plan

boundary

defect

1) outline

2) bCFT and dCFT interesting

3) aside dim'al analysis

2

2) symmetry and QFT

gauge symmetry - $SU(3) \times SU(2) \times U(1)$

global symmetry - $SU(2)$ flavor sym.

up and down quarks

discrete sym. - C, P, T

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spacetime sym.

special rel.

Poincaré group

relativistic QFT - marriage of special rel. + quant. mech.

- when can Poincaré be part of something bigger?

  - very limited

Coleman-Mandula ('67) - if one takes

$g \in$ Poincaré $\qquad h \in$ internal sym. group

$$[g, h] = 0$$

loopholes

a) conformal symmetry - proof involves S-matrix
- massless particles S-matrix problematic

b) SUSY - proof requires Lie algebra
- continuous groups
- SUSY instead Lie super-alg.

c) ~~discrete sym.~~ d) ~~spontaneously broken~~

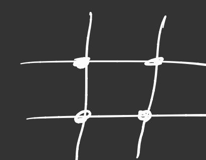
applications for CFT

critical phenomena – 2^{nd} order phase transition

– disc. in a derivative

at $T = T_c$ effective field theory description which is conformal

Ising model $\sigma_i = \pm 1$



$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

high T $\langle \sigma_i \rangle = 0$ $\longleftrightarrow$ low T $\langle \sigma_i \rangle \neq 0$

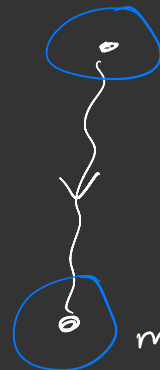
at $T = T_c$ 2^{nd} order phase transition

renormalization group

rules in QFT depend on energy scale

QCD

natural starting pt.'s for analyzing QFT more generally



quarks+gluons, UV, free theory

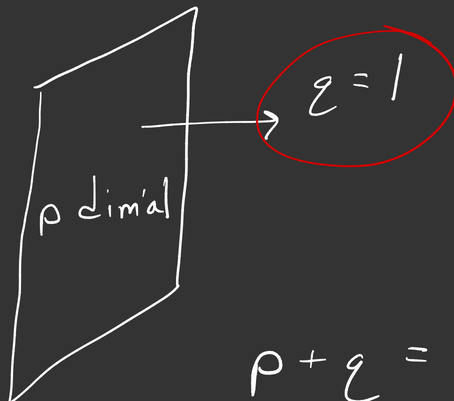
mesons, baryons, IR, gapped, trivial

generic fixed pt.'s of rel.'ic QFT are CFT's – interacting massless particles

string theory

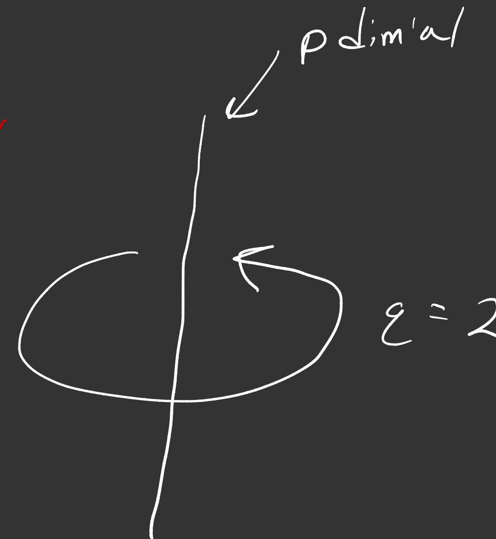
- QFT pt. like objects
- singular!
- replace pt.s w/ higher dim'al objects, e.g. strings
- traces 1+1 dim'al worldsheet
- 2d CFT on worldsheet

boundaries + defects



boundary

interface



$$p + q = d$$

defect dimension

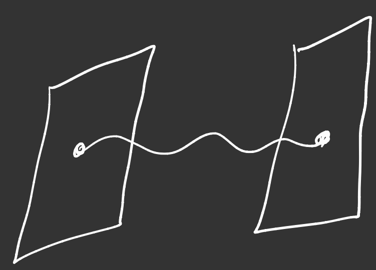
codimension

- break some but not all of conf. sym. group

- clearly important for critical systems in condensed matter
  - boundary of system
  - imperfections

- way of reading history through defects

  - D-branes

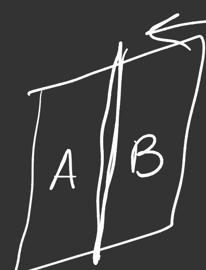


  - bry condi's for open strings
  - 2d CFT on worldsheet
  - seeded $2^{\underline{nd}}$ superstring revolution

- AdS/CFT correspondence - map between gravity and field theory
  - 2 of most important problems
    - quant. grav.
    - strong interactions of QFT
  - CFT lives on boundary of anti-de Sitter (AdS) space!

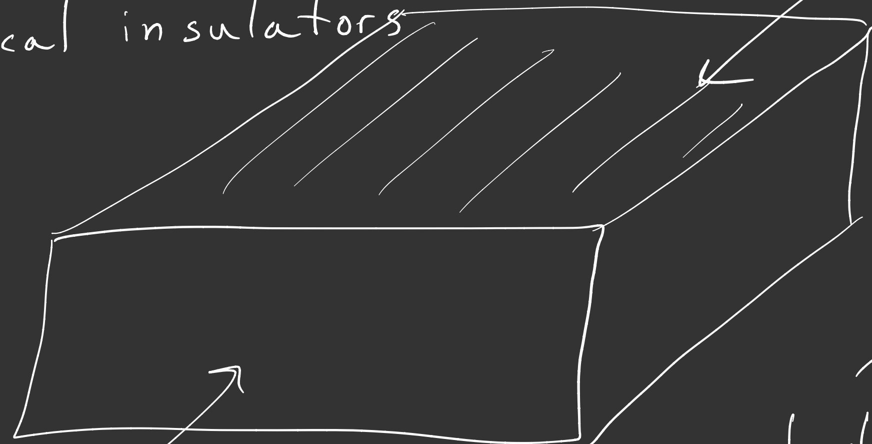
quantum information + entanglement

- entanglement as a resource to build a quantum computer
- it from bit (qubit) space-time as emerging from information
- in QFT, compute entanglement



inserting twist defect ($q=2$)

- topological insulators



protected massless surface states (discrete symmetries)

- experimental

outline

- dim'al analysis

Ch. 2 — Poincaré and conformal sym. group

Ch. 3 — constraints of conf'al sym. on correlation fn.s w/out defects and w/ defects

— Noether's Thm.

Ch. 4 - conf'al sym. in curved space

- stress tensor + displacement op.

- Ward id's $\longrightarrow$ $T^{\mu\nu}$ and D^i

Ch. 5 - two tools

- radial quantization

- OPE (operator product expansion)

conformal blocks - def. of CFT

Ch. 6 – conformal bootstrap (+ unitarity bounds)

– defects + bry.s

Ch. 7 – trace anomaly $T^{\mu}{}_{\mu} = 0$ classically

$T^{\mu}{}_{\mu} \neq 0$ QM'ally

Ch. 8 – two examples

– ϕ^4 theory w/ bry

– mixed dim'al QED

dim'al analysis

- solve problems might otherwise have no clue where to begin

- speed of capillary waves

surface tension

$\sigma = 72.8 \text{ mN m}^{-1}$

$\sim \text{kg s}^{-2}$

density	ρ	kg m^{-3}
gravity	g	m s^{-2}

$$\rho^a \sigma^b g^c \sim v$$

$a + b = 0$	kg
$-3a + c = 1$	m
$-2b - 2c = -1$	s

$$a + b = 0 \qquad \text{kg}$$

$$-3a + c = 1 \qquad \text{m}$$

$$-2b - 2c = -1 \qquad \text{s}$$

$$a \sim \rho$$

$$b \sim \sigma$$

$$c \sim g$$

$$-6a - 2b = 1 \longrightarrow (-6+2)a = 1$$

$$a = -\frac{1}{4}$$

$$b = \frac{1}{4}$$

$$c = \frac{1}{4}$$

$$v \sim \left(\frac{\sigma g}{\rho} \right)^{1/4} \sim 16 \text{ cm s}^{-1}$$

here we set $\hbar = c = 1$

one dimensionful quantity $\quad \text{mass} = \frac{1}{\text{length}}$

<u>example</u>

$$S = -\frac{1}{2}\int d^d x \left((\partial_\mu \phi)(\partial^\mu \phi) + m^2 \phi^2 \right)$$

S dimensionless $\quad e^{iS}$

$$\Longrightarrow \quad \phi \sim (\text{mass})^{\frac{d}{2}-1}$$

$$\Delta_\phi = \frac{d-2}{2}$$

$$m \sim (\text{mass})$$

what happens w/ $g\phi^n$ g small

perturbation theory

small compared to what?

$$\lambda = \frac{g}{E^{d - n\Delta_\phi}}$$ is dimensionless

char. energy scale

$$\underline{\text{pert}}\ \underline{\text{theory}} = \sum_{j=0}^{\infty} c_j \lambda^j$$

sign of $d-n\Delta_\phi$ important $\quad \lambda = \dfrac{g}{E^{d-n\Delta_\phi}}$

$d-n\Delta_\phi > 0 \Rightarrow \lambda$ small when E large

large when E small

relevant - importantat low E

$d-n\Delta_\phi < 0 \Rightarrow \lambda$ large for large E

λ small for small E

irrelevant

$d-n\Delta_\phi = 0$ classically marginal loop corrections $\Delta_\phi \neq \frac{d-2}{2}$

$\hookrightarrow$ but if it were $\Delta_\phi = \frac{d-2}{2}$

scale invariance

$\hookrightarrow$ closer to CFT

problem

$$S = -\frac{1}{2}\int d^d x \left((\partial\phi)^2 + g\,\phi^n \right)$$

- for what pairs of (n, d) does

$$d - n\,\Delta_\phi = 0\,?$$

$$d - n\,\frac{d-2}{2} = 0$$

$$\left[\begin{array}{ll} d = 4 & \frac{d-2}{2} = 1 \\ 4 - n = 0 & \end{array}\right.$$

$$n = 4$$

$$g\phi^4$$

$$\left[\begin{array}{ll} d = 3 & \frac{d-2}{2} = \frac{1}{2} \\ 3 - \frac{n}{2} = 0 & n = 6 \end{array}\right.$$

$$g\phi^6$$

problem

$$S = -\int d^d x \left(\frac{i}{2} \bar{\psi} \not{\partial} \psi + g(\bar{\psi}\psi)^n \right)$$

. what pairs (d, n) yield classically marginal g?

$$\Delta_\psi = \frac{d-1}{2}$$

$$\left[\begin{array}{l} d = 2 \qquad \Delta_\psi = \frac{1}{2} \\ d - 2n\Delta_\psi = 0 \\ 2 - 2n \cdot \frac{1}{2} = 0 \\ n = 2 \end{array} \right.$$

Thirring

$$g \, \bar{\psi}\psi\bar{\psi}\psi$$

b) $\phi \bar{\psi} \psi$ $\qquad \phi^2 \bar{\psi} \psi$

Yukawa

classically marginal?

problem

$$S = -\int d^d x \left(\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{i}{2} \bar{\psi} \gamma^\mu (\partial_\mu + ig A_\mu) \psi \right)$$

interaction

- scaling dimension of g?
- special about d = 4?

problem

$$S = -\frac{1}{2}\int d^dx\, (\partial_\mu \phi)(\partial^\mu \phi) + \int d^px \; g\,\phi^n \; \delta^{(q)}(x)$$

planar defect

- find triples (d, p, n) for which g is classically marginal

$p = d-2$; $\quad 0 = p - n\Delta_\phi = d-2 - n\,\frac{d-2}{2}$

$\Rightarrow \; n = 2$ (exactly marginal)

$$\left[\int d^{d-2}x \; g\,\phi^2 \; \delta^{(2)}(x) \right.$$

important entanglement entropy

$g\phi^4$

case 2

$p = d-1;$ $\quad 0 = p - n\Delta_\phi = d-1-n\,\dfrac{d-2}{2}$

$d=4 \qquad 3-n=0 \qquad n=3 \qquad g\phi^3$

$d=3 \qquad 2-n\,\dfrac{1}{2}=0 \qquad n=4 \qquad g\phi^4$

superpotential W $\qquad V = \left(\dfrac{\partial W}{\partial \phi}\right)^2 \qquad gW$

$g\phi^6$

$$\int d^d x \left[(\partial\phi)^2 + gV \right] + \int d^{d-1}x \; gW$$

$$d=3 \qquad W = \phi^4 \qquad V = \left(\frac{\partial W}{\partial \phi}\right)^2 = \phi^6$$

$$d=6 \qquad W = \phi^3 \qquad V = \left(\frac{\partial W}{\partial \phi}\right)^2 = \phi^4$$