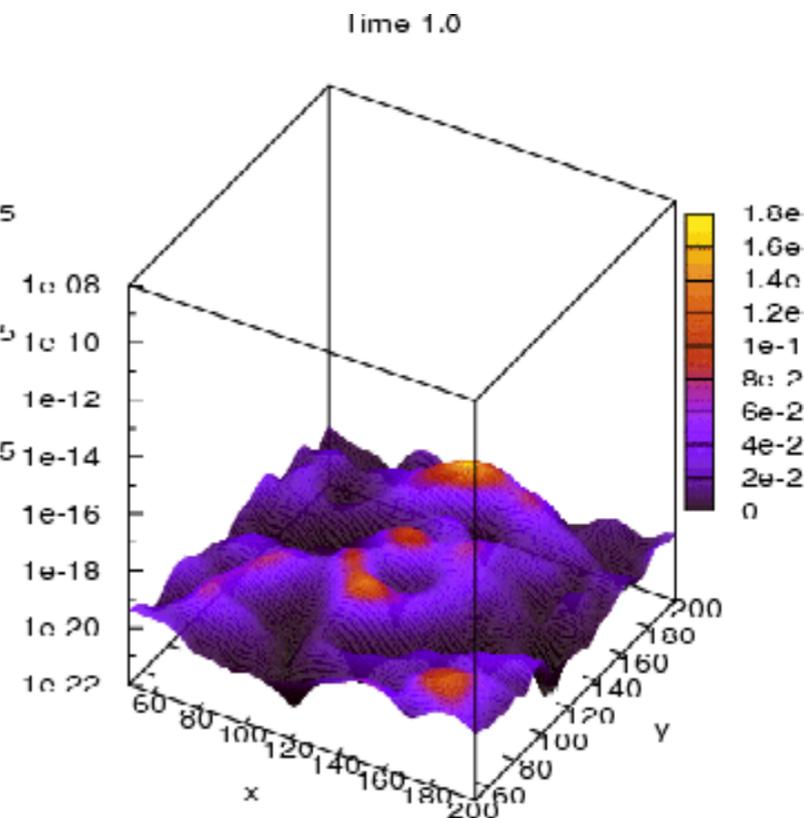
Hybrid Preheating

$$V(\phi, \chi) = \frac{\lambda}{4} (|\chi|^2 - v^2)^2 + \frac{1}{2} |\chi|^2 \phi^2 + V(\phi)$$

Animation by
Alfonso Sastre



Higgs



GW (Energy density)

INFLATIONARY PREHEATING

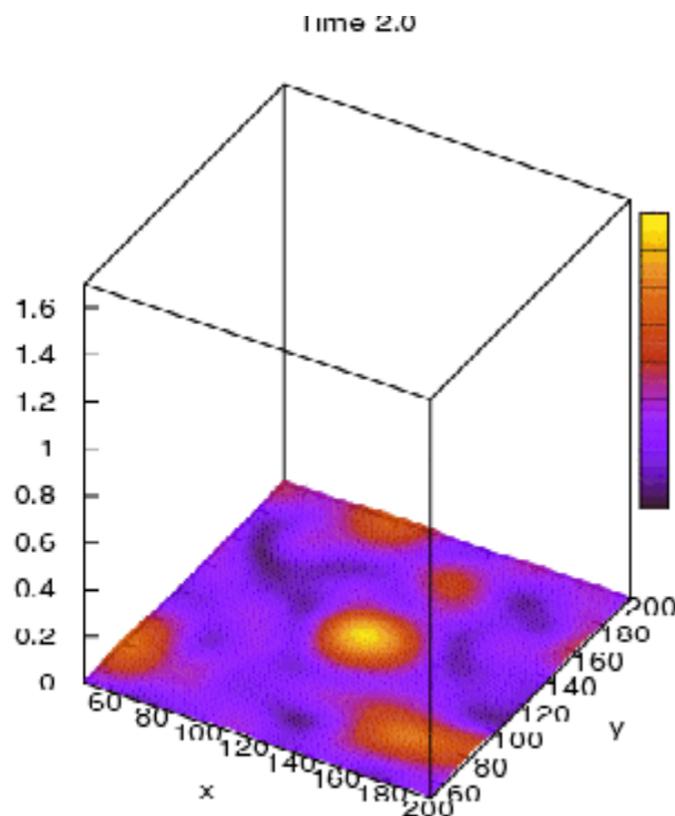
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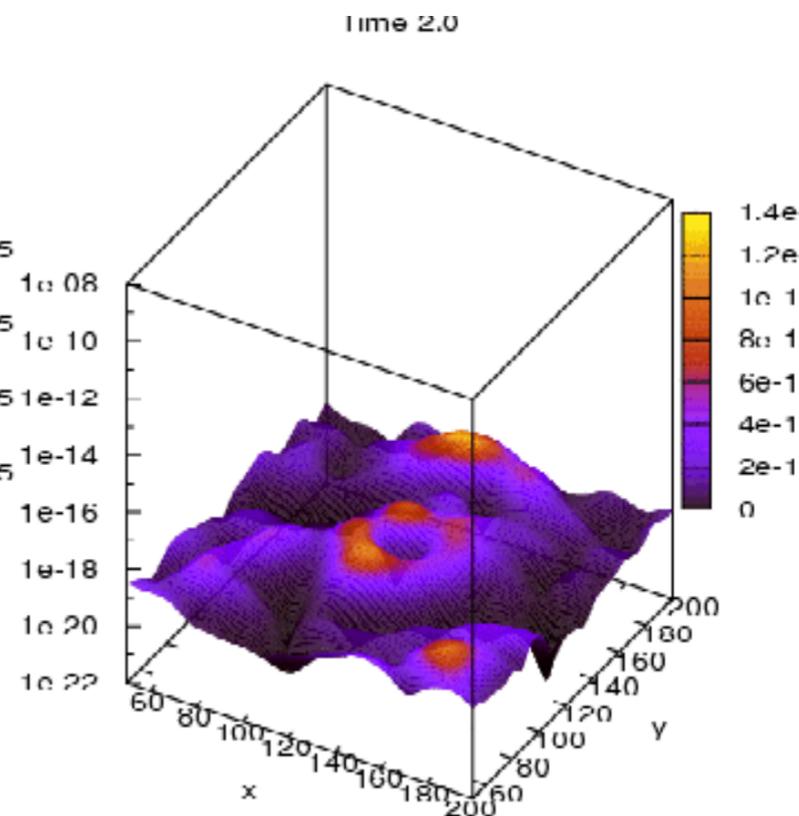
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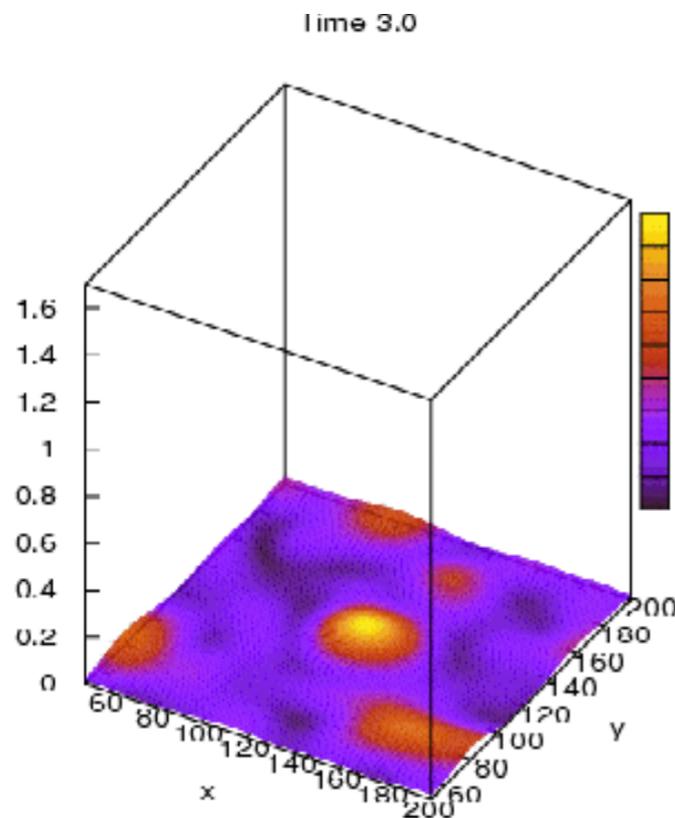
Lattice Simulations: Dynamics

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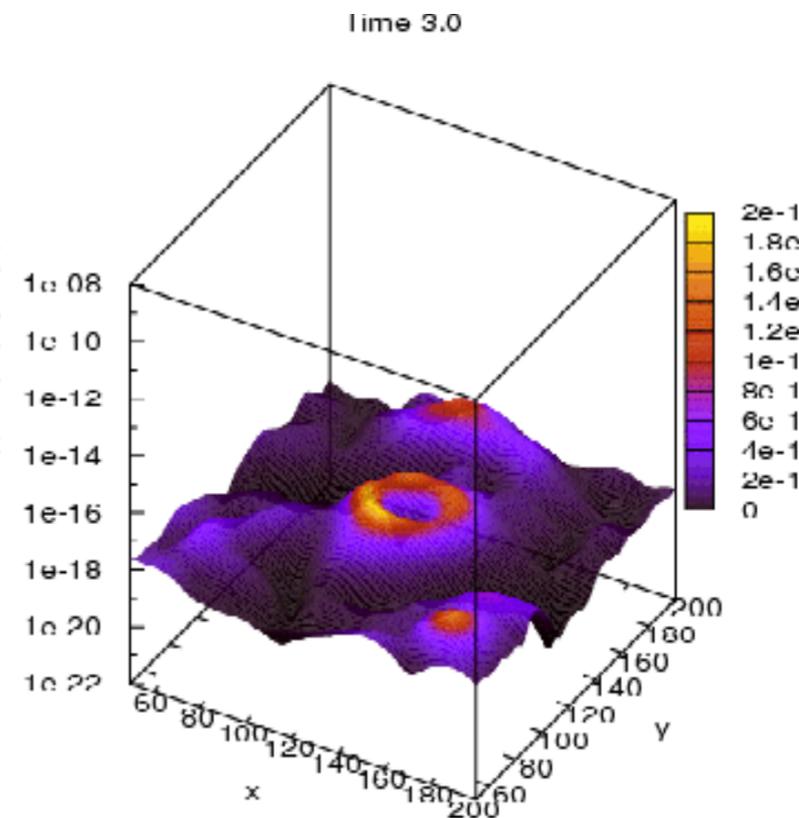
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GW (Energy density)

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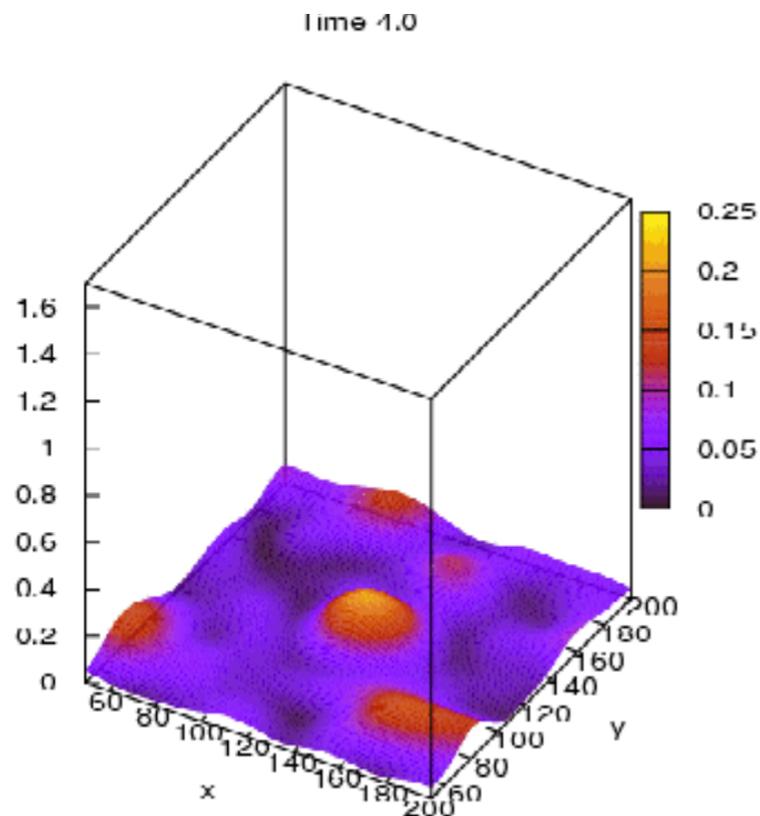
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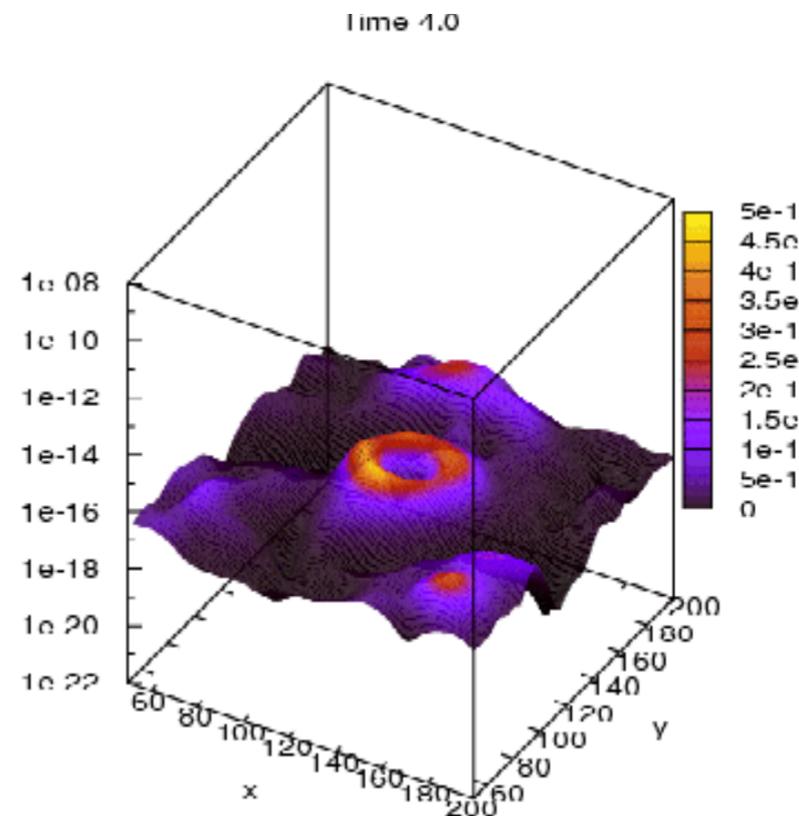
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GW (Energy density)

INFLATIONARY PREHEATING

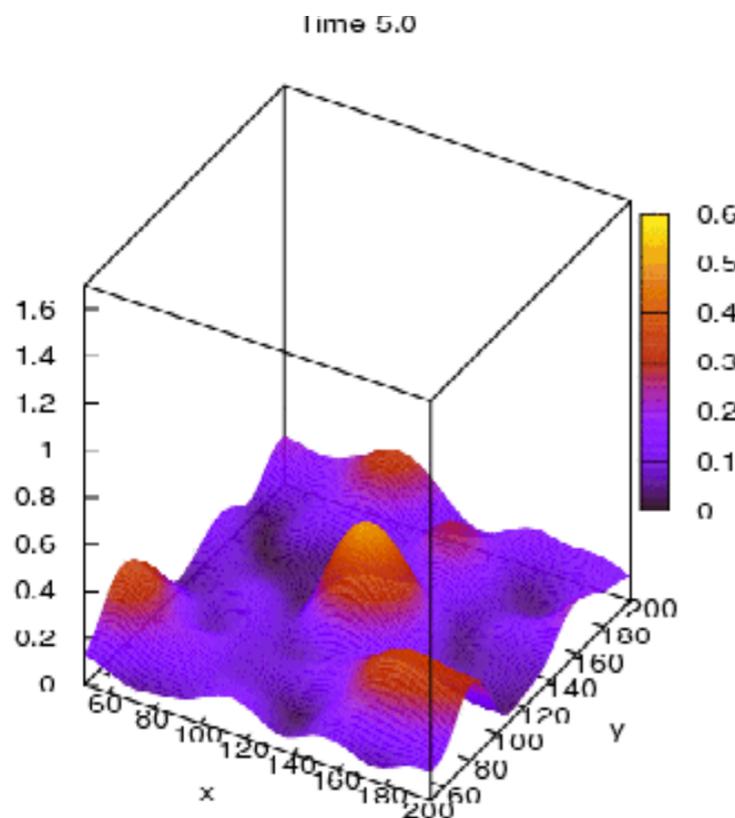
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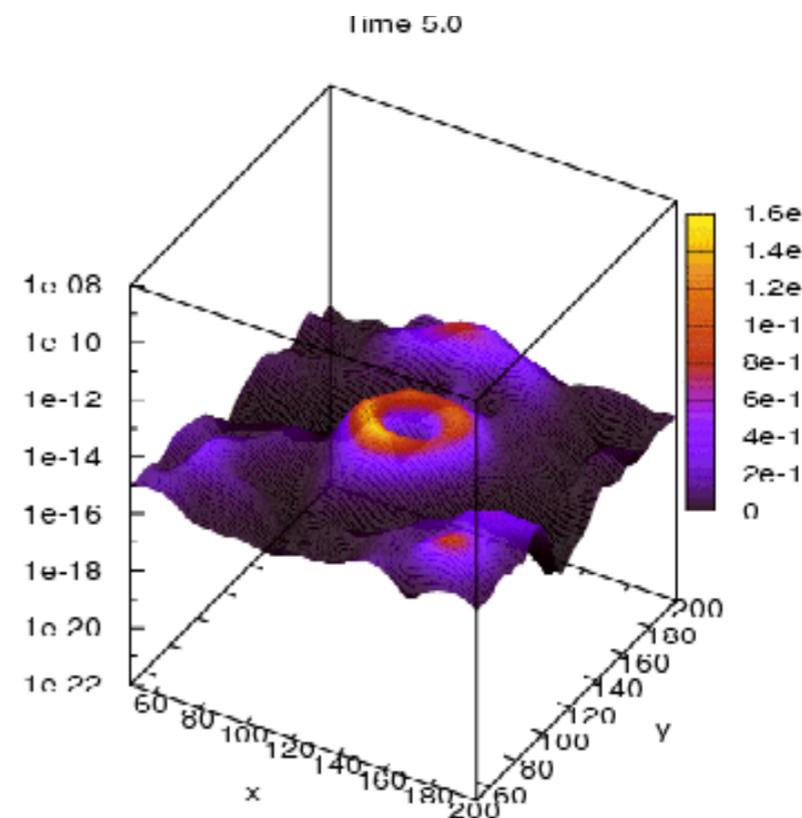
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Higgs



GW (Energy density)

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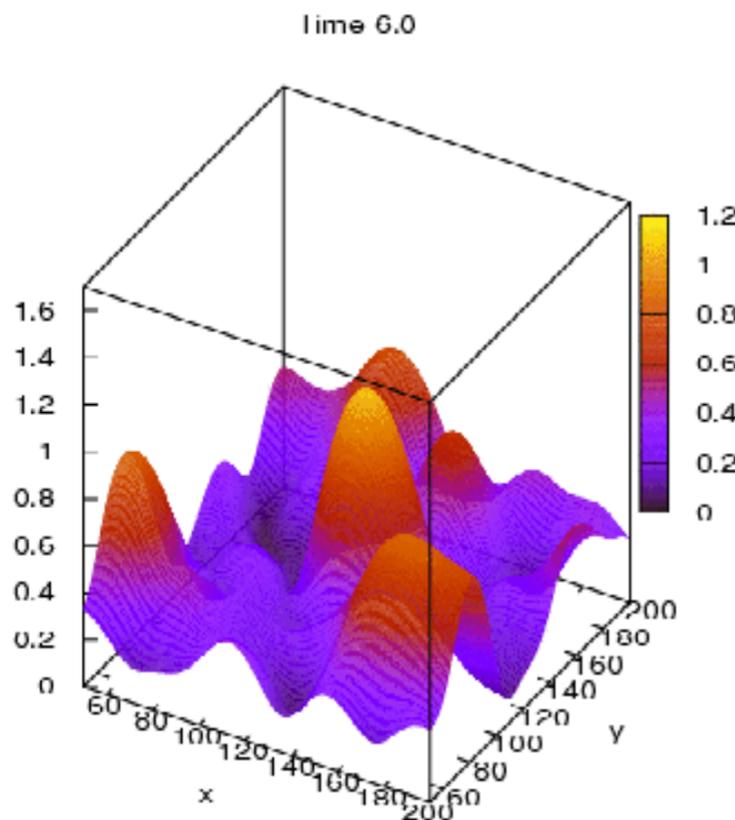
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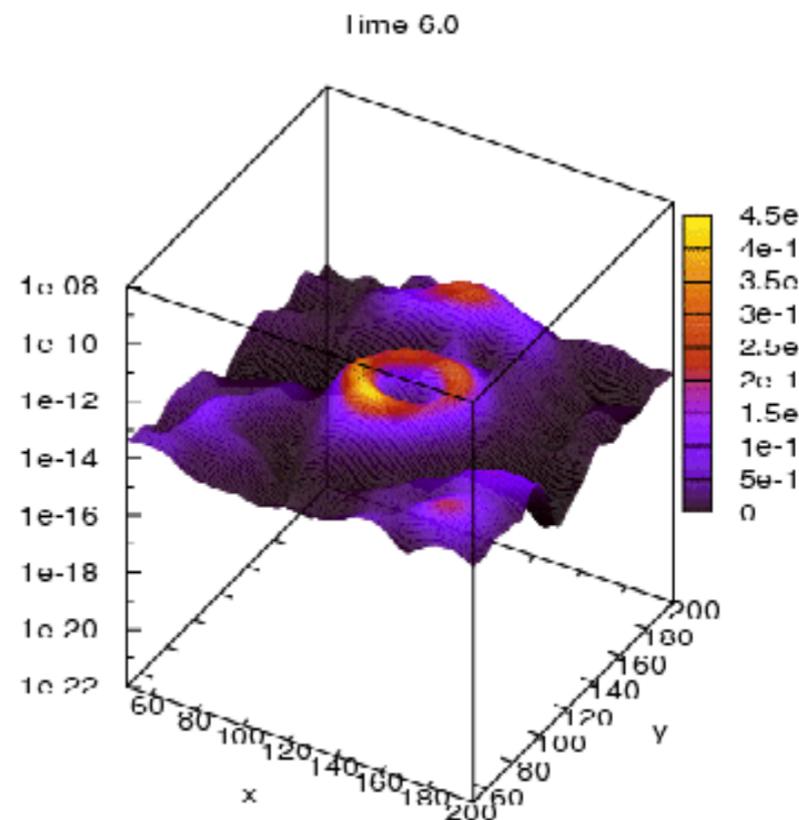
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Higgs



GW (Energy density)

INFLATIONARY PREHEATING

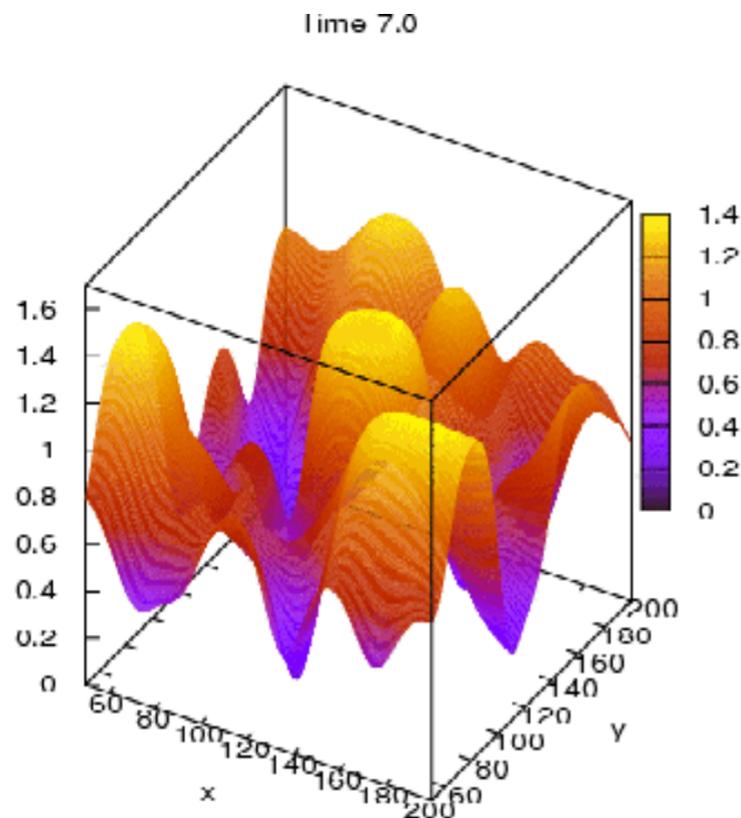
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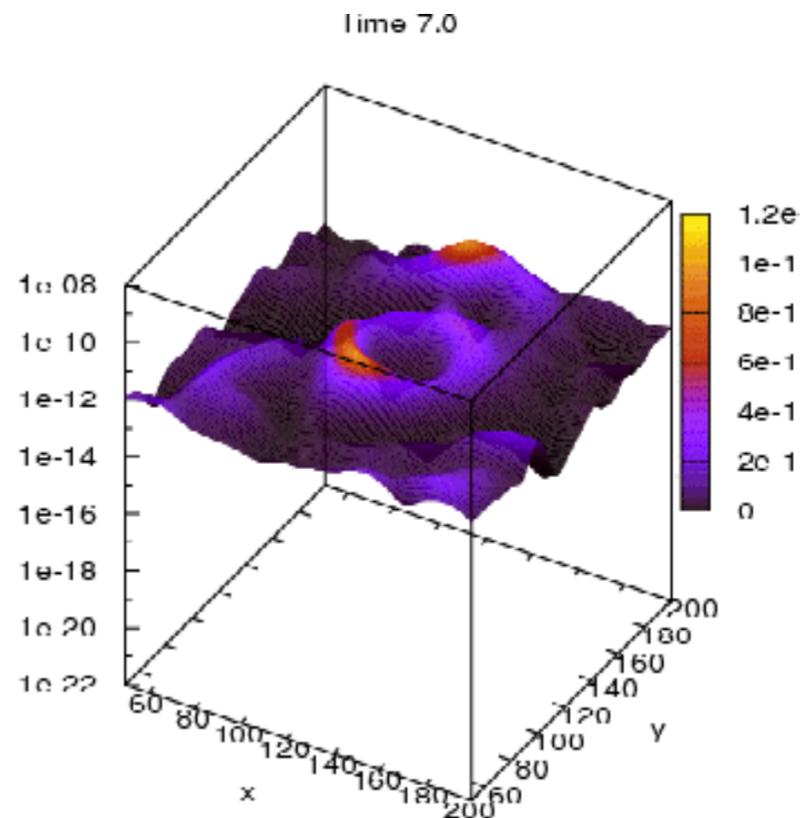
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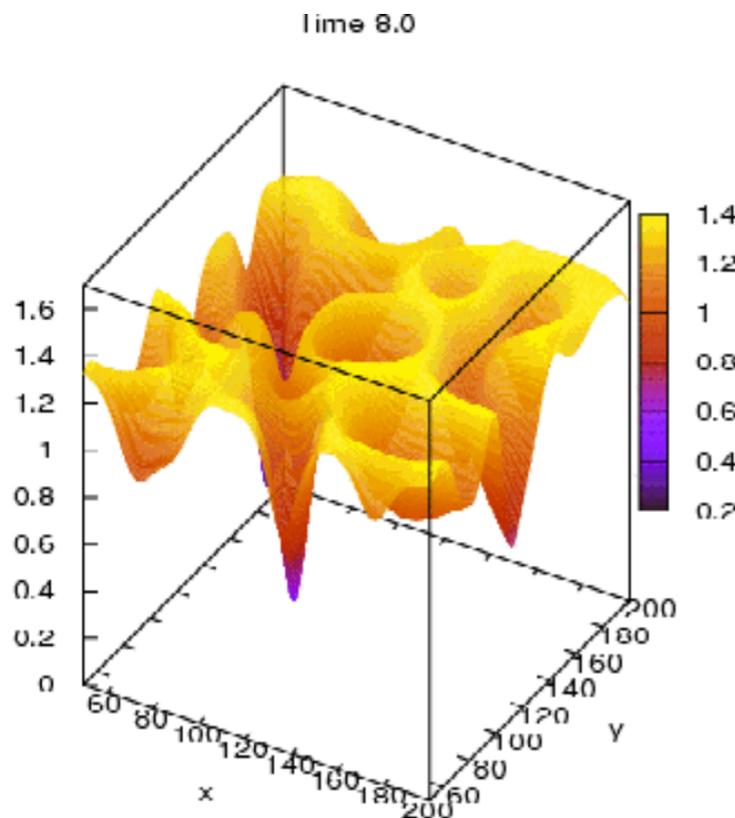
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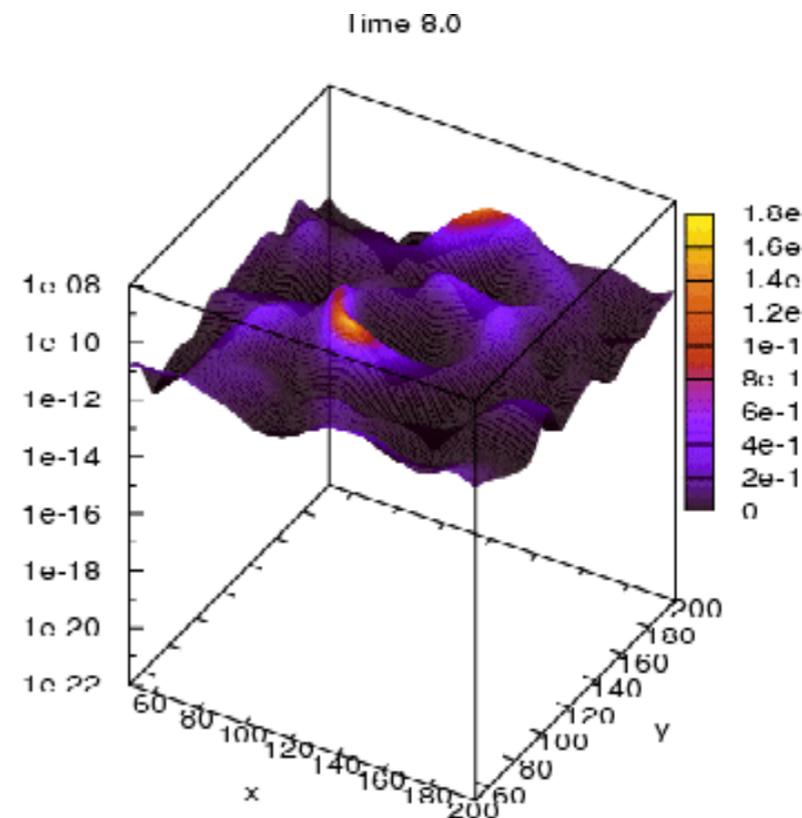
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GW (Energy density)

INFLATIONARY PREHEATING

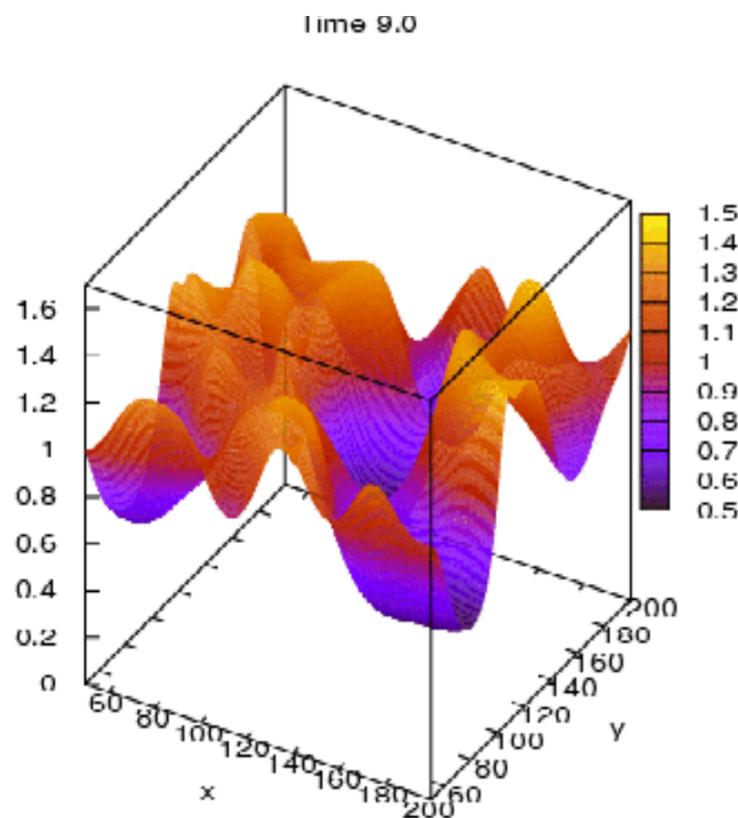
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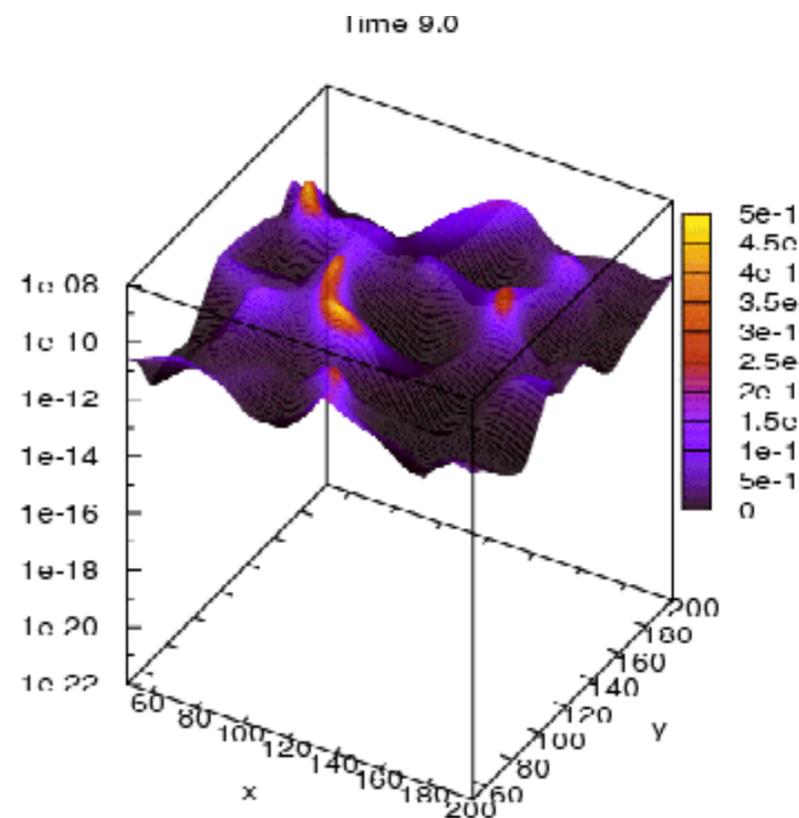
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GW (Energy density)

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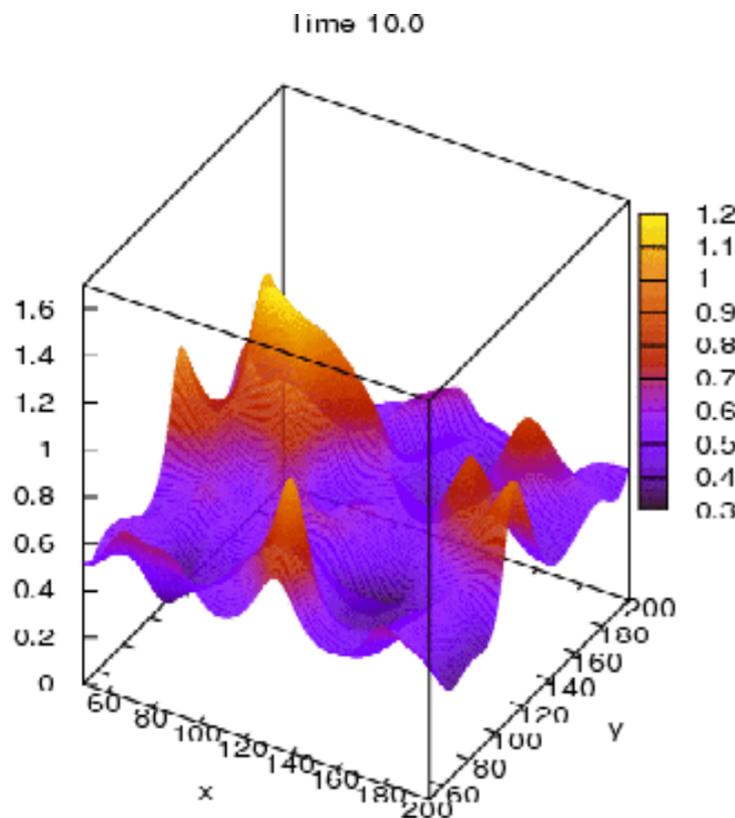
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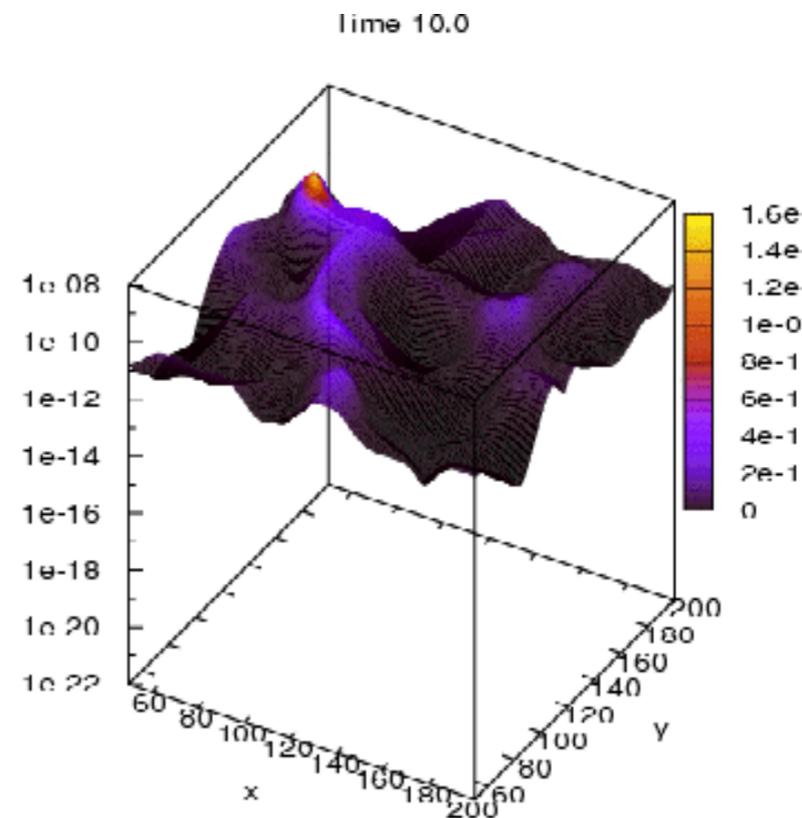
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Higgs



GW (Energy density)

INFLATIONARY PREHEATING

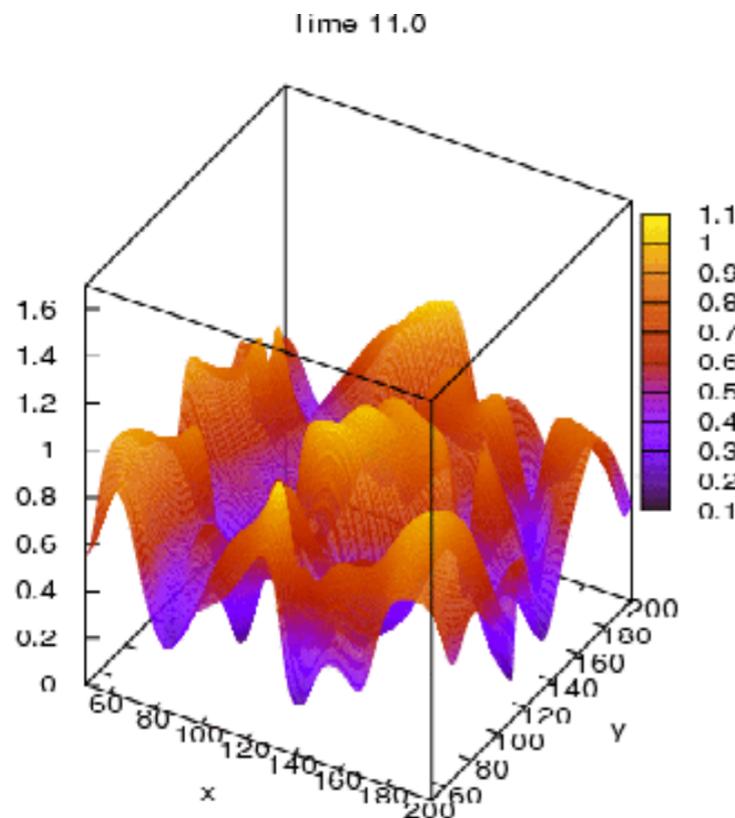
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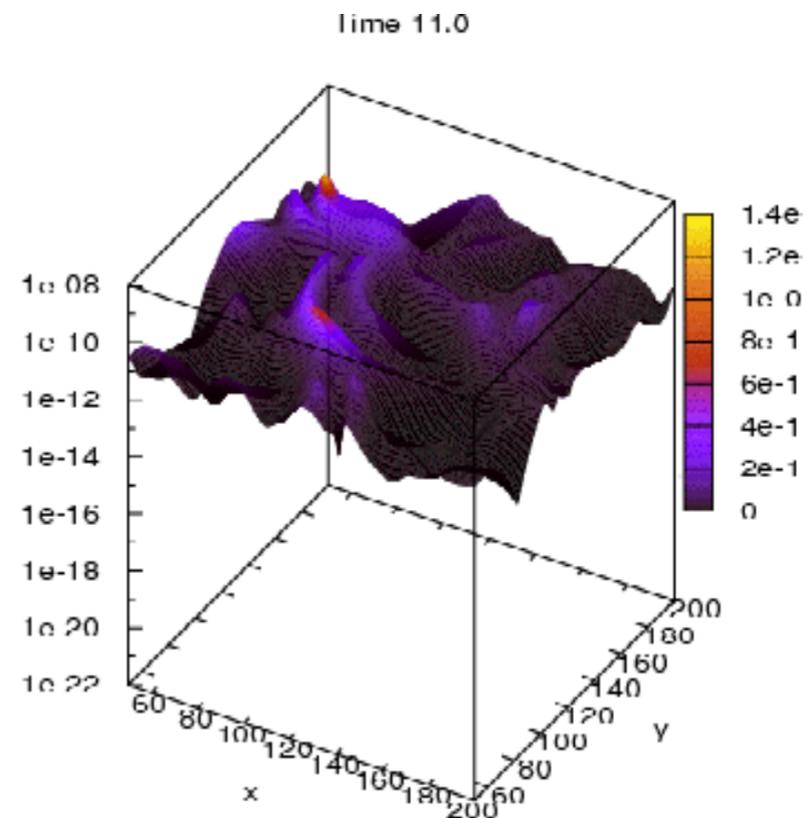
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Animation by
Alfonso Sastre



Higgs



GW (Energy density)

INFLATIONARY PREHEATING

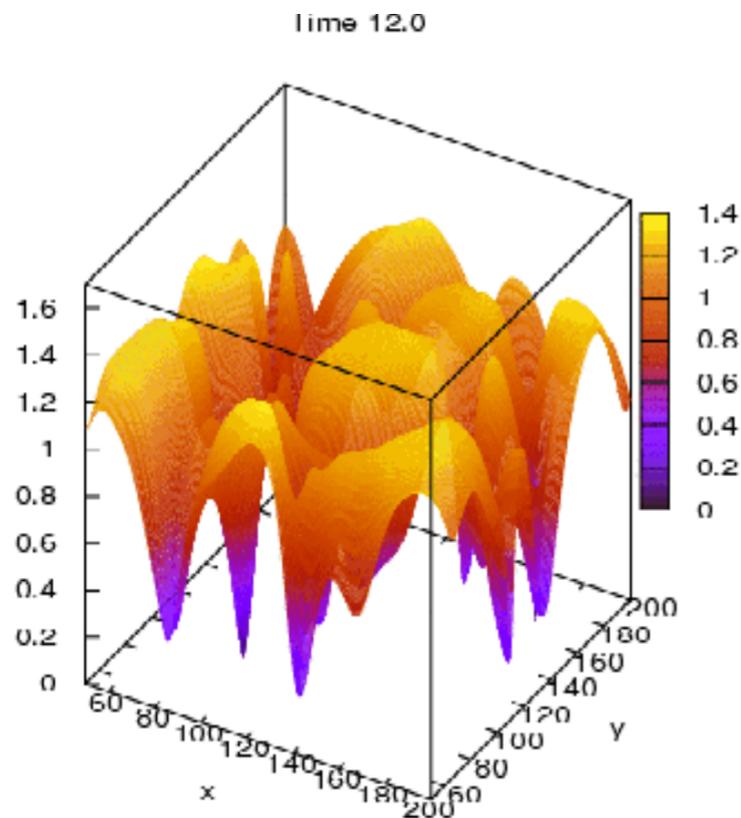
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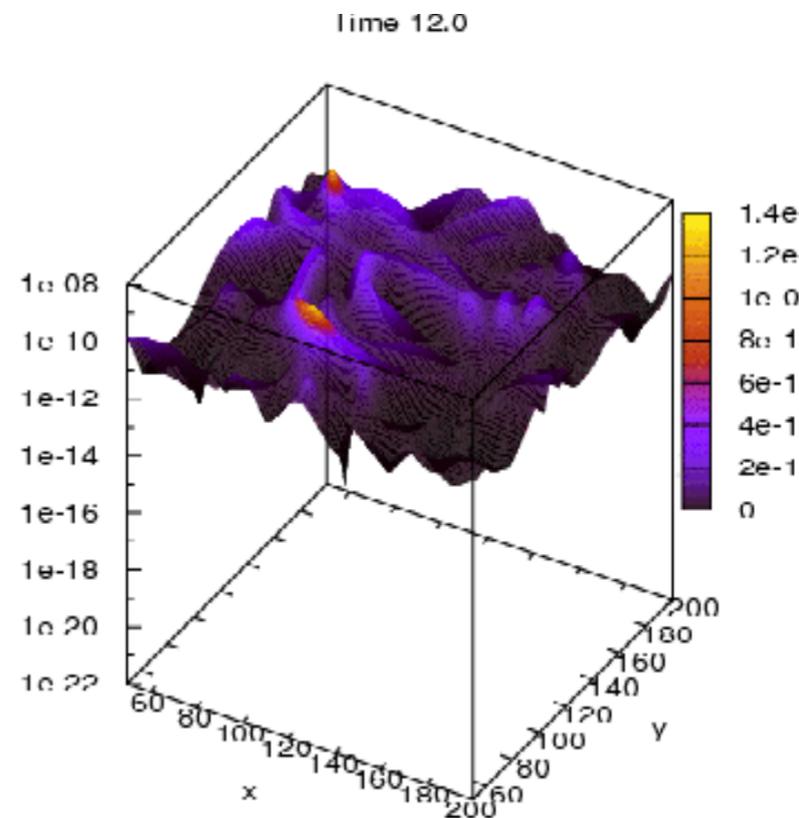
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Animation by
Alfonso Sastre



Higgs



GW (Energy density)

INFLATIONARY PREHEATING

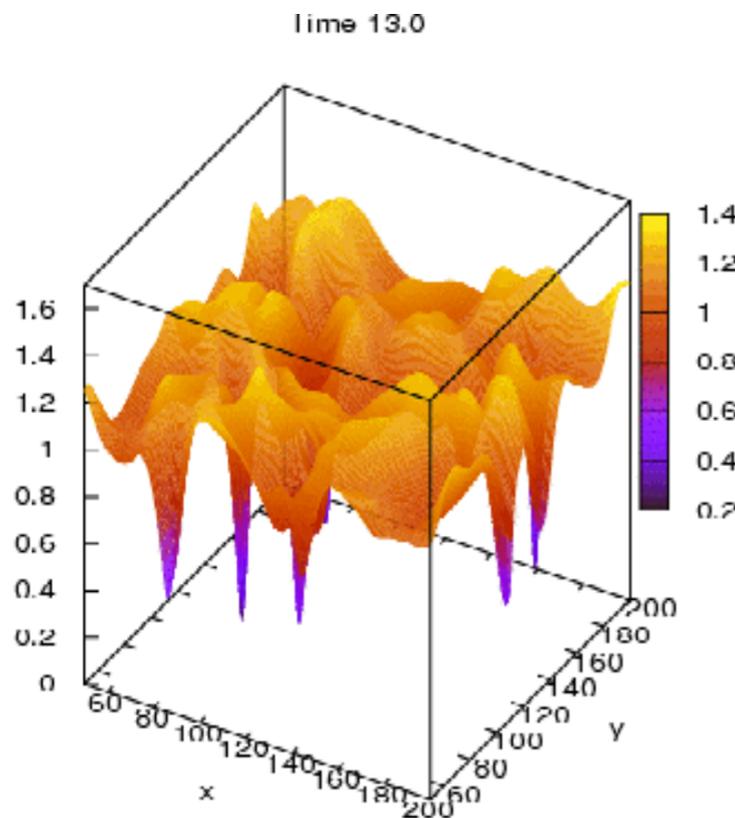
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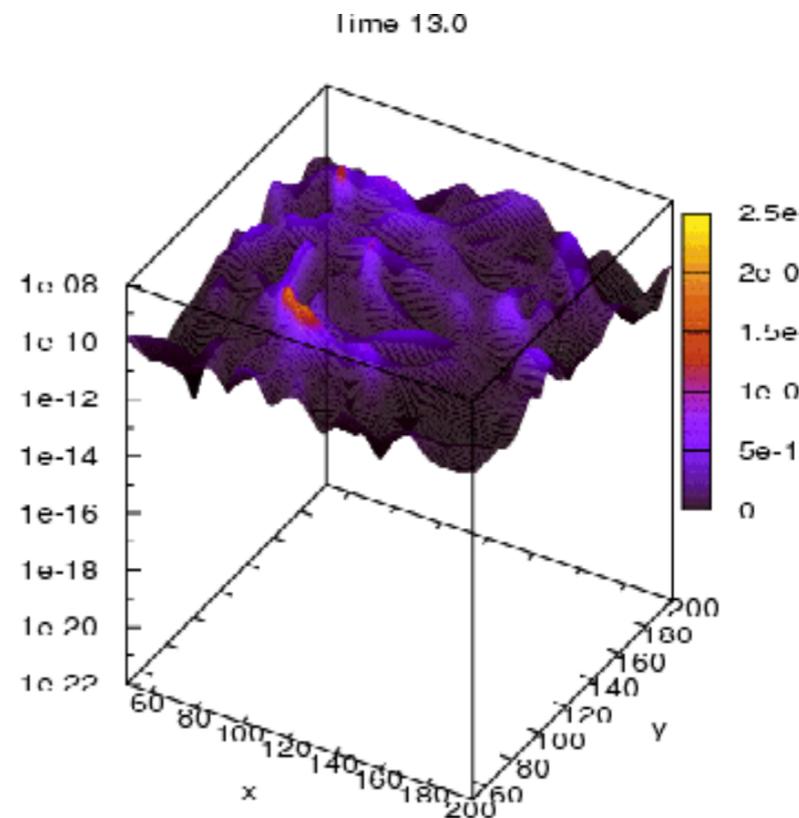
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GW (Energy density)

INFLATIONARY PREHEATING

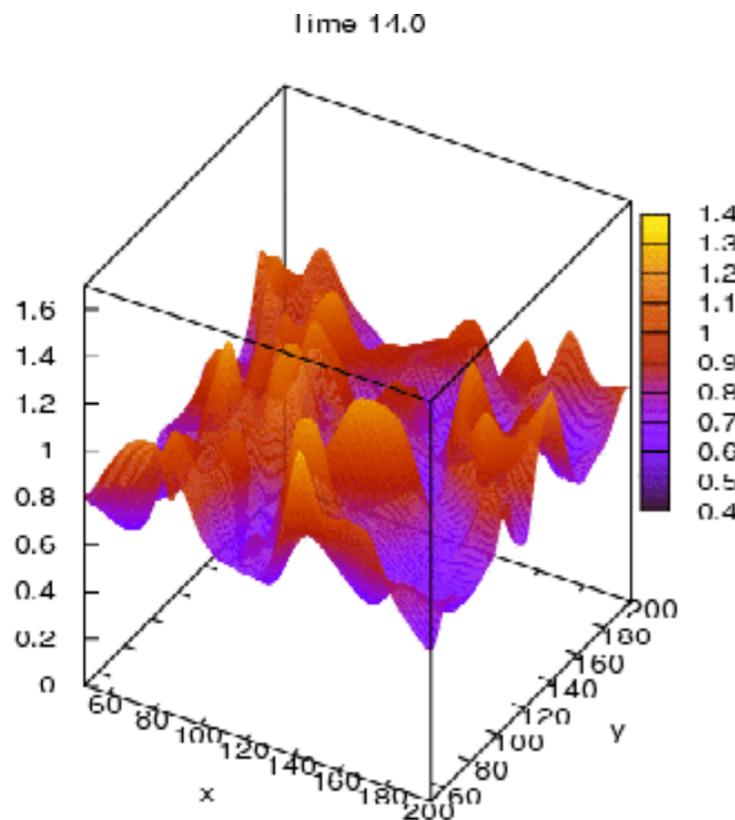
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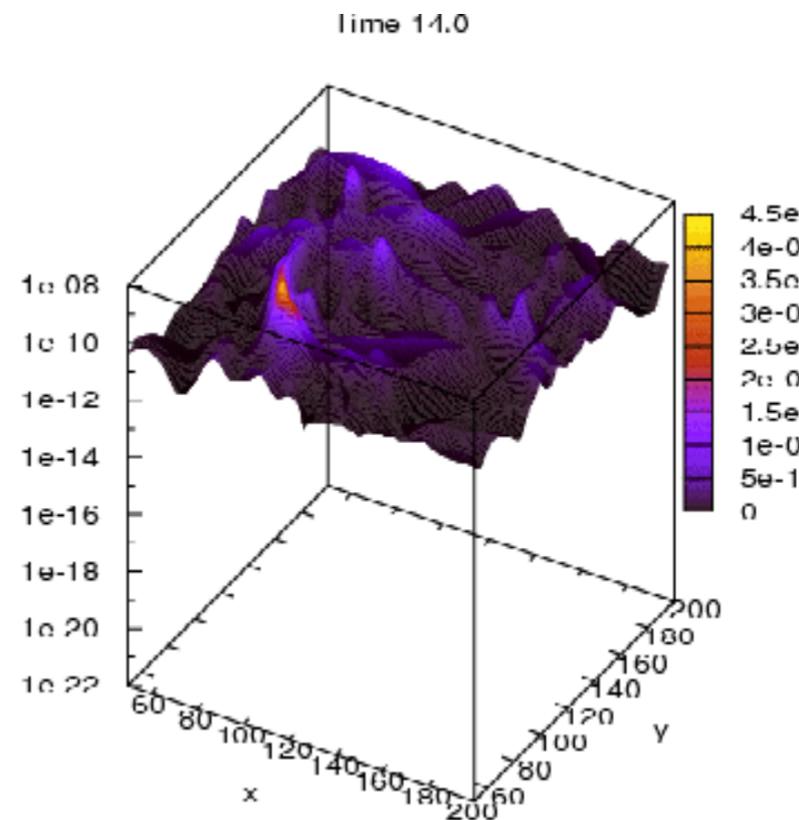
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Higgs



GW (Energy density)

INFLATIONARY PREHEATING

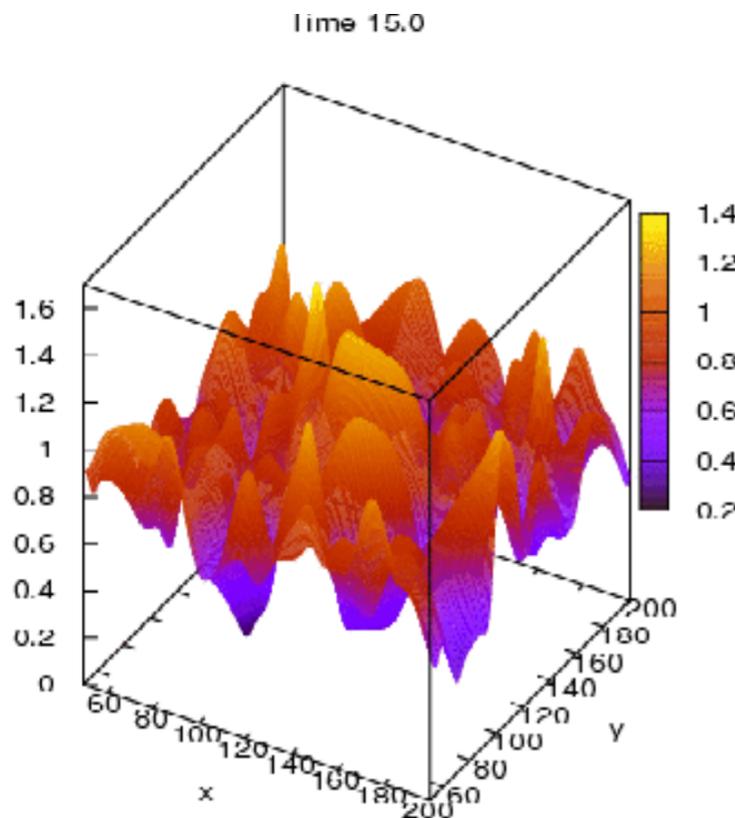
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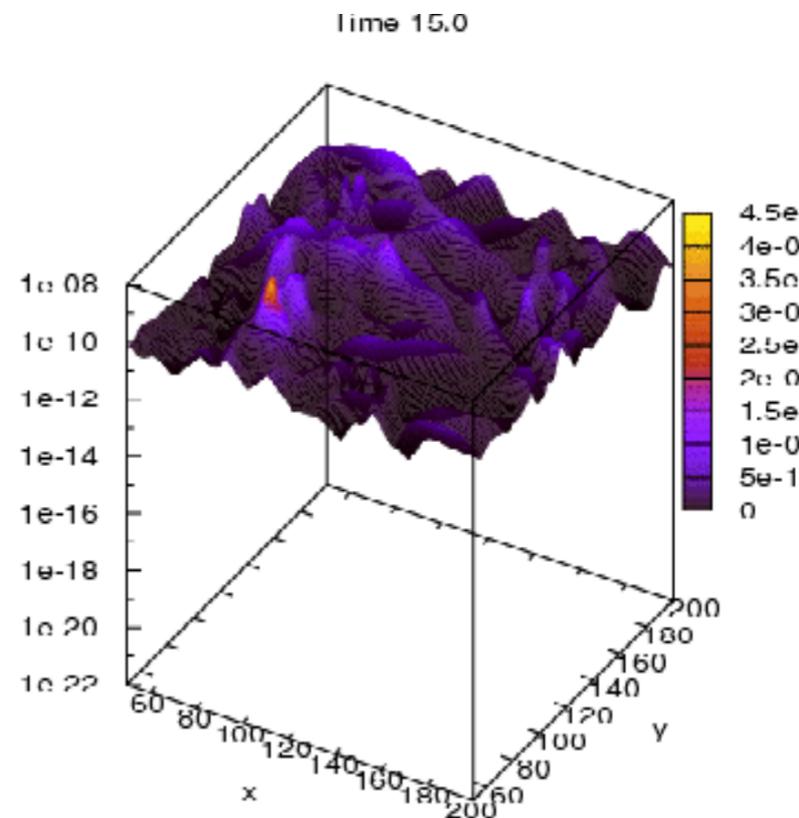
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Animation by
Alfonso Sastre



Higgs



GW (Energy density)

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics

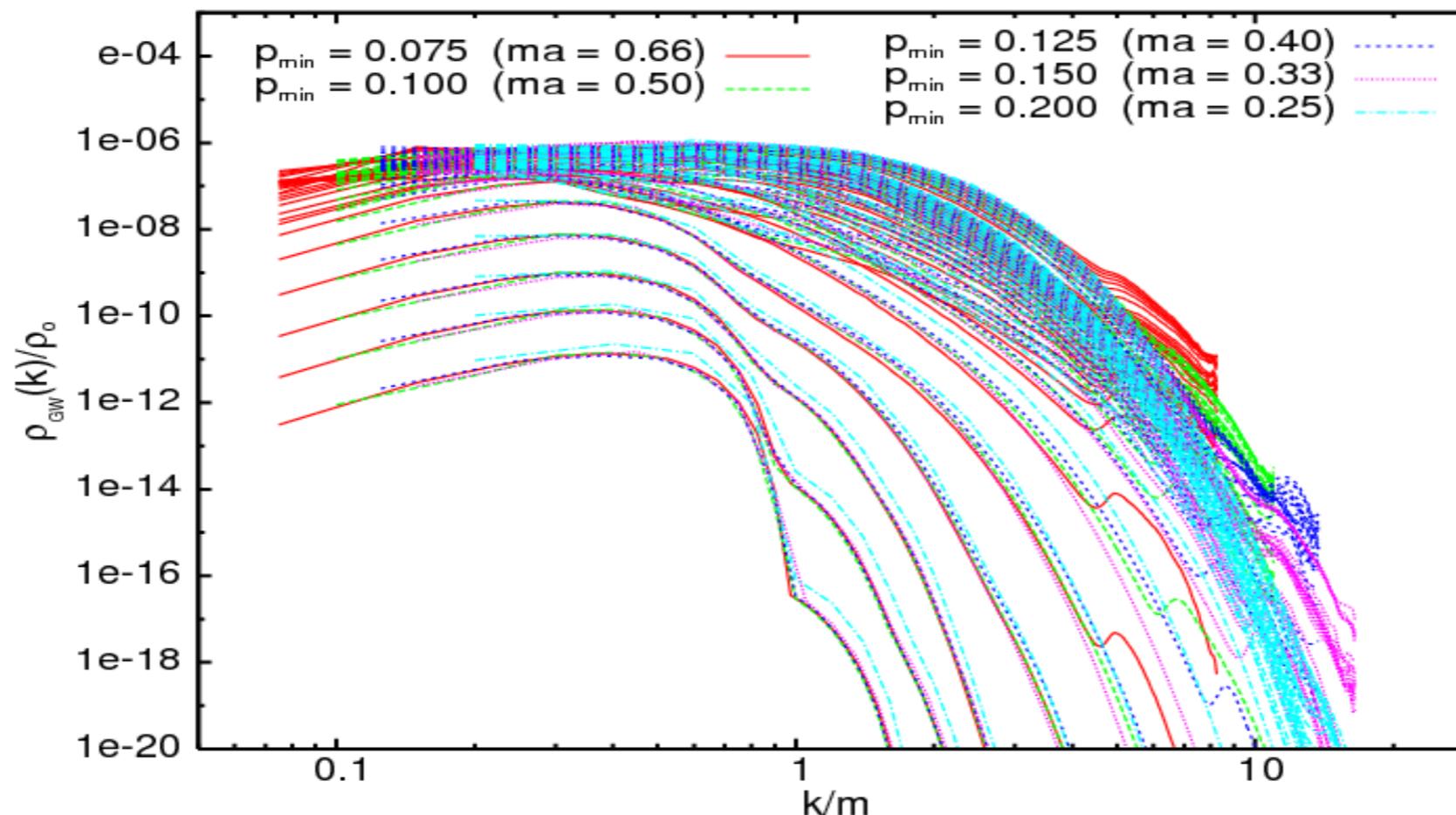
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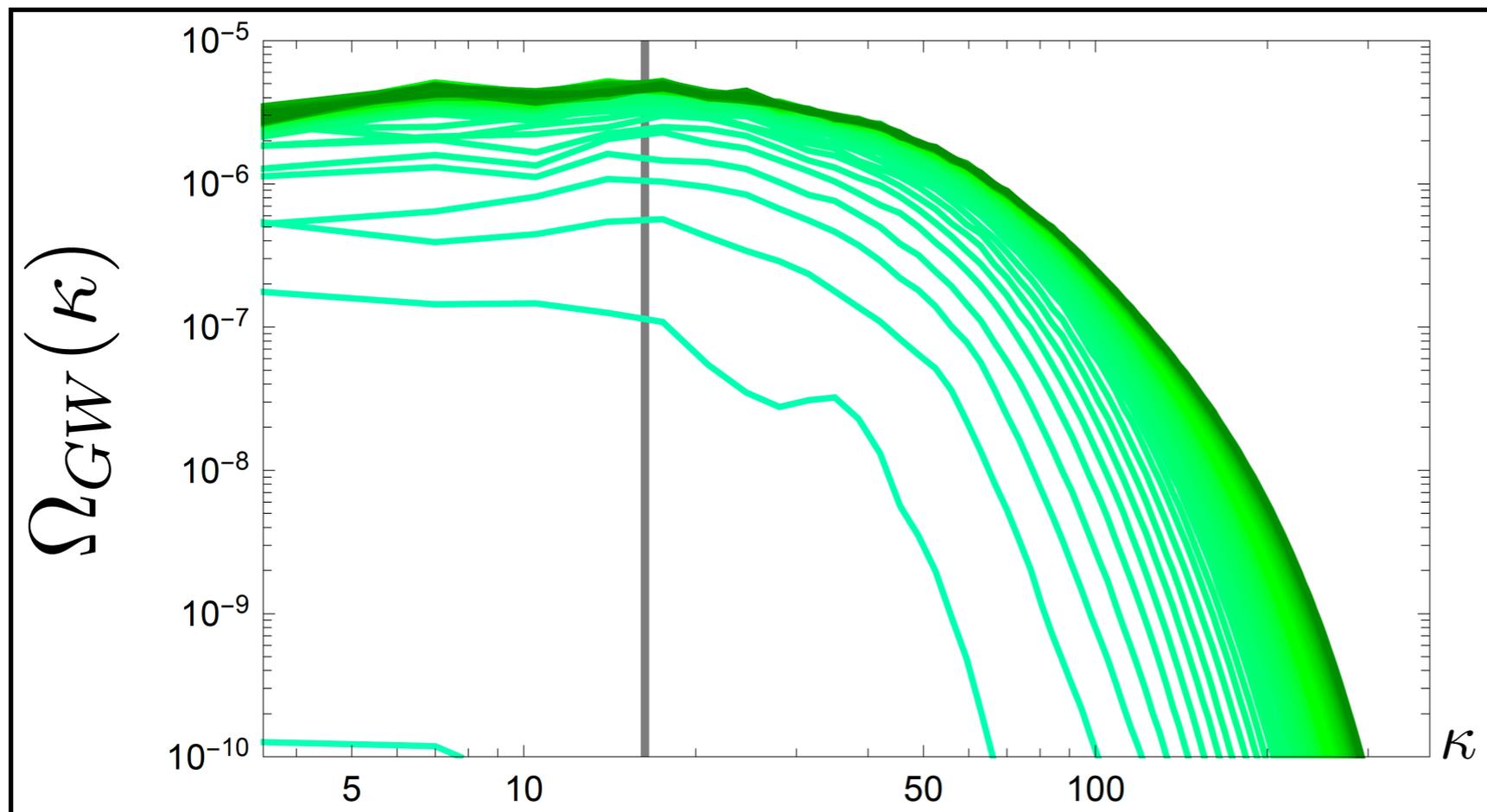
3 stages: **Exp. Instabilities** → **Non-linearities** → **Relaxation**

$$\frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}}{d \log k}$$



GW Spectrum

Parameter Dependence (Peak amplitude)



(DGF, Torrentí JCAP 2017)

GW Spectrum

Parameter Dependence (Peak amplitude)

Monomial Models: $\Omega_{\text{GW}}^{(o)} \sim A^2 \frac{\omega^6}{\rho m_p^2} q^{-1/2}$

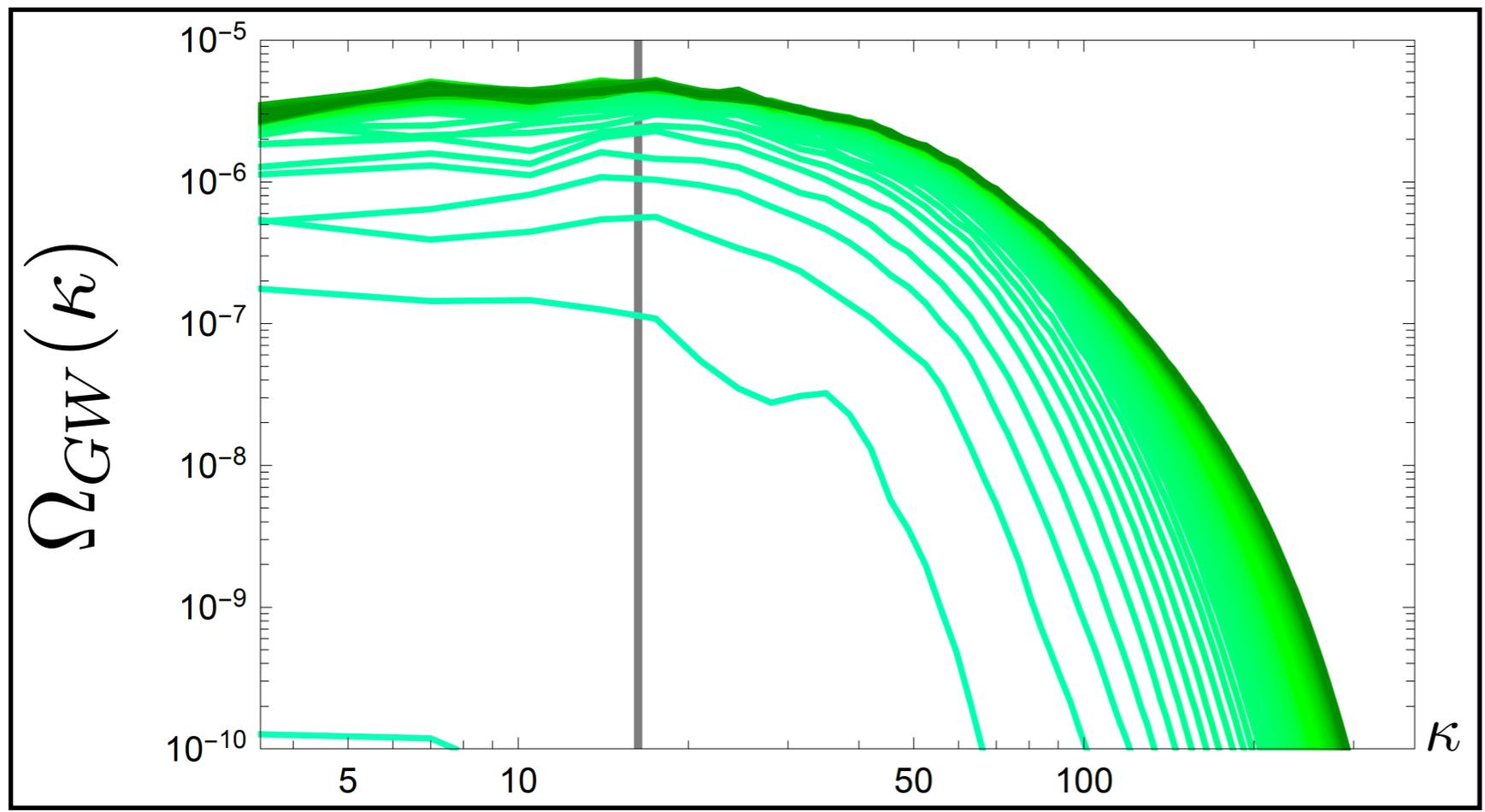
$\omega^2 \equiv V''(\Phi_I)$

$q \equiv \frac{g^2 \Phi_i^2}{\omega^2}$

Resonance Param.

$k_p \propto q^{2/3}$

Peak Position



(DGF, Torrentí JCAP 2017)

GW Spectrum

Parameter Dependence (Peak amplitude)

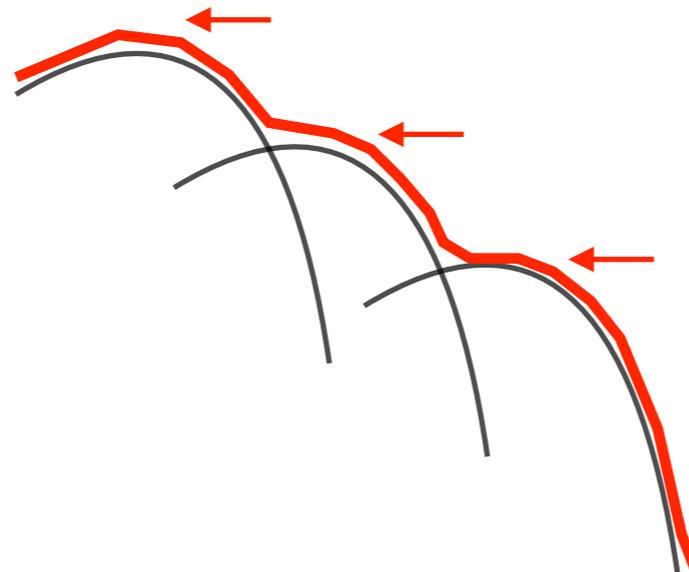
Monomial Models: $\Omega_{\text{GW}}^{(o)} \sim 10^{-9}$,
Large amplitude !

GW Spectrum

Parameter Dependence (Peak amplitude)

Monomial Models: $\Omega_{\text{GW}}^{(o)} \sim 10^{-9}$,
Large amplitude !

$\Omega_{\text{GW}} \propto q^{-1/2}$ \longrightarrow **Spectroscopy of particle couplings ?**

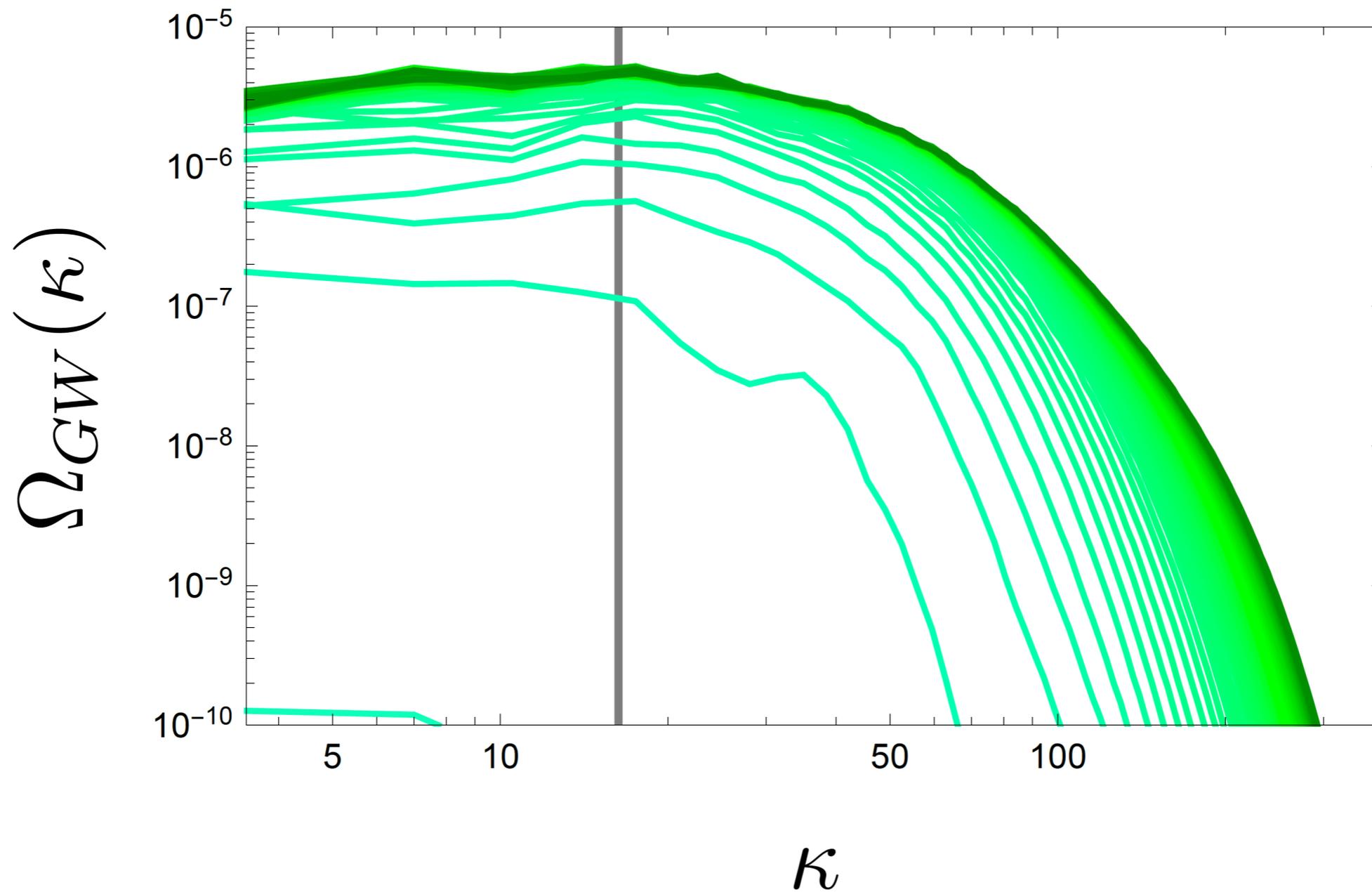


**different couplings
... different peaks ?**

GW Spectroscopy

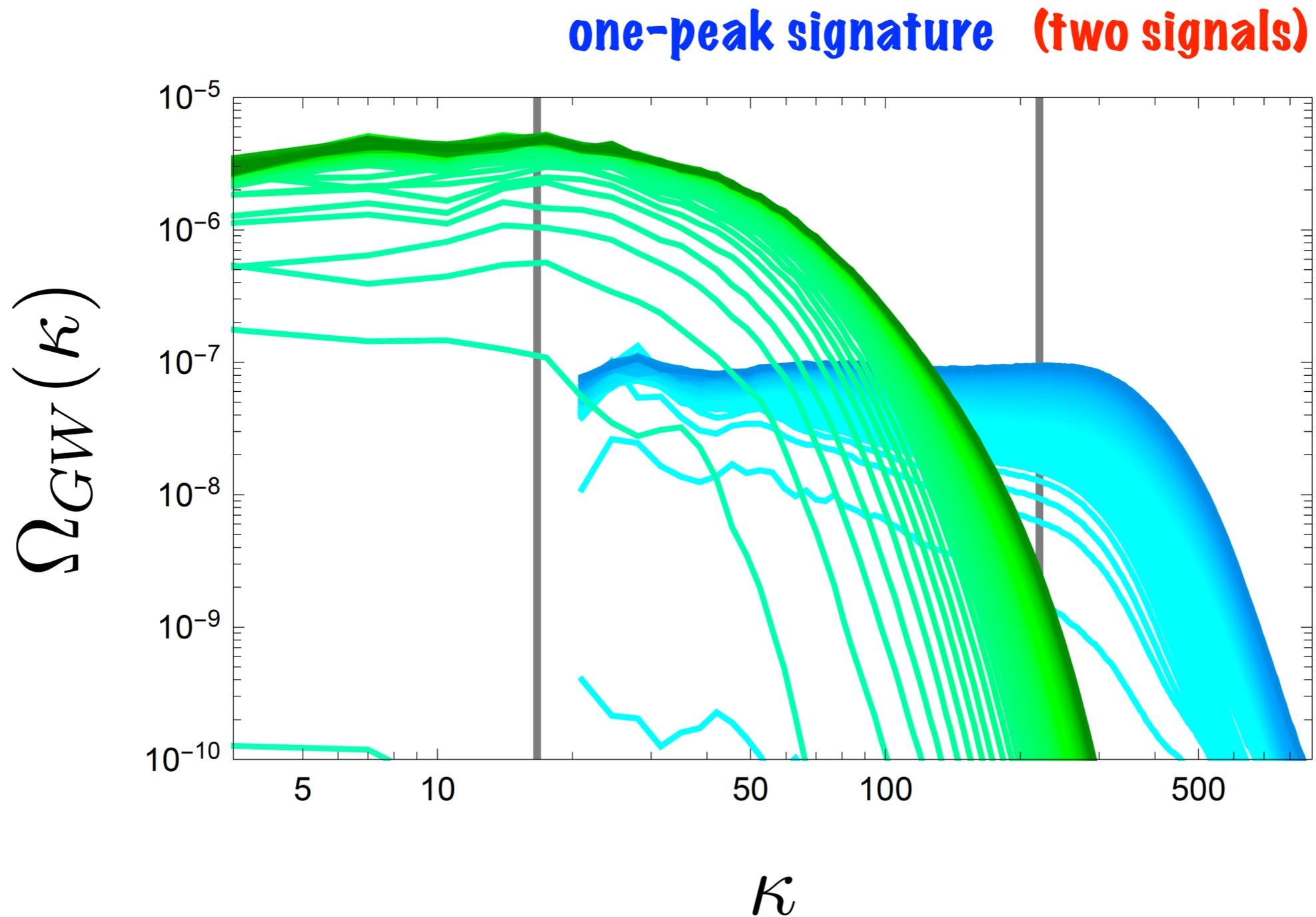
Parameter Dependence (Peak amplitude)

one-peak signature



GW Spectroscopy

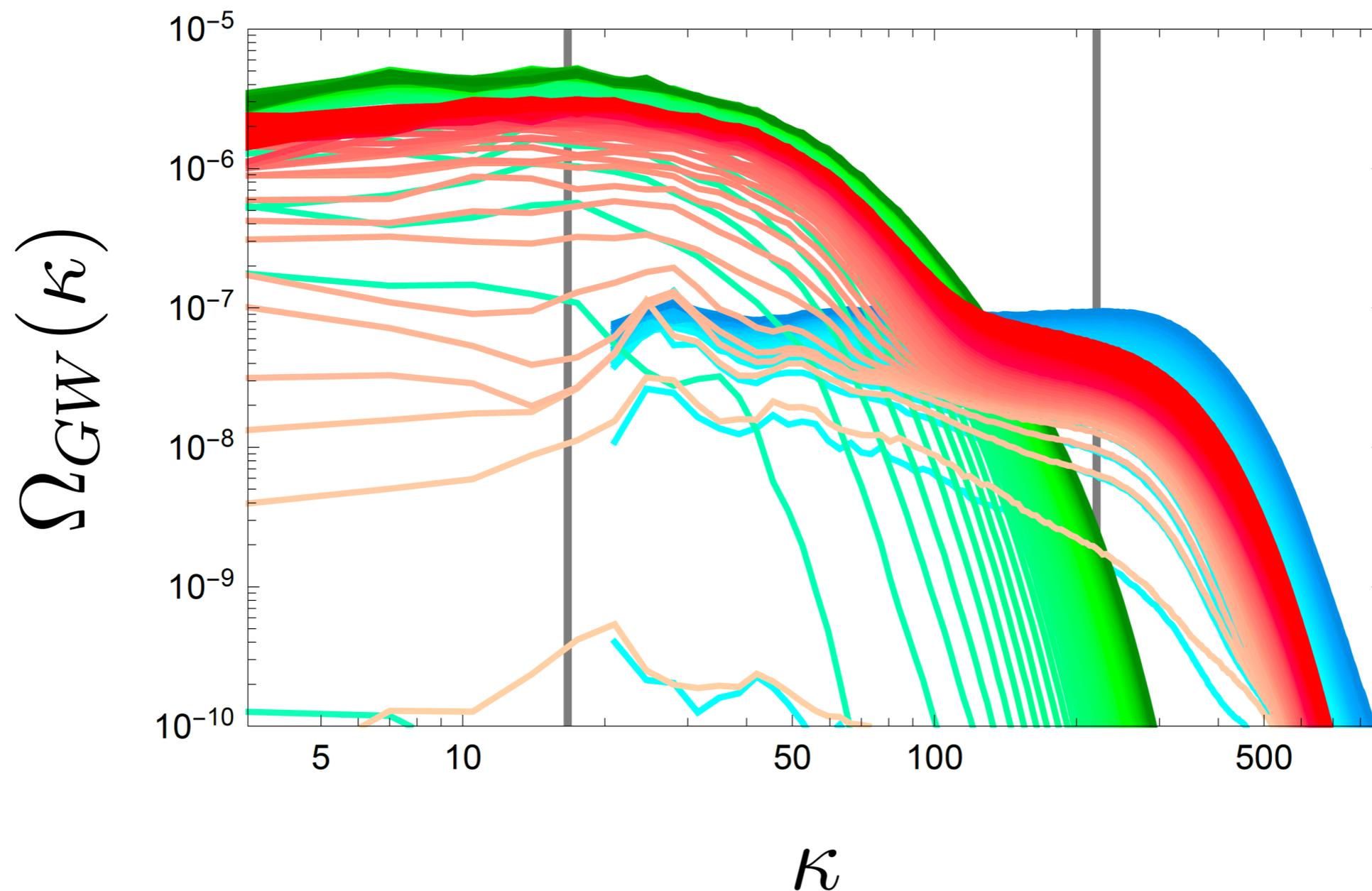
Parameter Dependence (Peak amplitude)



GW Spectroscopy

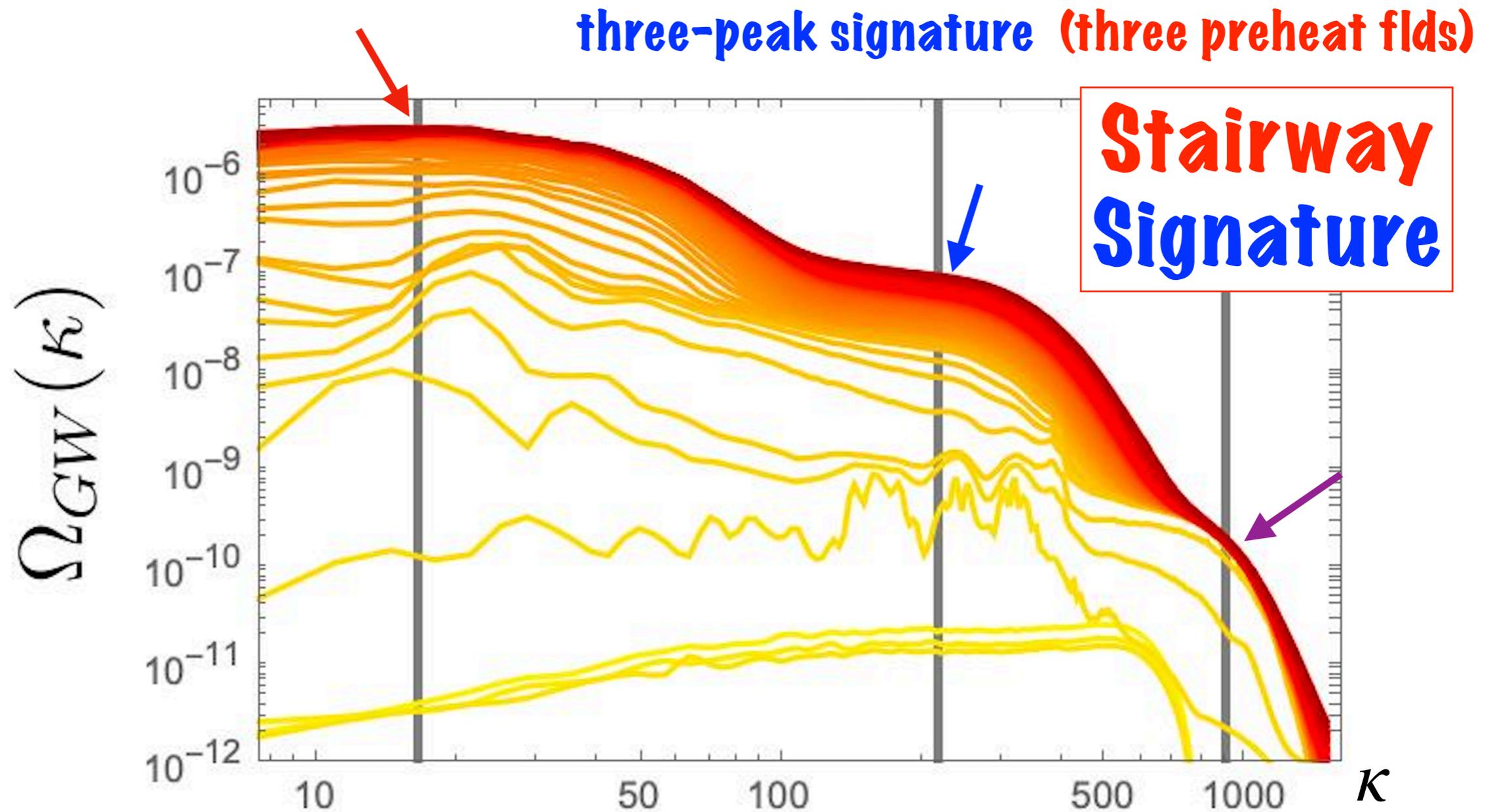
Parameter Dependence (Peak amplitude)

two-peak signature (two preheat flds)



GW Spectroscopy

Parameter Dependence (Peak amplitude)

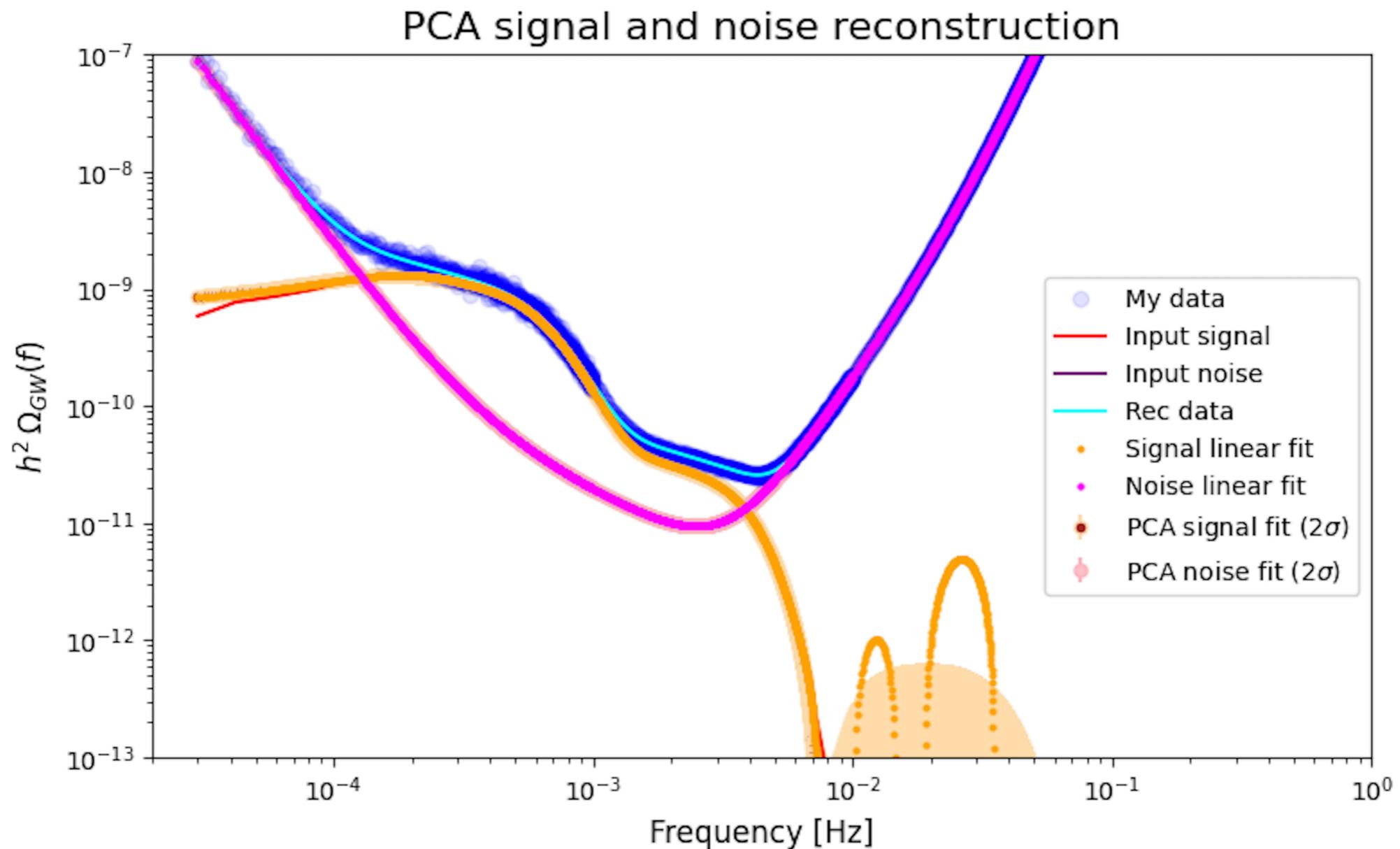


DGF, Florio, Loayza, Pieroni (work in progress)

GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA



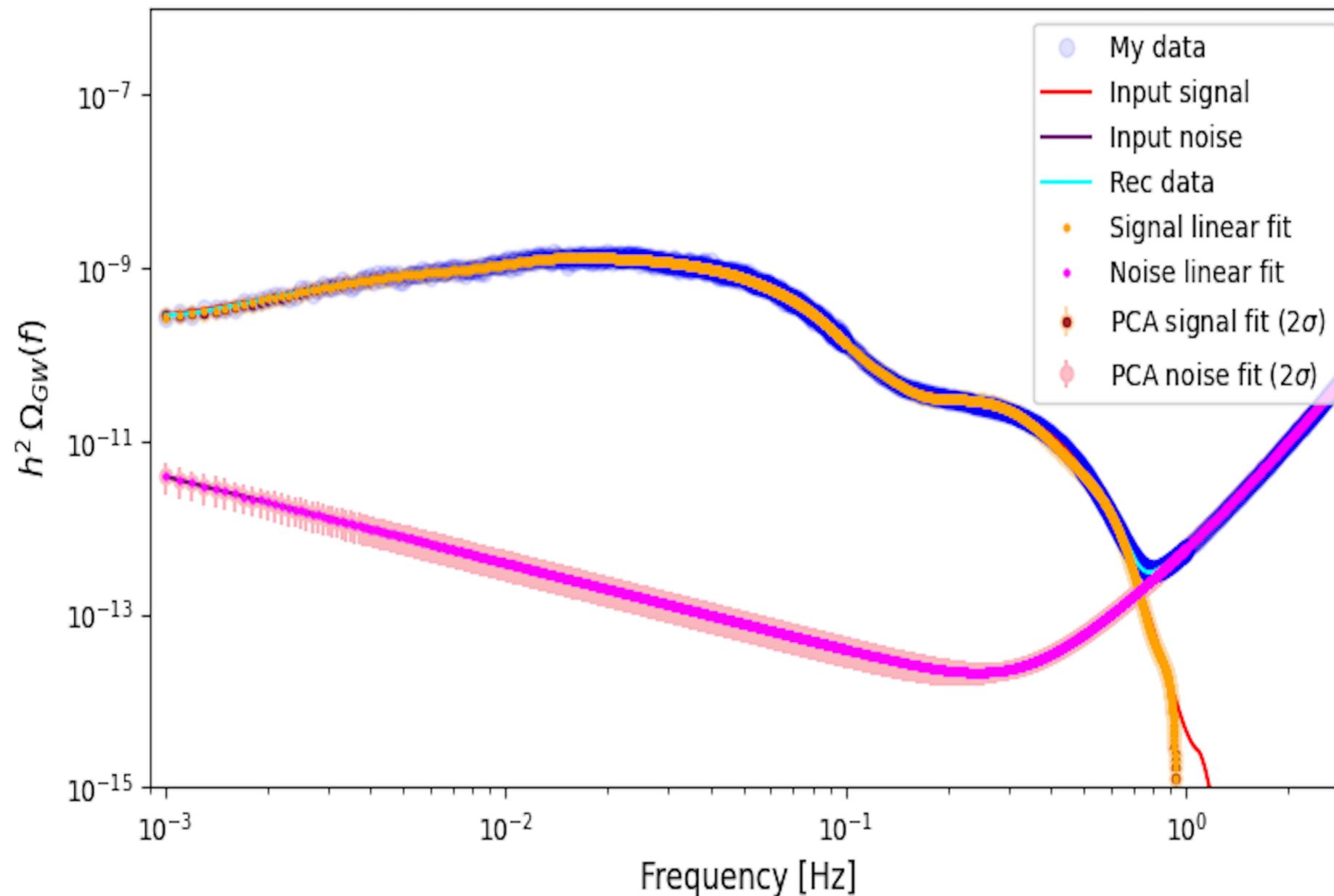
Note: Put by hand at LISA frequencies

GW Spectroscopy

Reconstruction (2-peak signal)

@ BBO

PCA signal and noise reconstruction

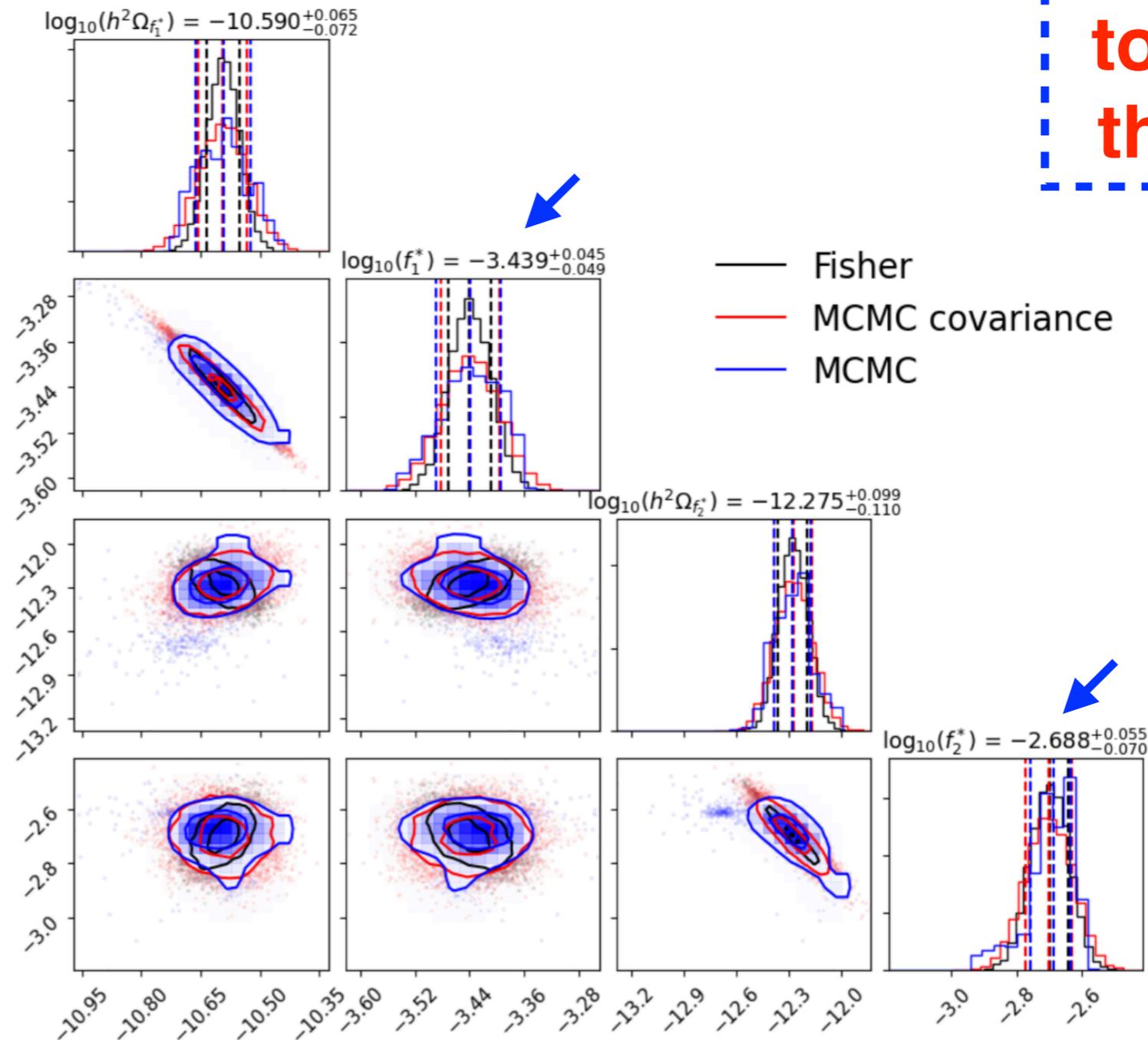


Note: Put by hand at BBO frequencies

GW Spectroscopy

LISA Reconstruction (2-peak signal)

**Couplings
to better
than 1%**



GW Spectroscopy

Reconstruction (2-peak signal)

Monomial Models: $\Omega_{\text{GW}}^{(o)} \sim 10^{-9}$, $f_o \sim 10^8 - 10^9$ Hz
Large amplitude! **... at high Frequency!**

Very unfortunate... no detectors there!



GW Spectroscopy

Reconstruction (2-peak signal)

Monomial Models: $\Omega_{\text{GW}}^{(o)} \sim 10^{-9}$, $f_o \sim 10^8 - 10^9$ Hz
Large amplitude! **... at high Frequency!**

Our example serves as proof of principle!

Very unfortunate... no detectors there!



INFLATIONARY (p)REHEATING (pRH)

Parameter Dependence (Peak amplitude)

Hybrid Models: $\Omega_{\text{GW}}^{(o)} \propto \left(\frac{v}{m_p}\right)^2 \times f(\lambda, g^2)$, $f_o \sim \lambda^{1/4} \times 10^9 \text{ Hz}$

INFLATIONARY (p)REHEATING (pRH)

Parameter Dependence (Peak amplitude)

Hybrid Models: $\Omega_{\text{GW}}^{(o)} \sim 10^{-11}$

Large amplitude !
(for $v \simeq 10^{16}$ GeV)

INFLATIONARY (p)REHEATING (pRH)

Parameter Dependence (Peak amplitude)

Hybrid Models: $\Omega_{\text{GW}}^{(o)} \sim 10^{-11}$, @ $\left\{ \begin{array}{l} f_o \sim 10^8 - 10^9 \text{ Hz} \\ \lambda \sim 0.1 \\ \text{(natural)} \end{array} \right.$

Large amplitude!
(for $v \simeq 10^{16}$ GeV)

INFLATIONARY (p)REHEATING (pRH)

Parameter Dependence (Peak amplitude)

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$\lambda \sim 0.1$
(natural)

$\lambda \sim 10^{-28}$
(fine-tuning)

INFLATIONARY (p)REHEATING (pRH)

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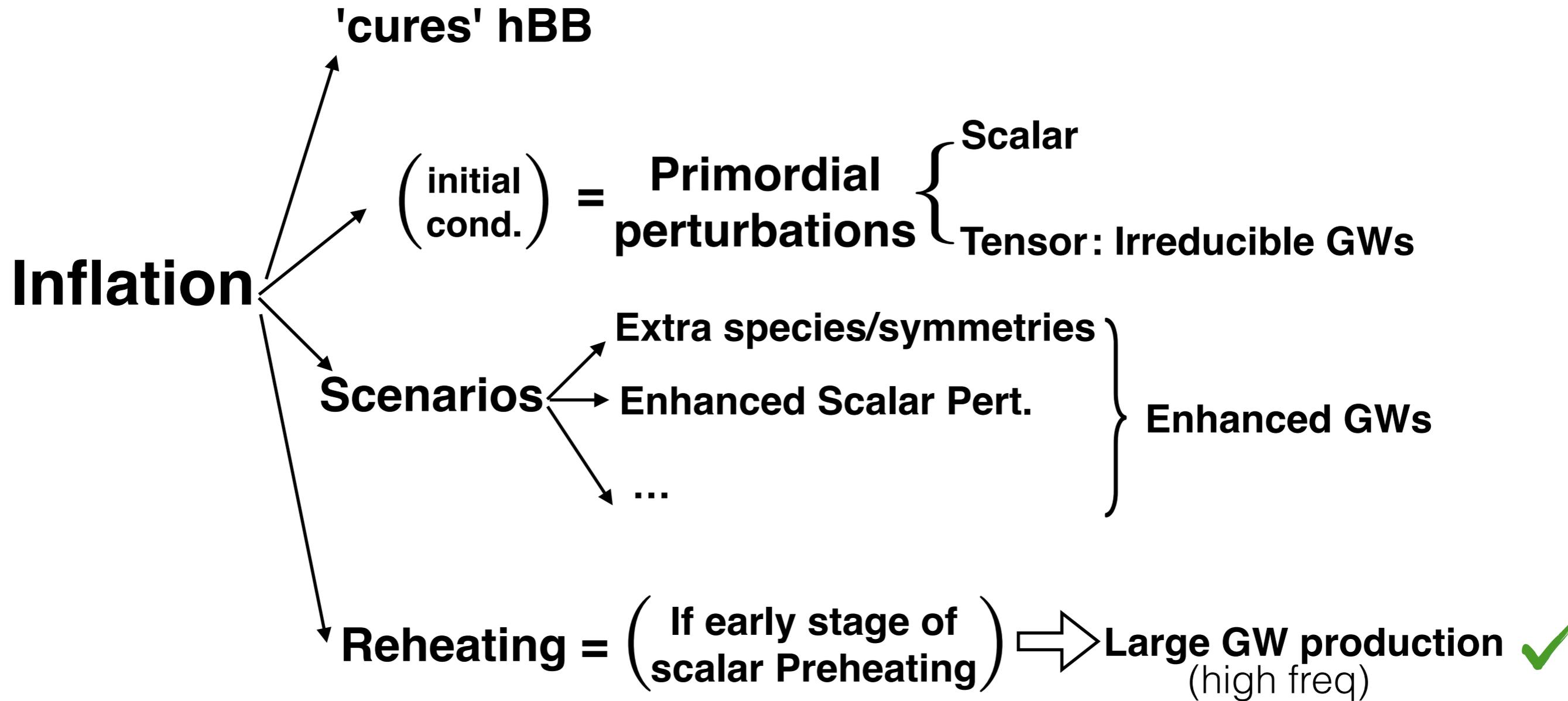
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$\lambda \sim 10^{-28}$
(fine-tuning)

realistically speaking ...



INFLATIONARY COSMOLOGY



OUTLINE

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

2) GWs from Inflation ✓

3) GWs from Preheating ✓

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

OUTLINE

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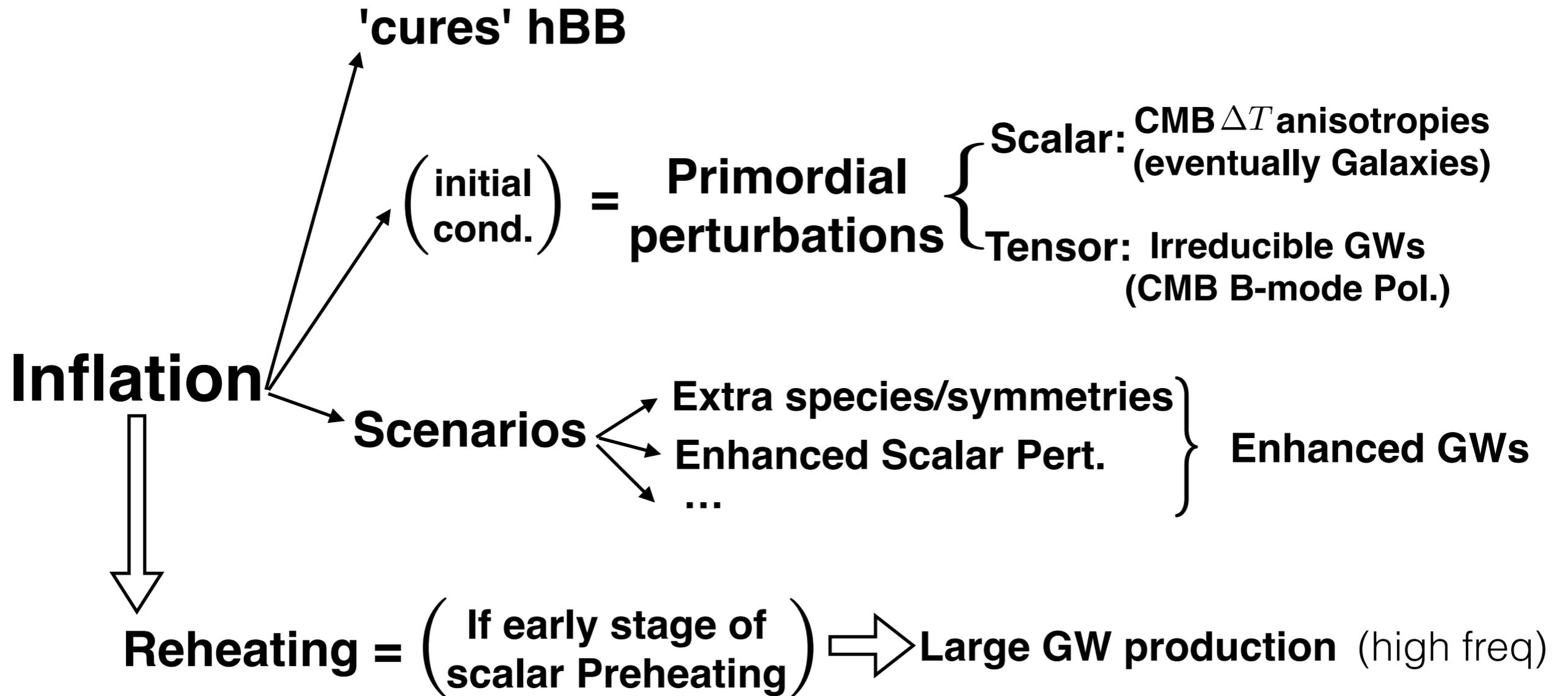
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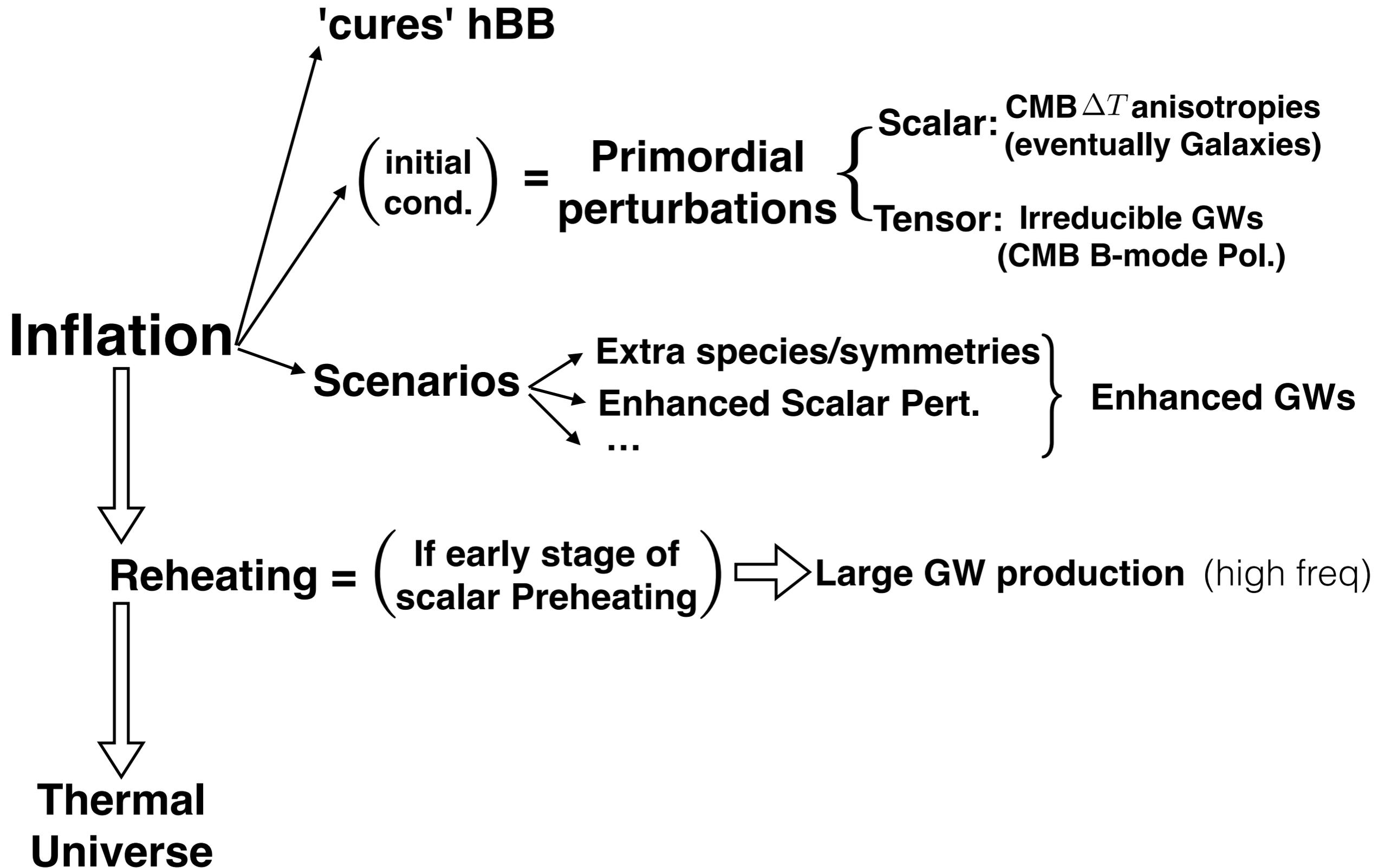
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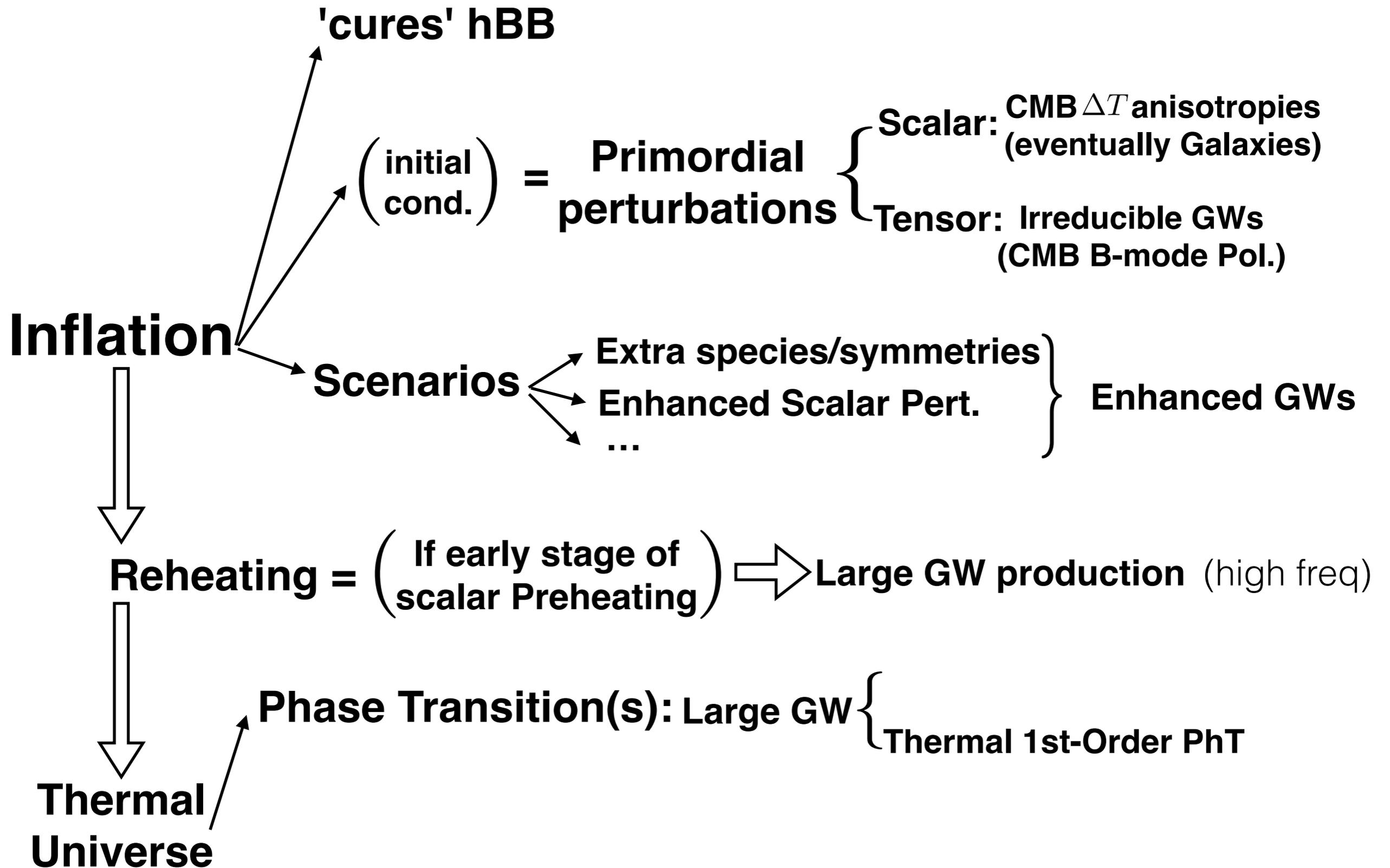
EARLY UNIVERSE



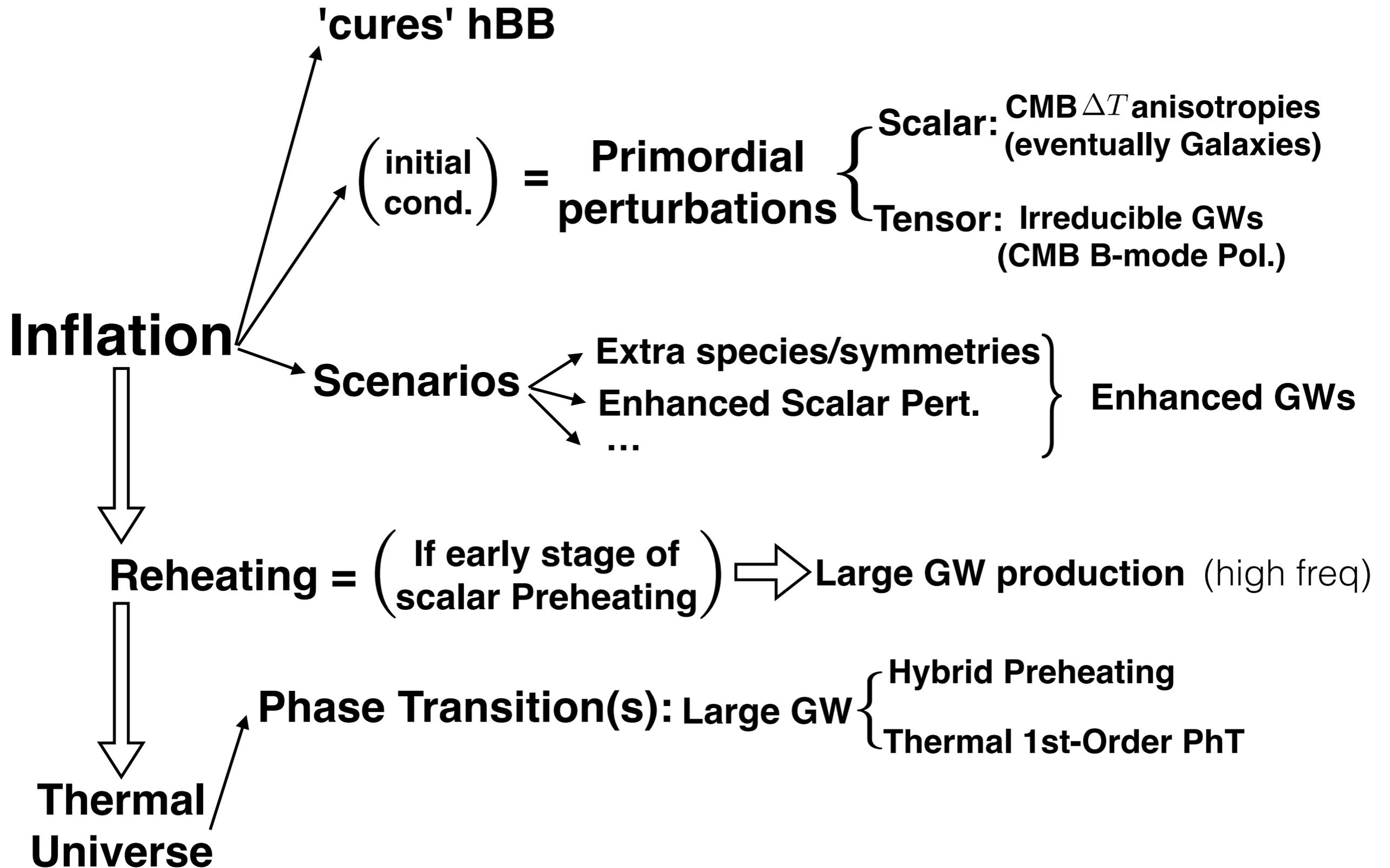
EARLY UNIVERSE



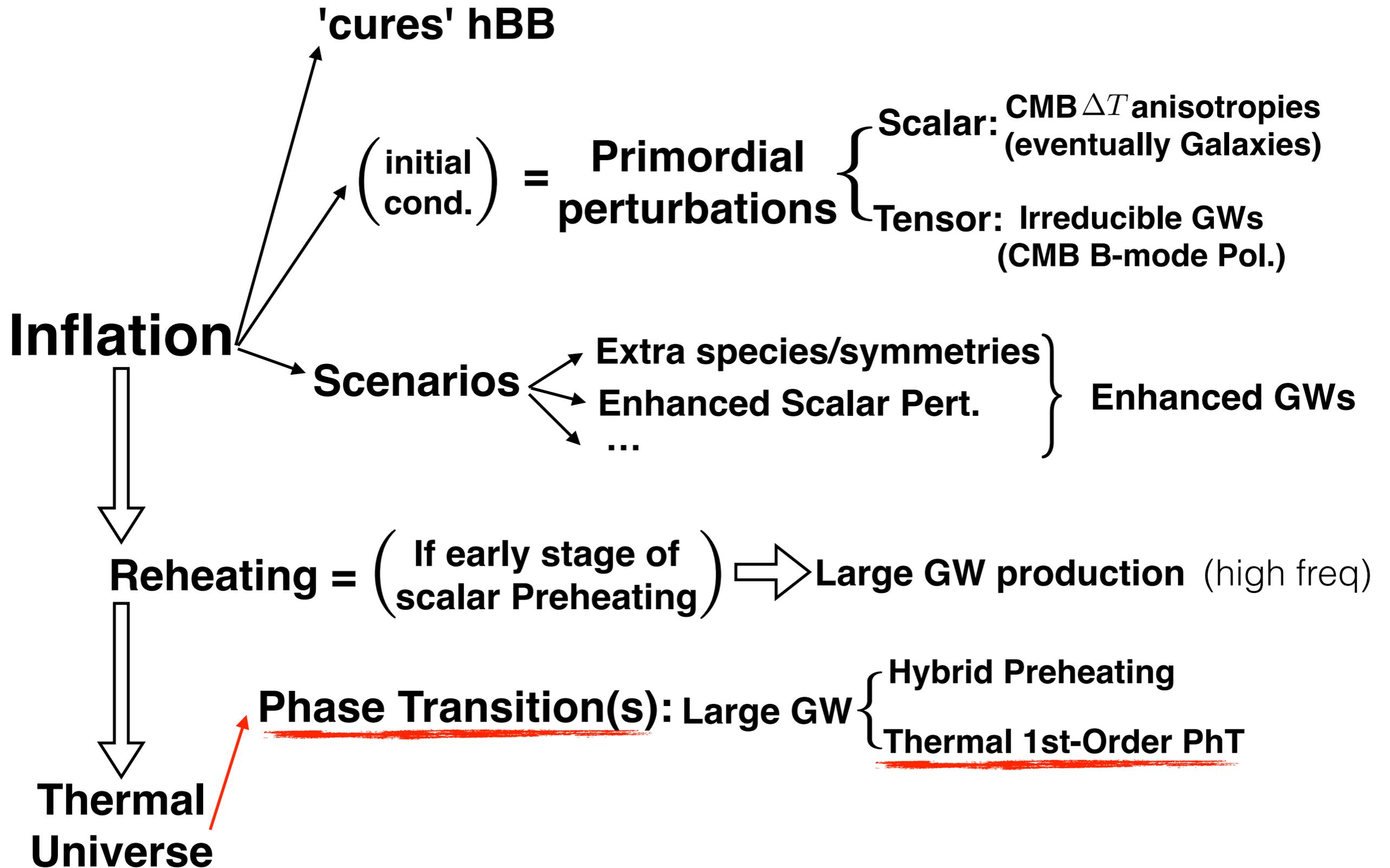
EARLY UNIVERSE



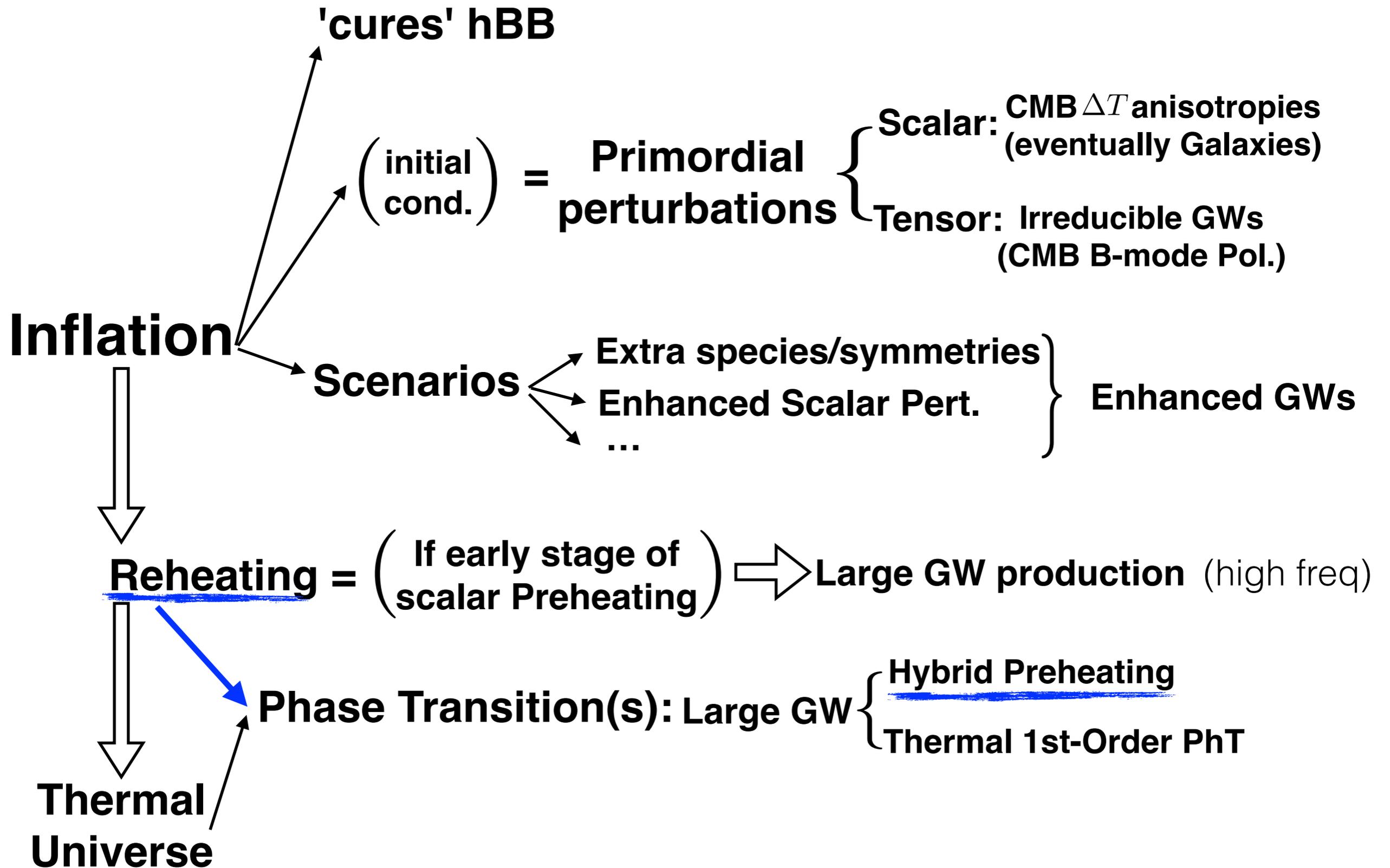
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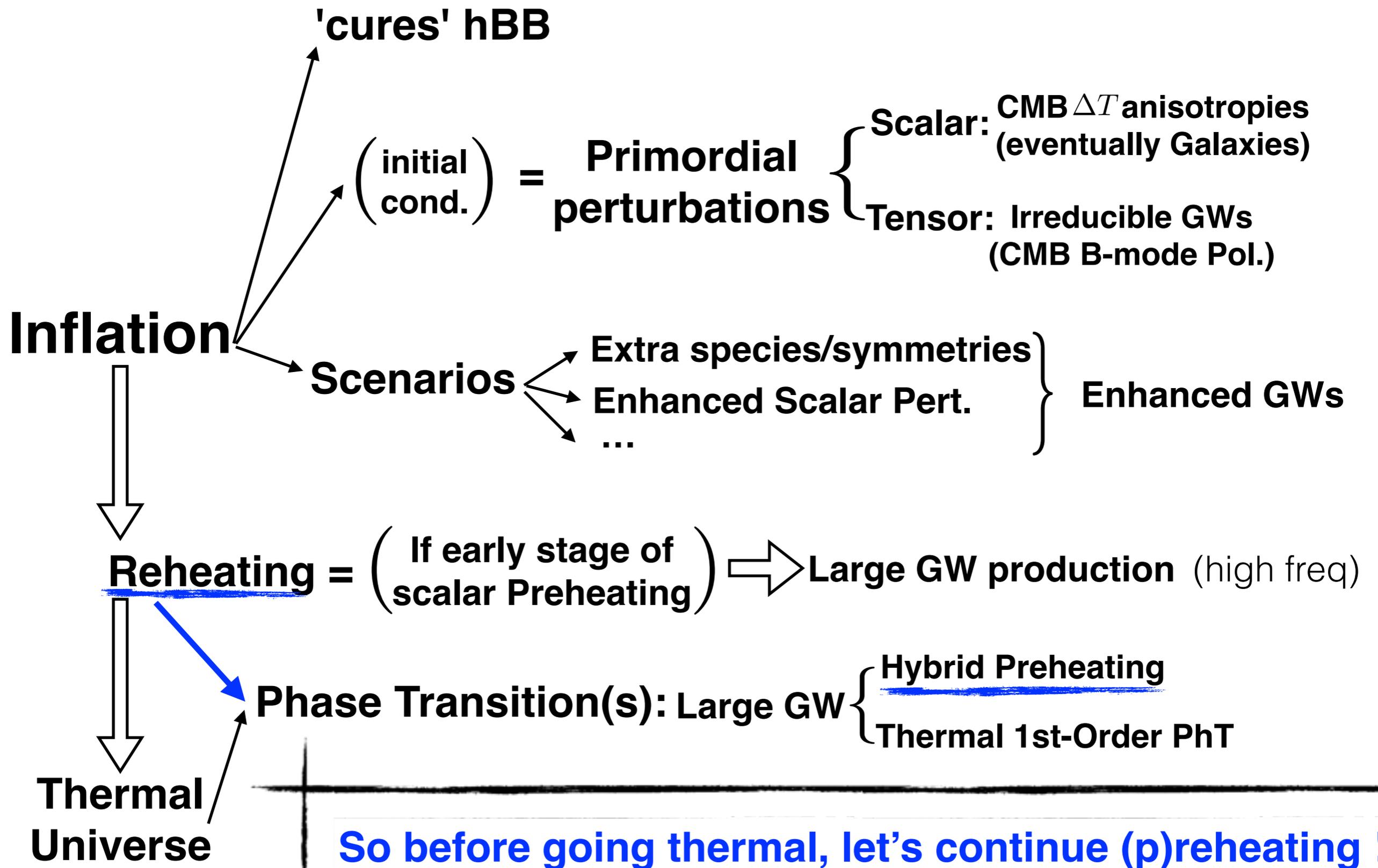
EARLY UNIVERSE



EARLY UNIVERSE



EARLY UNIVERSE



OUTLINE

1st Bloc

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2nd Bloc

2) GWs from Inflation ✓

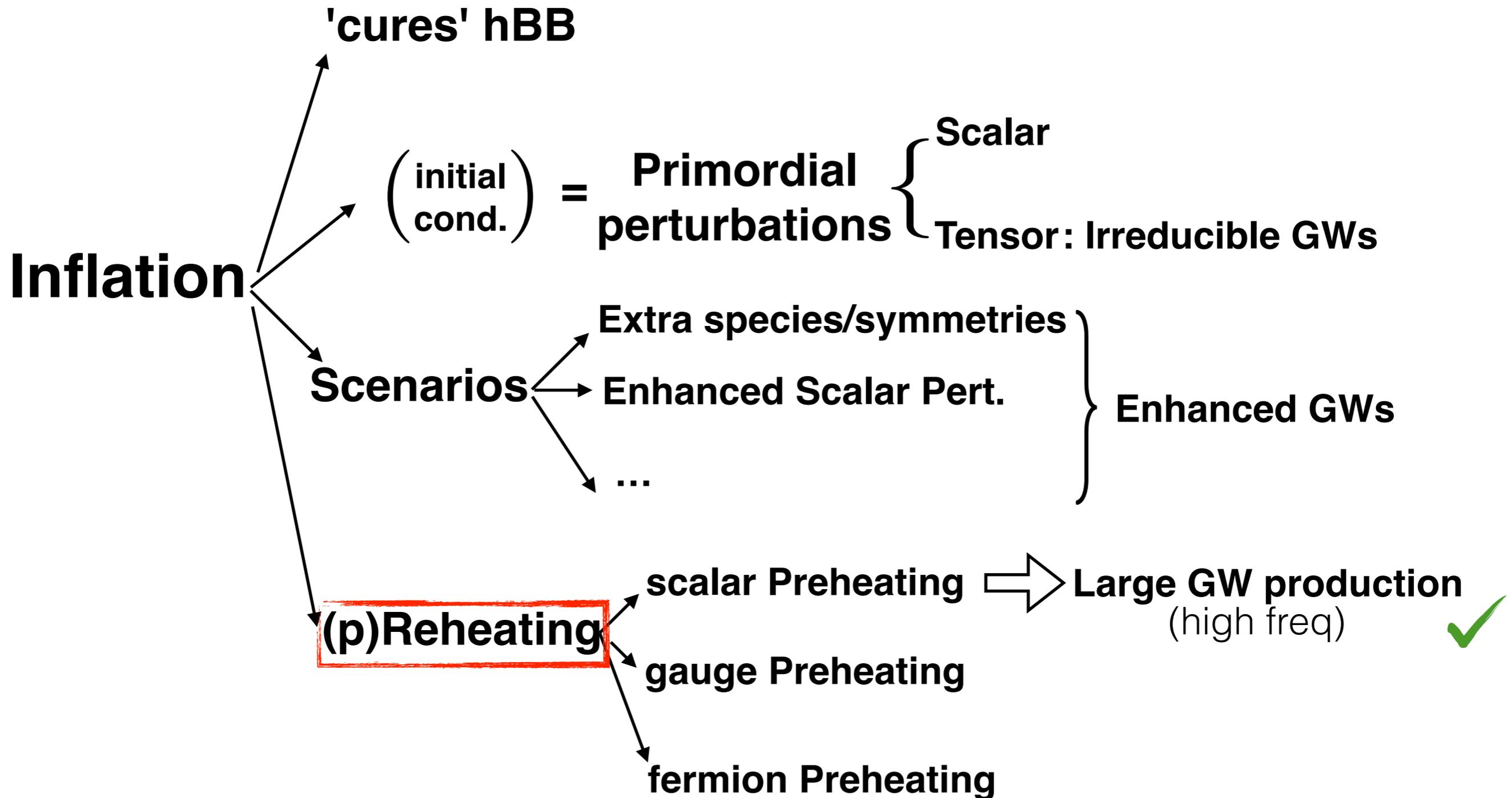
3) GWs from **Preheating**

3rd Bloc

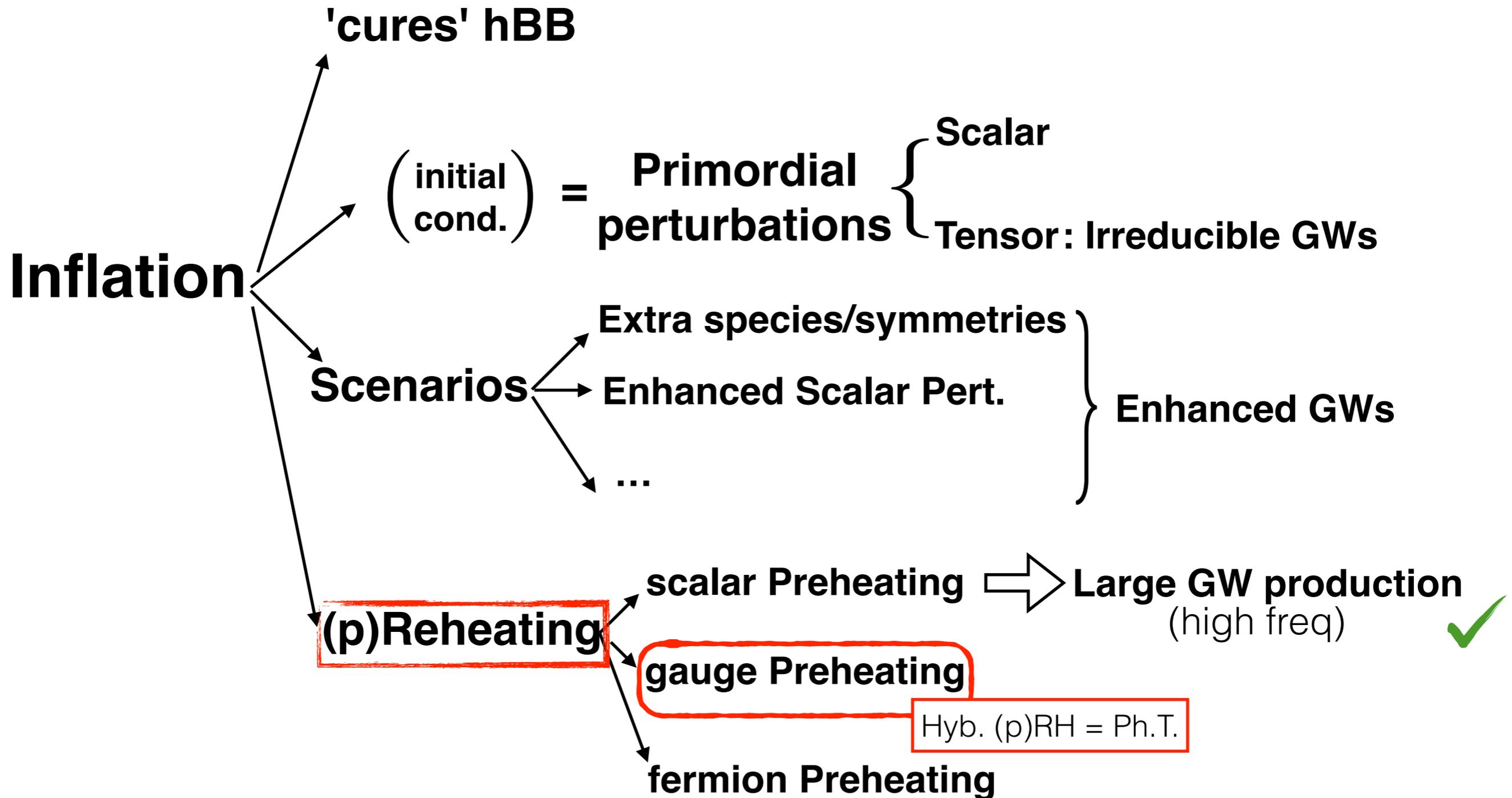
4) GWs from Phase Transitions

5) GWs from Cosmic Defects

INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



GAUGE (P)REHEATING

Hybrid Preheating = Higgs+Inflaton model

inflaton mass coupling

Inflaton: $\ddot{\phi}(t) + (\mu^2 + g^2|\chi|^2)\phi(t) = 0$

Higgs: $\ddot{\chi}_k + (k^2 + m^2 \left(\frac{\phi^2}{\phi_c^2} - 1\right) + \lambda|\chi|^2)\chi_k = 0$

Self-coupling V.E.V.

$m = \sqrt{\lambda}v$

$\phi_c \equiv m/g$ Critical value

GAUGE (P)REHEATING

Hybrid Preheating = Higgs+Inflaton model

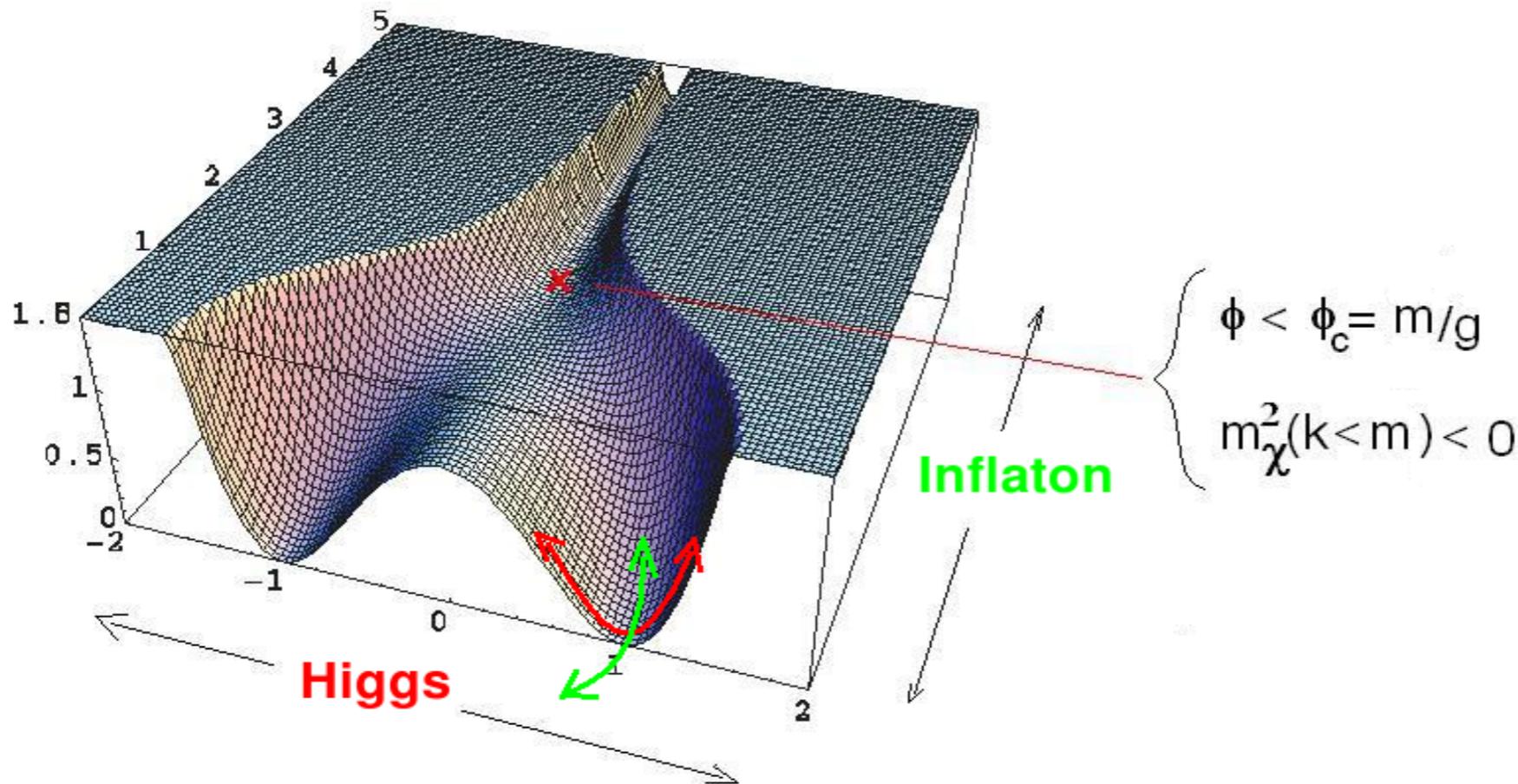
$$\left. \begin{array}{l} \text{Inflaton: } \ddot{\phi}(t) + (\mu^2 + g^2|\chi|^2)\phi(t) = 0 \\ \text{Higgs: } \ddot{\chi}_k + \left(k^2 + m^2 \left(\frac{\phi^2}{\phi_c^2} - 1\right) + \lambda|\chi|^2\right)\chi_k = 0 \end{array} \right\} \begin{array}{l} m = \sqrt{\lambda}v \\ \phi_c \equiv m/g \end{array}$$

GAUGE (P)REHEATING

Hybrid Preheating = Higgs+Inflaton model

$$\left. \begin{array}{l}
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Hybrid Preheating = Phase Transition

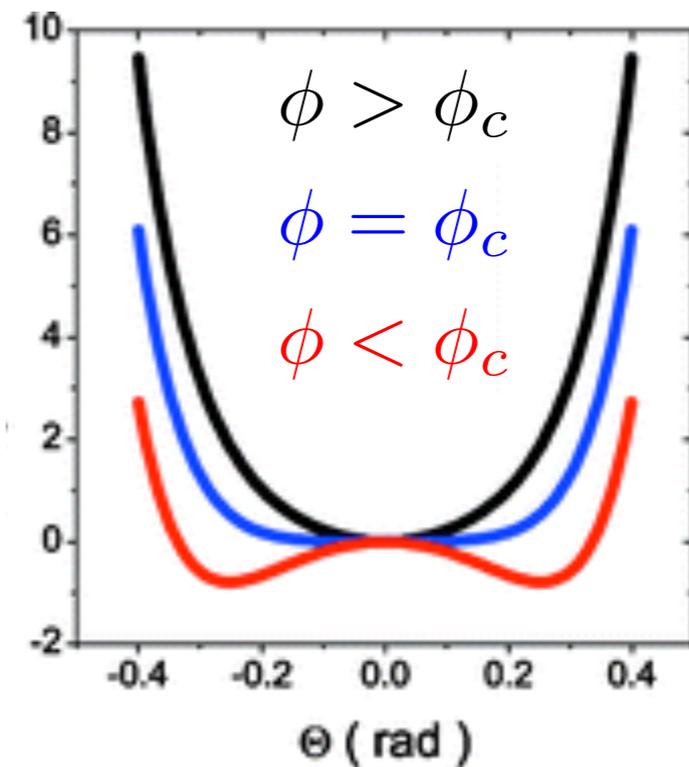
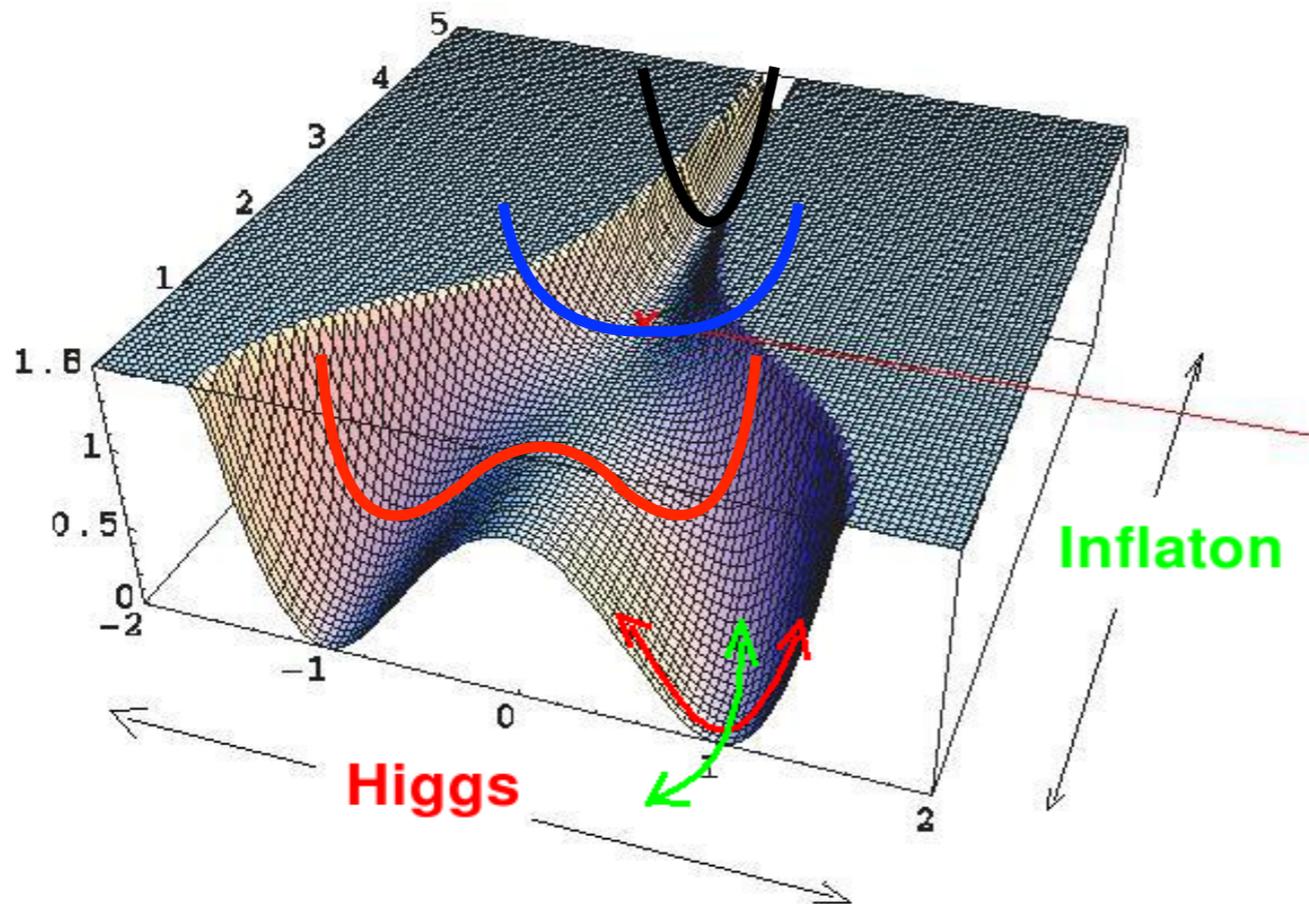


GAUGE (P)REHEATING

Hybrid Preheating = Higgs+Inflaton model

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Hybrid Preheating = Phase Transition

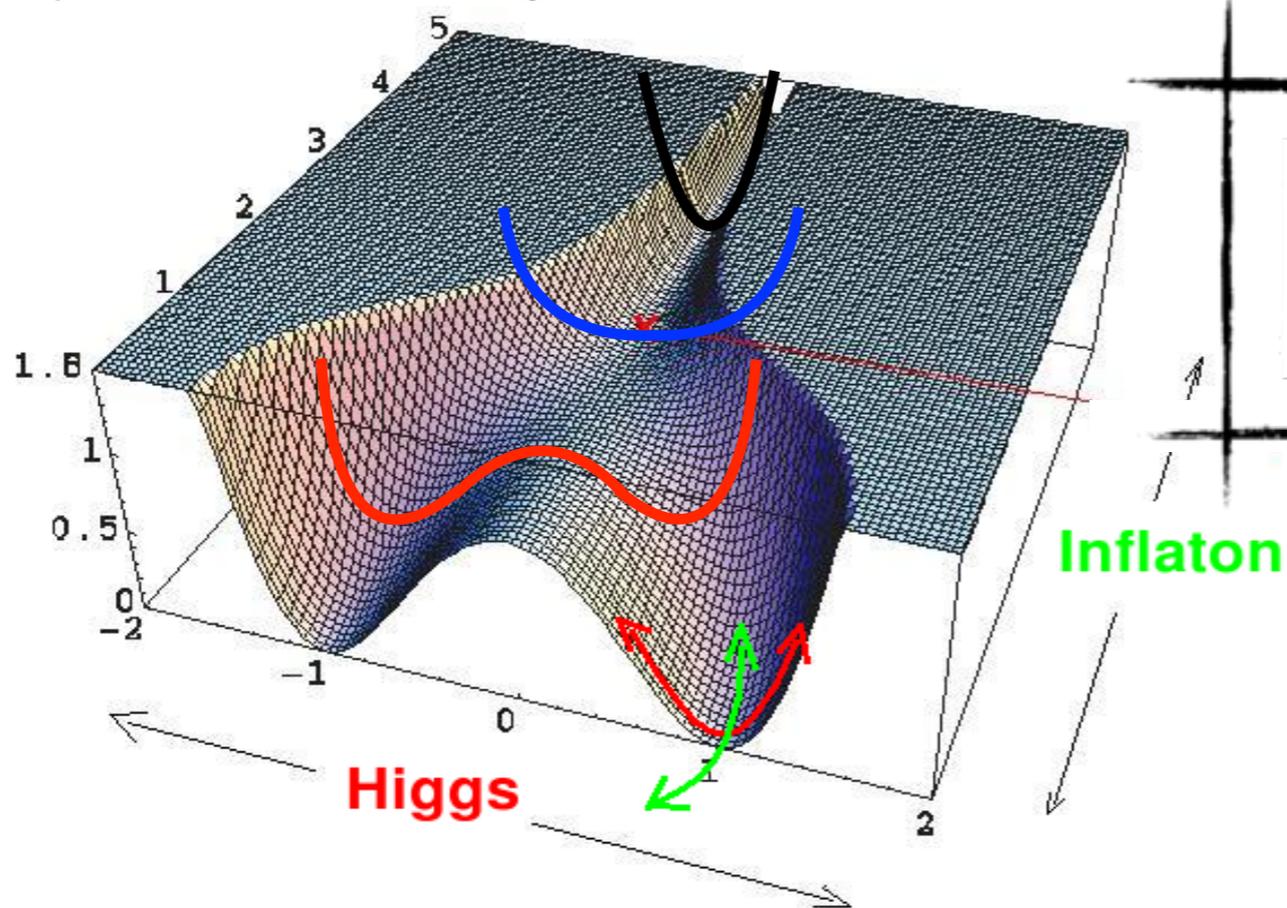


GAUGE (P)REHEATING

Hybrid Preheating = Higgs+Inflaton model

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 \end{array} \right\} \begin{array}{l}
 (k < m = \sqrt{\lambda}v) \\
 \chi_k, n_k \sim e^{\sqrt{m^2 - k^2}t}
 \end{array}$$

Hybrid Preheating = Phase Transition



**It is a Phase transition !
by Tachyonic Instability**

$$\langle \chi \rangle = 0 \rightarrow \langle \chi \rangle = v$$

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$V(\phi, \chi) = \frac{\lambda}{4} (\phi^2 - v^2)^2 + \frac{g^2}{2} \phi^2 \chi^2 + \frac{1}{2} m^2 \chi^2$$

Just to confuse you a little bit: now $\begin{cases} \chi : \textit{inflaton} \\ \Phi = \frac{\phi}{\sqrt{2}} : \textit{Higgs} \end{cases}$

GAUGE (P)REHEATING

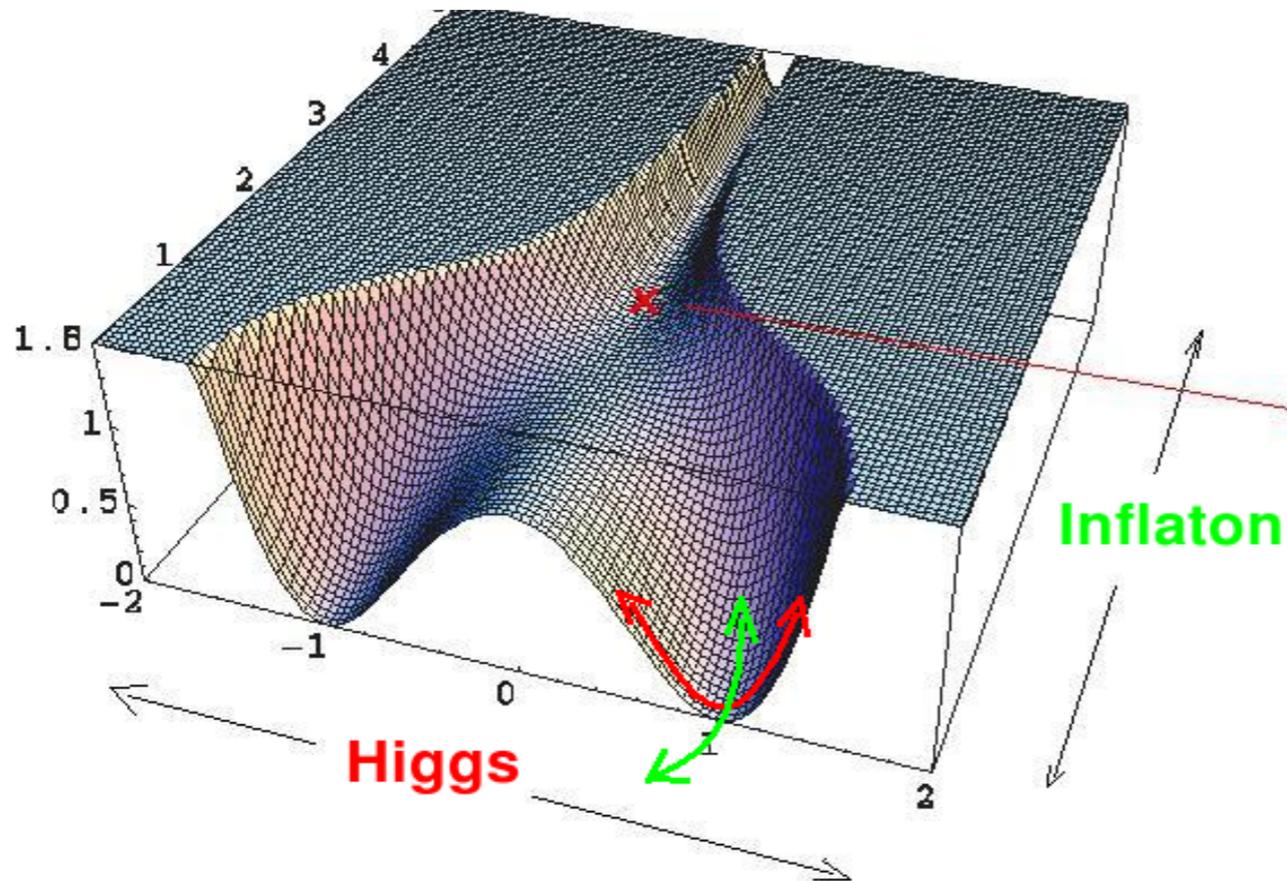
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GAUGE (P)REHEATING

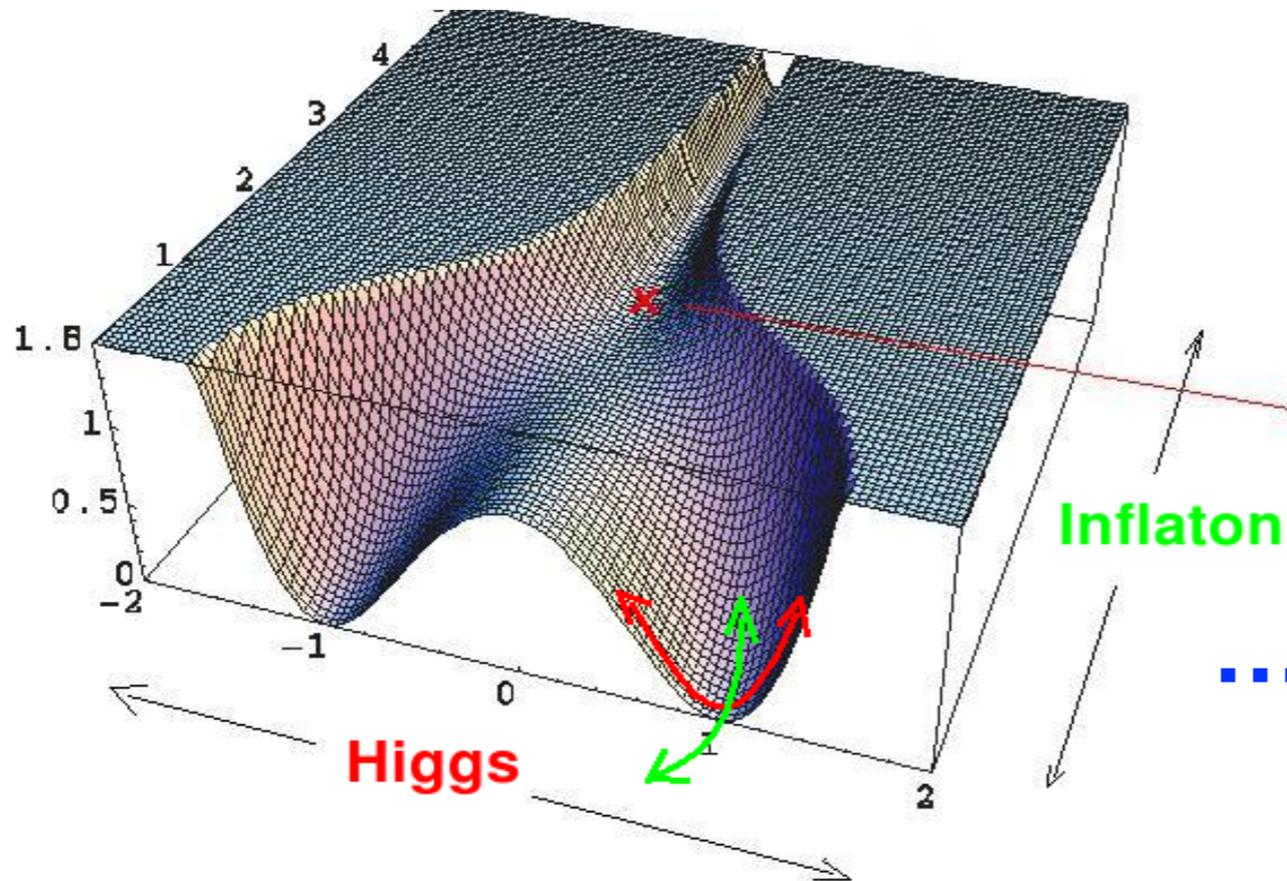
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... now there are gauge field(s) !

... so you excite the Higgs, you excite Gauge flds !

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

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EOM: $\left\{ \begin{array}{l} \text{Minkowski,} \\ \text{Temporal Gauge (A}_0=0) \end{array} \right.$

$$\begin{array}{rcl} \ddot{\varphi} - D_i D_i \varphi + V_{,\varphi^*} & = & 0 \quad \longrightarrow \text{SCALARS eom} \\ \ddot{A}_i - \partial_j \partial_j A_i + \partial_i \partial_j A_j & = & 2e^2 \text{Im} [\varphi^* D_i \varphi] \quad \longrightarrow \text{VECTORS eom} \\ \partial_i \dot{A}_i & = & 2e^2 \text{Im} [\varphi^* \dot{\varphi}] \quad \longrightarrow \text{GAUSS law} \end{array}$$

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

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$$\partial_i \dot{A}_i = 2e^2 \text{Im} [\varphi^* \dot{\varphi}] .$$

→ SCALARS eom

→ VECTORS eom

→ GAUSS law

GW EOM

$$\ddot{h}_{ij} - \partial_k \partial_k h_{ij} = 16\pi G \Pi_{ij}^{\text{TT}}$$

$$\Pi_{ij}^{\text{TT}} = [\partial_i \chi \partial_j \chi + 2 \text{Re} [D_i \varphi (D_j \varphi)^*] - B_i B_j - E_i E_j]^{\text{TT}}$$

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

EOM: { Minkowski,
Temporal Gauge ($A_0 = 0$)

$$\begin{aligned} \ddot{\varphi} - D_i D_i \varphi + V_{,\varphi^*} &= 0 && \rightarrow \text{SCALARS eom} \\ \ddot{A}_i - \partial_j \partial_j A_i + \partial_i \partial_j A_j &= 2e^2 \text{Im} [\varphi^* D_i \varphi] && \rightarrow \text{VECTORS eom} \\ \partial_i \dot{A}_i &= 2e^2 \text{Im} [\varphi^* \dot{\varphi}] . && \rightarrow \text{GAUSS law} \end{aligned}$$

GW EOM

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COVARIANT

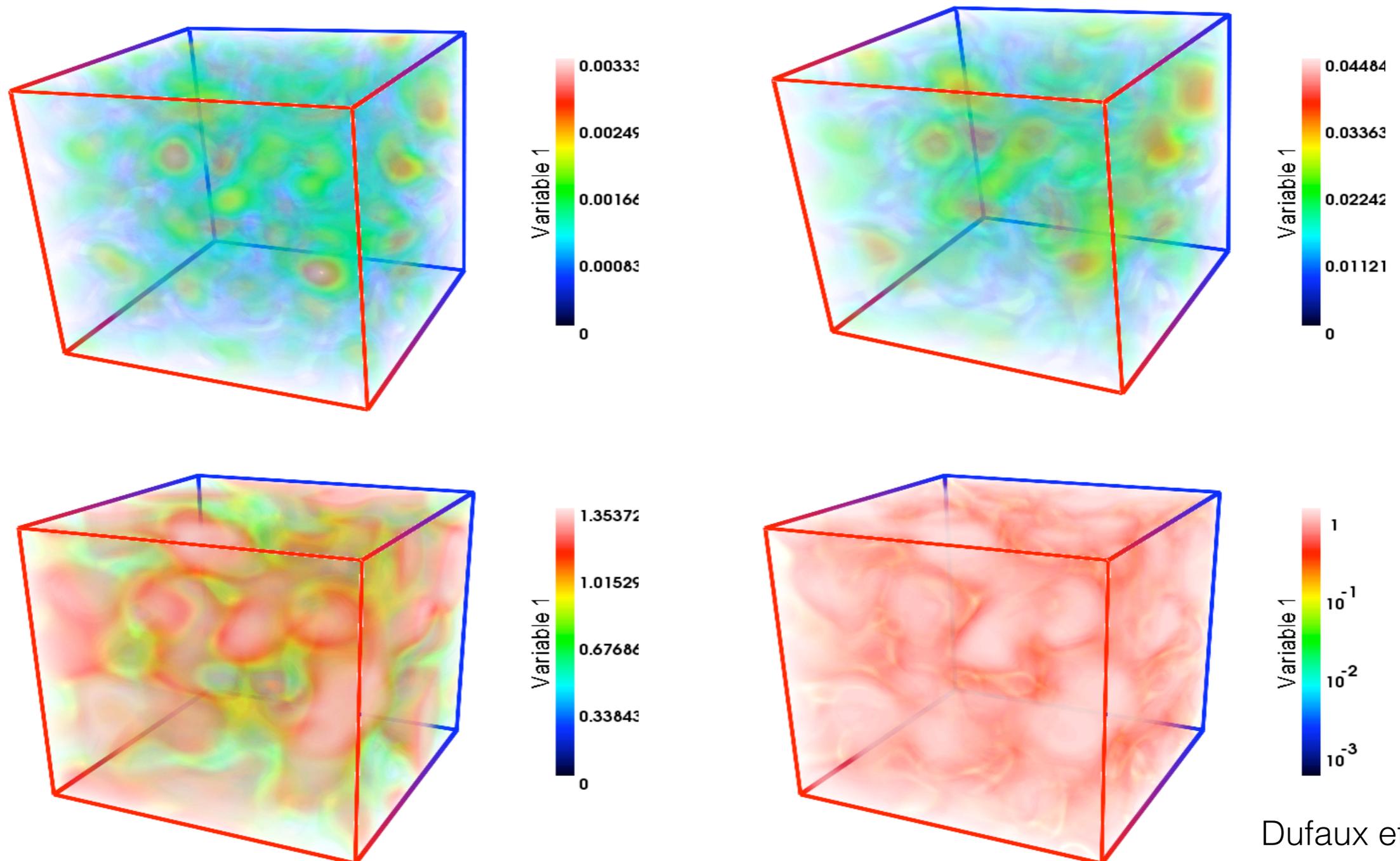
MAGNETIC

ELECTRIC

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

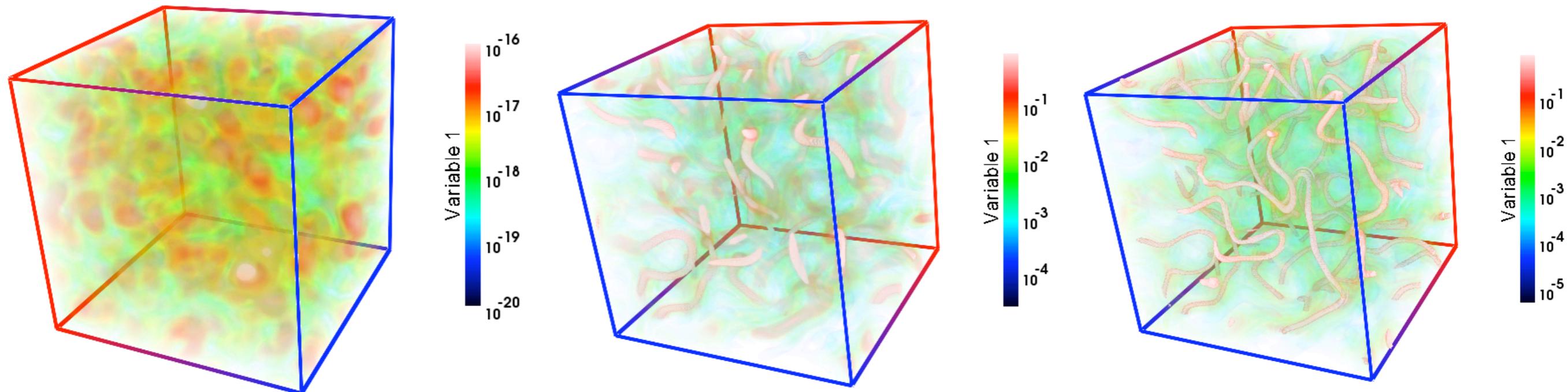
DYNAMICS OF THE HIGGS: $m_t = 5.5 \rightarrow m_t = 23$



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

DYNAMICS OF THE MAGNETIC FIELD: $mt = 5.5 \rightarrow mt = 17$



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

What's going on !?

Cosmic Strings are formed

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

What's going on !?

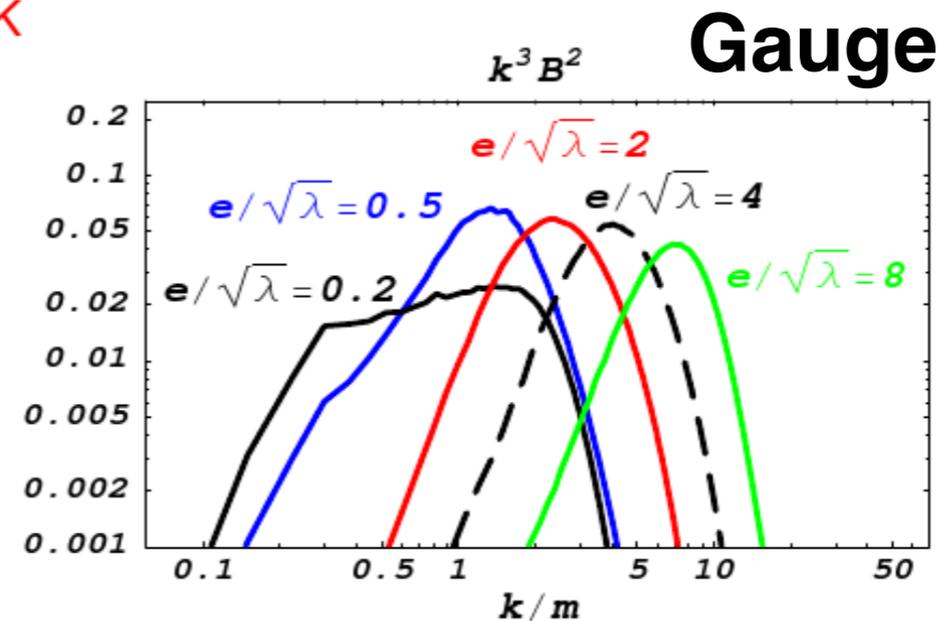
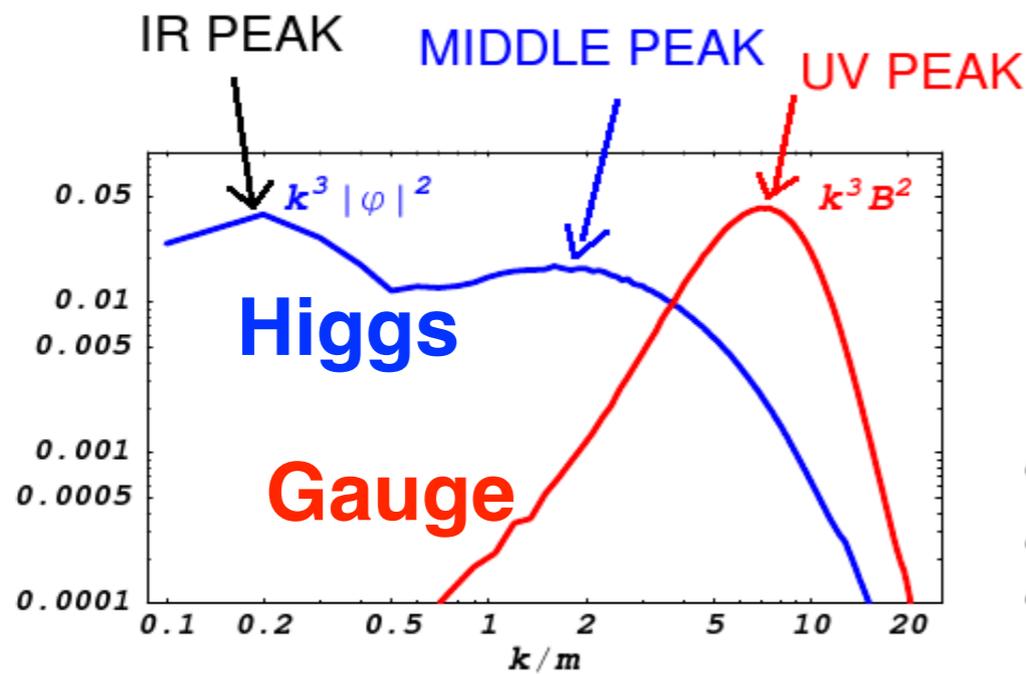
Cosmic Strings are formed

(Topological Defects \longrightarrow 5th Lecture)

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

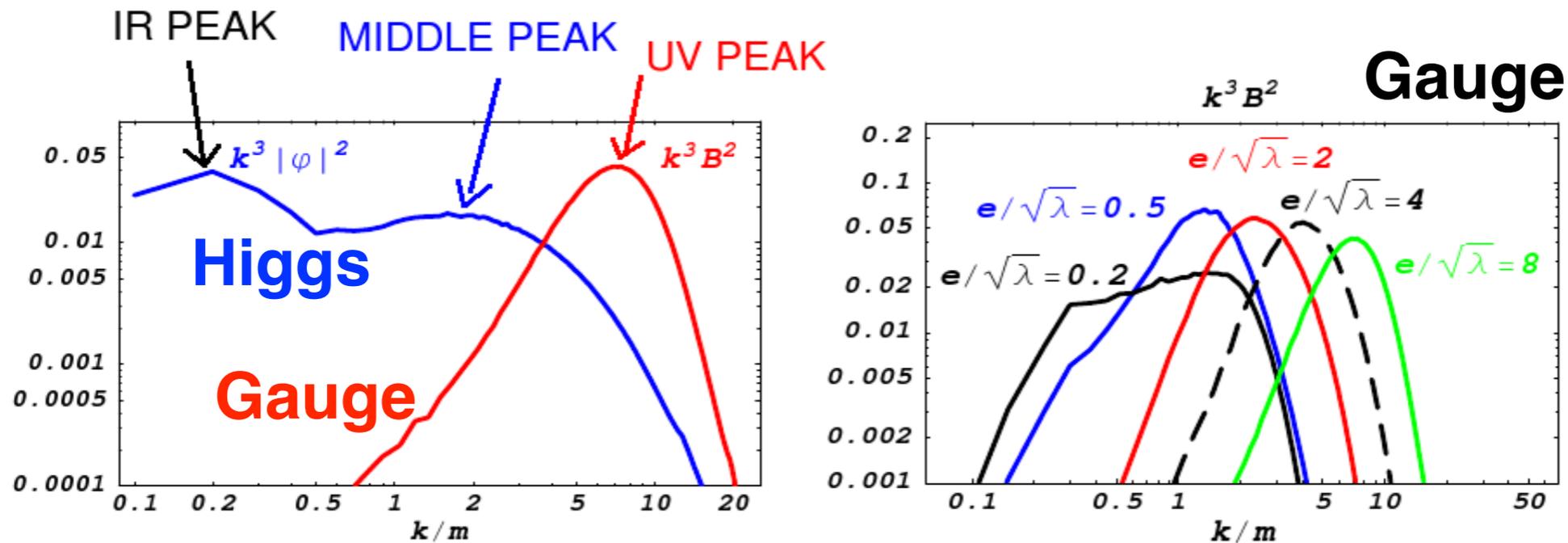
SCALARS AND VECTORS' SPECTRA:



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

SCALARS AND VECTORS' SPECTRA:



PARAMETERS ABELIAN-HIGGS Model: $m \equiv \sqrt{\lambda}v$, λ/g^2 , $e/\sqrt{\lambda}$, V_c

MIDDLE PEAK: $\left\{ \begin{array}{l} \text{Higgs mass} \\ (\text{Inflaton Velocity})^{1/3} \end{array} \right\} \rightarrow \text{Tachyonic Scale, Bubbles' Size}$

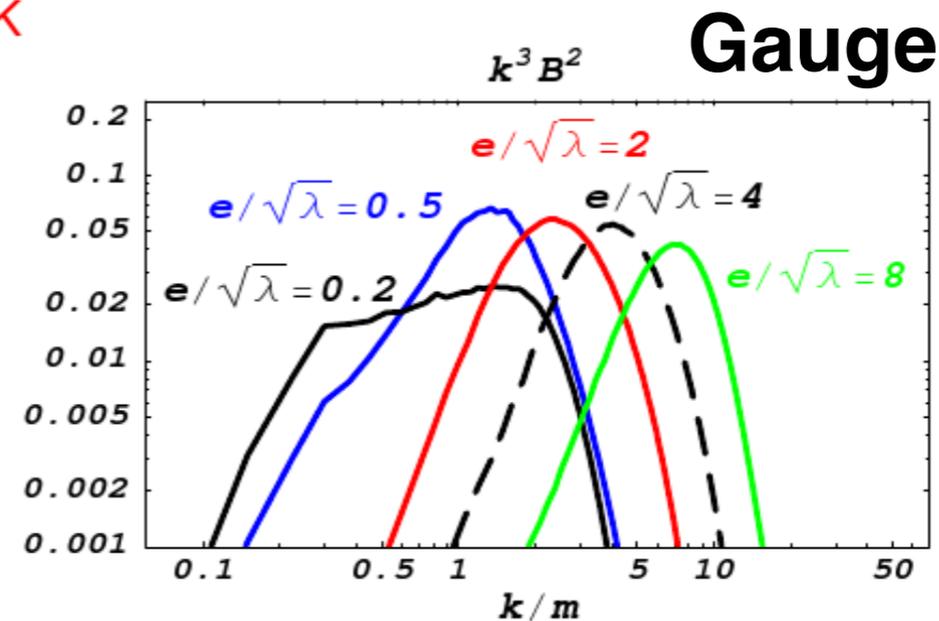
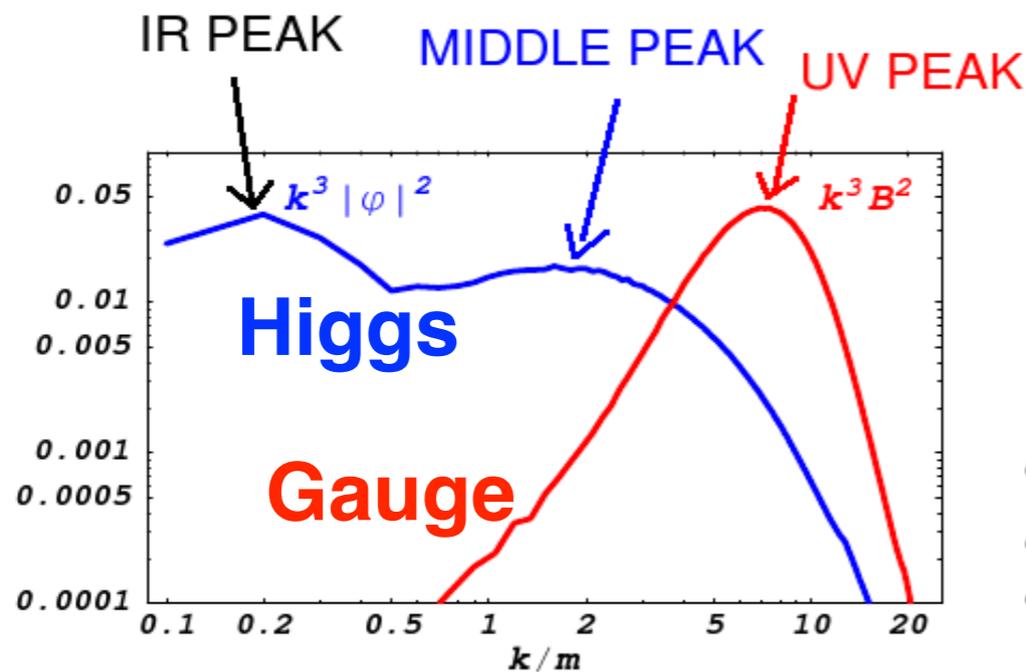
IR PEAK : Inflaton Velocity, Higgs+Inflaton Couplings (Dufaux et al 2009)

UV PEAK: Vector mass / Higgs Mass

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

SCALARS AND VECTORS' SPECTRA:



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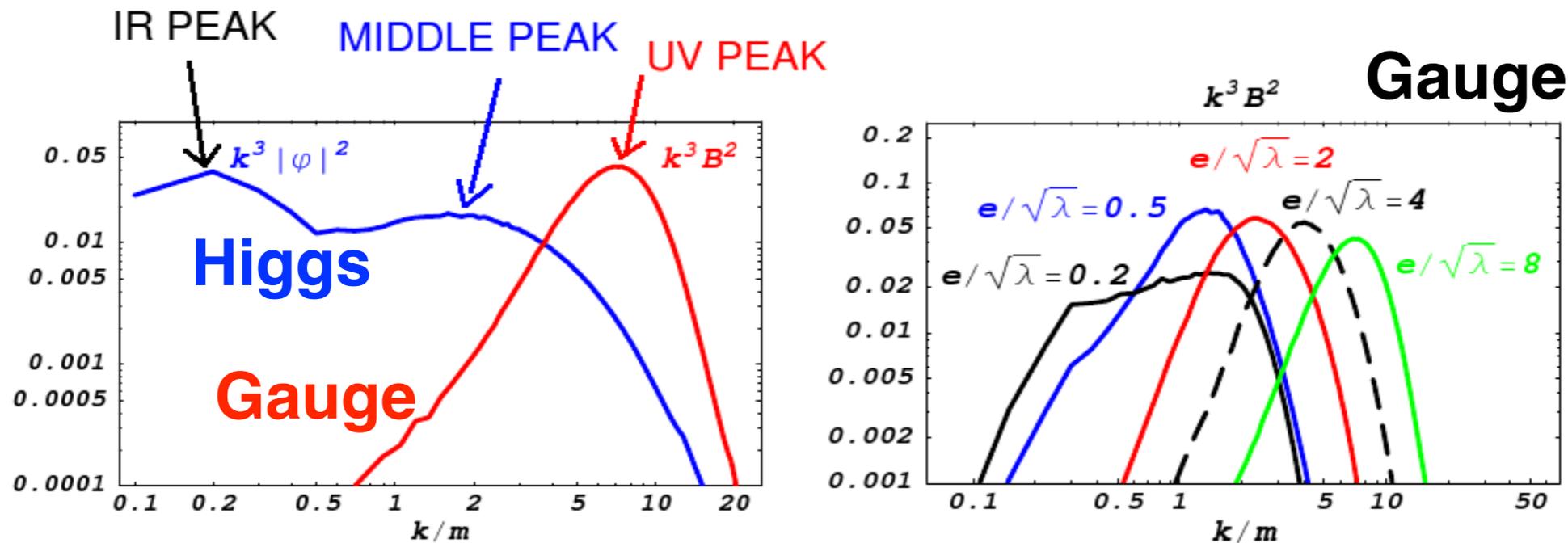
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GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

SCALARS AND VECTORS' SPECTRA:



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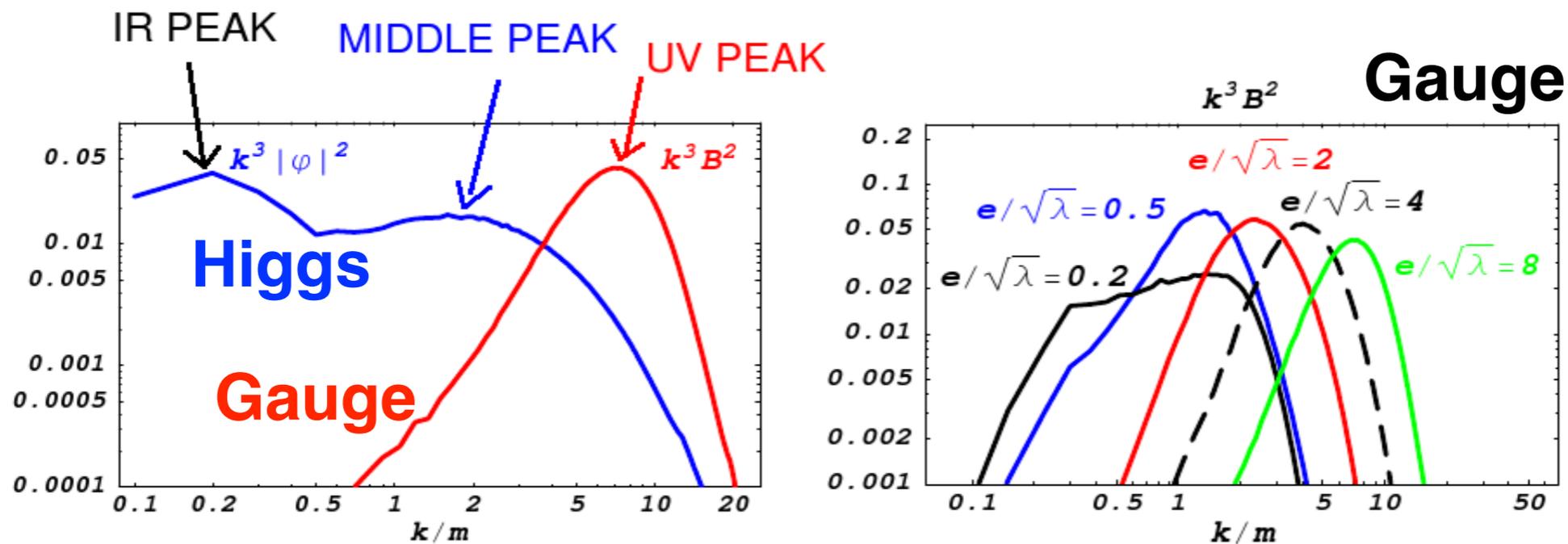
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GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

SCALARS AND VECTORS' SPECTRA:



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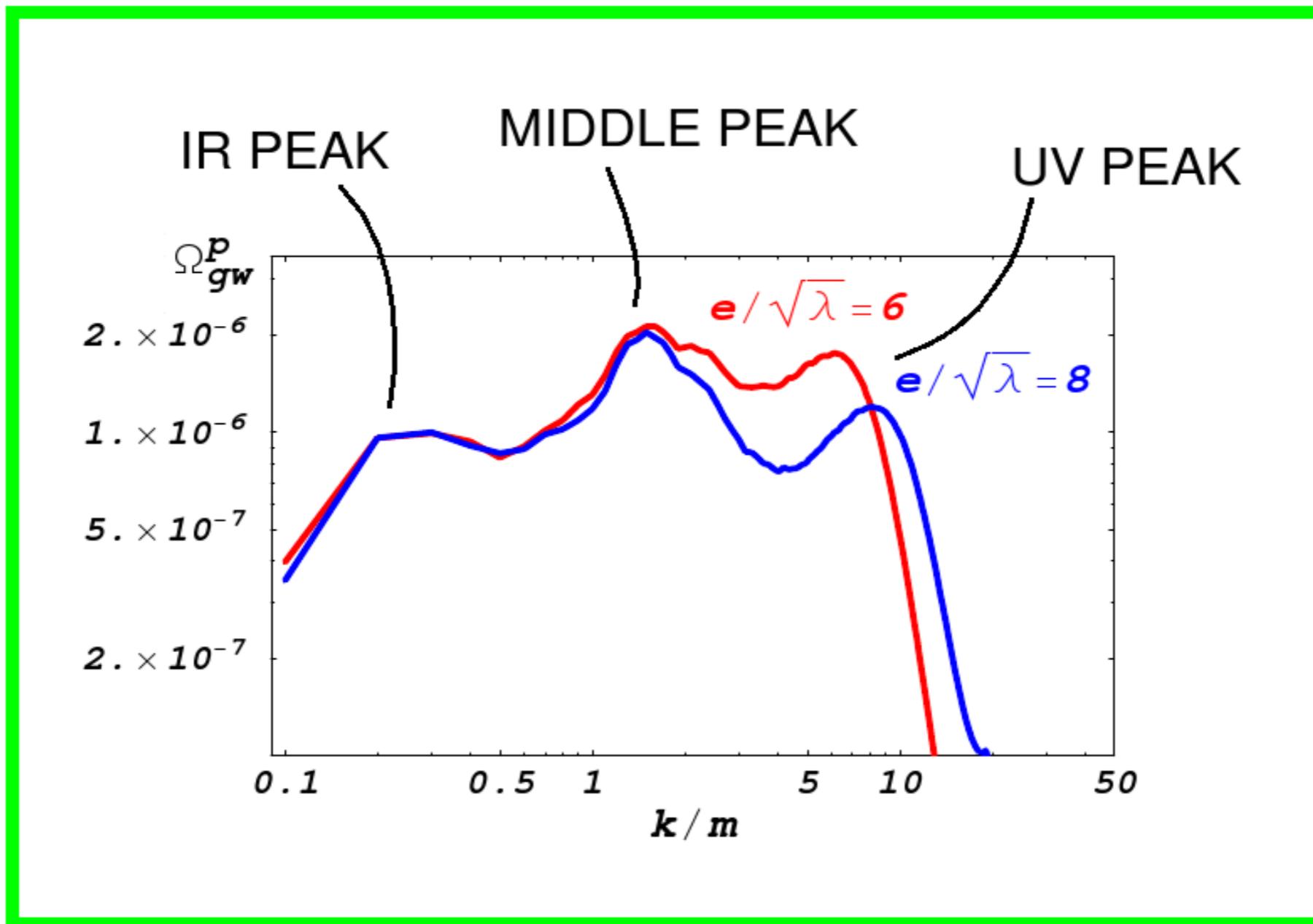
IR PEAK : Inflaton Velocity, Higgs+Inflaton Couplings

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GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

GRAVITATIONAL WAVES SPECTRA:

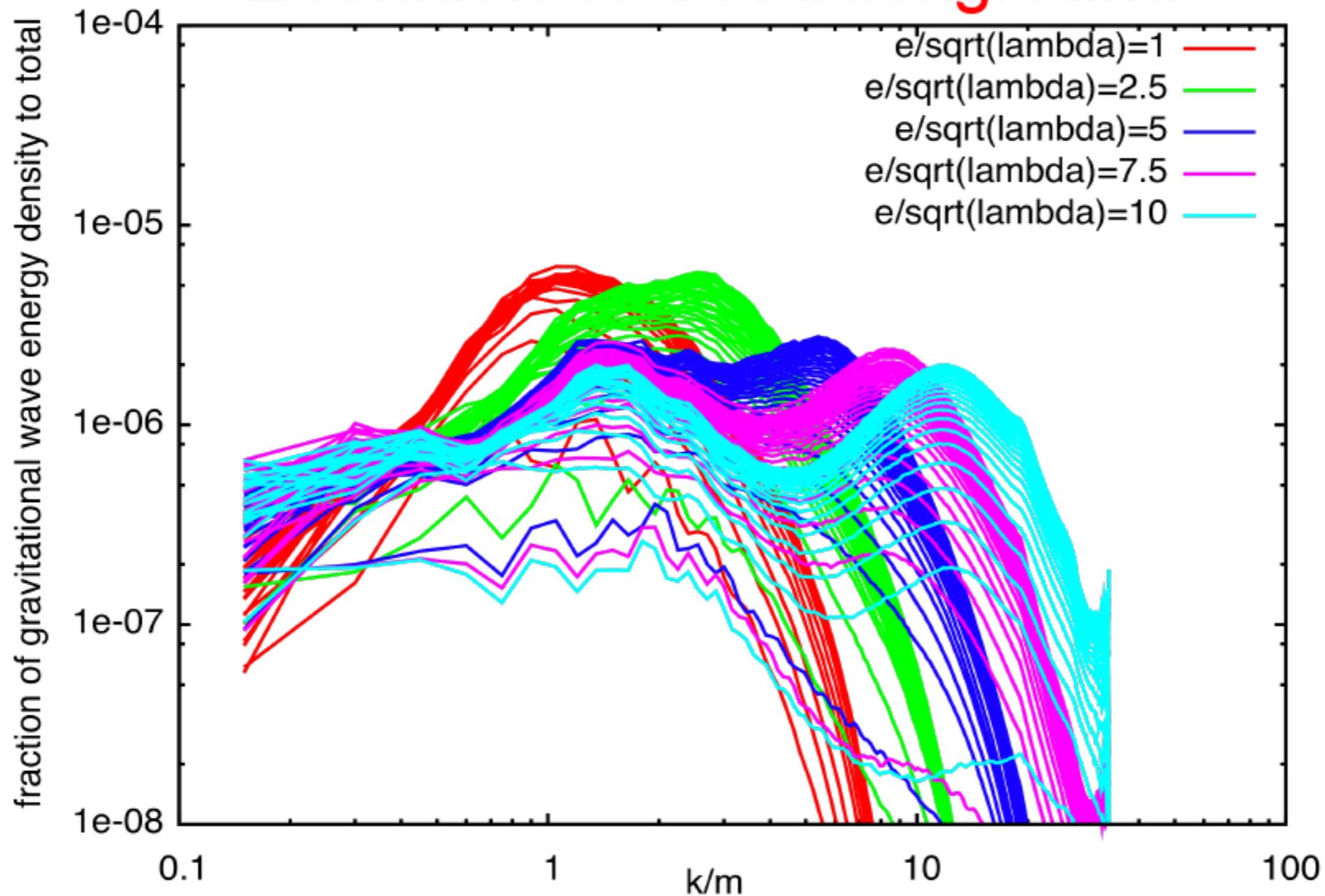


GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

GRAVITATIONAL WAVES SPECTRA:

Evolution of GW background



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

GW SPECTRA: ANALYTICS:

IR and Middle Peaks' Amplitude: $F(g, \lambda, V_c)$

UV peak Amplitude: Lattice Simulations

$$f_1 \lesssim f(g, \lambda, V_c) \quad (\text{IR peak})$$

$$f_2 \approx \lambda^{1/4} 10^{11} \text{ Hz} \quad (\text{Middle peak})$$

$$f_3 \approx \frac{e}{\sqrt{\lambda}} \lambda^{1/4} 10^{11} \text{ Hz} \quad (\text{UV peak})$$

RED-SHIFTED
FREQUENCIES

$$f(g, \lambda, V_c)$$

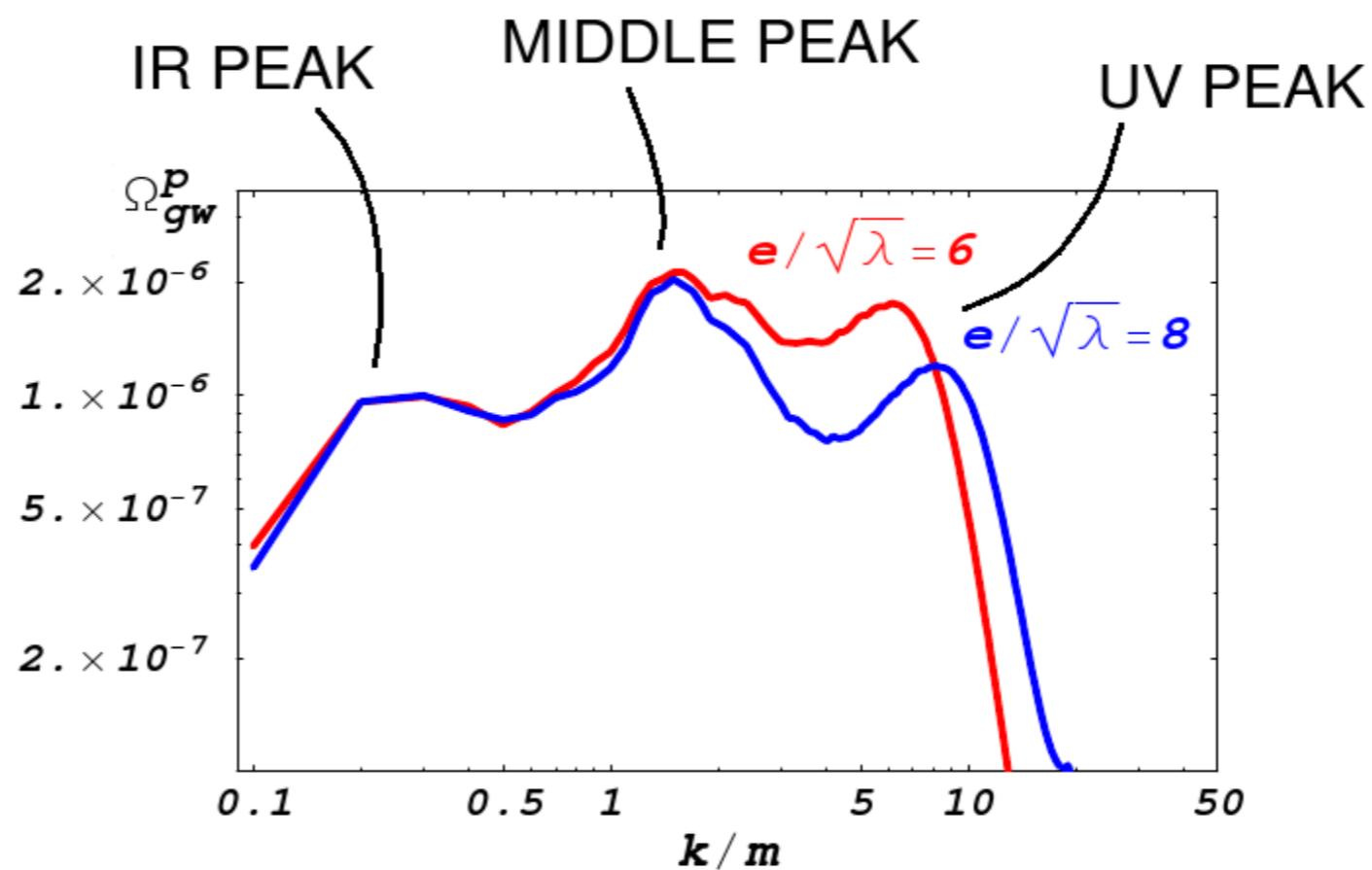


Dufaux et al '09

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks !
(particle physics spectroscopy)



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks !
(particle physics
spectroscopy)

$$\Omega_{\text{GW}}^{(o)} \sim 10^{-11},$$

Large amplitude(s) !

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks !
(particle physics
spectroscopy)

$$\Omega_{\text{GW}}^{(o)} \sim 10^{-11}, \quad @ \quad f_o \sim 10^8 - 10^9 \text{ Hz}$$

Large amplitude(s) ! ... but at high Frequency !

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks !
(particle physics
spectroscopy)

$$\Omega_{\text{GW}}^{(o)} \sim 10^{-11}, \quad @ \quad f_o \sim 10^8 - 10^9 \text{ Hz}$$

Large amplitude(s) ! ... but at high Frequency !

Very unfortunate... no high frequency detectors !



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks !
(particle physics
spectroscopy)

$$\Omega_{\text{GW}}^{(o)} \sim 10^{-11}, \quad @ \quad f_o \sim 10^8 - 10^9 \text{ Hz}$$

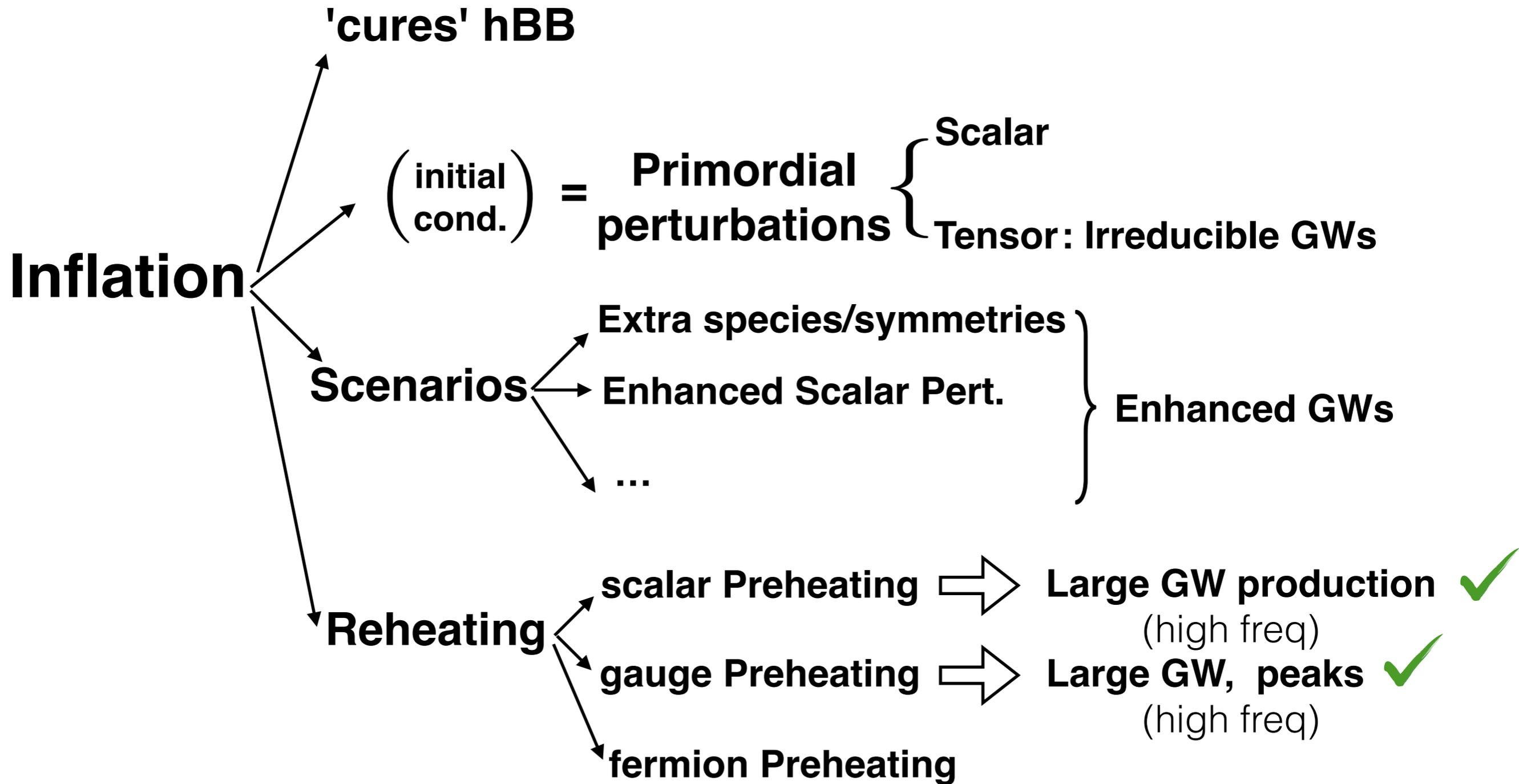
Large amplitude(s) ! ... but at high Frequency !

We Should look for this effect at low-freq models !

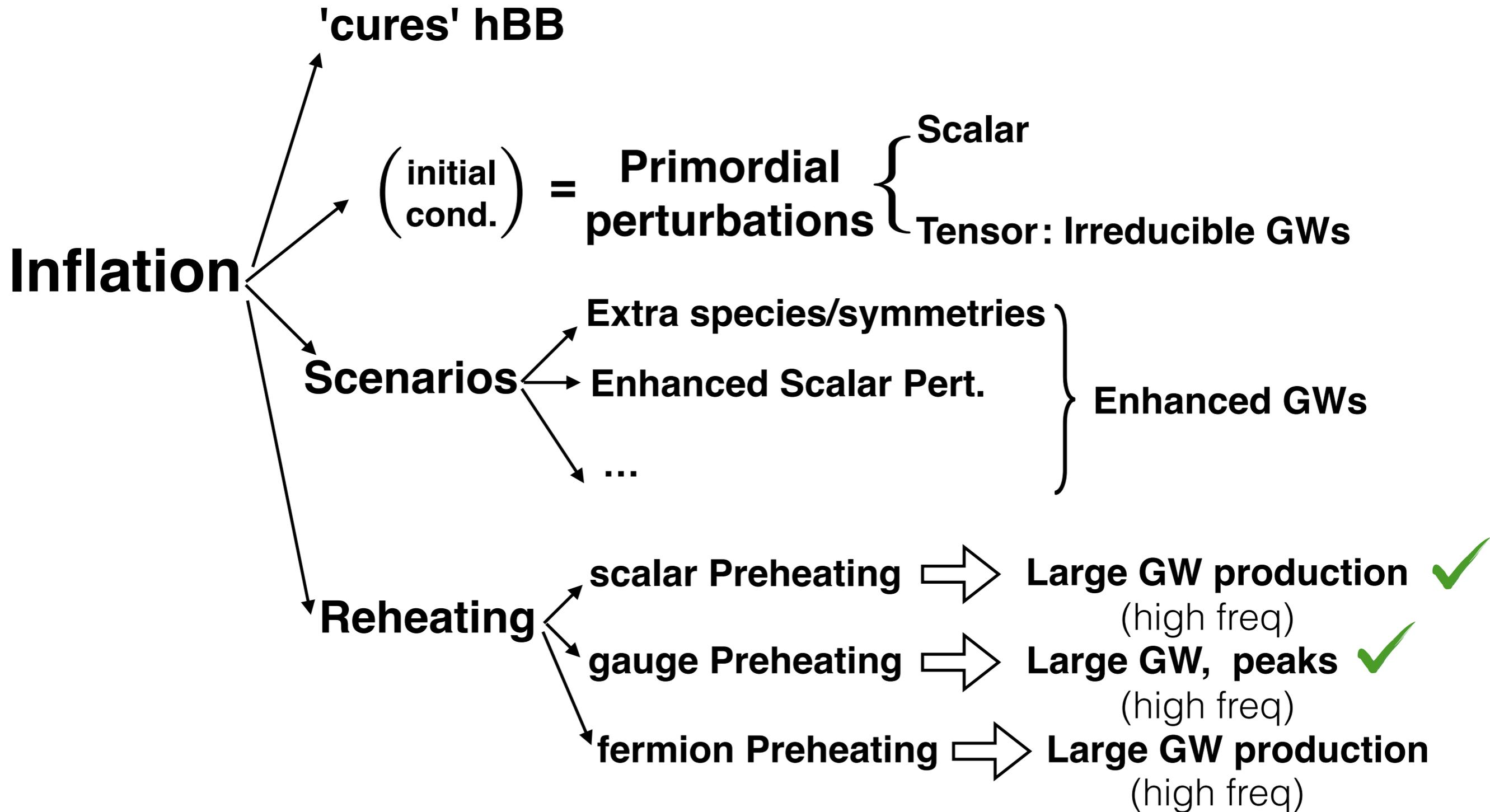
Very unfortunate... no high frequency detectors !



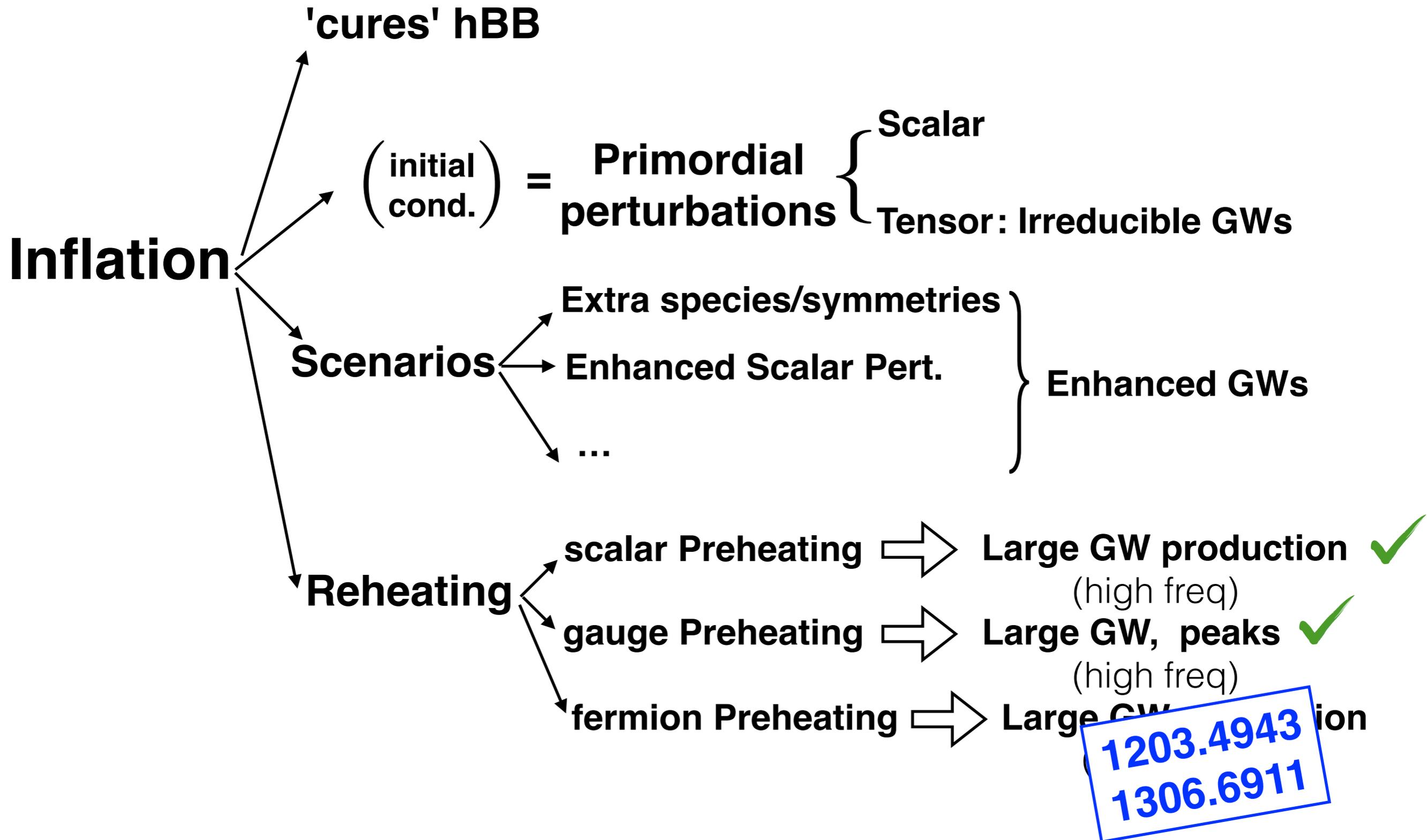
INFLATIONARY COSMOLOGY



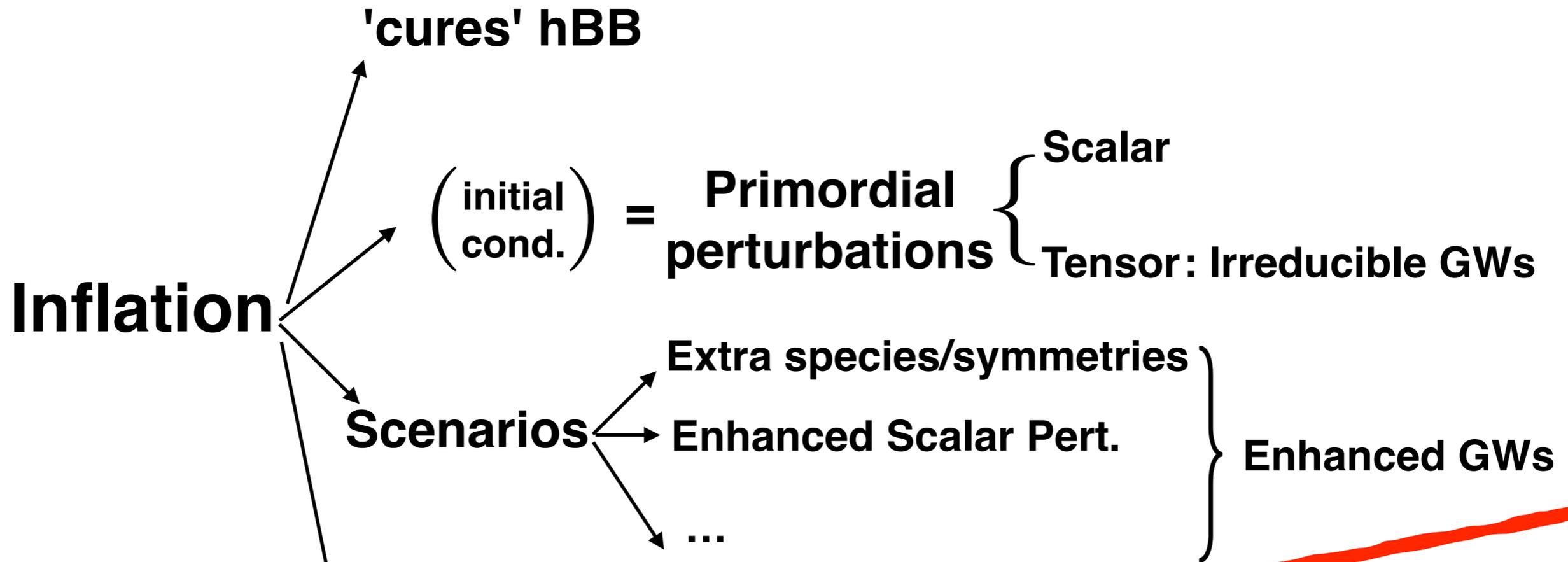
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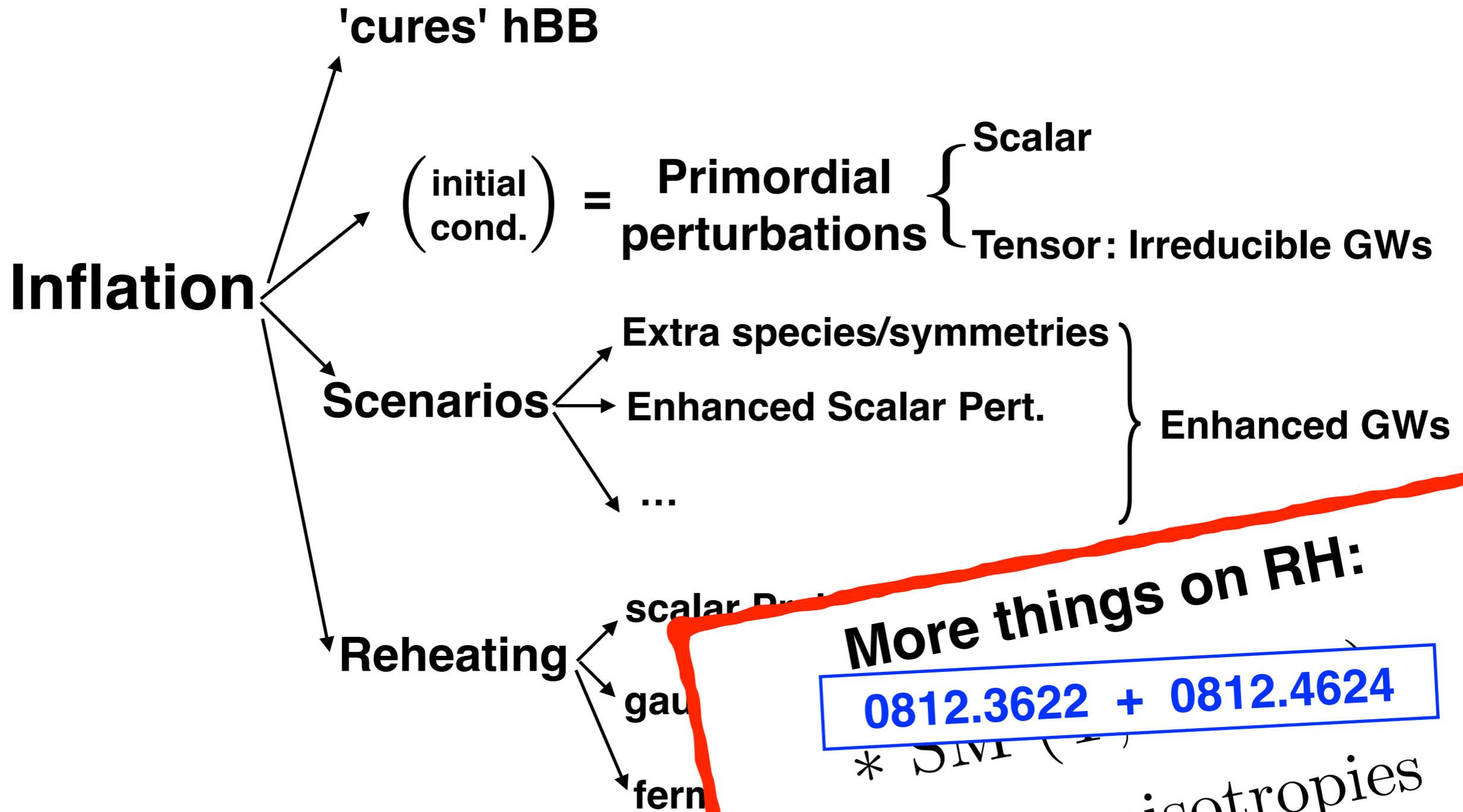


More things on RH:

* SM (Φ, Ψ_a, A_μ)

* GW anisotropies

INFLATIONARY COSMOLOGY



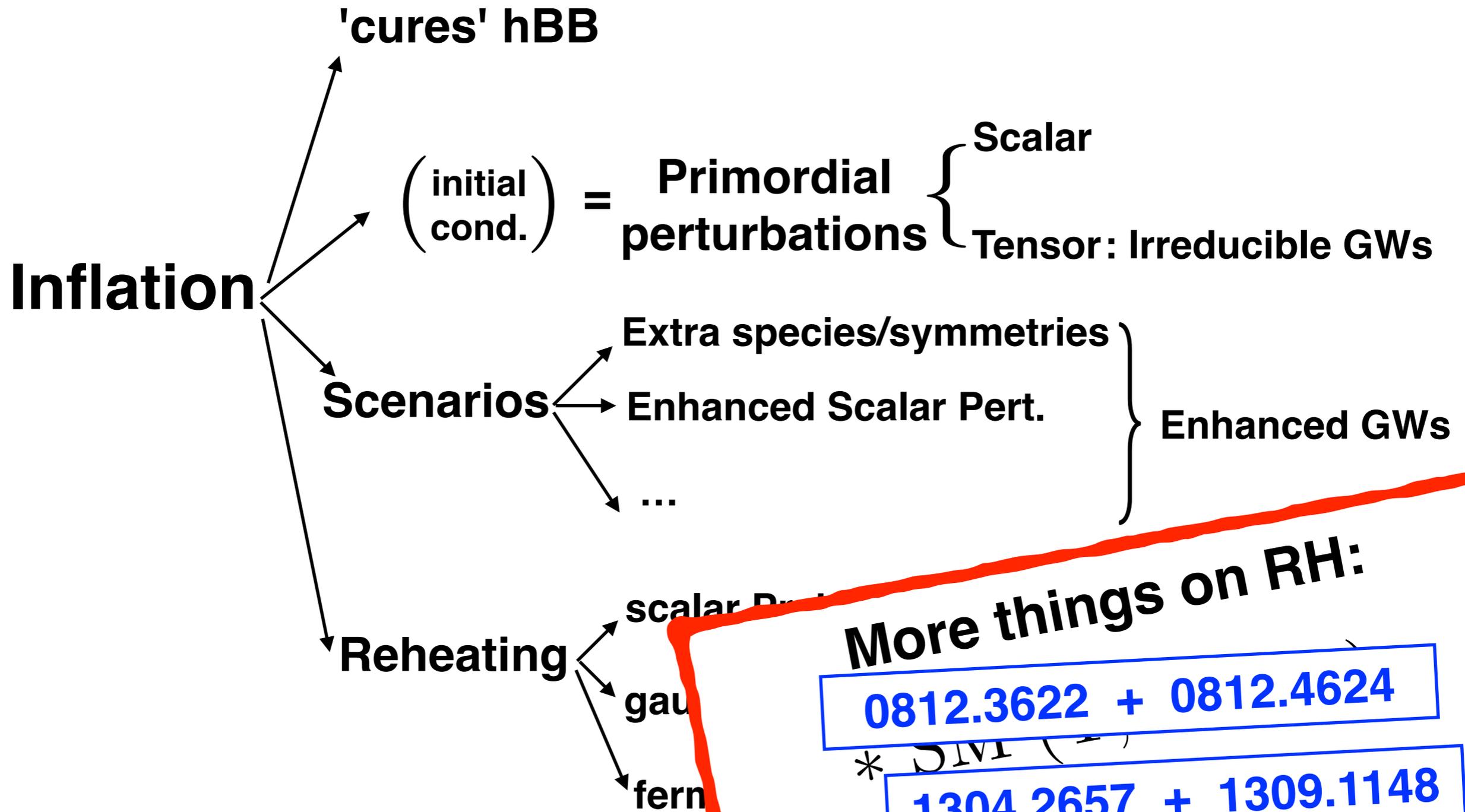
More things on RH:

0812.3622 + 0812.4624

* Δ_{LVI} ()

* GW anisotropies

INFLATIONARY COSMOLOGY



More things on RH:

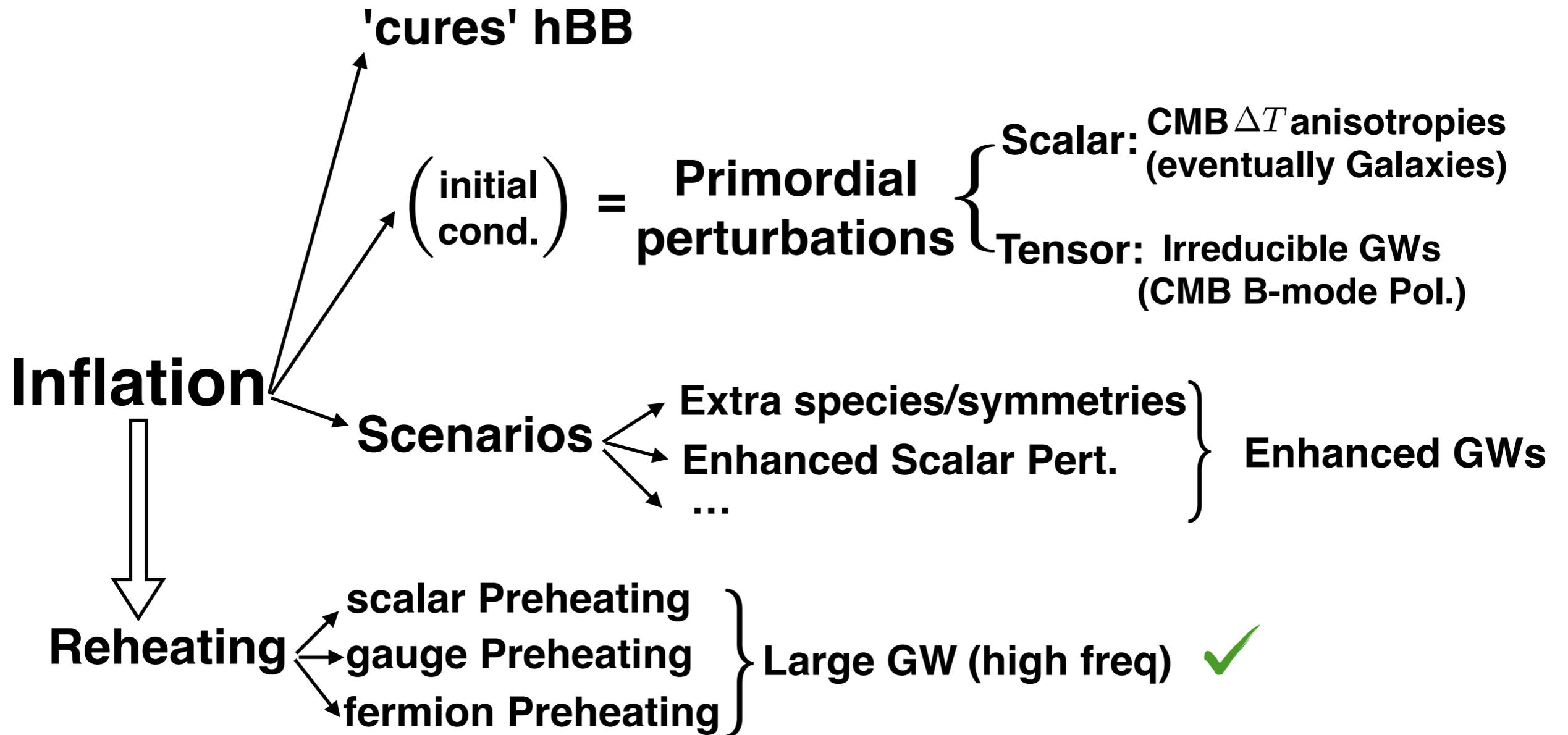
0812.3622 + 0812.4624

1304.2657 + 1309.1148

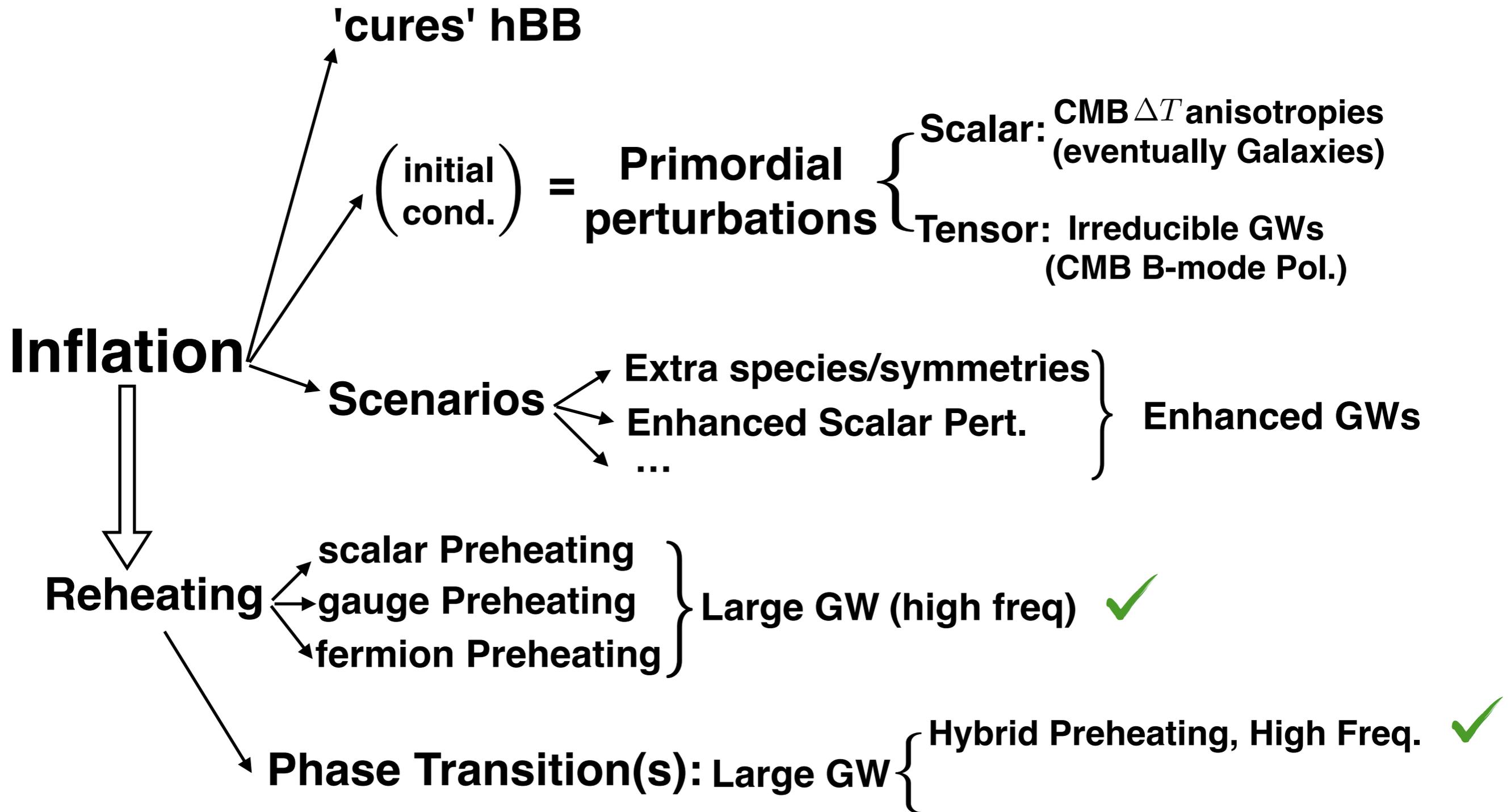
* DIVE ()

* GVV

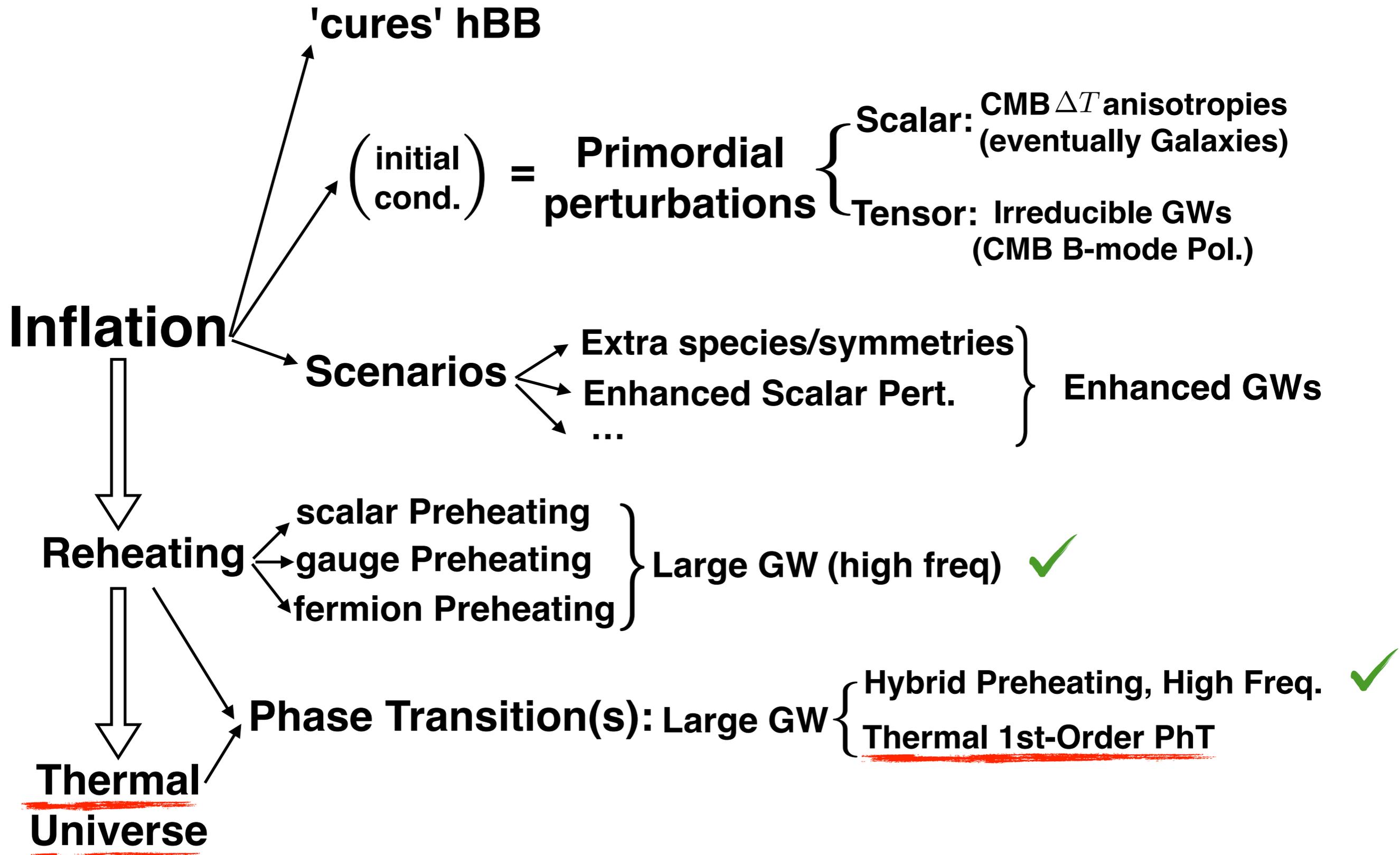
EARLY UNIVERSE



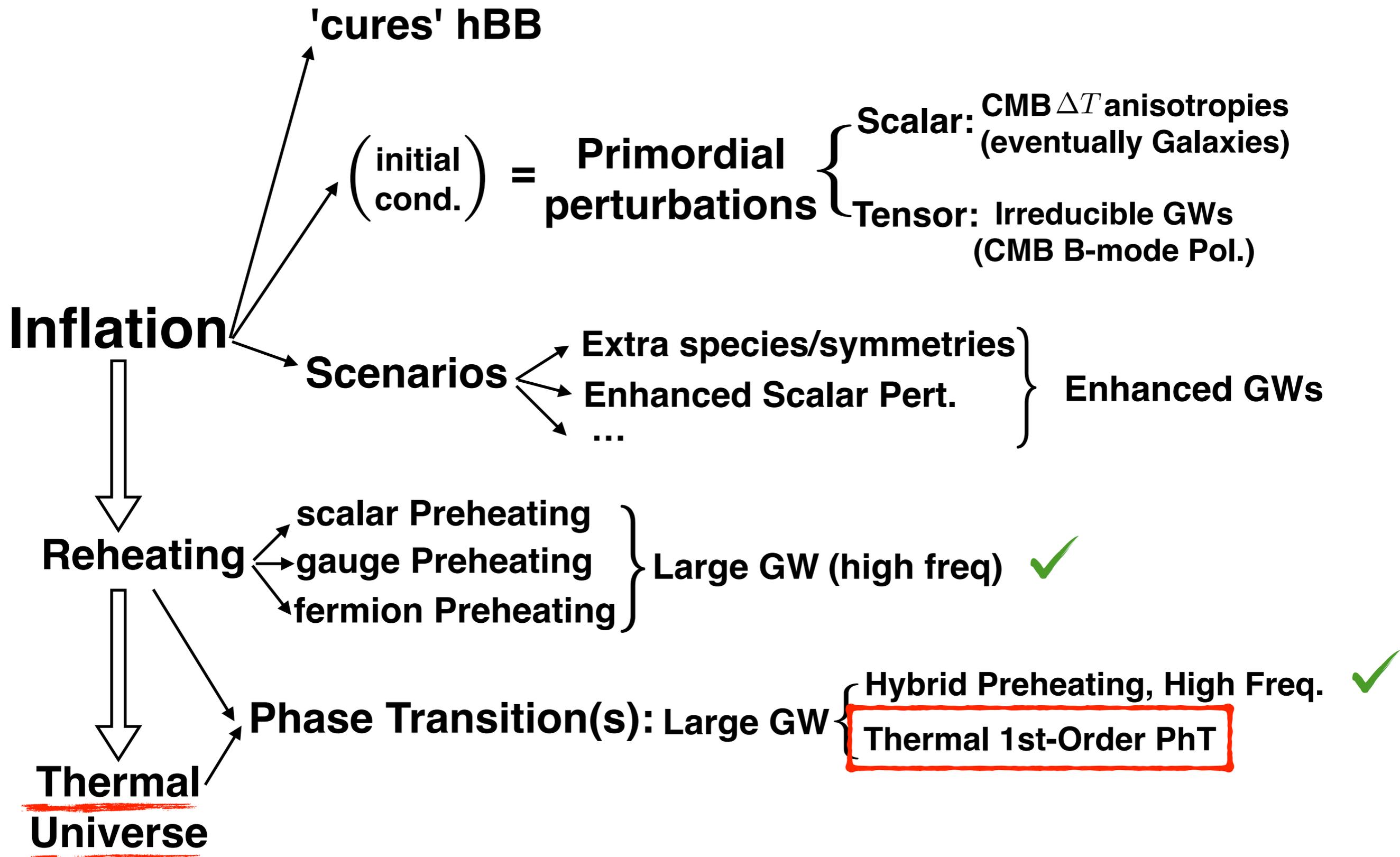
EARLY UNIVERSE



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GWs from first order phase transitions

* **GW causal source**: cannot 'operate' beyond the **horizon**

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$$\epsilon_* \leq 1$$

parameter characterising source

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$$f_c \stackrel{\text{@ Today}}{\uparrow} = f_* \stackrel{\text{@ Emission time}}{\uparrow} \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1 \text{ TeV}} \text{ Hz}$$

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@ Today ↑
@ Emission time ↑

for

$$\epsilon_* \simeq 10^{-2}$$

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LISA Freq !

for

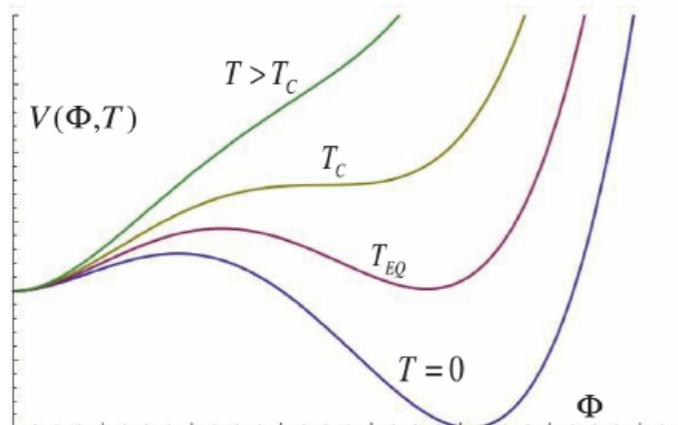
$$\epsilon_* \simeq 10^{-2}$$

$$T_* \simeq 1 \text{ TeV} \sim \text{EW scale !}$$

GWs from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

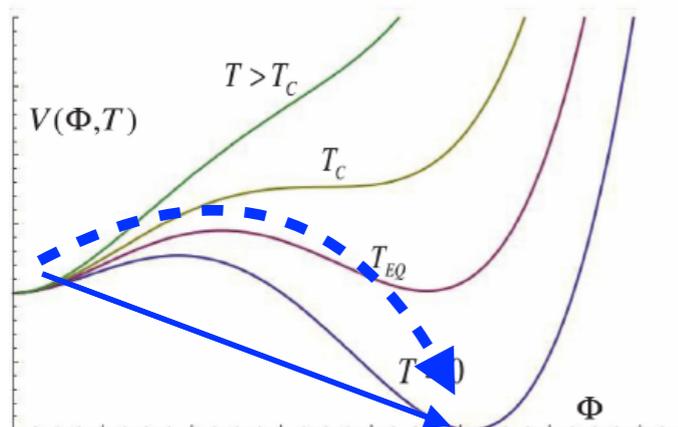
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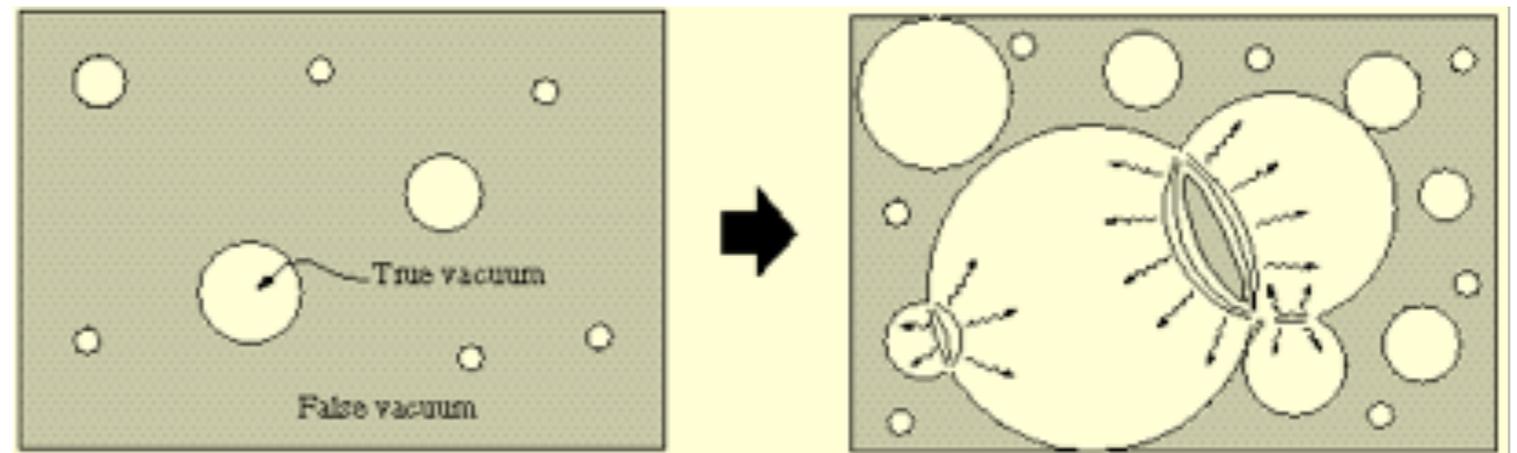
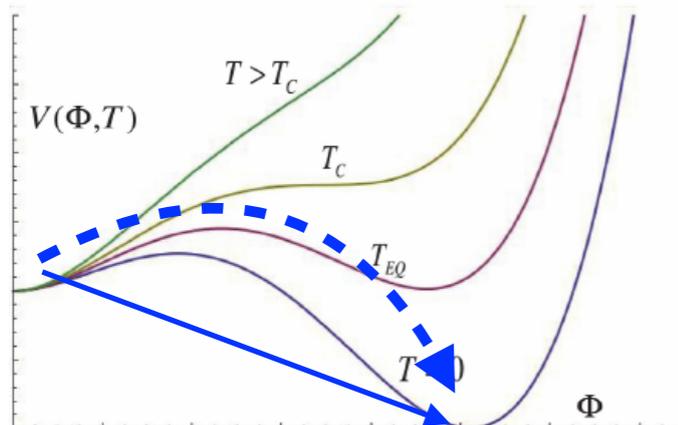


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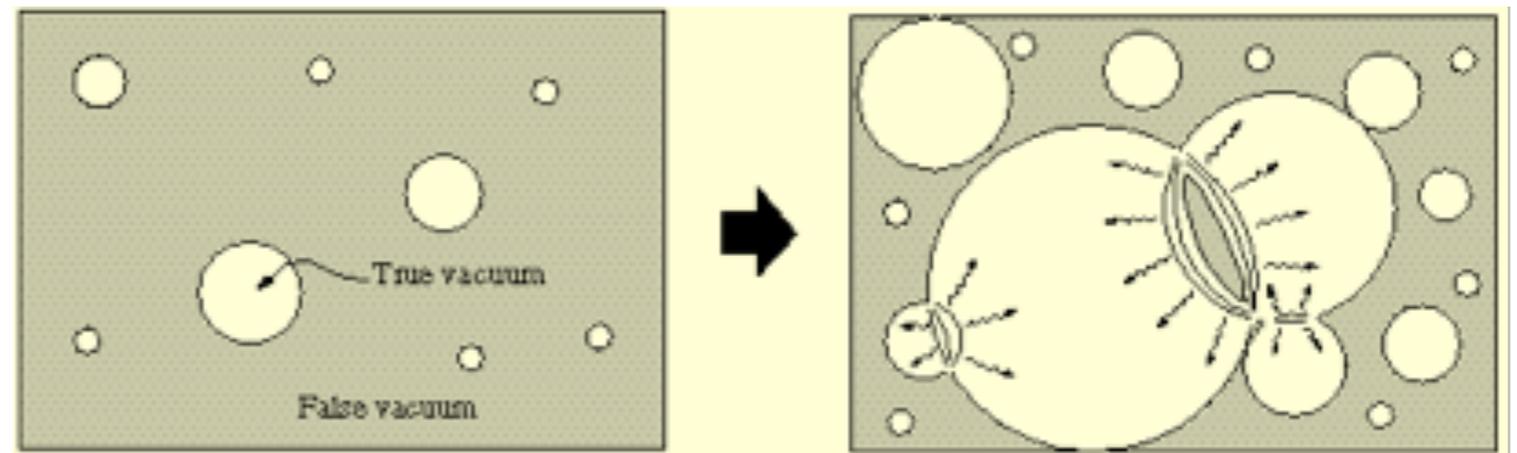
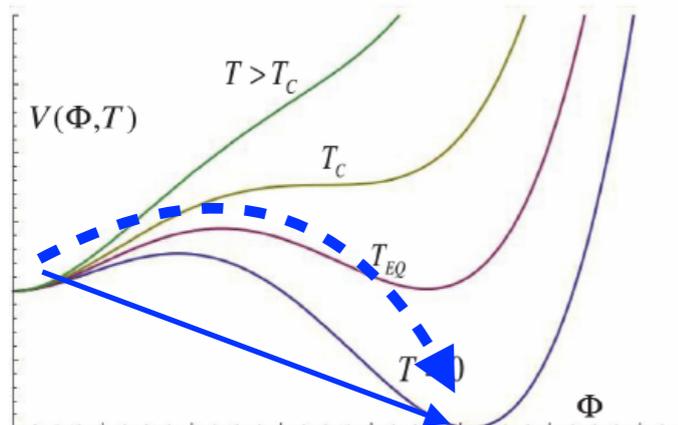


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source: Π_{ij} tensor
anisotropic stress

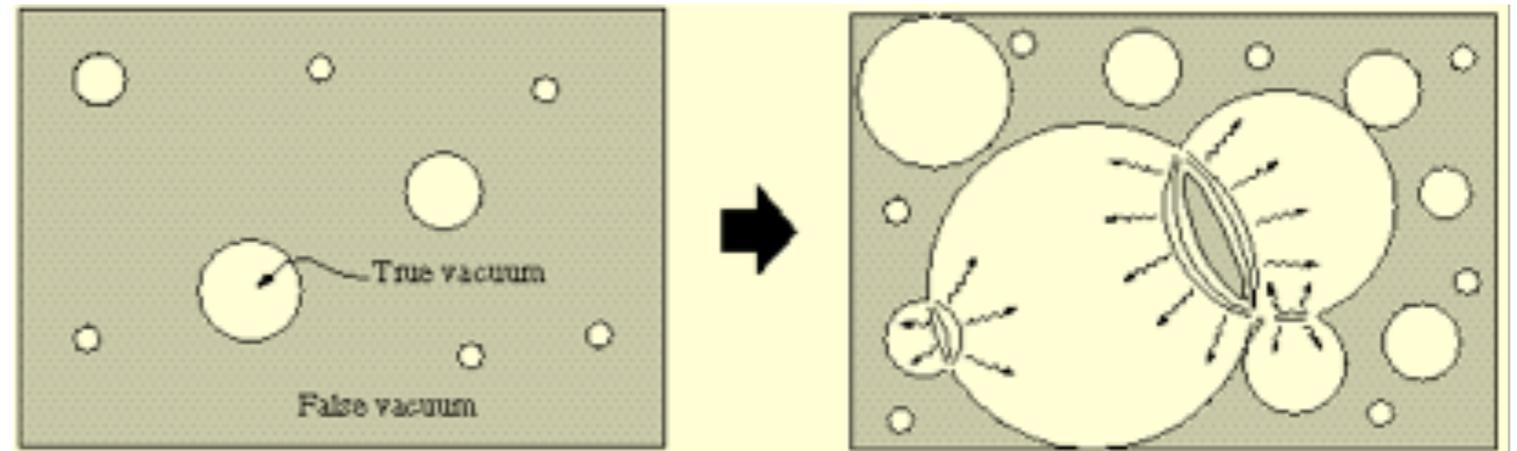
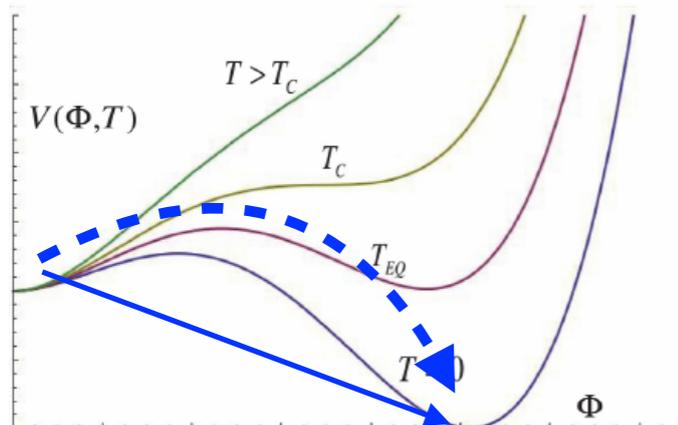
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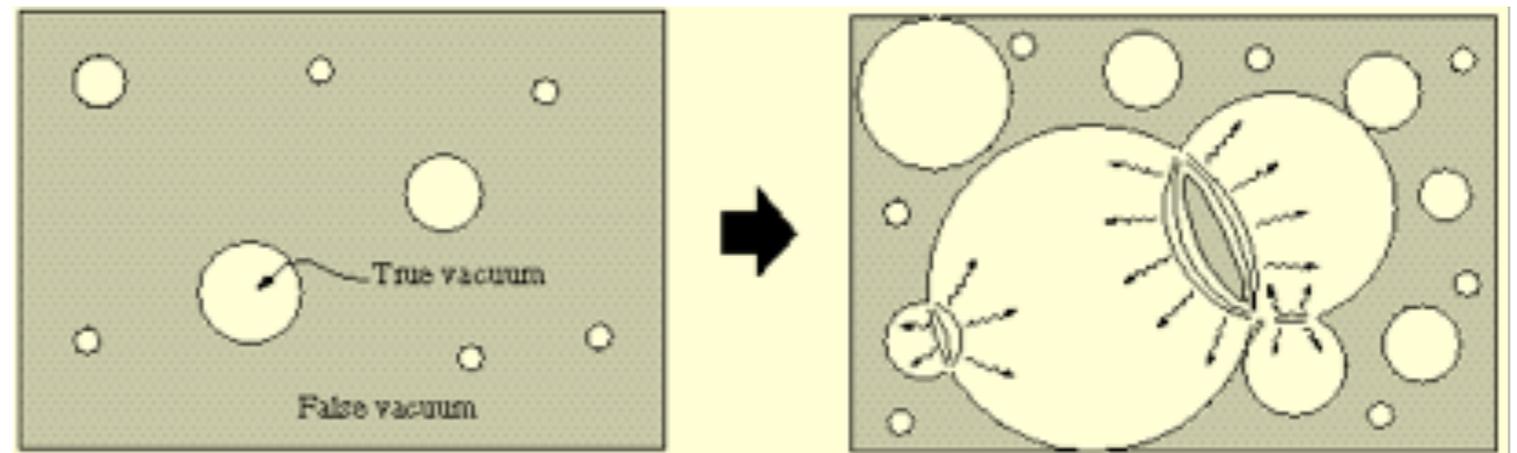
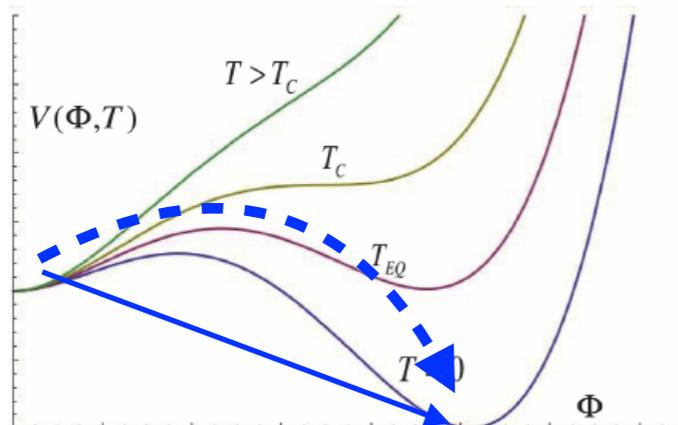
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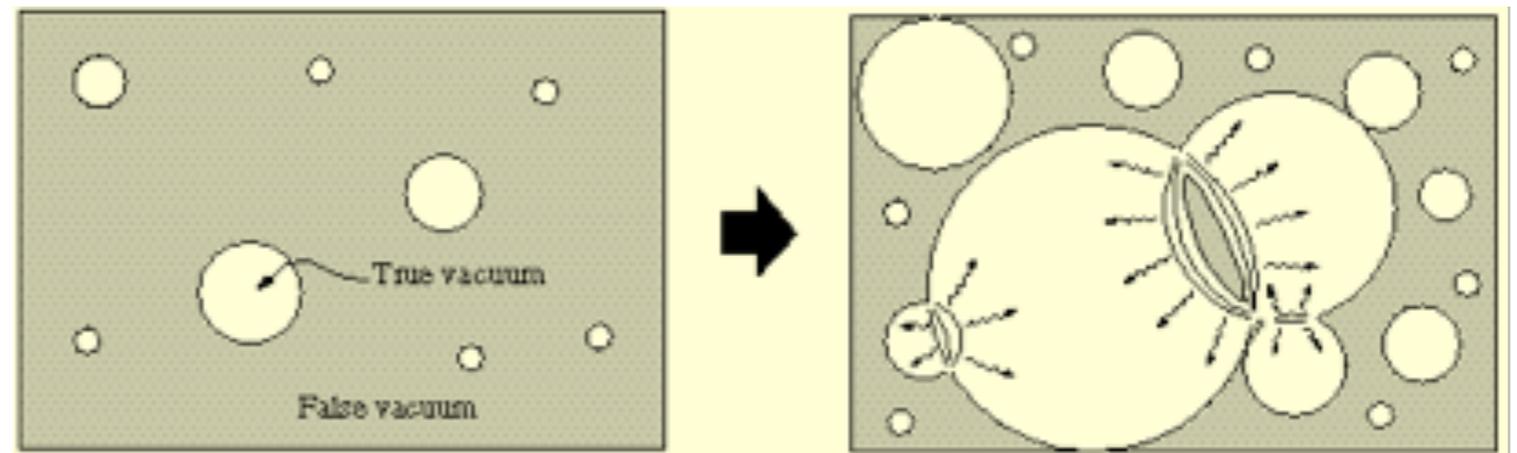
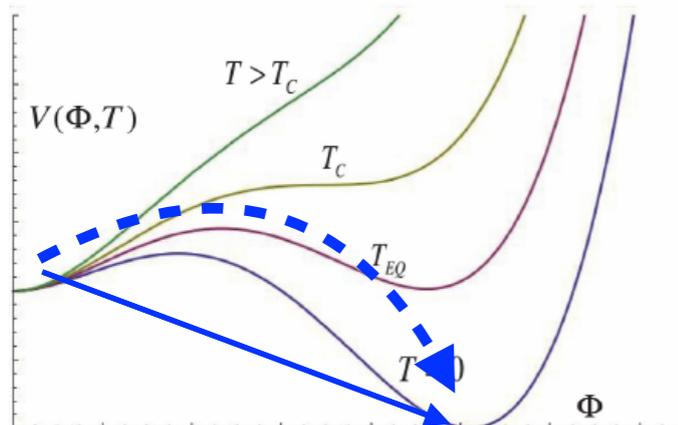
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Technically

source: Π_{ij} tensor
anisotropic stress

$$\Pi_{ij} \sim \partial_i \phi \partial_j \phi$$

$$\Pi_{ij} \sim \gamma^2 (\rho + p) v_i v_j$$

$$\Pi_{ij} \sim \frac{(E^2 + B^2)}{3} - E^i E^j - B^i B^j$$

What is ϵ in 1st Order PhT's?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1 \text{ TeV}} \text{ Hz}$$

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BUBBLE COLLISION

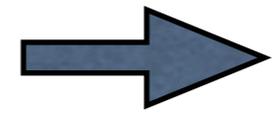
**SOUND WAVES AND
MDH TURBULENCE**

Parameters determining the GW spectrum

Freq.
(today)

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$$\frac{\beta}{H_*}, \quad v_b, \quad T_*$$

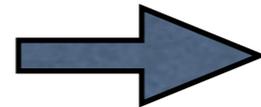
Parameter List
(not independent)

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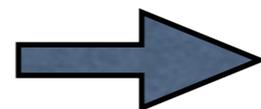


Parameter List
(not independent)

$$\frac{\beta}{H_*}, \quad v_b, \quad T_*$$

Amplitude
(today)

$$\Omega_{\text{GW}} \sim \Omega_{\text{rad}} \epsilon_*^2 \left(\frac{\rho_s^*}{\rho_{\text{tot}}^*} \right)^2$$



$$\frac{\rho_s^*}{\rho_{\text{tot}}^*} = \frac{\kappa \alpha}{1 + \alpha}$$

$$\alpha = \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}^*}$$

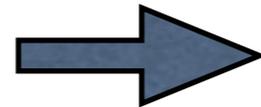
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Amplitude
(today)

Ω_{GW}

not most general!

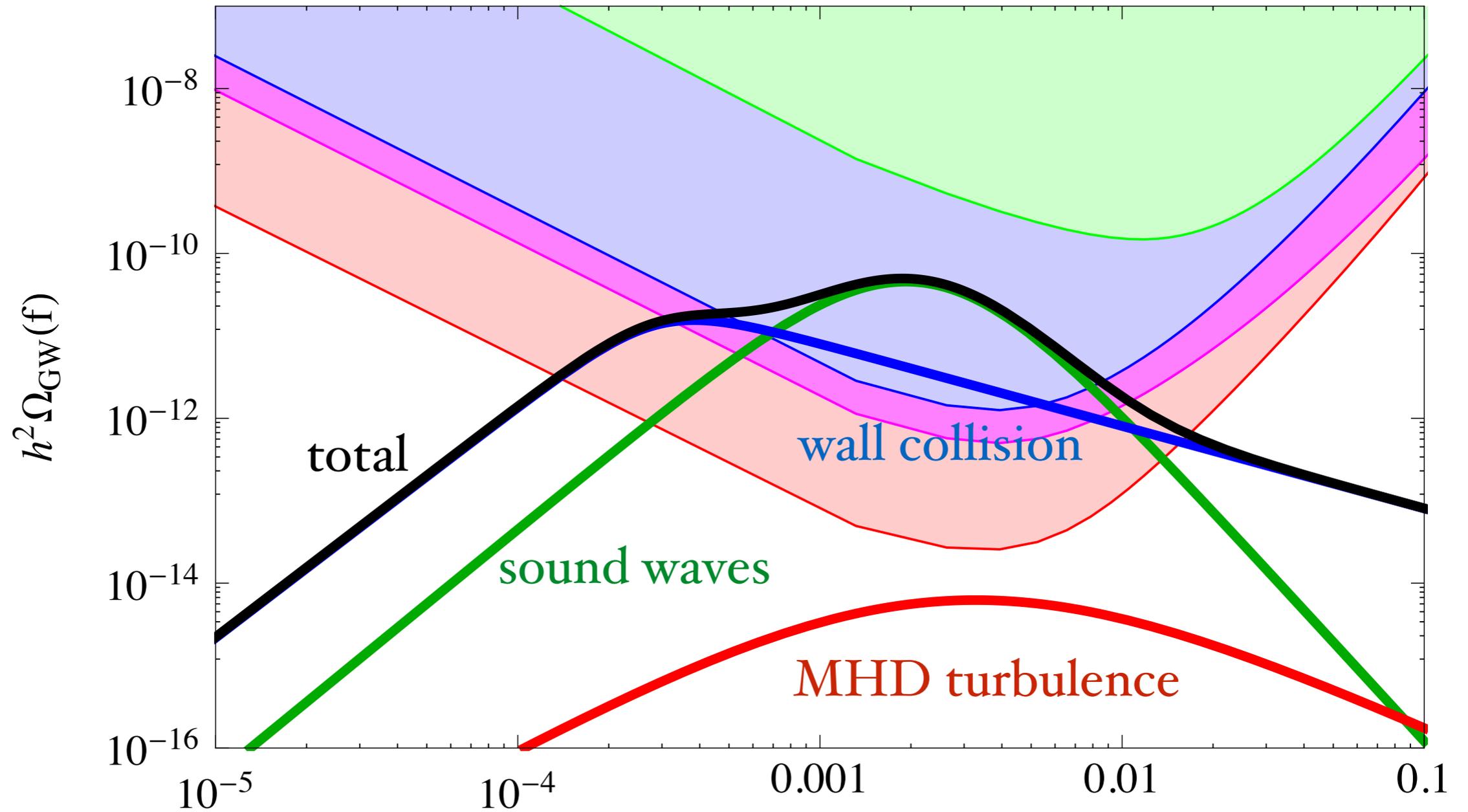
Hydro-dynamical effects & turbulence separate treatment



$$\alpha = \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}^*}$$

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Example of spectrum



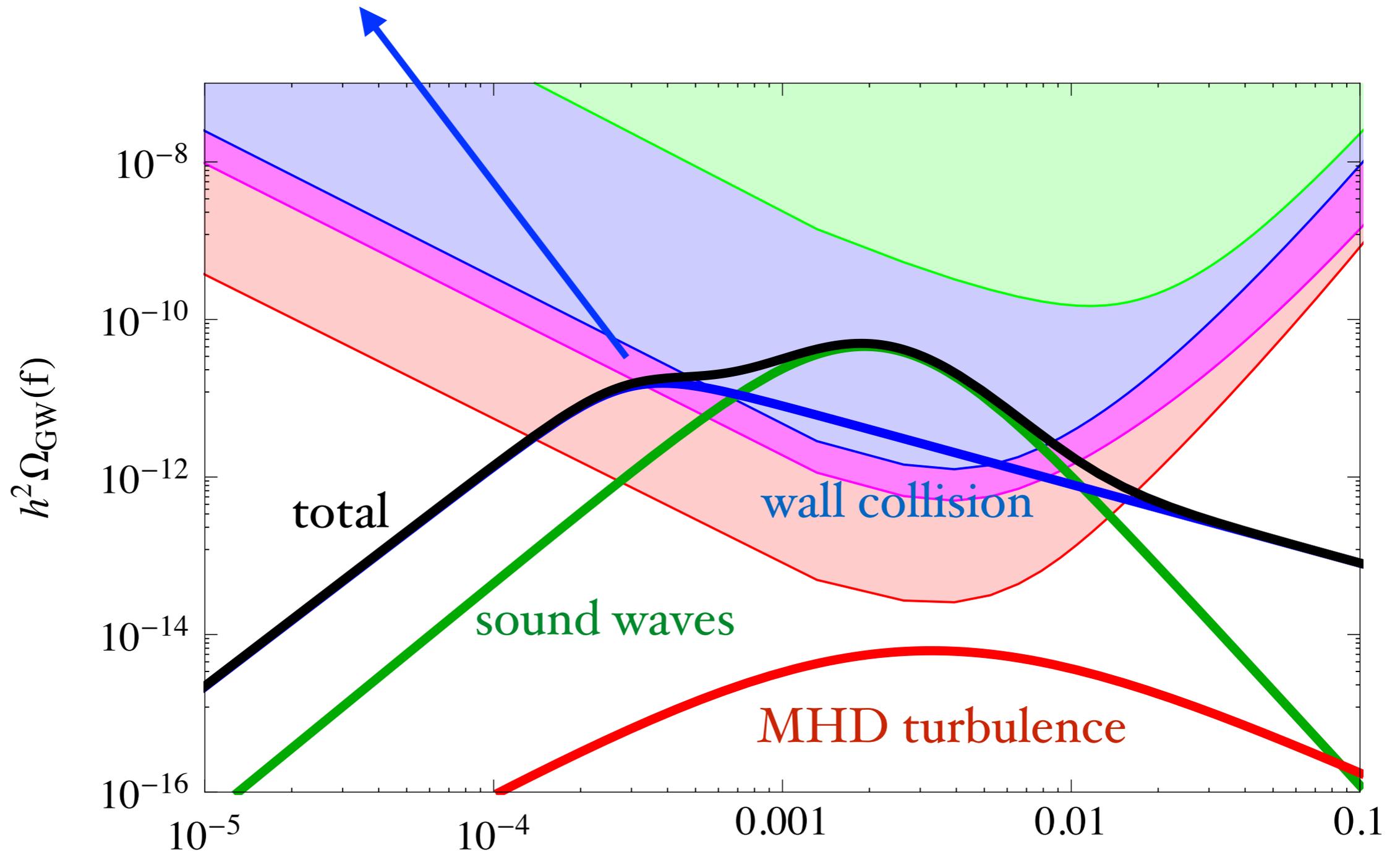
Caprini et al,
arXiv:1512.06239

f [Hz]

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peak of bubble collisions



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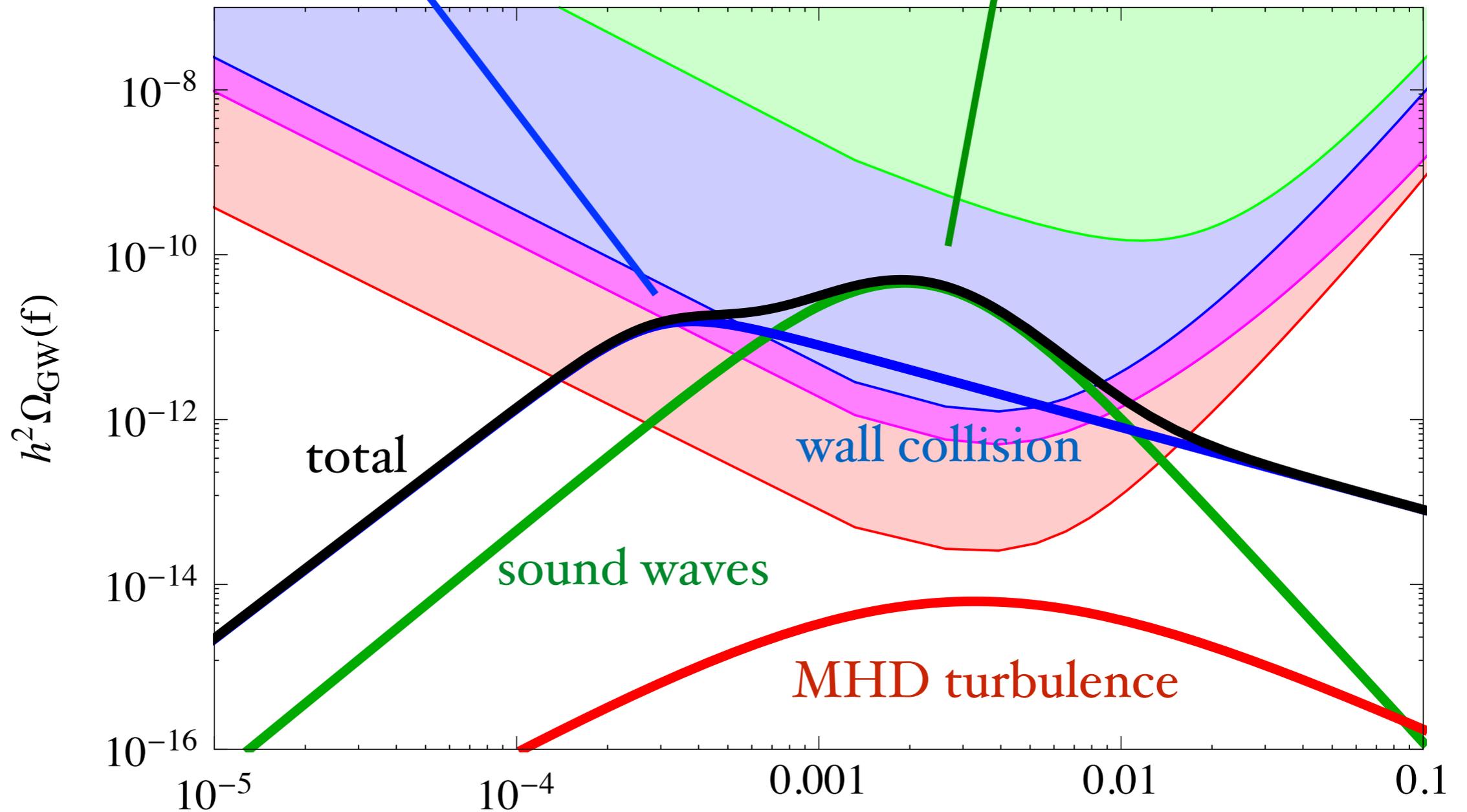
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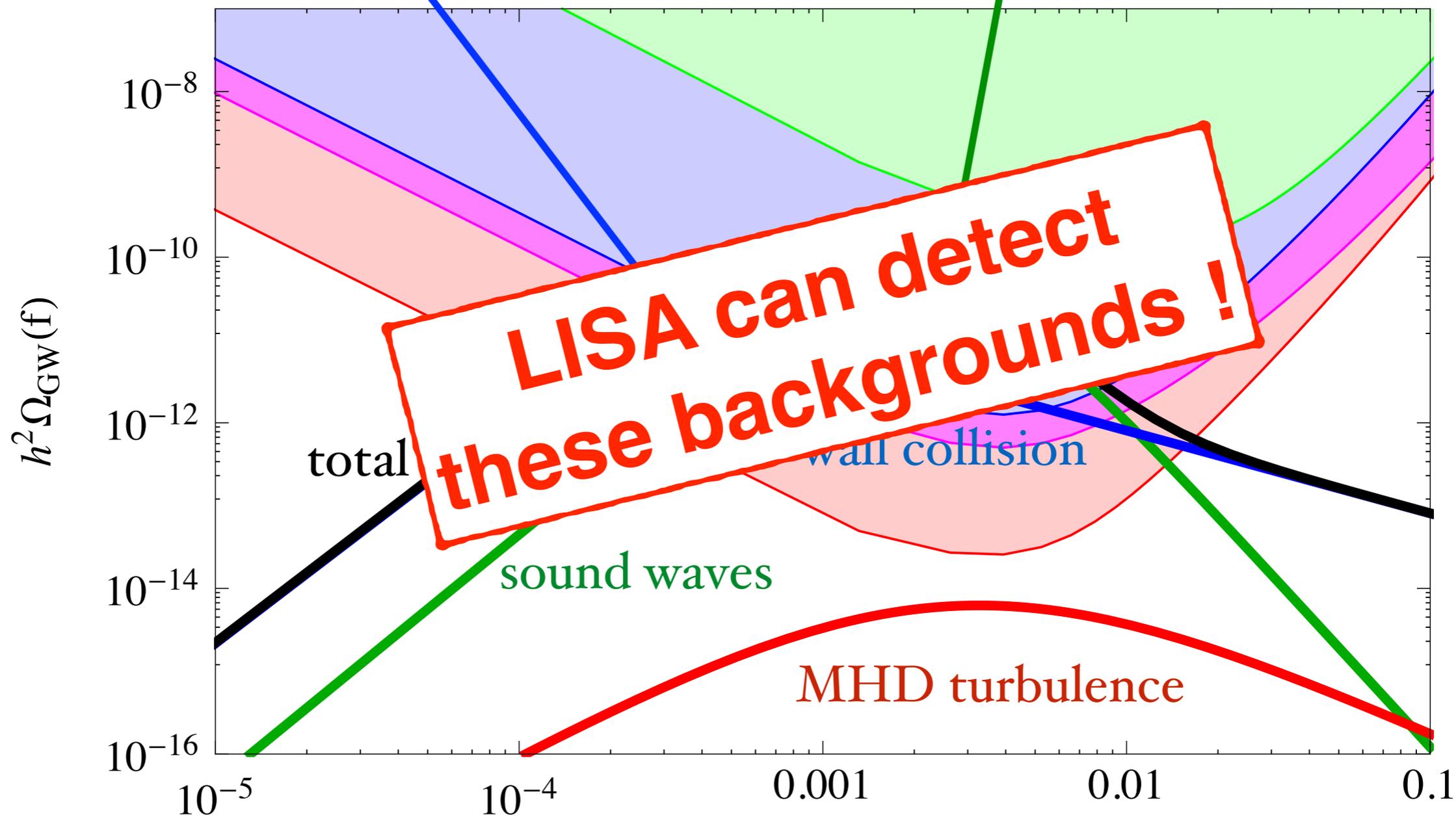
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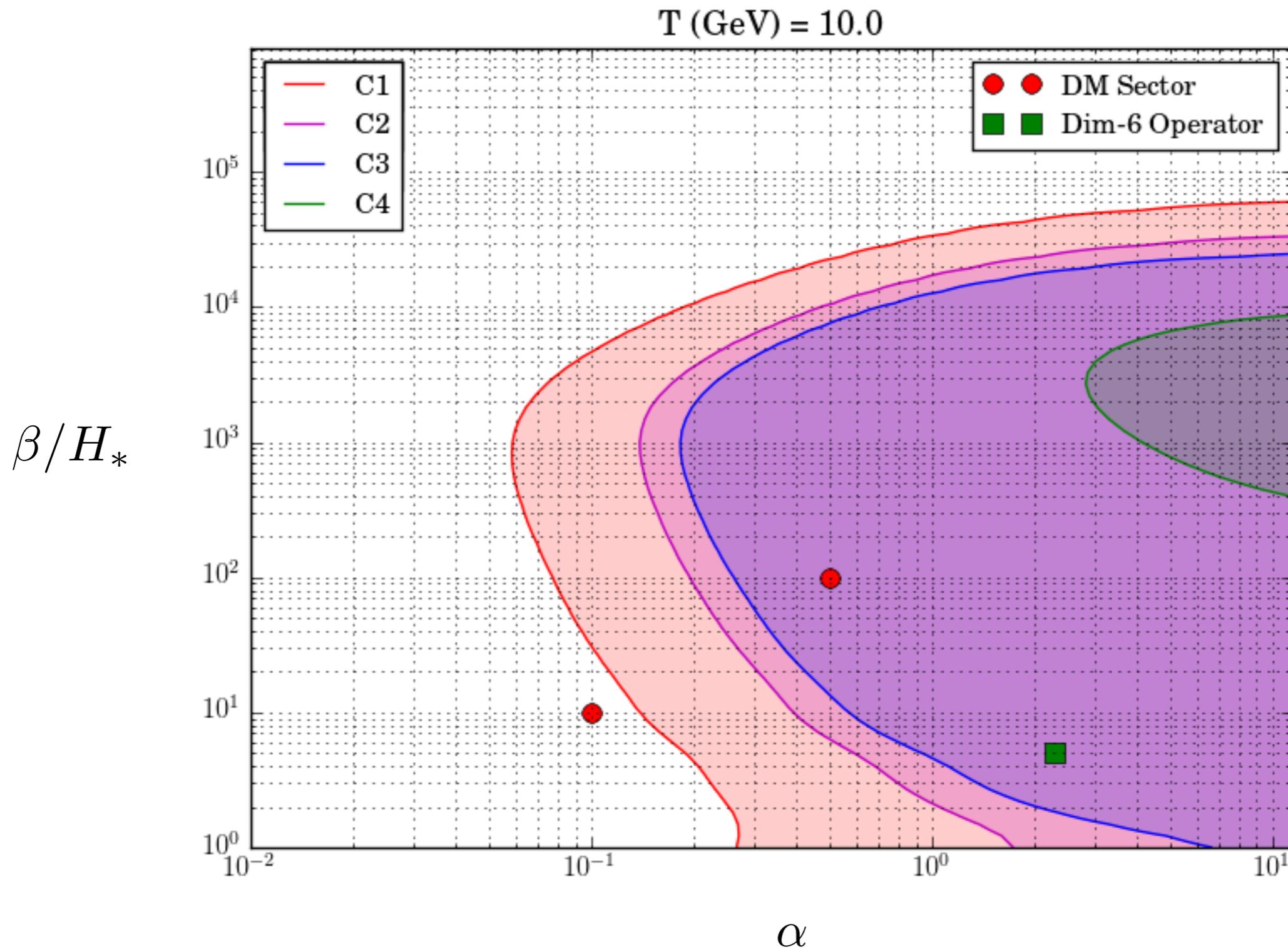


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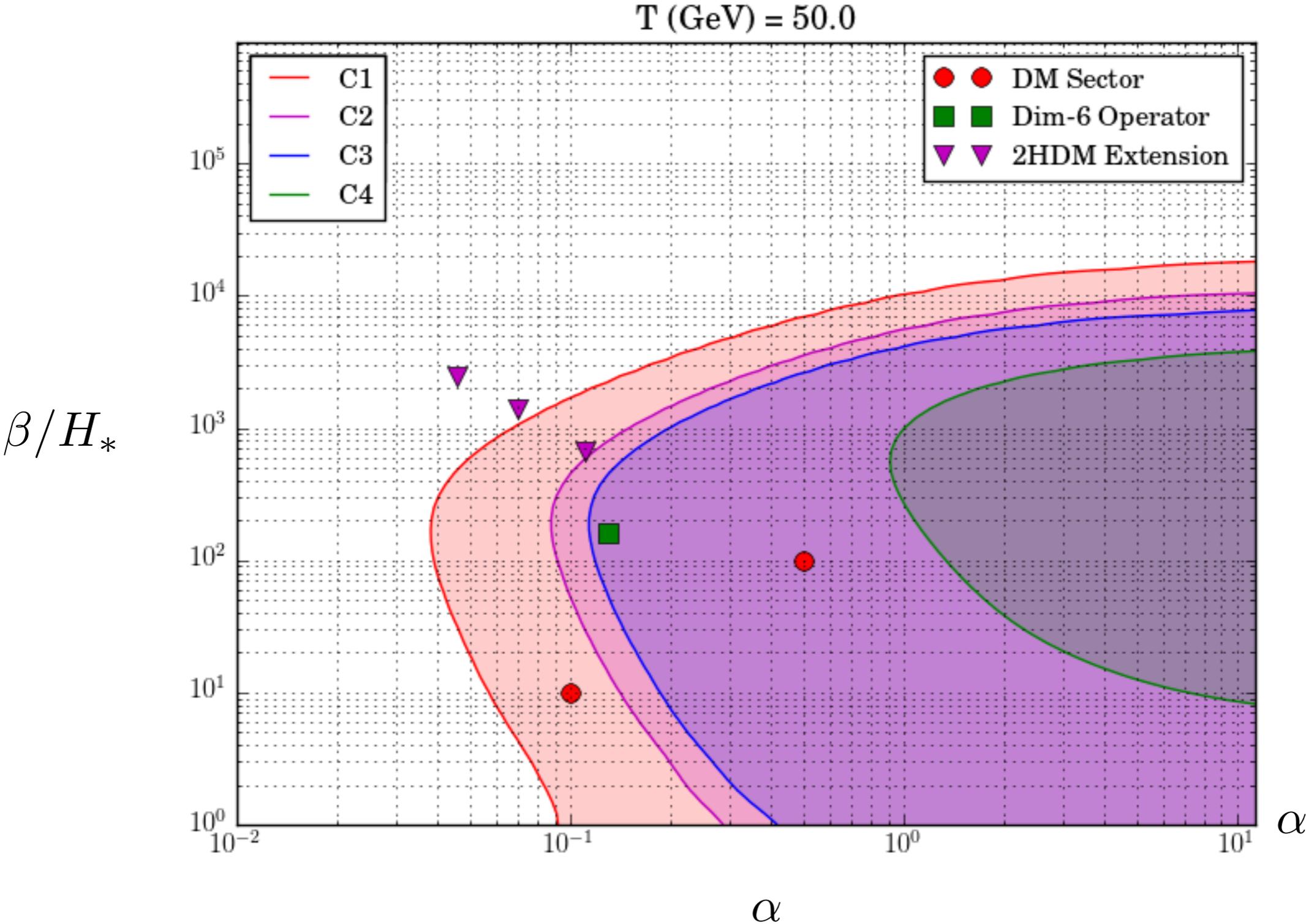
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Detection prospects for LISA



Caprini et al, 2015/2016 (LISA 1PhT working group)

Detection prospects for LISA



Evaluation of the signal

- **bubble collisions: analytical** and **numerical** simulations
Huber, Konstandin '08 Cutting, Hindmarsh et al 2018, ...

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- **MDH turbulence: analytical** evaluation
Kosowsky et al '07, Caprini et al '09, Niksa et al '18
numerical Pol et al 2019

Models for EWPT and beyond

- **LISA** sensitive to energy scale **10 GeV - 100 TeV !**
(mHZ)

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Models for EWPT and beyond

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- simple

Cosmology and Particle Physics interplay!
Connections with baryon asymmetry & dark matter

(e.g. Grojean et al 2015)

scalars (e.g. Kozackuz et al 2013)

dimension six operator (e.g. Grojean et al 2004)

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- sin

Conn

Problem: LHC is putting great pressure over BSM scenarios

Interplay!
& dark matter

(Schwaller et al 2015)

(Kuznetsov et al 2013)

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Conn

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**Interplay!
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(Carroll et al 2015)

(Kuznetsov et al 2013)

(Gondolo et al 2004)

- ... and beyond the EWPT

- Dark sector: provides DM

(Schwartz)

**GW → new probe of BSM physics!
(complementary to particle colliders)**

PhT

in the dilaton/radion
like models (Randall and Servant 2015)

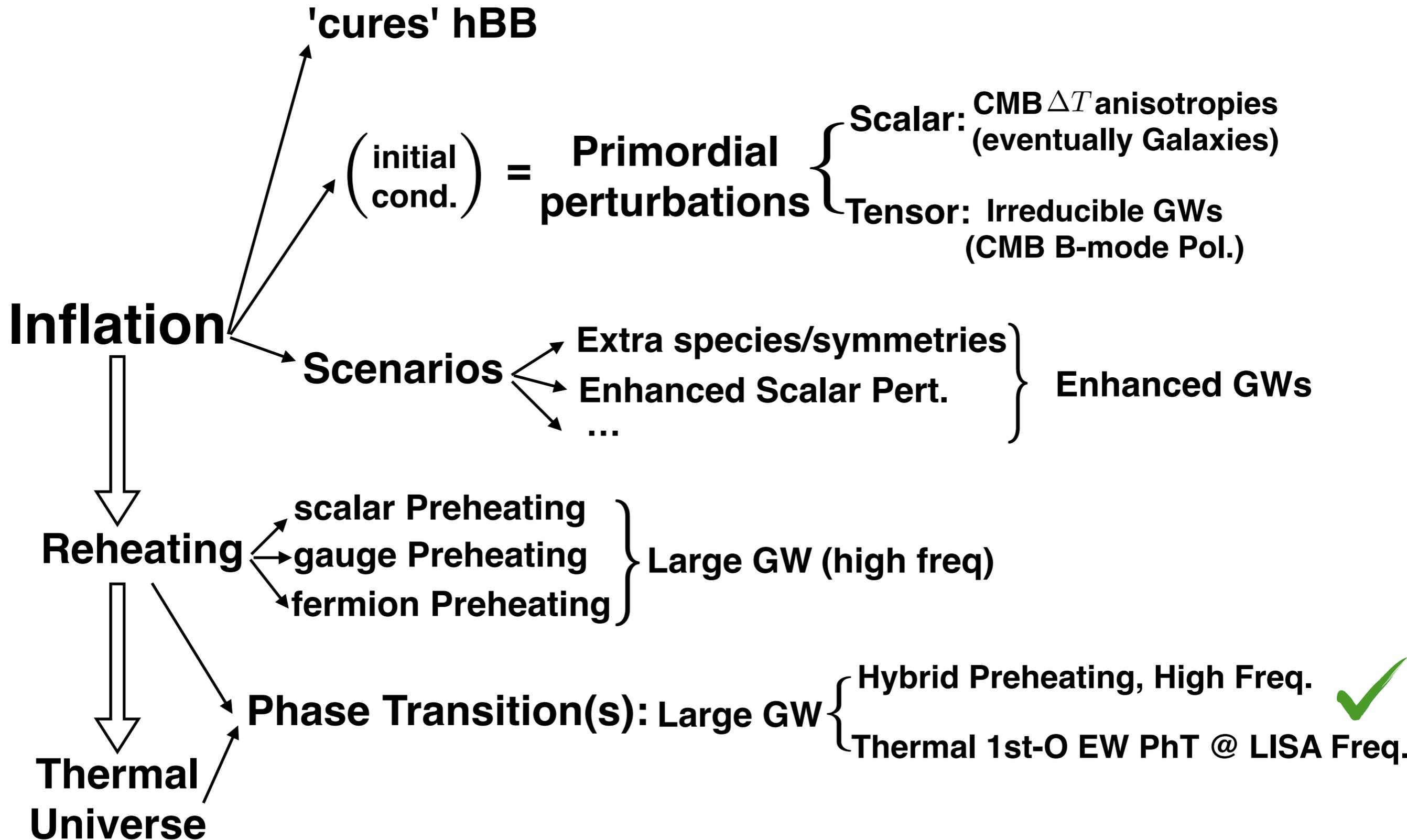
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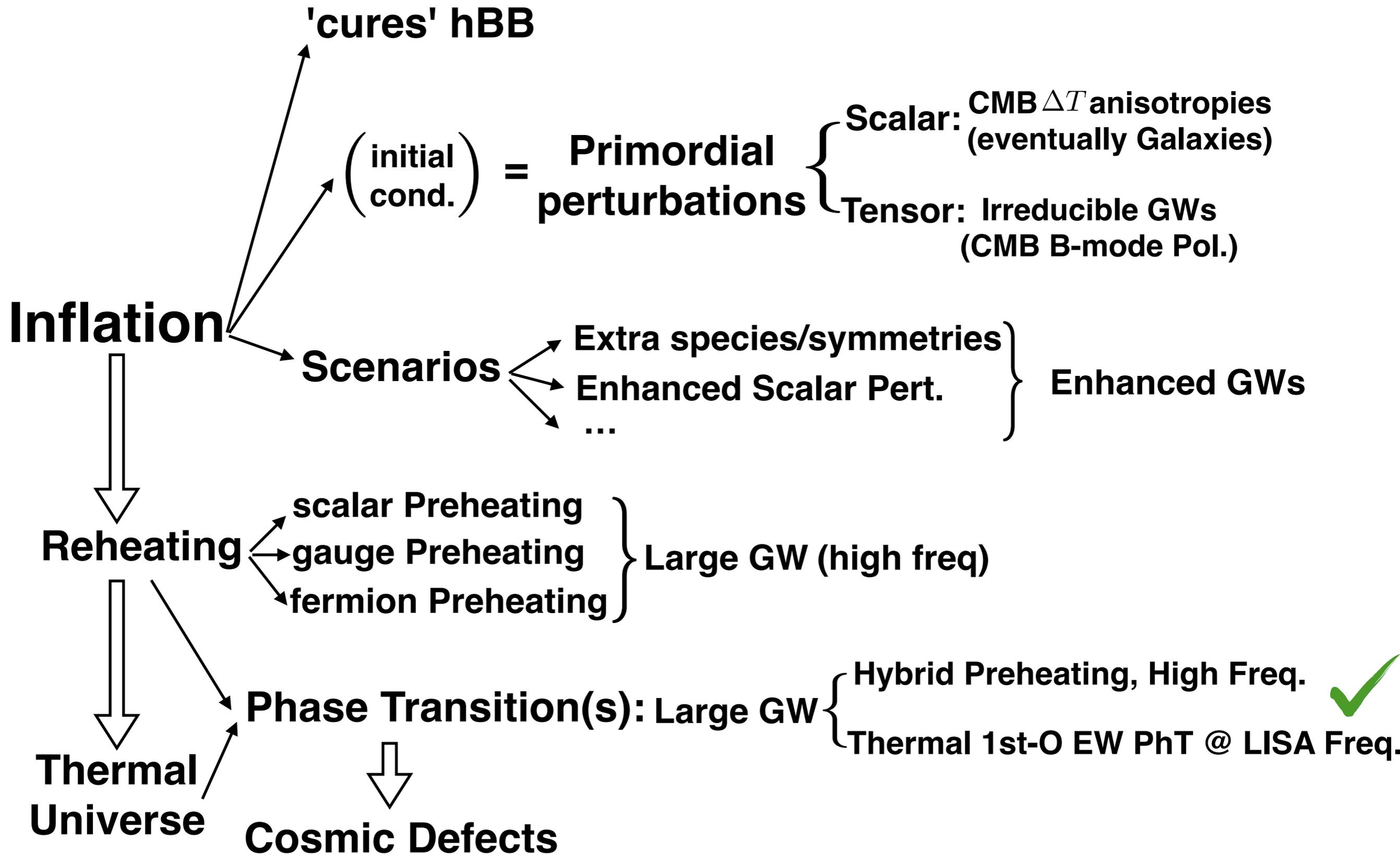
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- * Assuming LHC does not rule out models, LISA/ET can detect/constrain significant fraction of Param Space
- * Predictions depend on many assumptions (particularly in sound waves), so is our modelling correct?
- * Even if we detect it, then we infer α and β , but what BSM model is behind? **not univocal !**

EARLY UNIVERSE



EARLY UNIVERSE



EARLY UNIVERSE

