String Theory

Exercise 5.1:

Consider a 26-dimensional space-time with metric

$$ds^2 = G_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}(dx^{25} + A_\mu dx^\mu)^2$$

where $x^{25} \sim x^{25} + 2\pi R$. Show that the 26-dimensional action

$$S_{26} = \frac{1}{16\pi G_N^{(26)}} \int d^{26}x\sqrt{-G}\left(R^{(26)} + 4\partial_\mu\phi\partial^\mu\phi\right)$$

after integrating over x^{25} and doing a Weyl transformation, becomes

$$S_{25} = \frac{1}{16\pi G_N^{(25)}} \int d^{25}x\sqrt{-g}\left(R^{(25)} + F_{\mu\nu}F^{\mu\nu} + \partial_\mu\sigma_0\partial^\mu\sigma_0 + 4\partial_\mu\phi_0\partial^\mu\phi_0 + \text{KK tower}\right)$$

where σ_0 and ϕ_0 are the zero modes. Find the relation between the 26 and the 25-dimensional Newton constants.

Exercise 5.2:

Show that for the closed string compactified on a circle at the self-dual radius $R = \sqrt{\alpha'}$ there are 4 extra massless vectors and 8 extra massless scalars coming from states with non-zero momentum and/or winding number along the circle.

Exercise 5.3:

What are the conditions on $\psi_\pm$ that one has to impose in order for the spinor in 2 dimensions $\psi = \begin{pmatrix}\psi_- \\ \psi_+\end{pmatrix}$ to be Weyl?

Exercise 5.4:

Given a spinor $\xi^{(\mathbf{s})}$ in $D = 2k + 2$

$$\xi^{(\mathbf{s})} = (\Gamma^{k+})^{s_k+1/2}...(\Gamma^{0+})^{s_0+1/2}\xi$$

where ξ is such that $\Gamma^{a-}\xi = 0$, how do the positive and negative chirality conditions read as a condition on $s_0, s_1, ..., s_k$?

Exercise 5.5:

Show the equality

$$S^{\text{f}} = -\frac{T}{2}\int d^2\sigma\left(\bar{\psi}^\mu\rho^\alpha\partial_\alpha\psi_\mu\right) = i\,T\int d^2\sigma\left(\psi_-^\mu\partial_+\psi_{\mu-} + \psi_+^\mu\partial_-\psi_{\mu+}\right)$$