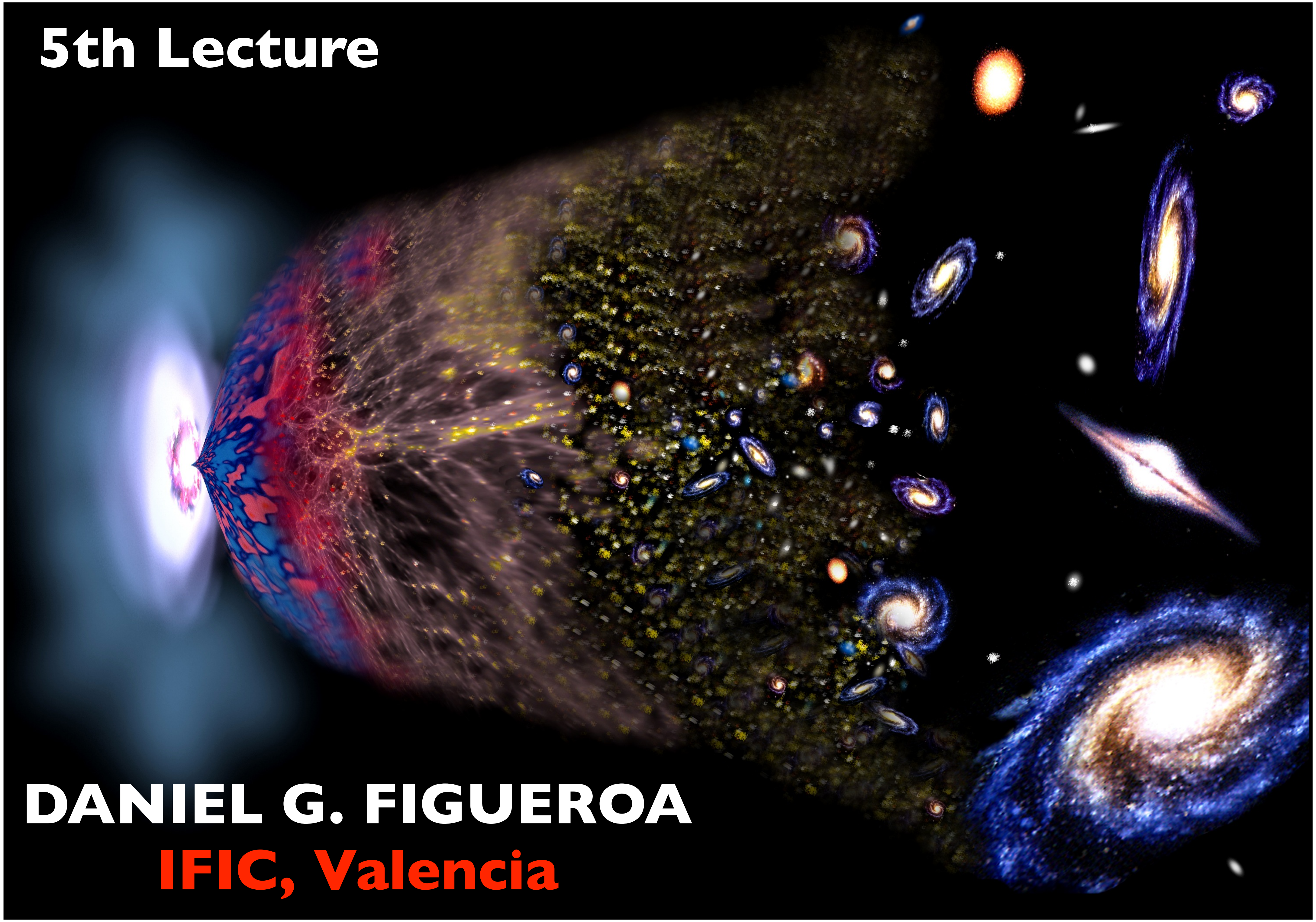
GRAVITATIONAL WAVE
— BACKGROUNDS —



5th Lecture

DANIEL G. FIGUEROA

IFIC, Valencia

GGI LECTURES ON THE THEORY OF FUNDAMENTAL INTERACTIONS — 2022 Program (3rd week)

OUTLINE

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

2) GWs from Inflation ✓

3) GWs from Preheating ✓

3rd Bloc

4) GWs from Phase Transitions ✓

5) GWs from Cosmic Defects

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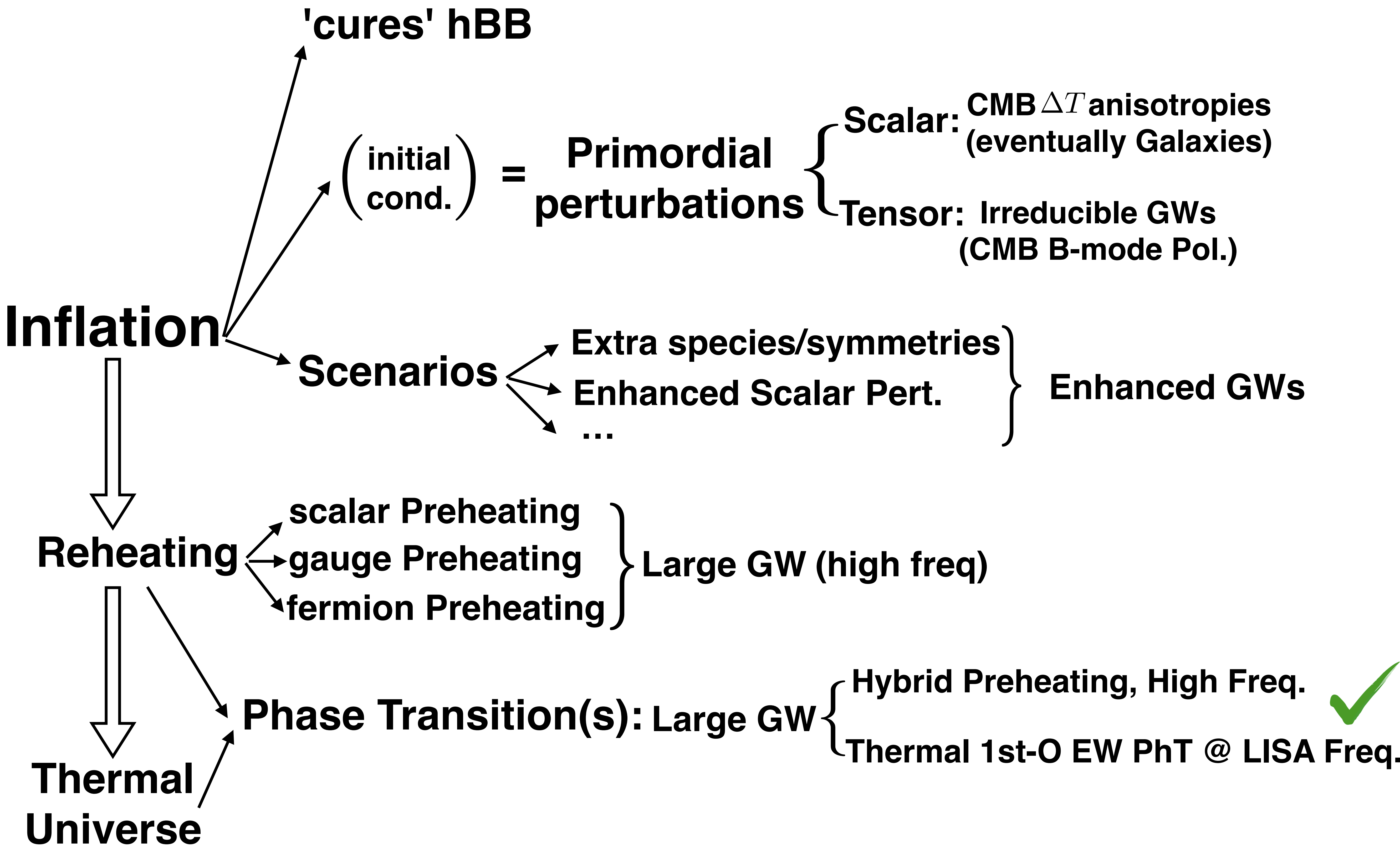
EARLY UNIVERSE



'cures' hBB

$\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

Scalar: CMB ΔT anisotropies (eventually Galaxies)

Tensor: Irreducible GWs (CMB B-mode Pol.)

Inflation

Scenarios

Extra species/symmetries

Enhanced Scalar Pert.

...

Enhanced GWs

Reheating

scalar Preheating

gauge Preheating

fermion Preheating

Large GW (high freq)

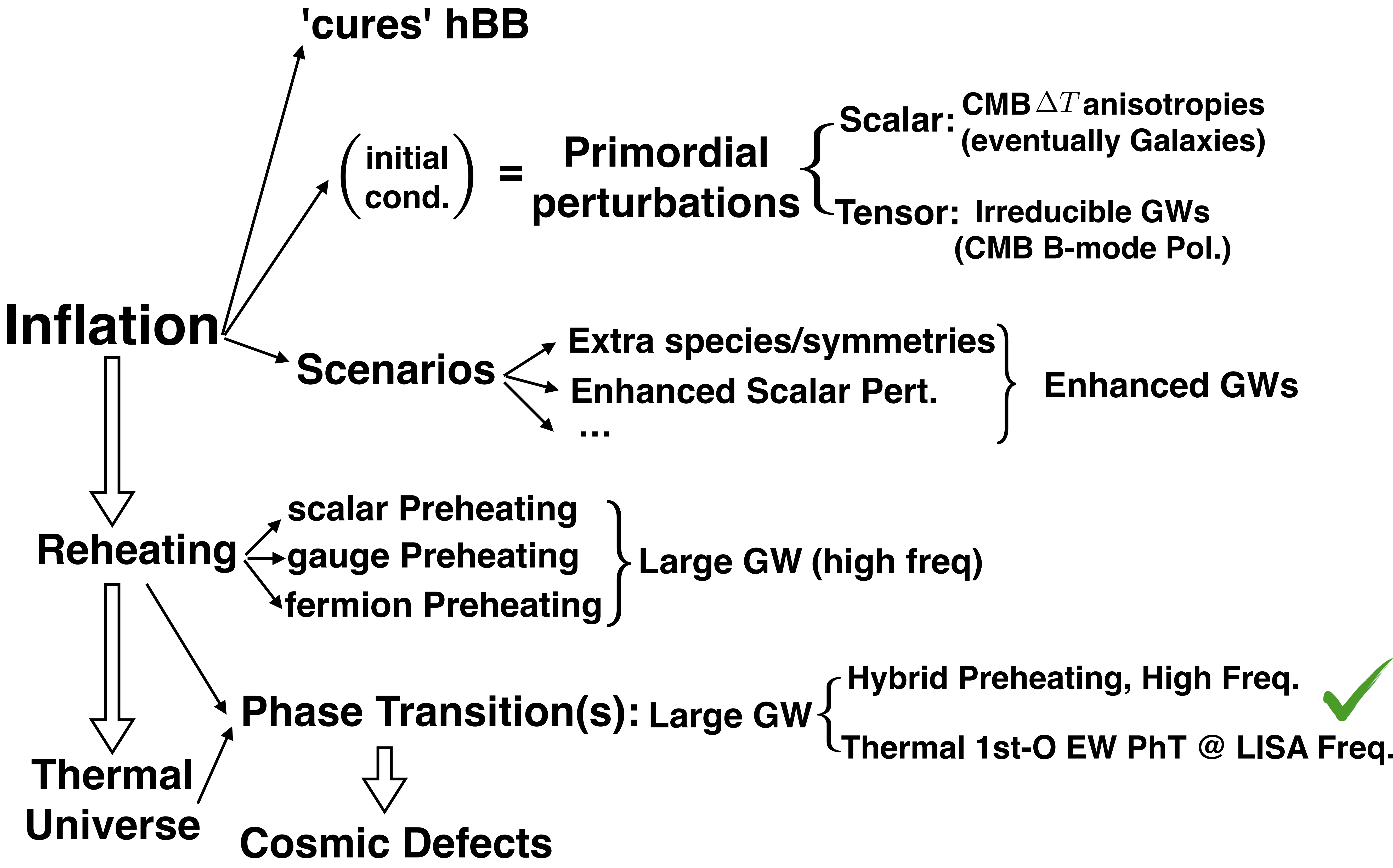
Thermal Universe

Phase Transition(s): Large GW

Hybrid Preheating, High Freq. ✓

Thermal 1st-O EW PhT @ LISA Freq.

EARLY UNIVERSE



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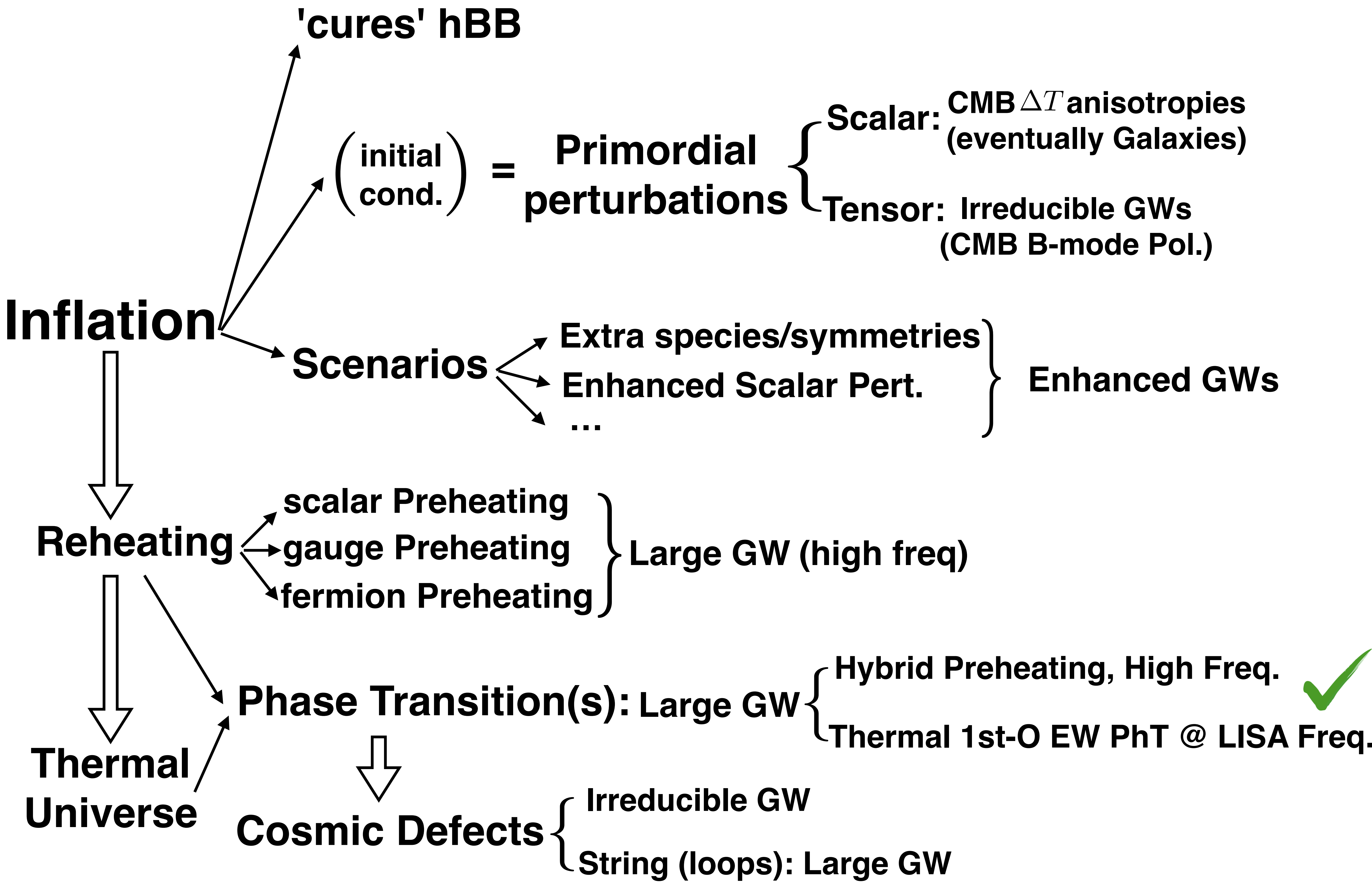
Cosmic Defects

EARLY UNIVERSE



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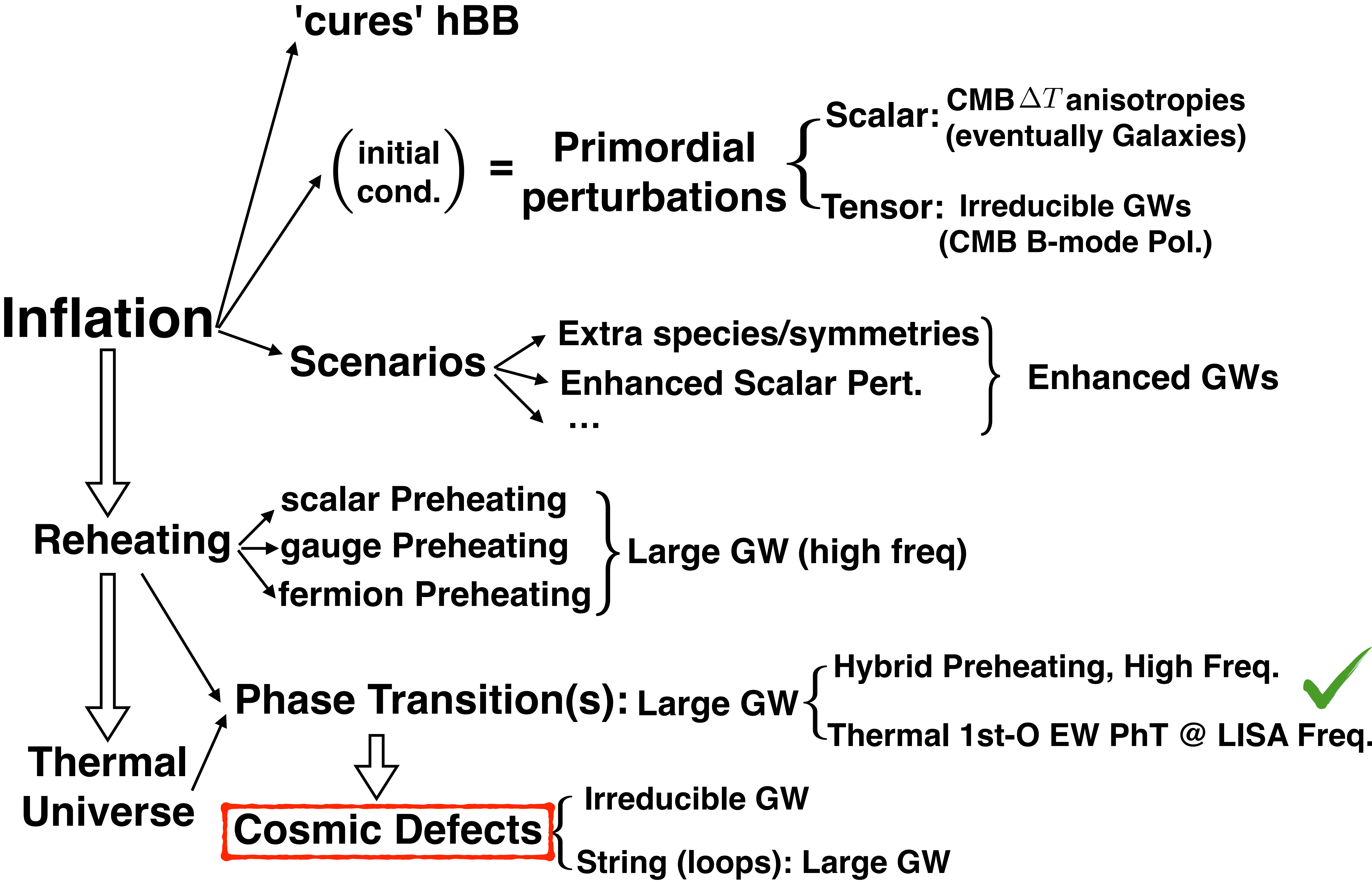
EARLY UNIVERSE



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Cosmic Defects

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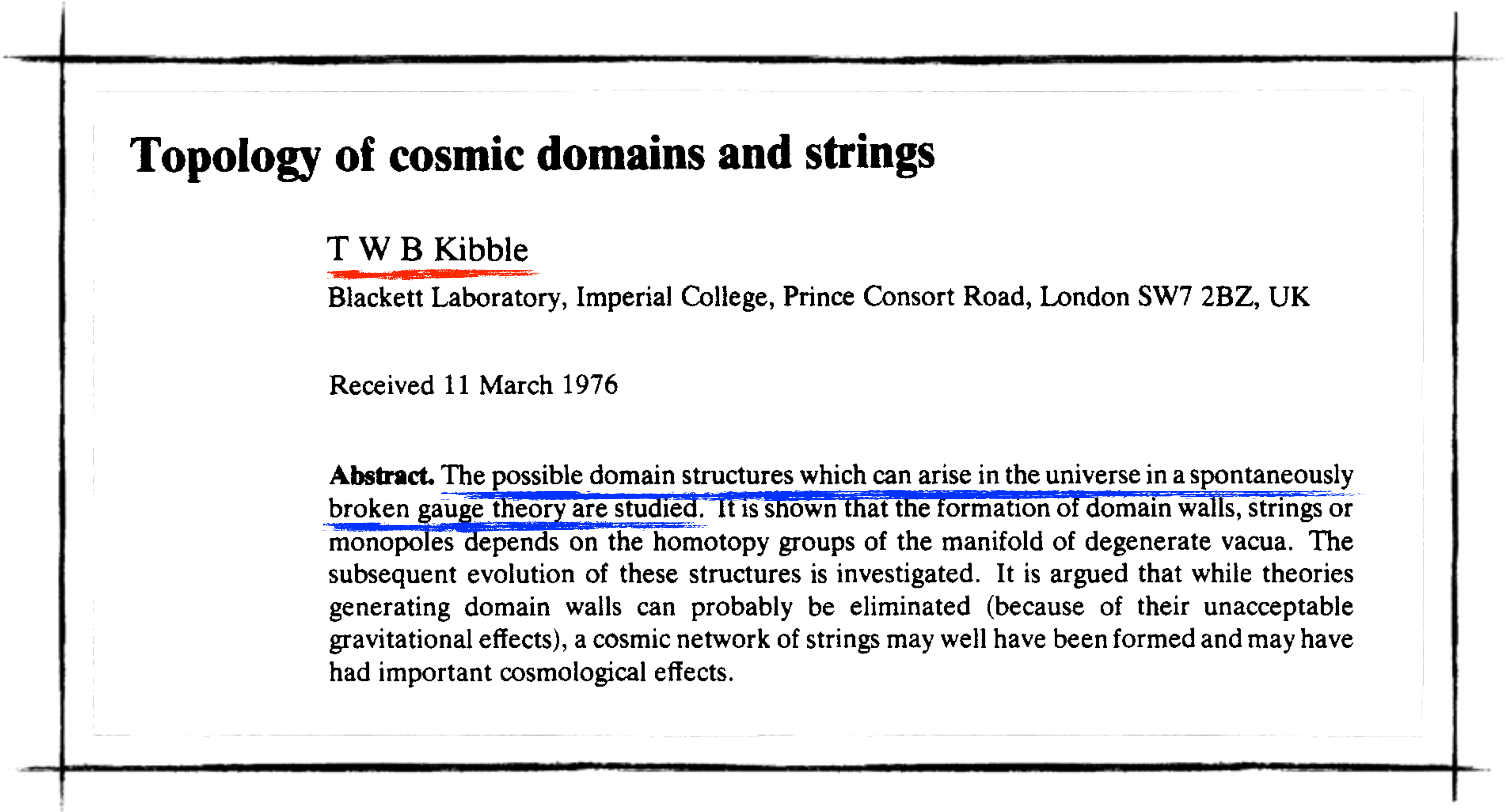
Cosmic Defects

Aftermath product of a Ph.T.

Introduction to Cosmic Defects



Topology of cosmic domains and strings

T W B Kibble
Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, UK

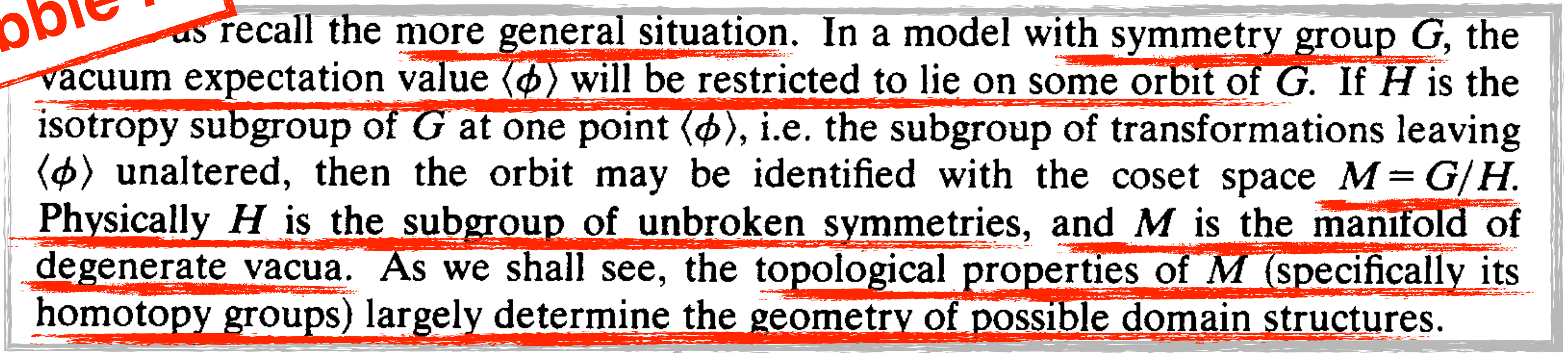
Received 11 March 1976

Abstract. The possible domain structures which can arise in the universe in a spontaneously broken gauge theory are studied. It is shown that the formation of domain walls, strings or monopoles depends on the homotopy groups of the manifold of degenerate vacua. The subsequent evolution of these structures is investigated. It is argued that while theories generating domain walls can probably be eliminated (because of their unacceptable gravitational effects), a cosmic network of strings may well have been formed and may have had important cosmological effects.

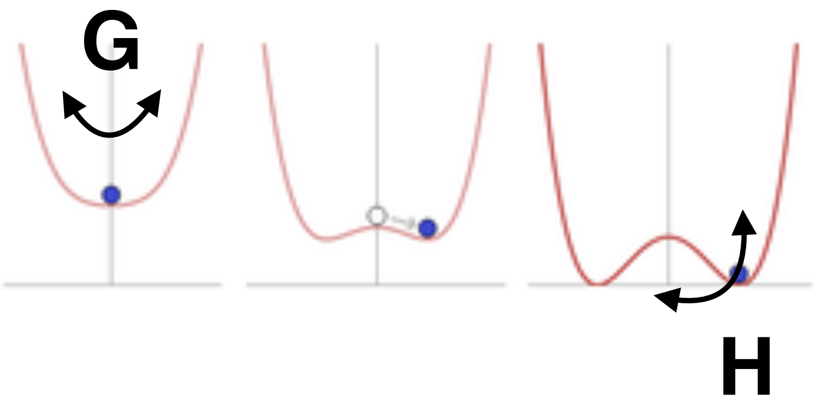
Kibble pioneered the study of topological defect generation in the early universe.

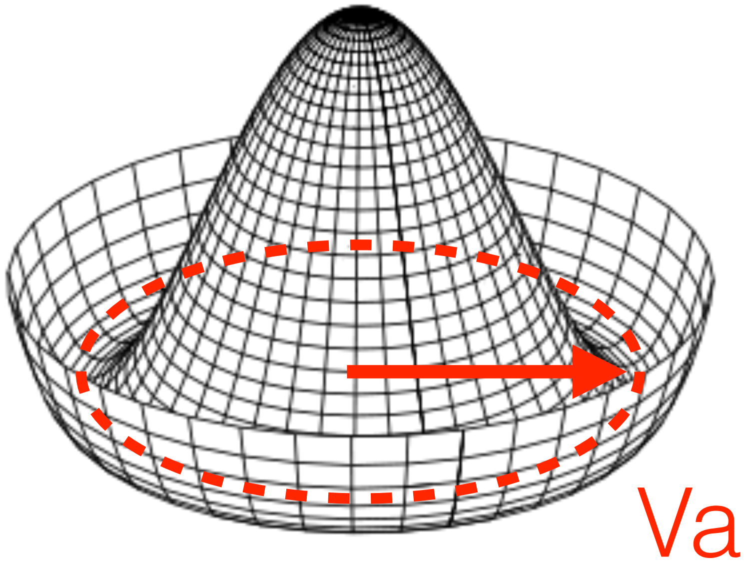
Introduction to Cosmic Defects

Kibble'76



us recall the more general situation. In a model with symmetry group G, the vacuum expectation value $\langle\phi\rangle$ will be restricted to lie on some orbit of G. If H is the isotropy subgroup of G at one point $\langle\phi\rangle$, i.e. the subgroup of transformations leaving $\langle\phi\rangle$ unaltered, then the orbit may be identified with the coset space $M = G/H$. Physically H is the subgroup of unbroken symmetries, and M is the manifold of degenerate vacua. As we shall see, the topological properties of M (specifically its homotopy groups) largely determine the geometry of possible domain structures.

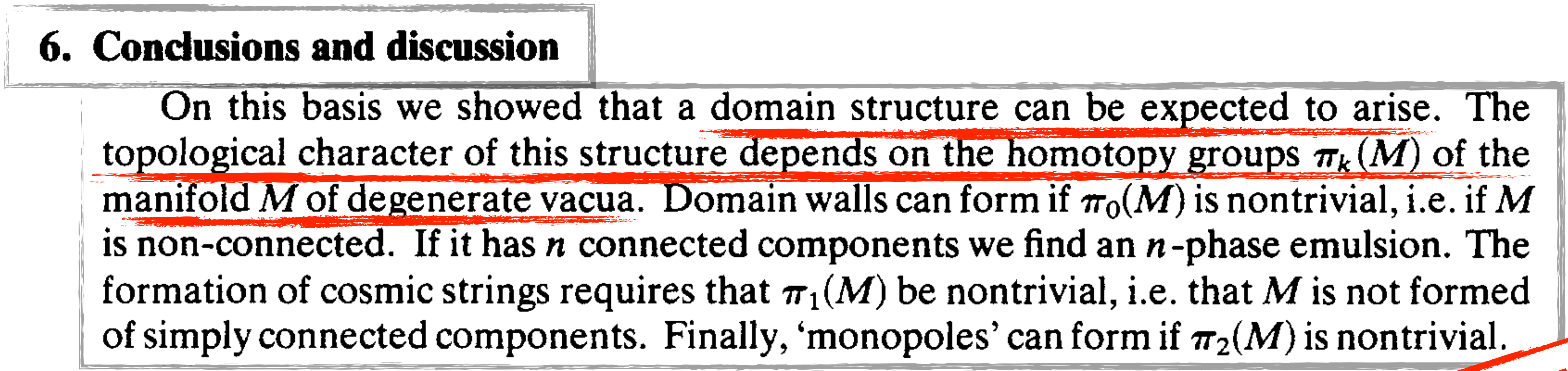




M = G/H

Vacuum Manifold

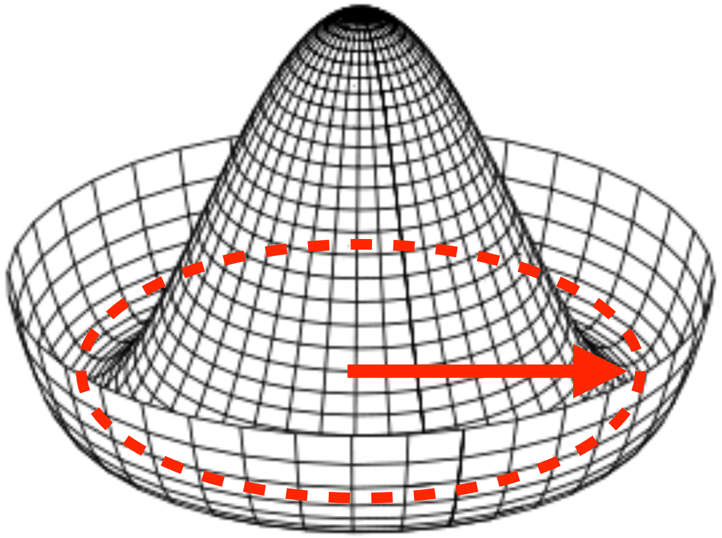
Introduction to Cosmic Defects



6. Conclusions and discussion

On this basis we showed that a domain structure can be expected to arise. The topological character of this structure depends on the homotopy groups $\pi_k(M)$ of the manifold M of degenerate vacua. Domain walls can form if $\pi_0(M)$ is nontrivial, i.e. if M is non-connected. If it has n connected components we find an n-phase emulsion. The formation of cosmic strings requires that $\pi_1(M)$ be nontrivial, i.e. that M is not formed of simply connected components. Finally, 'monopoles' can form if $\pi_2(M)$ is nontrivial.

Kibble'76



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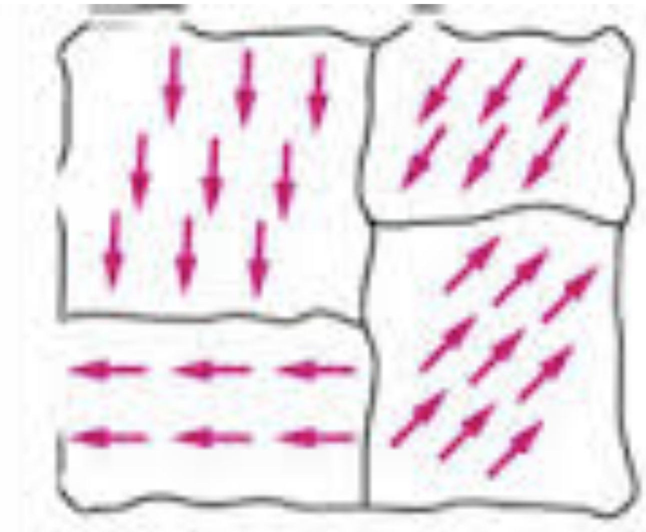
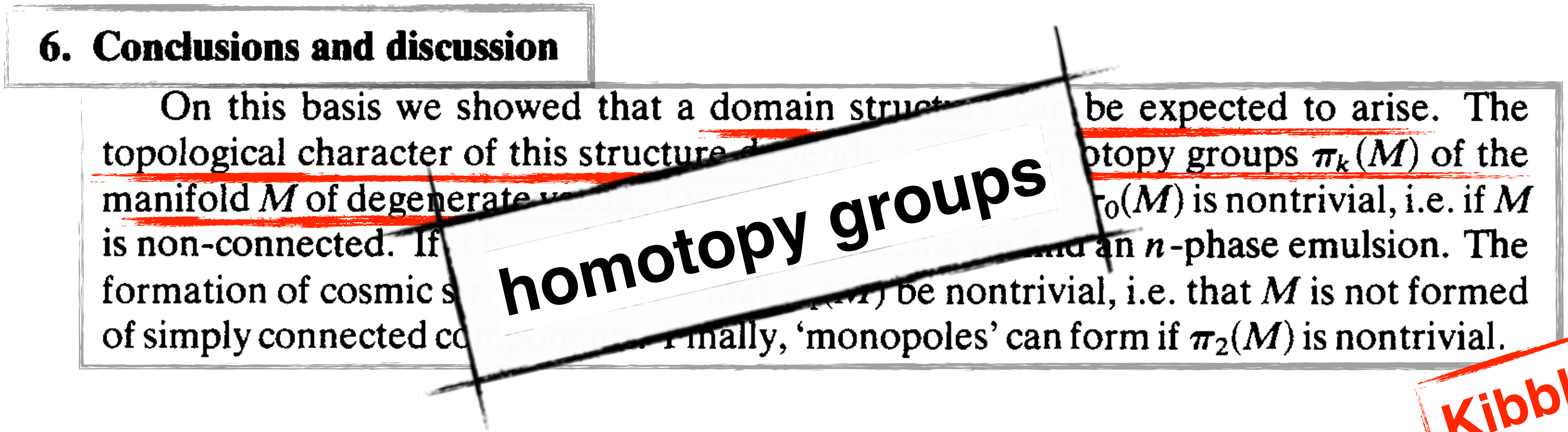


Introduction to Cosmic Defects



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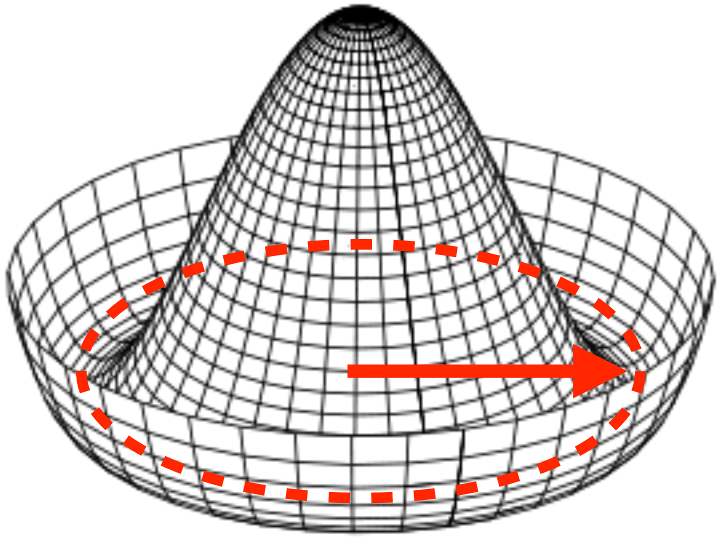
homotopy groups

Kibble'76



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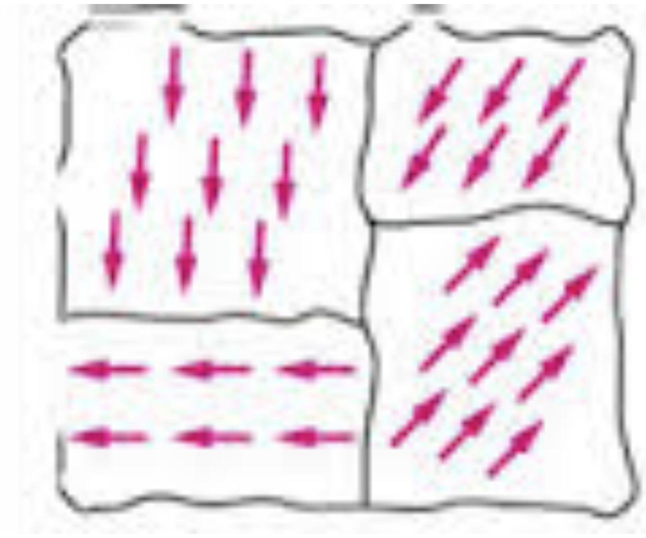
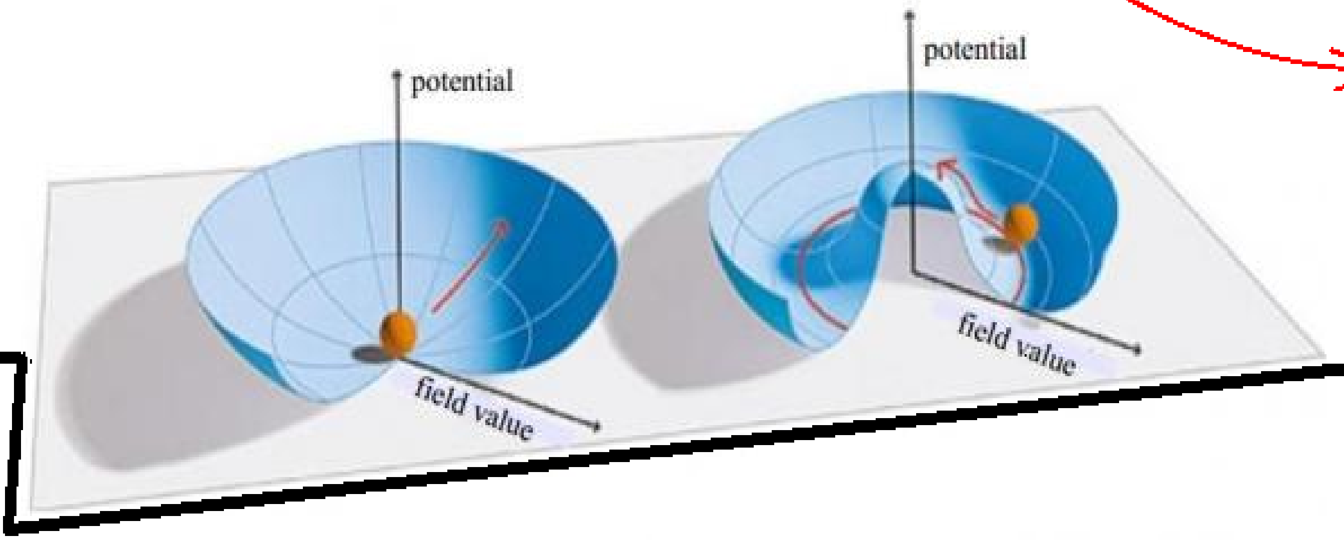


Introduction to Cosmic Defects

(1st Order, 2nd Order, Cross-Over)

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2|\Phi|^2T^2 & \text{(THERMAL)} \\ g^2|\Phi|^2\chi^2 & \text{(FIELD INT.)} \end{cases}$$



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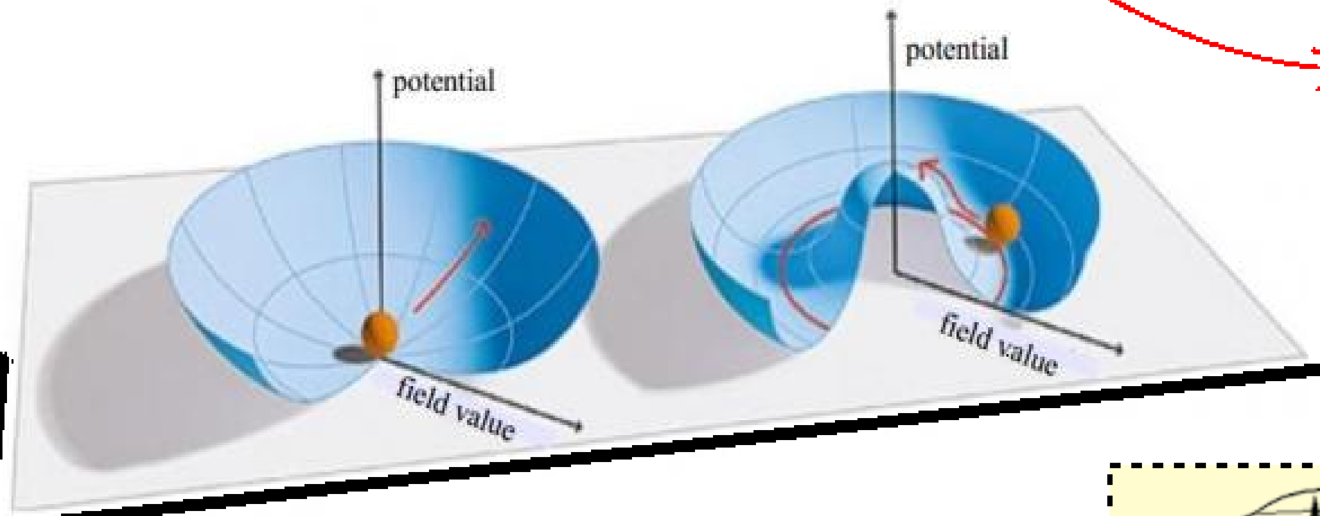

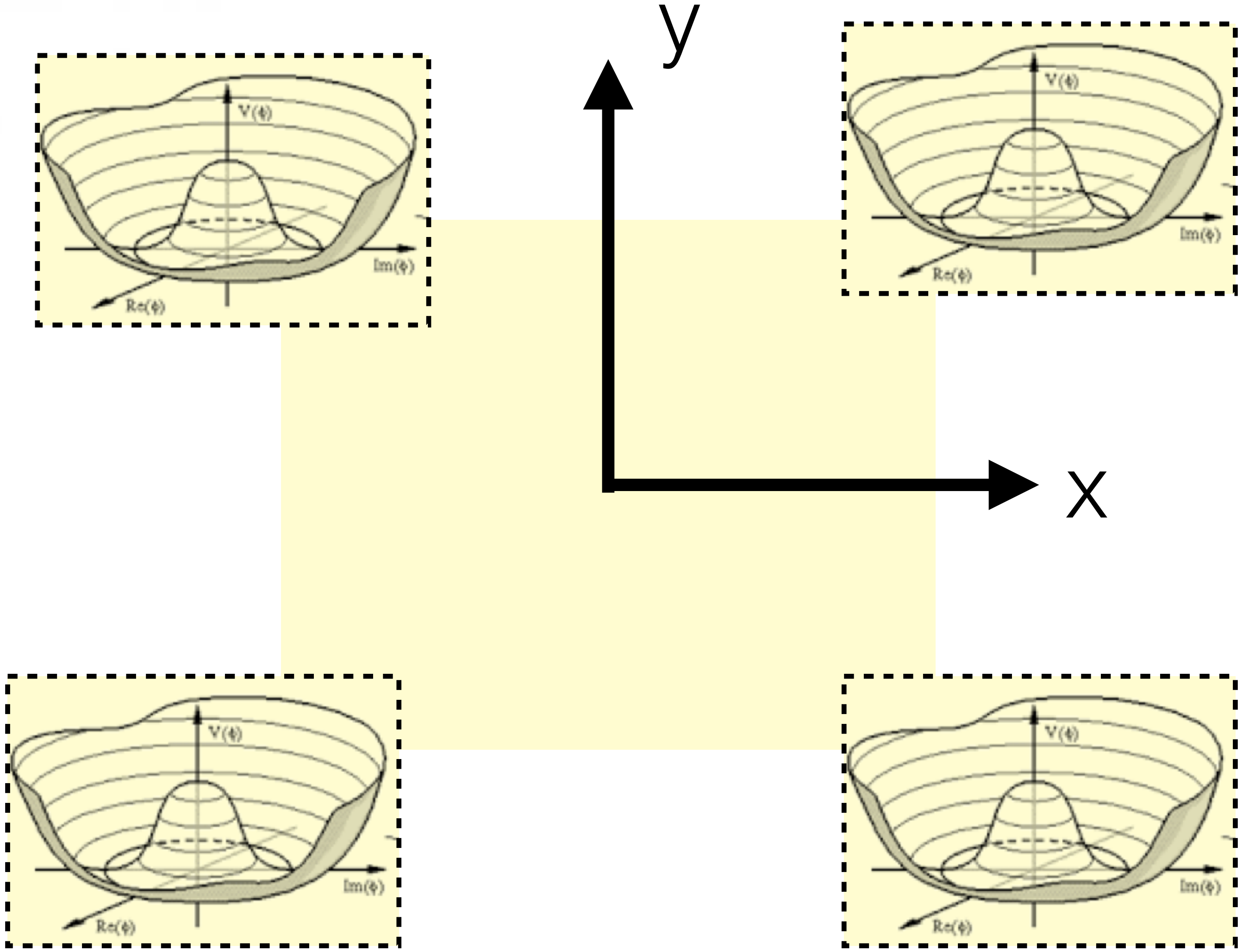

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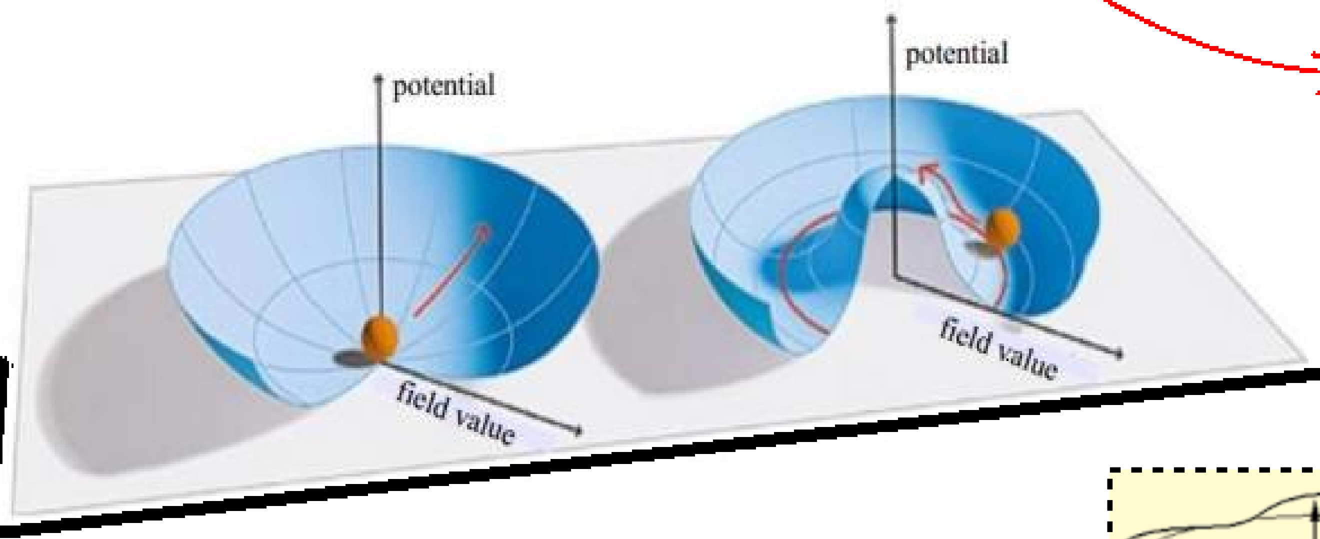

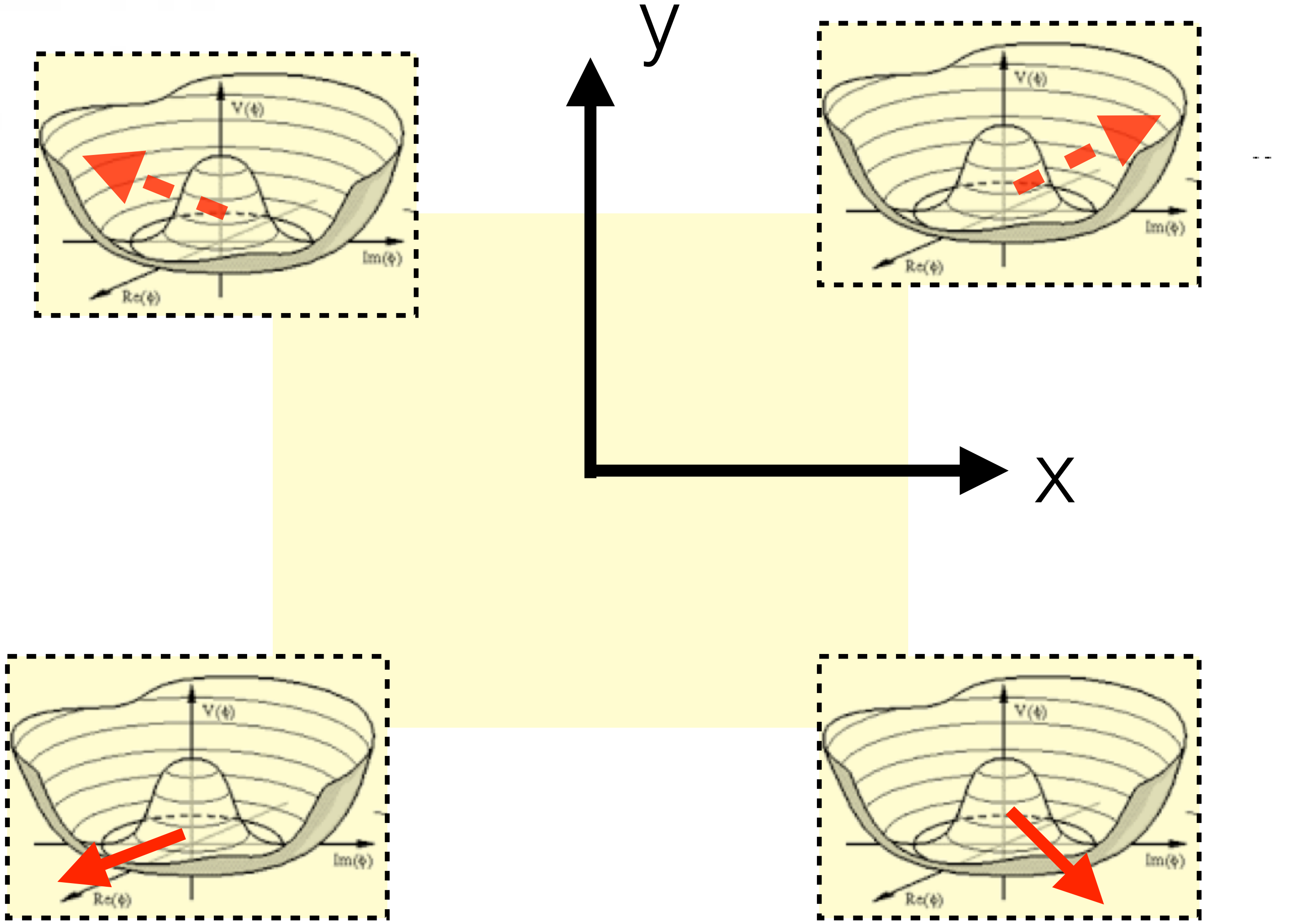

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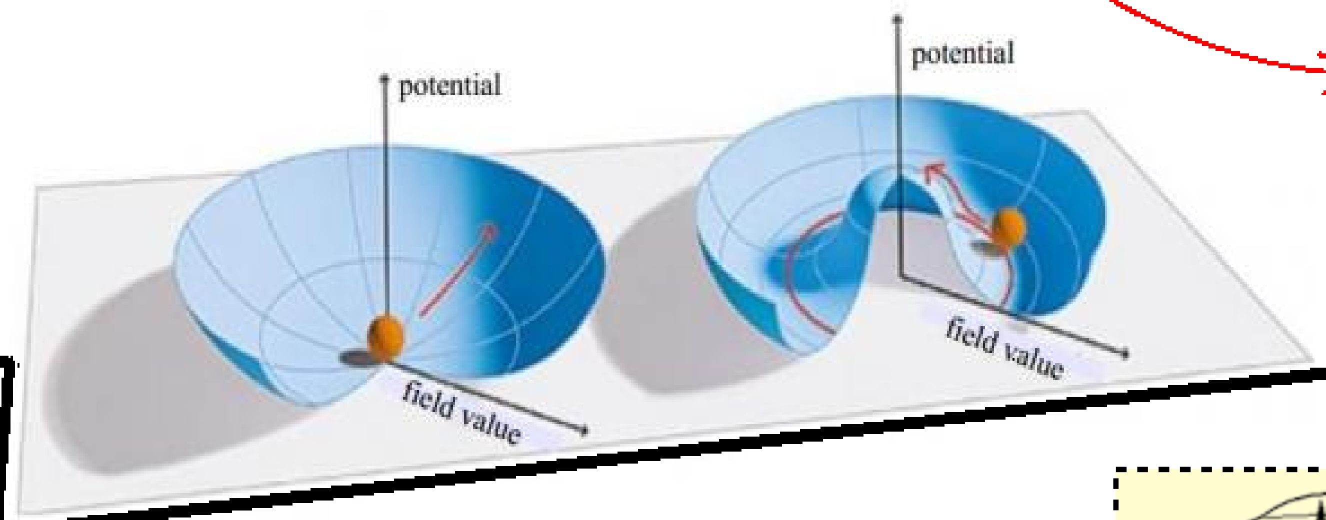

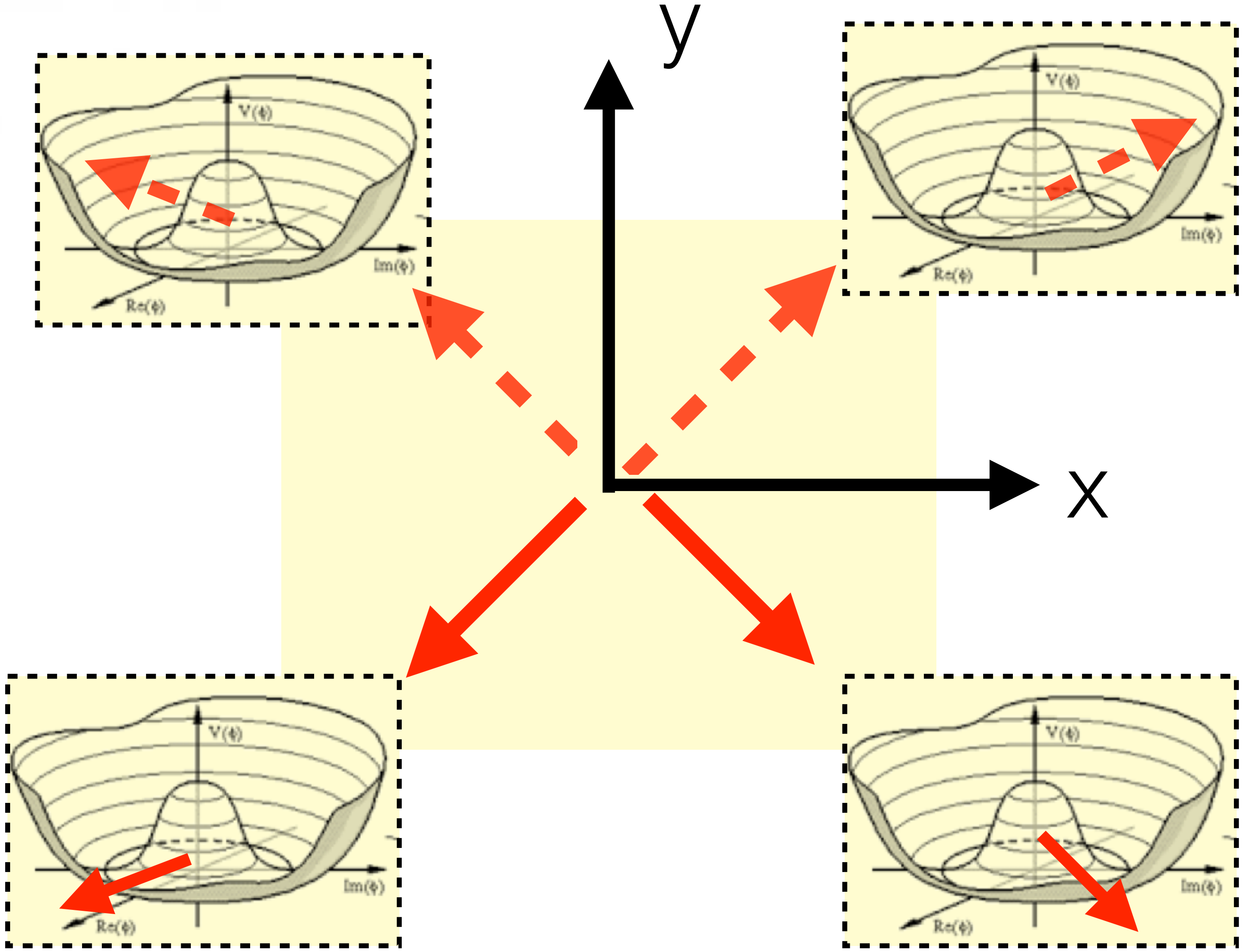

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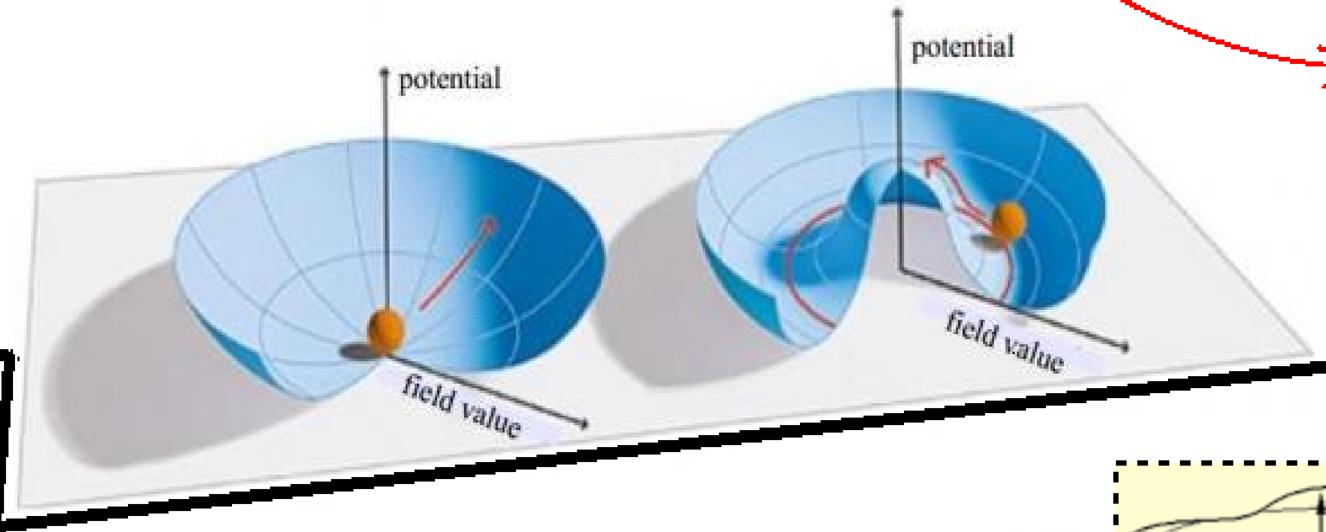

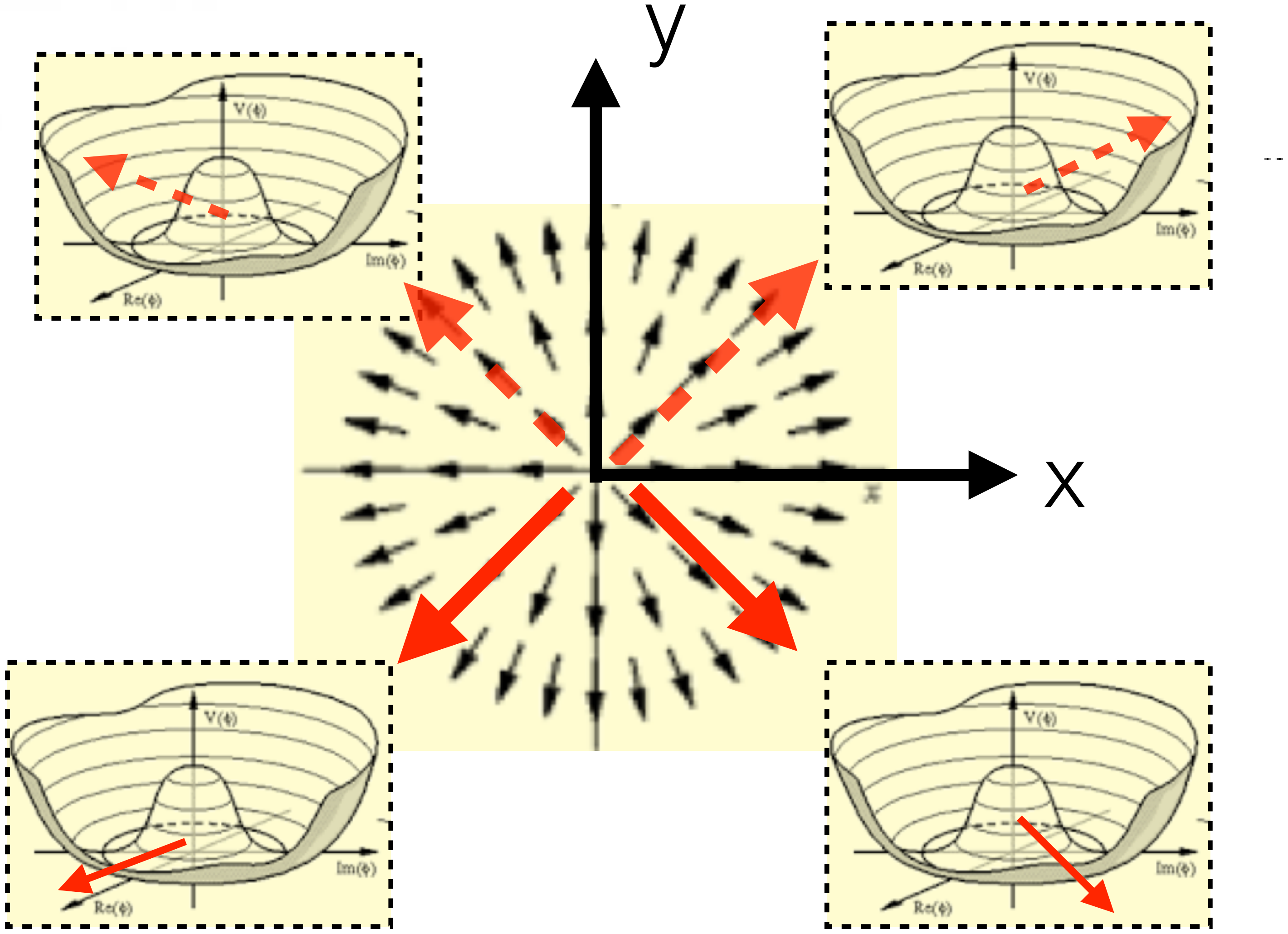

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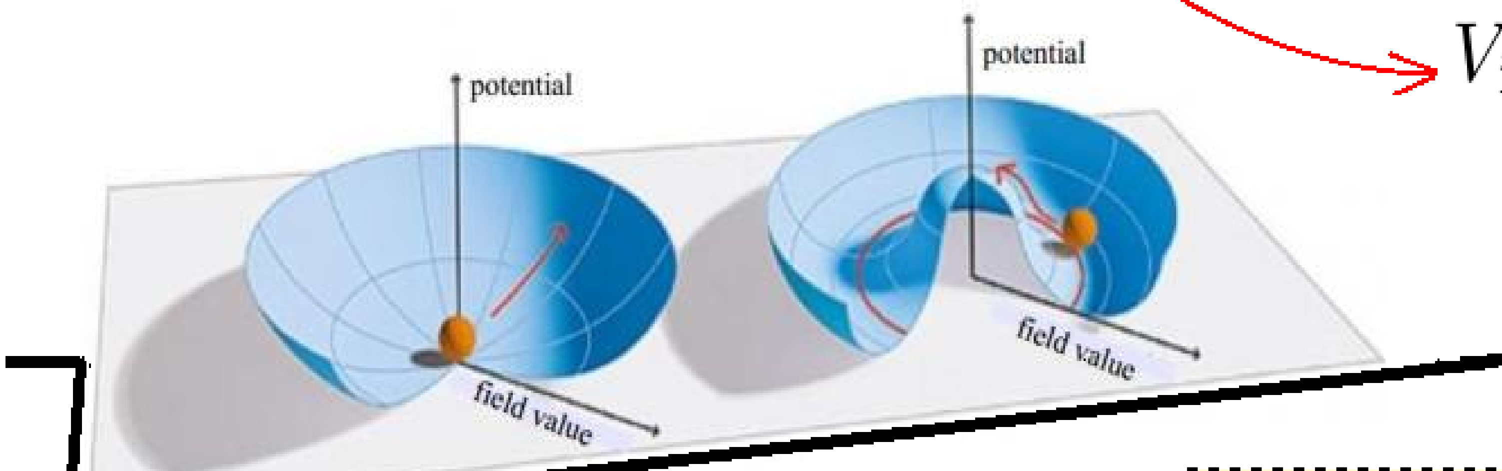

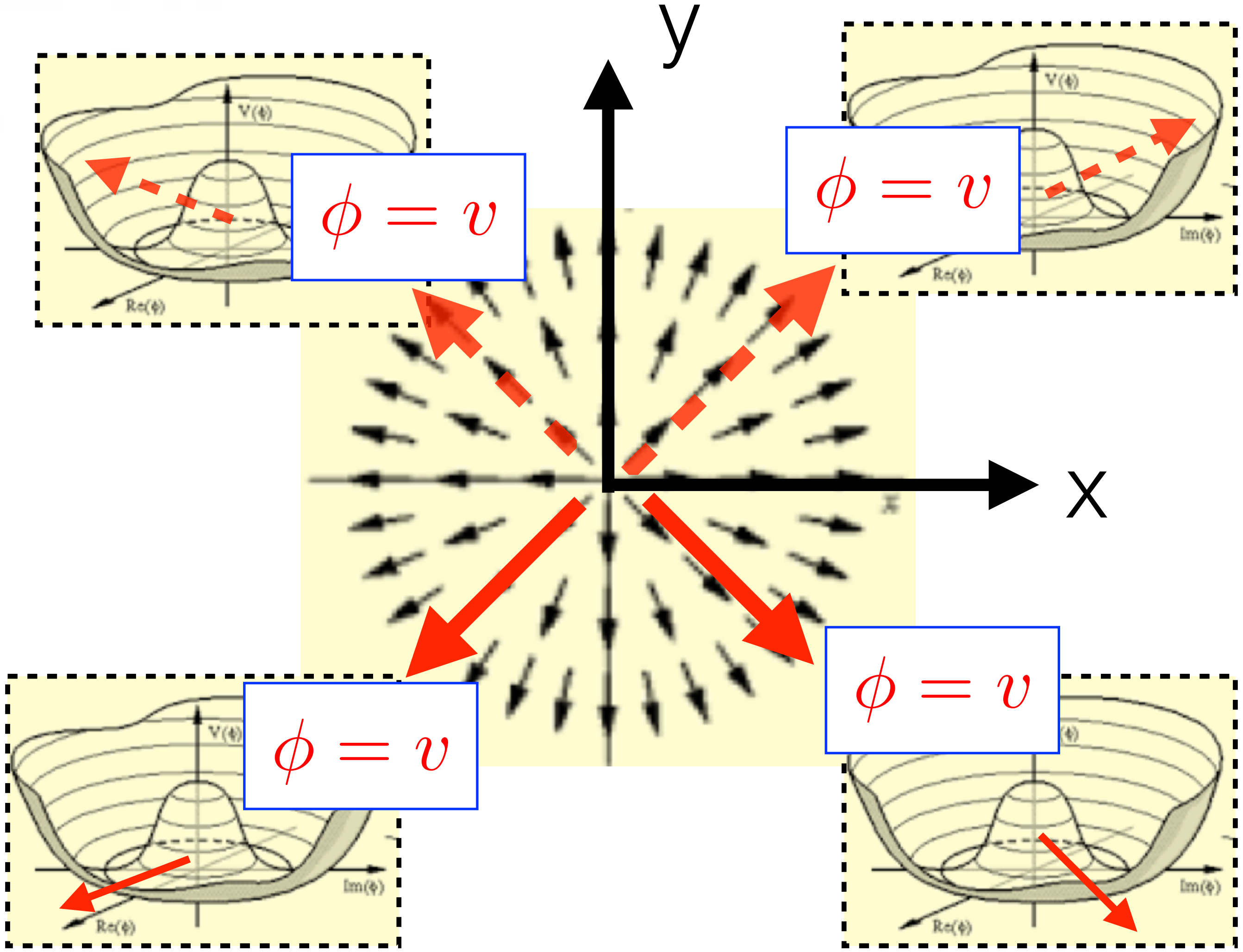

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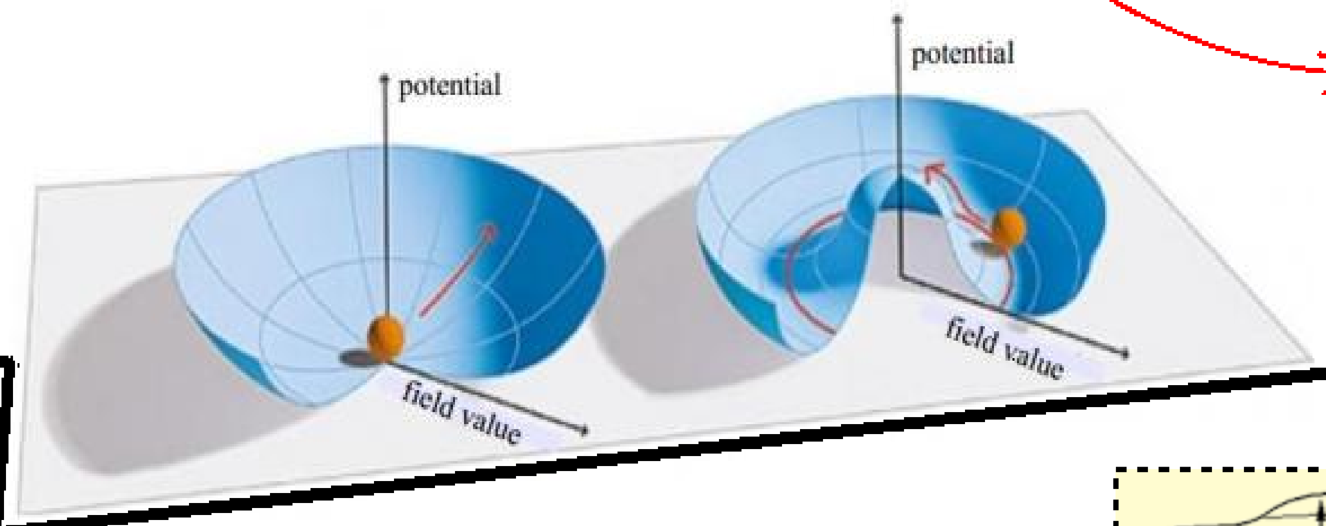

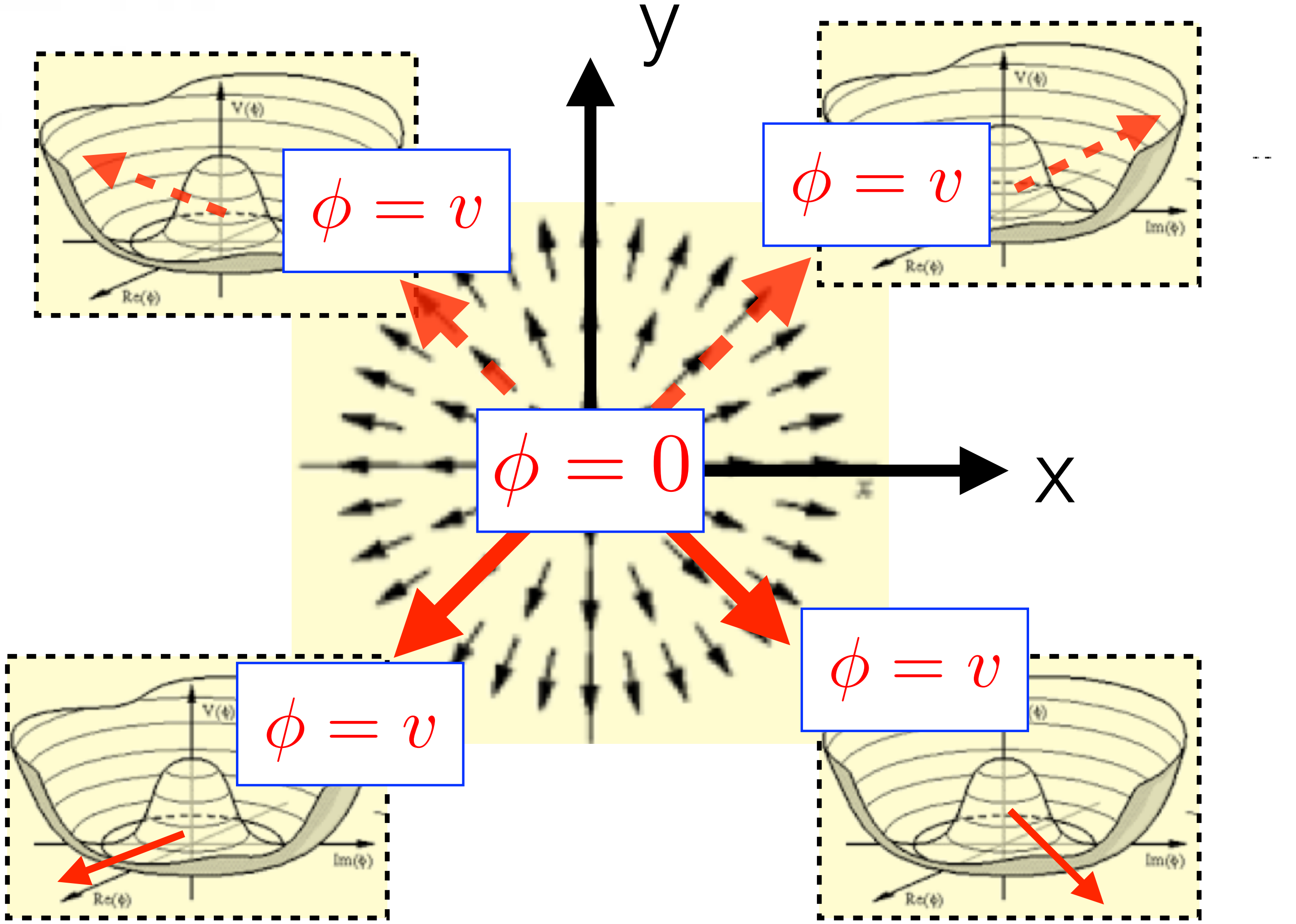

$\phi = v$

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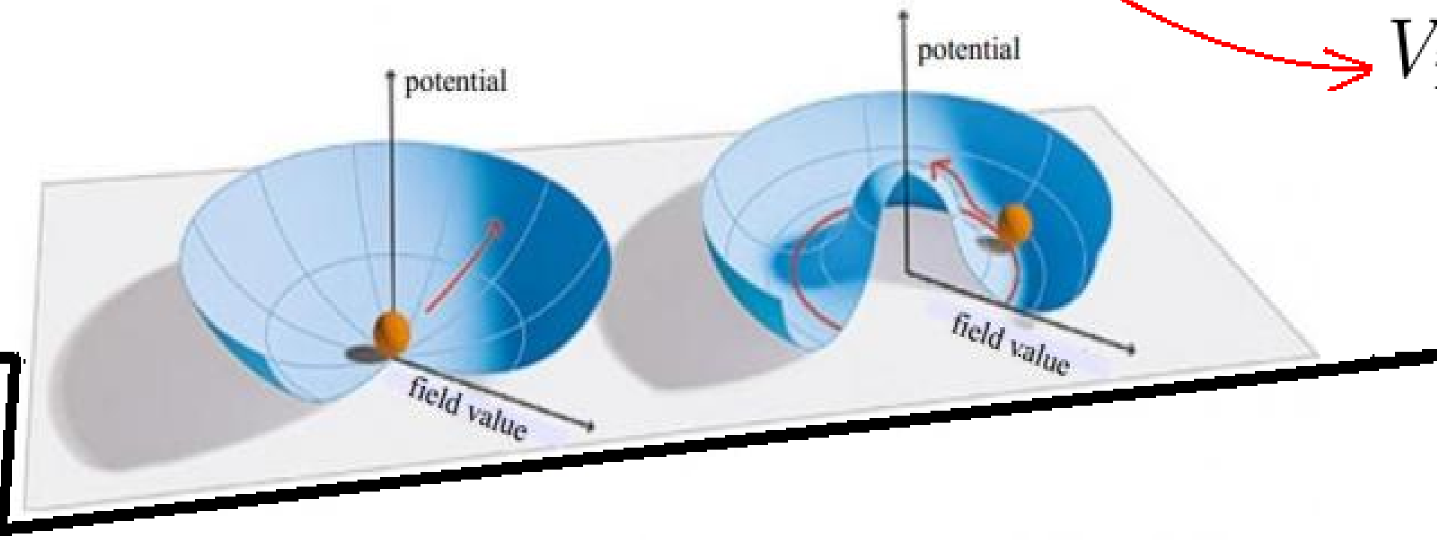

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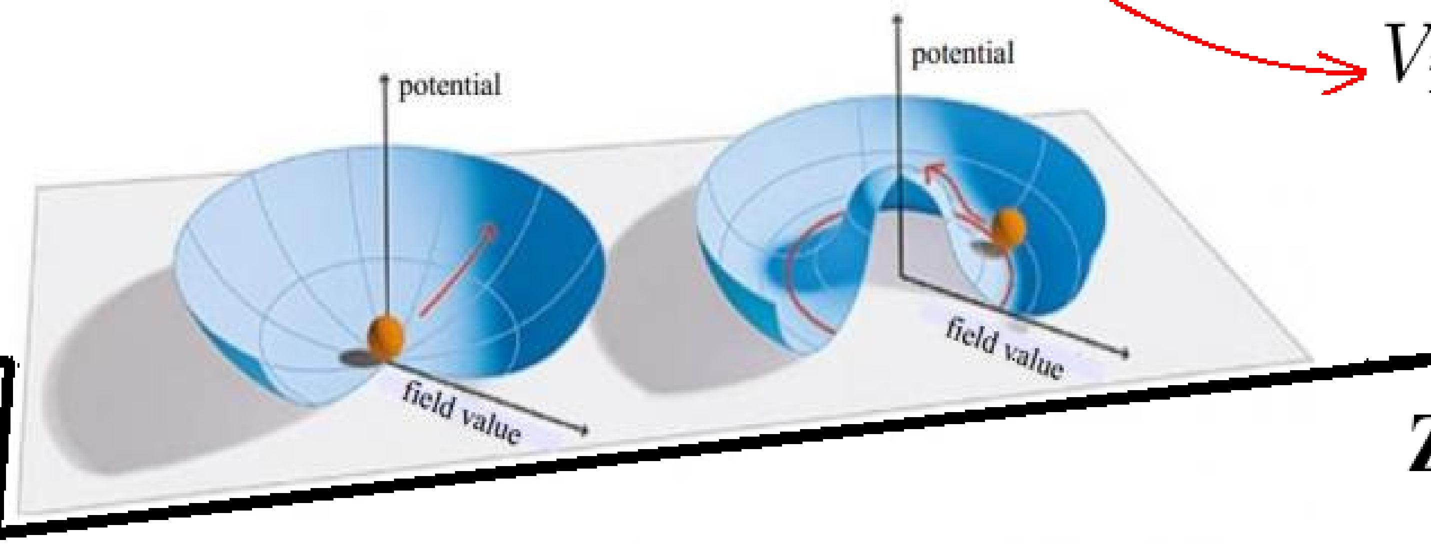
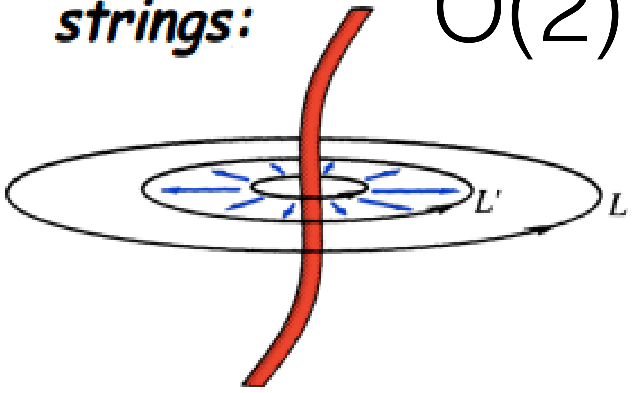
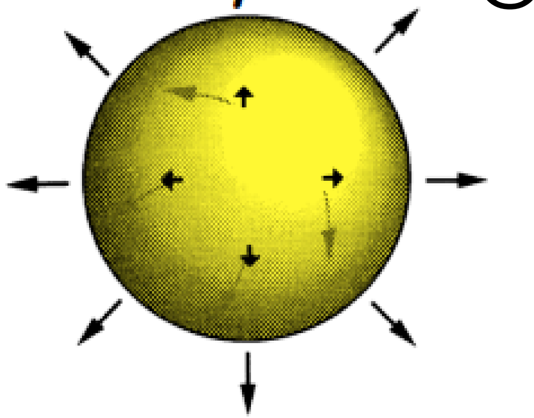


ZOOLOGY:

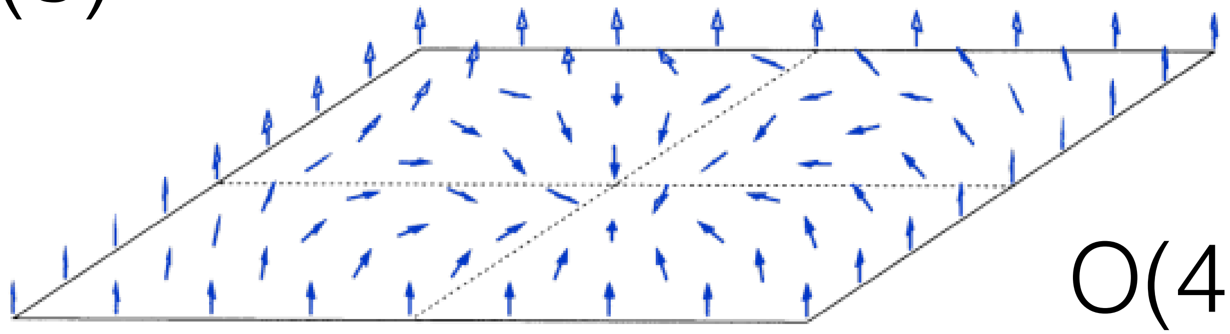
strings: O(2)



monopoles: O(3)



texture: (non-topological) O(4)



MICRO-PHYSICS → COSMIC DEFECTS

(M = G/H)

Introduction to Cosmic Defects

$$\text{DEFECTS: Aftermath of PhT} \rightarrow \begin{cases} \begin{cases} \mathrm{Domain\ \ Walls} \\ \mathrm{Cosmic\ Strings} \\ \mathrm{Cosmic\ Monopoles} \end{cases} \\ \mathrm{Non-Topological} \end{cases}$$

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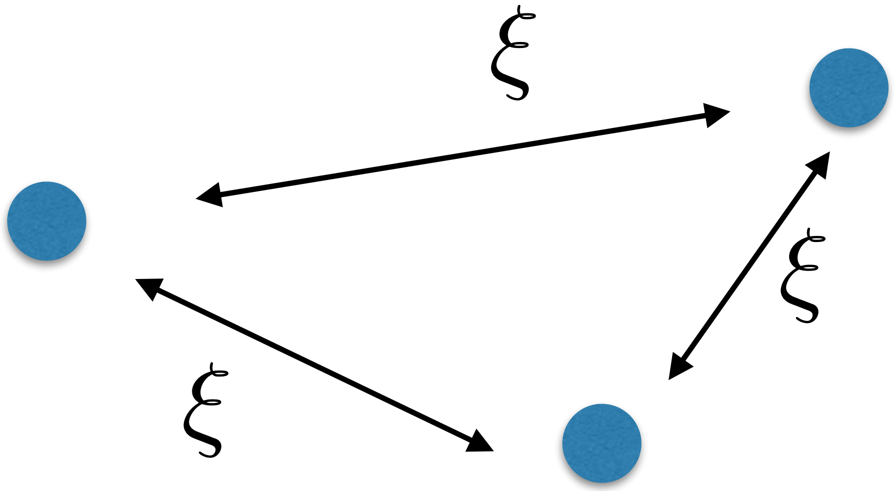
CAUSALITY & MICROPHYSICS ⇒ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$



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CAUSALITY & MICROPHYSICS $\Rightarrow$ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$

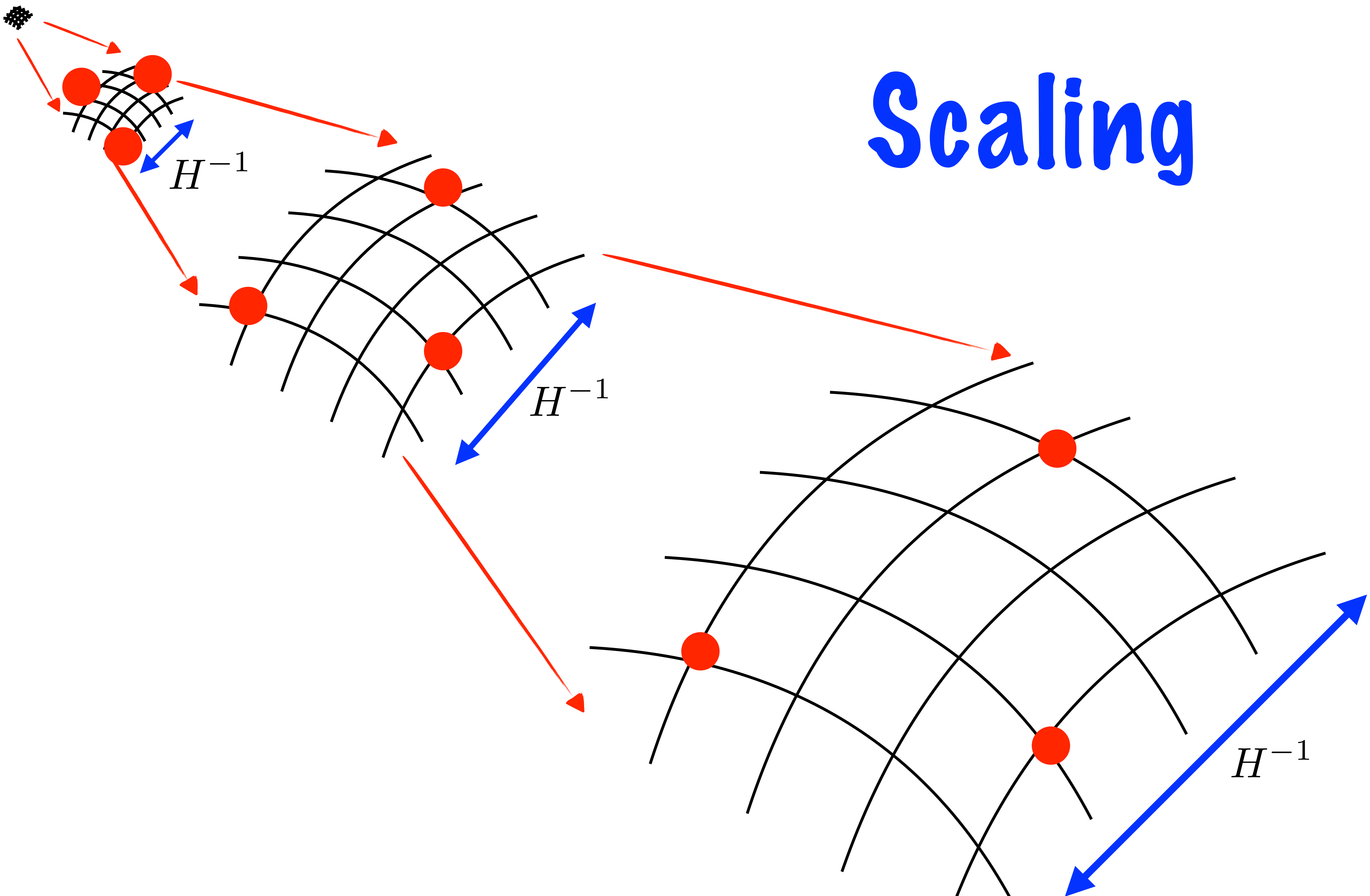
(Kibble' 76)

SCALING: $\lambda(t) = \text{const.} \quad \rightarrow \quad \lambda \sim 1 \quad \Rightarrow \quad k/\mathcal{H} = kt$

comoving momentum

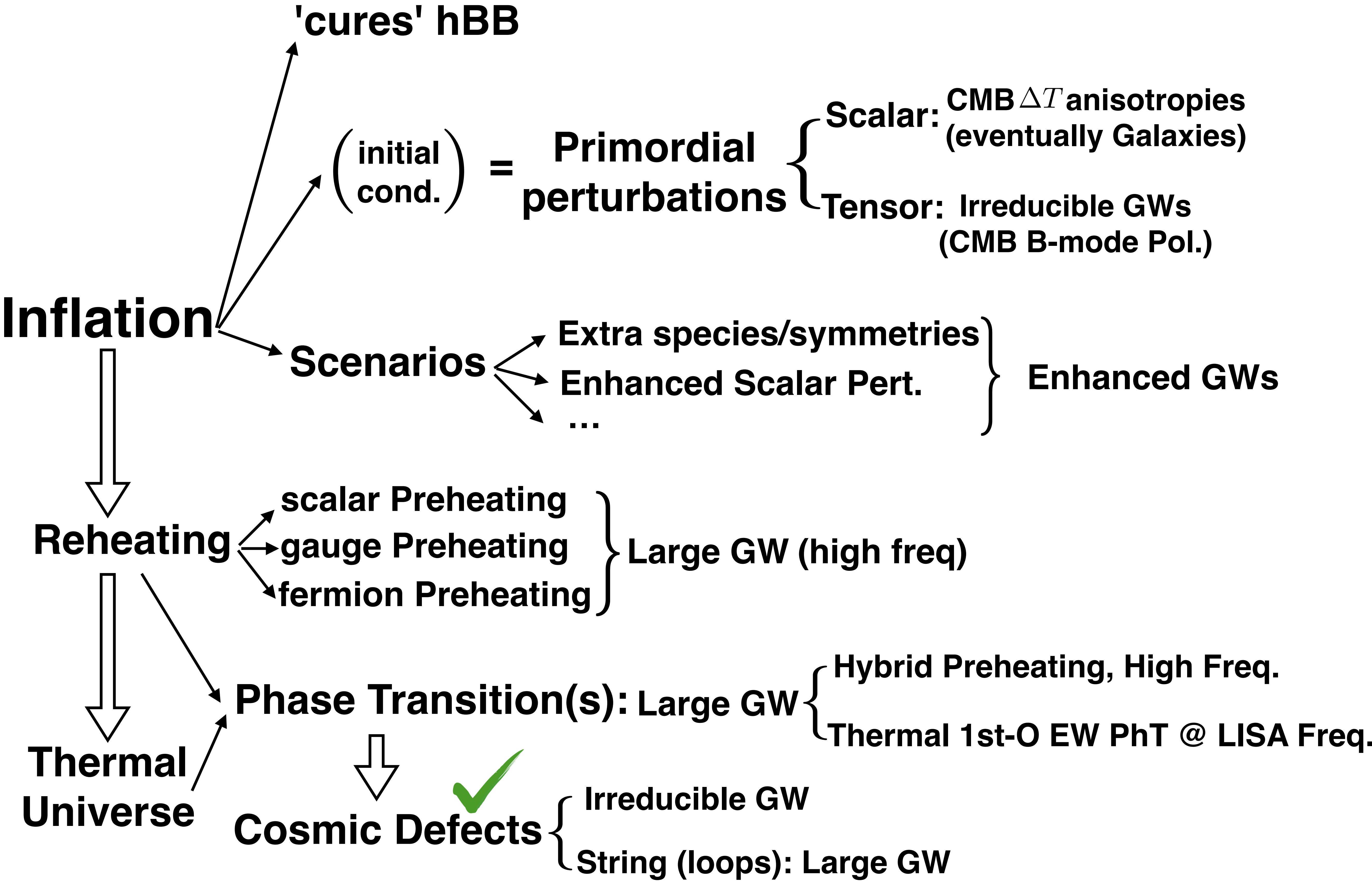
conformal time

Introduction to Cosmic Defects



Scaling

EARLY UNIVERSE



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Inflation

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Enhanced GWs

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- gauge Preheating
- fermion Preheating

Large GW (high freq)

Phase Transition(s): Large GW

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Thermal Universe

Cosmic Defects ✓

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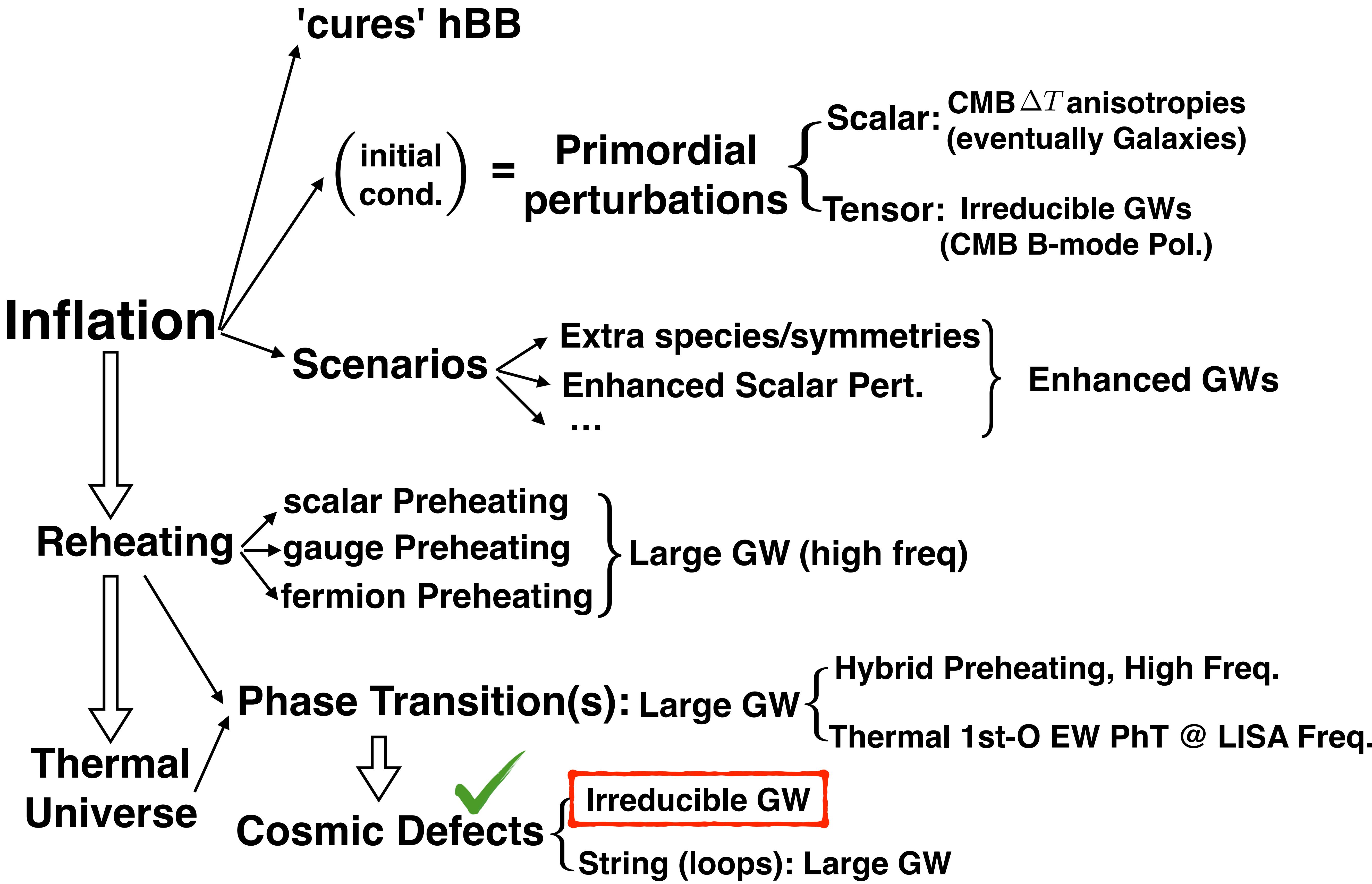
- String (loops): Large GW

EARLY UNIVERSE



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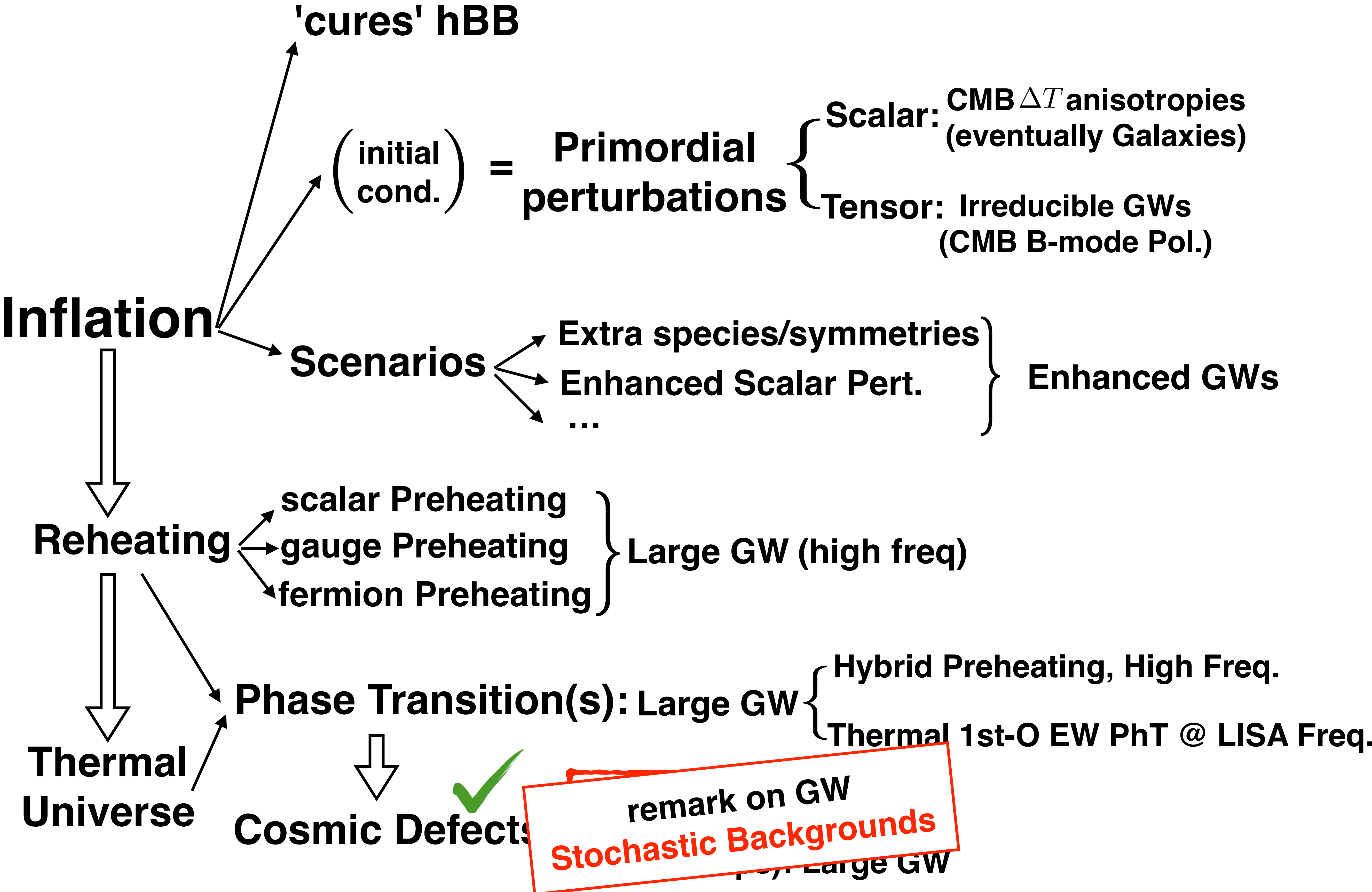

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Cosmic Defects ✓ ps): Large GW

remark on GW
Stochastic Backgrounds

Stochastic GW backgrounds

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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Define: $\bar h_{ij}\,(\mathbf{x},t) \;=\; a(t)h_{ij}\,(\mathbf{x},t)$

EOM: $\ddot{\bar h}_{ij}\,(\mathbf{x},t) - \left(\nabla^2 + \frac{\ddot a(t)}{a(t)}\right)\bar h_{ij}\,(\mathbf{x},t) = 16\pi G\, a(t)\Pi^{\rm TT}_{ij}\,(\mathbf{x},t)$

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$$\begin{aligned}\langle \mathcal{G}(\mathbf{k},t,t')\mathcal{G}(\mathbf{k},t,t'')\rangle_{T_k} &\equiv \frac{1}{T_k}\int_t^{t+T_k} d\tilde t\; \mathcal{G}(\mathbf{k},\tilde t,t')\mathcal{G}(\mathbf{k},\tilde t,t'') \\ &\underset{kt\,\gg\,1}{=} \frac{1}{2}(k^2+\mathcal{H}^2(t))\cos[k(t'-t'')]\end{aligned}$$

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$@k \gg \mathcal{H}$

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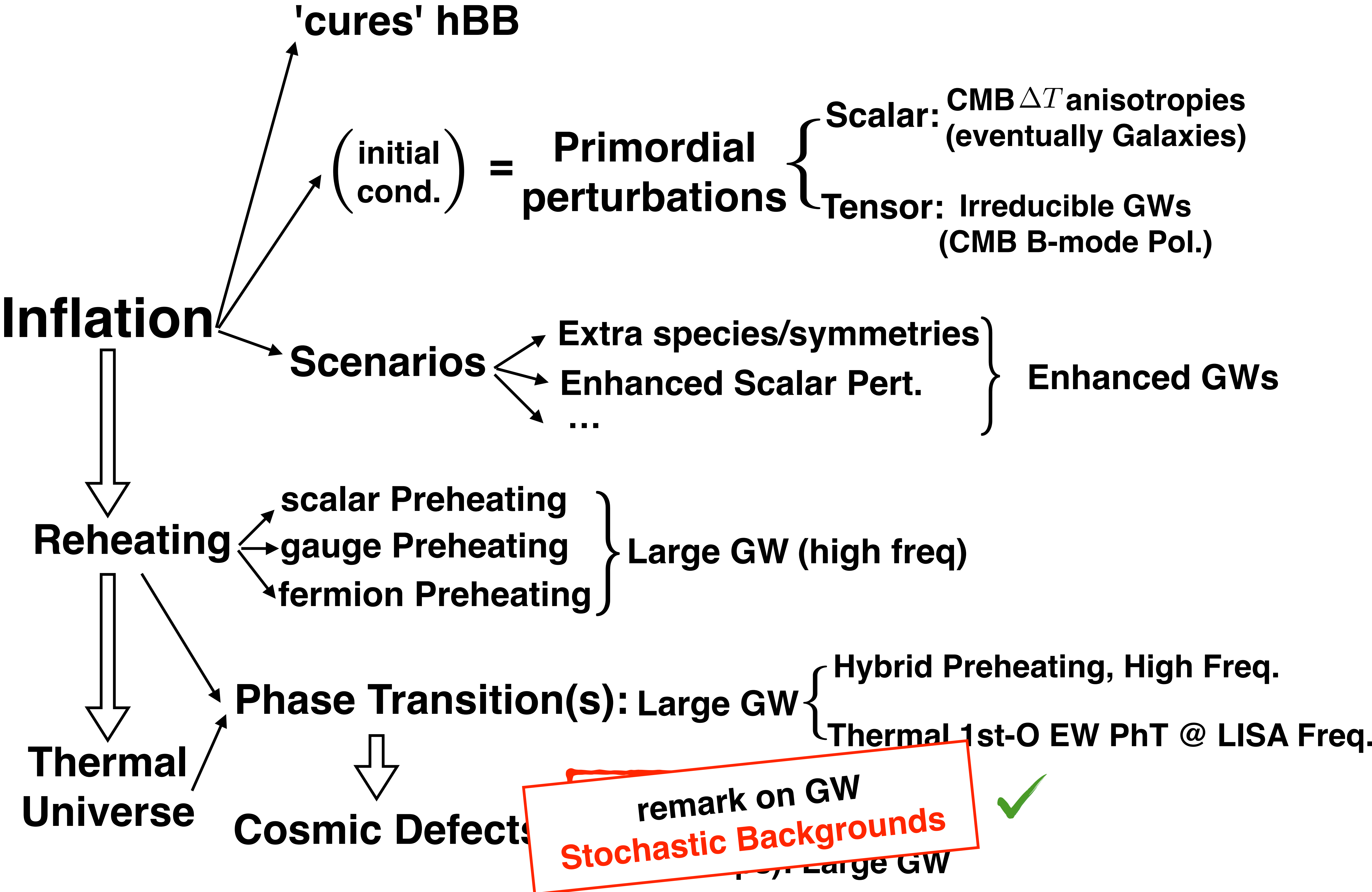


Inflation

- 'cures' hBB
- (initial cond.) = Primordial perturbations
  - Scalar: CMB ΔT anisotropies (eventually Galaxies)
  - Tensor: Irreducible GWs (CMB B-mode Pol.)
- Scenarios
  - Extra species/symmetries
  - Enhanced Scalar Pert.
  - …
  - Enhanced GWs

Reheating

- scalar Preheating
- gauge Preheating
- fermion Preheating
- Large GW (high freq)

Thermal Universe

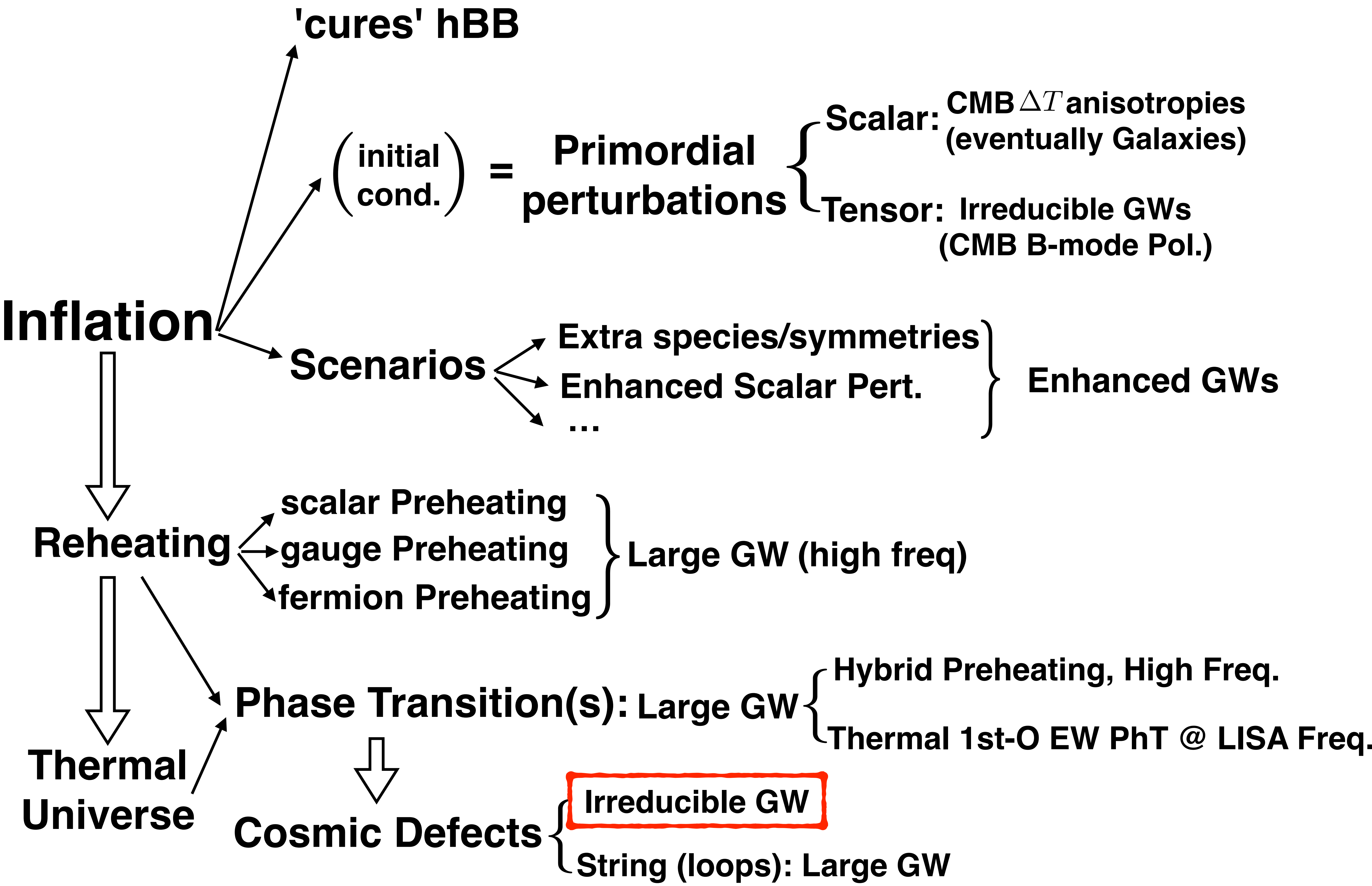
Phase Transition(s): Large GW

- Hybrid Preheating, High Freq.
- Thermal 1st-O EW PhT @ LISA Freq.

Cosmic Defects … Large GW

remark on GW
Stochastic Backgrounds ✓

EARLY UNIVERSE



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- String (loops): Large GW

GWs from a scaling network of cosmic defects

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

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(Unequal Time Correlator)

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GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2a^4(t)}\int dt_1dt_2\;a(t_1)a(t_2)\;\cos(k(t_1-t_2))\;\Pi^2(k,t_1,t_2)$$

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Comoving Scale — Conformal Time

GWs from a scaling network of cosmic defects

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SCALING

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SCALING

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$$\frac{d\rho_{\rm GW}}{d\log k}(\underset{\substack{\uparrow\\ \text{Comoving}\\ \text{Scale}}}{k},\underset{\substack{\uparrow\\ \text{Conformal}\\ \text{Time}}}{t}) \propto \frac{k^3}{M_p^2a^4(t)}\int dt_1dt_2 \quad \overset{\text{Expansion}}{\underset{\textbf{Rad. Dom}}{t_1t_2}} \quad \cos(k(t_1-t_2))\,\overset{\text{UTC}}{\underset{\textbf{SCALING}}{\frac{\rm V^4}{\sqrt{t_1t_2}}U(kt_1,kt_2)}}$$

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Rad. Dom **SCALING**

GWs from a scaling network of cosmic defects

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Rad. Dom **SCALING**

GWs from a scaling network of cosmic defects

VEV

Scaling @ RD

GW today:

$$\boxed{\Omega^{(o)}_{GW} \equiv \frac{1}{\rho_{\rm c}^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \frac{32}{3}\left(\frac{\rm V}{M_p}\right)^4 \Omega^{(o)}_{\rm rad}\; F_U\;, \quad (\text{SCALE INV.!})}$$

Defect type

$$F_U \equiv \int_0^x dx_1 dx_2 \sqrt{x_1 x_2}\, cos(x_1 - x_2) U(x_1, x_2)$$

DGF, Hindmarsh, Urrestilla, PRL 2013

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∀ PhT (1st, 2nd, ...), ∀ Defects (top. or non-top.)

DGF, Hindmarsh, Urrestilla, PRL 2013

GWs from a scaling network of cosmic defects

Total GW Spectrum

energy scale

constants

$$\boxed{h^2\Omega_{\rm GW}^{(\rm o)} = h^2\Omega_{\rm rad}^{(\rm o)}\left(\frac{V}{M_p}\right)^4\left[F_U^{(\rm R)} + F_U^{(\rm M)}\left(\frac{k_{\rm eq}}{k}\right)^2\right]}$$

RD

$$F_U^{(\rm R)} \equiv \frac{32}{3}\int_0^x dx_1 dx_2\,(x_1x_2)^{1/2}\cos(x_1-x_2)\,U_{\rm RD}(x_1,x_2)$$

MD

$$F_U^{(\rm M)} \equiv \frac{32}{3}\frac{(\sqrt{2}-1)^2}{2}\int_{x_{\rm eq}}^x dx_1 dx_2\,(x_1x_2)^{3/2}\cos(x_1-x_2)\,U_{\rm MD}(x_1,x_2)$$

DGF, Hindmarsh, Lizarraga, Urrestilla, PRD 2020

More on GW from Defect Networks

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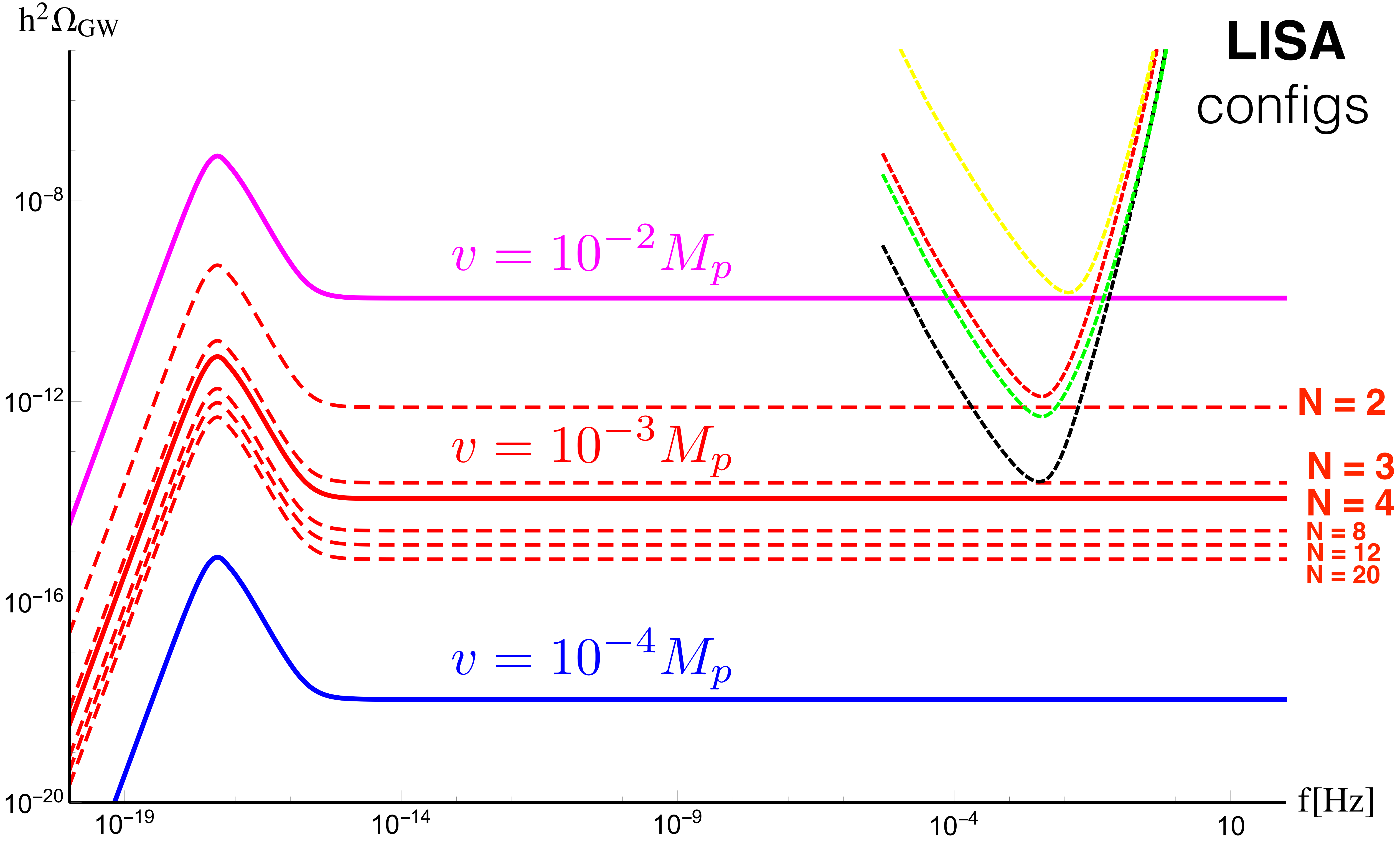


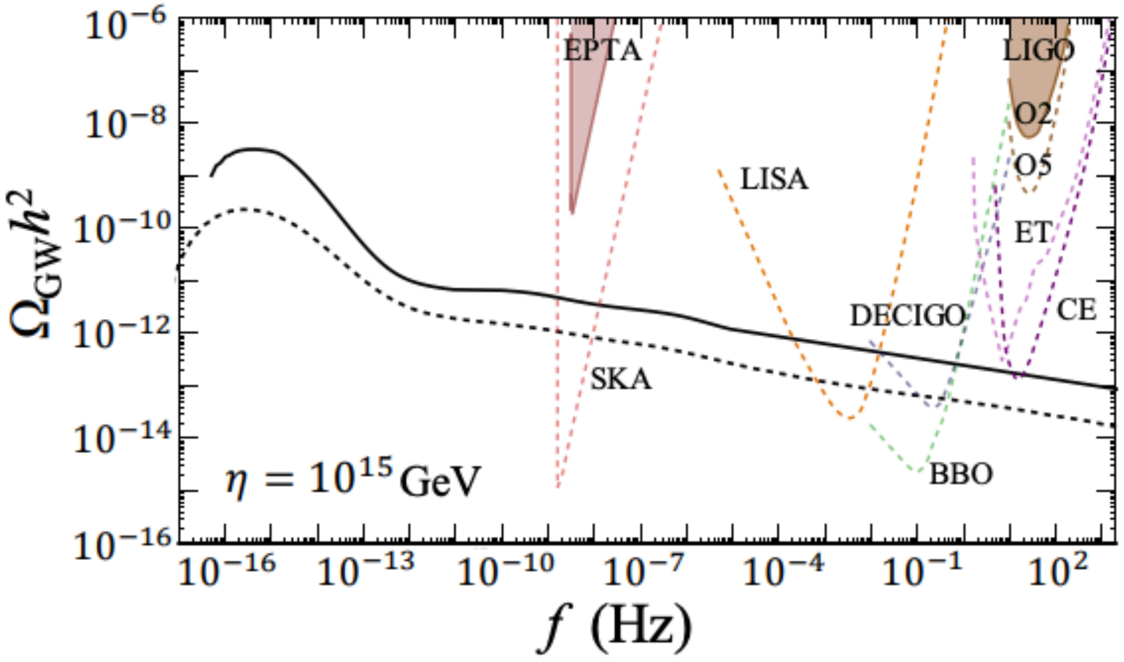
DGF, Hindmarsh, Lizarraga, Urrestilla, PRD 2020

Other works on GWs from Global String Networks

Don’t agree with scale invariance !

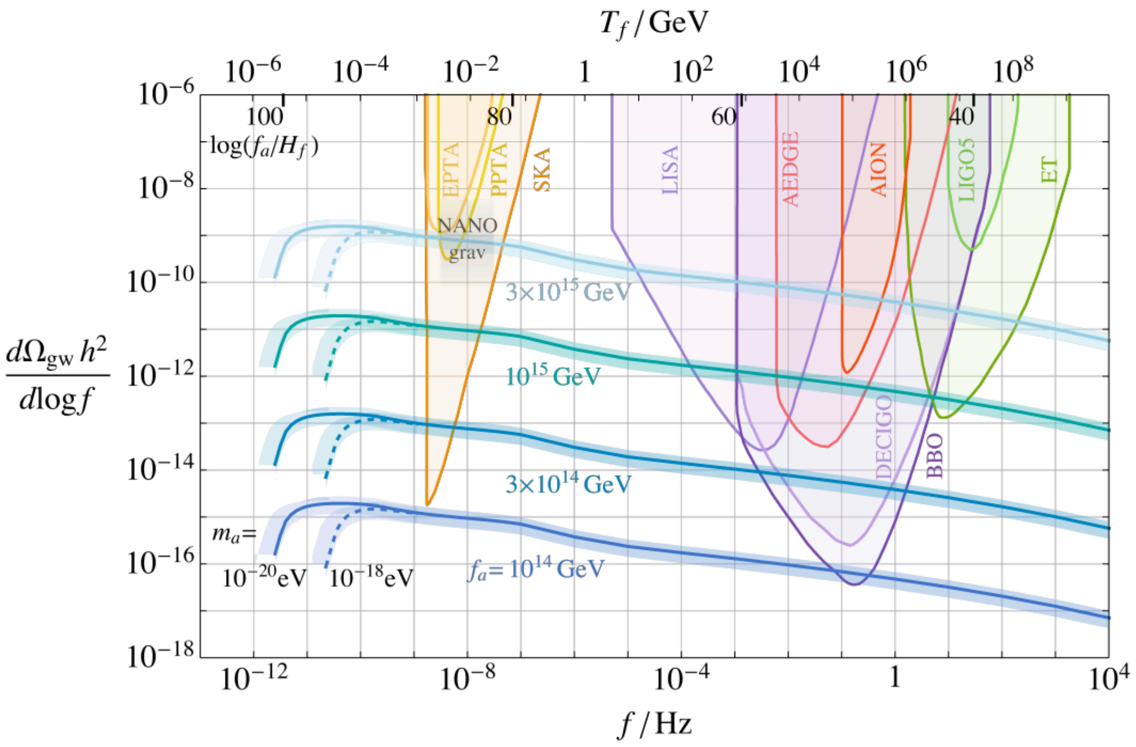
e.g. Chang & Cui

PDU 29 (2020) 100604 • e-Print: 1910.04781 [hep-ph]



JCAP 06 (2021) 034 • e-Print: 2101.11007 [hep-ph]

e.g. Gorghetto et al



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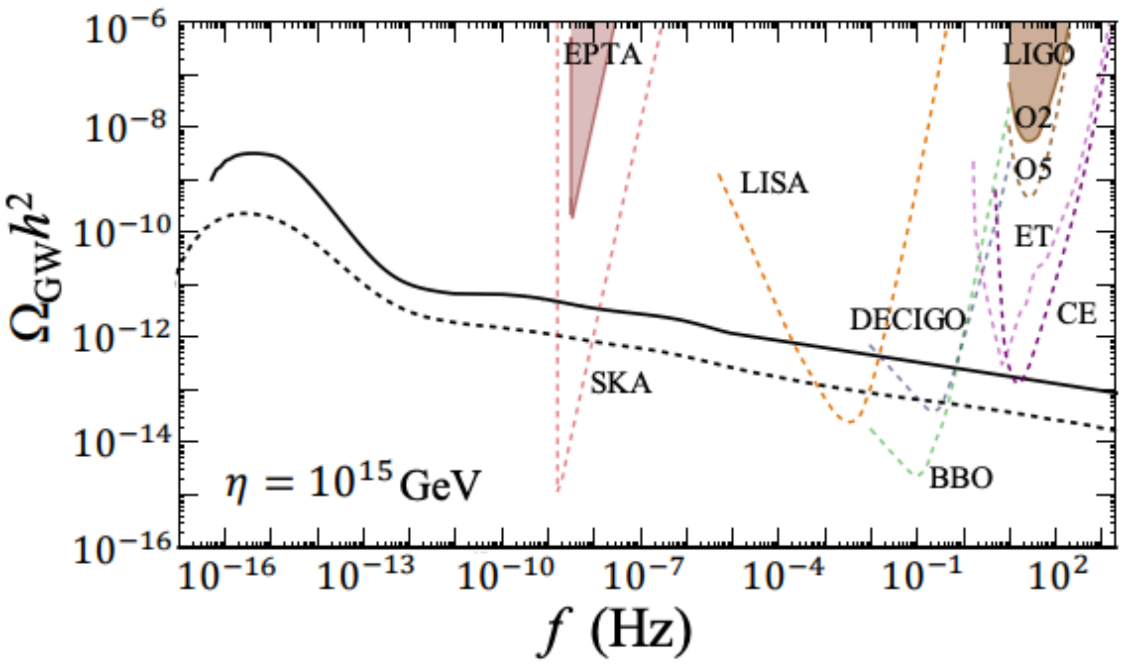

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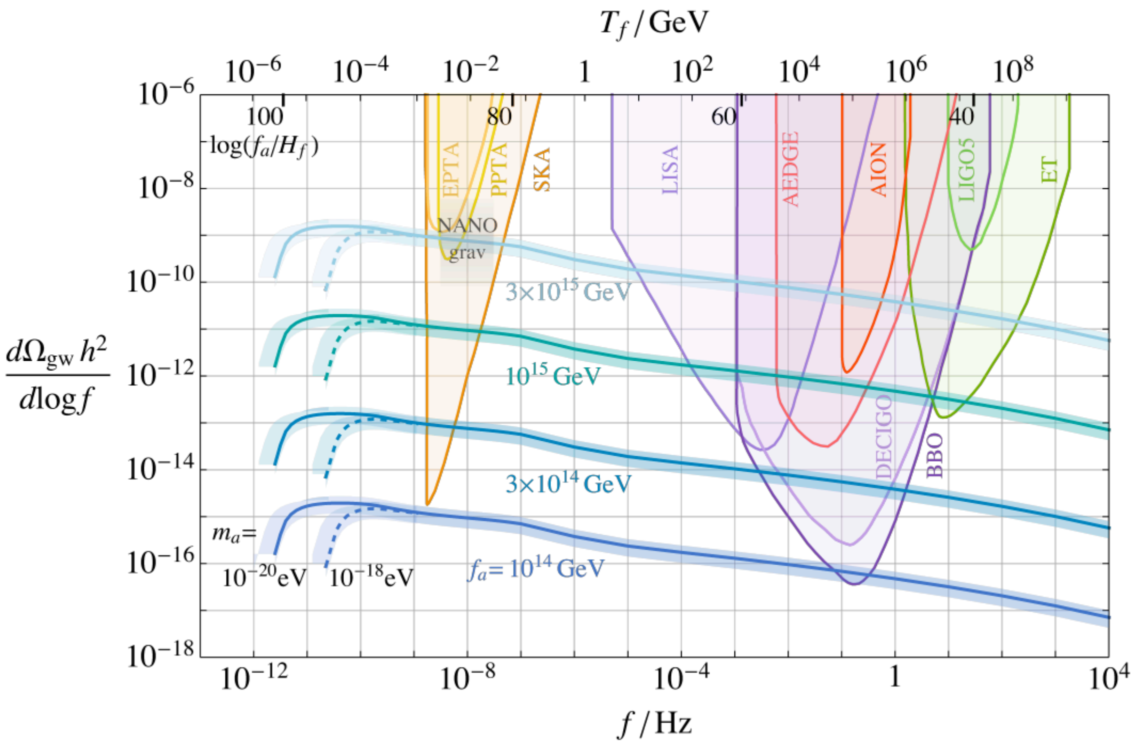

This is a hot topic because it can probe axions as DM

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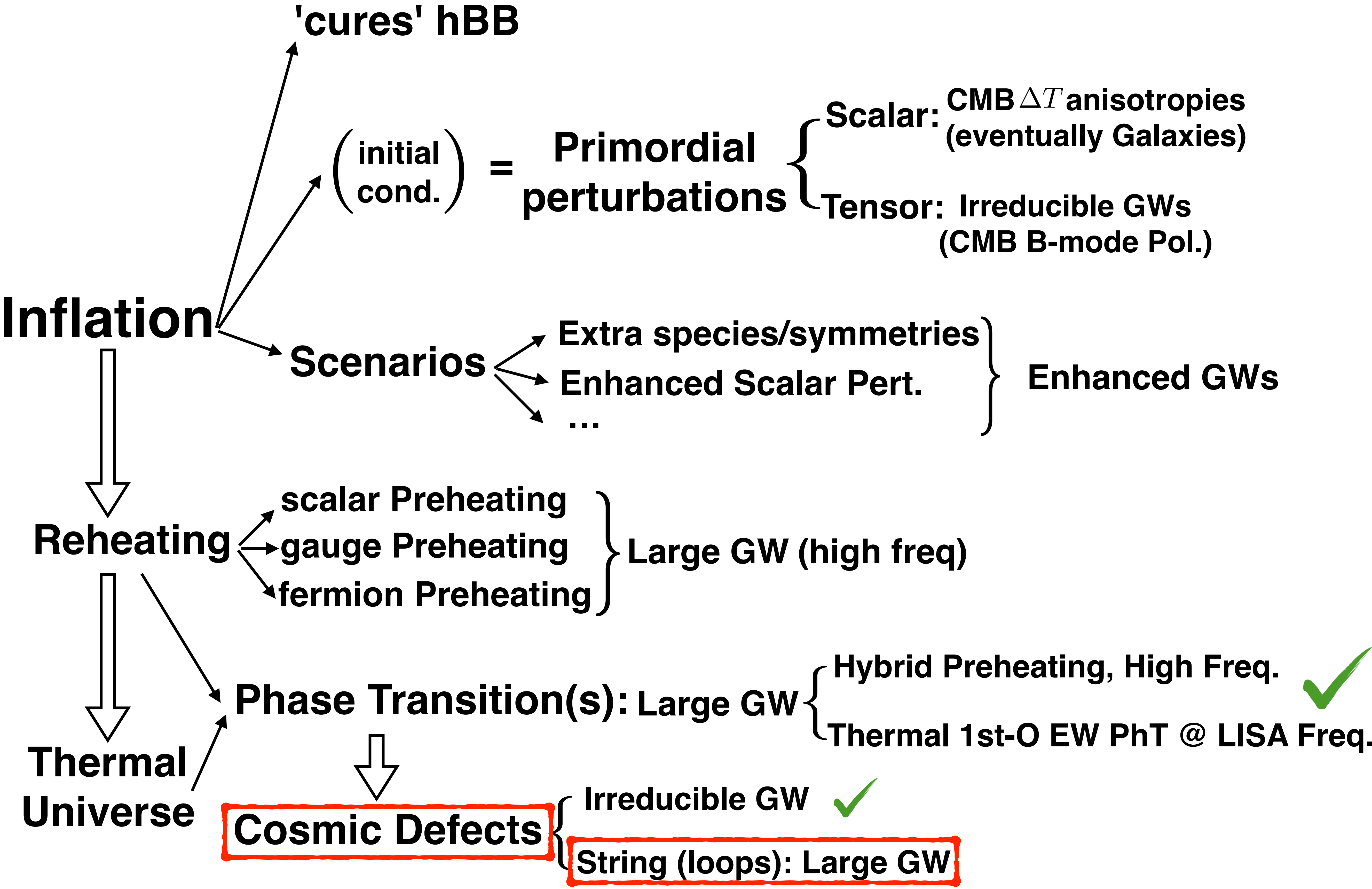

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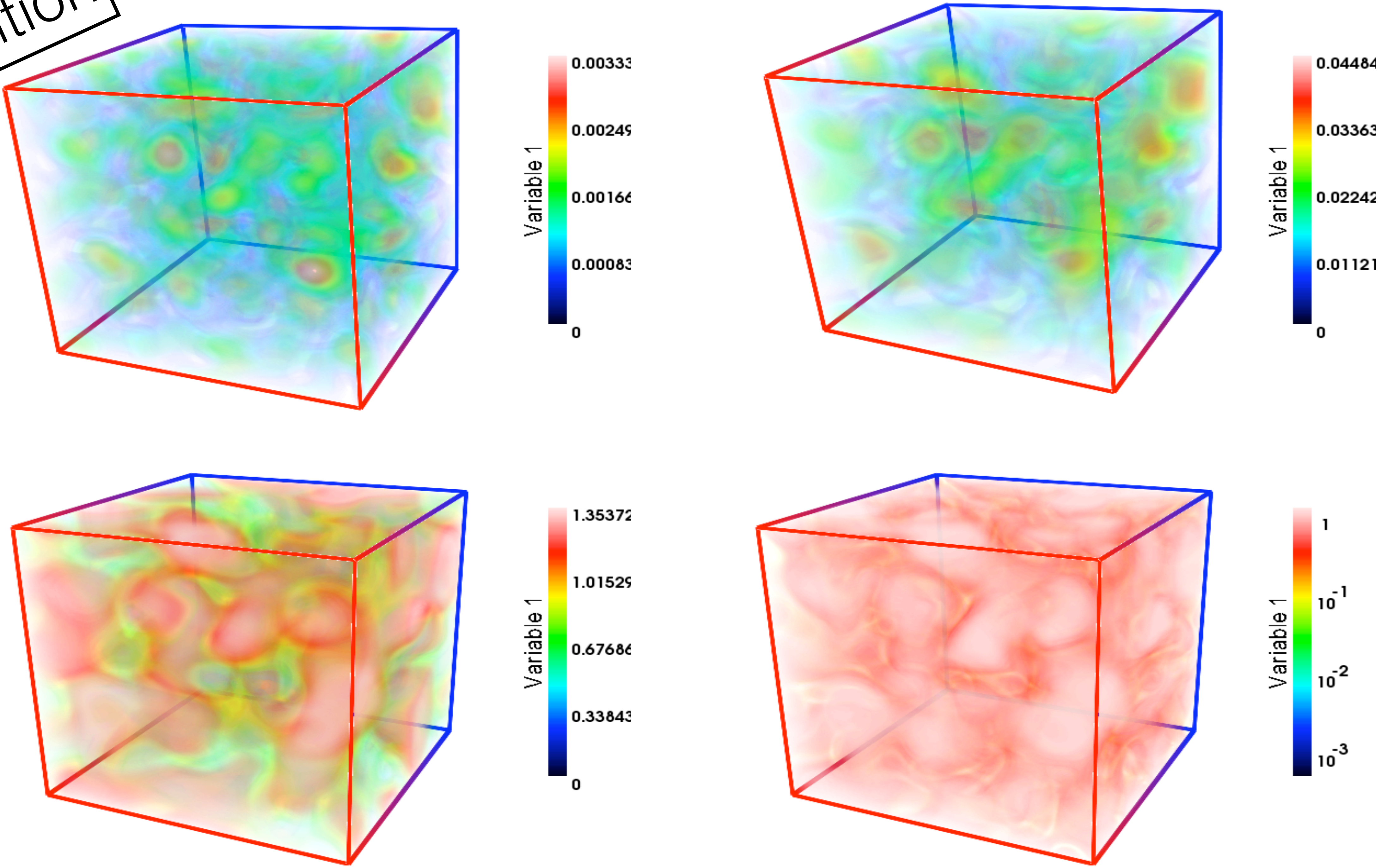
So when do cosmic strings form ?

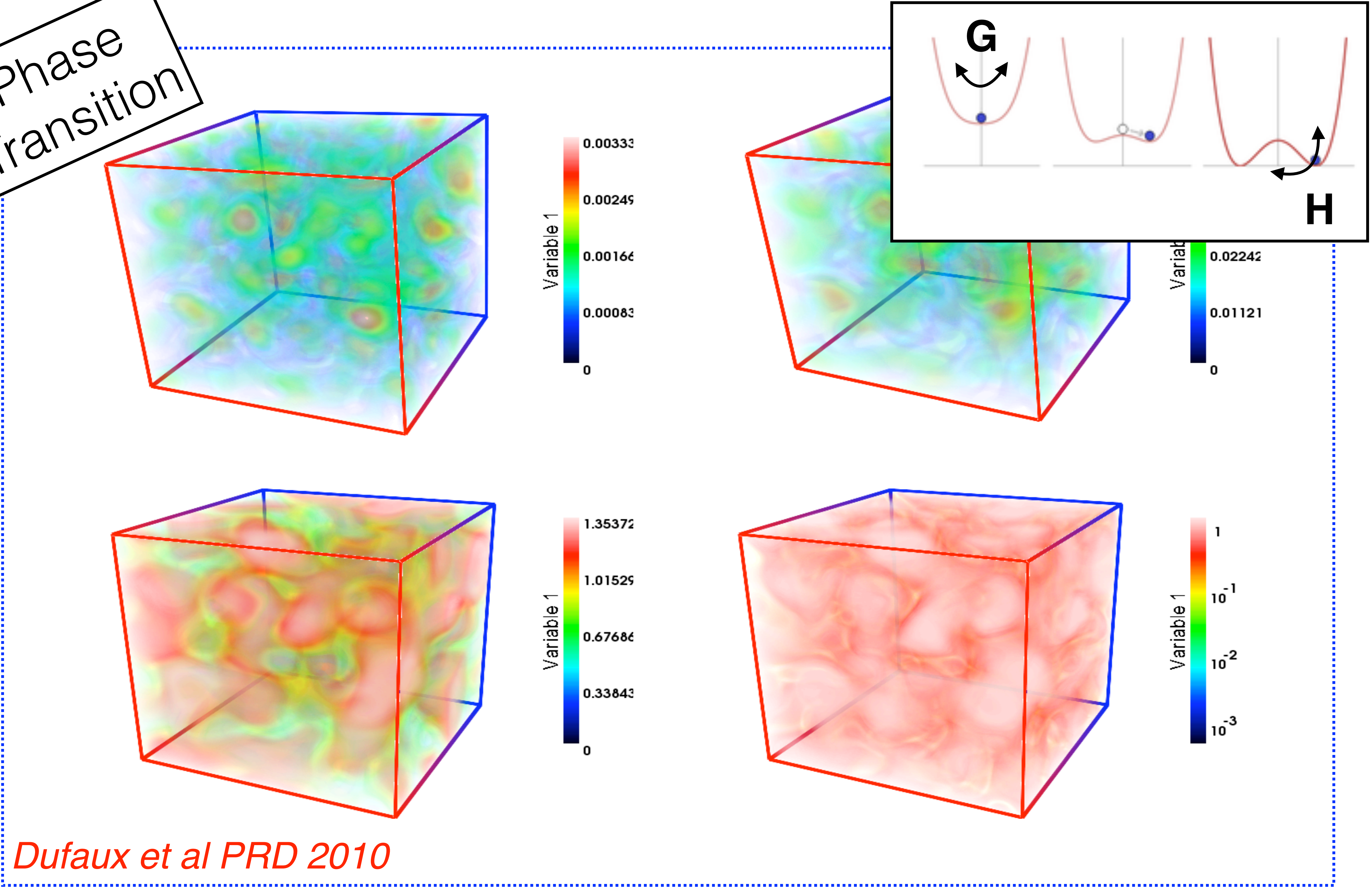
Phase
Transition



Dufaux et al PRD 2010

So when do cosmic strings form ?

Phase Transition



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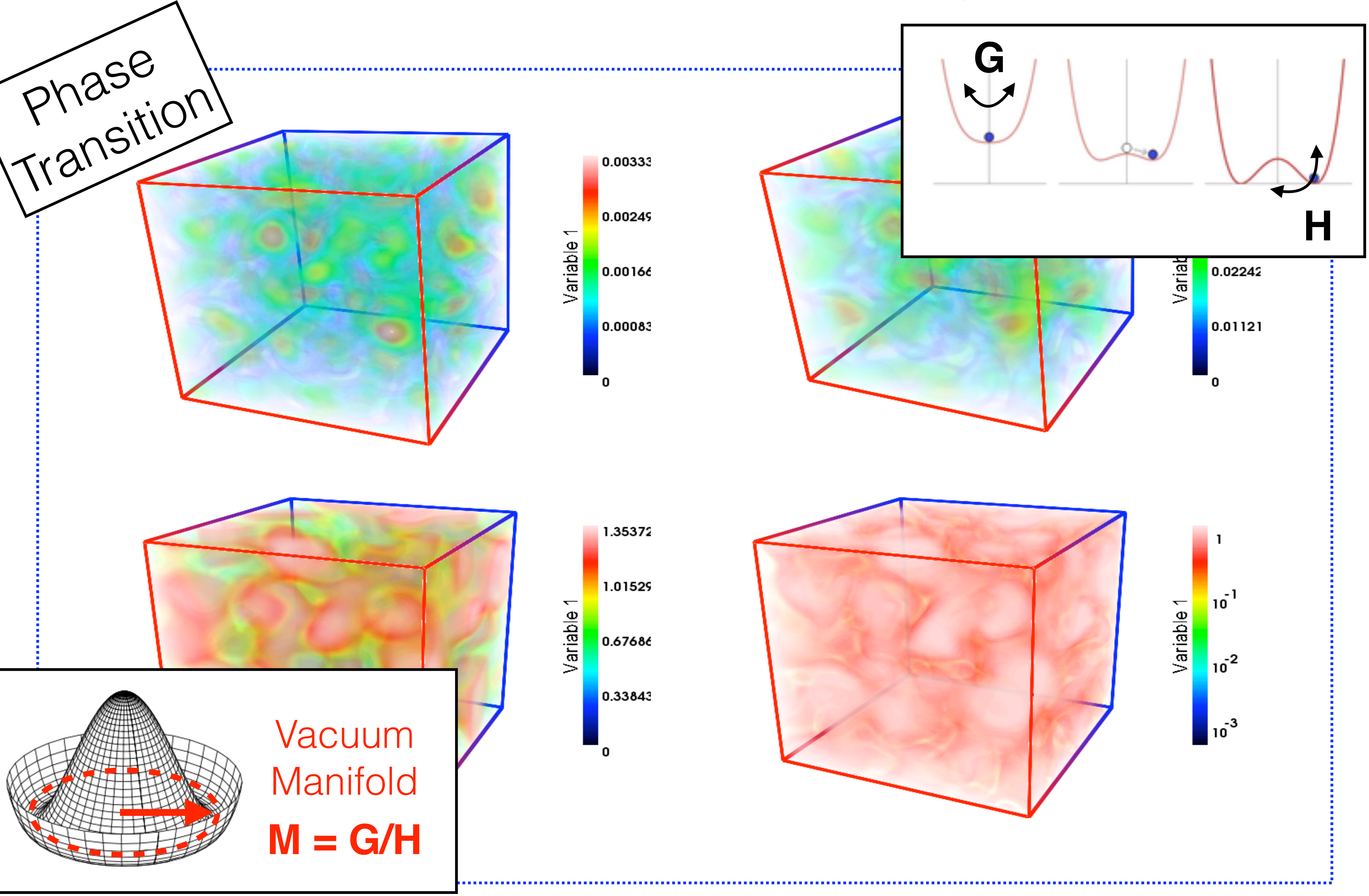
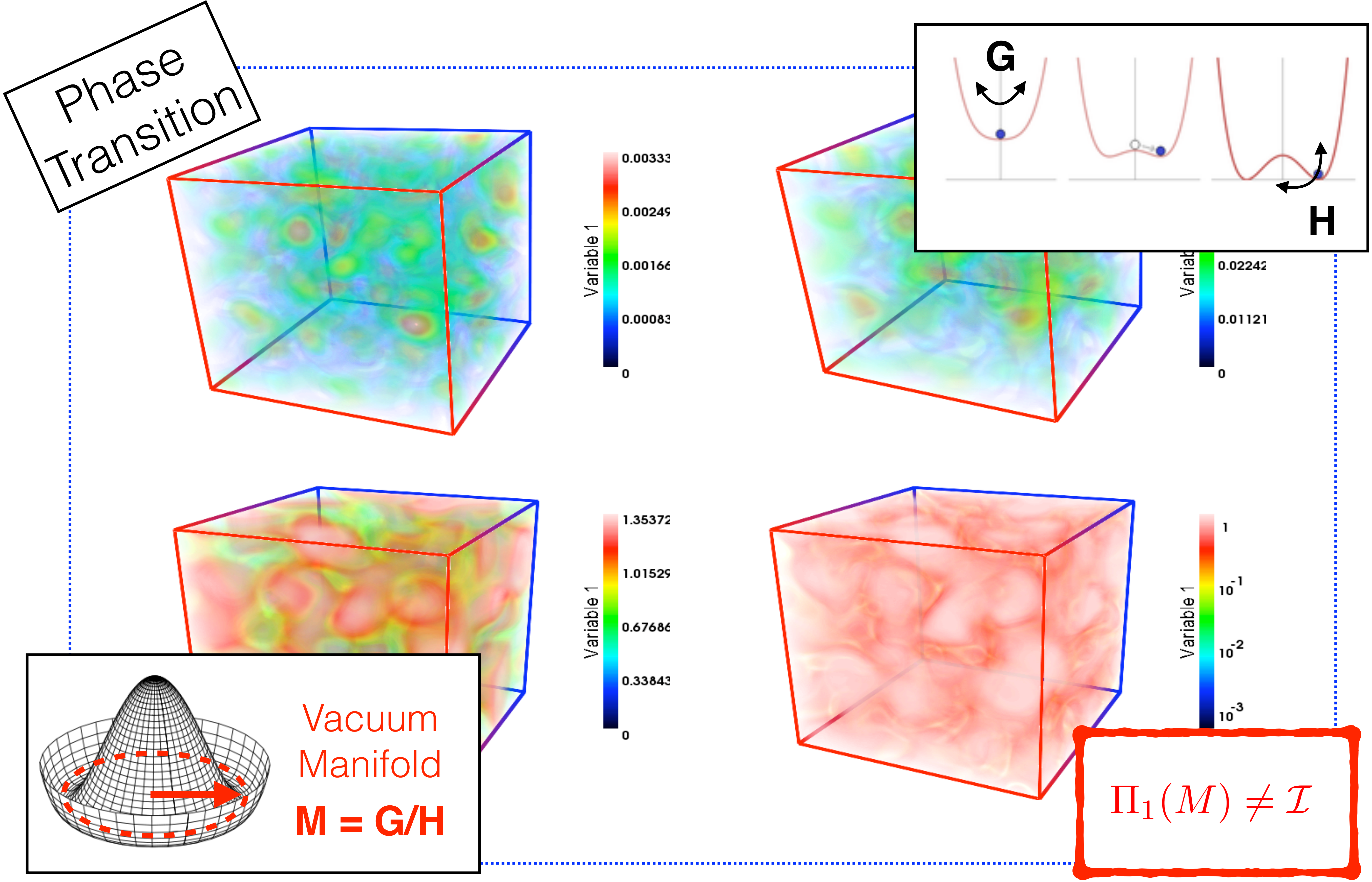


Vacuum Manifold

M = G/H

So when do cosmic strings form ?

Phase Transition



Vacuum Manifold

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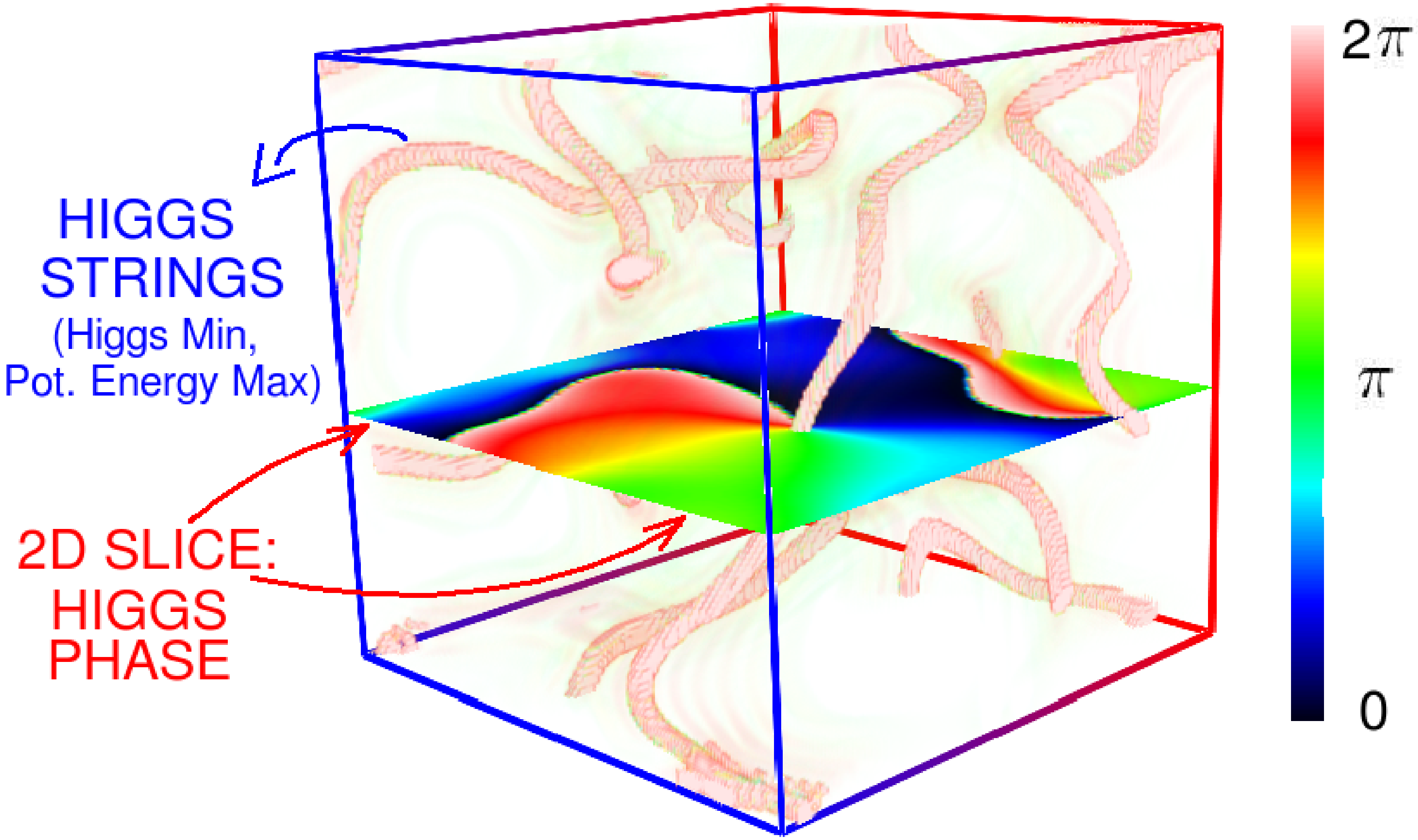
$$\Pi_1(M) \neq \mathcal{I}$$

So when do cosmic strings form ?

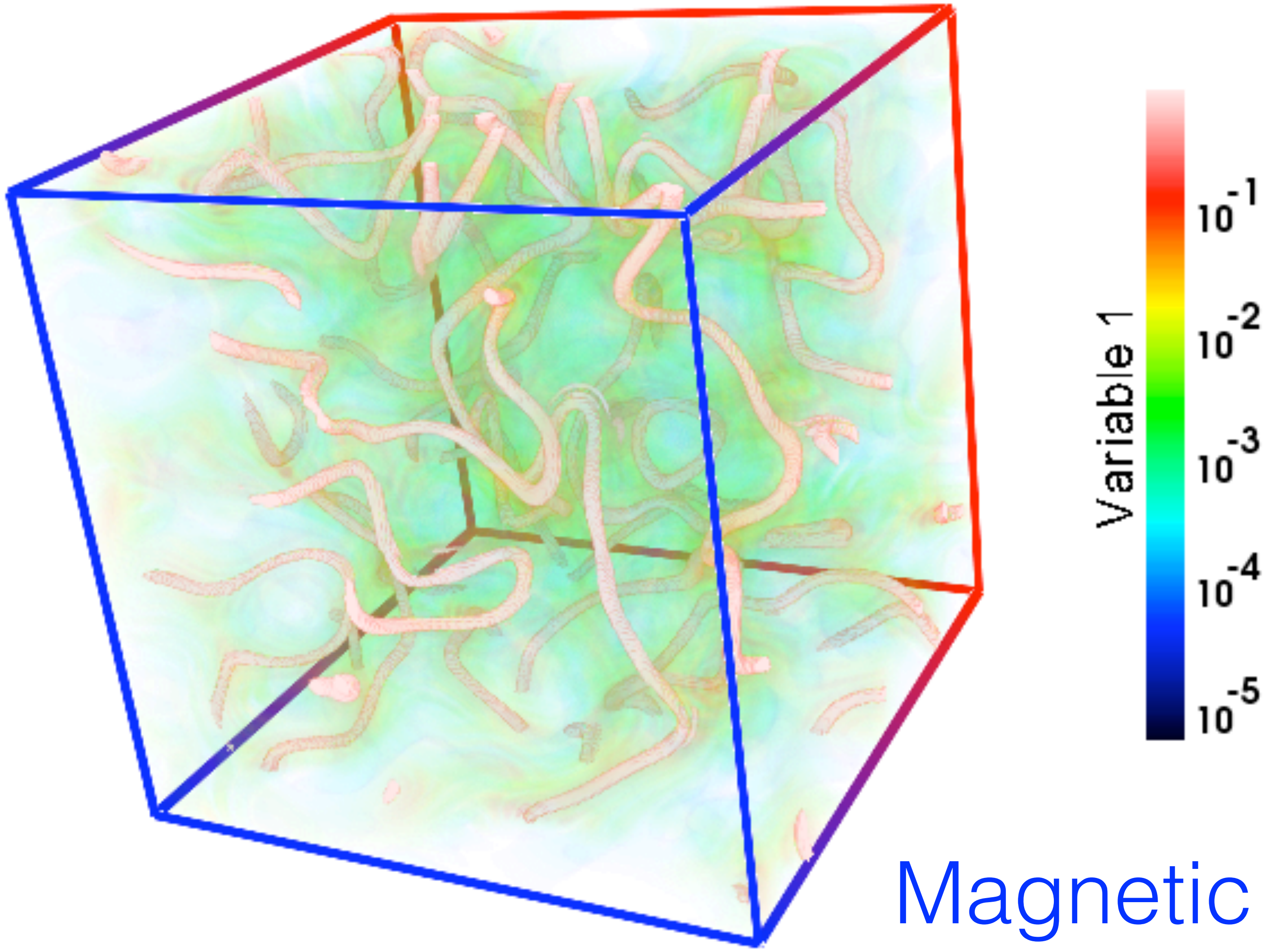
Cosmic strings form !



Dufaux et al PRD 2010

So when do cosmic strings form ?

Intensity of magnetic energy density



Magnetic Field
energy density

Dufaux et al PRD 2010

IF Defects are Cosmic Strings …

Further emission of GWs ! (Vilenkin ’81)

IF Defects are Cosmic Strings …

Intercommutation

Loops are formed !

IF Defects are Cosmic Strings ...

Loops are formed !

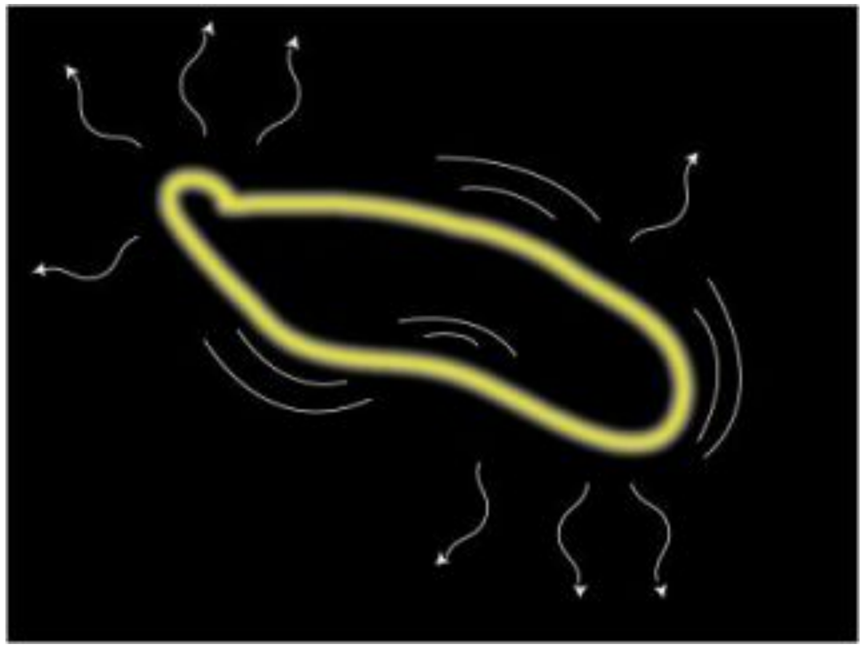


Image Credit: Google

Gravitational Waves emitted !

(releasing the loops' tension)

Cosmic Strings Network: Loop configurations

Cosmic string loop (length *l*) oscillates under tension μ

➡ **emits GWs in a series of harmonic modes**

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and many others !

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↓

"extra" emission on top of Irreducible background (only for strings)

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expansion history

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(Nambu-Goto simulations)

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(Nambu-Goto simulations)

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(Nambu-Goto simulations)

GW power emission

$$\propto 1/(fl)^{\boxed{q+1}}$$

features

(kinks,cusps,…)

Modeling

Model I

Analytical approach (parametric dependences)

Model II

Blanco-Pillado,
Olum,
Shlaer

Model III

Lorenz,
Ringevald,
Sakellariadou

Direct fit to Nambu-GOTO simulations in expanding universe

e-Print: **arXiv:1909.00819 [astro-ph.CO]**

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Calibrated via simulations (allows extrapolation regime)

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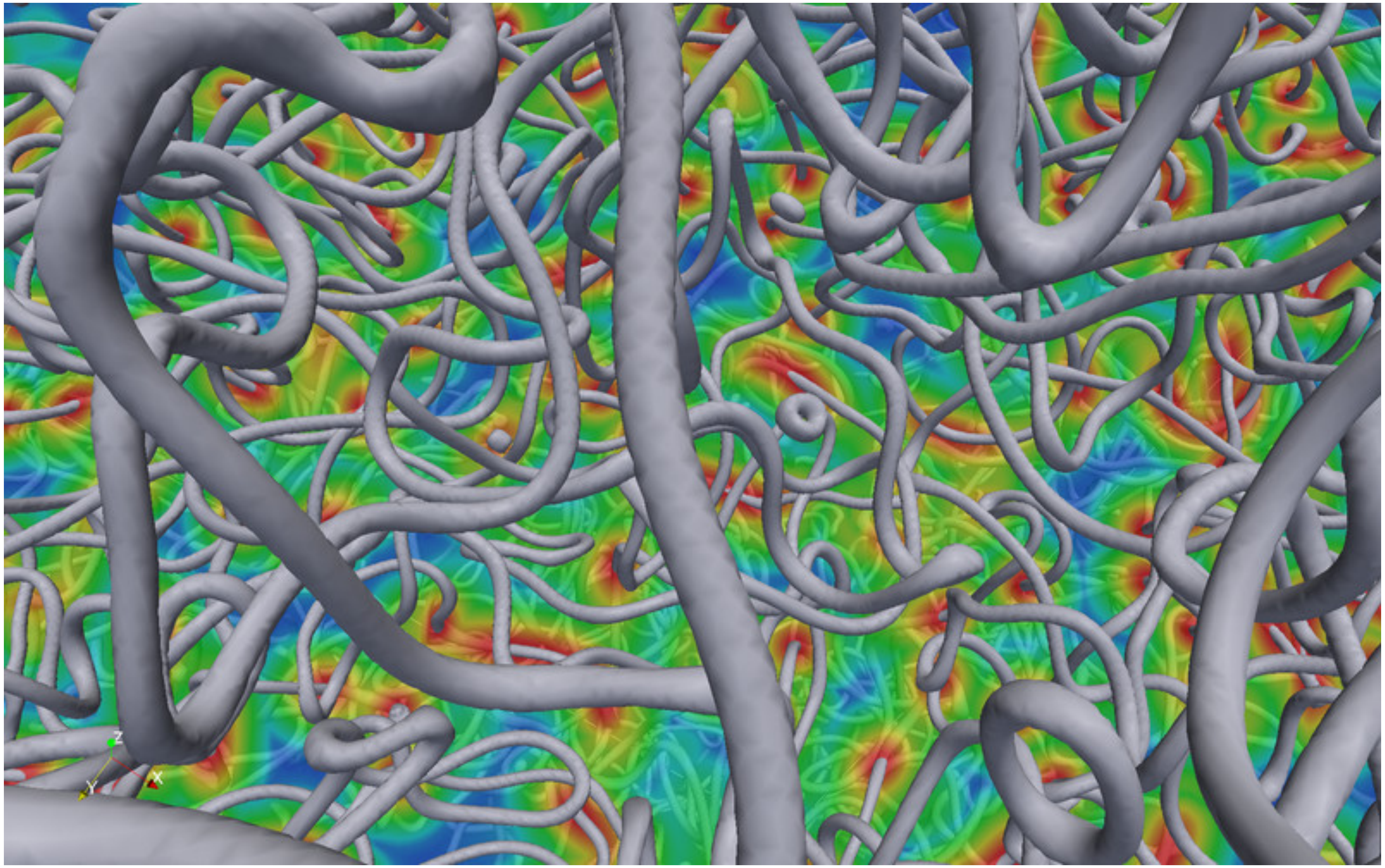
Modeling

What about lattice simulations ?



(Image: David Daverio)

Modeling

What about lattice simulations ?

Abelian-Higgs Simulations

* Loops formed ! … but decay into scalar/gauge fields

* If loops disappear… then no GW ?

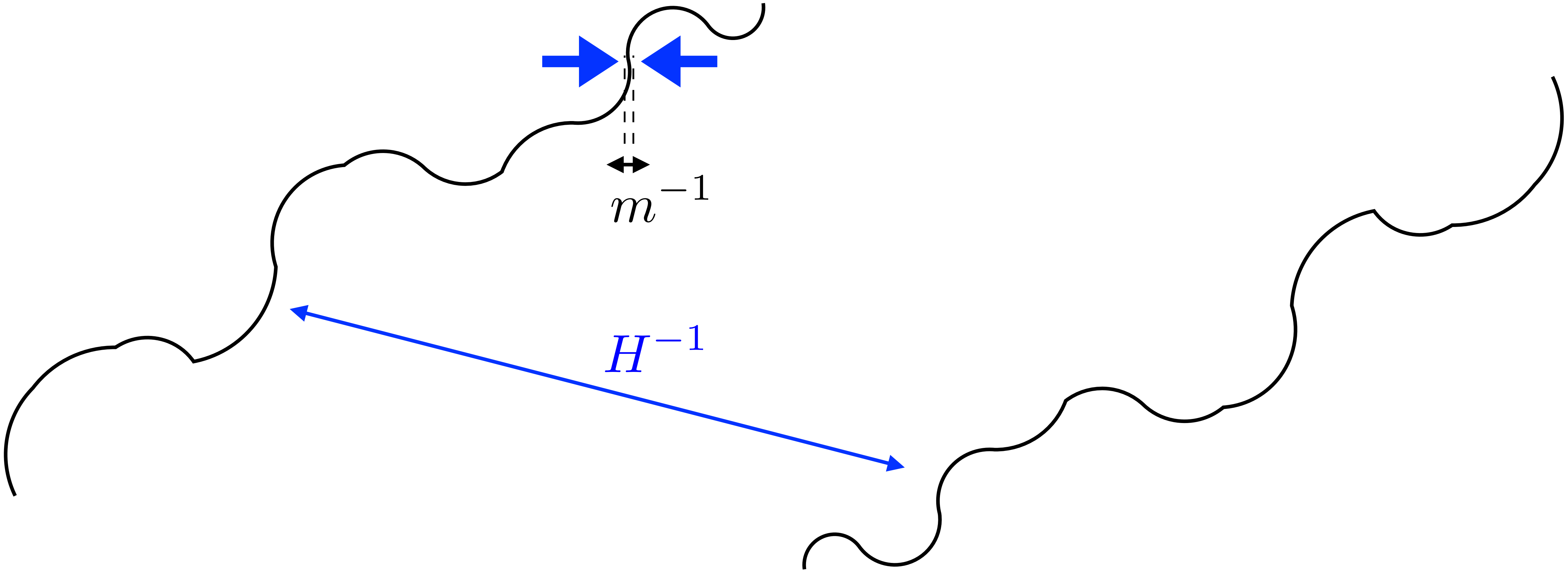
* There is an **irreducible GW emission** from the long string network, but negligible vs NG loop GW emission

So … next results based on Nambu-Goto strings !

Modeling

Nambu-Goto

Infinitely thin: $H^{-1} \gg m^{-1}$



Cosmic strings loops: GW background

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features

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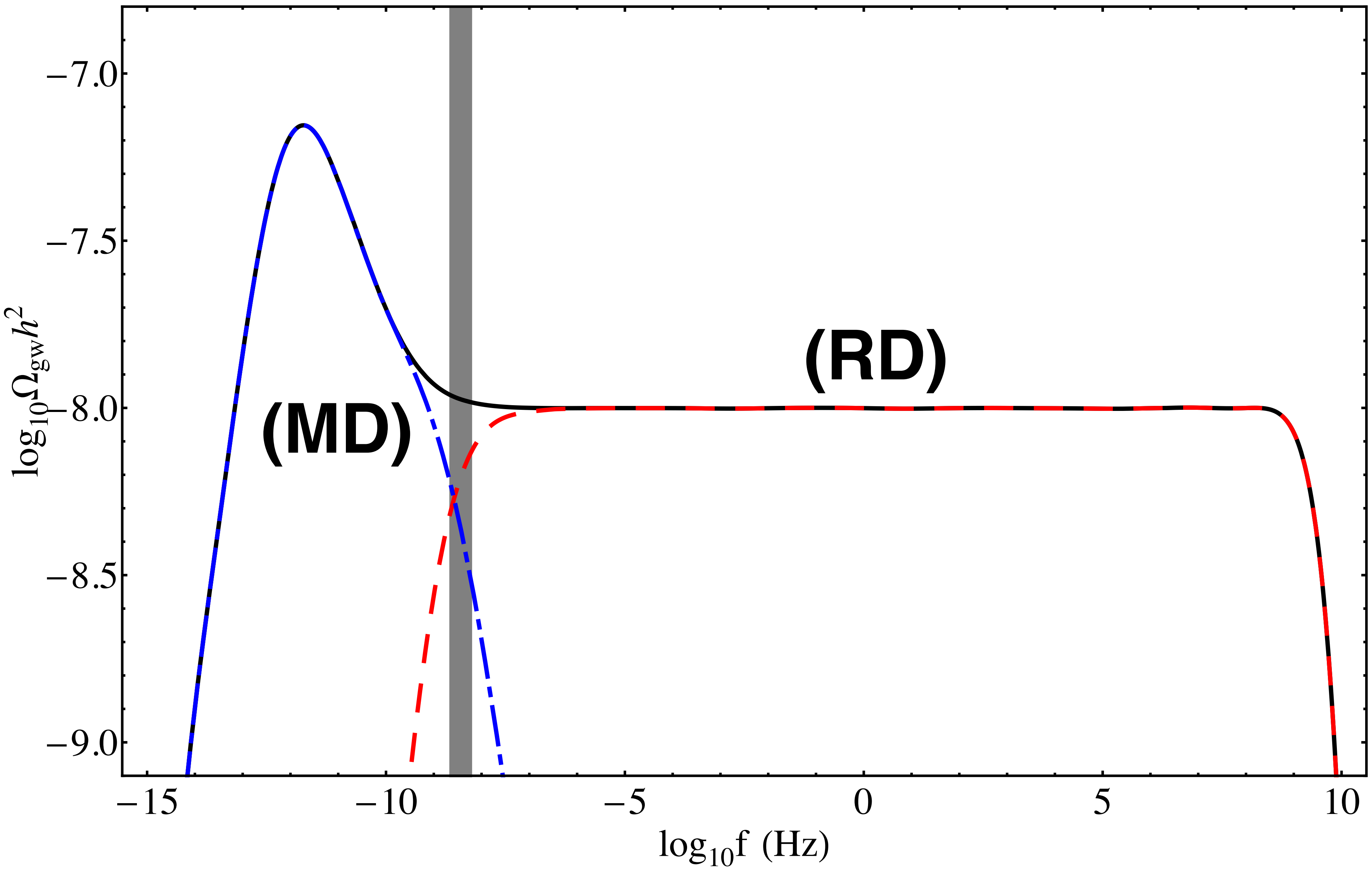
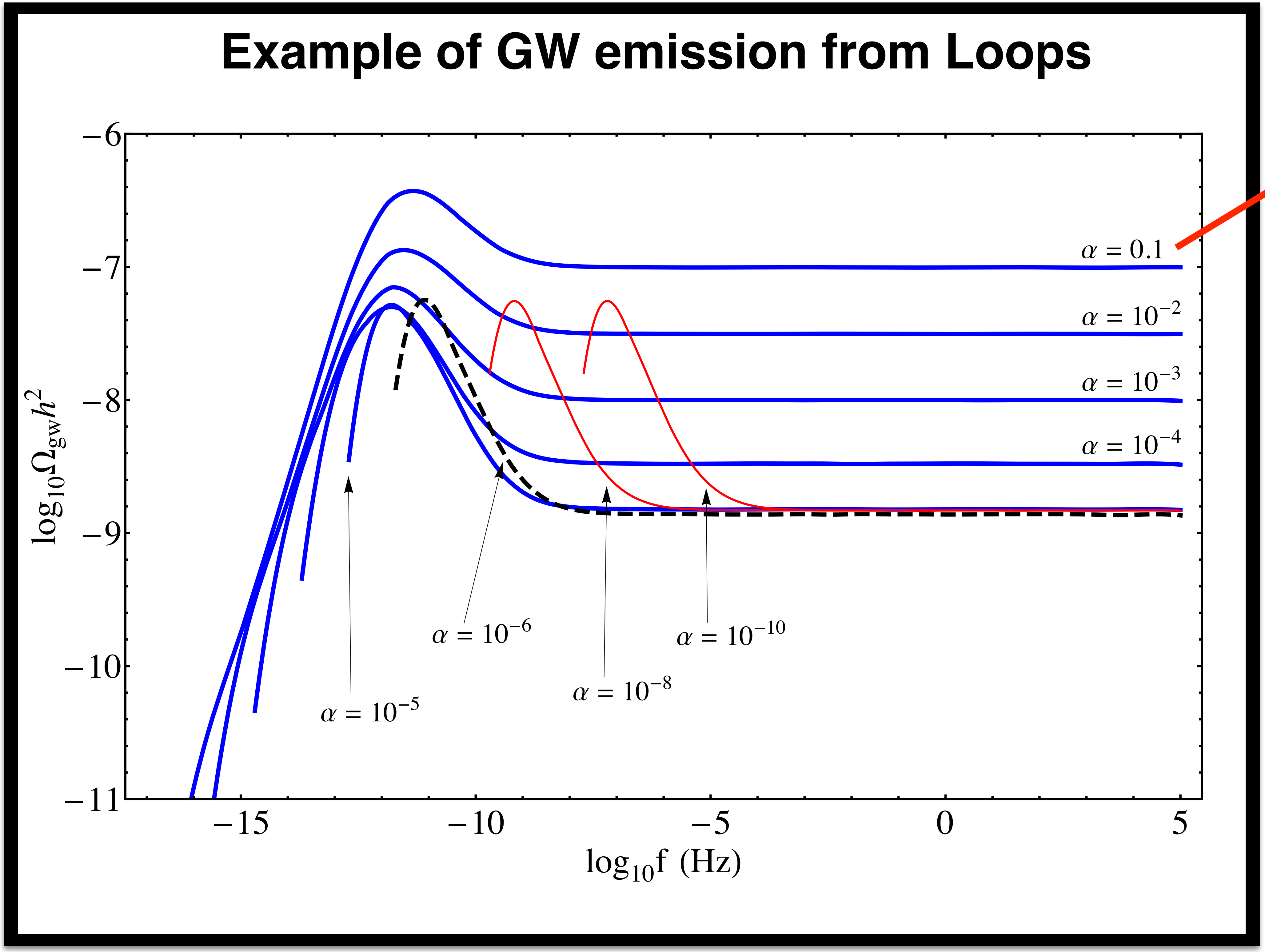
Cosmic strings loops: GW background

Example of GW emission from Loops



e.g. Sanidas et al 2012

Cosmic Strings Network: Loop configurations



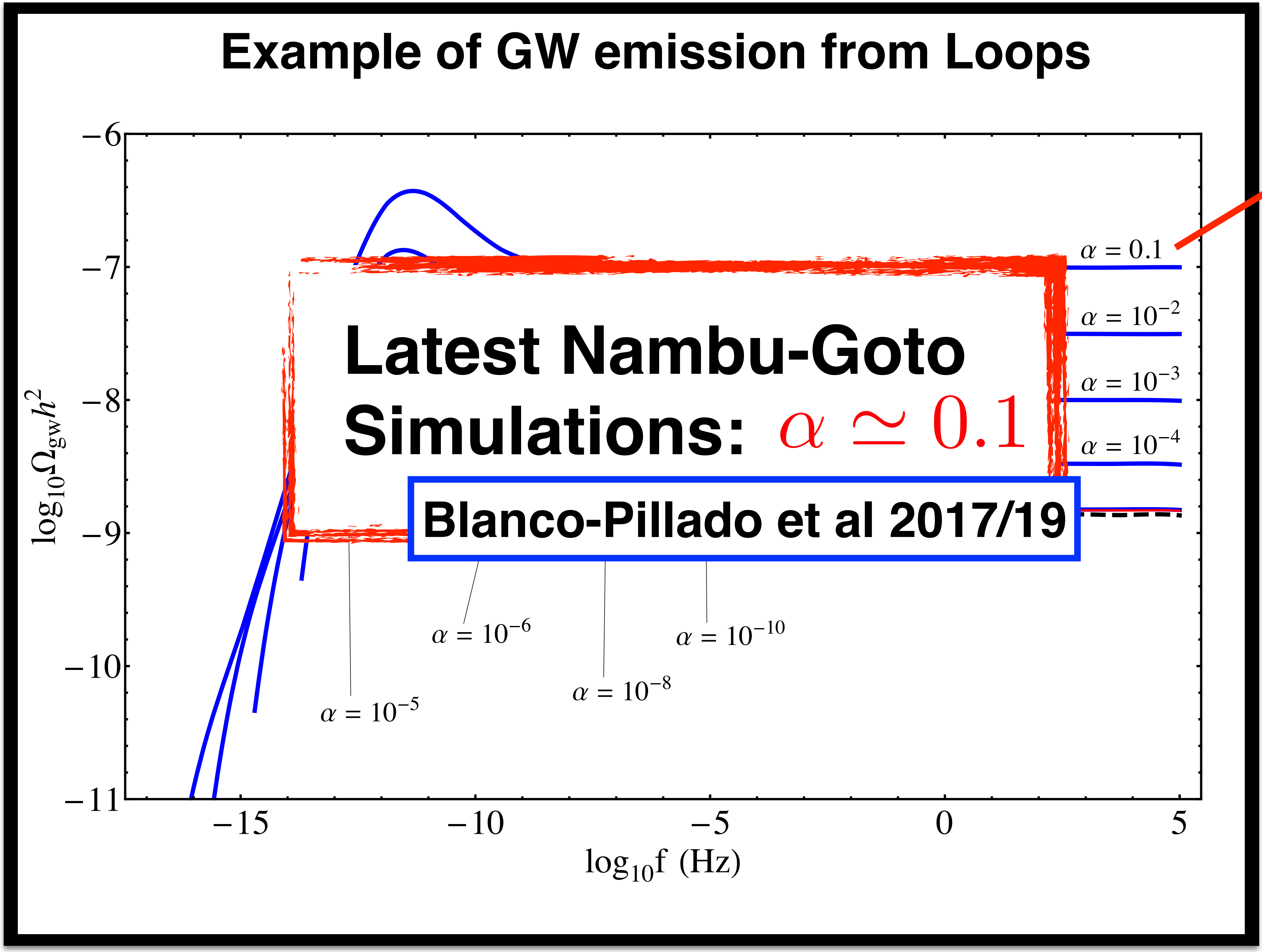
α

loop size

(relative to horizon)

Sanidas et al 2012

Cosmic Strings Network: Loop configurations



Latest Nambu-Goto Simulations: $\alpha \simeq 0.1$

Blanco-Pillado et al 2017/19

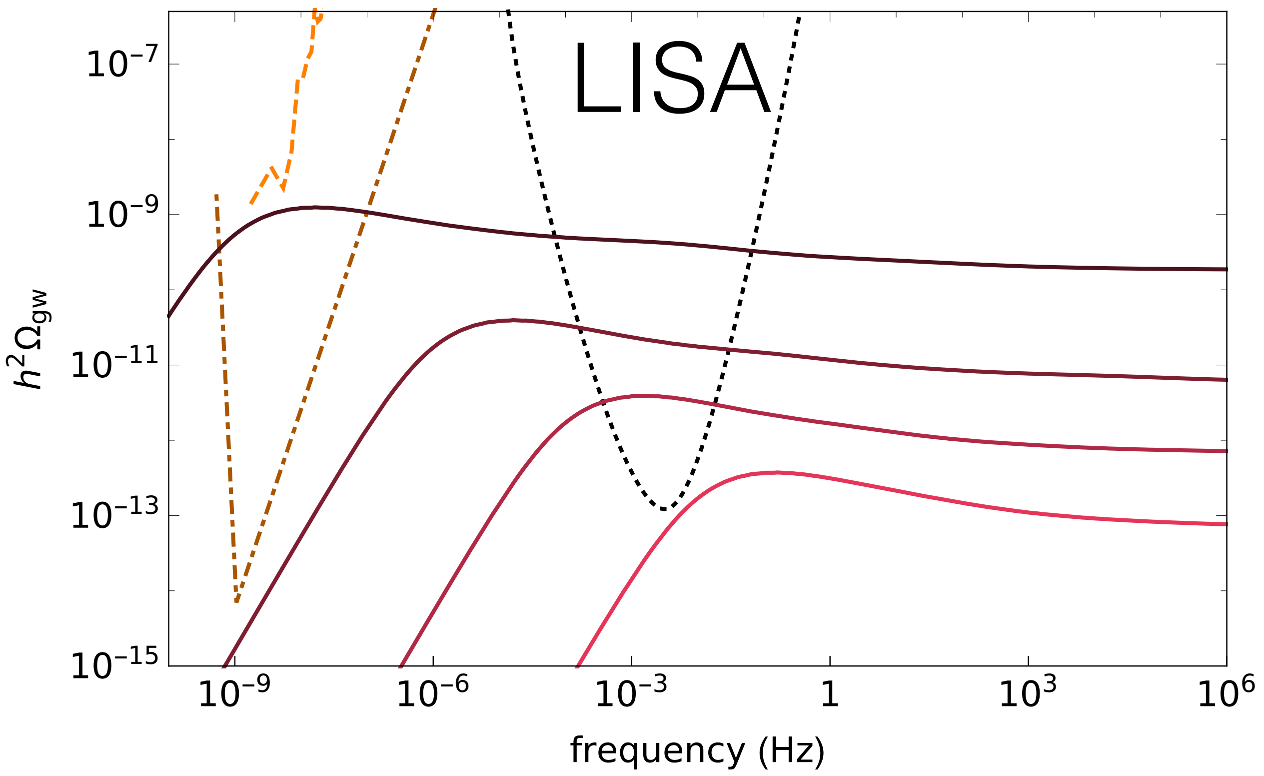
α

loop size

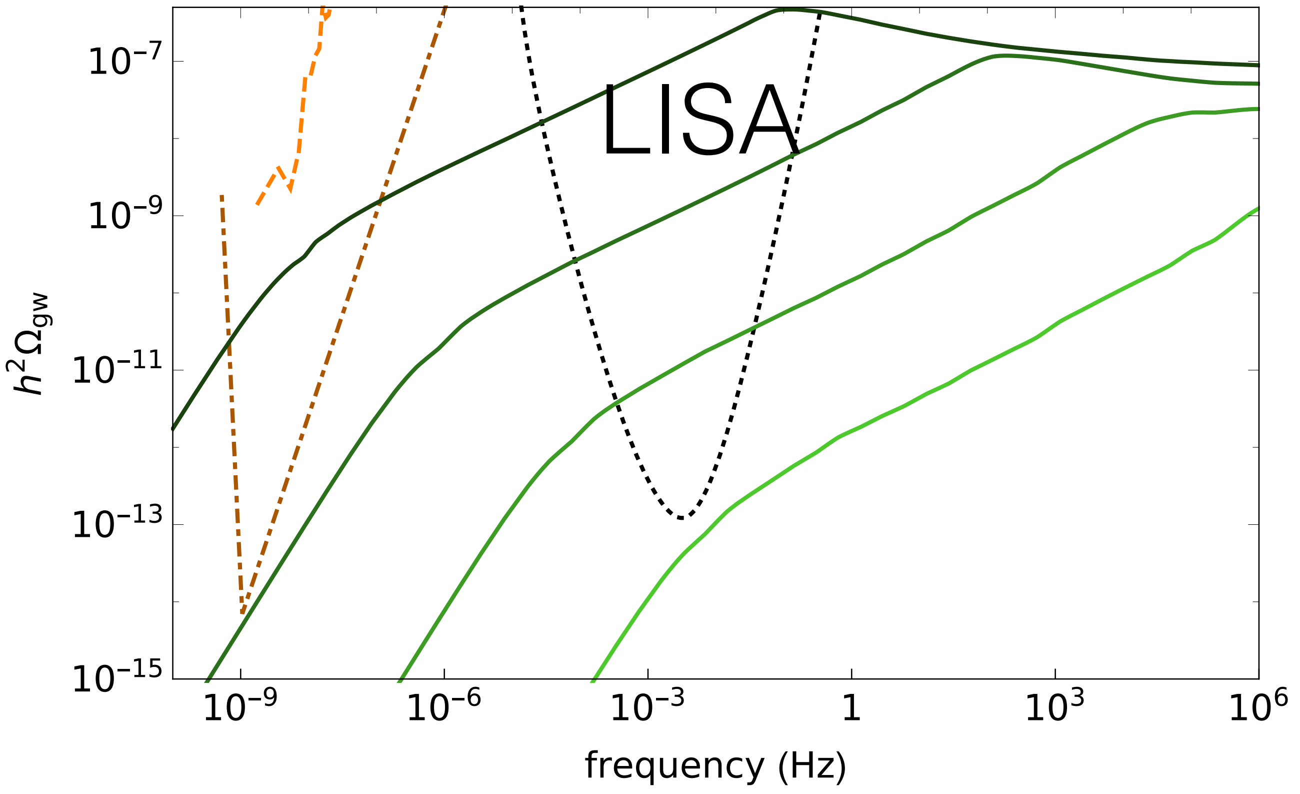
(relative to horizon)

Model II (BOS) vs Model III (LRS)

Model II



Model III



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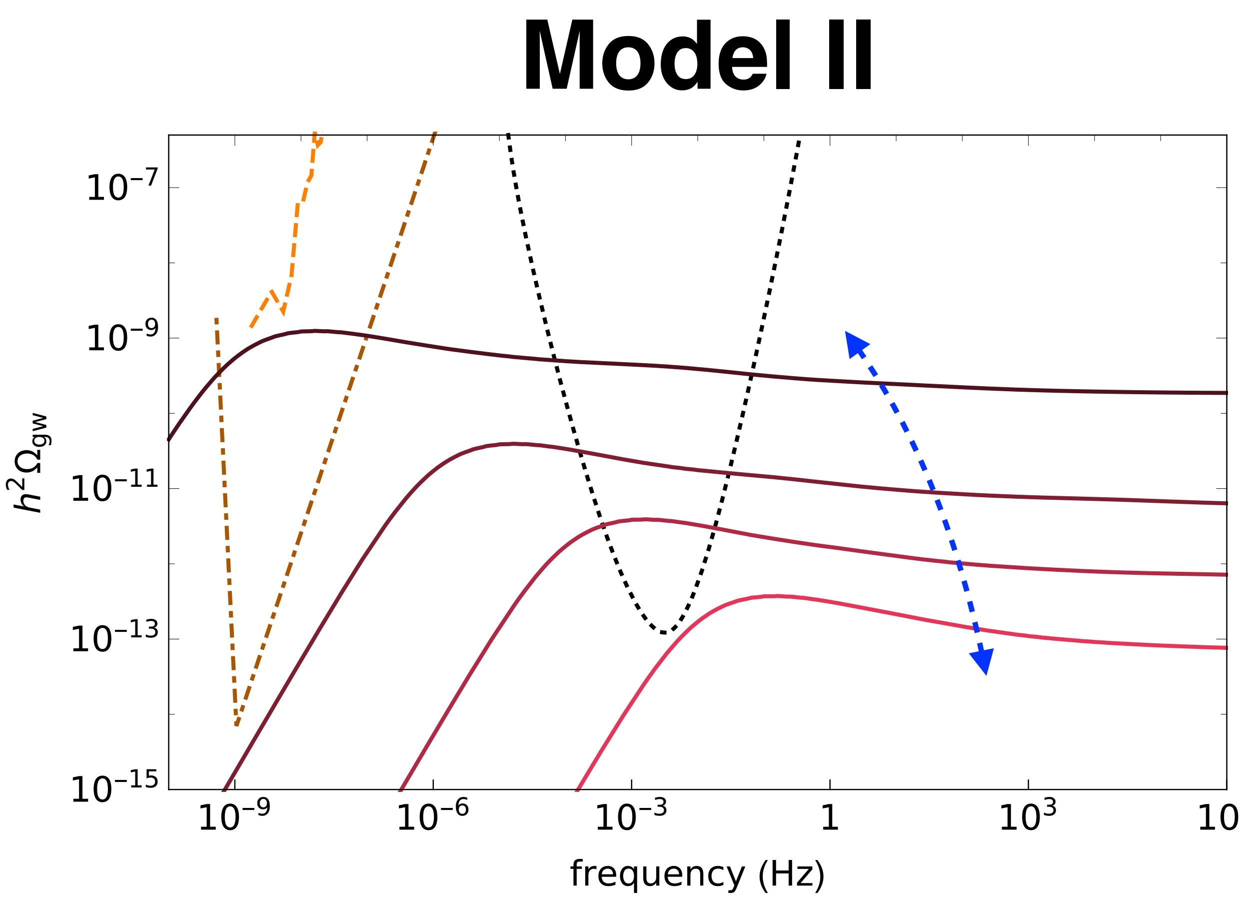

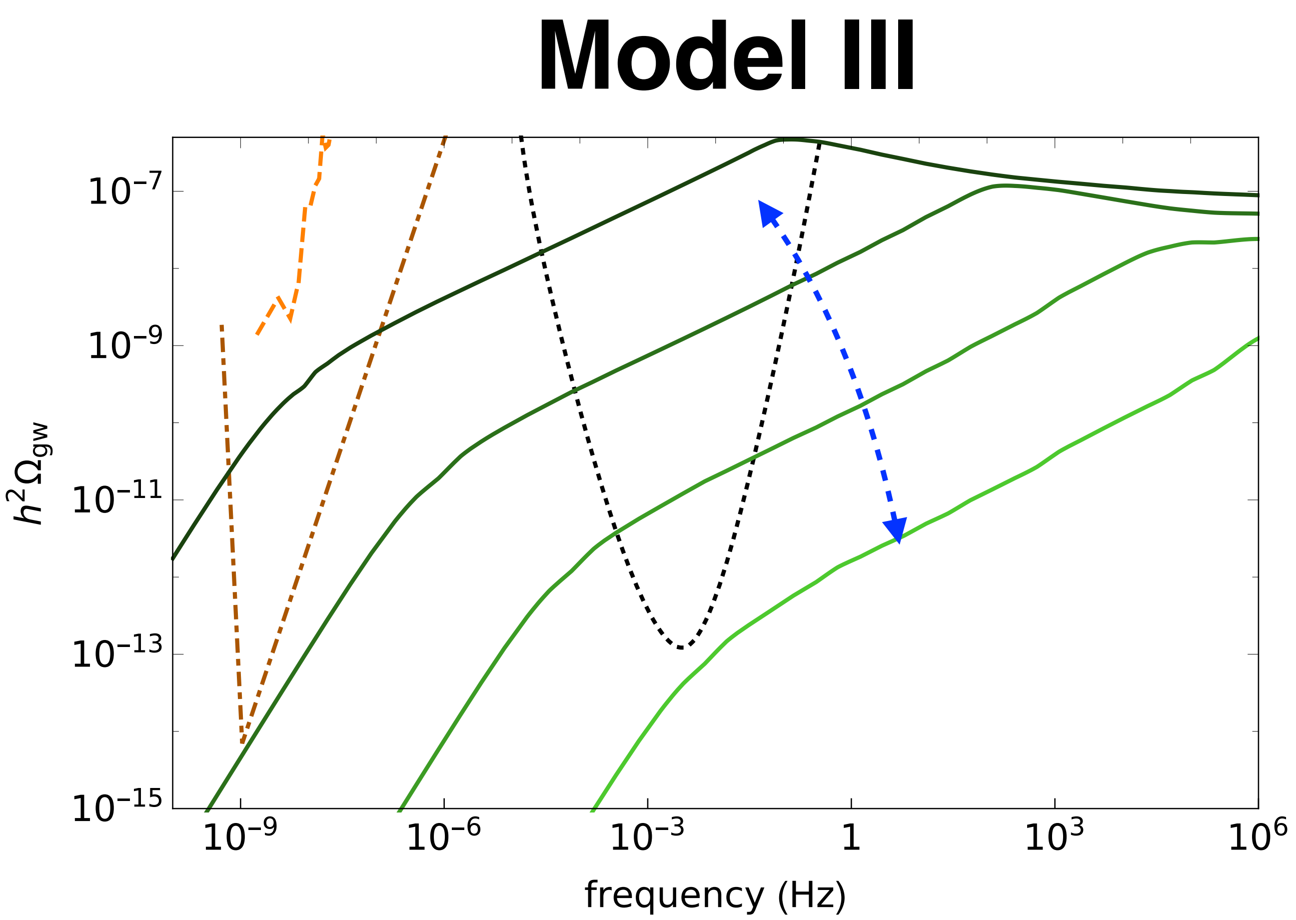

$$G\mu \sim 10^{-11} - 10^{-17}$$

@ LISA: Very large parameter space !

Model II (BOS) vs Model III (LRS)

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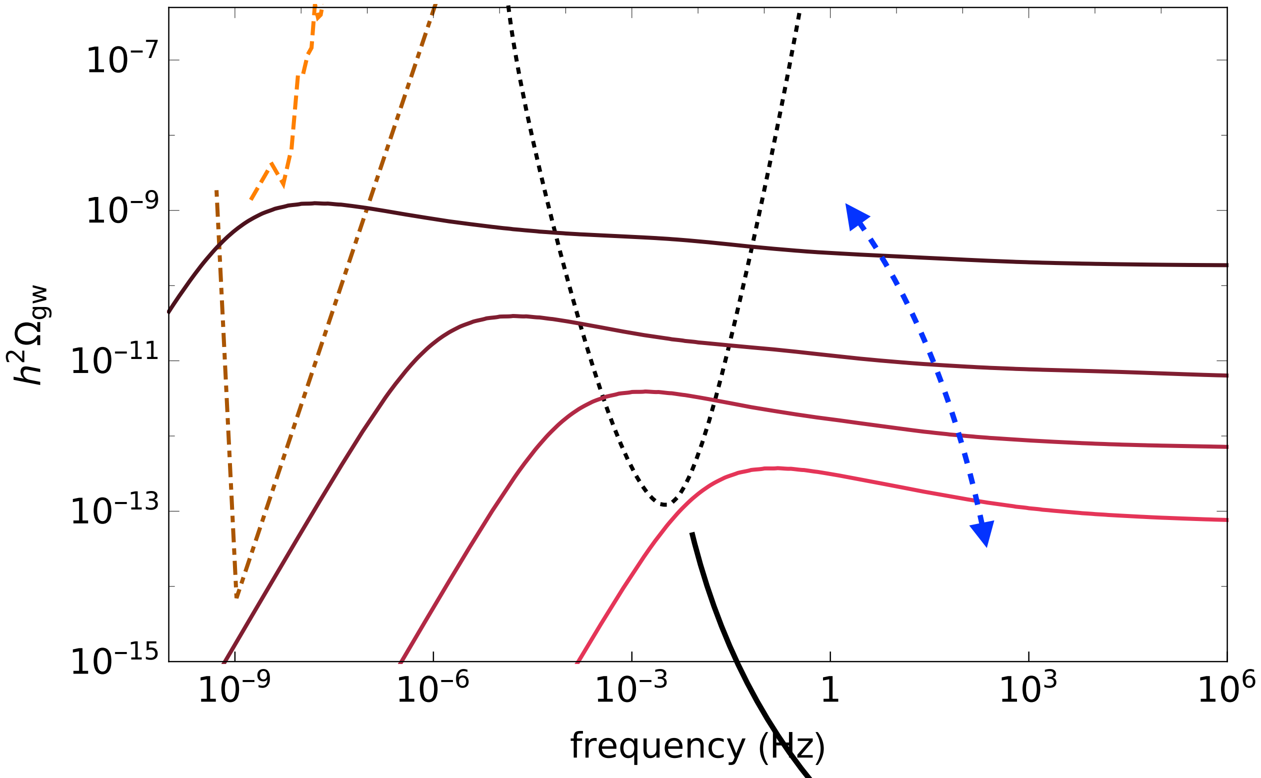

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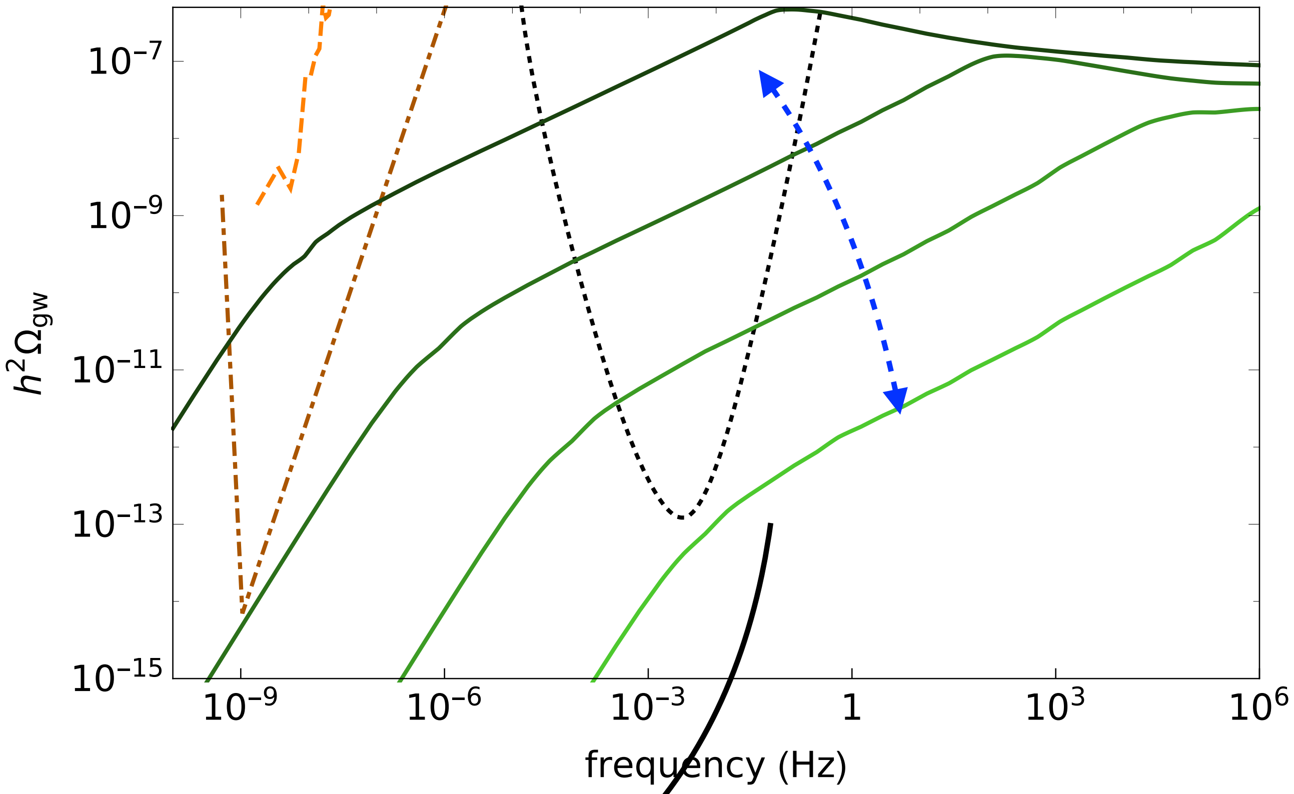

$$G\mu \gtrsim 10^{-17}$$

@ LISA: Very large parameter space !

GW background constrained by LISA

$$\boxed{G\mu \gtrsim 10^{-17} \quad (v \gtrsim 10^{10}\ \mathrm{GeV})}$$

CMB	PTA (today)	PTA (future)
$G\mu \sim 10^{-7}$	$G\mu \sim 10^{-11}$	$G\mu \sim 10^{-14}$

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(!)

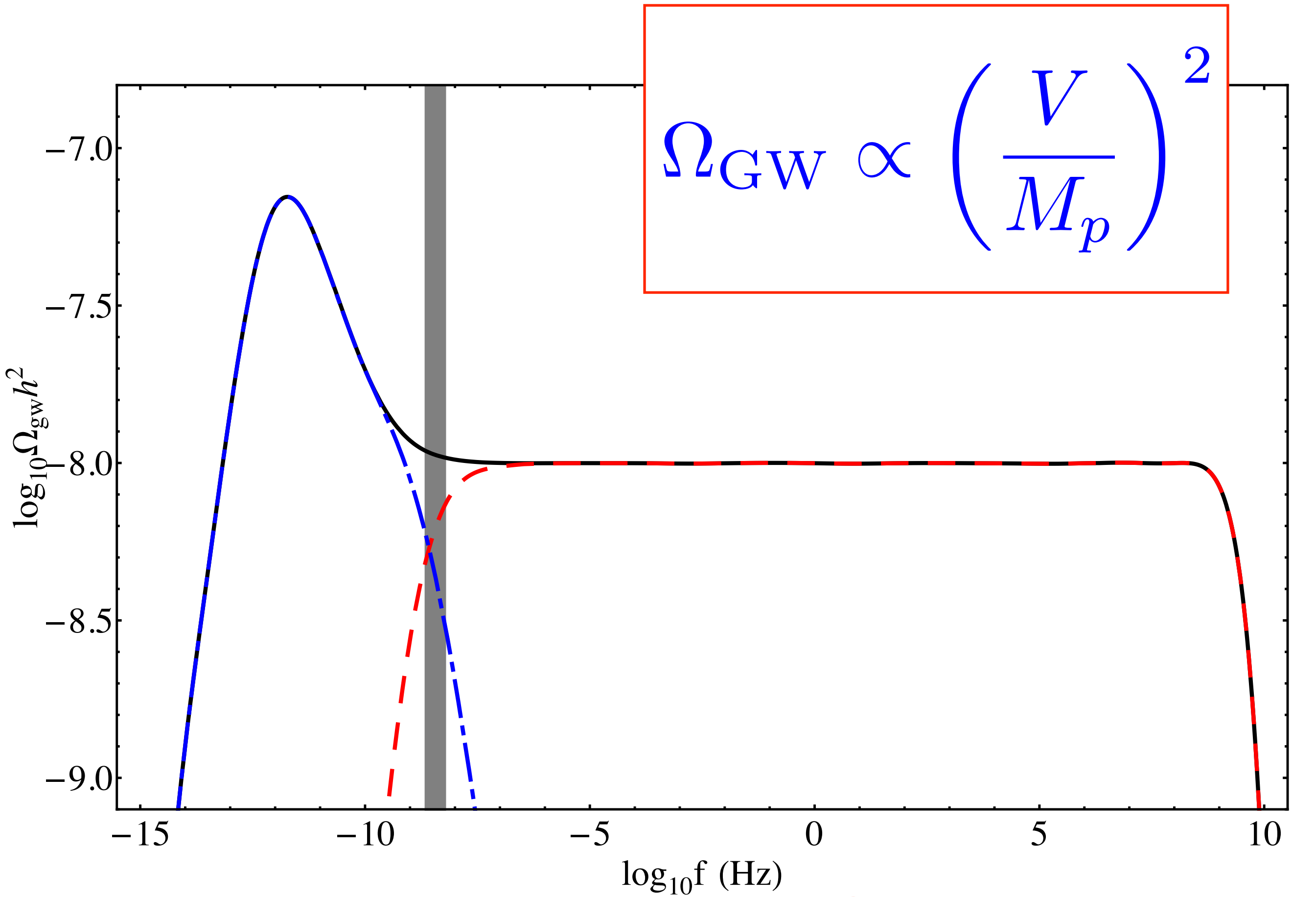
LISA $\left\{ \right.$

- **Best constraints on Comic Strings**
- **(actually only way to obtain them)**
- **Discovery, or stringent constraints**

Cosmic Strings Network: Loop configurations

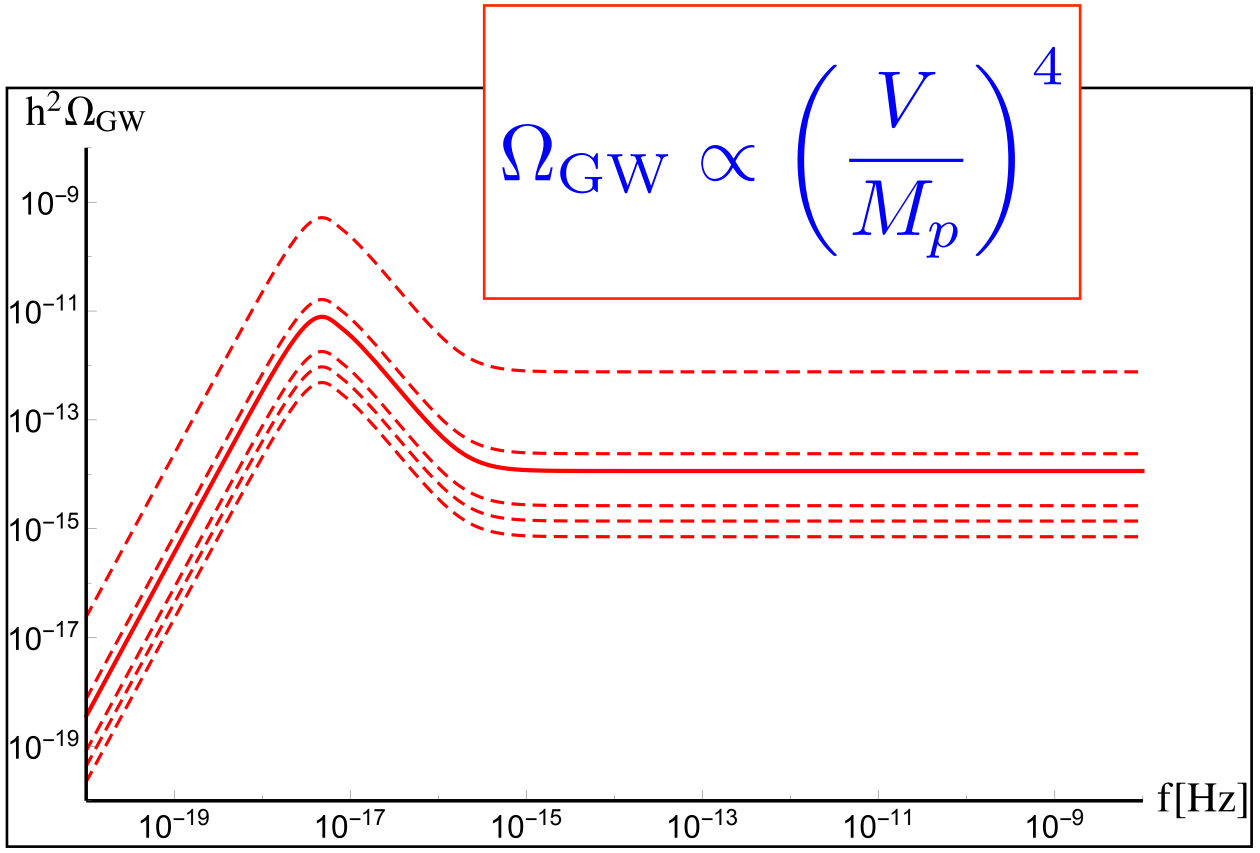
GW from string loops $\neq$ GW from "Infinite"-Strings

(particular emission) **(irreducible emission)**



$$\Omega_{\rm GW} \propto \left(\frac{V}{M_p}\right)^2$$

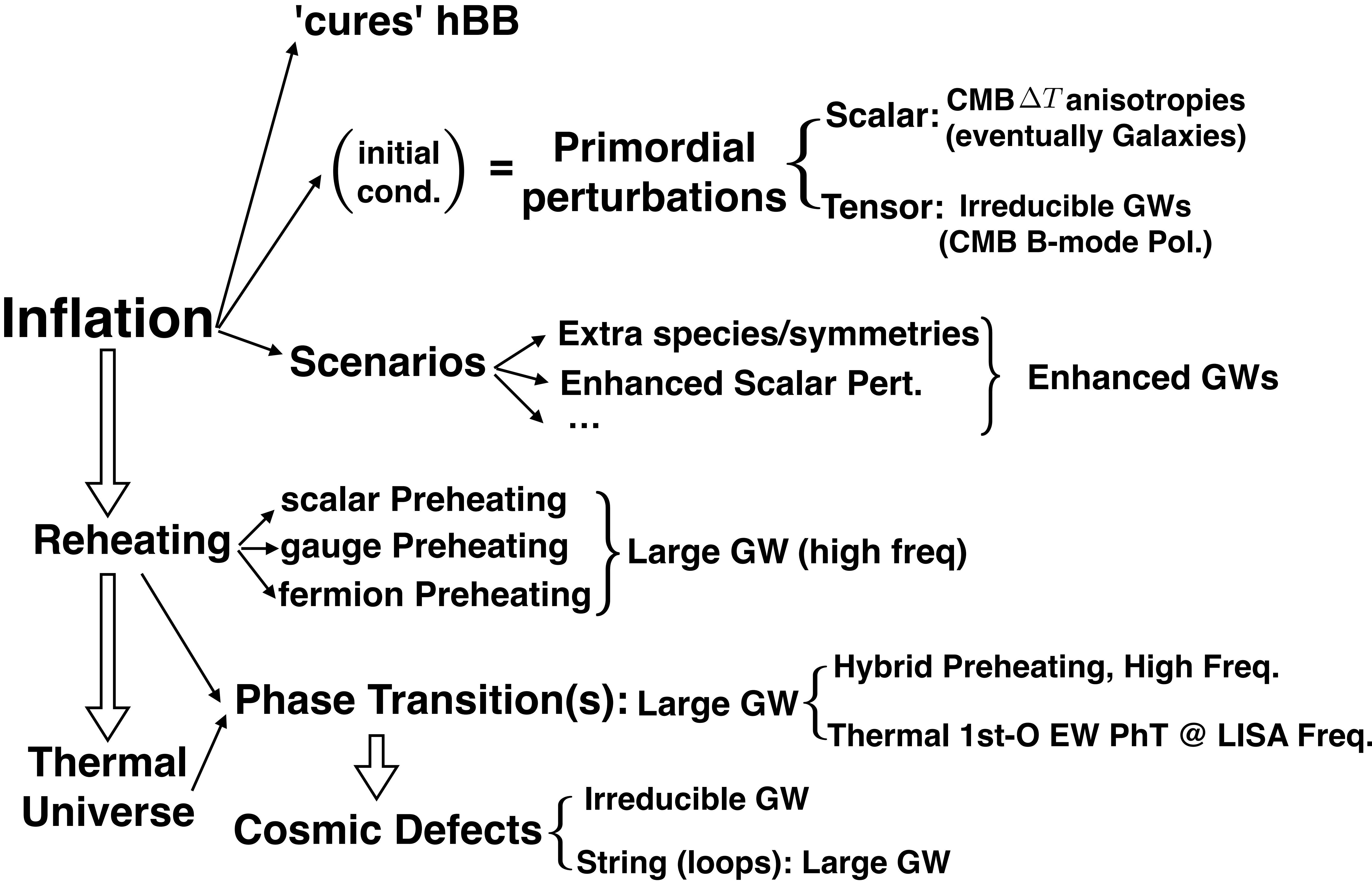
Vilenkin, Vachaspati, Bouchet, Siemens et al, Sanidas et al, Blanco-Pillado et al, … 1981 - 2020



$$\Omega_{\rm GW} \propto \left(\frac{V}{M_p}\right)^4$$

DGF, Hindmarsh, Lizarraga, Urrestilla, work in progress 2013-2020

EARLY UNIVERSE in GWs ✓



'cures' hBB

$\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

- Scalar: CMB ΔT anisotropies (eventually Galaxies)
- Tensor: Irreducible GWs (CMB B-mode Pol.)

Inflation

Scenarios
- Extra species/symmetries
- Enhanced Scalar Pert.
- …

Enhanced GWs

Reheating
- scalar Preheating
- gauge Preheating
- fermion Preheating

Large GW (high freq)

Phase Transition(s): Large GW
- Hybrid Preheating, High Freq.
- Thermal 1st-O EW PhT @ LISA Freq

Thermal Universe

Cosmic Defects
- Irreducible GW
- String (loops): Large GW

EARLY UNIVERSE in GWs ✓

'cures' hBB

(initial)

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OUTLINE

Gravitational Wave Backgrounds

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

2) GWs from Inflation ✓

3) GWs from Preheating ✓

3rd Bloc

4) GWs from Phase Transitions ✓

5) GWs from Cosmic Defects ✓

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Early Universe

OUTLINE

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Early Universe

Late Universe

6) Astrophysical Background(s) ← IF there is time …

OUTLINE

Gravitational Wave Backgrounds

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2) GWs from Inflation ✓

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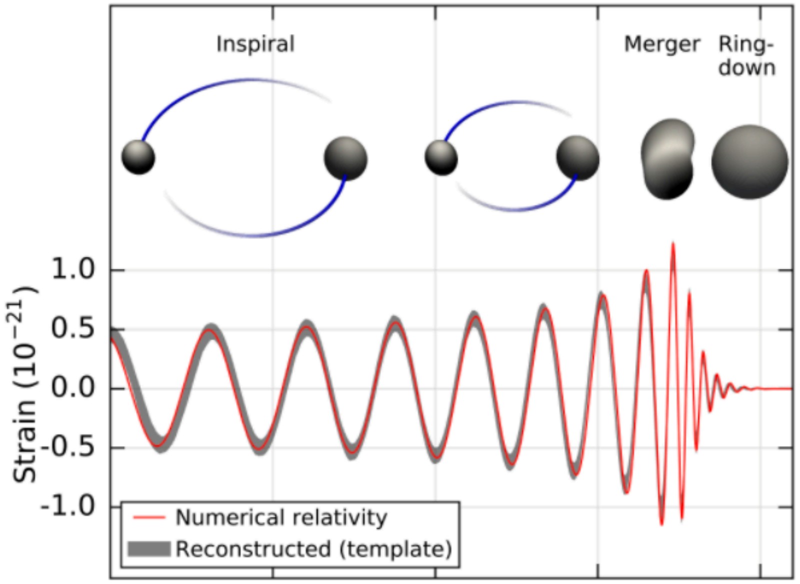
Early Universe

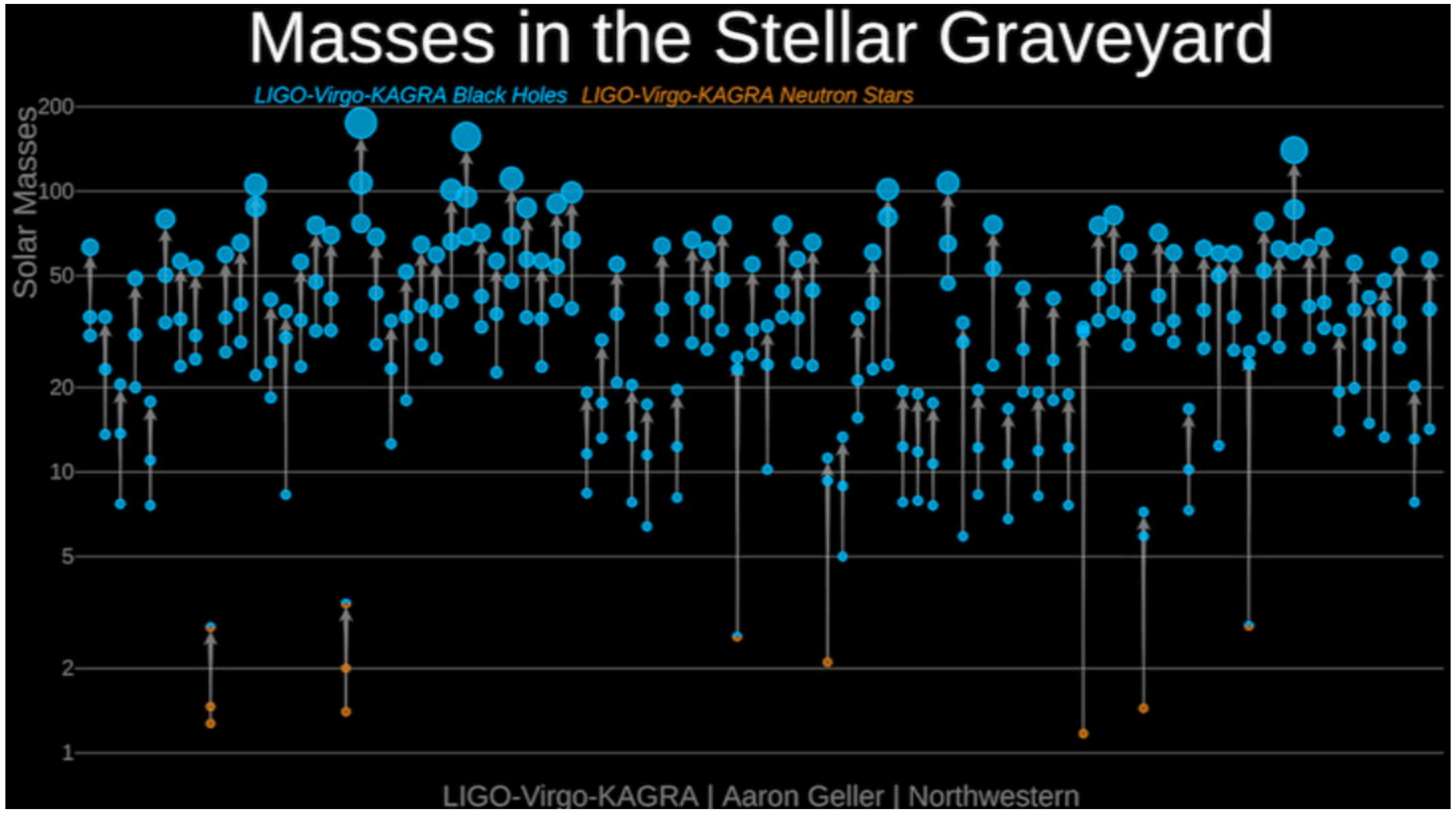
Late Universe

6) Astro Background and Observations

Late Universe $(0 \leq z \lesssim 10)$



LIGO/VIRGO
2015-now



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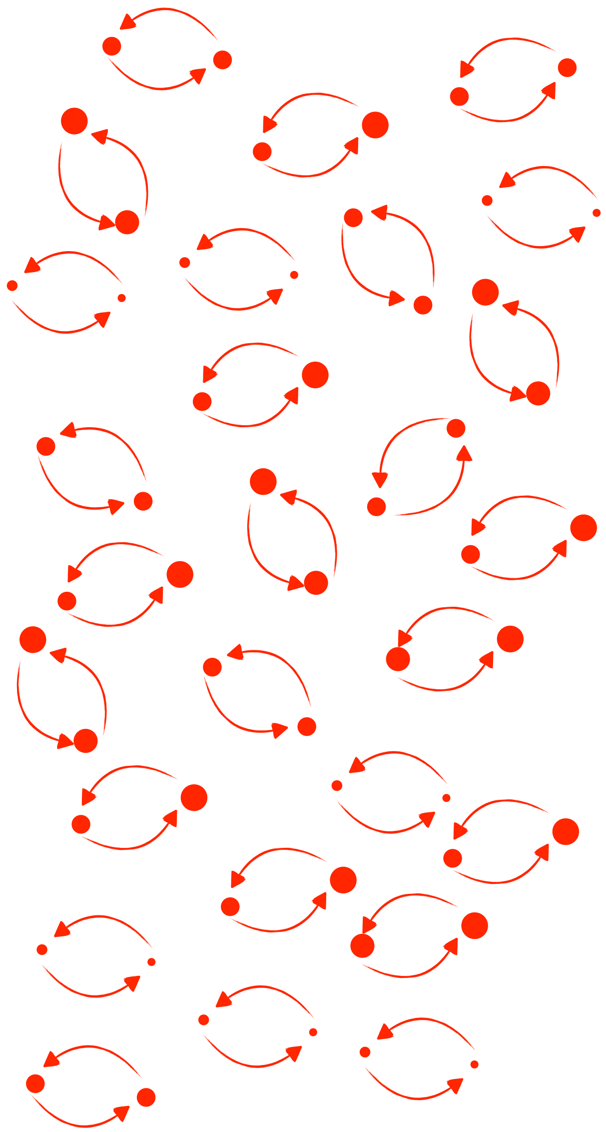

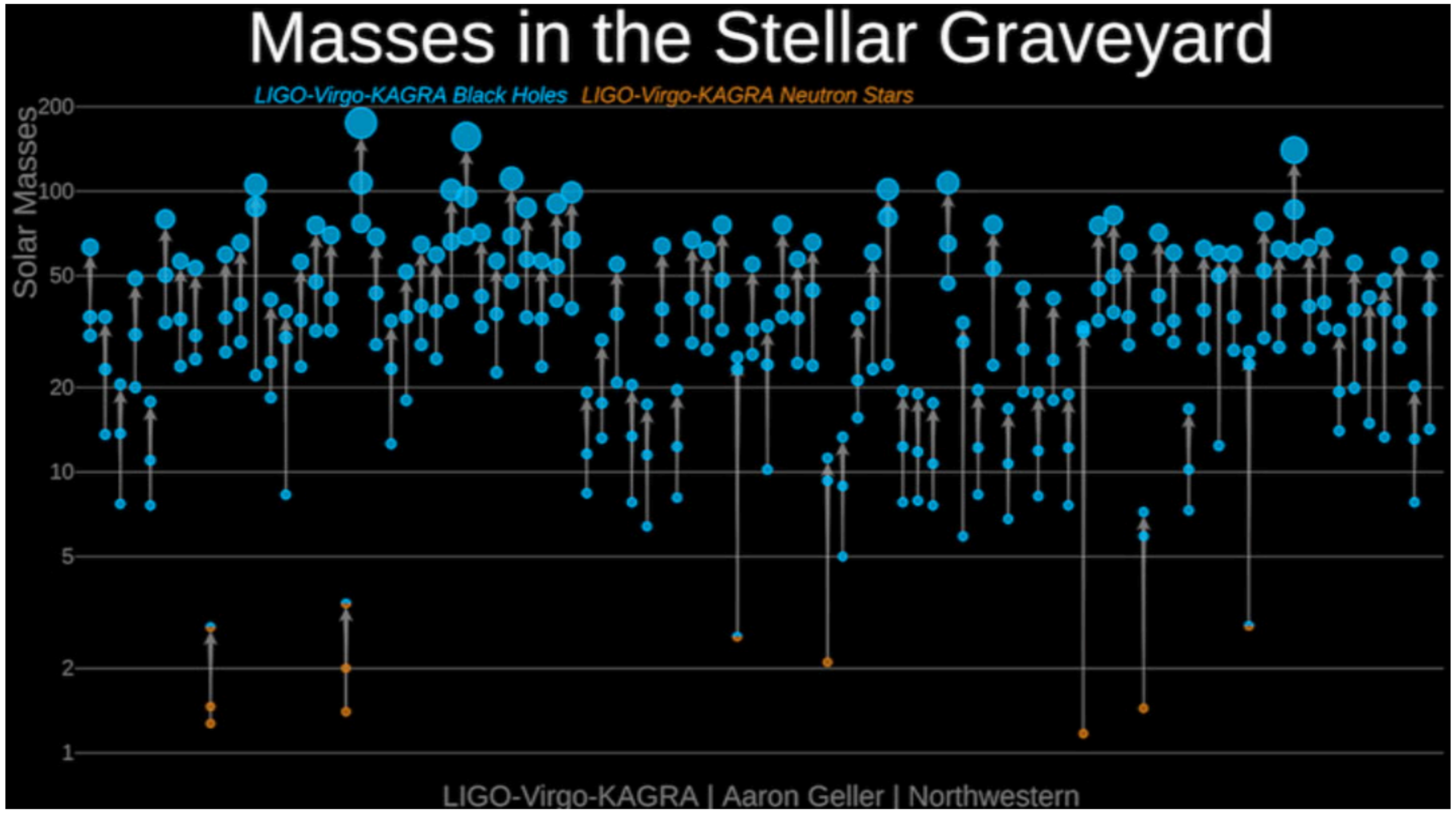

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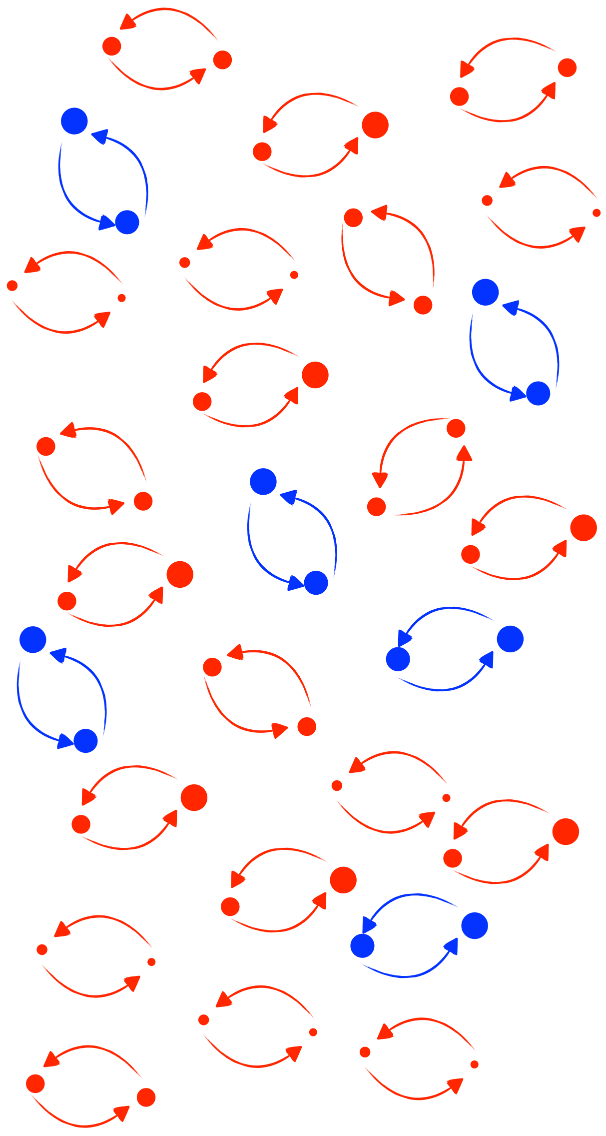
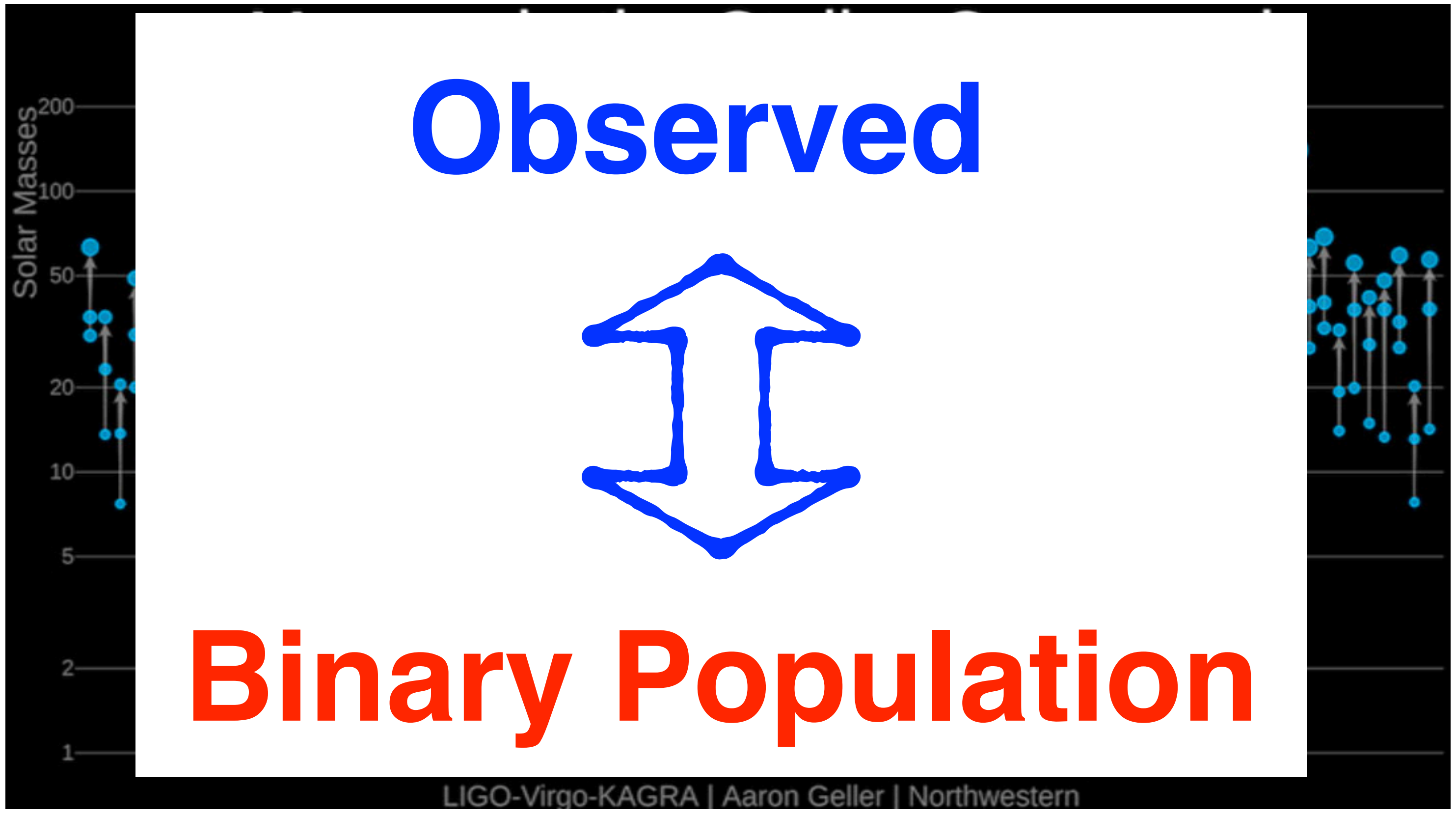




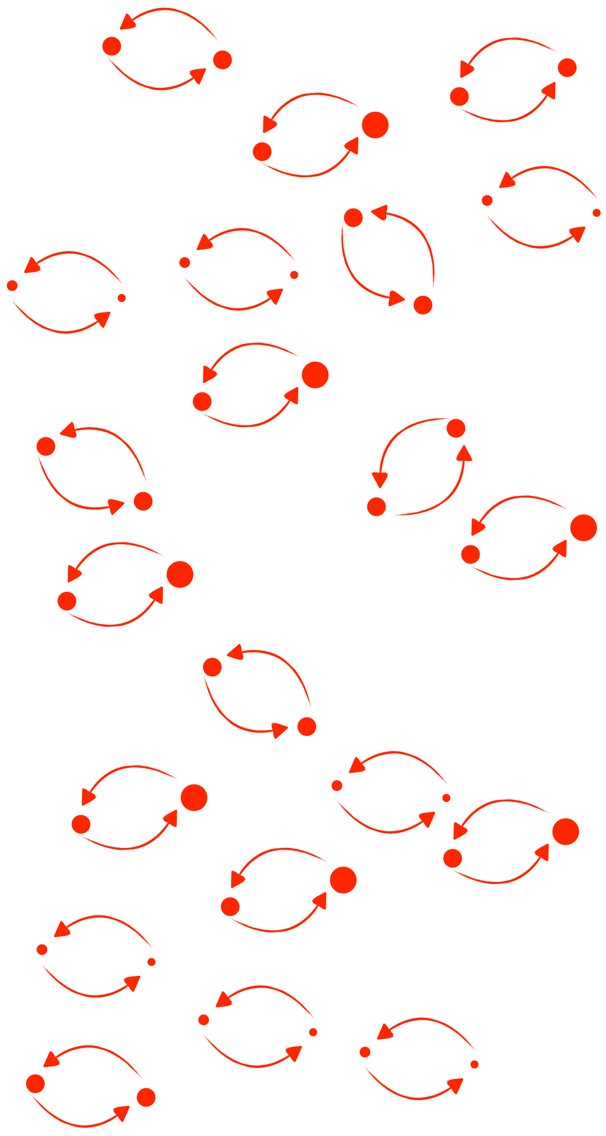
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Observed

⇕

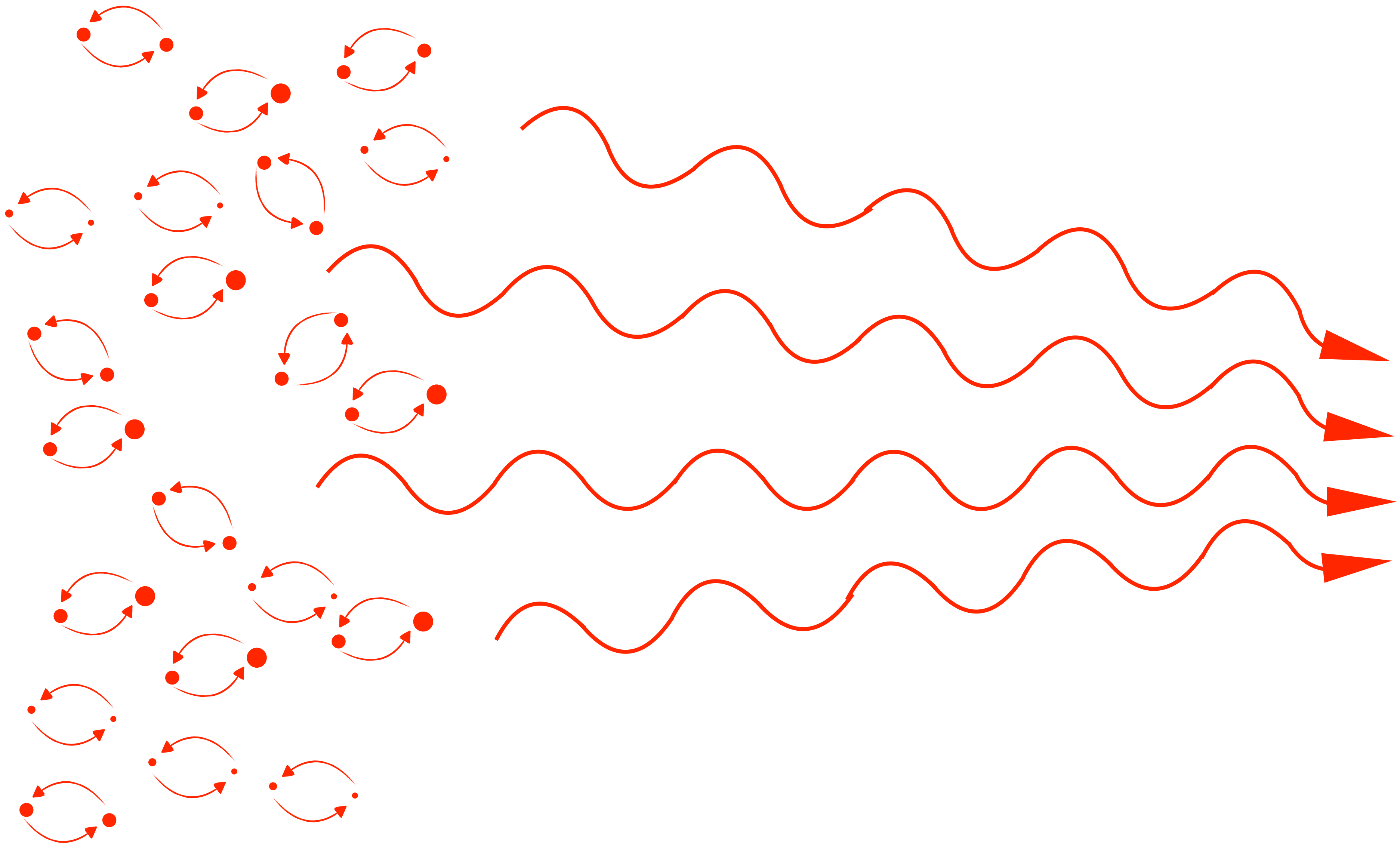
Binary Population

Late Universe $(0 \leq z \lesssim 10)$



Mostly unresolved binaries !

Late Universe $(0 \le z \lesssim 10)$



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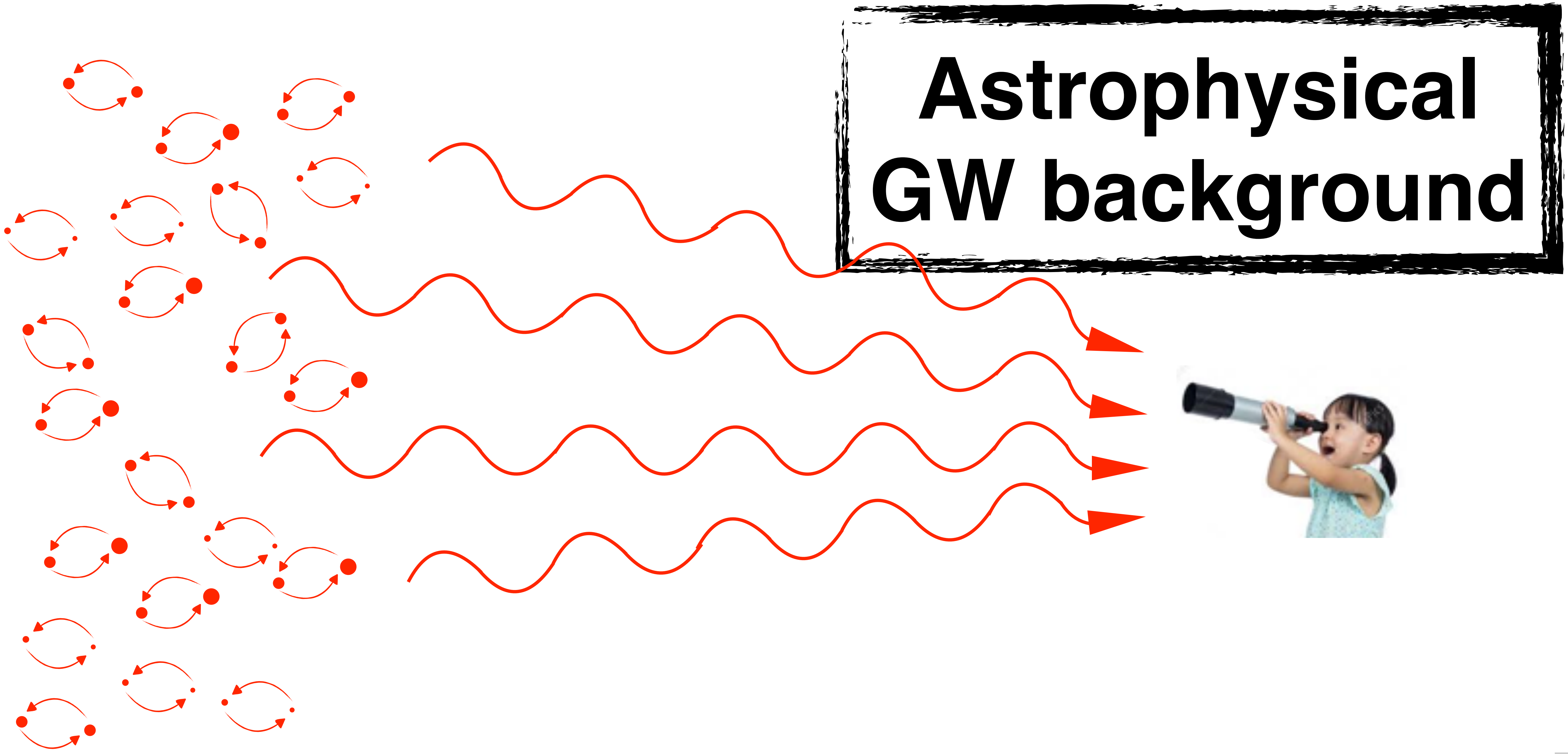
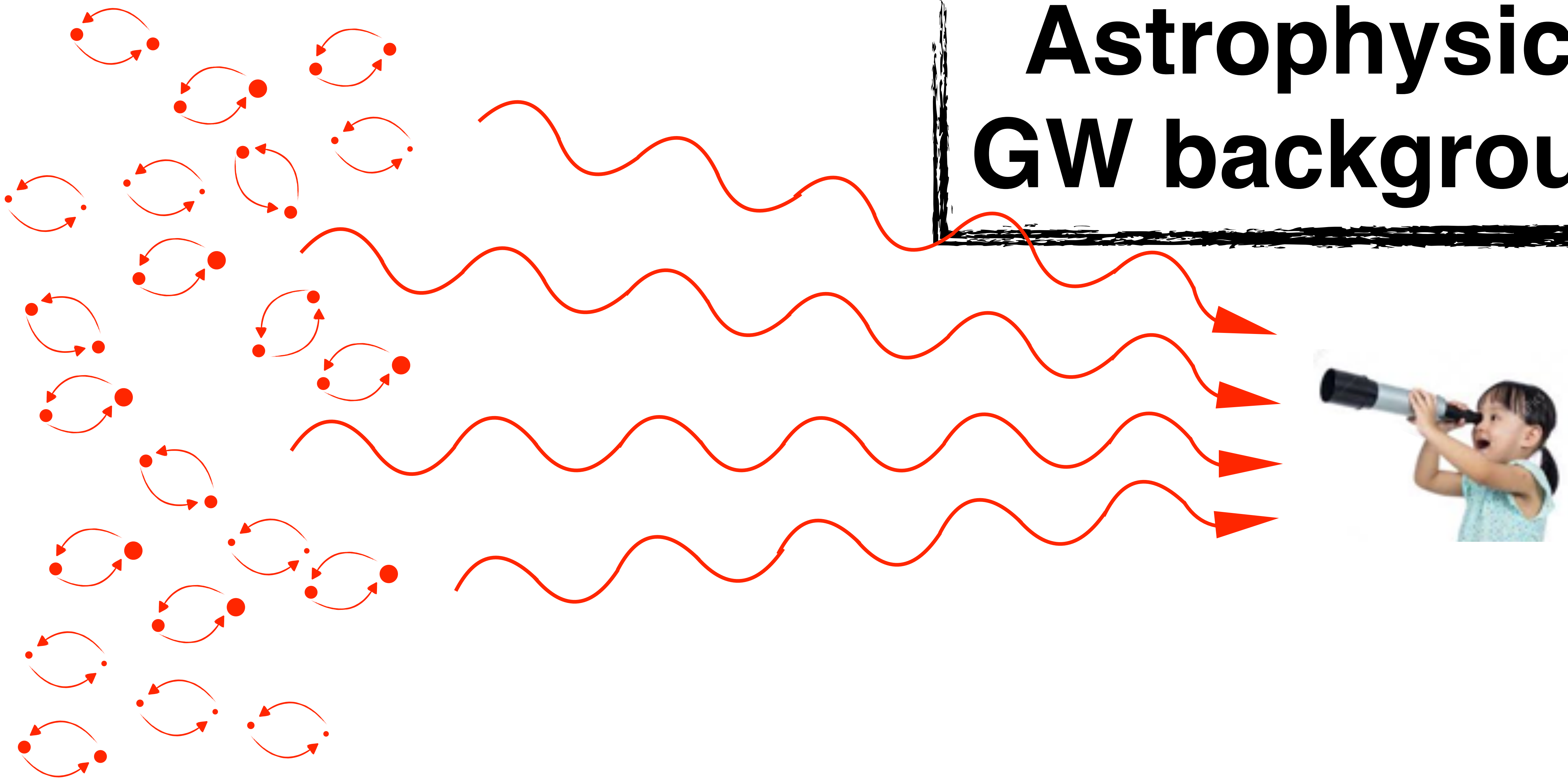


Astrophysical GW background

Late Universe

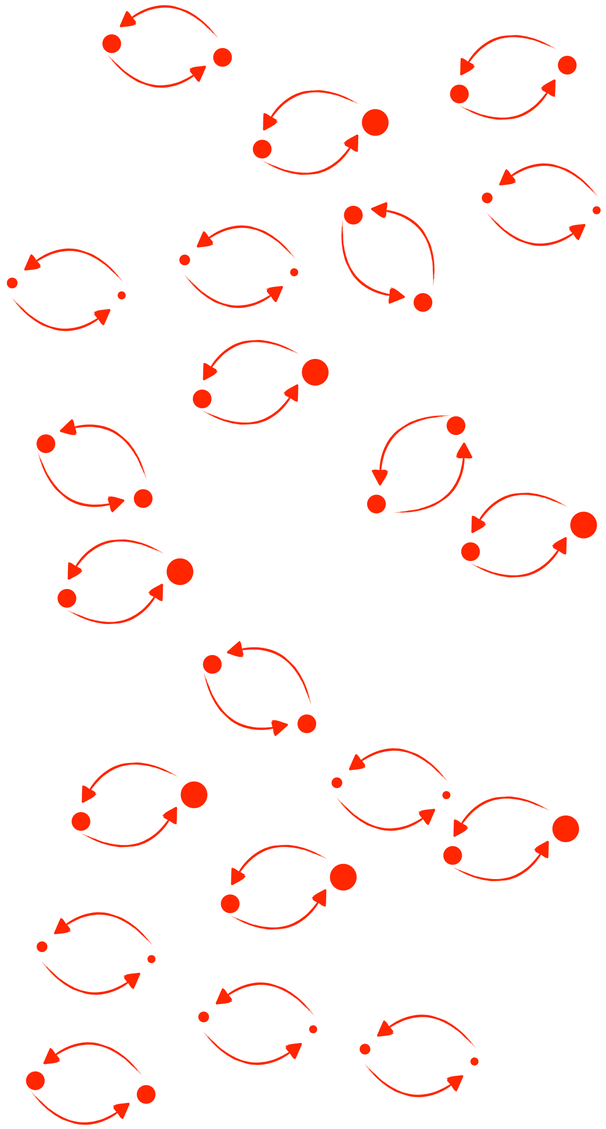
$$(0 \leq z \lesssim 10)$$



Black Holes
Neutron Stars
White Dwarfs

Astrophysical GW background

Late Universe $(0 \leq z \lesssim 10)$



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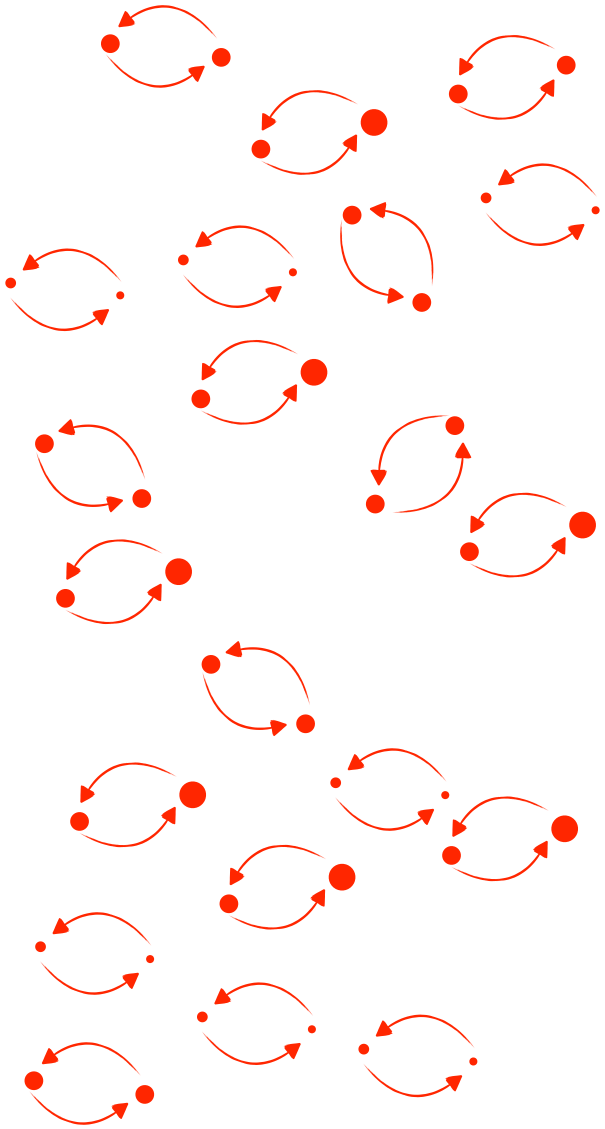

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Characteristic strain

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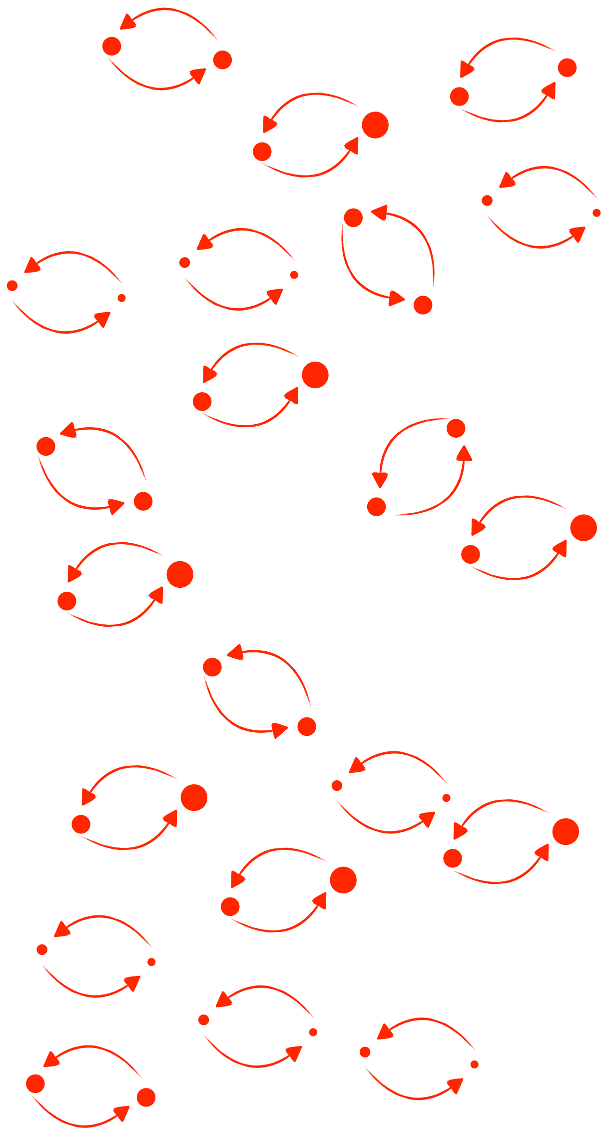

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For binary population: $\frac{dn}{dz}$ $\left(\textbf{comoving number density} \right)$

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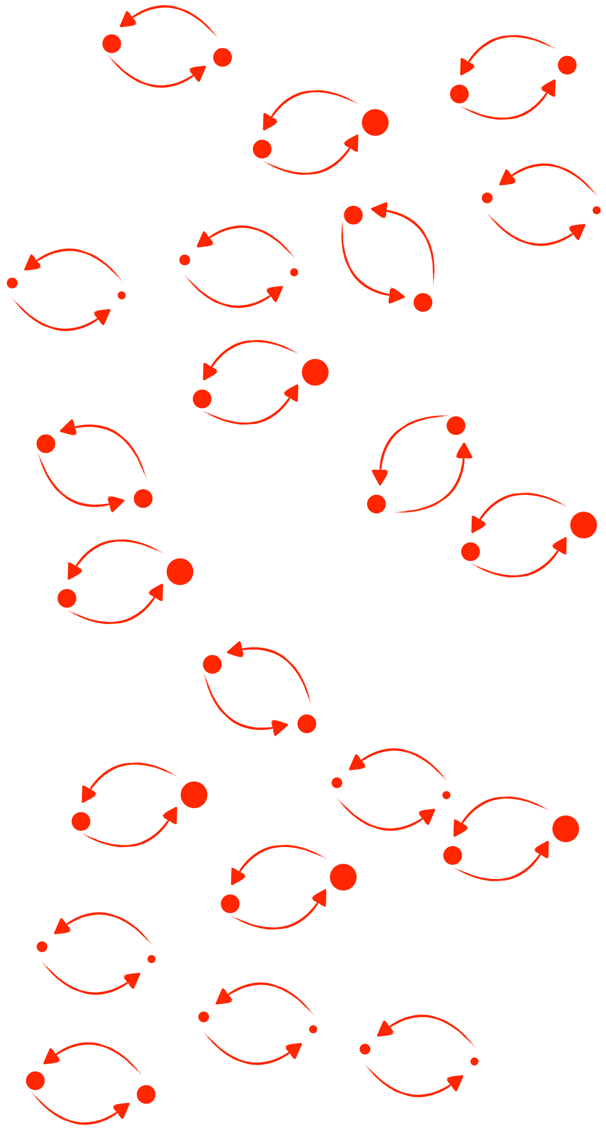

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$$\boxed{h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2}\int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r=f(1+z)}}$$

Late Universe $(0 \leq z \lesssim 10)$

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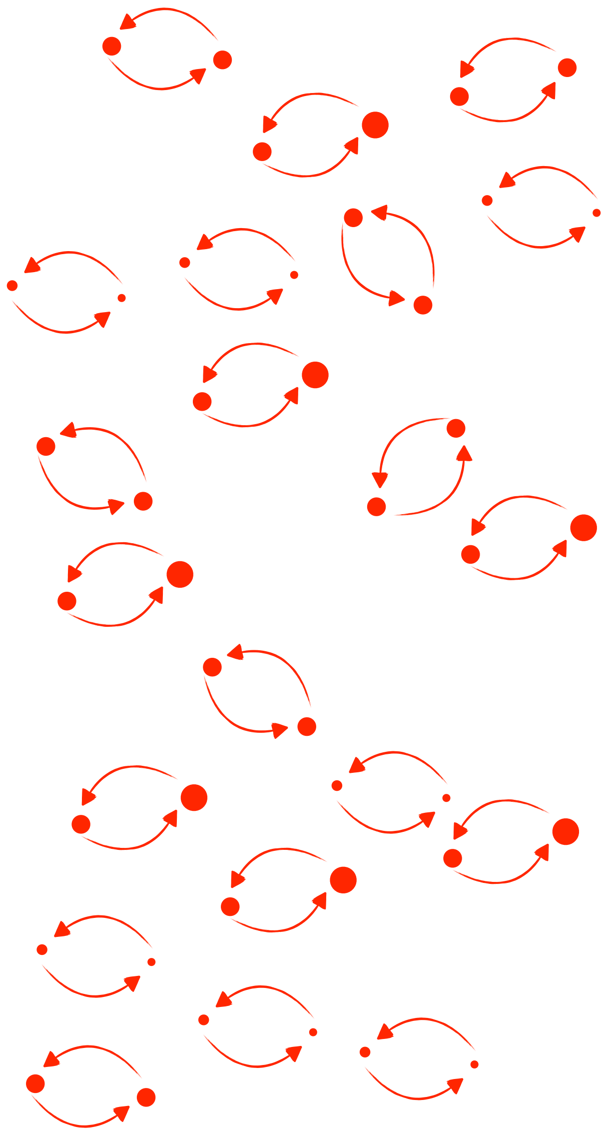

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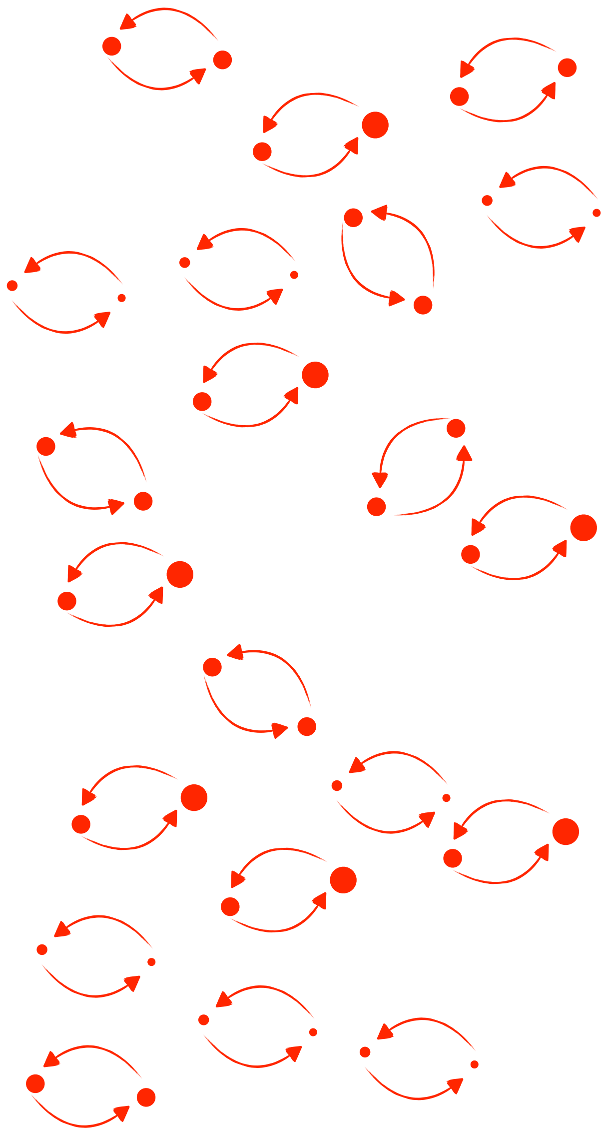
E.S. Phinney
astro-ph/0108028

$\downarrow$

source-frame energy spectrum

Late Universe $(0 \leq z \lesssim 10)$



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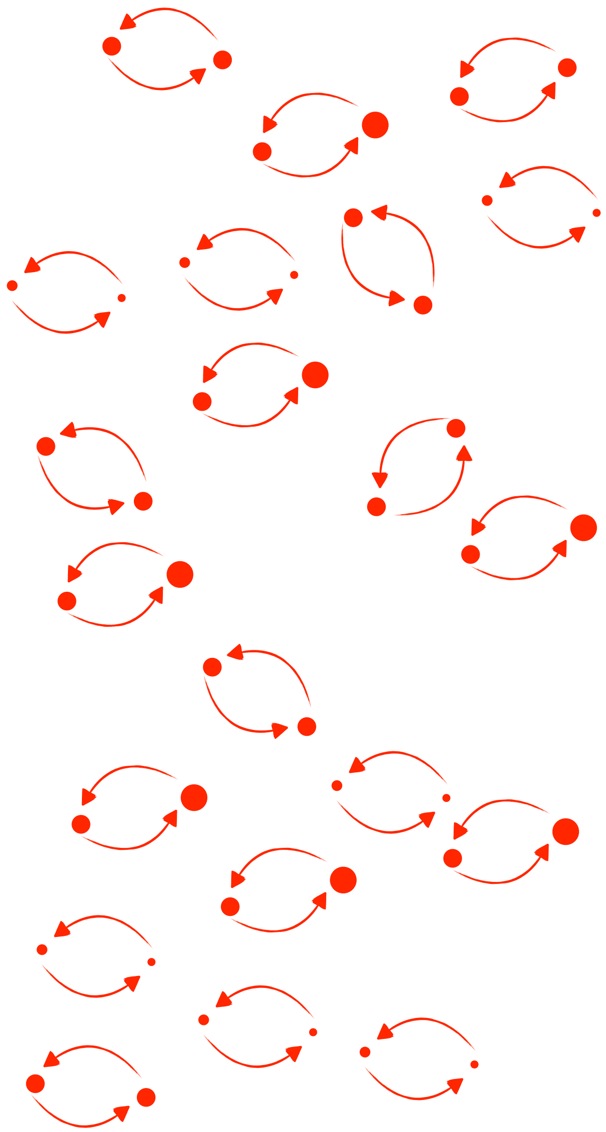
↑

Source frame frequency

Late Universe $(0 \leq z \lesssim 10)$



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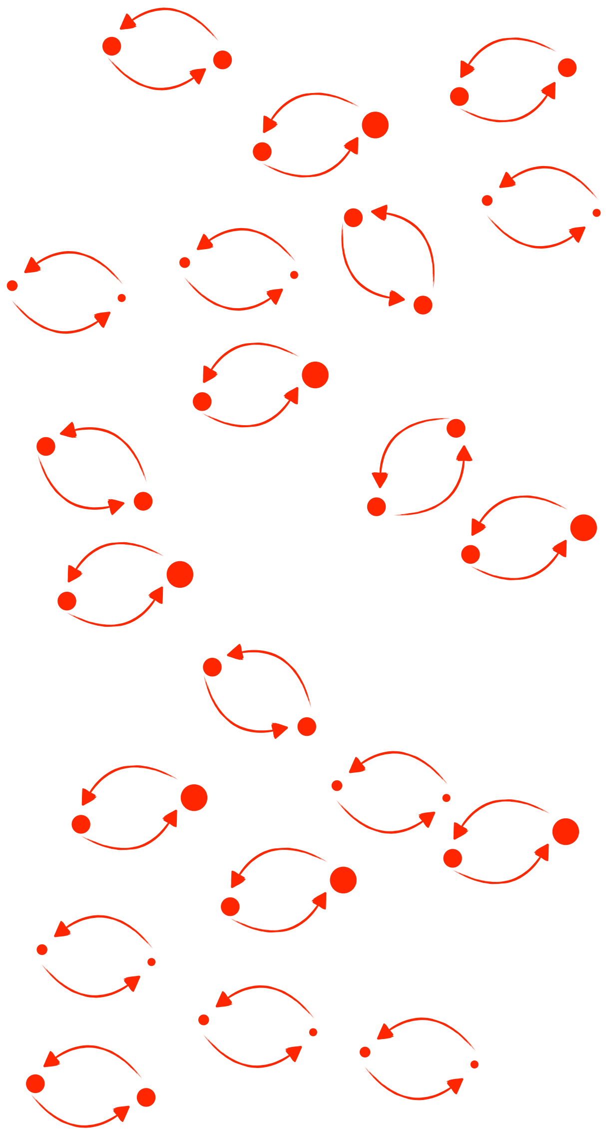
$$\frac{dE_{\rm GW}}{df_r} = \frac{\pi}{3}\frac{1}{G}\frac{(G\mathcal{M})^{5/3}}{\pi^{1/3}f_r^{1/3}}\Bigg|_{f_r=f(1+z)}$$

Chirp mass $\mathcal{M} = (m_1+m_2)^{2/5}\left(\frac{m_1 m_2}{m_1+m_2}\right)^{3/5}$

Late Universe $(0 \leq z \lesssim 10)$



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Using:

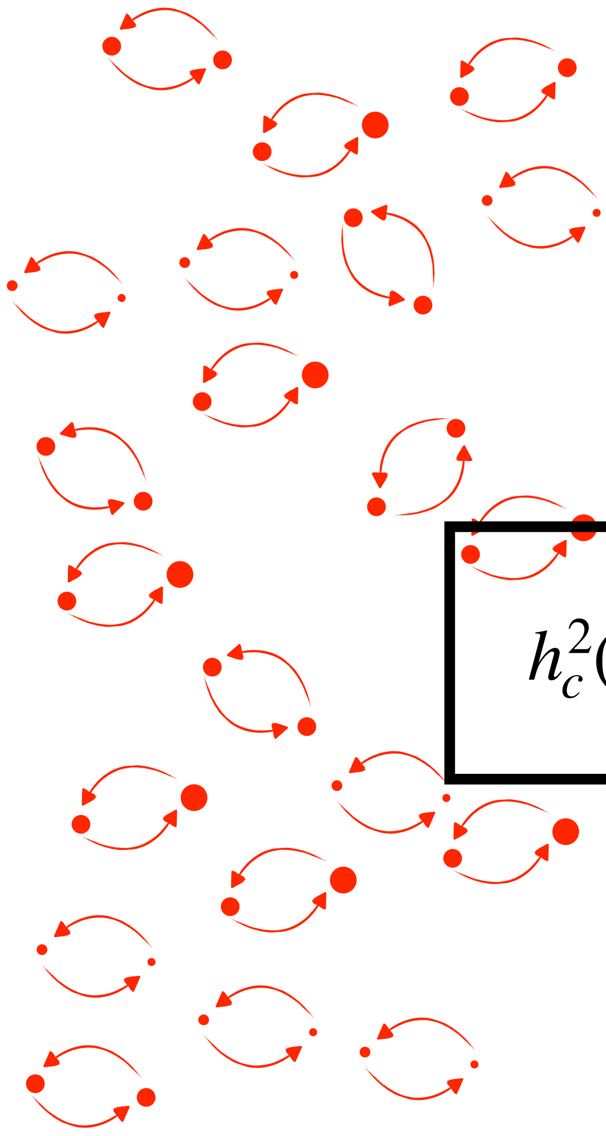
$$\frac{dn}{dz} \equiv \frac{d\mathcal{N}}{dz dm_1 dm_1 dV} \equiv R(z)\; p(m_1, m_2)\; \frac{dt_r}{dz}$$

Merging Rate

Mass function (distribution)

$$\frac{1}{(1+z)H(z)}$$

Late Universe $(0 \leq z \lesssim 10)$



$$\Omega_{\rm GW}(f)=\frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f}=\frac{2\pi^2}{3H_0^2}f^2h_c^2(f)\,,$$

$$h_c^2(f)=\frac{4}{\pi}\frac{G}{c^2}f^{-2}\int_0^\infty dz\frac{dn}{dz}\frac{1}{1+z}f_r\frac{dE_{\rm GW}}{df_r}\Big|_{f_r=f(1+z)}$$

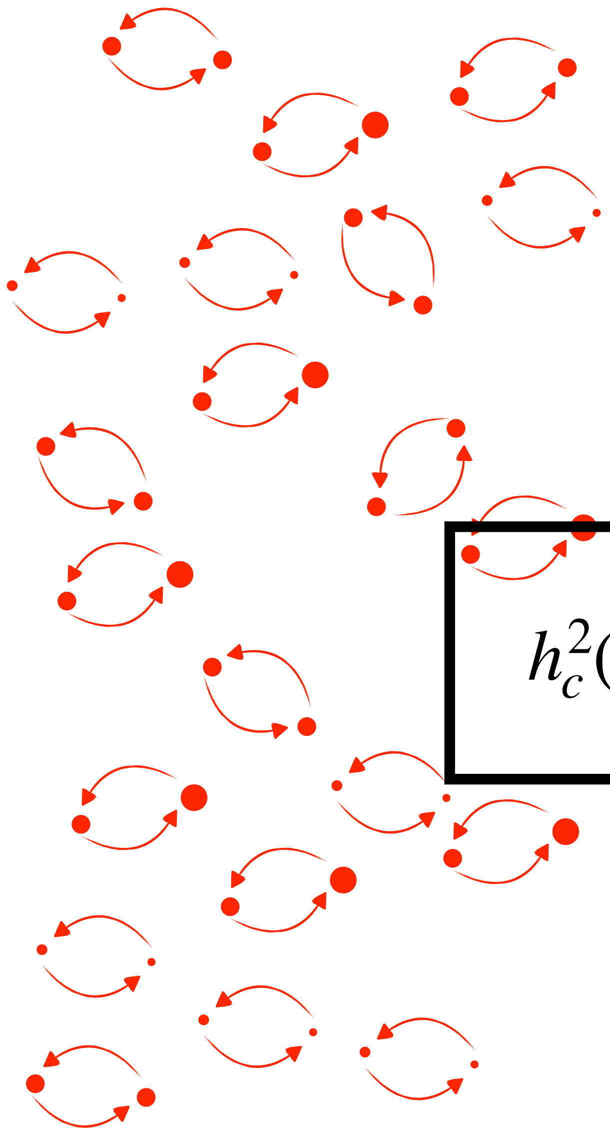
$$\boxed{h_c^2(f)=\frac{4G^{5/3}f^{-2/3}}{3\pi^{1/2}c^2}\int dm_1dm_2p(m_1,m_2)\int_0^{z_{\rm max}}dz'R(z')\frac{\mathcal{M}^{5/3}(m_1,m_2)}{(1+z)^{1/3}}\frac{dt_r}{dz}}$$

Late Universe $(0 \leq z \lesssim 10)$

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)\,,$$

$$h_c^2(f) = \frac{4}{\pi}\frac{G}{c^2} f^{-2} \int_0^\infty dz \frac{dn}{dz}\frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\bigg|_{f_r = f(1+z)}$$

$$\boxed{h_c^2(f) = \frac{4G^{5/3} f^{-2/3}}{3\pi^{1/2}c^2}\int dm_1 dm_2 p(m_1, m_2)\int_0^{z_{\rm max}} dz' R(z') \frac{\mathscr{M}^{5/3}(m_1, m_2)}{(1+z)^{1/3}}\frac{dt_r}{dz}}$$

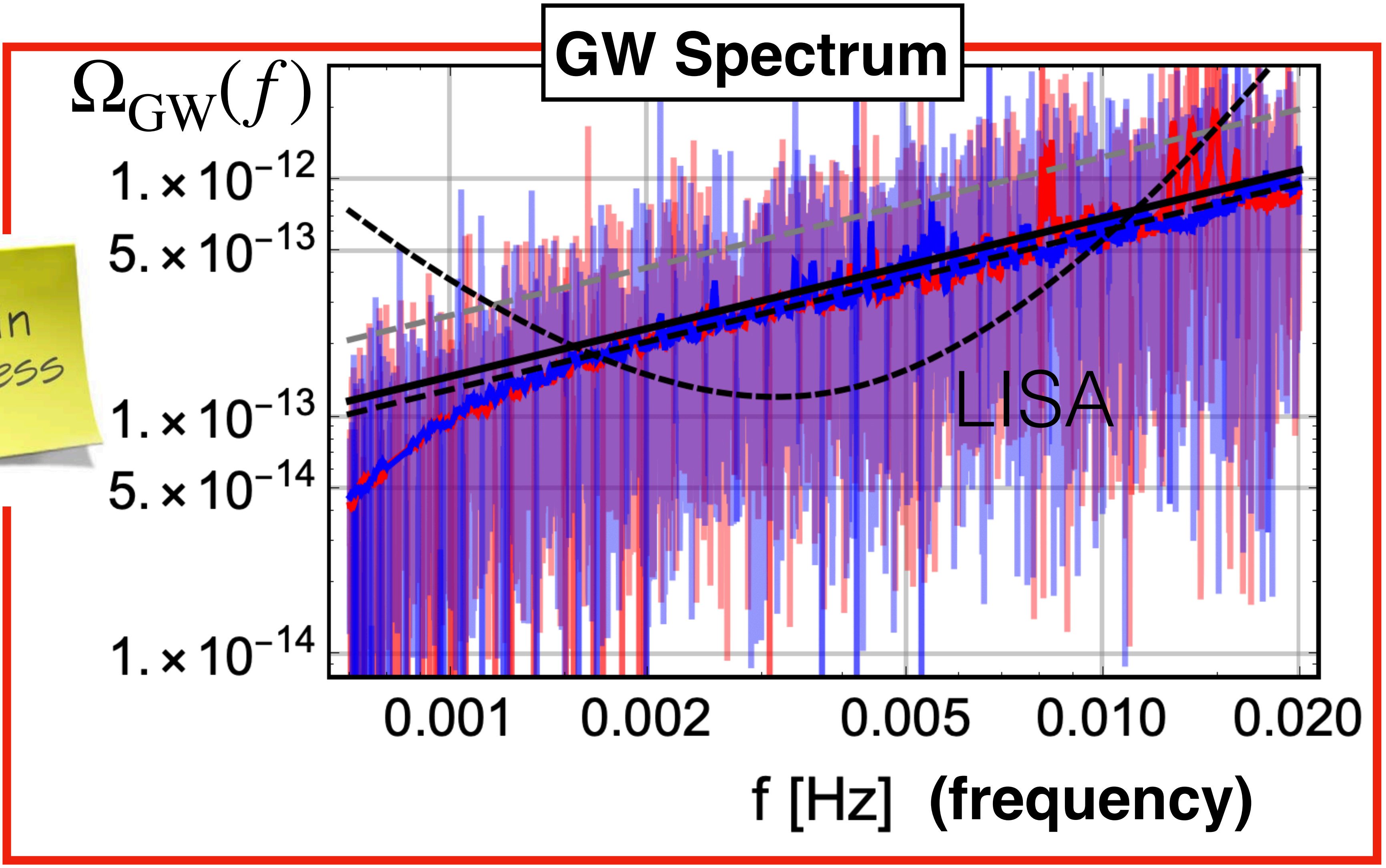


e.g. BH Binaries
(from LIGO/VIRGO)

$$R_{\rm BHB}(z) = R_{0.2}\frac{(1+z)^{2.7}}{1 + ((1+z)/2.9)^{5.6}}$$

Example: Stellar mass Black Hole population
(for LISA collaboration)

Work in Progress



[Babak, Caprini,
DGF, Karnesis,
Nardini, Marcoccia,
Pieroni, Ricciardone,
Sesana, Torrado]

Summary & Perspective

Gravitational Wave Backgrounds

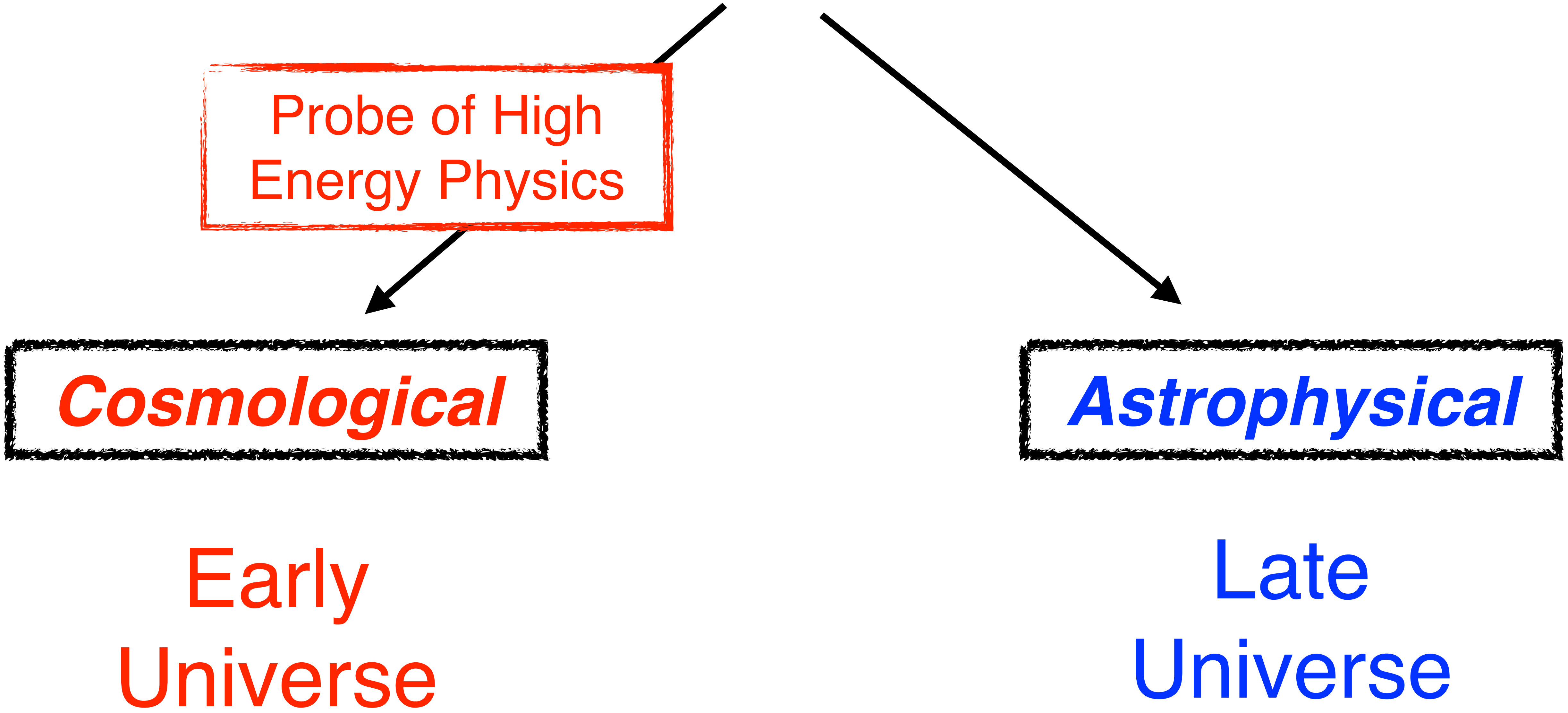
Cosmological

Early
Universe

Astrophysical

Late
Universe

Gravitational Wave Backgrounds



Probe of High Energy Physics

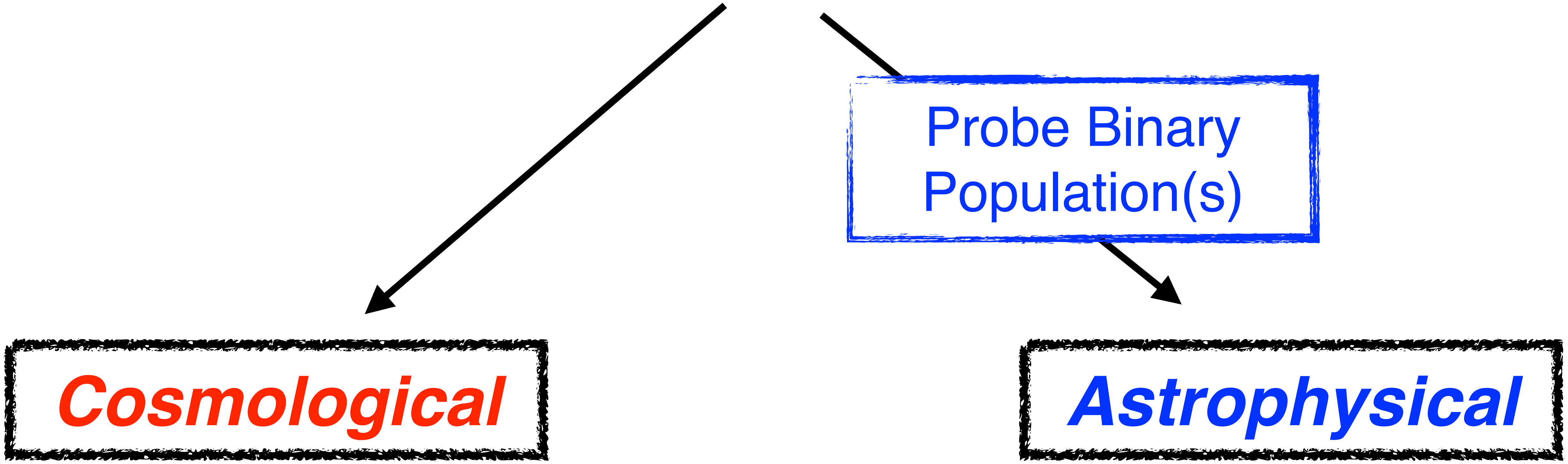
Cosmological

Early Universe

Astrophysical

Late Universe

Gravitational Wave Backgrounds



Probe Binary Population(s)

Cosmological

Astrophysical

Early Universe

Late Universe

Gravitational Wave Backgrounds

HOLY GRAIL

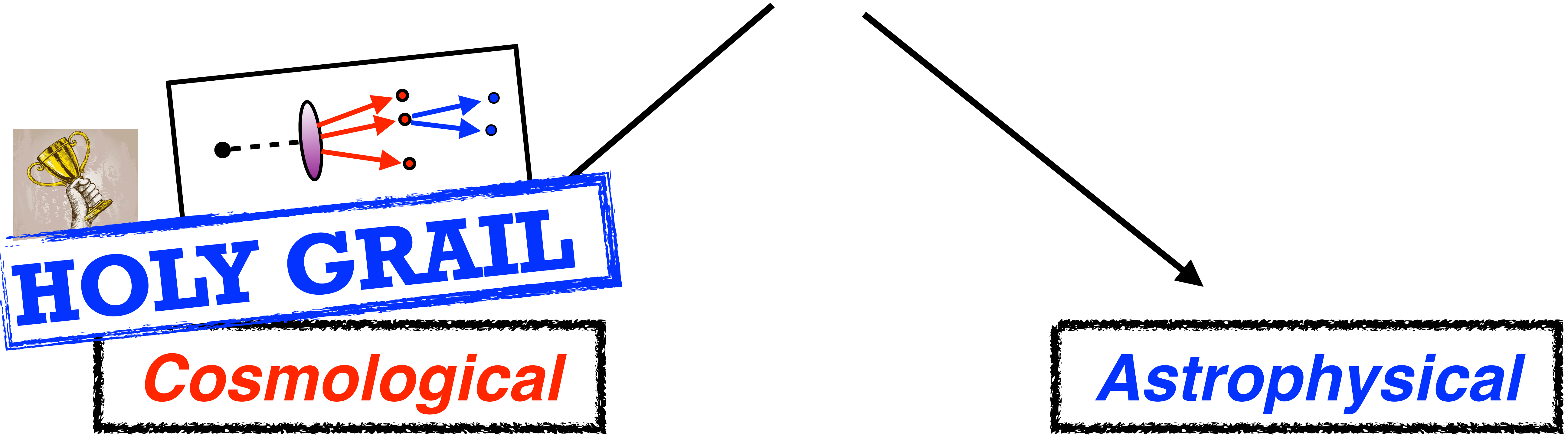
Cosmological

Early
Universe

Astrophysical

Late
Universe

Gravitational Wave Backgrounds



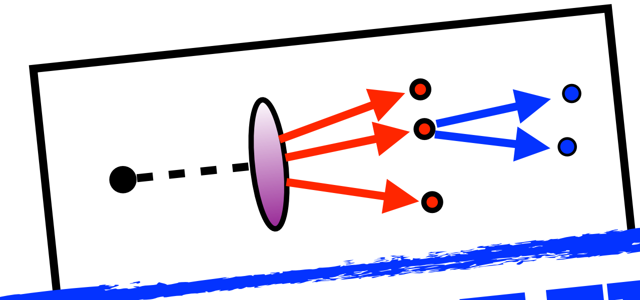
Cosmological

Astrophysical

Early
Universe

Late
Universe

Gravitational Wave Backgrounds



HOLY GRAIL

Cosmological

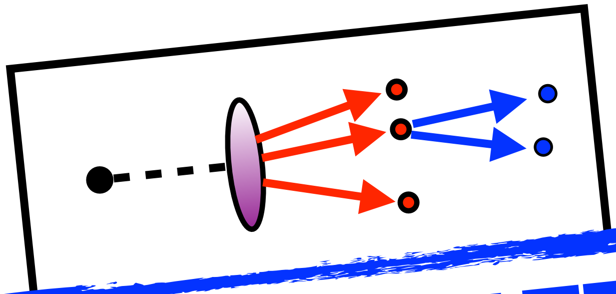
Early
Universe

GUARANTEED

Astrophysical

Late
Universe

Gravitational Wave Backgrounds



HOLY GRAIL

Cosmological

Early Universe

GUARANTEED

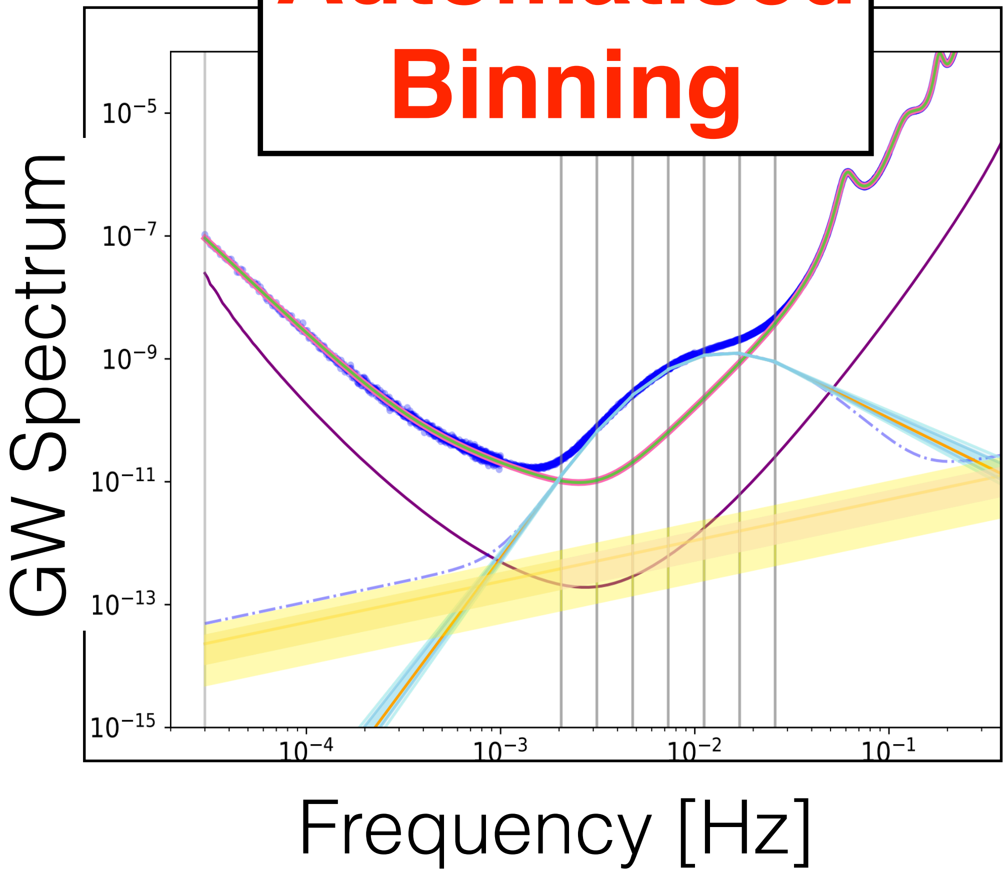
Astrophysical

Late Universe

Foreground

Signal Analysis

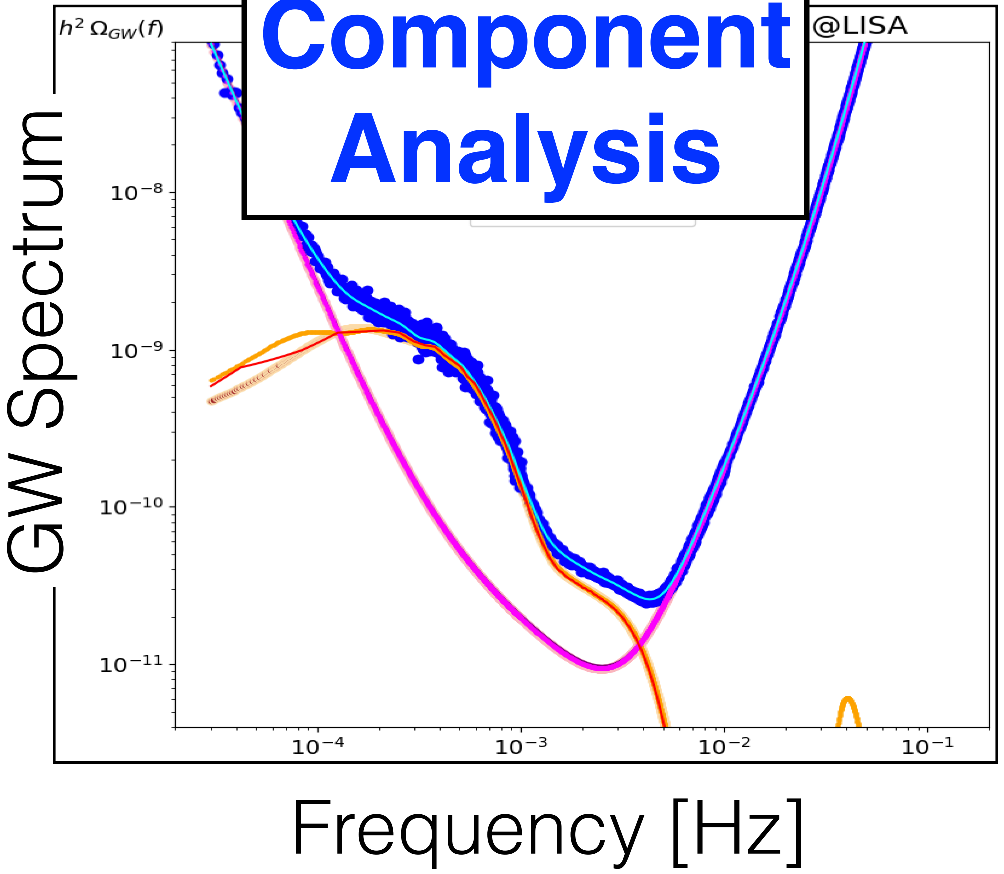
POWER SPECTRUM RECONSTRUCTION



Automatised Binning

Code `SGWBinner`

(Caprini et al [1906.09244](1906.09244))



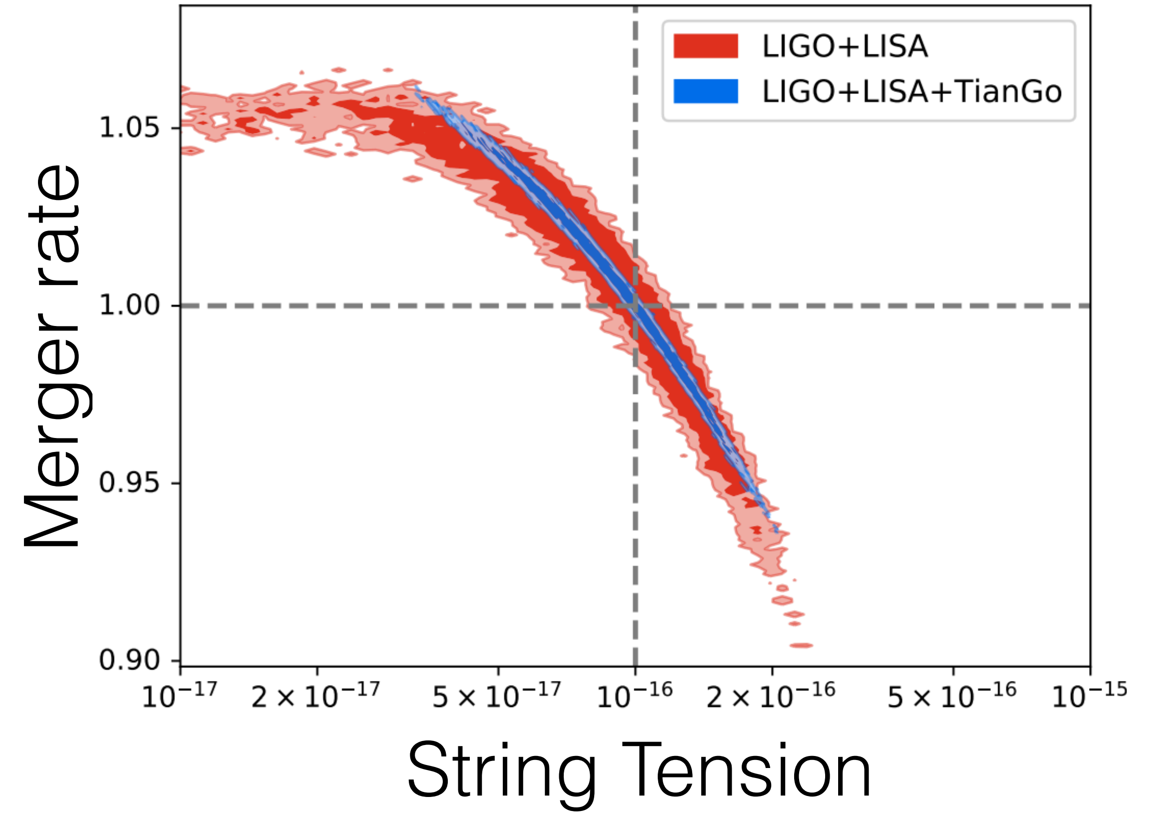
Principal Component Analysis

Pieroni & Barausse [2004.01135](2004.01135))

Signal Analysis

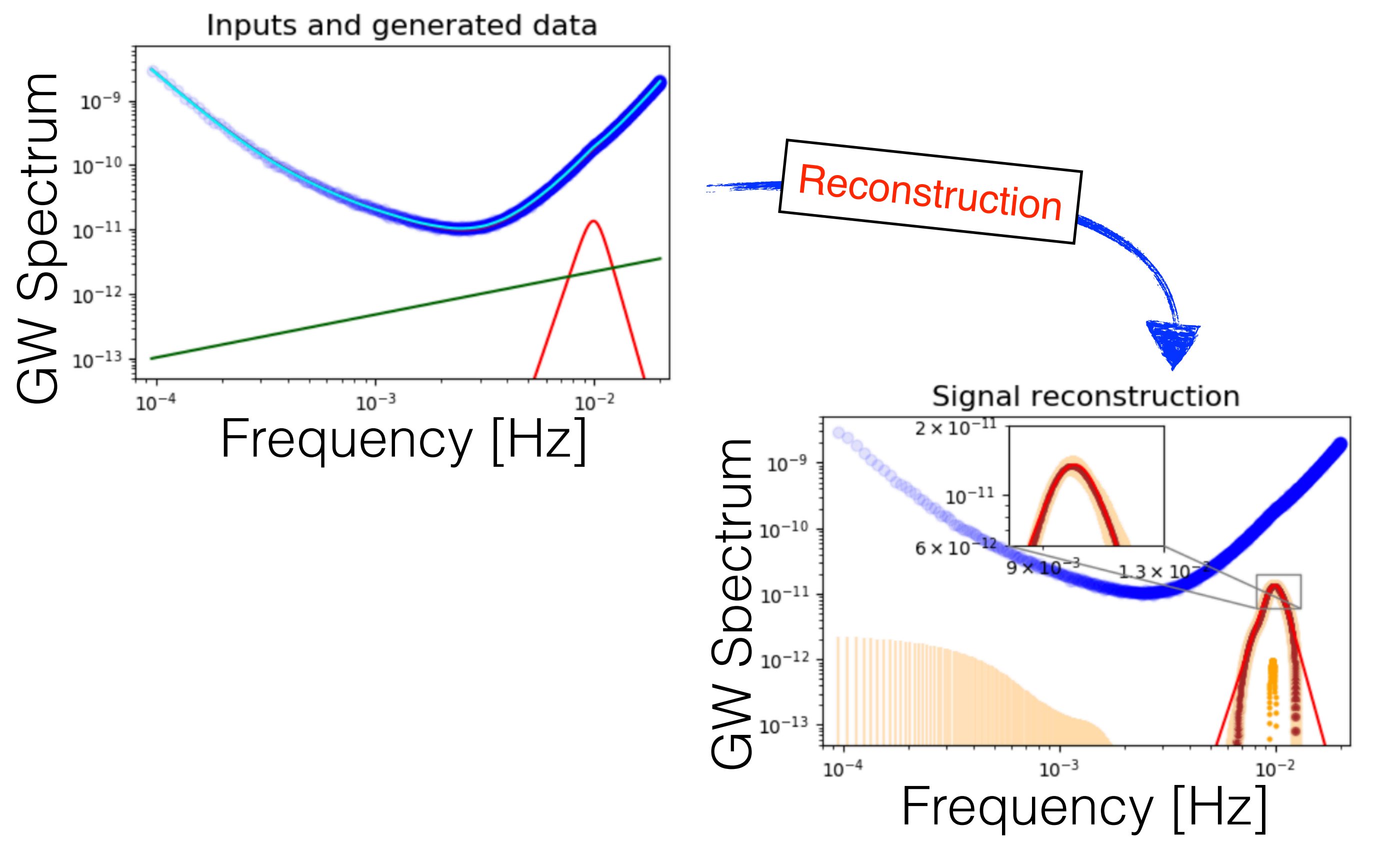
SIGNAL SEPARATION

Combining experiments



(from 2012.07874)

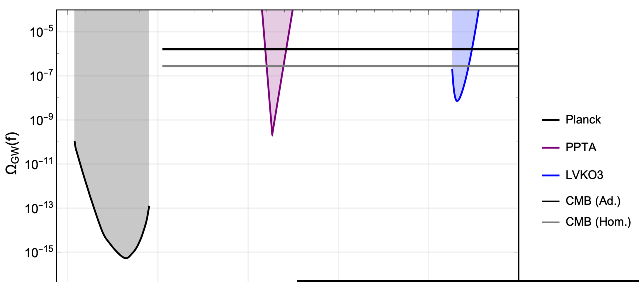
Reconstruction over foreground



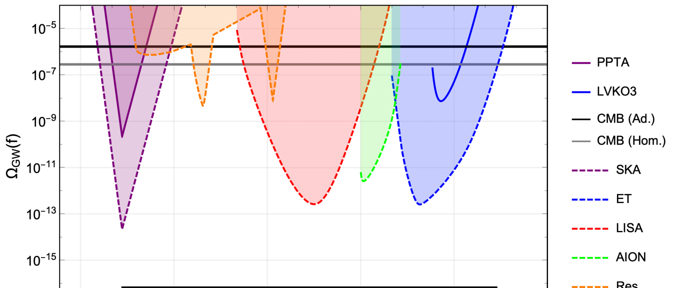
(from 2004.01135)

EXPERIMENTS

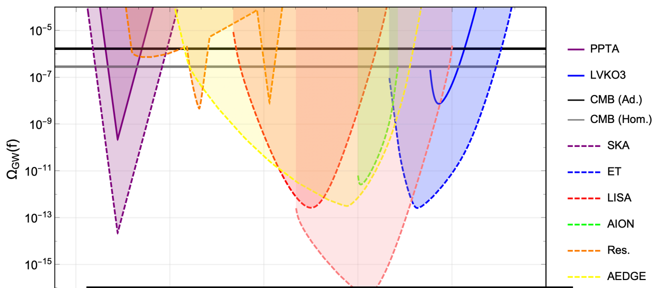
CURRENT



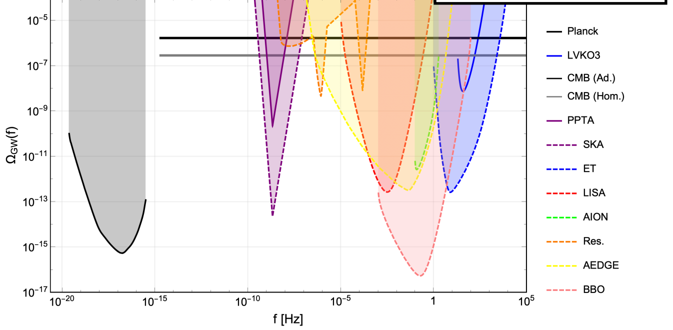
PLANNED



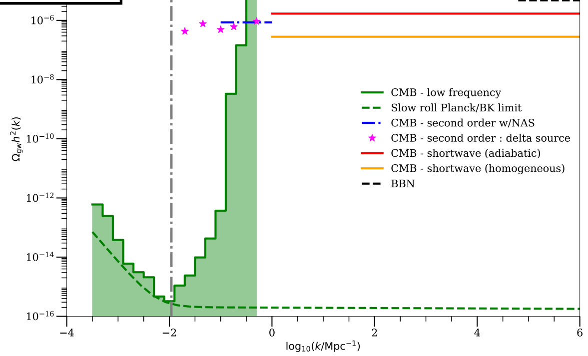
PLANNED II



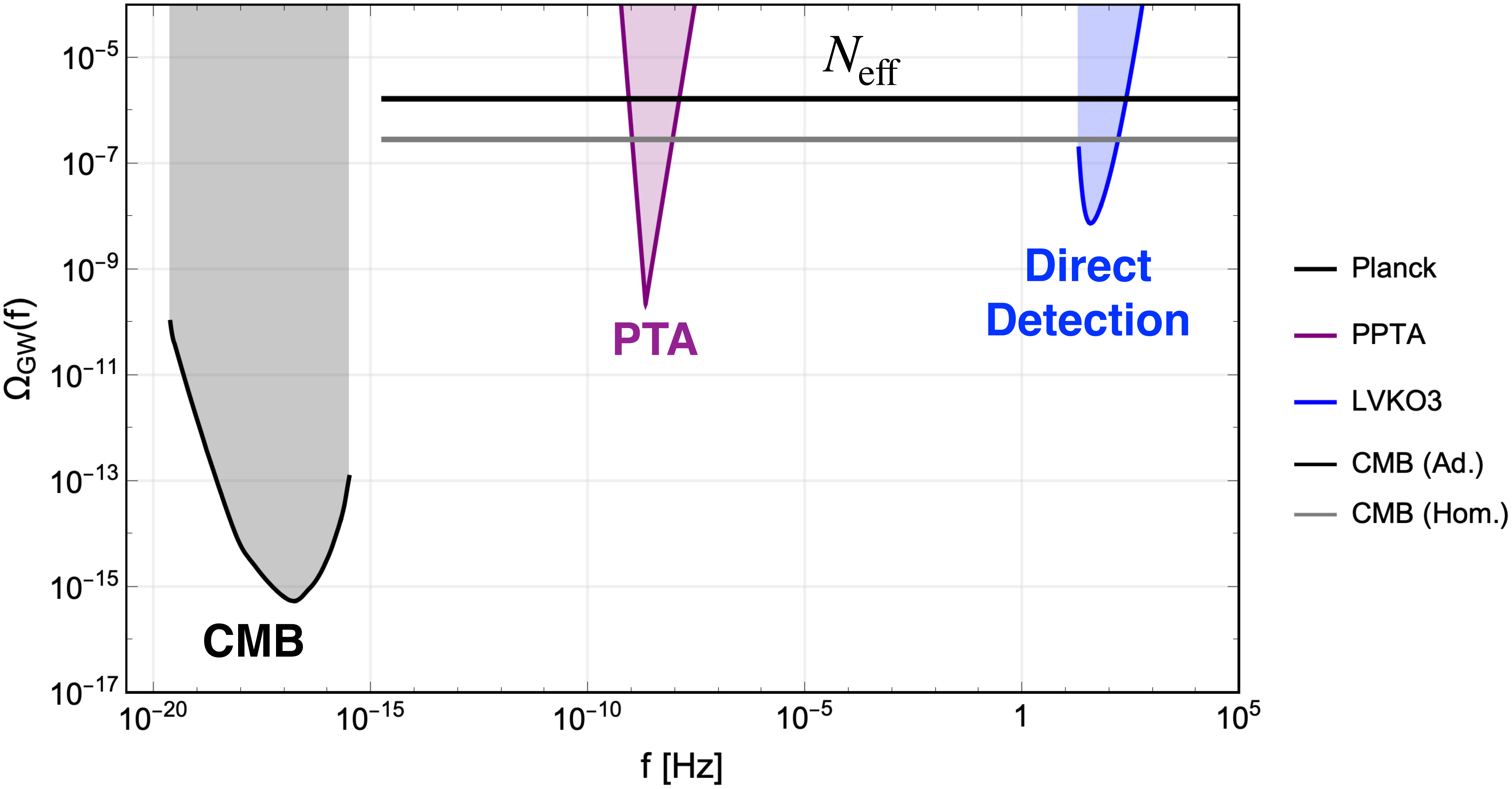
ALL



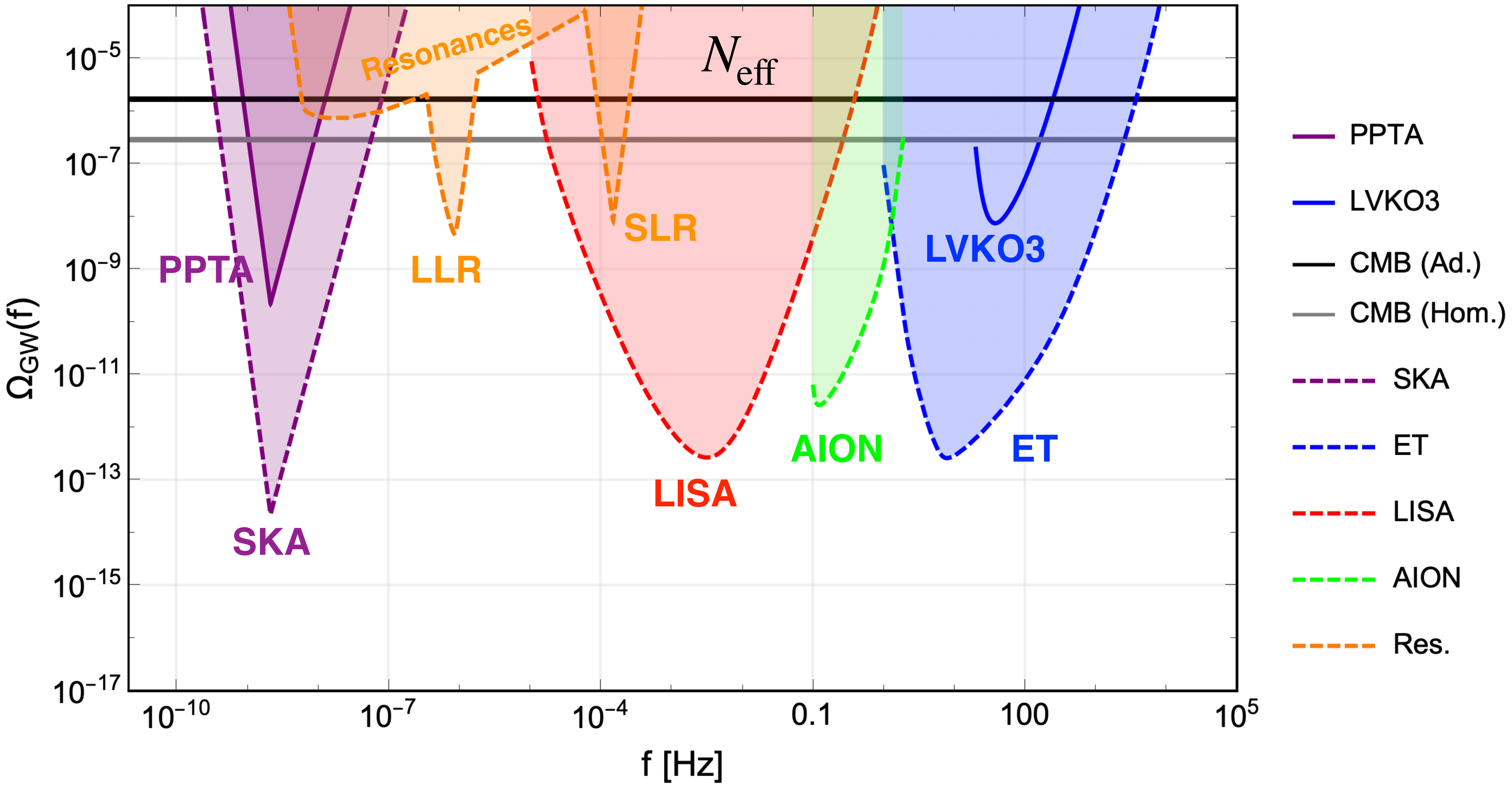
CMB



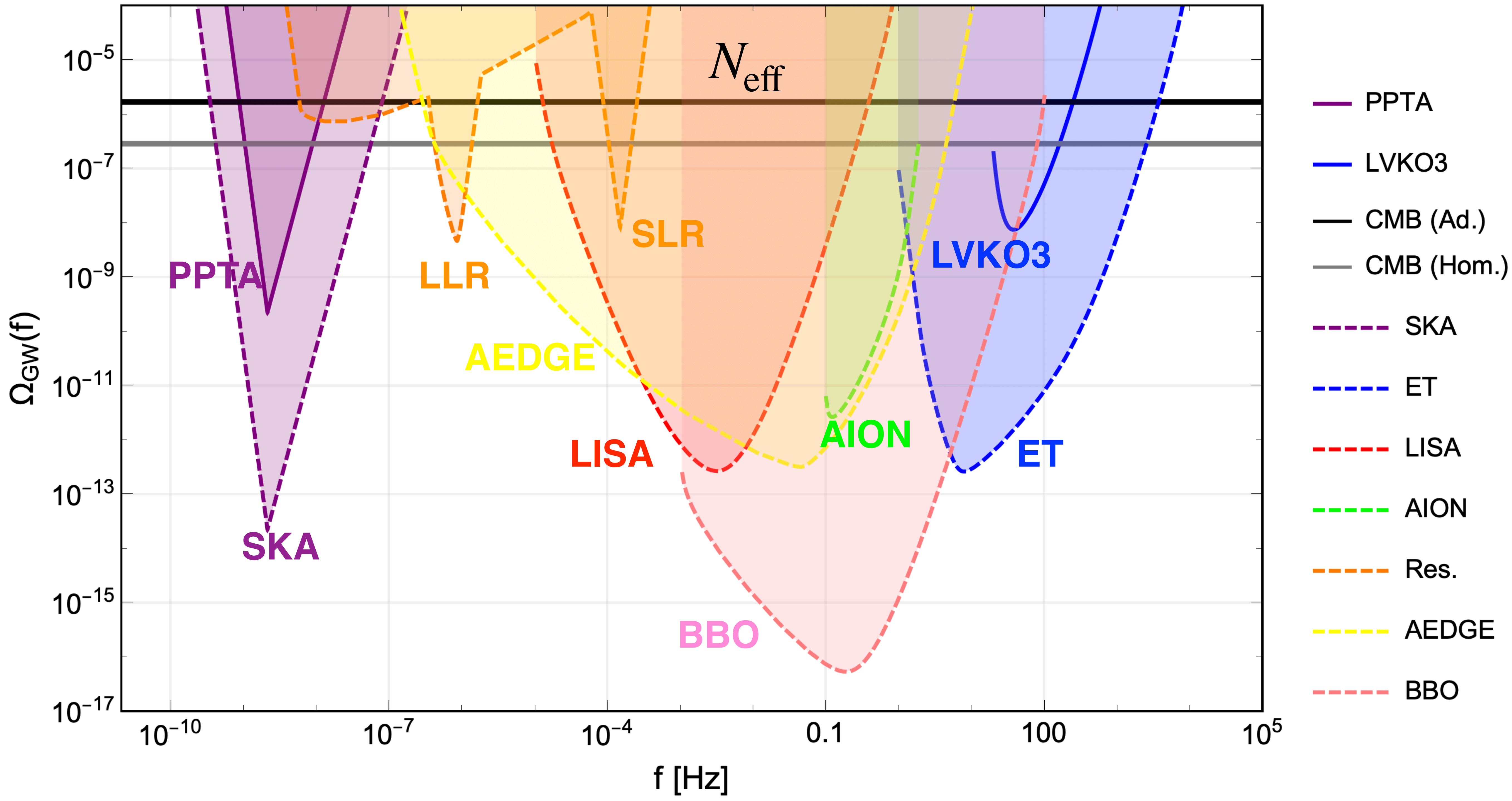
Current Constraints



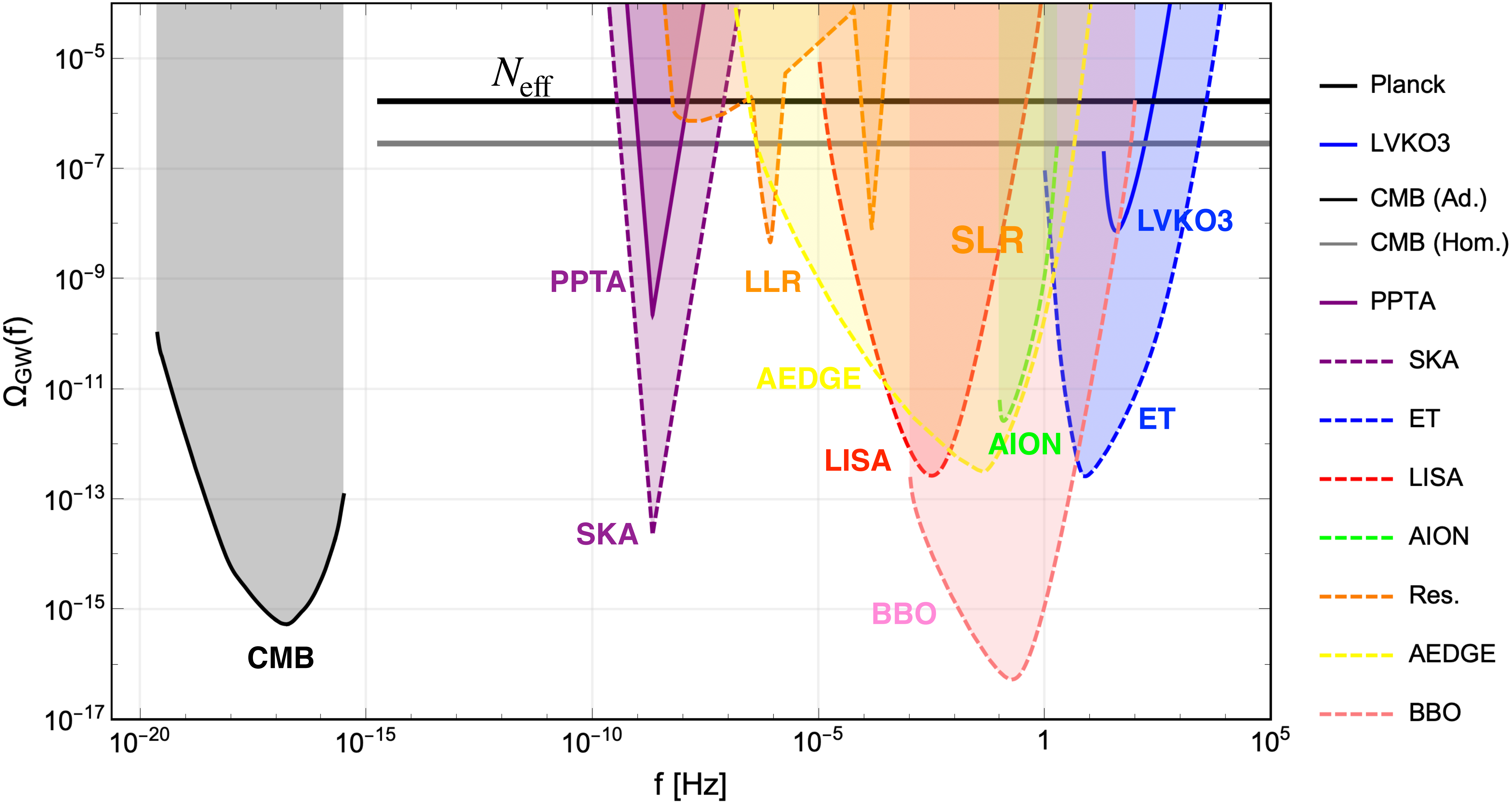
Current & Planned Direct Detection



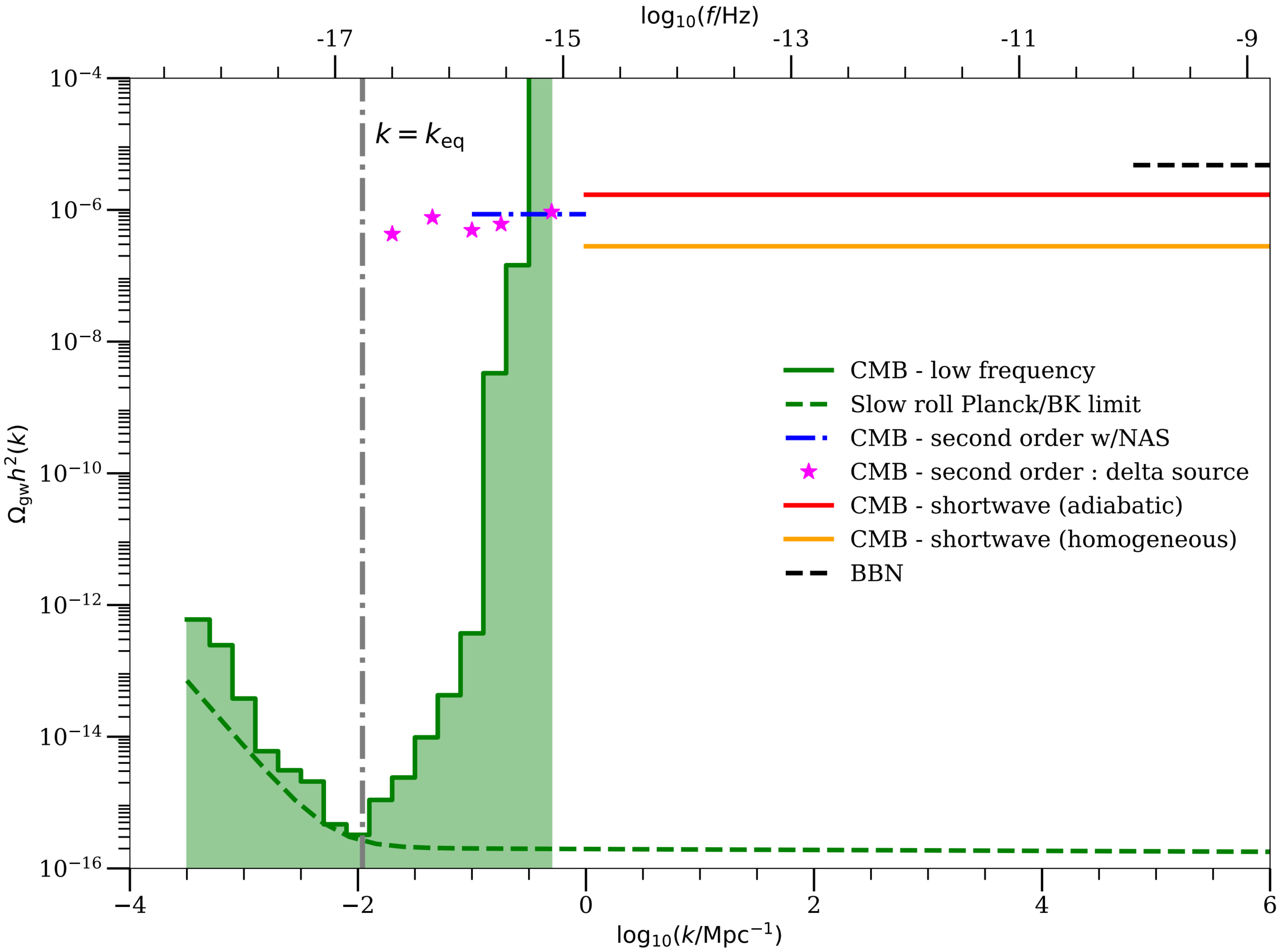
Current & Planned Direct Detection II



Full Experimental Landscape

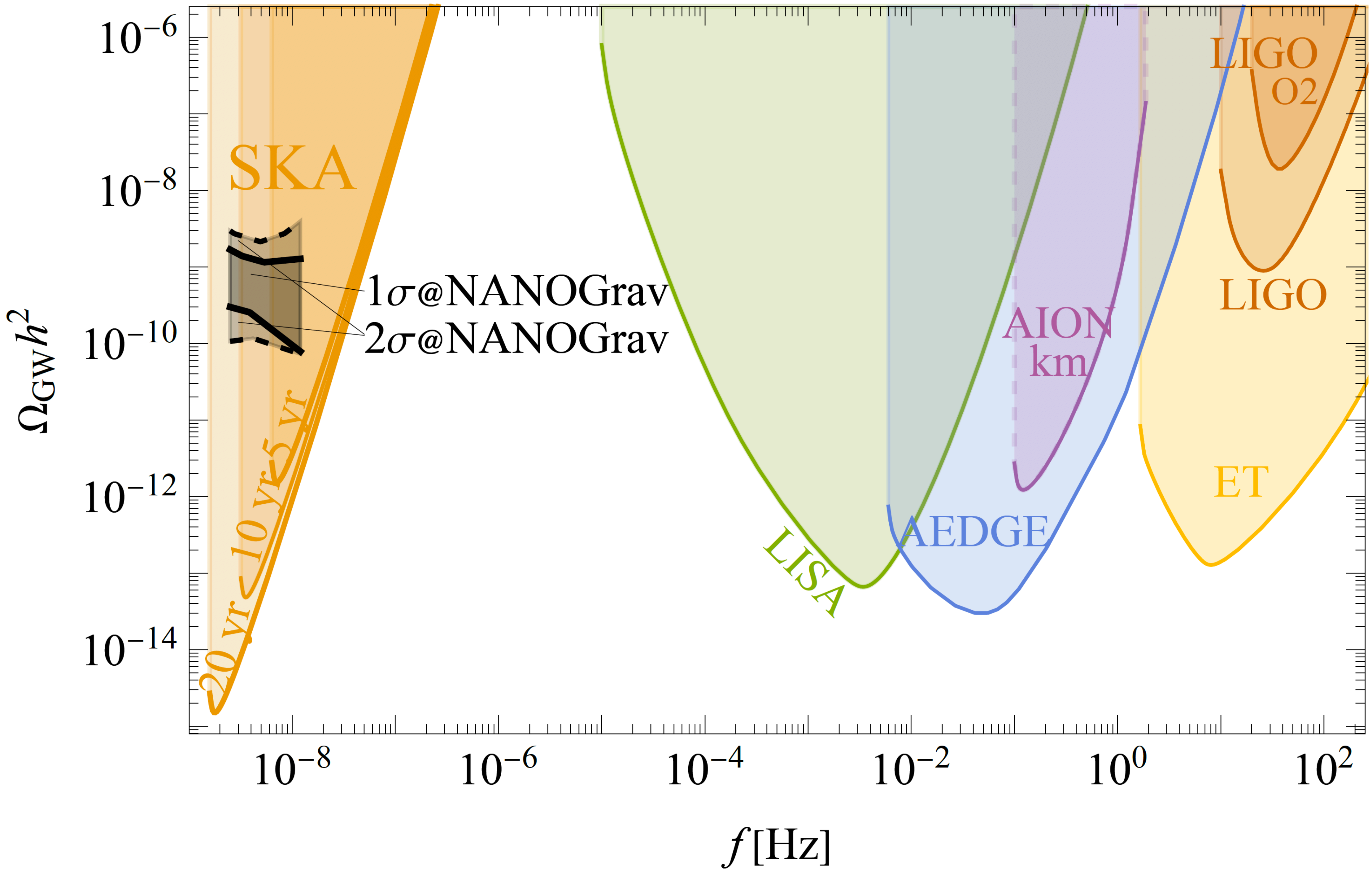


CMB Latest Analysis

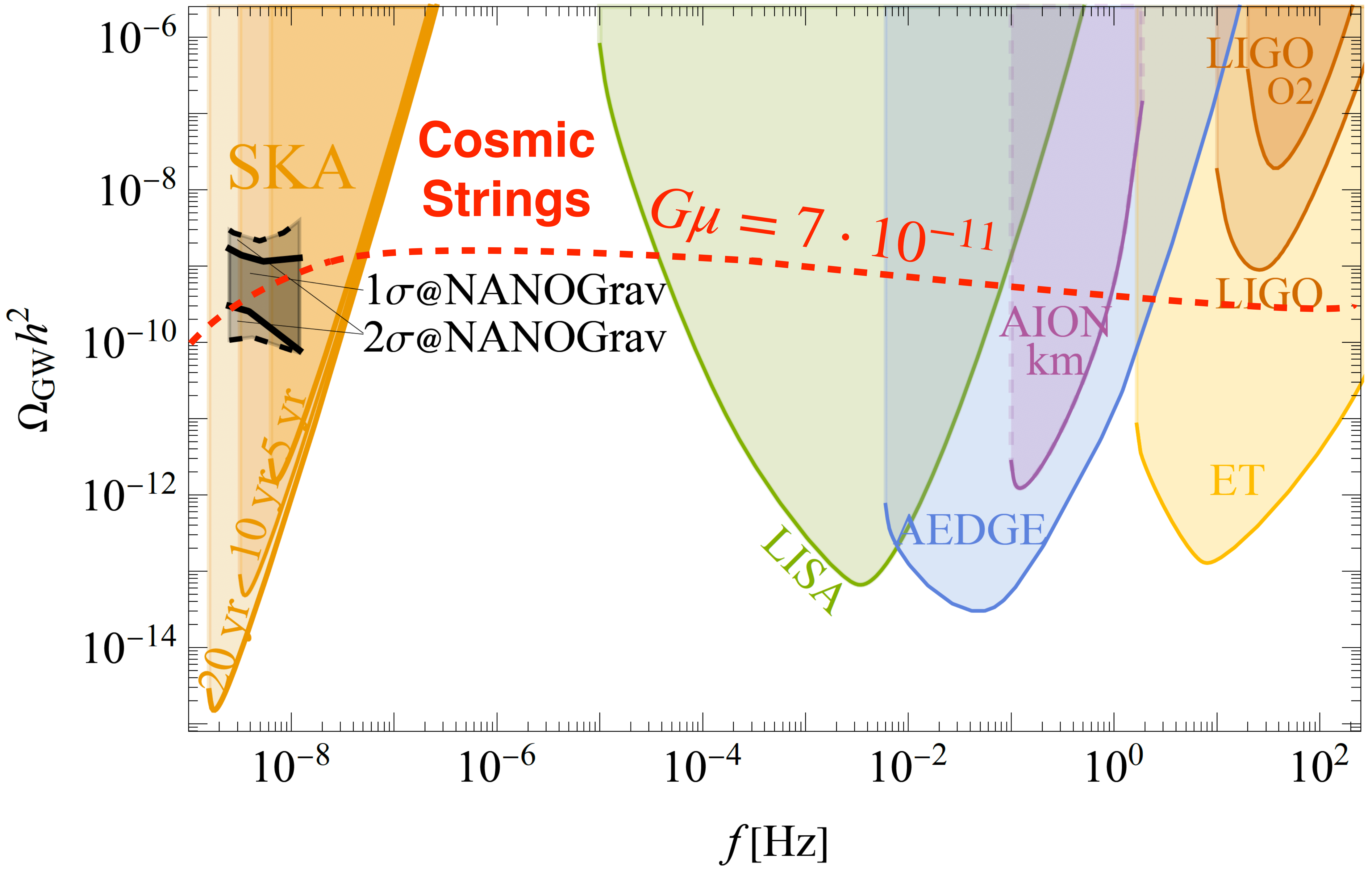


(from Copeland et al 2004.11396)

Latest PTA


SKA
1σ@NANOGrav
2σ@NANOGrav
5 yr
10 yr
20 yr
LISA
AEDGE
AION km
LIGO O2
LIGO
ET
$\Omega_{\rm GW}h^2$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
f [Hz]
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}

Latest PTA


SKA
Cosmic Strings
$G\mu = 7 \cdot 10^{-11}$
1σ@NANOGrav
2σ@NANOGrav
20 yr
10 yr
5 yr
LISA
AEDGE
AION km
ET
LIGO O2
LIGO
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
$\Omega_{\rm GW}h^2$
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
$f\,$[Hz]

OUTLINE

Gravitational Wave Backgrounds

1st Bloc

1) Cosmology/GR + GW def. ✓

2nd Bloc

2) GWs from Inflation ✓

3) GWs from Preheating ✓

3rd Bloc

4) GWs from Phase Transitions ✓

5) GWs from Cosmic Defects ✓

Early Universe

Late Universe

6) Astro Background and Observations ✓

OUTLINE

Gravitational Wave Backgrounds

1) Cosmology/GR + GW def.

Early Universe

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

Late Universe

6) Astro Background and Observations

OUTLINE

Gravitational Wave Backgrounds

complicated,
but robust

1) Cosmology/GR + GW def.

Early Universe

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3) GWs from Preheating

4) GWs from Phase Transitions

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Late Universe

6) Astro Background and Observations

OUTLINE

Gravitational Wave Backgrounds

complicated,
but robust

1) Cosmology/GR + GW def.

Early
Universe

2) GWs from Inflation

Intensive search
at the CMB.
Extra ingredients
= enhance GW

3) GWs from Preheating

High amplitude,
unlike detection

4) GWs from Phase Transitions

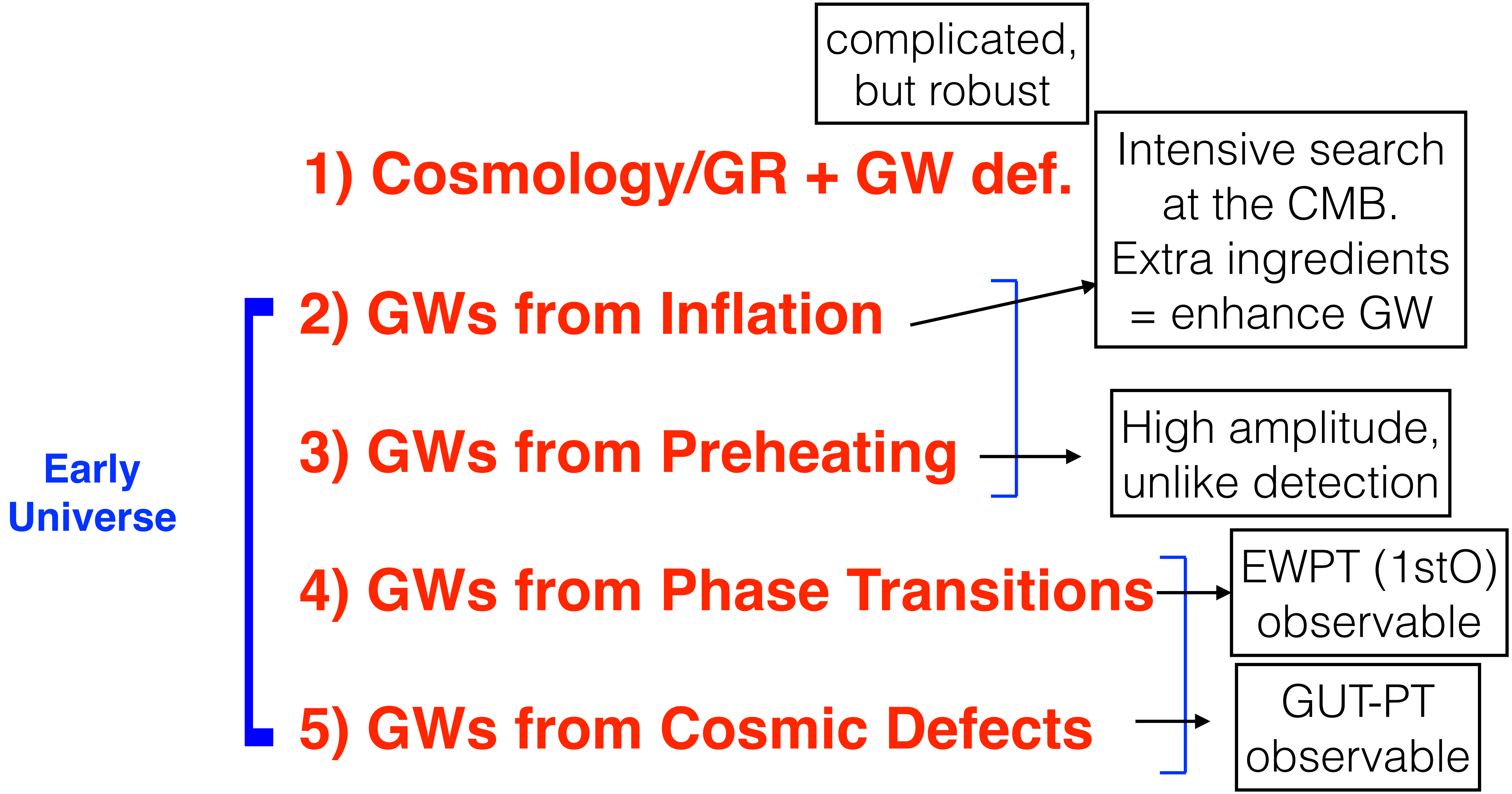
5) GWs from Cosmic Defects

Late Universe

6) Astro Background and Observations

OUTLINE

Gravitational Wave Backgrounds



complicated, but robust

1) Cosmology/GR + GW def.

Early Universe

2) GWs from Inflation

Intensive search at the CMB. Extra ingredients = enhance GW

3) GWs from Preheating

High amplitude, unlike detection

4) GWs from Phase Transitions

EWPT (1stO) observable

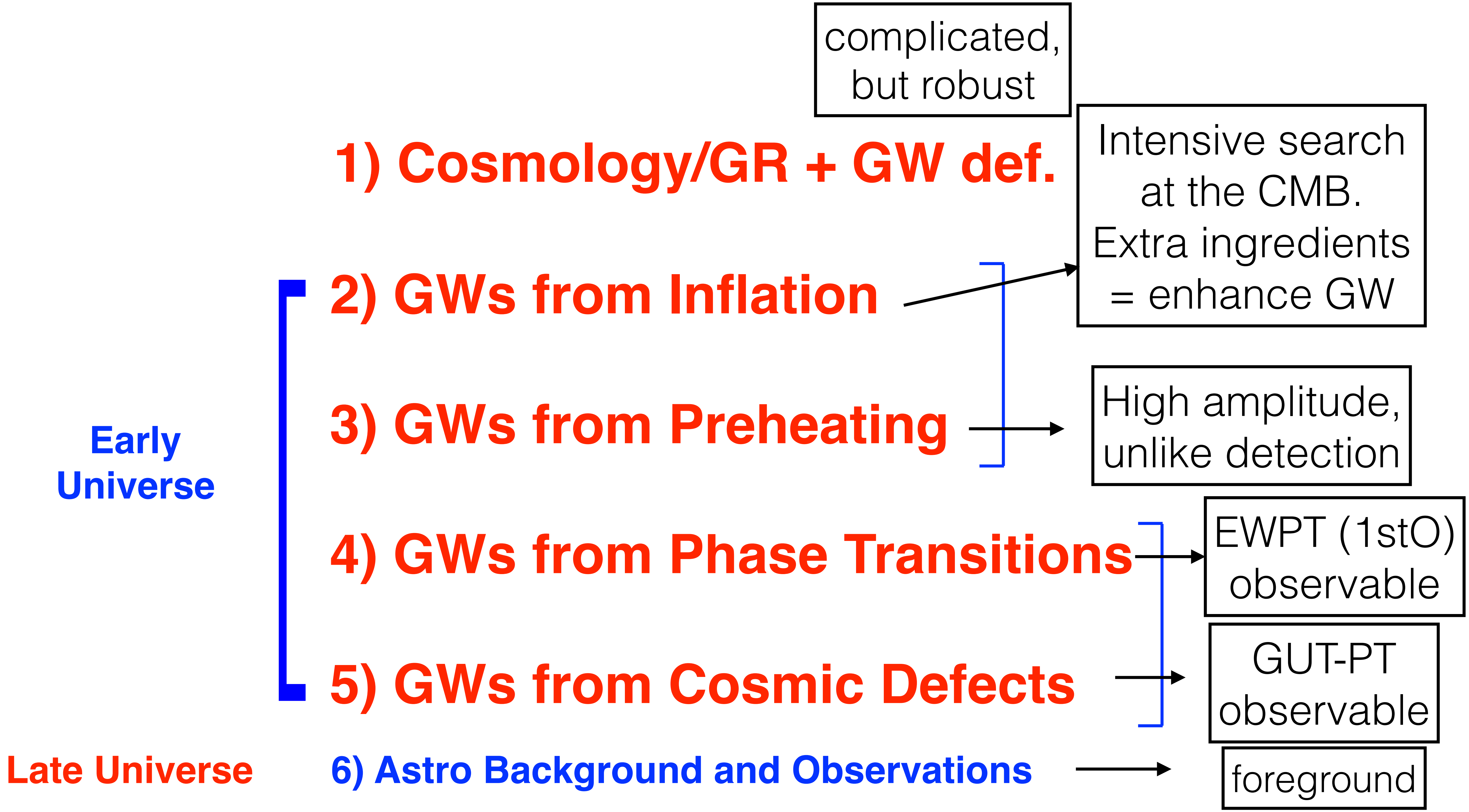
5) GWs from Cosmic Defects

GUT-PT observable

Late Universe **6) Astro Background and Observations**

OUTLINE

Gravitational Wave Backgrounds



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EWPT (1stO)
observable

5) GWs from Cosmic Defects

GUT-PT
observable

Late Universe

6) Astro Background and Observations

foreground

Propaganda, Part I

Review on Gravitational Waves from the Early Universe

Caprini & Figueroa

arXiv:1801.04268

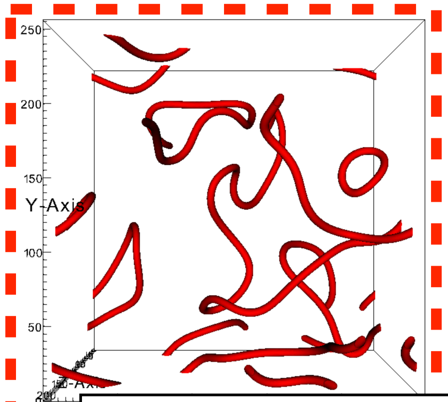
Propaganda, Part II

For you early universe numerics …

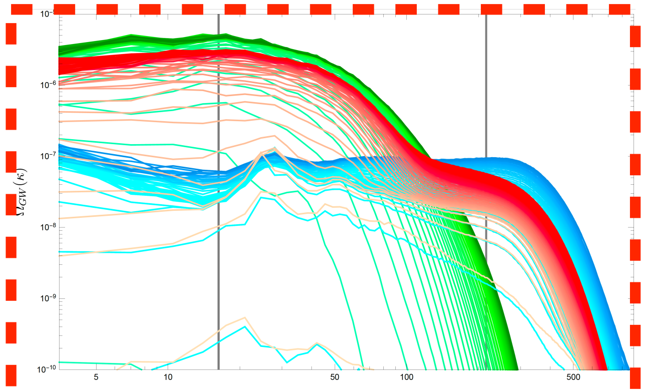
CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg, arXiv: 2102.01031

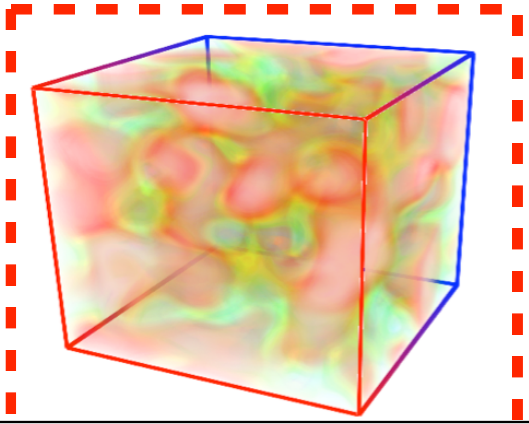
('GW computation' module about to be available)



Cosmic Strings



Grav. Wave emission



Non-linear dynamics

?

Whatever you want !

CosmoLattice

http://www.cosmolattice.net/

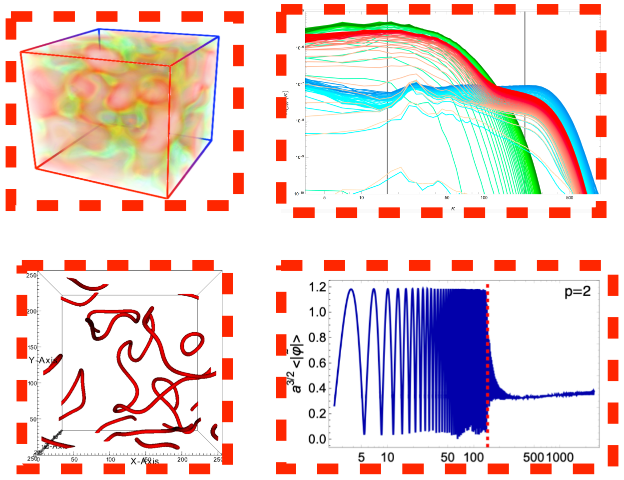
Physical Problem

- * Init Conditions
- * Eqs. of Motion

CosmoLattice

- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\delta\mathcal{O}(\delta t^n)$
- * Choose Param: g, m, ...
- * Choose Observables

Output



CosmoLattice

http://www.cosmolattice.net/

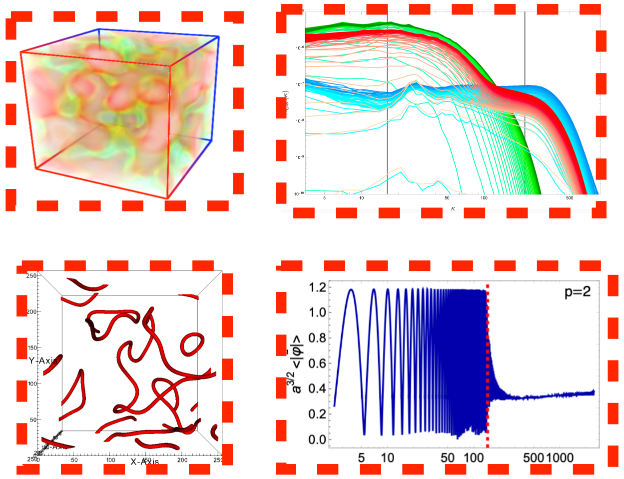
Physical Problem

- * Init Conditions
- * Eqs. of Motion

CosmoLattice

- *** New Physical Problem**
- * Choose Lattice: dt, N, dx
- * Choose Algorithm $\delta\mathcal{O}(\delta t^n)$
- * Choose Param: g, m, ...
- * Choose Observables

Output



CosmoLattice

http://www.cosmolattice.net/

Physical Problem

- * Init Conditions
- * Eqs. of Motion

CosmoLattice

- * **New Physical Problem**
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- * Choose Algorithm $\delta\mathcal{O}(\delta t^n)$
- * Choose Param: g, m, …
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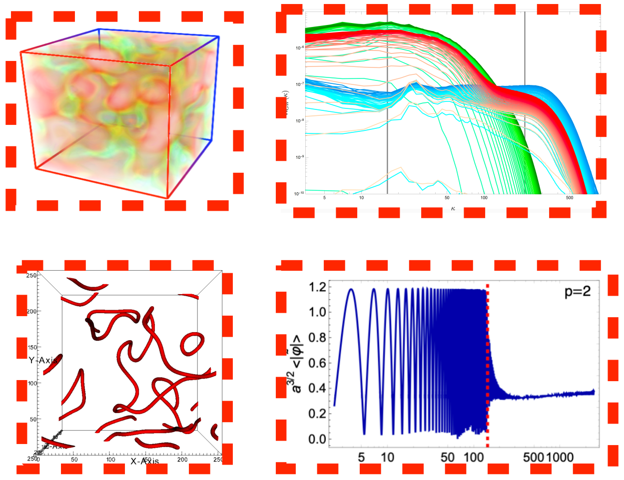
Output



CosmoLattice : platform for field theories,

You choose the problem to solve !

CosmoLattice

http://www.cosmolattice.net/

- *CosmoLattice* **(current public version) :**
  - **Scalar-gauge dynamics** [**U(1) & SU(2) interactions**]
  - **Multi-dimensional Parellization (you write serial !)**
  - **Symplectic Integrators** $\delta\mathcal{O}(\delta t^2) - \delta\mathcal{O}(\delta t^{10})$
  - **Modular, Symbolic language, Field algebra**

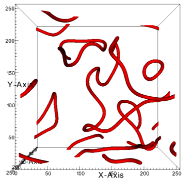
CosmoLattice

http://www.cosmolattice.net/

- *CosmoLattice* (current public version) :
  - **Scalar-gauge dynamics [U(1) & SU(2) interactions]**
  - **Multi-dimensional Parellization (you write serial !)**
  - **Symplectic Integrators** $\delta\mathcal{O}(\delta t^2)$ - $\delta\mathcal{O}(\delta t^{10})$
  - **Modular, Symbolic language, Field algebra**

- *CosmoLattice* (package upgrades towards 2022/beyond) :

  - ✓ **Gravitational waves** $\Box h_{ij} = 2\Pi_{ij}^{\rm TT}$
  - ✓ **Axion-like couplings** $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$
  - ✓ **Non-minimal coupling** $\xi\phi^2 R$
  - ✓ **Cosmic String Networks**



  - …

+

  - **Non-Canonical Kinetic Terms**
  - **Non-symplectic integrators** ✓
  - **Arbitrary Initial Conditions** ✓
  - **Fluid Dynamics**
  - …

Coming (hopefully) this Summer/Fall …

CosmoLattice

School 2022 ?

Coming (hopefully) this Summer/Fall …

CosmoLattice

School 2022 ?

@Valencia:

Now …

~ 800-ish slides afterwards…

Now …

~ 800-ish slides afterwards…

Thanks for your attention !

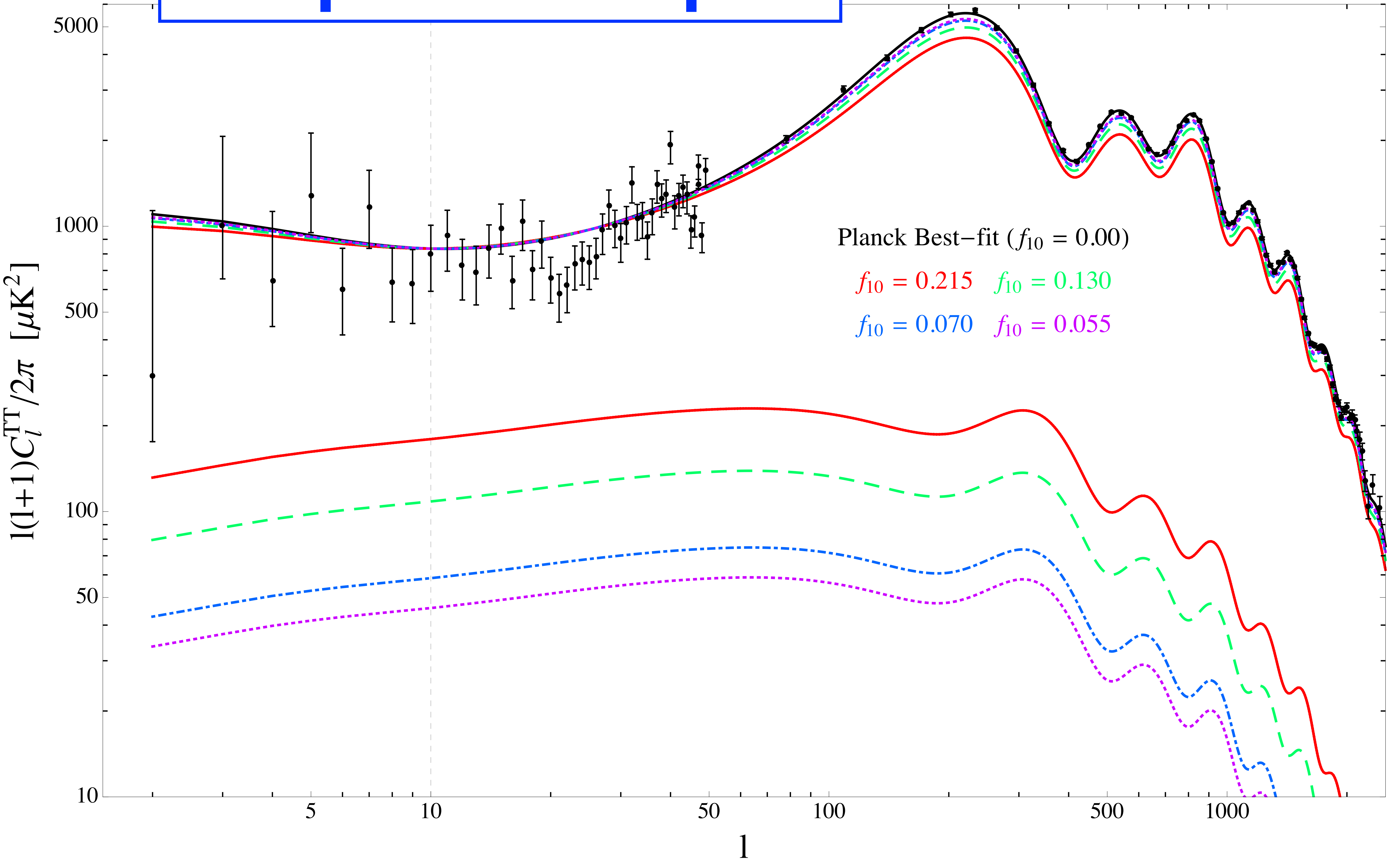
BACK SLIDES

CMB anisotropies due to defects

CMB Defects (Back) SLIDES

Cosmic Microwave Background

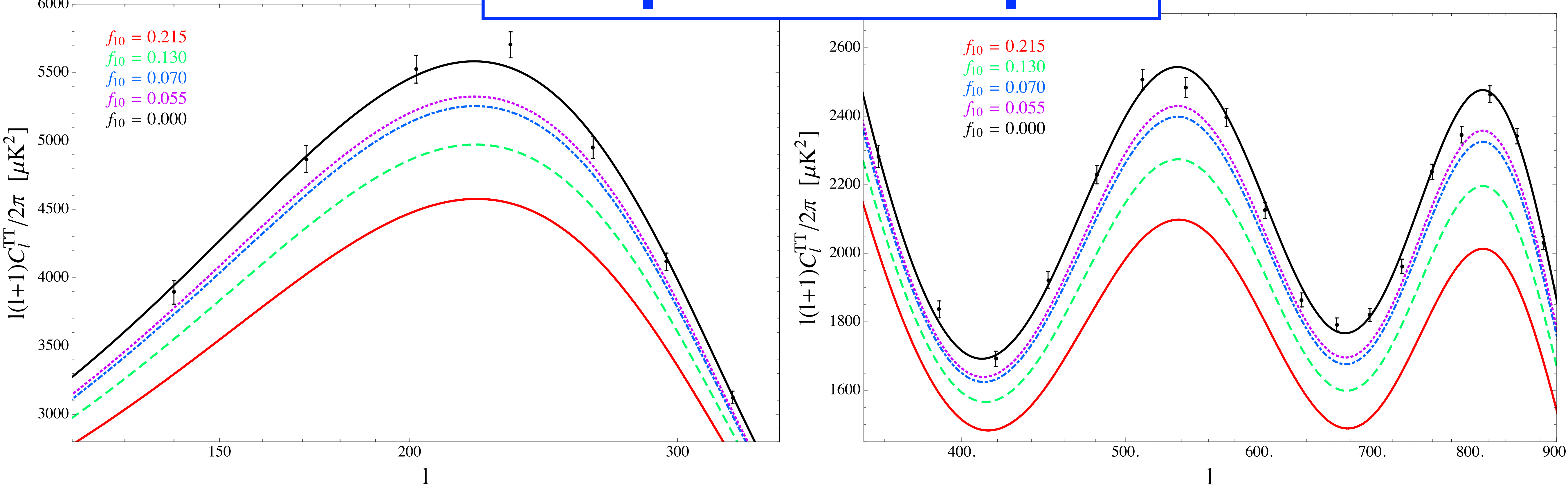
Temp-anisotropies



Durrer et al, JCAP 2014

Cosmic Microwave Background

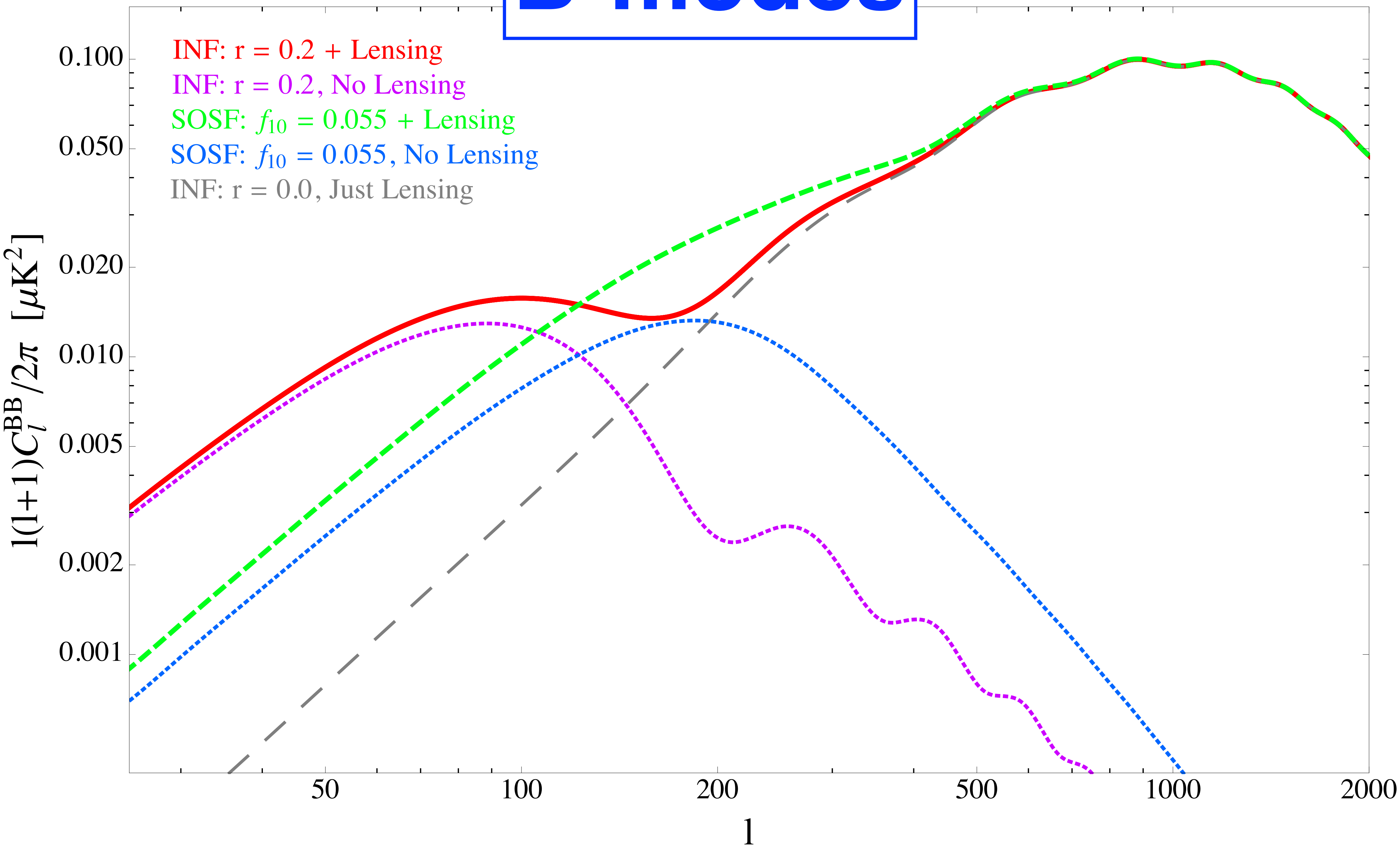
Temp-anisotropies



Durrer et al, JCAP 2014

Cosmic Microwave Background

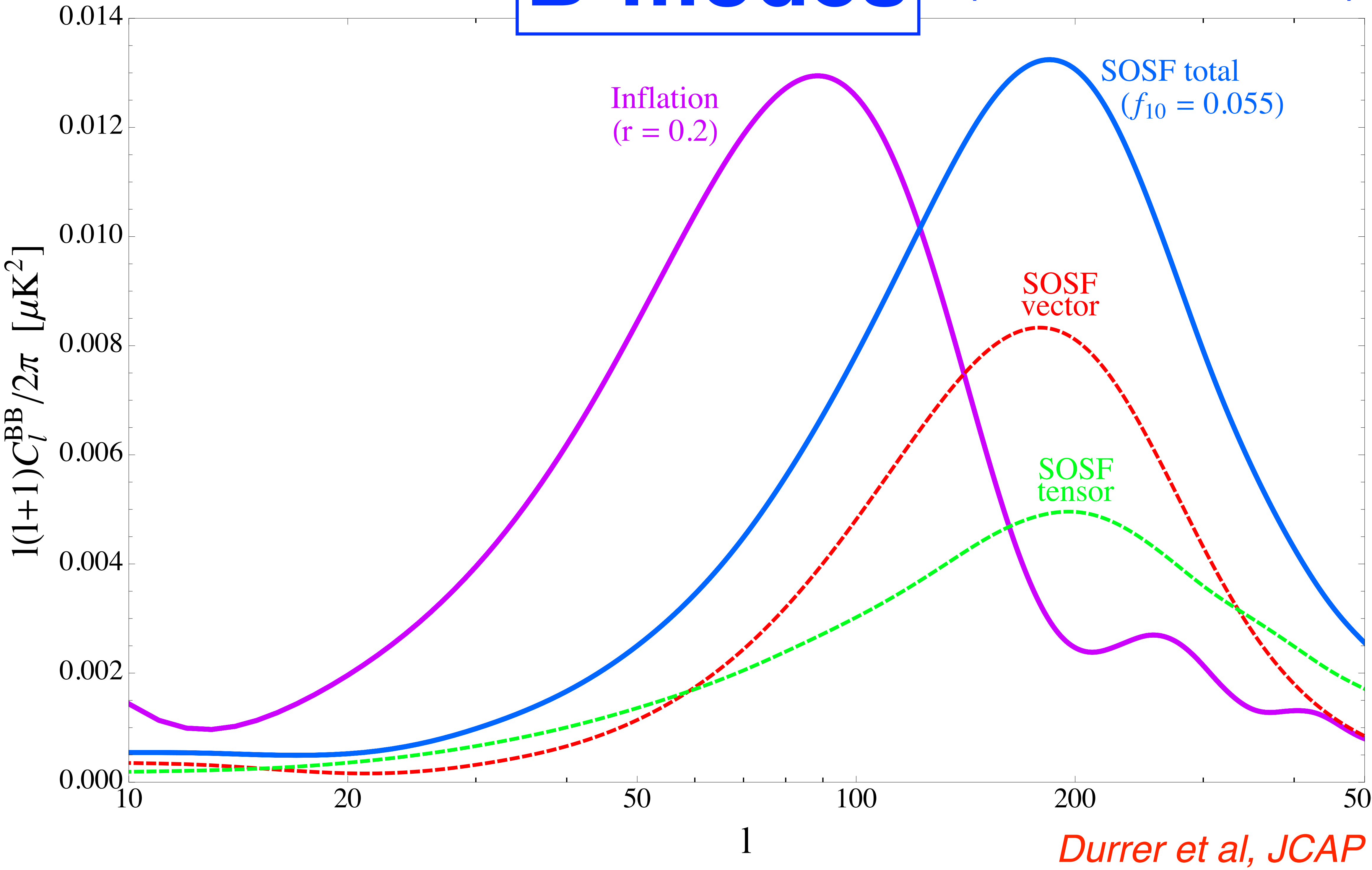
B-modes



Durrer et al, JCAP 2014

Cosmic Microwave Background

B-modes (SOSF = Defects)



Durrer et al, JCAP 2014