1. Coherent states

Coherent states are defined as the eigenstate of the destruction operator a introduced in lecture 2:

$$a|\phi\rangle = \phi|\phi\rangle. \tag{1}$$

Based on this definition, show that

(a)
$$|\phi\rangle = \exp\left\{-\frac{|\phi|^2}{2} + \phi a^\dagger\right\}|0\rangle, \tag{2}$$
where the state has been normalized such that $\langle\phi|\phi\rangle = 1$;

(b)
$$\int \frac{\mathrm{d}\phi^*\mathrm{d}\phi}{\pi}|\phi\rangle\langle\phi| = \mathbb{I}, \tag{3}$$
where $\mathbb{I}$ is the identity in the Fock space and the integral is done on the whole complex plane of values of ϕ.

(c)
$$\langle\phi|\phi'\rangle = \exp\left\{-\frac{|\phi|^2}{2} - \frac{|\phi|^2}{2} + \phi^*\phi'\right\}. \tag{4}$$

(d)
$$\langle\phi|n\rangle = (\phi^*)^n \mathrm{e}^{-|\phi|^2/2}, \tag{5}$$
where $|n\rangle$ is the state with n particles.

2. Branching and decay process with lethal competition

Consider the branching and decay process with lethal competition which we studied in problem 2 (d) of set n. 1, i.e.,

$$A \xrightarrow{\sigma} A + A, \quad A \xrightarrow{\mu} \emptyset, \quad \text{and} \quad A + A \xrightarrow{\lambda} A. \tag{6}$$

Determine the total Hamiltonian H of the corresponding evolution in the Fock space of states and the corresponding coherent-state path integral.