BSM + Naturalness

An irreverent and biased introduction

Real subject is model building.
When to do?
What does success look like?
How to go about model building?

When? Occam's Razor ⇒ new things
are usually bad!

1) SM and Data disagree
e.g. neutrino mass
W boson mass
H_0 tension

2) Theory motivated : Solves a theoretical
problem

$m_p << M_{Pl}$, Reductionism - Newton vs.
Celestial sphere
self energy of electron in classical mechanics

3) Experiment motivated: Design a model for an experiment

many discoveries are accidents e.g. muon

The focus of these lectures will be point 2.

What does success look like?

Depends on goal!
if 1), 3) then success is a model that is not excluded and explains new data
can be seen in new experiment

if theory issue: Solve a problem, elegant
can be tested on a reasonable time scale.

Please don't be 1001th model of DM production / GWs that has no or identical signals.

how to model build

1) find a problem (theory or issue with another model)
2) write a model that solves it

Theory problem in the SM (nonexhaustive list)

focus of these lecture [
cosmological constant
Higgs mass
strong CP

Flavor – CKM matrix & quark mass & ν
Inflation
Dark matter : what's it? How to get it? $\rho_{DM} = 5\rho_{SM}$
baryogenesis

Problems with Model – do not underestimate!
Think of famous model builders, most of them spent time fixing problems with other people's models.

e.g. MSSM

a) fewer parameters $\sim 120 \Rightarrow O(10)$
gauge mediation, gaugino mediation, ...

b) coincidence μ, B_μ problem

Model builders are good at finding problems and fixing problems.

eg. Rattazzi famous destroyer and builder
started bootstrap program to prove a friend wrong.

Throughout lectures I will constantly point out problems/issues. These are "opportunities"!!!
they are not just complaints.

Okay lets get started with theory motivation.
other than ~~theory~~ reductionism, it might not be obvious what does this mean.

Starting Point : Dimensional Analysis
sometimes called Naturalness

Example 1) Drop a ball, sometime later you hear it hit the floor.

How far away is the floor?

$$d \sim g t^2 \quad \Rightarrow \text{ get distance up to } O(1) \text{ number}$$

but if you missed physics, you can get the wrong answer if you measure and see a different answer, you suspect new physics e.g. air resistance

$$\cancel{d \sim \frac{bt}{mg}} \qquad d \sim mgt/\overset{\uparrow}{b}$$

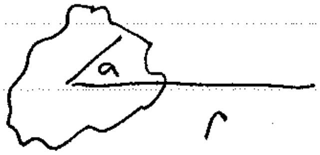
coefficient of air resistance

Example 2) frequency of a pendulum

$$\omega \sim \sqrt{g/L}$$

Example 3) Measure multipole expansion of an object

What is the size of the object?



$$V = \frac{1}{r} \sum_{l,m} \left(\frac{a}{r}\right)^{l} C_{lm} Y_{lm}(\hat{r})$$

$$\nearrow$$

analogous to EFT

$$V = \frac{1}{r}\left(\frac{b}{r} Y_{10}(\hat{r}) + \frac{d}{r^2} Y_{20}(\hat{r}) + \dots \right)$$

$b \approx a C_{10}$ $\quad$ $C_{10}, C_{20} \sim O(1)$ same number

$d \sim a^2 C_{20}$

$$a \approx b/d$$

can fail if symmetries are such that one is abnormally small

Example 4) Assuming point-like electron, E&M
when does one of these assumption fail?

electron ~ ball $\quad E = m_e c^2 = m_{bare} + e^2/R$

if E&M unimportant $m_e \sim m_{bare}$

important $m_e \sim e^2/R$

so $R \gtrsim e^2/m_e$ something breaks

What actually happens? @ $R \sim 1/m_e$ QM fuzzes you out and $V \neq 1/r$

add  so that $V \sim \alpha m_e$

We will do QFT next, so how to do dimensional analysis in QFT?

Dimensions right: Energy/distance when something happens $\Lambda = 1/r$

Spurions: fermion mass $m\psi\psi^c$

$$m \to e^{2i\theta} m \qquad \psi, \psi^c \to e^{-i\theta}\psi, \psi^c$$

so that $\delta m = \# m + 0 \cdot \Lambda$
removes dependence on Λ up to logs

Boson mass: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$

$$S = \int \frac{d^4p}{(2\pi)^4} \frac{1}{2} \tilde{\phi}(-p)(p^2 - m^2)\tilde{\phi}(p)$$

ϕ real so $\tilde{\phi}^*(p) = \tilde{\phi}(-p)$

spurion: $\tilde{\phi}(p) \to e^{i\theta(p)}\tilde{\phi}(p)$

ϕ still real as long as $\theta(p) = -\theta(-p)$

if $\theta = a_\mu p^\mu$ $\Rightarrow$ translation by a_μ $\delta\phi = a_\mu \partial^\mu \phi$

$\theta = a_{\mu\nu\rho}\, p^\mu p^\nu p^\rho \Rightarrow \delta\phi = a_{\mu\nu\rho}\, \partial^\mu \partial^\nu \partial^\rho \phi$

$\vdots$

any odd p

Say $\mathcal{L} = \lambda \phi^4 \rightarrow \int d^4p_1\, d^4p_2\, d^4p_3\, d^4p_4 \;\; \lambda(p_1 \cdots p_4)\, \delta^4(\sum p)$

$$\tilde{\phi}(p_1)\, \tilde{\phi}(p_2)\, \tilde{\phi}(p_3)\, \tilde{\phi}(p_4)$$

$$\lambda_{(p_1 \cdots p_4)} \rightarrow \lambda\, e^{-i[\theta(p_1)+\theta(p_2)+\theta(p_3)+\theta(p_4)]}$$

$$m^2(p_1,p_2) \rightarrow m^2(p_1,p_2)\, e^{-i[\theta(p_1)+\theta(p_2)]}$$

$$\delta m^2 \propto \lambda(p_1, p_2, p, -p) \cdot \underbrace{1/R^2}_{\Lambda^2} \leftarrow \text{units}$$

<u>Exercise</u> $\delta m^2 \sim y^2 \Lambda^2 , e^2 \Lambda^2$

normally use diagrams to short cut this analysis

Example 5) charged pion

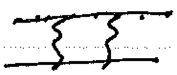
$$\delta m^2_{\pi^+} \sim e^2 \Lambda^2 \sim e^2/R^2 \text{ if E\&M is important}$$

$R \overset{\rightarrow}{\sim} e/m_\pi$ $\Rightarrow$ accurately predicts both ρ meson and size

more an accident

$$V \sim \frac{e^2}{R} - \frac{e^2}{R} e^{-mR}$$

Example 6) prediction of the charm quark mass

$m_{K_L} - m_{K_S} \sim$  $\sim m_K \frac{f_K^2 \Lambda^2}{m_W^4}$

got charm quark mass to $O(1)$

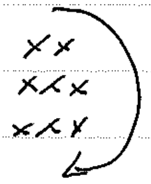
Example 7) proton mass

Why is $m_p \ll M_{Pl}$? Dimensional transmutation

$m_p \sim M_{Pl} e^{-8\pi^2/g^2}$ proton doesn't exist for $E \gtrsim m_p$

Robustly Predicts $m_p \ll M_{Pl}$ even though to get observed m_p need to tune g even more than before

Example 8) Fermi Predicts Schwinger Pair Production



electron goes in circles in a B field

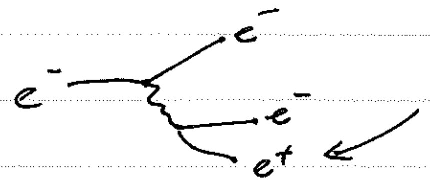
Larmor radiation $\omega_L \sim \gamma^2 \frac{eB}{m}$

negative energy is weird

$\omega_\gamma < \gamma m$ $\Rightarrow$ something happens when $\gamma e B \sim m^2$

or $eE_{eff} \sim m^2$ which is Schwinger!

In this case



not predicted or seen yet!

stimulated Schwinger in a B field

Example 9) axions

axions couple to neutron edm $\frac{a}{f}\frac{n^{\dagger}\sigma^{\mu\nu}n}{\Lambda_{QCD}}F_{\mu\nu}$

or analysis as before

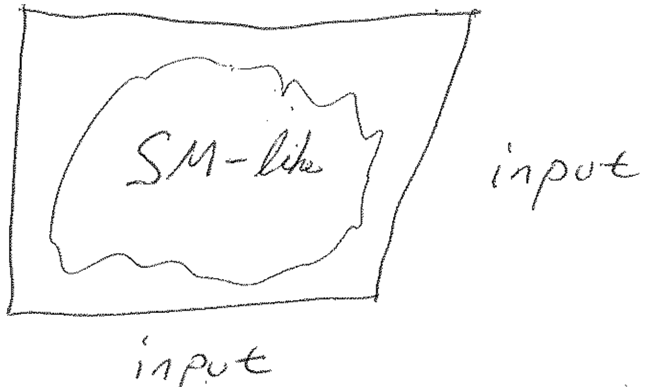
$$\delta m_a^2 \sim \frac{\Lambda^5}{f_a^2\Lambda_{QCD}^2} \leq m_{a,obs}^2 \sim \frac{\Lambda_{QCD}^4}{f_a^2}$$

$\Rightarrow \Lambda < \Lambda_{QCD}$ so $r_{Proton} \sim 1/\Lambda_{QCD}$ which is what we see.

Summary of philosophy

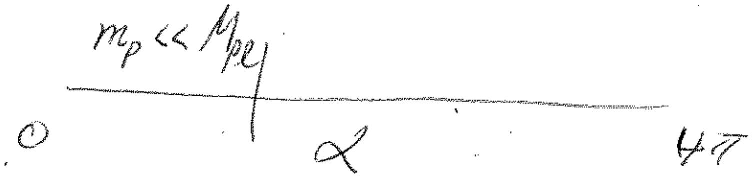
Top down

what do we want in a theory?

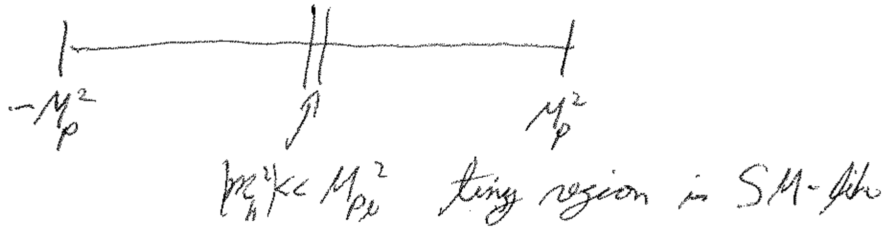


not well defined, dim. analysis sort of implies linear prior / distribution

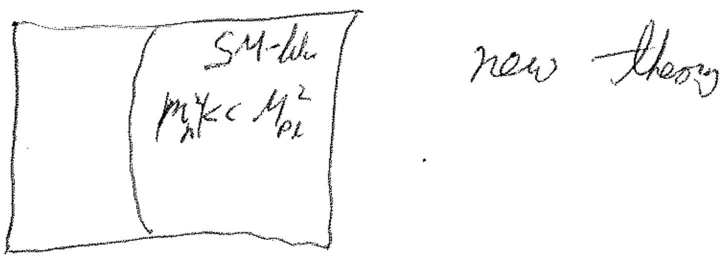
e.g. QCD



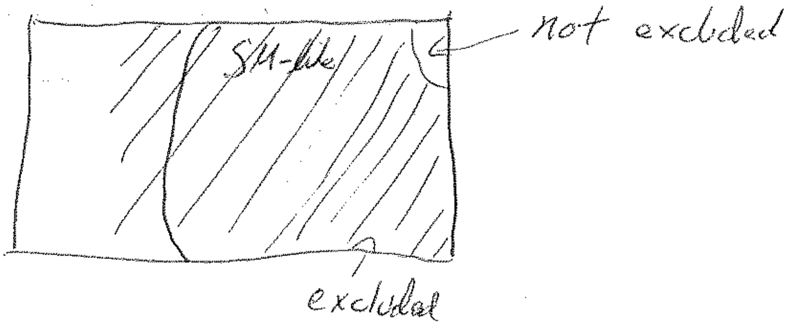
unhappy with SM



goal



actually want more - see SUSY

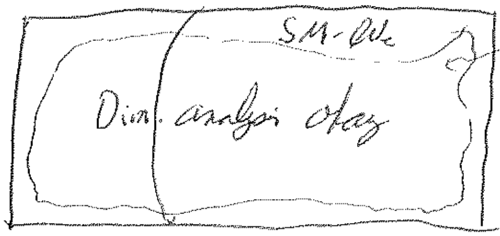


what does it look like?

$$\delta m_h^2 = \cancel{100}(10^8 GeV)^2 - (10^8 GeV)^2 = -(100 GeV)^2$$

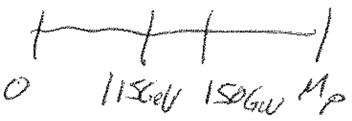
why are cancelation bad? manifestation of dim analysis failing. cancelation good if symmetry forces it.

really want



Care about overlap.

small non-excluded parameter space okay



small region of non-excluded higgs mass

Why is the overlap currently excluded?
C.C., m_h, etc.

say C.C. $\Lambda^4_{c.c.} \sim (\text{meV})^4$

dim. analysis say must be some reason meV scale is important (doesn't say why)

Dim. analysis says something in theory says meV is important. But we've explored up to TeV scale! TeV >> meV so very likely any theory where meV is important has been explored.

e.g. SUSY → changes E&M → $m \gtrsim$ TeV

Fat graviton → SM unchanged up to TeV
gravity changed at meV
okay?

Now Higgs.

Dim. analysis ← [scale] ~ m_h/y, m_h/g, $m_h/\sqrt{\lambda}$ ~ 100 GeV – 10 TeV

a bit generous with O(1) numbers ~ $O(10^4)$

if 10 TeV – haven't necessarily explored it (e.g. flavor has)
collider hasn't

so depending on O(1) numbers may be okay
what appears at [scale] may be okay.
or living outside of Dim. analysis is okay.

Currently all "old" models (age > 10 years)
O(1) doesn't help, & at [scale] stuff too easy to see.

Note: we are dropping highest bar of O(1) fraction of parameter space being SM-like. come back to it after a non-excluded, dim. analysis okay, theory of Higgs mass
c.c.
strong CP.

Cosmological Constant

energy density of vacuum : $\rho_{obs} \sim (2\times10^{-3}eV)^4$

ala proton mass $\rho_{guess} \sim M_p^4$

Like it or hate it there is a prediction for ρ that was experimentally verified.

Anthropics : Weinberg - need stars for life
Steve Weinberg - "Anthropic Bound on the Cosmological Constant"

$\rho_{c.c.} \lesssim \rho$(stars first formed)

Back then Quasars @ $z_0 = 4.5$

$\rho_{c.c.} \lesssim \rho_c \Omega_m (1+z)^3 \sim 60\, \rho_{obs}$

now galaxies @ $z = 8.6$

$\rho_{c.c.} \lesssim 300\, \rho_{obs}$

Solves the wrong problem! Says why $\rho_{c.c.} \lesssim M_p^4$ not how it happened.

e.g. proton mass needs $m_p \sim GeV$ but $m_p \sim M_p e^{-8\pi^2/g^2}$ is a nice mechanism for how.

So how does the c.c. get small?

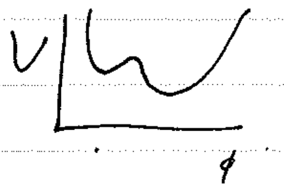
1) Brute Force$^{+\text{anthropics}}$: Only fully consistent solutions
- is a solution to naturalness problems in context of atomic lines
- bonus - seen before,

If at first you fail, try try try again

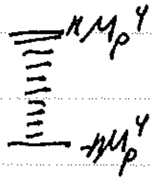
$$\mathcal{L} = \sum_i^{n} -m^2\phi^2 + \kappa\phi^3 + \lambda\phi^4$$

$\uparrow$ $\nearrow$

$\sim M_p$



2 vacua per scalar w/ $\Delta V \sim M_p^4$



evenly distributed $\Delta V \sim nM_p^4/N$

$N = \#$ of vacua $= 2^n$

to get observed $N \sim M_p^4/\rho_{obs} \sim 10^{120} \sim 2^{n}$

if $n \sim 400$ scalars
- a) avoids measure problem if just 1 vacua is right
- b) only mathematically "sound" solution
- c) seen before

Currently best & only option ☺

Since only option lets critique it

A). Measure Problem : very unlikely only 1 vacua $\rho < \rho_{c.c.}$

afflicts all multiple vacua solutions

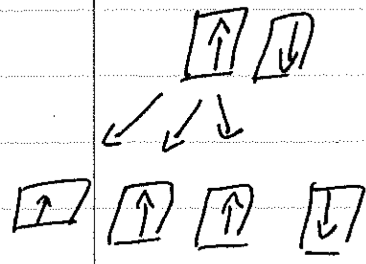
How do you define probability in Quantum GR?

Normally see more of something ⇒ more likely
Experimentally not true in Quantum GR. ~~E~~ CMB is
many causally disconnected regions of space all with the
same T. Clearly Quantum Gravity causes vast amounts
of entanglement. So can't treat as independent.
Likely not 100% entanglement since not copies

~~e.g. 100% entanglement.~~

Toy model : 50% ↑ , 50% ↓ doesn't need to be

now expansion 100% entanglement ↙ entanglement could be correlation



50/50

75/25

not 66/33

now 0% entanglement

$\boxed{\uparrow}$ $\boxed{\downarrow}$ 50/50

$\boxed{\uparrow}$ $\boxed{\downarrow}$ $\boxed{\uparrow}$ $\boxed{\downarrow}$ 50/50

so observationally neither, can't volume weight

no idea what to do?!! State of the art
making up answers

another maybe issue are Boltzmann Brain
Treat during inflation in static patch. So non-zero
probability that you fluctuated into being

B2) Not close enough to anthropic bound: are
very subject to measure problem

say ρ_a = anthropic bound

If flat distribution $P(\rho) = \frac{1}{\rho_a}$

so that $\int_0^{\rho_a} d\rho \, P(\rho) = 1$

be sloppy re
negative c.c.

be sloppy about negative cc.

$$\langle \rho \rangle = \int_0^{\rho_a} d\rho \, \rho \, P(\rho) = \frac{\rho_a}{2}$$

so expect ρ to be pretty close to ρ_a

What about a QCD-like solution?

$\rho = e^{-1/\alpha} M_p^4$ as before neglect negative c.c.

$$\alpha_a : \rho_a = e^{-1/\alpha_a} M_p^4 \Rightarrow \alpha_a = 1/\log(M_p^4/\rho_a)$$

$$\langle \rho_a \rangle = \int_0^{\alpha_a} d\alpha \, \rho \, P(\alpha) \approx \frac{\text{[illegible]}}{\alpha_a} e^{-1/\alpha_a} M_p^4 = \frac{\rho_a}{\log(M_p^4/\rho_a)}$$

$$1 = \int_0^{\alpha_a} d\alpha \, P(\alpha) \Rightarrow P(\alpha) = 1/\alpha_a$$

so guess $\rho \sim \frac{\rho_a}{300}$ which is what we observe!

No other good ideas for getting c.c. small!

Interesting 1/2 ideas or excluded attempts.

2) SUSY + $U(1)_R$

In SUGRA $V = e^K(K^{-1}_{\phi\bar\phi}|D_\phi W|^2 - 3|W|^2)$

$U(1)_R$; $W = 0$

Unbroken SUSY : $D_\phi W = 0$

Unless fine tuned cancelation SUSY breaking @ meV so selectron @ $m_e \pm$ meV

so very good idea but very excluded

We know $D_\phi W \gtrsim (\text{TeV})^2$

$$\Rightarrow \rho_{SUSY} \sim \text{TeV}^4 \sim 10^{60} \rho_{obs}$$

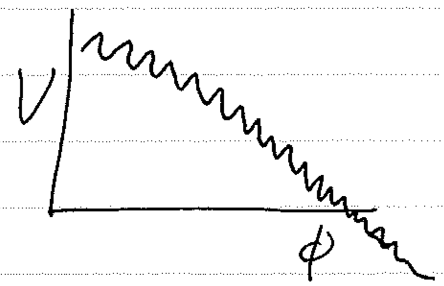
3) Relaxation : Historical

$$\begin{array}{ccc} \phi = C \neq 0 & \not\Rightarrow & \phi = C \neq 0 \\ d_e \neq 0 & & d_e \neq 0 \end{array}$$

L.F. Abbott - "A Mechanism for Reducing The Value of the C.C."

Starting attempt : arguably still the best attempt

have a potential



start near top and tunnel down eventually reach $V \approx 0$ if $\Delta V \lesssim (meV)^4$

"Relax" to c.c = 0!

Problems : a) Reheat temperature small + non-uniform

$T_{RH} \lesssim (\Delta V)^{1/4}$ also very non-uniform as opposed to our observed universe.

b) goes past $V_{c.c.} = 0$ so to what extent is it an actual relaxation mechanism?

infinities

c) Many vacua so it has eternal inflation and the measure problem → no idea about probabilities

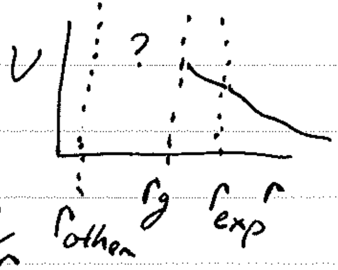
All known relaxation attempts have these problems and typically more, such as not being mathematically consistent.

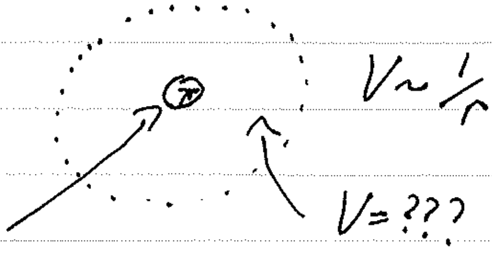
hep-th/0306106

4) Fat Graviton: actually no contradiction yet. Problem comes from assuming $V=\frac{GmM}{r}$ beyond what is experimentally valid.

if UV cutoff of gravity was $1/\text{meV} \sim 0.1$ mm $\sim r_g$ then no issue.

By analogy it looks like





other probes indicate $r_e \ll r_g$

could be like pion if pion was small & ρ still existed

Experimentally we have not tested gravity @ 0.1 mm scale, but we are close! off shell probes of gravity are hard

Intriguing idea: no idea how to make it precise.

EW Hierarchy Problem

Guess scale @ which Higgs radius comes into play or when potential changes form

using previous rules, we find

$\Lambda^2 \sim m_h^2/\lambda, \; m_h^2/g^2, \; m_h^2/y^2$

$\Lambda \sim 100 - 10^3$ GeV

first reaction, explored 100 GeV scale well but not TeV scale so lots of parameter space available

So lets reframe the question: Is there a model where the Higgs mass is calculable and not excluded?

Lots of models so lets categorize them

We have previously seen solutions and new solutions.

generic / boring : Technicolor
composite Higgs
Relaxation
Human Stupidity / got symmetries wrong
Historical solutions
anthropics
CFT : AdS/CFT

non-generic / aliens / government conspiracy : SUSY
Extra dimensions
Little Higgs
Twin Higgs
XXX Higgs
Measure Based Solutions
UV/IR mixing

Lets do a quick review of the aliens to see why people care

SUSY : Fermionic extra dimension

see e.g. hep-ph/9709356

Only possible extension to the Lorentz group other than CFTs.

Lots of very nice theoretical properties

predicts $m_h \leq m_Z$ so excluded by data also gives $\delta m_h^2 \sim \frac{6 y_t^2 m_{\tilde t}^2}{16\pi^2}$ ⇒ stops between 500-600GeV

also excluded

hep-ph/9803315

Extra Dimensions $d > 4$?

small so we haven't seen them yet,
or we can't travel in them (confined to a brane)

New particles come from $p_5 = \frac{n\pi}{L}$

$$E^2 - p^2 - p_5^2 = m^2 \qquad m_{eff}^2 = m^2 + \frac{n^2\pi^2}{L^2}$$

called KK modes, infinite tower of particles

In models $\frac{6 y_t^2 m_{\tilde t}^2}{16\pi^2} \sim \delta m_h^2$

so m_{KK} also @ 500-600 GeV ⇒ excluded by colored particle searches.

hep-ph/0506256

Twin Higgs : make new particles LHC-phobic

step 1) remove all gauge charges from $m_{\tilde t}$

$$\mathcal{L} = y H_1 Q_1 u_1^c + y H_2 Q_2 u_2^c - \lambda (H_1^2 + H_2^2 - f^2)^2 + O \cdot (H_1^4 + H_2^4)$$

$\mathbb{Z}_2$ symmetry : different charge

Impose SO(4) symmetry - Higgs is pseudo-Goldstone

$H_1 = f \sin h/f \approx \frac{f+h}{\sqrt{2}}$ $\mathbb{Z}_2$ symmetric min

$H_2 = f \cos h/f \approx \frac{f-h}{\sqrt{2}}$

so $\mathcal{L} = \frac{y}{\sqrt{2}} h (Q_1 u_1^c \mp Q_2 u_2^c)$ so equal coupling of higgs to a different set of uncharged quarks

so properties of the Higgs/EW sector are what exclude this.

People Like this because you need to tune it less as Higgs properties are more poorly measured than searches for colored particles.

Lets look @ the theories we've seen before and rank them by how often they're solutions to naturalness problems.

1 & 2 : Compositeness & Relaxation

3 : Human Stupidity / We are missing physics / CFT

4 : Technicolor / Maybe unfair since no classical analog

5 : Historical Solutions

6 : Anthropics

all better than aliens!

Let's go over these solutions in the order in which they were invented.

Technicolor : QCD breaks EW symmetry.

What if W/Z bosons get a mass from a QCD'?
Excluded by discovery of the Higgs boson

Composite Higgs :

Compositeness is 1 of the two most common solutions to a naturalness problem. Classically it is like before where we estimate size of object.

examples : size of proton from dipole
size of non-sphericity of a cow
" " phonon
" " Landau-Ginzburg scalar
...

idea : what if Higgs were a pion?

If you scale up pion results at odds with
EW & precision Higgs

also top partner?

Old Anthropics

can't ignore! Why do we live in a part of the universe where matter exists when most of the universe is empty?
Need matter to think

as before anthropics explains value not how
Atomic Principle - hep-ph/9707380
if $v < \Lambda_{QCD}$: Sphalerons turn all baryons into neutrinos so no proton/neutron & atoms

$\Lambda_{QCD} < v < 5v_0$: a okay

$5v_0 < v < 10^3 v_0$: neutron is too heavy

$m_n - m_p - m_e > Q_{binding} \Rightarrow$ all nuclei decay

$10^3 v_0 < v$: $uuu = \Delta^{++}$ more stable than p,n
so universe decays into Δ^{++}

so either all $\Delta^{++} e^- e^-$ or $p^+ e^-$
helium hydrogen
no complicated atoms.

$V \gtrsim 1.2\, V_0$ deutron unbound so no nucleosynthesis

start of nucleosynthesis is $p^+ + n \to D^+ + \gamma$

afterwards $D + (p \text{ or } D) \to$ Helium, Tritium $\to$ other

old anthropic is usually brute force + anthropics.

<u>New Anthropics</u>

What if anthropics for the c.c. affects the Higgs boson mass?

2 general approaches 1) more vacua when $\langle h \rangle = v$

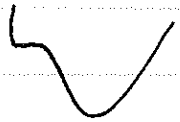
so more ~~vacua~~ likely that c.c. = correct value when $\langle h \rangle \sim v$. $\langle h \rangle \sim v$ changes fine scanning

2) What if $\langle h \rangle \sim v$ changes coarse scanning so that you can now ~~[illegible]~~ hit $V \sim 0$

Lets write toy models which do this

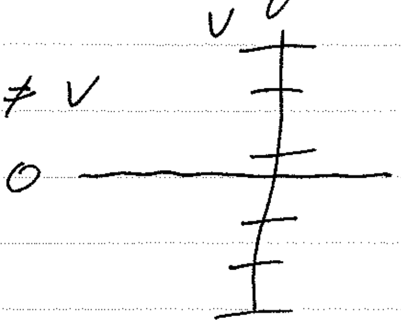
1) $V = -\frac{m^2}{2}|\phi|^2 + \lambda|\phi|^4 + \kappa\,\phi\,H_u H_d$, N scalars ϕ

if $\langle H_u H_d\rangle = 0$ $V =$  2 identical vacua

if $\langle H_u H_d\rangle \gtrsim \frac{m^3}{\sqrt{\lambda}\,\kappa}$  1 vacua

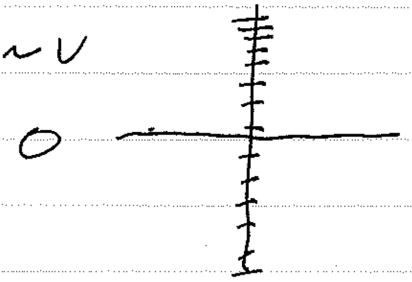
only if $\langle H_u H_d\rangle$ small do you get 2 vacua to do a fine scan

$\langle H_u H_d\rangle \neq v$



coarse scan not close to zero

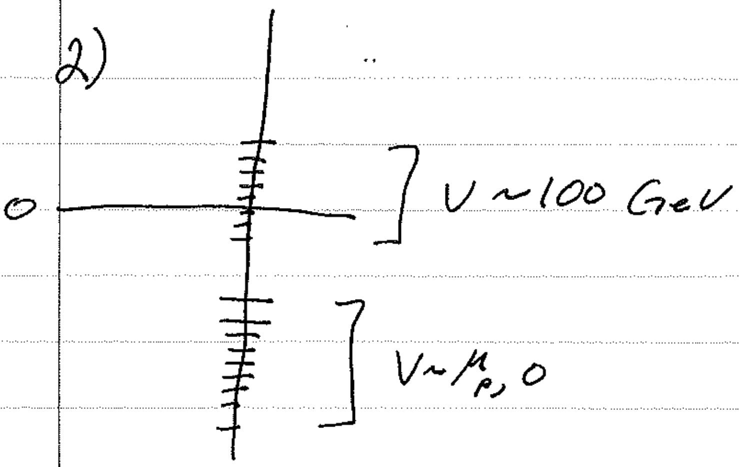
$\langle H_u H_d\rangle \sim v$



fine scan with 2^N new vacua

hep-ph/2012.04652

2)



Main issues :

a) rewriting a full UV model

b) If you're using anthropic + brute force for c.c.
Why not also for Higgs?
Almost no additional cost.

c) No example of anthropic + Brute force?

d) Measure problem w/ multiple vacua?

Historical Solution

Think experimentalist's lab : lots of finetuned things, function of environment + history

Nnaturalness : take inspiration from nuclear transitions

many different transitions, some are accidentally tuned

hep-ph/1607.06821

1st pass : N scalars w/ random mass + interaction

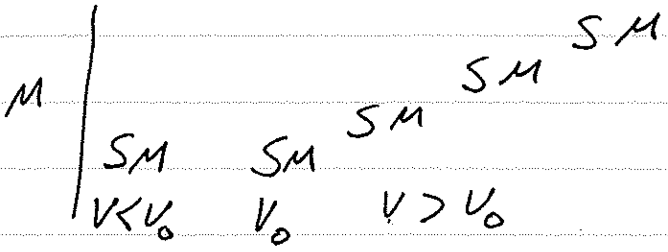
$$V = \sum \frac{1}{2} m_{ij}^2 \phi_i \phi_j + K_{ijk} \phi_i \phi_j \phi_k + \lambda_{ijkl} \phi_i \phi_j \phi_k \phi_l$$

What happens to any number of initial particles?
ends up in the lightest particle!

lightest has $M/\sqrt{N}$ where M = typical mass scale
Solves the problem! observers see & are made of the lightest scalar because its all that is around.
Due to history!

Now lets do Higgs
a) Lots of Higgs $\Rightarrow$ W,Z boson $m_W >> m_h$

b) Lots of SM-like



1) $v < v_0$ has stuff lighter than us
2) lots of massless particles in all sectors $N_{eff} >> 1$

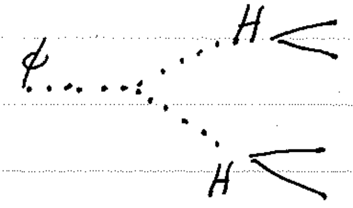
so this history is out. need new history.

New history : Reheating occurs via reheaton ϕ

ϕ couples equally to all sectors via lowest dimensional operator

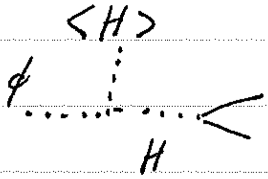
$V = K\phi \sum_i H_i H_i^\dagger$ assume $m_\phi^2 <$ all m_H^2

if $m_H^2 > 0$



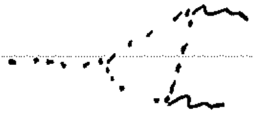
$\Gamma \sim \frac{K^2 m_\phi^7}{m_H^8}$

if $m_H^2 < 0$



$\Gamma \sim \frac{K^2}{m_H^2} m_\phi$

if $m_H^2 > 0$



$\Rightarrow \Gamma \sim \frac{K^2}{m_H^4} m_\phi^3$

so we see that we reheat to the lowest $V \neq 0$ Higgs sector aka us!

Main issues :

a) $N_{eff} \neq 0$ still need a bit of an accident

b) low T_{RH} MeV $< T_{RH} < 100$ GeV

b') $T_{max} > T_{RH}$ issue

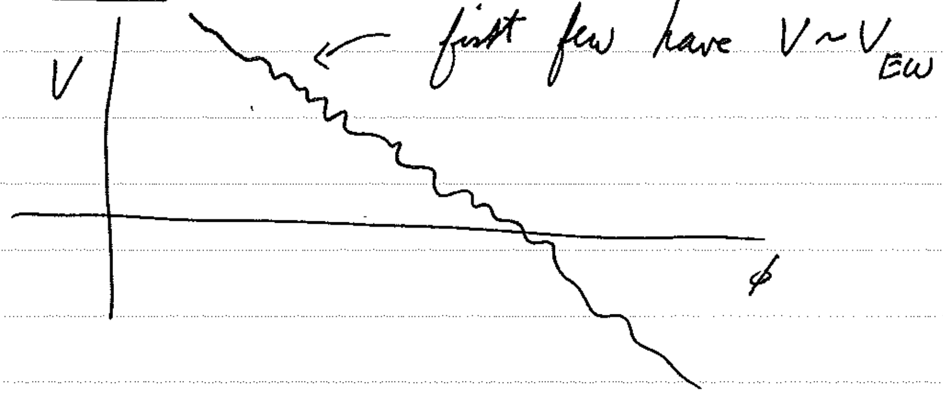
c) μ problem $m_\phi \sim m_h^{lightest}$

d) N large , $N \sim \Lambda^2/m_H^2$

Note all issues are experimental or aesthetic, good news!!

Relaxion

hepph/1504.07551



history : If you start to the left you will end up in a vacuum with $v \sim v_{EW}$ for a long time before tunnelling down. Needs friction though!
scans during inflation

Note : not a relaxation mechanism
True min has $v = \Lambda$

$$V = (\Lambda^2 + K\phi)|H|^2 + \lambda|H|^4 + K\phi\Lambda^2 + \frac{\phi}{f} G\tilde{G}$$

$$\Downarrow$$

$$y v \Lambda_{QCD}^3 \cos(\phi/f)$$

Choose parameters so that

if $v < v_{EW}$ $\qquad K\Lambda^2 > \frac{y v \Lambda_{QCD}^3}{f}$

so no min

if $v > v_{EW}$ $\qquad K\Lambda^2 < y v \Lambda_{QCD}^3 / f$

so there is a minimum

Constraints on parameters

a) assume $H \sim$ const since you scan $\Delta m_H^2 \sim \Lambda^2$
$\Delta\rho \sim \Lambda^4$ so $H \doteq \Lambda^2/M_p$

b) select correct V $\qquad K\Lambda^2 f \sim m_\pi^2 f_\pi^2$

c) Classical beats Quantum, no eternal inflation
$\Delta\phi$ in Hubble $\sim V'/H^2 \gtrsim H$

Note $\Delta\phi \sim N \cdot \dot{\phi}/H \sim N K\Lambda^2/H^2 \doteq \Lambda^2/K$ ← to scan Higgs mass

$N \doteq H^2/K^2$

Experimental Input $\quad f \doteq 10^9$ GeV
$\quad K\Lambda^2 \sim 10^{-13}\,\text{GeV}^3 \rightarrow H < 10^{-4}$ GeV
$\Lambda^2 < M_p H \Rightarrow \Lambda < 10^7$ GeV $\rightarrow K \sim 10^{-27}$ GeV
$N \doteq 10^{62}$

Main issues

a) 10^{62} e-foldings of inflation
hard to realize

b) $H_{\text{Inflation}} < 10^{-4}$ GeV, very low scale inflation

c) $\Delta\phi \doteq 10^{41}$ GeV $\gg M_p, f$

have some expectation that $\Delta\phi > M_p$ is hard

$\Delta\phi > f$ hard, need clockwork

$f_1 \sim 10^{32} f_2$

d) eternal inflation other vacua w/ c.c. > 0

e) Measure problem - did you really help?
w/ eternal inflation why are you early?
you could be later. Can't say how much you helped.

f) Make c.c. problem really hard.
randomly chose our vacuum to have c.c. = 0
real coincidence!

a & b solved by another friction mechanism
e.g. particle production

c use clockwork (multi axion models)

d - f conceptual problems.
maybe correlate w/ reheating?

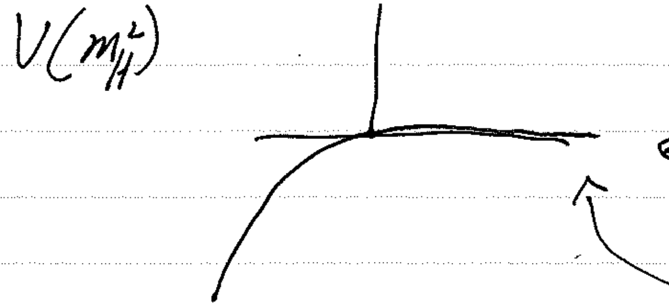
Relaxation hep-ph/ 1802.10093

with compositeness most common solution in classical physics

dipole moment of a capacitor
charge density rearranges itself onto a sphere so
dipole, quadrupole are all 0

dipole moment of CO_2

Relax m_H^2 to zero : Only one model!



issues to fix
a) max not min
b) flat
c) scan m_H^2 only hard
$\kappa \phi v^2 \ll \kappa \phi \Lambda^2$

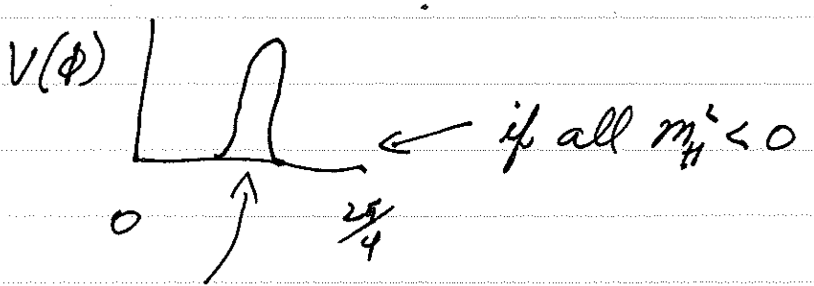
$\Rightarrow$ tilt + flip sign $\mathbb{Z}_N$ model

math simplest in $\mathbb{Z}_4$

4 Higgs bosons / SMs $H_i \to H_{i+1}$
non-linear $\mathbb{Z}_4$ $\phi/f \to \phi/f \mp 2\pi/4$

$$V = [\Lambda^2 + K^2\cos(\phi/f)]\,|H_1|^2 \quad + \lambda \sum_i |H_i|^4$$
$$[\Lambda^2 + K^2\sin(\phi/f)]\,|H_2|^2$$
$$[\Lambda^2 - K^2\cos(\phi/f)]\,|H_3|^2$$ no cross terms to make
$$[\Lambda^2 - K^2\sin(\phi/f)]\,|H_4|^2$$ life simple

$K^2 > \Lambda^2$ so Λ^2 not full UV cutoff



if $m_H^2 > 0$ so we have flipped the sign

now tilt → other potential for ϕ

UV contribution.

$$\phi/f \to \phi/f + 2\pi/N \Rightarrow (K^2)^N \cos(N\phi/f)$$

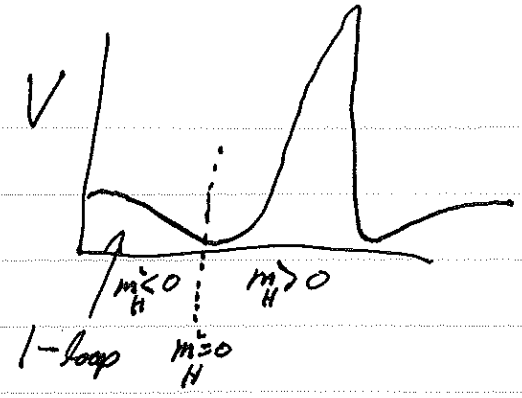
so UV contribution suppressed in large N limit

only non-polynomial thing can survive

1-loop : since $\log(K^{2N}\cos\frac{N\phi}{f}) \to \cos\frac{N\phi}{f}$

$$V \approx \sum_i |H_i|^4 \beta_\lambda \log |H_i|^2/\Lambda^2$$

$$\sim \frac{\beta_\lambda K^4}{N^2}\cos(N\phi/f)$$



m_H^2 @ min. $V' \approx -\frac{K^4\beta_\lambda}{Nf} + \overbrace{\frac{K^2}{f}\frac{m_H^2}{\lambda}\phi}^{\text{from } \frac{m_H^4}{\lambda}} = 0$

$\Rightarrow m_H^2 \sim \frac{K^2}{N}\underbrace{\lambda\beta_\lambda}_{1/100}$

smaller β_λ and λ, the smaller the mass!

given SM values win in m_H^2 by 100
so 100 GeV → TeV which is dicey

↑

Main issues

~~0)~~ $m_H^2>0$ not $m_H^2<0$
can be solved
1) abit of an odd structure
2) only 1 model!

Would be really odd if this was the only model.

Really odd that the most common naturalness solution has only 1 model when applied to the Higgs

Human Stupidity / We are missing something

first lets clarify what people say.

Dimensional analysis : Experimental fact, like Occam's razor, F=ma

For the LHC, people really care about O(1) numbers.

take $\frac{y}{\sqrt{2}}\phi\psi\psi^c$ 

$$\delta m_\phi^2 = -\frac{y^2}{2}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left(\frac{1}{\not p}\frac{1}{\not p}\right) = \frac{-y^2\Lambda^2}{8\pi^2}$$

what this means is that m_ϕ is not calculable. Dim Reg, UV cutoff are not physical so don't take any physical intuition from it.

Best version I know of : Assume UV theory where m_ϕ is calculable and divergence is cancelled by a single new particle w/ mass $m_{\tilde\psi}$

$$\delta m^2 = \frac{-y^2}{4\pi^2}\int p^3dp\left(\frac{1}{p^2}-\frac{1}{p^2+m_\psi^2}\right) = \frac{-y^2 m_{\tilde\psi}^2}{8\pi^2}$$

can get log if you want. So in this UV theory

$m_\phi^2 = m_0^2 - \frac{y^2 m_{\tilde\psi}^2}{8\pi^2}$ fine tuned if they cancel for no reason

$\Rightarrow m_{\tilde\psi}^2 \leq 8\pi^2 m_\phi^2 / y^2$.

$m_{\tilde\psi} \leq 500\,\text{GeV}$ if applied to Higgs, True for SUSY

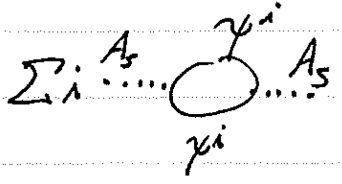
Let's use examples to see how much we should believe in these statements

1) Breaks assume 1 partner assumption

5D circle with 5D massless fermion coupled to a U(1) gauge boson

$S = 2\pi R \int d^4x \sum_n \bar\Psi^{(n)} \left(i\partial\!\!\!/ - m - i\left(\underset{\substack{\uparrow \\ \partial_5\ \text{piece}}}{\frac{n}{R}} - g_5 A_5\right)\gamma_5\right)\Psi^{(n)}$

$m_{\text{eff}}^2 = (n/R)^2 + m^2$



See hep-th/0508134 for details + math

1^{st} KK mode

a) Naive guess from before $\delta m_{A_5}^2 = \frac{-4g^2}{16\pi^2}\left(\frac{1}{R}\right)^2$

b) actual answer $\delta m_{A_5}^2 = \frac{-4g^2}{16\pi^2}\left(1/R\right)^2 \underbrace{\frac{6\zeta(3)}{4\pi^2}}_{0.2}$

so m_{KK}^2 is $\sqrt{5}$ times larger than expecting
clearly can violate by $O(1)$

Note - can see why in position space but not momentum space!

2) Another Multiple Partner example

Nima's g-2 example - hep-ph/2106.01373

consider $\mathcal{L} = \frac{e D^2 e^c}{\Lambda_1} + \frac{e D^2 e^c H^\dagger H}{\Lambda_2^3}$

close Higgs loop $\frac{\tilde{m}^2}{16\pi^2}\frac{1}{\Lambda_2^3} \lesssim \frac{1}{\Lambda_1}$

Now UV complete w/ 2 particles $\begin{matrix} S, N \\ S^c, N^c \end{matrix}$

$$\delta\mathcal{L} = e H^\dagger (y_L \ \ 0)\begin{pmatrix} S^c \\ N^c \end{pmatrix} + e^c H^- (S \ N)\begin{pmatrix} 0 \\ y_R \end{pmatrix}$$

$$+ (S N)\begin{pmatrix} m_S & 0 \\ \underset{\uparrow}{y_V' h_0} & m_L \end{pmatrix}\begin{pmatrix} S^c \\ N^c \end{pmatrix}$$

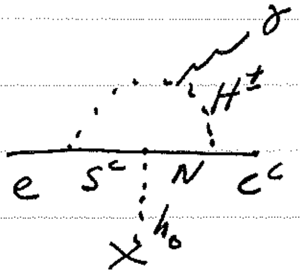
M, y_L, y_R spurions of $SU(2)_L \times SU(2)_R$

If analytic result

$$e D^2 e^c \; (y_L \;\; 0)\, M^{-1} \begin{pmatrix} 0 \\ y_R \end{pmatrix} = e D^2 e^c \; (y_L \;\; 0) \begin{pmatrix} 1/m_S & 0 \\ \frac{-y_\nu h_0}{m_S m_L} & 1/m_L \end{pmatrix} \begin{pmatrix} 0 \\ y_R \end{pmatrix}$$

$= 0$

1-loop result

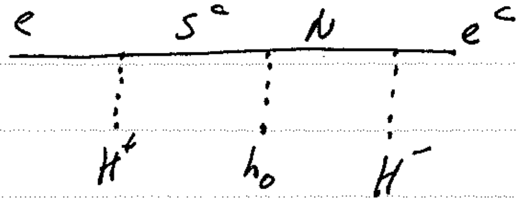


is analytic

none-zero @ 2-loop due to logs

if you diagonalize looks like $(y_L \; y_L') \begin{pmatrix} 1/m_H & \\ & 1/m_L \end{pmatrix} \begin{pmatrix} y_R \\ y_R' \end{pmatrix}$

$$1/\Lambda_1 \sim \frac{y^2}{(16\pi^2)^2} \frac{y_\nu h_0}{M^2} \log()$$



$$(y_L \;\; 0) \frac{1}{\underset{\parallel}{M M^\dagger M}} \begin{pmatrix} 0 \\ y_R \end{pmatrix}$$

$$y_\nu^2 h_0^2 m_S + m_S^3$$

same

$$y_\nu^3 h^3 + y_\nu h (m_L^2 + m_S^2) \qquad m_L (y_\nu^2 h_0^2 + m_L^2)$$

so $\quad 1/\Lambda_2^3 \sim \frac{y^2 y_\nu h}{M^4}$

Violates $\frac{m^2}{16\pi^2} \frac{1}{\Lambda_2^3} \leq \frac{1}{\Lambda_1}$ by a loop factor

3) Basis dependence

$\delta\mathcal{L} = \frac{\phi^2}{f^2} F^2$, no electrons

$\delta m_\phi^2 \sim \frac{\Lambda^4}{16\pi^2 f^2}$

field redefinition $\phi A \Rightarrow A$

$\delta\mathcal{L} = F^2 + (\partial\phi)....$

now $\phi \to \phi + \text{const}$ is a symmetry so that $\delta m^2 = 0$

3') $\phi F\tilde{F}$

$\delta m^2 = 0$

EM duality

$\phi F^2 + \phi F\tilde{F}$

$\delta m^2 \neq 0$

4) Multiple Portals on Steroids

Miss IR symmetries

Let's UV complete $(m + y\phi)\psi\psi^c$

if $m_{\tilde\psi}^2 \lesssim m_\phi^2/y^2 < m$ should have already seen

$\mathbb{Z}_N$ again

$$\mathcal{L} = \sum_j \left(m + yf\cos\left(\frac{\phi}{f} + \frac{2\pi j}{N}\right)\right) \psi_j \psi_j^c$$

$$\sum_i \cdots \bigcirc \Rightarrow \delta m_\phi^2 \approx \left(\frac{yf}{m}\right)^N \frac{m^4}{f^2}$$

linear tuning can give exponential confusion

can see from symmetry $\Phi = f e^{i\phi/f}$

$\Phi \to \Phi e^{i2\pi/N}$

$\delta V \sim (y\Phi)^N / m^{N-4}$ by symmetry if you can Taylor Series potential in $y\Phi$.

$m_\phi^2 \sim y^N$ not y^2 so you are parametrically wrong because you missed an IR symmetry

hep-ph/2306.12489

5) Lets look @ $V = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4$

another example of missed IR symmetries

guess $m_{NP}^2 \sim m_\phi^2/\lambda$ from 

UV complete into $V = \varepsilon^4 \cos(\phi/f)$

$m_\phi^2 \sim \varepsilon^4/f^2 \qquad \lambda \sim \varepsilon^4/f^4$

guess new particles at f

UV completion $U(1)/\mathbb{Z}_N$ via $\Phi \to \Phi e^{2\pi i/N}$

$$V = -\frac{1}{2}m^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 + \Phi^N/\Lambda^{N-4}$$

$\langle\Phi\rangle \simeq m/\sqrt{\lambda} \equiv F$ take $\lambda \sim 1$

so that $m_\rho \sim F$

$f = F/N$ goldstone boson potential

$$V = \frac{f^N}{\Lambda^{N-4}} \cos\left(N\phi/F\right)$$

$m_{guess} \sim f \sim F/N$ so guessed parametrically the wrong scale.

but dimensional analysis is never parametrically wrong. What happened? actually something does happen @ this scale.

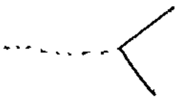
to more resemble the EW hierarchy problem lets consider a 'Yukawa version of this theory.

$\mathcal{L} = \sqrt{2}\, y f \sin \frac{\phi}{2f} \bar{\Psi}\psi \Rightarrow$ 

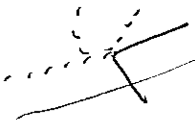
$m_{guess} \sim m_\phi / y$

~~Let~~ calculate m_ϕ

1) ~~physical yukawa~~



~~iy~~



~~$iy \left[-\frac{1}{2}\left(\frac{1}{2f}\right)^2 \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2} \right] = iy$~~

Lets calculate m_ϕ

$\|$

$\cos\frac{\phi_0}{f} \nearrow$

$$\delta V = \frac{y^2 f^2}{2\pi^2}\int\frac{dx^2}{X^4}\left(1-\frac{D(x)}{4f^2}+\frac{1}{2}\left(-\frac{D(x)}{4f^2}\right)^2+\frac{1}{3!}\left(-\frac{D(x)}{4f^2}\right)^3+\dots\right)$$

gives usual divergence, we even choose dim reg.

$$= \frac{y^2 f^3 \cos(\frac{\phi_0}{f})}{2\pi^2}\int\frac{dx^3}{X^4}\, e^{-\frac{1}{4f^2}D(x)} = \frac{y^2 f^2}{2\pi^2}\int\frac{dx^3}{X^4}\, e^{-\frac{1}{16\pi^2 X^2 f^2}}$$

$$= 8y^2 f^4 \cos \phi_0/f$$

finite mass without new particles!

Idea that $\Lambda \leq$ new particle mass is completely wrong! what happens at scale predicted by naturalness is that higher loop diagrams are important.

EW Hierarchy problem is a vibrant and interesting area to work on. Lots of ideas and opportunities. Very young person dominated field because old people gave up.

Strong CP

What is the value of the term $\frac{\theta G\tilde{G}}{32\pi^2}$?

$$\mathcal{L} = \frac{\theta G\tilde{G}}{32\pi^2} + q\mathcal{M}q^c \qquad q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \mathcal{M} = \begin{pmatrix} m_u e^{i\theta_u} & \\ & m_d \end{pmatrix} e^{i\theta_a}$$

spontaneous breaking of $\langle qq^c \rangle \neq 0 = U$

	$SU(3)$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)_A$
A_μ	adj				
q	$\square$	$\square$		1	1
q^c	$\bar{\square}$		$\bar{\square}$	-1	1
$\mathcal{M}$		$\bar{\square}$	$\square$		-2
θ					$\theta \to \theta - 2\alpha$

lets write the most general $\mathcal{L}$ consistent with symmetries.

$$\mathcal{L} = f_\pi^2 \, \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + a f_\pi^3 \, \mathrm{Tr}(\mathcal{M}U)$$

$$+ \, b e^{i\theta} f_\pi^4 \det U$$

$$U = e^{i\frac{T^a \pi^a}{\sqrt{2} f_\pi}}$$

Mass for η' $\Rightarrow$ $\mathcal{L} = \frac{1}{2} m_{\eta'}^2 \left(\eta' - \frac{\theta f_\pi}{\sqrt{2}}\right)^2 + \ldots$

$\langle \eta' \rangle = \frac{\theta f_\pi}{\sqrt{2}}$ also let $\langle \underset{\substack{\shortparallel \\ \phi}}{\pi^0} \rangle \neq 0$

$$U = e^{\frac{i\theta}{2}} \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$$

$$V = -a f_\pi^3 \, \mathrm{Tr}\left[\begin{pmatrix} m_u e^{i\theta_u} & 0 \\ 0 & m_d e^{i\theta_d} \end{pmatrix} U \right] + h.c.$$

$$= -2a f_\pi^3 \left[m_u \cos\left(\phi + \frac{\bar\theta}{2}\right) + m_d \cos\left(\phi - \frac{\bar\theta}{2}\right) \right]$$

$$\bar\theta = \theta + \theta_u + \theta_d$$

$$\Rightarrow \tan\phi = \frac{m_u - m_d}{m_u + m_d} \tan\frac{\bar\theta}{2}$$

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\frac{\bar\theta}{2}}$$

$$m_\pi^2(\theta) = a f_\pi \sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos\bar\theta}$$

from pion mass sum rules see θ small
Current strongest constraint is the eDM of Hg
which gets contribution from nucleons.

future might be eDM of proton.

now $\theta \lesssim 10^{-10}$

Solutions

massless up quark
broken P or CP
axion

Massless up quark

main issue excluded

if $m_u = 0$ rotate away $\bar{\theta}$, θ_u ill defined

Parity or CP

set $\bar{\theta} = 0$ in UV, arrange corrections to be small.

SM correction $\beta_{\bar{\theta}} = \arg \mathrm{Tr}\left[f(Y_u, Y_d) \right]$

write first non zero thing $\beta_{\bar{\theta}} \propto Y_u^6 Y_d^6 g^2$
g^2 there to see CP violation in CKM matrix

Main issue here: how to naturally set up
$\theta_{CKM} \sim \mathcal{O}(1) \quad \bar{\theta} \approx 0$

Parity: $SU(2)_L \leftrightarrow SU(2)_R$

$$Q_L \leftrightarrow Q_R^\dagger$$
$$H_L \leftrightarrow H_R^\dagger$$
$$L_L \leftrightarrow L_R^\dagger$$

$\bar{\theta} = 0$ while $y_u \dfrac{Q_L H_L Q_R H_R}{\Lambda_u} + y_d \dfrac{Q_L H_L^\dagger Q_R H_R^\dagger}{\Lambda_d}$

so $y_u = y_u^\dagger \qquad y_d = y_d^\dagger$

$\bar{\theta} = \theta + \arg\det y_u + \arg\det y_d = 0$

but θ_{CKM} unconstrained!

issues: Only in simplest models does there exist no additional phase. Jump through hoops to suppress them.

any BSM will have large running of $\bar{\theta}$ if not careful

Parity breaking easily introduces $\bar{\theta} \neq 0$ e.g. if phase of $v_R \neq 0$

Higher Dim. operators can easily reintroduce problems.

complex scalar Φ ⇒ breaks CP

Charge Parity : new quarks ψ, ψ^c $U(1)_Y = \pm 1/3$

$$\mathcal{L} = \mu \psi \psi^c + Y^{ij} H Q_i d_j^c + A^i \psi d_i^c \Phi$$

$$\mathcal{M} = \begin{pmatrix} \mu & A^i \langle \Phi \rangle \\ \begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} & m_D \end{pmatrix}$$

Det $\mathcal{M}$ = real   but complex entries.

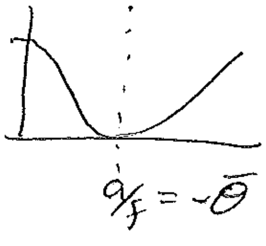
same issues as parity but always has extra phase. so a bit worse of a starting point.

Axion : what if θ was dynamical?

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f} \frac{G\tilde{G}}{32\pi^2}$$

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\bar{\theta} + a/f}{2}\right)}$$

min @ $\underset{+\frac{a}{f}}{\bar{\theta}} = 0!$



Main issue: how do you actually get only
this coupling?

sensitive to HDimension operators
Quality problem

actually so few problems, people have moved on
to other topics such as

ways to make axions dark matter
make it easier to find
large/smaller couplings and mass
new ways to look for it.