String Theory

Exercise 3.1:

a) Show that the following open string state is null

$|\phi>= L_{-1}|\chi_0>$, where $L_m|\chi_0>=0$; $\alpha^\mu_m|\chi_0>=0\,\forall m>0$ and $(L_0-a+1)|\chi_0>=L_0|\chi_0>=0$

b) What are the conditions on a_μ and $\tilde{a}_\mu$ such that the following closed string state is null

$$|\phi>=(a_\mu\,\alpha^\mu_{-1}\tilde{L}_{-1}+\tilde{a}_\mu\,\tilde{\alpha}^\mu_{-1}L_{-1})|\chi_0>$$

where $|\chi_0>$ satisfies the conditions above, both on the left moving and the right-moving sectors.

Exercise 3.2:

Show that the following state is spurious

$$|\psi>=\sum_{n=1}^{\infty}L_{-n}|\chi_n>$$

where $|\chi_n>$ is an eigenstate of L_0 with eigenvalue $a-n$

Exercise 3.3:

Consider an open string state of the form

$$|\phi>=e_\mu\,(A\,\alpha^\mu_{-2}+B\,\alpha_0\cdot\alpha_{-1}\alpha^\mu_{-1})\,|0,k>$$

where A and B are constants. Determine the conditions on A,B and e_μ so that $|\phi>$ satisfies the physical-state conditions. Compute its norm.

Exercise 3.4:

Consider an open string state of the form

$$|\phi>=\left(A\,\alpha_{-1}\cdot\alpha_{-1}+B\,\alpha_0\cdot\alpha_{-2}+C\,(\alpha_0\cdot\alpha_{-1})^2\right)|0,k>$$

where A,B and C are constants. Determine the conditions on A,B,C so that $|\phi>$ satisfies the physical-state conditions. Compute its norm.

Exercise 3.5:

Construct the spectrum of the open string at level $N=2$. Do it first by constructing the possible states, and demanding the physical-state conditions. What is this state in terms of representations of $SO(D-1)$? Redo the exercise but now in light cone gauge.

Exercise 3.6:

Construct the spectrum of the open string in light-cone gauge at level $N = 3$. How many states are there in terms of D? See what this number can come from in terms of representations of $SO(D-1)$. What would be the corresponding physical states?