

String Theory

Exercise 6.1:

Find the energy momentum tensor as the Noether current corresponding to worldsheet translation symmetry. For this, compute the variation of the action under a worldsheet translation $\delta\sigma^\alpha = a^\alpha$. The Lagrangian transforms by a total derivative of the corresponding current, in this case the energy momentum tensor

$$\delta\mathcal{L} = a^\alpha \partial_\beta T_\alpha^\beta$$

Exercise 6.2:

Find the supercurrent as the Noether current corresponding to worldsheet supersymmetry. For this, compute the variation of the action under a supersymmetry transformation with parameter ϵ , and find the current J^α from

$$\delta\mathcal{L} = \bar{\epsilon} \partial_\alpha J^\alpha$$

Exercise 6.3:

Compute the commutator of two supersymmetry transformations $[\delta_{\epsilon_1}, \delta_{\epsilon_2}]X^\mu$ and $[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\psi^\mu$. Show that it amounts to a translation on the worldsheet, and find the parameter of this translation.

Exercise 6.4:

Compute the zero point energy $E_0 = -a$ in the R and NS sector, using the regularization

$$Z_\alpha = \frac{1}{2} \sum_{n=0}^{\infty} (n + \alpha) \rightarrow -\frac{1}{24} + \frac{1}{4}\alpha(1 - \alpha), \quad \text{for } 0 \leq \alpha \leq 1$$

Exercise 6.5:

Construct the physical states of the open string in the NS and R sector up to the massless level.

Exercise 6.6:

a) Show that the following open string state in the NS sector is null

$$|\psi\rangle = G_{-1/2}|\chi\rangle, \quad \text{where } G_r|\chi\rangle = 0 \quad \forall r > 0 \quad \text{and} \quad (L_0 - a_{\text{NS}} + \frac{1}{2})|\chi\rangle = 0$$

b) Find the constant a such that the following open string state in the NS sector is null

$$|\psi\rangle = (G_{-3/2} + a G_{-1/2} L_{-1})|\chi\rangle$$

where χ is annihilated by $G_{r>0}$, as above, but now $(L_0 - a_{\text{NS}} + \frac{3}{2})|\chi\rangle = 0$.

c) Show that the following open string state in the R sector is null

$$|\psi\rangle = F_0 F_{-1} |\chi\rangle \quad \text{where} \quad F_n |\chi\rangle = L_n |\chi\rangle = 0 \quad \forall n > 0 \quad \text{and} \quad (L_0 - a_{\text{R}} + 1) |\chi\rangle = 0$$

Exercise 6.7:

Construct the open string states of mass $M^2 = 1/\alpha'$ both in the R and NS sector. How many states are there in each sector? (You can do the counting by working directly in light cone gauge)