Collider phenomenology

LECTURE PLAN

① COLLIDER PHENOMENOLOGY

- COLLIDING PARTICLES
- PARTON-MODEL FOR THE DRELL-YAN PROCESS
- BREIT-WIGNER & NWA

② QUANTUM CHROMO DYNAMICS

- A REVIEW OF QCD
- RADIATIVE CORRECTIONS to DY
- DGLAP EVOLUTION

③ RESUMMATION

- SOFT-COLLINEAR FACTORISATION, IRC SAFETY
- THE TRANSVERSE MOMENTUM OF THE Z BOSON

④ JET PHYSICS

- WHY JETS?
- JET DEFINITIONS

⑤ JET SUBSTRUCTURE

- GROOMING and TAGGING
- MACHINE LEARNING (briefly)

BIBLIOGRAPHY

QFT part: SCHWARTZ
PESKIN & SCHROEDER

PHENO PART:
- CAMPBELL, HUSTON, KRAUSS "the BLACK BOOK of QCD"
- ELLIS, STIRLING, WEBBER "QCD & COLLIDER PHYSICS"
- SM, SPANNOWSKY, SOYEZ "LOOKING INSIDE JETS"

not used in these lectures, but recommended

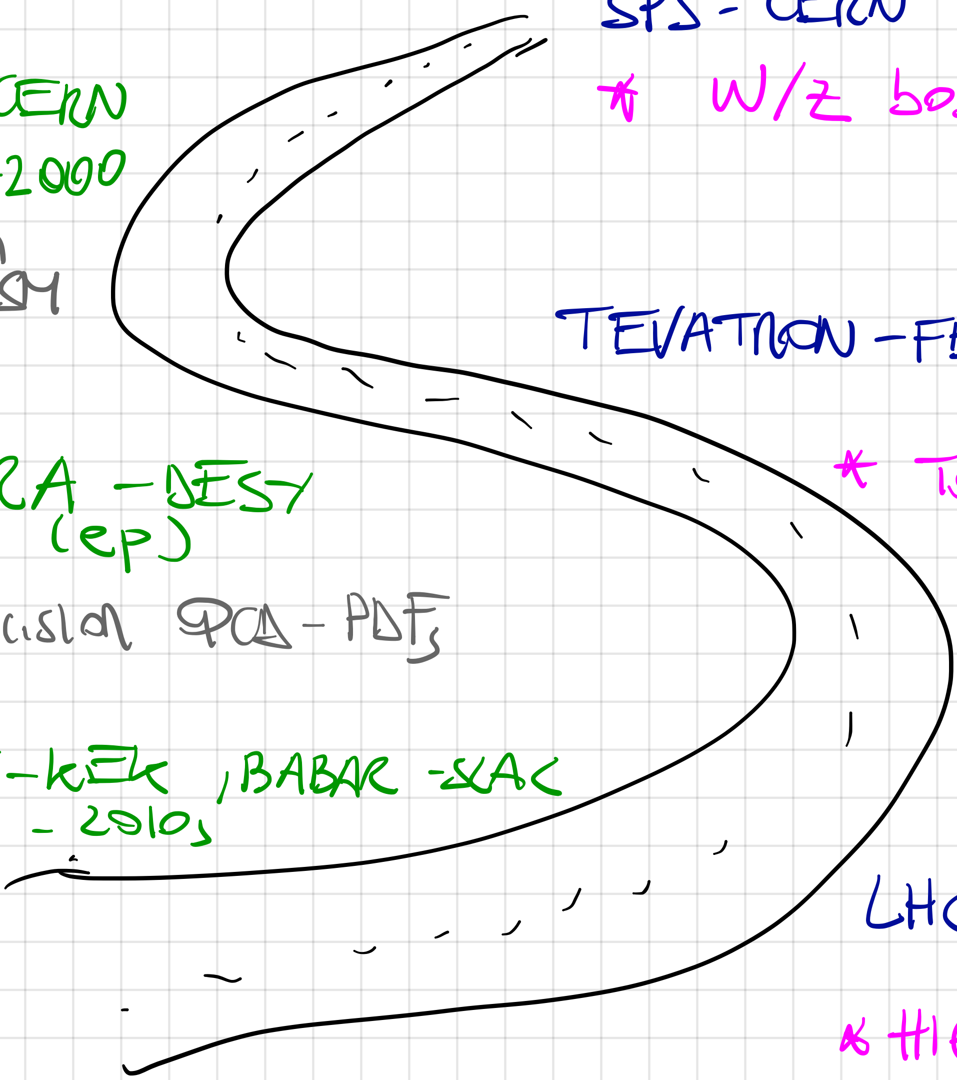
- COLLINS "FOUNDATIONS OF PERT. QCD"
- DOKSHITZER, KHOZE, MUELLER, TROYAN "BASICS OF PERT. QCD".

COLLIDING PARTICLES

- Believe it or not smashing particles is our best way (or one of the best ways) to understand the Universe. Why?

- heavy objects
- short-distance physics
- rare processes.

COLLIDING ... WHAT?



SPS - CERN 1981-84

* W/Z bosons

LEP - CERN 1989-2000

* precision test SM

TEVATRON - FERMILAB 1983-2011

* TOP QUARK

HERA - DESY (ep)

precision QCD - PDFs

BELLE - KEK, BABAR - SLAC 1990s - 2010s

precision flavour physics

LHC - CERN 2009-...

* HIGGS BOSON

- electro-positron (or ep) colliders usually employed for precision studies:
  - "clean environment" ie we collide elementary particles
  - strong interactions limited to the final state
  - limitation in energy

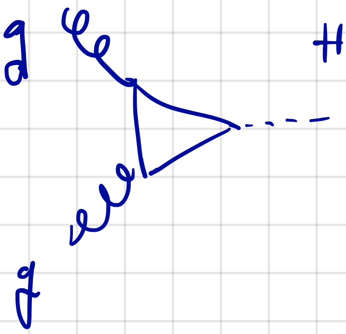
- proton-(anti) proton are discovery machines
  - higher energies
  - "messy environment" ie we collide composit particles.
  - strong interactions everywhere

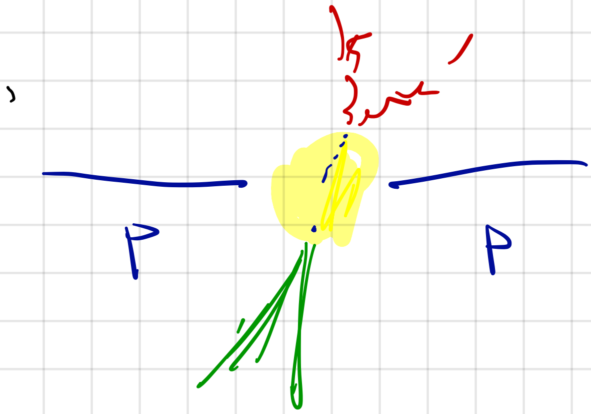
- STRONG INTERACTIONS dominate collider physics.

- A DEEP understanding of QCD is mandatory for collider phenomenology.

THE GAME WE PLAY

what theorists think | what phenomenologists think





what actually happens:

- bunches of 10^{11} protons are collided at collisions sites with a rate $\sim$ 40 MHz
- instantaneous luminosity: $\mathcal{L} = \dfrac{N_1 N_2 \nu}{A_{eff}} \simeq \dfrac{10^{22} \cdot 4 \cdot 10^{7}\,\text{Hz}}{(10^{-2}\,\text{cm})^2}$

$$\simeq 4 \cdot 10^{+33}\ \text{cm}^{-2}\text{s}^{-1}$$

peak $= 2 \cdot 10^{34}\,\text{cm}^{-2}\text{s}^{-1}$.

- integrated luminosity: $\int dt\, \mathcal{L}$

$$\boxed{1\ \text{cm}^2 = 10^{24}\ \text{b}}$$

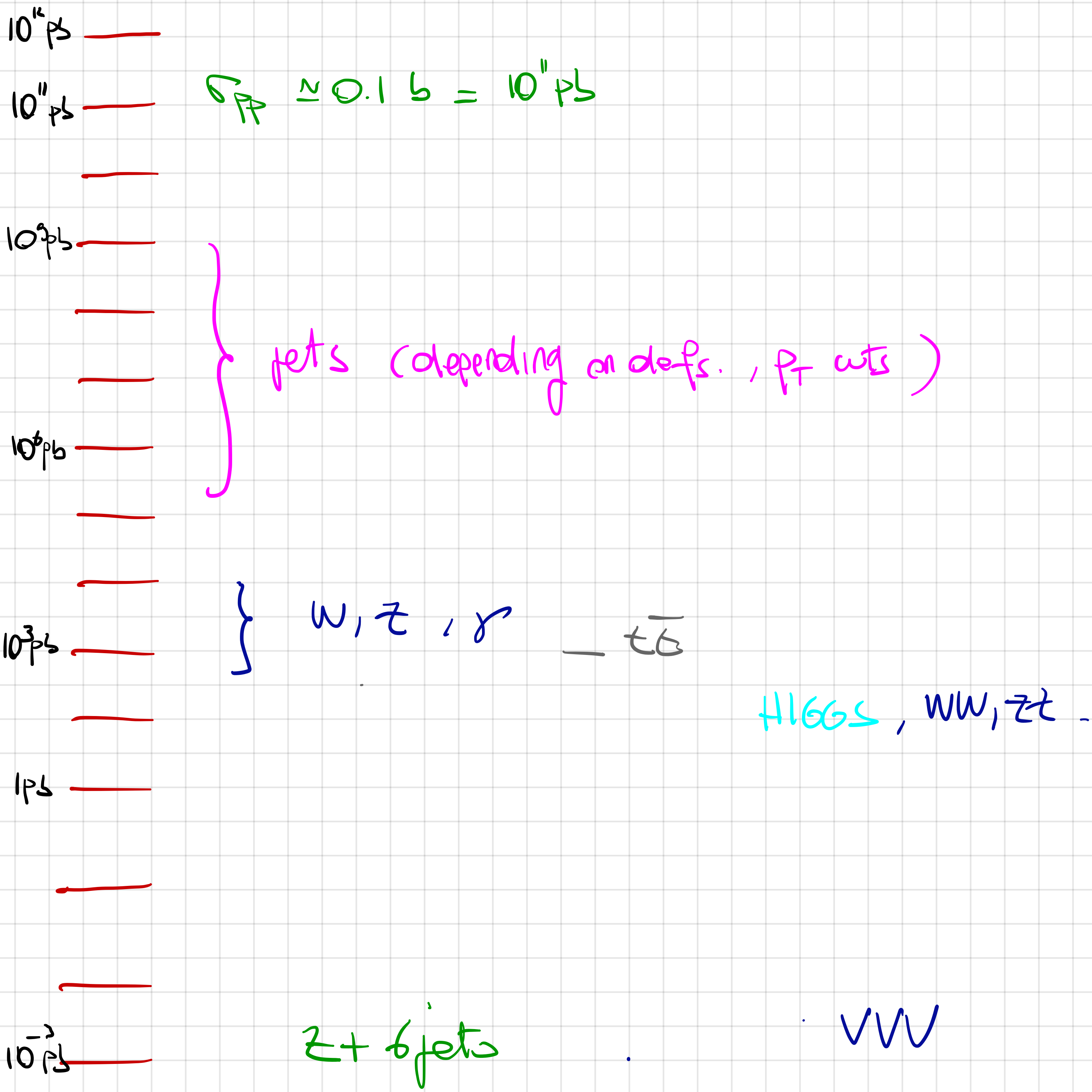
- total proton-proton x-section

$$\sigma_{pp} \simeq 4\pi (r_p)^2 = 4\pi\,(10^{-15}\,\text{m})^2$$
$$= 4\pi\,(10^{-13}\,\text{cm})^2 = 4\pi \cdot 10^{-26} \cdot 10^{24}\,\text{b}$$
$$\simeq 0.1\ \text{b}$$

So we have ~ 10^9 collisions per second!

or if you want $10^9 / 40\,\text{MHz} \sim 25$ collisions per bunch crossing.

- Most of this collisions are soft pp collisions: no H/Z/W etc.



- Huge range in x-sections, many different final states

- Background to each other and for new physics.

INTERESTING QUESTIONS of PHENO

- can we model all of this?
- if so, with what accuracy?
- can we design strategies to separate different processes?

↙ theoretically sound

↘ experimentally measurable.

the S-MATRIX

- Scattering matrix is an operator that maps initial states to final states.
- in s-p $\langle b| e^{-iH(T_f - T_i)} |a\rangle \xrightarrow[T_f - T_i \to \infty]{} \langle b|S|a\rangle$
- you'll have a dedicated set of lectures of this
- here we sketch some properties (and shortcomings)

PROPERTIES

a) S is unitary (conservation of probability)

$$1 = \sum_n |\langle n| S |a\rangle|^2$$

$$= \sum_n \langle a| S^\dagger |n\rangle\langle n| S |a\rangle = \langle a| S^\dagger S |a\rangle \quad \forall a$$

$$\Leftrightarrow \quad S^\dagger S = \mathbb{1} \quad \blacksquare$$

b) we often consider

$$S = \mathbb{1} + iT \qquad \text{(we separate off "nothing happens")}$$

$$S^\dagger S = \mathbb{1}$$

$$\Rightarrow (\mathbb{1} - iT^\dagger)(\mathbb{1} + iT) = \mathbb{1}$$

$$\Rightarrow -i(T - T^\dagger) = T^\dagger T$$

Let's consider matrix elements

$$\langle b | T | a \rangle = T_{ba} \; ; \quad \langle b | T^\dagger | a \rangle = T^*_{ab}$$

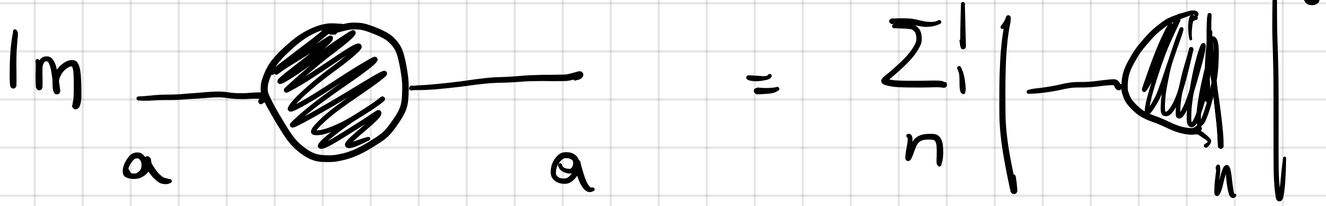
$$-i\left(T_{ba} - T^*_{ab}\right) = \sum_n\!' \, T_{bn} T^*_{na}$$

in particular if $a = b$

$$\boxed{2 \, \mathrm{Im}\, T_{aa} = \sum_n\!' \, |T_{an}|^2}$$

OPTICAL THEOREM

pictorially



EXERCISE:

Consider a theory with two scalar fields:

$$\mathcal{L} = -\frac{1}{2}\phi\left(\Box + M^2\right)\phi - \frac{1}{2}\pi\left(\Box + m^2\right)\pi + \frac{\lambda}{2}\pi^2\phi$$

a) compute the decay rate $\Gamma(\phi \to \pi\pi)$ at $O(\lambda^2)$ in the "standard" way $\left[\Gamma = \frac{\lambda^2}{32\pi M}\sqrt{1 - \frac{4m^2}{M^2}}\;\theta\,(M>2m)\right]$

b) repeat the calculation using the optical theorem.

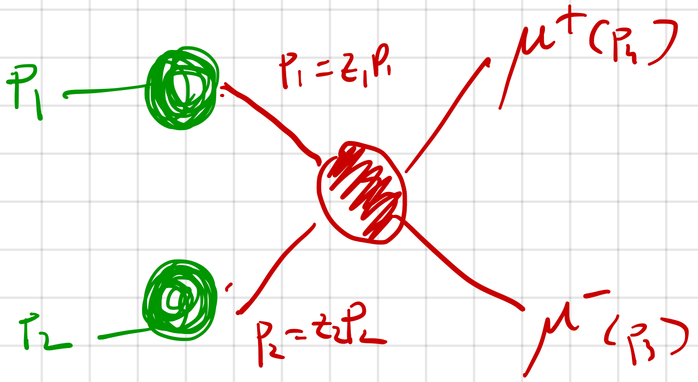
OUR FIRST PROCESS: DRELL-YAN

$$p\,p \to (\text{some intermediate state}) \to \mu^+\mu^-$$

- compute scattering cross-section as a function of $\mu^+\mu^-$ invariant mass. Q^2
- we would like to use perturbative field theory
- however protons are governed by non-pert. dynamics
- How do we make sense of S-matrix elements which are non-perturbative?
- We work at the x-section level and we state the parton-model inspired factorisation formula (more tomorrow)

PARTON-MODEL

- at high-energy the inelastic p-p collision can be described in terms of interactions between two partons (quarks), one from each proton, with momenta $p_1 = z_1 P_1$, $p_2 = z_2 P_2$.



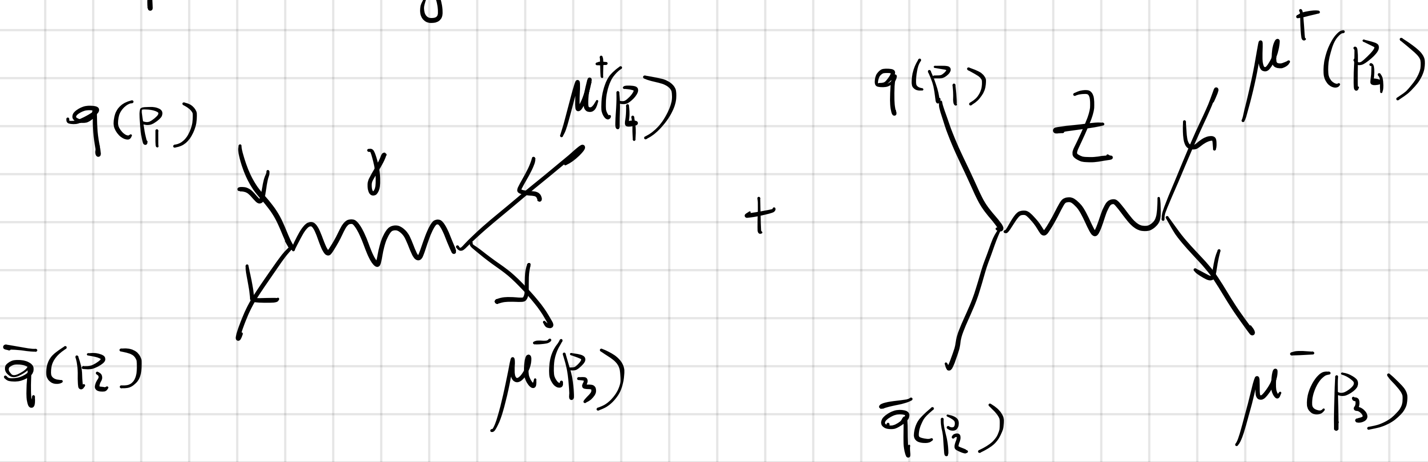
$$S = (P_1 + P_2)^2 = 2 P_1 \cdot P_2$$
$$Q^2 = (p_3 + p_4)^2$$
$$\tau = Q^2 / S$$

$$\sigma_{pp \to \mu^+\mu^-}(\tau, Q^2) \simeq \int_0^1 dz_1 \int_0^1 dz_2 \; f_q(z_1) \, f_{\bar q}(z_2) \; \hat\sigma_{q\bar q \to \mu^+\mu^-}\left(\frac{\tau}{z_1 z_2}, Q^2\right)$$

NOTE

- we work with x-sections rather than amplitudes
- the argument of $\hat\sigma$ is dictated by dimensional analysis and boost invariance
- cross-talks between the protons are neglected.

- PDFs must be fitted from experimental data (DIS but also DY)
- $\hat{\sigma}$ can be computed using perturbative techniques, eg Feynman diagrams:



$$\mathcal{M}_\gamma \quad + \quad \mathcal{M}_Z$$

$$\mathcal{M}_\gamma = (ie)^2 e_q \bar{u}(P_3)\gamma^\mu v(P_4) \frac{(-i g_{\mu\nu})}{q^2} \bar{v}(P_2)\gamma^\nu u(P_1)$$

$$\mathcal{M}_Z = \frac{(-ig)^2}{4\cos^2\theta_W} \bar{u}(P_3)\gamma^\mu(V_e - A_e\gamma_5)v(P_4) \frac{i\left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{m_Z}\right)}{q^2 - m_Z^2} \cdot \bar{v}(P_2)\gamma^\nu(V_f - A_f\gamma_5)u(P_1)$$

$$P_1^2 = P_2^2 = P_3^2 = P_4^2 = 0$$

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + 2\,\mathrm{Re}\,\mathcal{M}_\gamma^* \mathcal{M}_Z$$

· if you've never done it, do it by hand: it's a good exercise. Then check it with FEYNCALC.

[first useful trick $q^\mu q^\nu$ does not contribute for massless fermions]:

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \frac{d^3\vec{p}_3}{(2\pi)^3\, 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 (2E_4)} |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(P_1+P_2-P_3-P_4)$$

flux factor ($\frac{1}{2\hat{s}}$) — Lorentz-invariant phase-space — 4-momentum conservation

$$\hat{\sigma} = \frac{1}{2\hat{s}} \frac{1}{(2\pi)^2} \int \frac{d^3\vec{p}_3}{4E_3 E_4^*} |\mathcal{M}|^2 \delta(E_1+E_2-E_3-E_4)$$

degrees of freedom; ie independent variables in $|\mathcal{M}|^2$

$$2\cdot 4 \;-\; 2 \;-\; 4 \;=\; 2$$

on-shell (-2), mom. cons (-4), (angles ϕ, ϑ)

$$d^3\vec{p}_3 = d|\vec{p}_3|\, |\vec{p}_3|^2\, d\phi\, d\cos\vartheta$$

in the centre of mass frame

$$\begin{cases} E_1 + E_2 = E_3 + E_4 \\ \vec{0} = \vec{p}_3 + \vec{p}_4 \end{cases}$$

massless particles: $E_3 = |\vec{p}_3| = |\vec{p}_4| = E_4$

$E_1 = E_2 = \sqrt{\hat{s}}/2$

$$\hat{\sigma} = \frac{1}{2\hat{s}} \frac{1}{16\pi^2} \int_{-1}^{1} d\cos\vartheta \int_0^{2\pi} d\phi \int dE_3 \frac{E_3^2}{E_3 E_4} \delta(\sqrt{\hat{s}} - 2E_3) |M|^2(\vartheta, \phi)\Big|_{E_3 = E_4}$$

$$= \frac{1}{64\pi^2} \frac{1}{\hat{s}} \int_{-1}^{1} d\cos\vartheta \int_0^{2\pi} d\phi \, |M|^2(\vartheta,\phi)\Big|_{E_3 = E_4 = \frac{\sqrt{\hat{s}}}{2}}$$

[...]

$$\hat{\sigma}(\hat{s}) = \frac{4}{3} \frac{\alpha^2}{\hat{s}} \frac{\pi}{N_c} \left[e_q^2 - 2 e_q V_e V_f \frac{\kappa \hat{s}}{\hat{s} - m_Z^2} + (V_e^2 + A_e^2)(V_f^2 + A_f^2) \left(\frac{\kappa \hat{s}}{\hat{s} - m_Z^2} \right)^2 \right]$$

$$\boxed{\kappa = \frac{\sqrt{2}\, G_F m_Z}{4\pi\alpha}}$$

$$\Rightarrow \frac{d\hat{\sigma}}{dQ^2} = \hat{\sigma}(\hat{s})\, \delta(\hat{s} - Q^2)$$

$$\left[\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 = q^2 \equiv Q^2 \right]$$

→ Let's go back to our master formula

$$\frac{d\sigma}{dQ^2}(\tau, Q^2) = \int_0^1 dz_1 f(z_1) \int_0^1 dz_2 f(z_2) \frac{d\hat{\sigma}}{dQ^2}\left(\frac{\tau}{z_1 z_2}, Q^2\right)$$

$$\Rightarrow \frac{d\sigma}{d\Phi^2} = \int_0^1 dz_1 \int_0^1 dz_2 \, f(z_1)\, f(z_2)\, \hat{\sigma}\left(\frac{\tau}{z_1 z_2}, \Phi^2\right) \delta(z_1 z_2 S - \Phi^2)$$

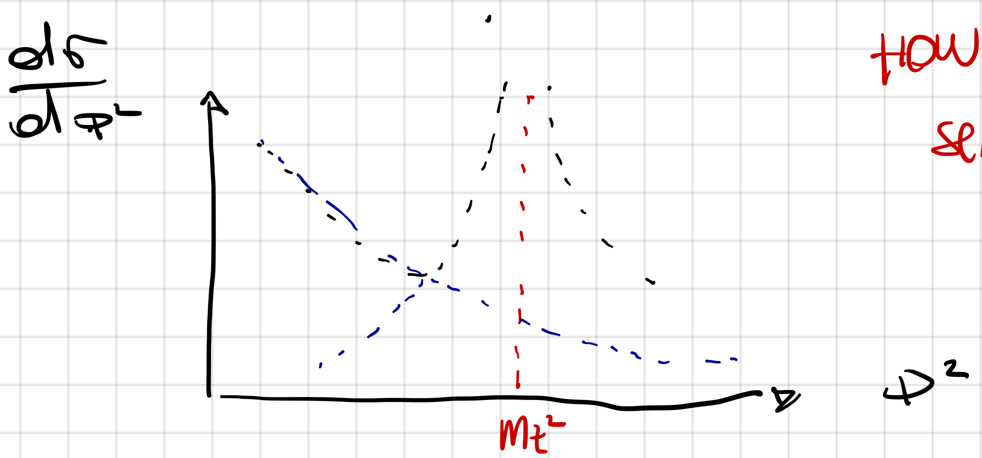
$$\left[\begin{array}{l} z_1 z_2 = z \\ z_1/z_2 = w \end{array} \Rightarrow \begin{array}{l} z_1 = \sqrt{zw} \\ z_2 = \sqrt{z/w} \end{array} \Rightarrow dz_1 dz_2 = |J|\, dw\, dz \right]$$

$$= \int_0^1 dz \int_0^1 \frac{dw}{w} \, f(\sqrt{zw})\, f(\sqrt{z/w})\, \hat{\sigma}\left(\frac{\tau}{z}, \Phi^2\right) \delta(z S - \Phi^2)$$

$$= \frac{1}{S} \, \hat{\sigma}(1, \Phi^2)\, F(\tau)$$

parton luminosity.

$$\Phi^4 \frac{d\sigma}{d\Phi^2} \sim \tau F(\tau)$$



How do we make sense of this behavior?

BREIT-WIGNER

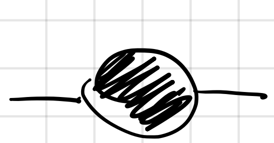
- Let's at a scalar for simplicity.
- The all-order renormalised two-point function takes the form:

$$iG(p^2) = \frac{i}{p^2 - m^2 + \Sigma'_i(p^2) + i\varepsilon}$$

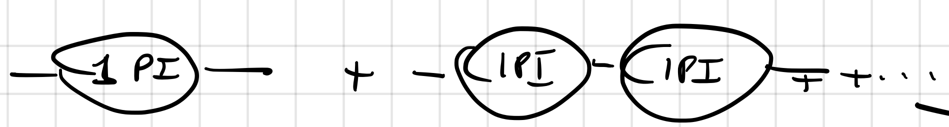
where
- m is the mass in some renormalisation scheme
- $i\Sigma'_i$ is the all-order 1-PI self-energy graph.
- the pole of the propagator is located at $p^2 = m_p^2 \Rightarrow m_p^2 - m^2 - \Sigma'_i(m_p^2) = 0$.

- If the particle we are considering is not stable, we can relate its decay rate to

$$\Gamma_{TOT} = \frac{1}{m_p} \mathrm{Im} \left[\quad \right] =$$



$$= \frac{1}{m_p} \mathrm{Im} \left[\quad + \quad + \dots \right]$$



$$= \frac{1}{m_p} \mathrm{Im}\, \Sigma'_i(m_p^2) + \dots$$

in a weakly coupled theory, where $\Gamma \ll m_p$ we can ignore the non-1PI contributions.

Thus, if the particle is not stable, Σ_i' develops an imaginary part and the pole mass can become complex

BREIT-WIGNER SCHEME: the pole mass is defined by

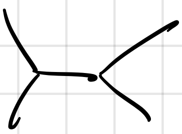
$$m_p^2 - m^2 + \mathrm{Re}\left[\Sigma_i'(m_p^2)\right] = 0$$

but working with complex masses is also possible.

· Thus, in the BW scheme we have

$$iG(p^2) = \frac{i}{p^2 - m_p^2 + i m_p \Gamma_{TOT}} \qquad (\Gamma_{TOT} \ll m_p)$$

· For instance, if we have a process driven by an s-channel diagram:

$\Gamma \sim \Big|$  $\Big|^2$

$$\sim \left| \frac{i g^2}{p^2 - m_p^2 + i m_p \Gamma} \right|^2 \sim \frac{g^4}{(p^2 - m_p^2)^2 + (m_p \Gamma)^2}$$

·· This gives rise to the famous Breit-Wigner distributions of cross-section about a resonance. The width of the distribution is given by the decay rate Γ.

decay rate <-> width.

NARROW WIDTH APPROXIMATION

$$\frac{1}{(p^2-m_p^2)^2+(m_p\Gamma)^2} \xrightarrow[\frac{\Gamma}{m_p}\to 0]{} \frac{\pi}{m_p\Gamma}\,\delta(p^2-m_p^2)$$

which allows us to separate production and decay process of a heavy state, thus greatly simplifying the calculations.

· What happens in the opposite limit $\Gamma \gg m_p$?

We have broader and broader resonances, and then we lose the concept of particle. E.g. glueballs states in QCD.

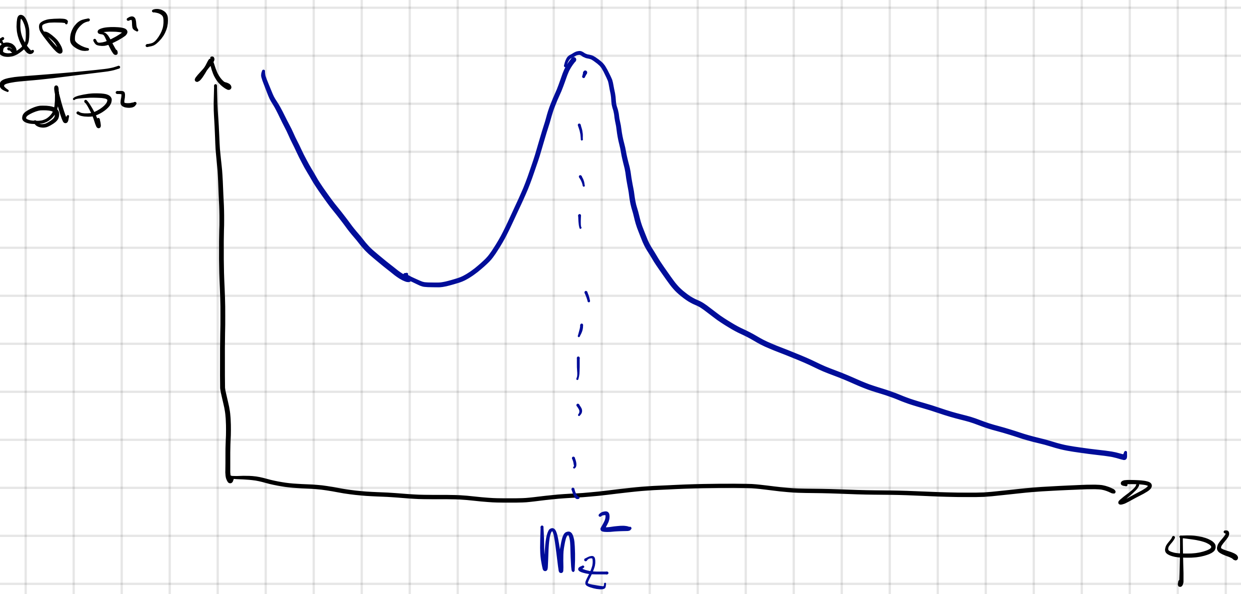
EXERCISE 1

$$\int_{-\infty}^{+\infty} dp^2 \frac{1}{(p^2-m_p^2)^2+(m_p\Gamma)^2} = \frac{1}{(m_p\Gamma)^2}\int_{-\infty}^{+\infty} dp^2 \frac{1}{1+\frac{(p^2-m_p^2)^2}{\Gamma^2 m_p^2}}$$

$$= \left[x = \frac{p^2-m_p^2}{\Gamma m_p}\right] = \frac{1}{m_p\Gamma}\int_{-\infty}^{+\infty} dx \frac{1}{1+x^2} = \frac{\pi}{m_p\Gamma}$$

Thus exploiting the Breit–Wigner propagator, we can re-write our DY spectrum as

$$\frac{d\hat{\sigma}}{dq^2} = \frac{4}{3}\frac{\alpha^2}{\hat{s}}\frac{\pi}{N_c}\left[q_f^2 - 2 q_f V_e V_f \frac{\chi_1(\hat{s} - m_Z^2)}{(\hat{s}-m_Z^2)^2 + m_Z^2\Gamma_Z^2} + (V_e^2 + A_e^2)(V_f^2 + A_f^2)\frac{(\chi_1 \hat{s})^2}{(\hat{s}-m_Z^2)^2 + m_Z^2\Gamma_Z^2}\right]\delta(\hat{s} - q^2)$$



- clear strategies for searching for new particles decaying into leptons, e.g. Z': look for bumps in the invariant mass spectrum.
- this strategy can be extended to hadronic final states.

- The Higgs boson was discovered in 2012 by looking for a (small) bump in the invariant mass distribution of two photons $m_{\gamma\gamma}$.

- Finally, if we look at the x-section in the vicinity of the Z-peak, we can use NWA:

$$\int_{(m_Z-\Delta)^2}^{(m_Z+\Delta)^2} d\varphi^2 \, \frac{d\hat{\sigma}}{d\varphi^2} = \int_{(m_Z-\Delta)^2}^{(m_Z+\Delta)^2} d\varphi^2 \, \hat{\sigma}(\hat{s})\, \delta(\hat{s}-\varphi^2)$$

$$= \int_{(m_Z-\Delta)^2}^{(m_Z+\Delta)^2} d\varphi^2 \, \frac{4}{3}\frac{\alpha^2}{\hat{s}}\frac{\pi}{N_c}\left[\frac{\sqrt{2}\,G_F m_Z^2}{4\pi\alpha}\right]^2 \hat{s}^2 (V_f^2+A_f^2)(V_e^2+A_e^2)$$

$$\frac{\pi}{m_Z \Gamma_Z}\, \delta(\hat{s}-m_Z^2)\, \delta(\hat{s}-\varphi^2)$$

$$= \frac{G_F m_Z^3}{6\sqrt{2}\,\pi}(V_e^2+A_e^2)\frac{1}{\Gamma_Z} \; \frac{G_F m_Z^2 \sqrt{2}}{N_c}(V_f^2+A_f^2)\,\delta(\varphi^2-m_Z^2)$$

$$= \hat{\sigma}_{q\bar{q}\to Z} \; BR_{Z\to\mu^+\mu^-}.$$