String Theory

Exercise 4.1:

In the four closed-string tachyon amplitude, take t fixed and vary s. Locate the poles of the aplitude in terms of s. What do these poles correspond to? Using the expansion

$$\Gamma(z) \approx \frac{1}{z+n}\frac{(-1)^n}{n!} \ , \quad \text{for z} \approx -\text{n} \ ,$$

see how the amplitude behaves close to these points. Compare to the corresponding Feynman diagram in QFT.

Exercise 4.2:

How does the four closed-string tachyon amplitude behave in the hard scattering limit $s \to \infty$, $t \to \infty$, s/t fixed? (Use that for large x, $\Gamma(x) \sim exp(xlnx)$).

Exercise 4.3:

Consider the action

$$S = -\frac{T}{2}\int d^2\sigma\sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu + \lambda \int d^2\sigma \sqrt{-h}\,\phi(X)$$

Show that if $\lambda \neq 0$, then $h_{\alpha\beta} = 0$.

Exercise 4.4:

Show that the Lagrangian in the action

$$S = -\frac{T}{2}\int d^2\sigma\, \epsilon^{\alpha\beta} B_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu$$

changes by a total derivative under the gauge transformation $B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\lambda_{\nu]}$. Is the action invariant under gauge transformations in the case of open strings?

Exercise 4.5:

Show that the space-time action in the string frame

$$S_{\rm st} = \frac{1}{2\kappa_0^2}\int d^D x\sqrt{-g}e^{-2\phi}\left(R - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\phi\partial^\mu\phi\right)$$

becomes the Einstein-frame action

$$S_{\rm st} = \frac{1}{2\kappa^2}\int d^D x\sqrt{-\tilde{g}}\left(\tilde{R} - \frac{1}{12}e^{-\frac{8\tilde{\phi}}{D-2}}H_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} - \frac{4}{D-2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\right)$$

by the redefinition

$$\tilde{g}_{\mu\nu} = e^{-\frac{4(\phi-\phi_0)}{D-2}}g_{\mu\nu} \ , \quad \tilde{\phi} = \phi - \phi_0$$

where ϕ_0 is a constant, and $\kappa = \kappa_0 e^{\phi_0}$. ($\tilde{H}^{\mu\nu\lambda}$ here means that the indices of $H_{\mu\nu\rho}$ have been raised with $\tilde{g}^{\mu\nu}$).