Black holes in imaginary time

jueves, 8 de abril de 2021 17:18

What does this have to do with bhs??

$t = -i\tau$

$$ds^2 = \left(1 - \frac{2M}{r}\right) d\tau^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega$$

$\tau \sim \tau + \beta$: we're going to see that not all β are allowed

Go near the "Euclidean horizon" at $r = 2M$

$$r = 2M + \xi^2 \qquad \xi^2 \ll 2M$$

$$ds^2 \simeq \frac{\xi^2}{2M} d\tau^2 + 8M\, d\xi^2 + 4M^2 d\Omega$$

$$8M\,\xi^2 = x^2$$

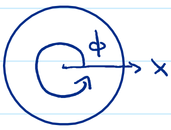
$$ds^2 \simeq +\kappa^2 x^2 d\tau^2 + dx^2 + 4M^2 d\Omega \qquad \kappa = \frac{1}{4M} \text{ : surface gravity}$$

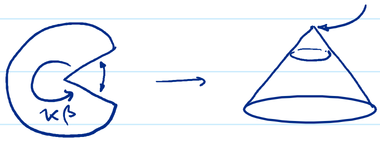
$$\rho^2 d\phi^2 + d\rho^2 \qquad x = \rho$$

$$\kappa\tau = \phi \qquad \tau = \frac{1}{\kappa}\phi$$

$$\tau \sim \tau + \beta \qquad \phi \sim \phi + \kappa\beta$$

Normally $\phi \sim \phi + 2\pi$. If instead $\kappa\beta < 2\pi$: conical singularity





distributional (delta-fn) curvature

$\kappa\beta = 2\pi$ for absence of conical singularities in the Euclidean geometry

$$\Rightarrow \quad \beta = \frac{2\pi}{\kappa} \qquad T = \frac{\kappa}{2\pi} \qquad T = \frac{\hbar\kappa}{2\pi} \text{ : Hawking Temperature}$$

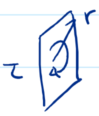
Since the geometry is periodic $\tau \sim \tau + \beta$, all correlation functions of a QFT in this geometry will also exhibit this same periodicity

$\Rightarrow$ QFT in a bh background is finite-temperature field theory at

$$T = T_H = \frac{\kappa}{2\pi}$$

The geometry of Euclidean Schwarzschild:

near $r = 2\pi$

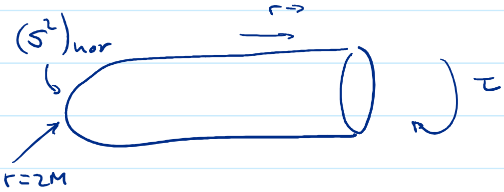


$\times S^2$

large r $\quad ds^2 \to d\tau^2 + dr^2 + r^2 d\Omega$

near $r = 2M$ τ  $\times S^2$

large r $ds^2 \to d\tau^2 + dr^2 + r^2 d\Omega$



$\times S^2$

There's nothing behind $r = 2M$

Geometry is completely non-singular and geodesically complete

Topology is $\mathbb{R}^2 \times S^2 \neq S^1 \times \mathbb{R}^3$