QFT at finite temperature

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In QFT we study correlation functions of products of field operators $\phi(x)$.
An important two-point correlation function is the Feynman propagator:

$$G(x_1, x_2) = \langle T(\phi(x_1)\,\phi(x_2))\rangle \qquad T: \text{Time ordered}$$

$$= \langle T(\phi(t_1)\,\phi(t_2))\rangle = \begin{cases} \phi(t_1)\phi(t_2) & t_1 > t_2 \\ \phi(t_2)\,\phi(t_1) & t_2 > t_1 \end{cases}$$

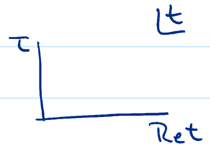
Usually $\langle\ \rangle$ refers to expectation values in the vacuum state.
We want to study it in a state of thermal equilibrium at temperature $T = 1/\beta$.

Then $\langle O\rangle_\beta = \mathrm{Tr}\left(\frac{e^{-\beta H}}{Z}O\right)$

$$G_\beta(t_1, t_2) = G_\beta(t) = \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta H}T(\phi(t_1)\,\phi(t_2))\right)$$

$t = t_2 - t_1$

Complex time plane



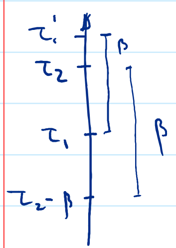
$t = -i\tau$: imaginary time

Heisenberg picture evolution of operators:

$$O(t_0 + t) = e^{iHt}\,O(t_0)\,e^{-iHt}$$

$$O(\tau_0 + \tau) = e^{H\tau}\,O(\tau_0)\,e^{-H\tau}$$

Take $\tau_2 - \beta < \tau_1 < \tau_2$ $\qquad \beta = 1/T$



$$\tau_1' = \tau_1 + \beta$$

$$\phi(\tau_1') = \phi(\tau_1 + \beta) = e^{\beta H}\,\phi(\tau_1)\,e^{-\beta H}$$

$$\tau_2 - \tau_1' = \tau_2 - \tau_1 - \beta = \tau - \beta$$

$$G_\beta(\tau_1', \tau_2) = G_\beta(\tau - \beta) = \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta H}T(\phi(\tau_1')\,\phi(\tau_2))\right)$$

$\tau_1' > \tau_2$

$$= \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta H}\,\phi(\tau_1')\,\phi(\tau_2)\right)$$

$$= \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta H}\,e^{\beta H}\,\phi(\tau_1)\,e^{-\beta H}\,\phi(\tau_2)\right)$$

$$= \frac{1}{Z}\, \mathrm{Tr}\left(e^{-\beta H}\phi(\tau_2)\,\phi(\tau_1)\right) \qquad \tau_2 > \tau_1$$

$$= \frac{1}{Z}\, \mathrm{Tr}\left(e^{-\beta H}\, T\left(\phi(\tau_2)\,\phi(\tau_1)\right)\right)$$

$$= G_\beta(\tau)$$

$$\Rightarrow \boxed{G_\beta(\tau-\beta) = G_\beta(\tau) \quad : \quad \text{periodic in imaginary time} \quad \tau \sim \tau+\beta}$$

KMS (Kubo, Martin, Schwinger) <u>condition for thermal equilibrium</u> in QFT: <u>correlation functions are periodic in the imaginary time direction</u>

The KMS condition is the definition of stationary thermal equilibrium

Then, for studying QFT at finite temperature we consider the QFT in

$$ds^2 = \underbrace{d\tau^2}_{S^1} + \underbrace{dx^2 + dy^2 + dz^2}_{\mathbb{R}^3} \qquad \tau \sim \tau + \beta$$

$$S^1 \times \mathbb{R}^3$$