DARK MATTER ② - GGI School 22 :

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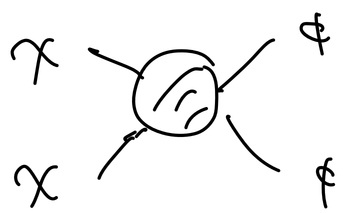
Outline:
- 2→2 WIMP
- 2→2 Beyond WIMP
- 3→2 SIMPs, Cannibal, ELDER...

WIMP :

Alternative back of the envelope: redshift to T_{eq} (~0.8 eV)

$$\langle \sigma v \rangle = \frac{\alpha_{eff}^2}{m_\chi^2}$$



@ matter-radiation eq. : $\rho_{matter}^{eq} = \rho_\chi^{eq} + \rho_b^{eq} = \rho_\gamma^{eq}$

$$\rho_\chi \simeq 5\rho_b \Rightarrow \rho_\chi^{eq} \simeq \rho_\gamma^{eq}$$ [neglects O(1)]

$$n_\chi^{FO} \sim n_\chi(eq)\left(\frac{T_F}{T_{eq}}\right)^3 \underset{\substack{\uparrow \\ NR \\ \rho = nm}}{\sim} \frac{\rho_\chi(T_{eq})}{m_\chi}\frac{T_F^3}{T_{eq}^3} \sim$$

$$\underset{\substack{\uparrow \\ (eq)}}{\sim} \frac{\rho_\gamma(T_{eq})}{m_\chi}\frac{T_F^3}{T_{eq}^3} \underset{\substack{\uparrow \\ \rho_\gamma \sim T^4}}{\sim} \frac{T_F^3\, T_{eq}}{m_\chi} \underset{\substack{\uparrow \\ x = \frac{m}{T}}}{\sim} \frac{T_{eq}\, m_\chi^2}{x_F^3}$$

Compare to F.O. condition (*) :

$$\Gamma_{2\to2} \sim n_\chi \langle\sigma v\rangle \sim H \sim \frac{T^2}{M_{pl}}$$

$$\Gamma_{2\to2} \sim \frac{T_{eq}\, m_\chi^2}{x_f^3}\,\frac{\alpha_{eff}^2}{m_\chi^2} \sim H_f \sim \frac{T_f^2}{M_{pl}} \sim \frac{m_\chi^2}{x_f^2 M_{pl}}$$

$$\Rightarrow \quad m_\chi \simeq \alpha_{eff}\sqrt{T_{eq} M_{pl}} \sim \alpha_{eff}\cdot(30\ \mathrm{TeV})$$

If $\alpha_{eff} \sim 10^{-2}$, weak scale emerges!

Coincidence of scales! T_{eq}, M_{pl}.

If $\alpha_{eff} \ll 10^{-2}$, $m_\chi \ll$ EW scale. Light dark matter.

Another way to write: $\langle\sigma v\rangle \simeq \dfrac{\alpha_{eff}^2}{m_\chi^2} \sim \dfrac{1}{T_{eq} M_{pl}}$

unitarity bound: $\alpha_{eff} \lesssim 4\pi \rightarrow m_\chi \lesssim 300$ TeV

[Griest & Kamionkowski, 1989]

$\exists$ ways to evade - thermal relics w/ mass that exceeds this by $\lesssim 12$ orders of magnitude!

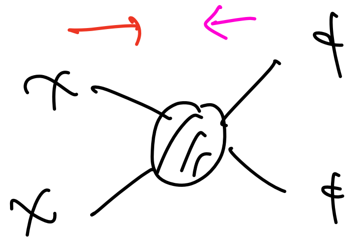
NR chain $\leftarrow$ [Super Heavy Thermal DM, Kuflik & Kim PRL 1906.00981]

Zombies $\leftarrow$ [Zombies, Kramer, ... PRL 2003.04900]

$m_{DM} \gtrsim$ keV & above ($\lesssim 300$ TeV + caveat)

FORBIDDEN CHANNELS: [Griest & Seckel 1991]

[Ruderman & D'Agnolo PRL 1505.07107]



$m_\chi < m_\phi$

time

@ $T=0$: process forbidden

But the early universe is a thermal environment! Can happen by living off of the tail of the distribution.

See: Boltzmann eq:

$$\frac{\partial n_\chi}{\partial t} + 3 n_\chi H = - n_\chi^2 \langle \sigma v \rangle_{\chi\chi \to \phi\phi} + n_\phi^{eq\,2} \langle \sigma v \rangle_{\phi\phi \to \chi\chi}$$

Trick - use detailed balance! In eq - RHS $= 0$

$$\Rightarrow \quad \langle \sigma v \rangle_{\chi\chi \to \phi\phi} = \underbrace{\langle \sigma v \rangle_{\phi\phi \to \chi\chi}}_{\text{ordinary, } \frac{\alpha_{eff}^2}{m_\chi^2}} \cdot \frac{(n_\phi^{eq})^2}{(n_\chi^{eq})^2}$$

$$\Rightarrow \quad \frac{\partial n}{\partial t} + 3Hn = - \langle \sigma v \rangle_{\phi\phi \to \chi\chi} \left(n_\chi^2 e^{-\frac{2\Delta m}{T}} - n_\phi^{eq\,2} \right)$$

F.O.: $\Gamma_{\chi\chi \to \phi\phi} \sim H$

$$n_\chi \frac{\alpha_{eff}^2}{m_\chi^2} \cdot e^{-2\Delta x} \sim \frac{T^2}{M_{pl}} \sim \frac{m_\chi^2}{x_f^2 M_{pl}}$$

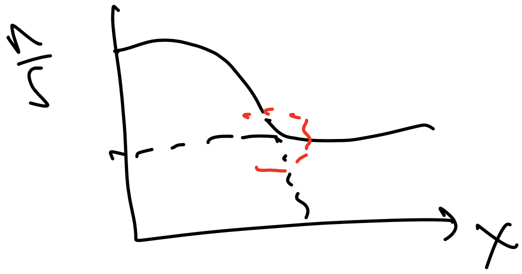
only difference from WIMP eq.

$$\Rightarrow \quad m_\chi \sim \alpha_{eff} \cdot \sqrt{T_{eq} M_{pl}} \cdot e^{-\Delta x} \qquad \ll \text{WIMP}$$

exponentially smaller masses than WIMP.

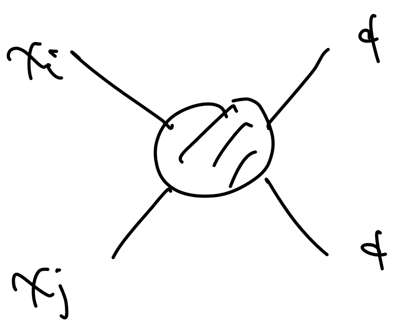
$m_\chi \gtrsim$ keV & above

Note: – F.O. picture looks exactly the same:

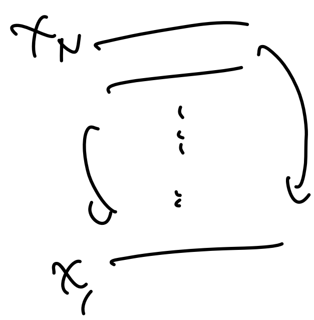


– like with $x_F \sim 20$ still.

CO-ANNIHILATIONS: [Griest & Seckel 1991]



where tower of particles co-participate @ FO.



Spectrum of N dark states lightest one is DM.

In general, described by N coupled B.E.s.

Trick: assume rapid exchange $\chi_i \leftrightarrow \chi_j \Rightarrow \mu_i = \mu_j$

(found in chemical eq. w/ itself)

$$\Rightarrow \frac{n_i}{n_j} \simeq \frac{n_i^{eq}}{n_j^{eq}}$$ removes $N-1$ eqs.

Consider $n = \sum_i n_i$ total number density.

$$\Rightarrow \frac{dn}{dt} + 3nH = -\langle\sigma_{eff} v\rangle (n^2 - n_{eq}^2)$$

$$\langle\sigma v\rangle_{eff} = \sum_{ij} \frac{(n_i^{eq})(n_j^{eq})}{(n^{eq})^2} \langle\sigma_{ij} v\rangle$$

$\Rightarrow$ Identical to WIMP otherwise - so use

$$\langle\sigma_{eff} v\rangle \sim \frac{1}{T_{eq} M_{pl}} \,!$$

The relic needn't be dominated by the lightest particle! In particular, abundance can be set entirely by interacting of completely other particles w/o even involving the lightest one!

Abundance can be independent of the mass of the DM!

Happens in Supersymmetry (SUSY).

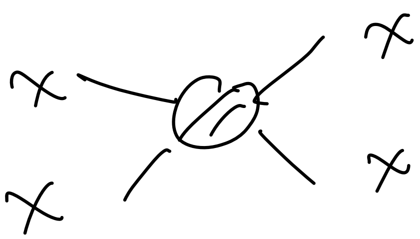
3→2 - SIMPs : [YH, Kuflik, Volansky, Wacker, PRL 1402.5143]

(on beach!! :))

DM = lightest state in nearly secluded dark sector.

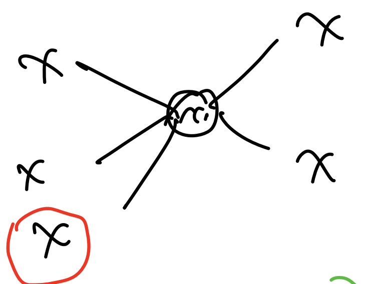
What if what's most important is how DM int' w/ itself?

Self int' of DM set relic abundance?



doesn't change the # density

⇒ first process that does:



3→2

(note: DM is scalar for 3→2)

Let's use our estimating tools:

$$\left[\ \partial_t n + 3nH = -\langle \sigma v^2 \rangle_{3\to 2}\left(n_\chi^3 - n_\chi^2 n_\chi^{eq}\right)\ \right]$$

F.O. happens when $\Gamma_{3\to 2} \sim H$

$$\Gamma_{3\to2} \sim \underbrace{n_\chi^2}\ \underbrace{\langle\sigma \underline{\underline{v}}^2\rangle_{3\to2}}$$

└ - if I am a DM particle, need to meet 2.

└ v^2 - flux $(nv)^2$, standard notation for collision term.

$$\langle\sigma v^2\rangle_{3\to2} \equiv \frac{\alpha_{\rm eff}^3}{m_\chi^5}$$

Take $n_\chi(\mathrm{FO})$ from redshift to T_{eq} trick:

$$n_\chi^{\rm FO} \sim \frac{T_{eq}\, m_\chi^2}{x_f^3}$$

$$\Rightarrow \frac{T_{eq}^2\, m_\chi^4}{x_f^6} \cdot \frac{\alpha_{\rm eff}^3}{m_\chi^5} \overset{!}{\sim} \frac{T^2}{M_{pl}} \sim \frac{m_\chi^2}{x_f^2 M_{pl}}$$

$$\Rightarrow \underline{\underline{m_\chi \sim \alpha_{\rm eff}\,(T_{eq}^2 M_{pl})^{1/3}}} \sim \underline{\underline{\alpha_{\rm eff}\cdot(100\ \mathrm{MeV})}}$$

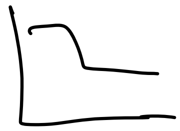
"generalized geometric mean M_{pl}, T_{eq}".

If $\alpha_{\rm eff}\sim1$, strong scale emerges!

Strongly Interacting Massive Particle. "SIMP".

"The SIMP Miracle".

much lighter mass than before. very diff' motivating!

Similar f.o. picture 

Note: $x_F \sim 20$ still, f.o. when NR:

$$n_\chi \sim (mT)^{3/2} e^{-m/T}$$

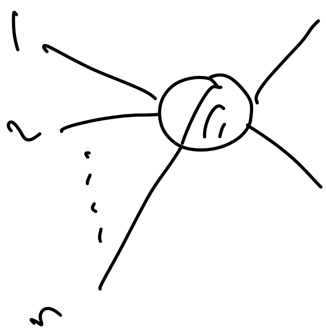
$$n_\chi^2 \sim (mT)^3 e^{-2m/T}$$

$$n_\chi^2 \langle \sigma v^2 \rangle_{3\to 2} \sim H$$

$$\Rightarrow x_F \sim \ln(\text{params}) \sim 20$$

More generally, $n \to 2$ interacting: (for sym', or fermions, $\cancel{3\to 2}$)

$n \to 2$



self int

$$\langle \sigma v^{n-1} \rangle_{n\to 2} = \frac{\alpha^n}{m_\chi^{2+3(n-2)}}$$

See how - detailed balance: forward = backward in eq:

$$(n^{eq})^n \langle \sigma v^{n-1} \rangle_{n\to 2} - (n^{eq})^2 \langle \sigma v \rangle_{2\to n} = 0$$

$$\Rightarrow \langle \sigma v^{n-1} \rangle_{n\to 2} = \underbrace{(n^{eq})^{2-n}}_{[3(2-n)]} \underbrace{\langle \sigma v \rangle_{2\to n}}_{[-2]} \sim \underline{\underline{m^{-[2+3(n-2)]}}}$$

$\Gamma_{n\to 2} \sim n_\chi^{n-1} \langle \sigma v^{n-2} \rangle_{n\to 2} \sim H$

(each n_χ @ f_0 has 1 power of T_{eq})

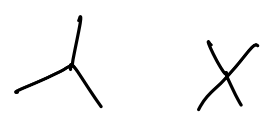
$\Rightarrow \quad m_\chi \sim \alpha \left(T_{eq}^{n-1} M_{pl} \right)^{\frac{1}{n}}$

e.g. $n=4$; $4\to 2$: $m_\chi \sim \alpha \cdot (100 \text{ keV})$

$n\to 2$ less familiar than $2\to 2$ – Toy Model

(later see Dark Sectors)

3→2 Toy Model – $\mathbb{Z}_3$: single scalar ϕ : χ^3, $|\chi|^4$

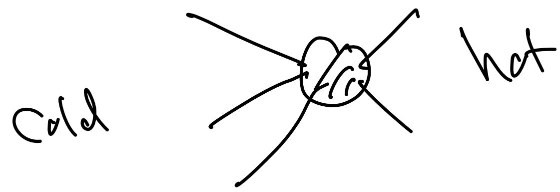


3 vertices – inspiration for parametrization of "α^3" for χ sec

3×3pt or 3pt + 4pt for instance.

Been cheating you!

Implicitly been assuming 1 temp' for the entire system.

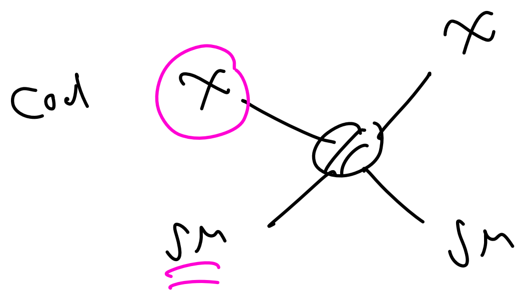


$3 \to 2$ pumps heat into the system.

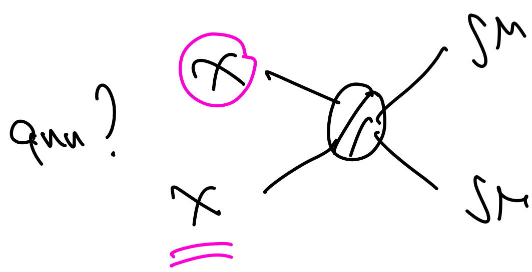
Need to be able to cool, to dump entropy!

Can be either to other light state or SM.

Exchange Heat w/ SM:



$\Gamma_{kin} \sim n_{SM} \underbrace{\langle \sigma v \rangle}$



$\Gamma_{ann} \sim n_X \underbrace{\langle \sigma v \rangle}$

can't be done - cool w/o ann?

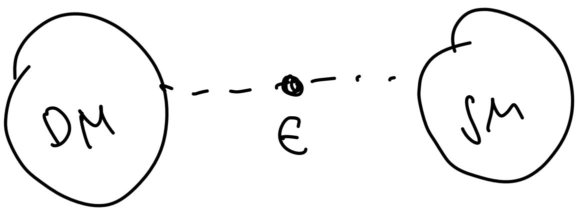
$$\frac{\Gamma_{ann}}{\Gamma_{kin}} \sim \frac{n_X}{n_{SM}} \sim e^{-m_X/T} \sim 10^{-8} \ll 1$$

scatter off of light SM species - γ, ν, e

⇒ Scatter off of light (abundant) SM species: e, γ, ν

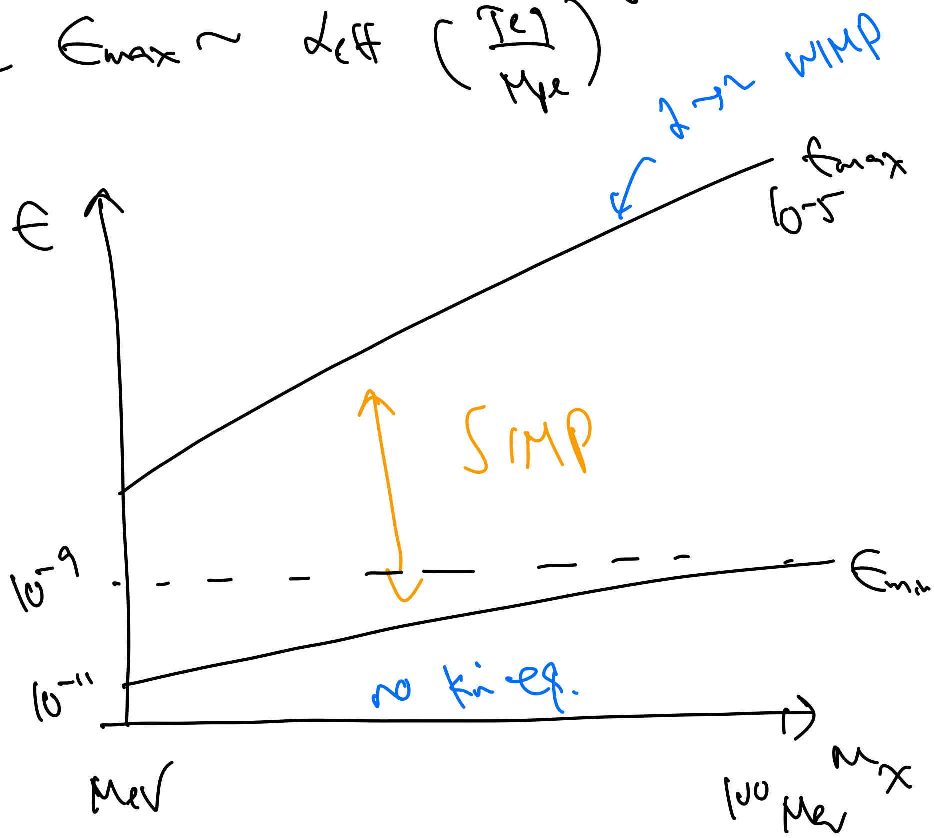
⇒ conditions: $\left.\frac{\Gamma_{kin}}{\Gamma_{3\to2}}\right|_{F.O.} \gtrsim 1$, $\left.\frac{\Gamma_{ann}}{\Gamma_{3\to2}}\right|_{F.O.} \lesssim 1$

parametrize SIMP-SM int': $\langle\sigma v\rangle_{kin} \sim \langle\sigma v\rangle_{ann} \equiv \frac{\epsilon^2}{m_\chi^2}$



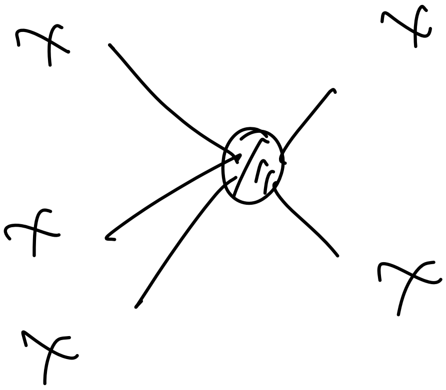
⇒ Range of ε where works:

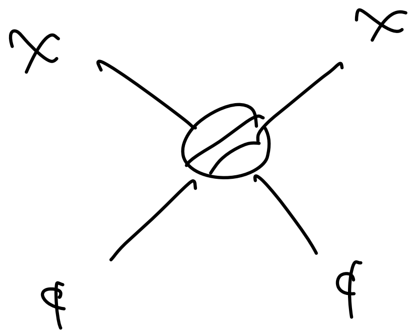
$$\begin{cases} \epsilon_{min} \sim \alpha_{eff}^{1/2} \left(\frac{T_{eq}}{M_{pl}}\right)^{1/3} \\ \epsilon_{max} \sim \alpha_{eff} \left(\frac{T_{eq}}{M_{pl}}\right)^{1/6} \end{cases}$$



as long as $\epsilon_{min} \le \epsilon \le \epsilon_{max}$, precise value doesn't matter.

SIMP





decouple 1st.

decouple 2nd
(active during f.o.)

what happens if order is reversed?

$\Rightarrow$ FIRST VIA CANNIBALS...