String Theory

Exercise 1.1:

Show that the point particle action S_0 is invariant under reparameterizations of the world-line by substituting $\tau' = f(\tau)$.

Exercise 1.2:

a) Show that in the non-relativistic limit the action S_0 has the usual non-relativistic form, kinetic energy minus potential energy, where the potential energy is the rest mass.
b) Show that in the non-relativistic limit the Nambu-Goto $S_{\mathrm{NG}} = S_1$ reduces to a kinetic term minus a potential term proportional to the length of the string. Show that the kinetic term comes only from the transverse velocity of the string. Calculate the mass per unit length.

Exercise 1.3:

Show that the equation of motion derived from $\tilde{S}_0$ is the geodesic equation

$$\ddot{X}^\mu + \Gamma^\mu_{\rho\lambda}\dot{X}^\rho \dot{X}^\lambda = 0$$

Exercise 1.4:

Find the explicit expression for the energy-momentum tensor for the Polyakov action using the definition

$$T_{\alpha\beta} = -\frac{2}{T}\frac{1}{\sqrt{-h}}\frac{\delta S_P}{\delta h^{\alpha\beta}}$$

and show it is traceless ($h^{\alpha\beta}T_{\alpha\beta} = 0$). Relate the trace $T^\alpha{}_\alpha$ to the variation of the action under a Weyl transformation $h_{\alpha\beta} \to e^{2\omega}h_{\alpha\beta}$. In D-dimensional flat space ($g_{\mu\nu} = \eta_{\mu\nu}$), relate the conservation law $\nabla^\alpha T_{\alpha\beta} = 0$ to the equations of motion for the fields X^μ.

Exercise 1.5:

Using the equation of motion for the metric $h^{\alpha\beta}$, show that the Nambu-Goto action and the Polyakov action are equivalent at the classical level.

Exercise 1.6:

Using the equation of motion for the metric $h^{\alpha\beta}$, show that the Nambu-Goto p-brane action is equivalent to the Polyakov action

$$S_{\mathrm{p-Pol}} = -\frac{T_p}{2}\int d^{p+1}\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu + \Lambda_p \int d^{p+1}\sigma\sqrt{-h}$$

where Λ_p is a cosmological constant that you have to choose appropriately.