A1. Conserved charges in gauge theories

- in the introduction, I mentioned that conserved charges in gauge theories (including GR) are necessarily quasi-local

- now, I will discuss a very general Lagrangian formalism = covariant phase space formalism for constructing conserved charges in gauge theories (see e.g. G. Compère's lecture notes 1801.07064 for a review + reference list)

- will start w/ a quick review of the construction of conserved charges in non-gauged theories

1. Global symmetries

- action $S = \int d^dx\, \mathcal{L}(\phi)$, ϕ = fundamental fields (can have spin → more indices)

- a global symmetry is characterized by a symmetry parameter $\varepsilon = const$, under which

$$\delta_\varepsilon S = \int d^dx\, \partial_\mu M^\mu(\varepsilon) \qquad \text{(0 up to bnd. terms)}$$

- under a general variation $\delta\phi$

$$\delta S = \int d^dx \Big(\underbrace{E}_{\text{e.o.m.}}\, \delta\phi + \partial_\mu \Theta^\mu(\delta\phi) \Big), \quad \forall \delta\phi$$

presymplectic potential / symplectic potential current density (Lee & Wald '89)

- then, the current $\boxed{J^\mu = \Theta^\mu(\delta_\varepsilon\phi) - M^\mu(\varepsilon)}$ = Noether current

  is conserved on-shell

Proof:

$$\delta_\varepsilon \mathcal{L} = \underbrace{E}_{0 \text{ on-shell}}\, \delta_\varepsilon\phi + \partial_\mu \Theta^\mu(\delta_\varepsilon\phi) = \partial_\mu M^\mu(\varepsilon) \Rightarrow E\,\delta_\varepsilon\phi = -\partial_\mu J^\mu \Rightarrow 0 \text{ on-shell}$$

Remarks :

- $\Theta^\mu(\delta\phi)$ is entirely determined by the Lagrangian, when integrating by parts to obtain the e.o.m. For example for $\mathcal{L} = \mathcal{L}(\phi, \partial_\mu\phi)$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\partial_\mu\phi = \left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\right)\right]\delta\phi + \partial_\mu\Big(\overbrace{\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\phi}^{\Theta^\mu(\delta\phi)}\Big)$$

- if the Lagrangian contains several $\neq$ fields ϕ_i, they should be summed over in the above expressions

Example : complex scalar $S = -\int d^dx\, \partial_\mu\phi\,\partial^\mu\phi^*$

- invariant under $\phi \to e^{i\varepsilon}\phi$, $\phi^* \to e^{-i\varepsilon}\phi^*$ $\Rightarrow$ $\delta_\varepsilon\phi = i\varepsilon\phi$, $\delta_\varepsilon\phi^* = -i\varepsilon\phi^*$

- $J^\mu = \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta_\varepsilon\phi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^*}\delta_\varepsilon\phi^* = -i\varepsilon\left(\phi\,\partial^\mu\phi^* - \phi^*\partial^\mu\phi\right)$ charge current

- easy to check it is conserved on-shell

- Noether currents are affected by the addition of total derivatives to the action, even if the e.o.m. are not.

- one can add improvement terms to the current, $J^\mu \to J^\mu + \partial_\nu K^{\mu\nu}$, $K^{\mu\nu} = -K^{\nu\mu}$ w/o affecting its conservation (e.g. for stress tensor in E&M, to make it symmetric)

- the formulae I gave hold equally well in curved space as in flat space. However, one should be careful that in curved space, Θ^μ, M^μ, J^μ are vector densities rather than vectors, i.e. $\Theta^\mu = \sqrt{g}\,\Theta^\mu_{vect}$, etc.. Then, the bnd. terms become

$$\partial_\mu\Theta^\mu = \partial_\mu\left(\sqrt{g}\,\Theta^\mu_{vect}\right) = \sqrt{g}\,\nabla_\mu\Theta^\mu_{vect.} \qquad \text{as expected}$$

- it will soon become more convenient to switch to form notation

Levi-Civita symbol $\pm 1, 0$

$$\mathcal{L} \to \mathcal{L}^{(d)} = \frac{1}{d!}\hat{\varepsilon}_{\mu_1\ldots\mu_d}\mathcal{L}\; dx^{\mu_1}\wedge dx^{\mu_2}\ldots dx^{\mu_d} \qquad \mathcal{L} = \sqrt{g}\,\mathcal{L}_{sc}$$

$J^\mu \rightarrow J^{(d-1)} = \frac{1}{(d-1)!} \underbrace{\hat{\varepsilon}_{\mu_1 \ldots \mu_{d-1}\alpha} J^\alpha}_{\varepsilon_{\mu_1\ldots}\ \alpha\ J^\alpha_{\text{vect}}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{d-1}}$ same for Θ, M

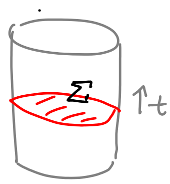
Levi-Civita tensor ←

$\Rightarrow \delta \mathcal{L}^{(d)} = E^{(d)} \delta_\varepsilon \phi + d\,\Theta^{(d-1)}(\delta\phi) = dM_\varepsilon^{(d-1)} \Rightarrow J^{(d-1)} = \Theta^{(d-1)}(\delta_\varepsilon \phi) - M_\varepsilon^{(d-1)}$ etc.

* d here is the exterior derivative : $\Omega_p \to \Omega_{p+1}$

- if the transformation parameters carry spacetime indices, they carry over (e.g. the Noether current associated w/ translations in the direction $\hat{x}^\nu$: $J^{\mu(\nu)} = T^{\mu\nu}$)

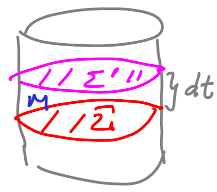
<u>Conserved charges</u> : given J^μ conserved, define $\boxed{Q = \int_\Sigma d^{d-1}x \sqrt{\gamma}\, n^\mu J_\mu^{\text{vect}}}$ where Σ is a spacelike codimension one surface w/ unit forward-pointing normal n^μ. & γ = det (induced metric on Σ)



- alternatively, $Q = \int_\Sigma J^{(d-1)}$

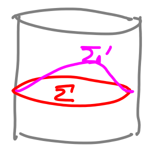
<u>Properties</u> :

i) $\frac{dQ}{dt} = 0$, provided no flux of J^μ through the boundary



since $\Delta Q = \int_\Sigma d^{d-1}x\, n_\mu J^\mu - \int_{\Sigma'} d^{d-1}x\, n_\mu J^\mu = \int_M d^d x\, \partial_\mu J^\mu = 0$

ii) Q is independent of the choice of Σ (same argument)



iii) Q is affected by improvement terms (if $K^{\mu\nu} \neq 0$ @ $\partial\Sigma$)

in form notation, $J^{(d-1)} \to J^{(d-1)} + dK^{(d-2)}$

↑ contributes to Q as $\int_{\partial\Sigma} K^{(d-2)}$

2. Local symmetries

- same setup as before, except that now the symmetry parameter is an arbitrary function $\varepsilon(x)$

- everything derived in the global case still holds, but more is true

- for simplicity, assume that $\delta_\varepsilon \phi = f(\phi)\,\varepsilon(x) + f^\mu(\phi)\,\partial_\mu \varepsilon(x)$ (e.g. E&M $\delta_\varepsilon A_\mu = \partial_\mu \varepsilon$)

- symmetry $\Rightarrow$ $\delta_\varepsilon S = \int d^d x \left[E\, \delta_\varepsilon \phi + \partial_\mu \Theta^\mu(\delta_\varepsilon\phi, \phi) \right] = 0$, $\forall\, \varepsilon(x)$ off-shell

$$= \int d^d x \left[E \left(f(\phi)\,\varepsilon(x) + f^\mu(\phi)\,\partial_\mu \varepsilon(x) \right) + \partial_\mu \Theta^\mu \right]$$

$$= \int d^d x \left[\varepsilon(x) \left(E\, f(\phi) - \partial_\mu \left(f^\mu(\phi)\, E \right) \right) + \partial_\mu \overbrace{\left(\Theta^\mu + \varepsilon f^\mu E \right)}^{\text{boundary terms}} \right] \quad \text{boundary terms } = 0 \text{ if } \varepsilon(x) \text{ has compact support}$$

$$= 0 \quad \forall\, \varepsilon(x) \quad \Rightarrow \quad \boxed{f(\phi)\, E - \partial_\mu \left(f^\mu(\phi)\, E \right) = 0} \quad \text{Noether identities}$$

- these manipulations also hold in curved space, but in order to avoid working w/ non-covariant quantities, it is better to pull out a $\sqrt{g}$ in front & replace $\partial_\mu \to \nabla_\mu$ & $\Theta^\mu \to \Theta^\mu_{\text{vect}}$.

- $\Rightarrow$ in theories w/ local "symmetries", $\exists$ off-shell relations (= Noether identities) between the equations of motion $\Rightarrow$ e.o.m not independent ($\exists$ constraints, # = # local symm.)

- this is of course nothing but the well-known fact that local / gauge "symmetries" are not "true" symmetries, but rather they encode the redundancy of the description. Fewer degrees of freedom than eqns $\Rightarrow$ off-shell relations

Example : E & M : $\delta_\varepsilon A_\nu = \partial_\nu \varepsilon \Rightarrow f = 0$, $f^\mu{}_\nu = \delta^\mu{}_\nu$, N.I : $\partial_\mu \left(f^\mu{}_\nu \underbrace{\partial_\rho F^{\rho\nu}}_{(\text{e.o.m})^\nu} \right) = \partial_\nu \partial_\rho F^{\rho\nu} = 0$

GR : $\nabla_\mu G^{\mu\nu} = 0$

Exercise: Prove this is the correct N.I. for GR + scalar field.

- let now $\varepsilon(x)$ be arbitrary (not necessarily of compact support).

$$E\,\delta_\varepsilon\phi = E\left(f(\phi)\,\varepsilon(x) + f^\mu(\phi)\,\partial_\mu\varepsilon(x)\right) = \partial_\mu\left(\underbrace{\varepsilon(x)\,f^\mu E}_{S^\mu}\right) + \Big[\underbrace{E\,f(\phi) - \partial_\mu(f^\mu E)}_{=0 \text{ by the Noether identities}}\Big]\,\varepsilon(x)$$

- S^μ = $\underbrace{\text{on-shell vanishing}}_{E\,\delta_\varepsilon\phi = \partial_\mu S^\mu}$ $\underbrace{\text{Noether current}}_{E\,\delta_\varepsilon\phi = \partial_\mu M^\mu(\varepsilon) - \partial_\mu\Theta^\mu(\delta_\varepsilon\phi) = -\partial_\mu J^\mu}$

Noether current

- N.B: for global symmetries $E\,\delta_\varepsilon\phi = -d(J^{\text{Noether}})$, but $J^{\text{Noether}} \neq 0$

- since $\partial_\mu S^\mu = -\partial_\mu J^\mu$, we must have

$$\boxed{S^\mu(\phi) = -J^\mu(\phi) - \partial_\nu Q^{\mu\nu}}$$

antisymmetric

Noether-Wald surface charge

i.e. on-shell, the Noether current J^μ associated w/ a local symmetry is a pure boundary term

- this is a general fact about gauge theories, where $\nexists$ local conserved currents associated to the "symmetry".

- however, Q in $J = -S - dQ$ is in general non-trivial, and will yield non-trivial quasi-local contributions to the charge $\int_\Sigma J$ if $Q \neq 0$ on $\partial\Sigma$

- moreover, Q is not arbitrary: it can be algoritmically constructed from the Lagrangian $\to S^\mu$ & $J^\mu = \Theta^\mu - M^\mu$ via the off-shell relation $S^\mu = -J^\mu - \partial_\nu Q^{\mu\nu}$
(∃ some ambiguities, discussed @ the end of this lecture)

- convenient: $Q^{\mu\nu} \leftrightarrow Q^{(d-2)} = \frac{1}{2(d-2)!}\,\varepsilon_{\mu_1\ldots\mu_{d-2}\alpha\beta}\,Q^{\alpha\beta}\,dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_{d-2}}$

Example : E & M + complex scalar

$$S = -\int d^dx \left[(\overbrace{\partial_\mu\phi - iA_\mu\phi}^{\mathcal{D}_\mu\phi})(\overbrace{\partial^\mu\phi^* + iA^\mu\phi^*}^{\mathcal{D}_\mu\phi^*}) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

invar. under $\quad \phi \to e^{i\varepsilon(x)}\phi \quad , \quad A_\mu \to A_\mu + \partial_\mu \varepsilon(x) \qquad M^\mu_\varepsilon = 0$

$\mathcal{J}^\nu$ = global current in presence of backgnd. A_μ (gauge invar)
$\tilde{}$matter

$$\delta S = \int d^dx \left\{ \left[\partial_\mu F^{\mu\nu} - \overbrace{i(\phi^*\mathcal{D}^\nu\phi - \phi\mathcal{D}^\nu\phi^*)}^{\mathcal{J}^\nu}\right]\delta A_\nu + E_\phi\,\delta\phi^* + E_{\phi^*}\,\delta\phi \right\}_{\text{e.o.m}}$$

$$- \partial_\mu\big(\underbrace{F^{\mu\nu}\delta A_\nu + \delta\phi\,\mathcal{D}^\mu\phi^* + \delta\phi^*\mathcal{D}^\mu\phi}_{-\Theta^\mu}\big)$$

gauge invar

- considering $\delta = \delta_\varepsilon$, the terms in the first paranthesis should yield the Noether identity, upon plugging $\delta A_\nu = \partial_\nu\varepsilon$ & integrating by parts (Check!)

- $$E^I\delta_\varepsilon\phi^I = \partial_\nu\Big[\underbrace{(\partial_\mu F^{\mu\nu} - \mathcal{J}^\nu_{matt})\varepsilon(x)}_{S^\nu}\Big] - \varepsilon(x)\underbrace{\partial_\nu}_{N.I.}\big(\partial_\mu F^{\mu\nu} - \cancel{\mathcal{J}^\nu_{matt}}\big) - \varepsilon(x)\cancel{\partial_\mu \mathcal{J}^\mu_{matt}}$$

- the Noether current is

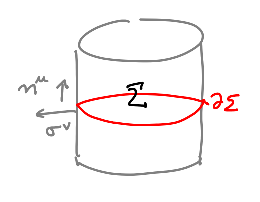
$$J^\mu = \Theta^\mu(\delta_\varepsilon\phi_i) - \overset{0}{M^\mu_\varepsilon} = -F^{\mu\nu}\partial_\nu\varepsilon + \varepsilon(x)\mathcal{J}^\mu_{matt}$$

$$= -\partial_\nu(\underbrace{F^{\mu\nu}\varepsilon}_{Q^{\mu\nu}}) + \varepsilon(x)(\underbrace{\partial_\nu F^{\mu\nu} + \mathcal{J}^\mu_{matt}}_{-S^\mu}) \qquad \text{as advertised}$$

* Note matter contr. to J^μ is as expected, but gauge field kinetic term contribution "conspires" to cancel it exactly @ local level

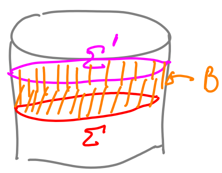
- in form notation $\quad Q^{\mu\nu} \to Q^{(d-2)} = \frac{1}{2(d-2)!}\varepsilon_{\mu_1\ldots\mu_{d-2}\alpha\beta}F^{\alpha\beta}\cdot\varepsilon\; dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_{d-2}} = \varepsilon(*F)$

- for $\varepsilon = $ const, $dQ = d*F = 0$ on-shell where no sources are present (e.g, near ∞), so the associated charge



$$Q = \int_{\partial\Sigma} Q^{(d-2)} = \int_{\partial\Sigma} dx^{d-2}\sqrt{\sigma}\; n_\mu \sigma_\nu Q^{\mu\nu} \quad \text{is } \underline{\text{conserved}} \quad = \text{total electric charge}$$

- conservation



no sources on the bnd.

$$\Delta Q = \int_{\partial\Sigma'} Q - \int_{\partial\Sigma} Q = \int_{B} dQ = \int_{B} d * F \overset{\downarrow}{=} 0$$

- for more examples, see e.g. G. Compère's lecture notes

! Note however that the $(d-2)$ form Q we just constructed <u>need not</u> in general satisfy $dQ=0$ on-shell, and thus <u>does not necessarily</u> lead to a <u>conserved charge</u>. $\underset{\nearrow}{J_\varepsilon} = -\underset{\nwarrow}{S_\varepsilon} - dQ_\varepsilon$

not generically 0   0 on-shell

Prove: $\{ d\omega = 0$ on-shell

- to obtain something naturally conserved, consider the variation

$$\delta J_\varepsilon = -\delta S_\varepsilon - d\delta Q_\varepsilon = \delta \Theta(\delta_\varepsilon\phi) - \underbrace{\delta M_\varepsilon}_{0 \text{ in E\&M}} = \underbrace{\omega(\delta\phi, \delta_\varepsilon\phi)}_{\text{(pre)symplectic current (density)}} + \underbrace{\delta_\varepsilon \Theta(\delta\phi)}_{0 \text{ b/c } \Theta \text{ gauge invar}}$$

- on-shell, $d\omega = 0$ (N.B $\delta S = d\Theta$ on-shell)

- $$\omega^\mu(\delta_1\phi, \delta_2\phi) \equiv \delta_1 \Theta^\mu(\delta_2\phi) - \delta_2\Theta^\mu(\delta_1\phi) = \delta F^{\mu\nu} \underset{\text{field-sp.}}{\wedge} \delta A_\nu + \delta\phi^* \wedge \delta\mathcal{D}^\mu\phi + \delta\phi \wedge \delta\mathcal{D}^\mu\phi^*$$

$$\Rightarrow \omega^\mu(\delta\phi, \delta_\varepsilon\phi) = \Big(\delta F^{\mu\nu} \underbrace{\delta_\varepsilon A_\nu}_{\partial_\nu\varepsilon} - \underbrace{\delta_\varepsilon F^{\mu\nu}}_{0}\delta A_\nu\Big) + \left.\begin{aligned} &i\big(\delta\phi^*\, \varepsilon\, \mathcal{D}^\mu\phi + \varepsilon\,\phi^*\,\delta\mathcal{D}^\mu\phi\big) \\ &+ i\big(-\delta\phi\,\varepsilon\,\mathcal{D}^\mu\phi^* - \varepsilon\,\phi\,\delta\mathcal{D}^\mu\phi^*\big)\end{aligned}\right\} \varepsilon\,\delta J^\mu_{matt}$$

falloff cond.

$\Rightarrow$ if $\varepsilon = \text{const}$ & J^μ_{matt} vanishes, then $\omega(\delta\phi, \delta_\varepsilon\phi) = 0 \Rightarrow d\,\delta Q_\varepsilon = 0$.

- in diff-invariant theories, a similar procedure yields $(d-2)$ form current differences that are manifestly conserved on-shell

Why covariant phase space formalism?

- Parallel : Covariant formalism | Point particle

$$\mathcal{L}(\phi, \partial_\mu \phi) \qquad\qquad \mathcal{L}(q, \dot q)$$

$$\delta \mathcal{L} = E\,\delta\phi + d\Theta \qquad\qquad \delta\mathcal{L} = \left(\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot q}\right)\delta q + \partial_t \Big(\underbrace{\frac{\partial \mathcal{L}}{\partial \dot q}\delta q}_{p\,\delta q}\Big)$$

- it follows that the (pre)symplectic potential current density $\Theta \Leftrightarrow p\,\delta q$

- & that the presymplectic form / current density $\omega = \delta_1 \Theta(\delta_2 \phi) - \delta_2 \Theta(\delta_1 \phi)$
$\Leftrightarrow \delta p \wedge \delta q$ $\leftrightarrow$ symplectic form on phase sp.

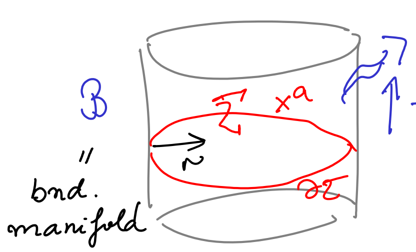
- construction of the phase sp., but in a fully covariant manner

- the pre-symplectic form $\tilde\Omega = \int_{\Sigma_\tau \text{ Cauchy surf.}} \omega$ is indep. of time & Σ on-shell ($d\omega = 0$)

- the presymplectic form thus obtained has zero modes due to gauge redundancy. These can be cured by projecting onto gauge orbit $\Rightarrow$ non-degenerate symplectic form Ω.

Example : E & M in a box (e.g. AdS) *eqns/signs not checked

$$\Theta^\mu = F^{\mu\nu}\delta A_\nu \quad \Rightarrow \quad \omega^\mu(\delta_1, \delta_2) = \delta_1(F^{\mu\nu}\delta_2 A_\nu) - \delta_2(F^{\mu\nu}\delta_1 A_\nu) = \delta F^{\mu\nu} \wedge \delta A_\nu$$



Notation: μ: all coord $= \{t, i \text{ (all spatial coord)}\}$ met. γ

$= \{r, \alpha \leftarrow \text{all bnd. coord}\}$ metric g

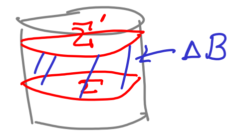
$= \{t, r, a\}$ coord on $\partial\Sigma$ met. σ

- the pre-symplectic form (i.e, the symplectic form before modding out the zero modes) is

$$\tilde{\Omega} = \int_\Sigma \omega = \int_\Sigma d^{d-1}x \sqrt{\gamma}\, n_t \omega^t = \int d^{d-1}x \sqrt{\gamma}\, n_t\, \delta F^{ti} \wedge \delta A_i$$

- $\tilde{\Omega}$ will only be conserved if we impose appropriate bnd. conditions @ $\mathcal{B} = \partial\Sigma \times \mathbb{R}_t$

- to find them, let us compute the flux of ω through $\mathcal{B}$, since



$$\tilde{\Omega}' - \Omega = \int_{\Sigma'} \omega - \int_\Sigma \omega = \int_{\Delta\mathcal{B}} \omega$$

$$\Delta\Omega = \int_{\Delta\mathcal{B}} d^d x^\alpha \sqrt{g}\; n_r \omega^r = \int_{\Delta\mathcal{B}} d^d x^\alpha \sqrt{g}\; n_r\, \delta F^{r\alpha} \wedge \delta A_\alpha = 0$$

$\hookrightarrow \Rightarrow \delta A_\alpha \big|_{\mathcal{B}} = 0$

- to have a well-defined phase space, impose Dirichlet bnd. cond.

- these bnd cond. restrict the gauge param $\varepsilon(x^\alpha, z) = \underset{\substack{\uparrow \\ \text{const}}}{\varepsilon_0} + O(z^{\#})$

- now that we have a well-defined phase space, let us study the degeneracies of $\tilde{\Omega}$ due to gauge redundancies

- zero modes of $\tilde{\Omega}$: $\forall$ gauge transf. $\delta A_\nu = \partial_\nu \varepsilon$ w/ $\varepsilon\big|_{\partial\Sigma} = 0$, since

$$\int_\Sigma d^{d-1}x \sqrt{\gamma}\, n_t\, \delta F^{ti}\, \partial_i \varepsilon = \int_\Sigma d^{d-1}x \sqrt{\gamma}\, \nabla_i (\delta F^{ti} \varepsilon) = \int_{\partial\Sigma} d^{d-2}x \sqrt{\sigma}\; \varepsilon\, \underset{\substack{\uparrow \\ \text{satisfies bnd. cond.}}}{\delta F^{tr}}\, n_t n_r$$

<u>Exercise</u> : Show that for AdS gauge field falloffs, this contribution is $\neq 0$ only if $\varepsilon(x^\mu)\big|_{z=0} \neq 0$

- all gauge transf. w/ parameter $\varepsilon(x^\mu)$ that vanishes on $\partial\Sigma$ are unphysical, as they do not affect the symplectic form (to mod out) = trivial gauge transformations

- gauge transf. whose param. $\varepsilon(x) \neq 0$ on $\partial\Sigma$ are physical symmetries (do affect the symplectic form) = large gauge transformations

- since the Dirichlet bnd. cond. on A_α restrict ε to be a const, ε_0, on B

allowed gauge tranf = ε_0 (const on B) + $\underbrace{\forall \text{ gauge transf. that} = 0 \text{ on } B}_{\text{trivial redundancies}}$

ε_0 ← large gauge transf (non-trivial) ← true symm. of E&M in a box (w/ Dirichlet bnd. cond)

- given the general connection between the symplectic form and the charge ($\omega(\delta_\varepsilon\phi, \delta\phi) + d\,\delta Q_\varepsilon = 0$), trivial gauge transformations will not carry any charge, while large gauge transformations will.

Remarks: 1 The fact that only $\varepsilon = \text{const.}$ is allowed on the bnd. B is a feature of the particular (Dirichlet) bnd. cond. we have imposed. $\exists$ examples where this requirement is relaxed $\Rightarrow$ more large symmetries. (e.g. in asympt flat spt, $\exists\ \infty$ # of symm)

2. In AdS (anti-de Sitter) spacetime, Dirichlet bnd. cond. are very natural. The fact that large gauge transf. of the bulk electro-magnetic theory coincide with global transf. on the boundary, B (b/c $\varepsilon = \text{const.}$ corresp. to a global symmetry) is simply indicative of the fact that the bulk gauge field is dual to a boundary current (so the "true" symmetry they are associated with is the same)