Partition functions from Euclidean path integrals

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Partition function of system at Temperature $T = 1/\beta$

$$Z(\beta) = \sum_n e^{-\beta E_n} = \mathrm{Tr}\, e^{-\beta H} = \sum_q \langle q | e^{-\beta H} | q \rangle$$

Path integral computation of Transition amplitudes:

$$\langle q^f_{t_2} | q^i_{t_1} \rangle = \langle q^f | e^{-i(t_2 - t_1) H} | q^i \rangle$$

$$= \int \mathcal{D}q \, e^{i \int_{t_1}^{t_2} dt \, \mathcal{L}[q, \dot{q}]} = \int \mathcal{D}q \, e^{i I[q, t_1, t_2]}$$

Consider periodic evolution in imaginary Time:

$$it = \tau \qquad |q^i\rangle = |q^f\rangle \qquad \tau_2 - \tau_1 = \beta$$

$$q(\tau) \qquad q(0) = q(\beta)$$

$$\sum_q \langle q | e^{-\beta H} | q \rangle = \int_{q(0)=q(\beta)} \mathcal{D}q \, e^{\int_0^\beta d\tau \, \mathcal{L}[q, i \frac{dq}{d\tau}]}$$

$$\Vert$$

$$Z(\beta) = \int_{q(0)=q(\beta)} \mathcal{D}q \, e^{-I_E[q]}$$

$$it \to \tau$$

$$(\partial_t q)^2 \to -(\partial_\tau q)^2$$

$$\mathcal{L}[q, i \tfrac{dq}{d\tau}] \equiv -\mathcal{L}_E[q, \tfrac{dq}{d\tau}]$$

$$i \int dt \, \mathcal{L} \to -\int d\tau \, \mathcal{L}_E$$

$$iI \to -I_E$$

Thus, <u>the path integral for periodic configurations of a system in imaginary Time with periodicity β, computes its partition function at Temperature β^{-1}.</u>