

Advanced GR : Conserved charges & black hole thermodynamics

Dec. 2021

- **goal** : advanced topics w/ view towards current research
 - assume good working knowledge of GR
 - review, if necessary, the notion of diffeomorphism, Einstein's eqns, black hole solutions (Schwarzschild, Kerr) and their properties

Outline : 2 main directions

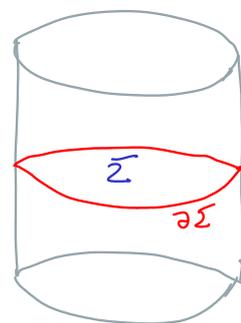
A. Conserved charges in gauge theories (start today)

- learn how to compute cons. charges (e.g. charge, energy, ang. mom.) in (in principle) arbitrary spacetimes using the **covariant phase space formalism** ← powerful & covariant ⇒ general
- ! in a **gauge theory** (local, rather than global symmetries $\varepsilon(\kappa)$ including GR), the cons. charges are **not** given as integrals of a local current, but instead they are **quasi-local** (associated w/ a subregion in spt.)

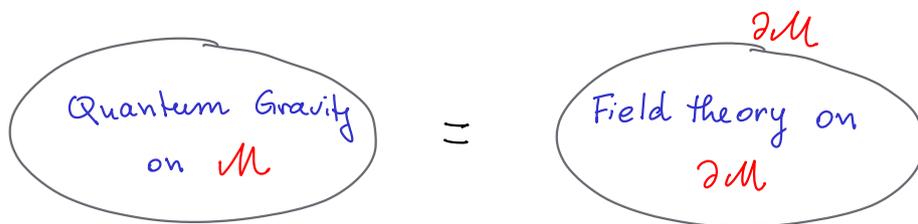
$$J_{\text{Noether}}^{\mu} = 0 + \nabla_{\nu} K^{\mu\nu} \quad \leftarrow \text{antisymm}$$

\uparrow
on-shell

$$Q = \int_{\partial\Sigma} d^{d-2}x K^{tr}$$



- notice that $Q \neq 0$ iff gauge parameter falls off slowly enough for the boundary contribution to be non-zero \Rightarrow in gauge theories, including GR, symmetries are **asymptotic symmetries**
- asympt. symmetries are **true** symmetries; they act non-trivially on phase space (even though gauge transf. \leftrightarrow redundancies of description, **large** gauge transformations are not).
- very interesting applications to **holography**



asympt. symm in gravity \leftrightarrow symmetries of the field theory

\hookrightarrow examples : AdS_3 & Minkowski $_4$ (time permitting)

B. Black hole thermodynamics

- very deep & fundamental relation between **gravitation**, **thermodyn** & **quantum theory**
- responsible for breakthrough in quantum gravity (holography)
- recent developments (QG \Leftrightarrow quantum info) are heavily based on this older subject of b.h. thermodynamics ('70's)

Plan

①. The four laws of black hole mechanics

- laws obeyed by geometric quantities associated to black holes that bear a remarkable mathematical resemblance to the laws of thermodynamics

- general proof & generalization due to Wald - Iyer $S_{BH} = \frac{A_H}{4G} + (\dots)$

② Converse of the 1st law : if entropy \propto area \Rightarrow Einstein eqns
(Jacobson)

\Rightarrow gravity not fundamental, but emergent

③ Geometry & the laws of entanglement dynamics (time permitting)