DQMP Department of Quantum Matter Physics

Tomonaga-Luttinger liquids from field theory to experimental realizations

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http://dqmp.unige.ch/giamarchi/

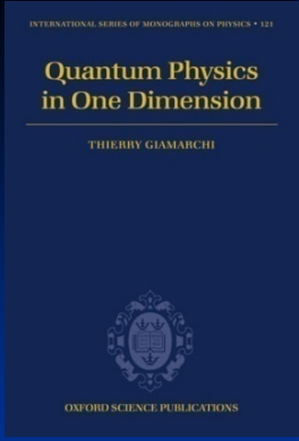
UNIVERSITÉ DE GENÈVE

FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

Science to shape the future of the Army ★ Army Research Office ★ 1951

MaNEP SWITZERLAND

References



TG, Quantum physics in one dimension, Oxford (2004)

TG in ``Understanding Q. Phase Transitions", Ed. L. Carr, CRC Press (2010)

M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

TG, C. R. Acad. Sci. 17 322 (2016)

Crash course on Fermi liquids

References on fermi liquids

- Basic course on many-body physics http://dqmp.unige.ch/giamarchi/local/people/thierry.giamarchi/pdf/many-body.pdf

- Lectures Les houches (Singapore) 2009, TG **arXiv:1007.1030**

- Lectures Les Houches 2010, A. Georges + TG **arXiv:1308.2684**

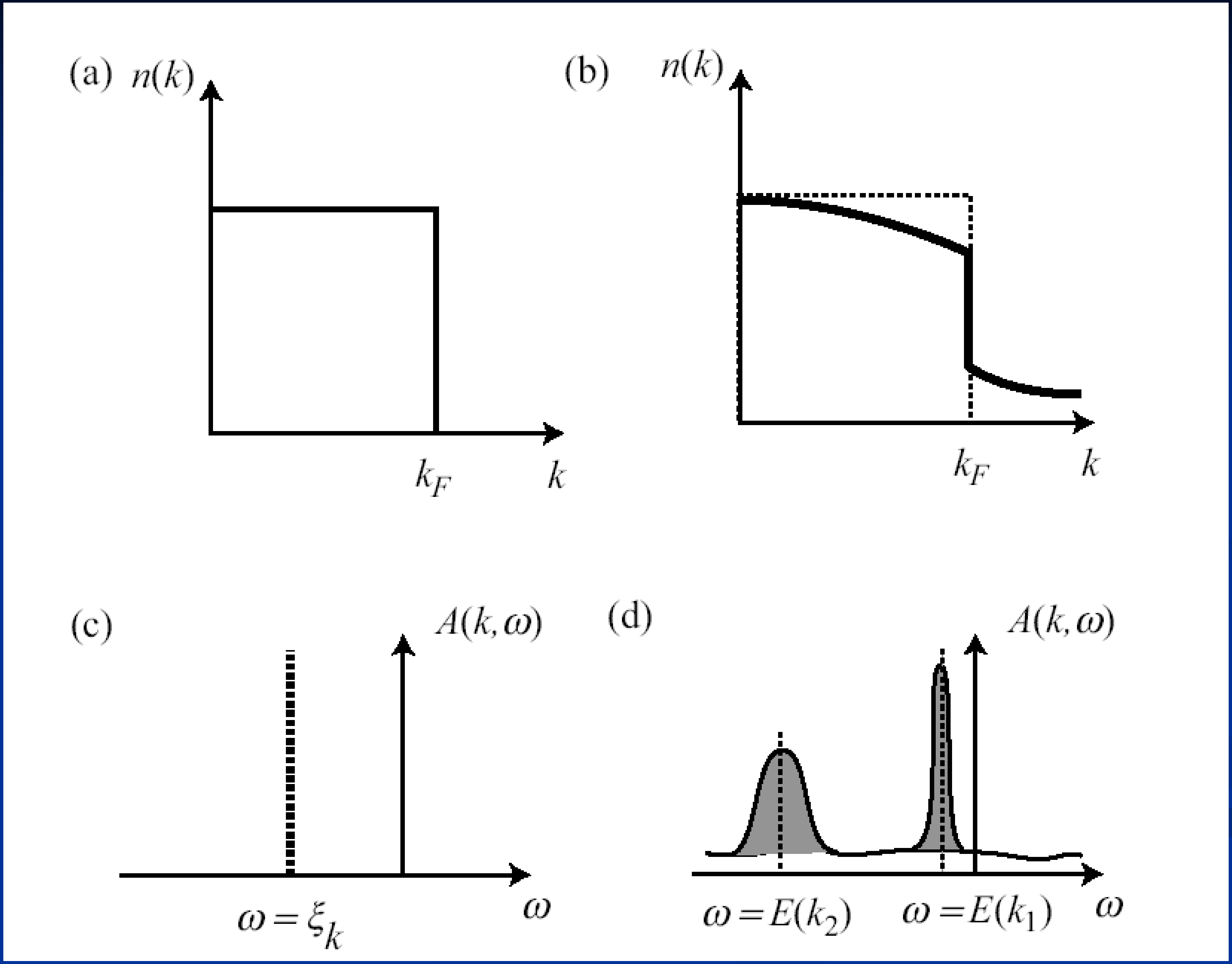
How to master materials ?

- Understood: free electrons
- Real systems : Coulomb interaction

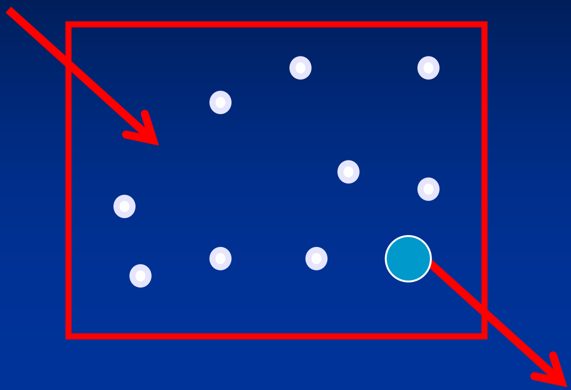
E » 10 000 K !

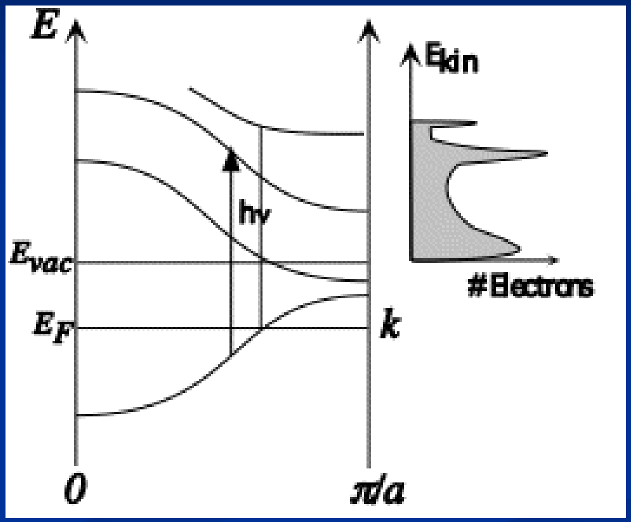
- Properties of realistic systems ?
- Free electron theory works quite well : Landau Fermi liquid

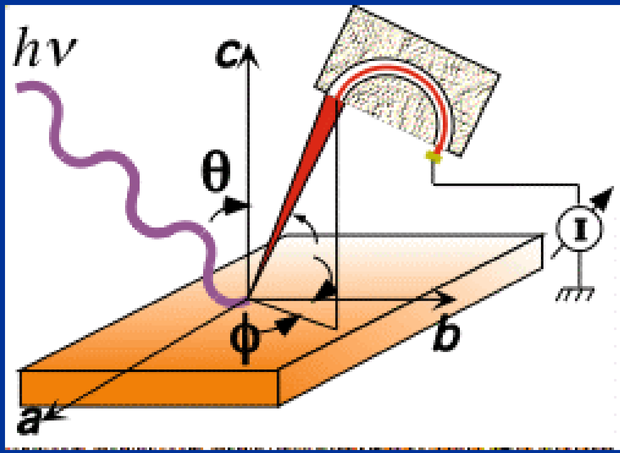
$$m \rightarrow m^*$$


(a)
n(k)
k_F
k
(b)
n(k)
k_F
k
(c)
A(k,ω)
ω = ξ_k
ω
(d)
A(k,ω)
ω = E(k_2)
ω = E(k_1)
ω

Photoemission

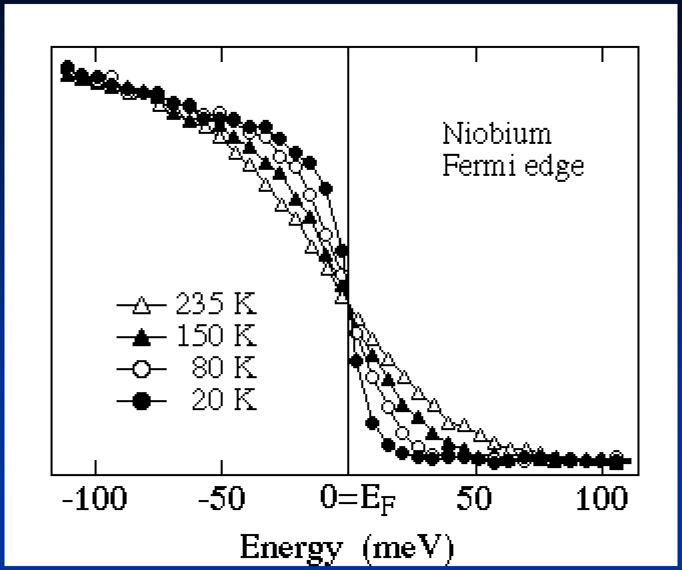




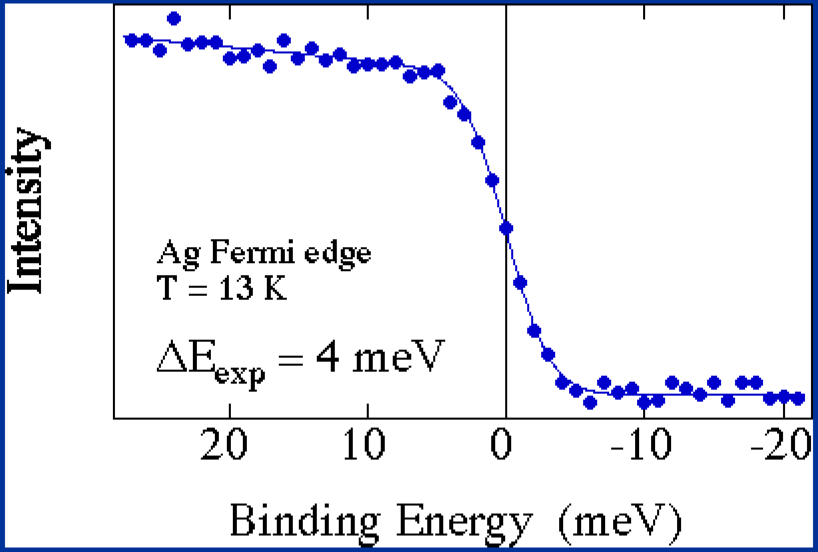


$$k_{//} = \sqrt{\frac{2mE_{kin}}{\hbar}}\sin\theta$$

Measures Spectral function $A(k,\omega)$


Niobium
Fermi edge
235 K
150 K
80 K
20 K
-100
-50
0=E_F
50
100
Energy (meV)

(M. Grioni)


Intensity
Ag Fermi edge
T = 13 K
ΔE_{exp} = 4 meV
20
10
0
-10
-20
Binding Energy (meV)

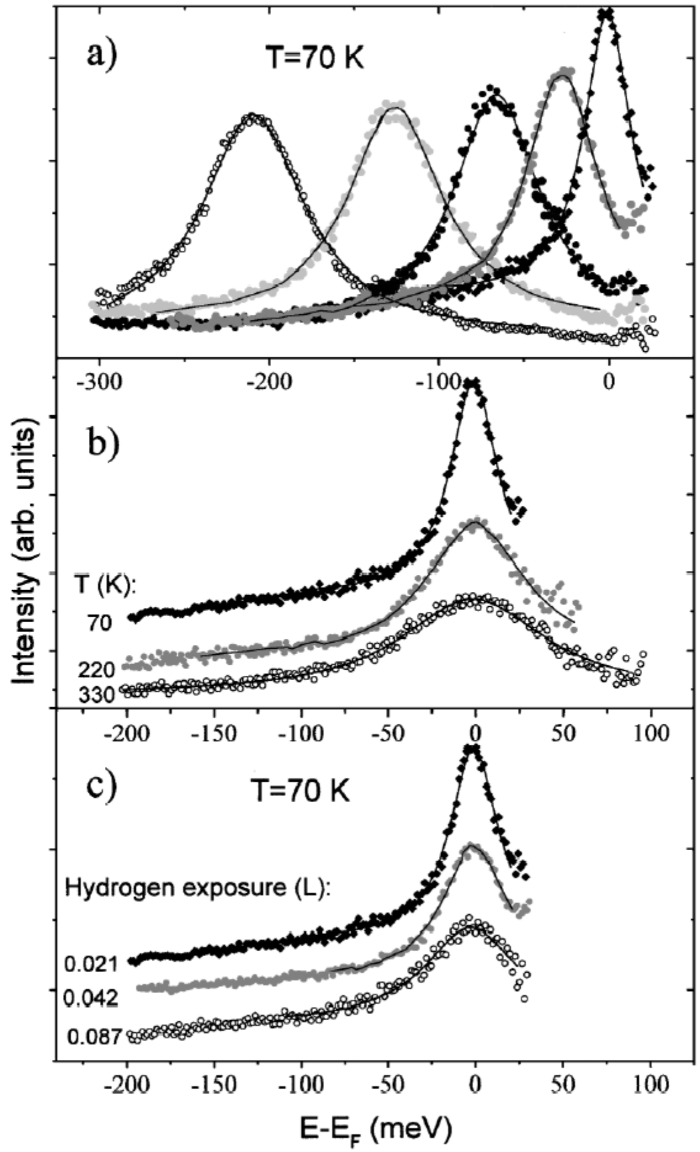


FIG. 2. Spectral intensity as a function of binding energy for constant emission angle, normalized to the experimentally determined Fermi cutoff. Data are symbols, while lines are fits to the Lorentzian peaks with a linear background. The dependence on the (a) binding energy, (b) temperature, and (c) hydrogen exposure is shown.

T. Valla et al. PRL 83 2085 (1999)

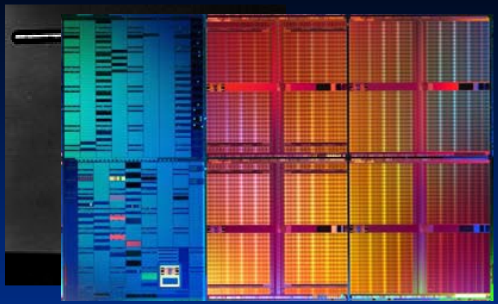
Mo(110) surface

Fermi liquid theory

- Shown perturbatively in U
- Much more general and robust

Element	m*/m	χ/χ_0
Nb	2	1
3He	6	20
Heavy fermion	100	100

- Transistor



1956

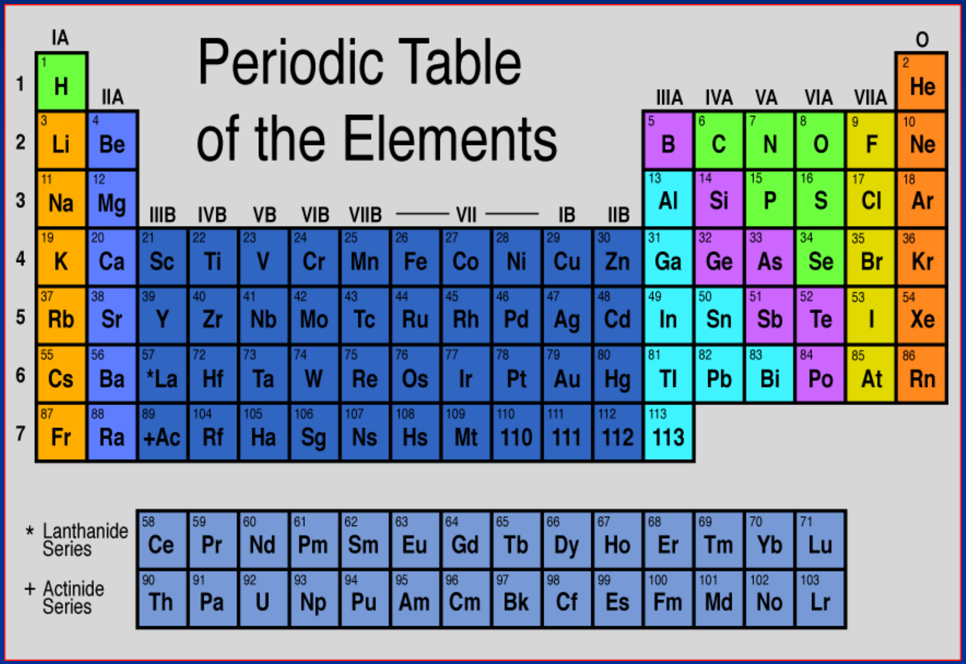
- Superconductivity

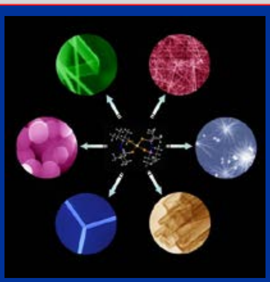
(1913),1972, 1973,1987,2003

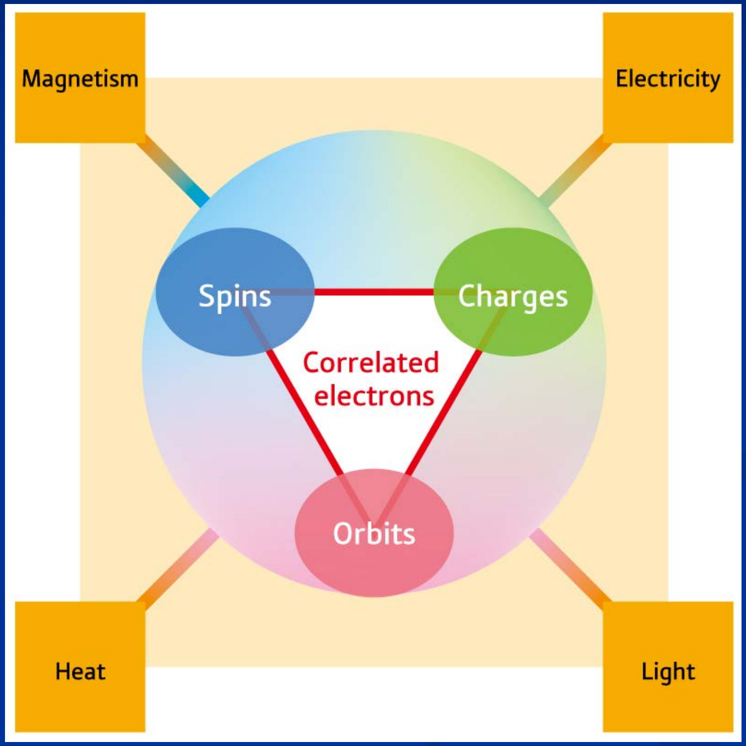
- Giant magnetoresistance

2007

Materials of the future

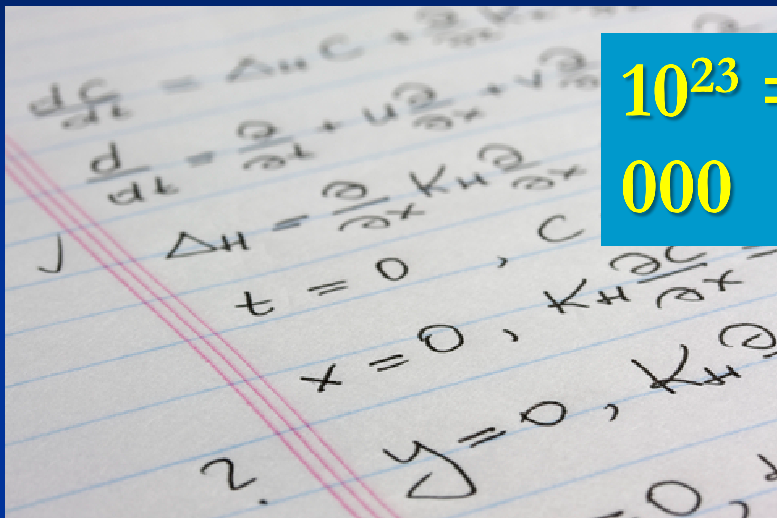






(Y. Tokura, Japan)

Need to understand interactions



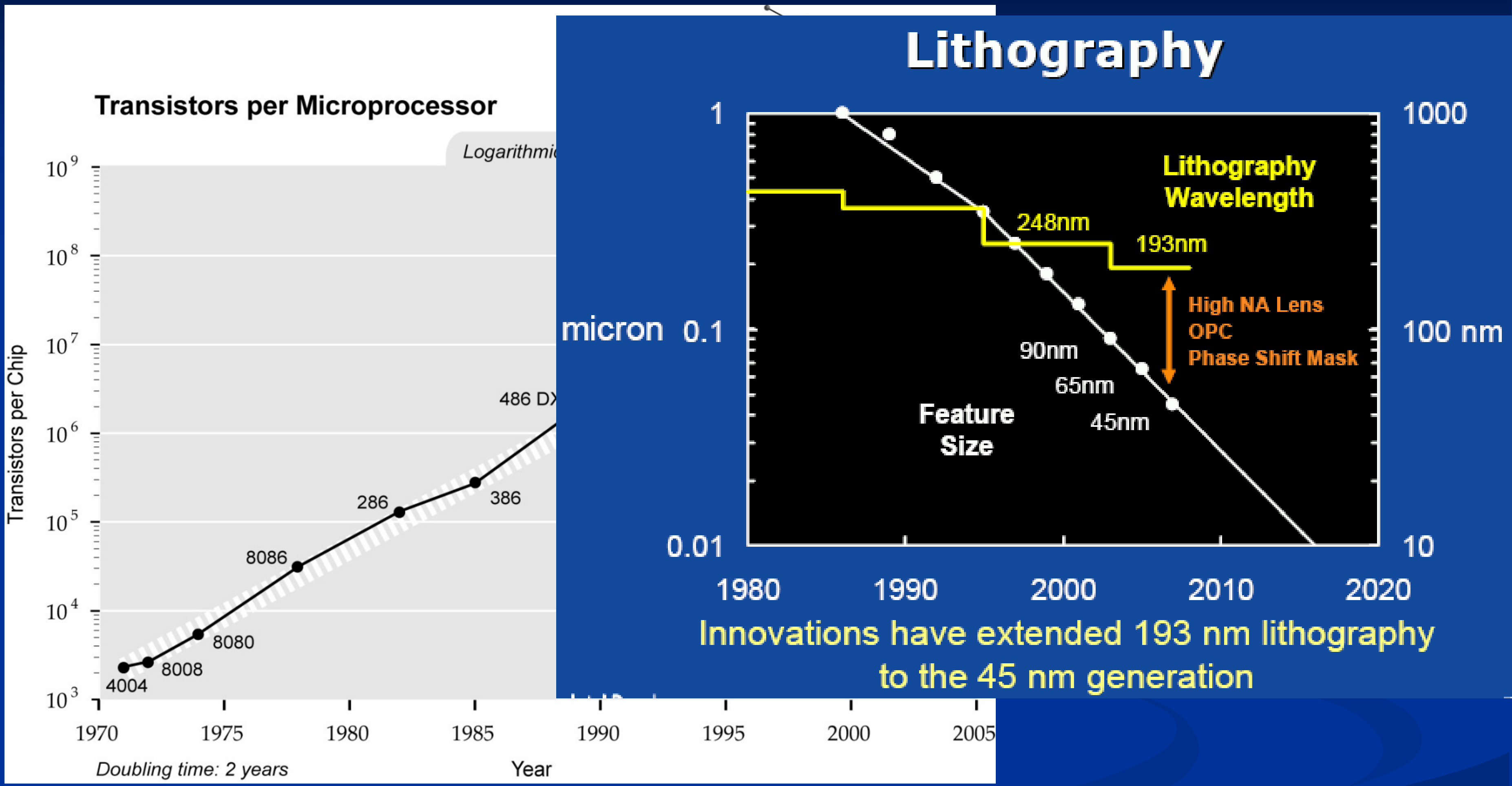
10^{23} = 100 000 000 000 000 000 000 000 000

Swiss supercomputing center (Mano); machines with 7 10^15 operations per second

Quantum nature of the problem :
numerical instabilities with classical computers

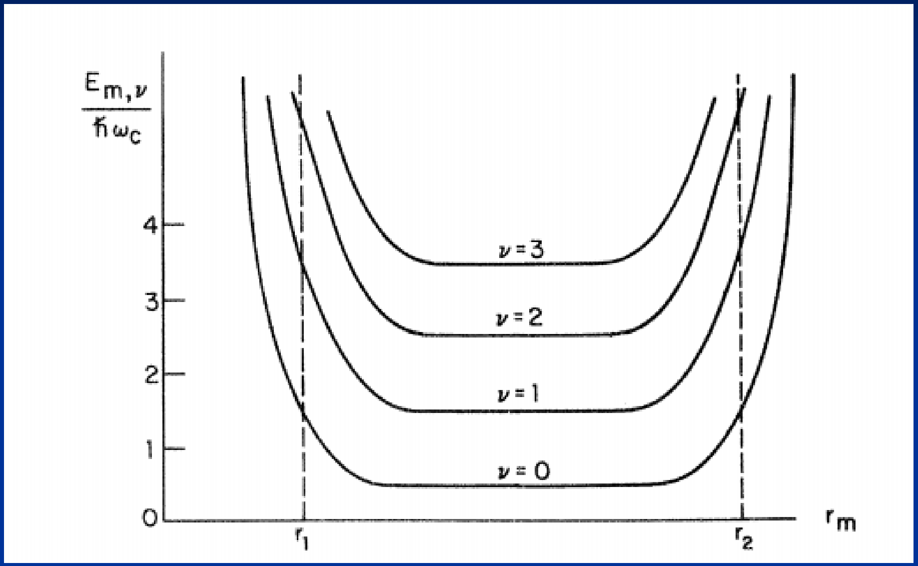
Less and less dimensions

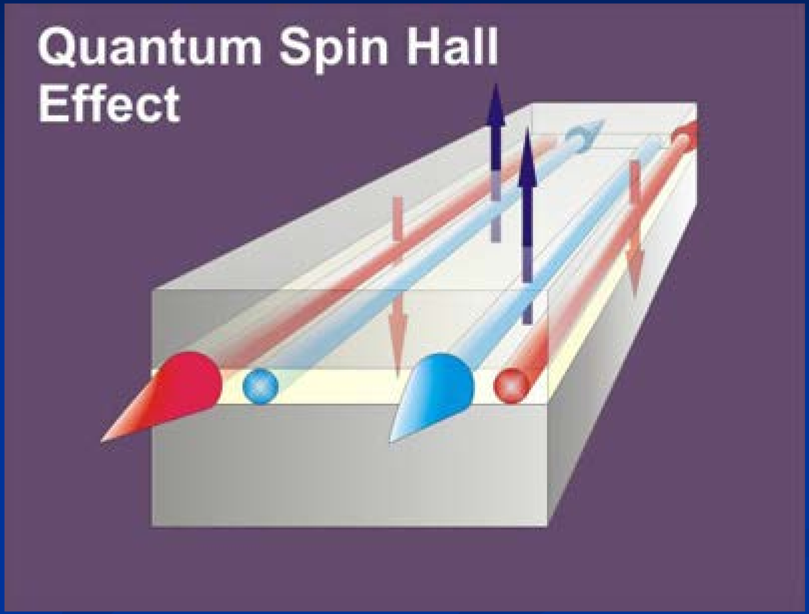
Future electronic



Need to worry about reduced dimensionality

Physics at the edge





Presence of edge
(B. I. Halperin)

Quantum hall effect
Topological insulators….

Why one dimension ?

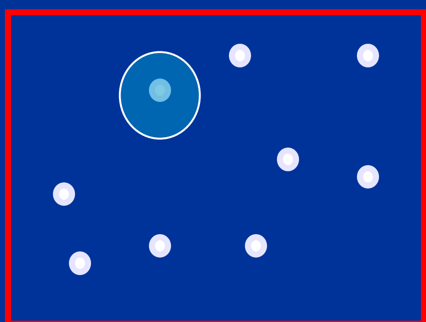
A good reason to work on 1D

However, my personal reason for working on one-dimensional problems is merely that they are fun. A man grows stale if he works all the time on the insoluble and a trip to the beautiful work of one dimension will refresh his imagination better than a dose of LSD.

Freeman Dyson (1967)

One dimension is specially interesting

- No individual excitation can exist (only collective ones)

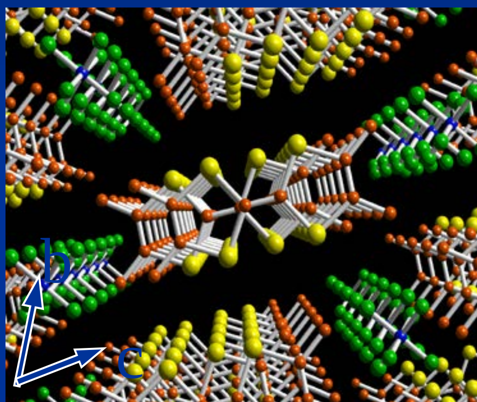


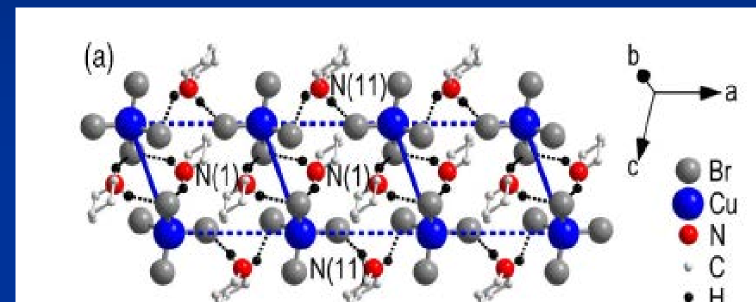
- Strong quantum fluctuations

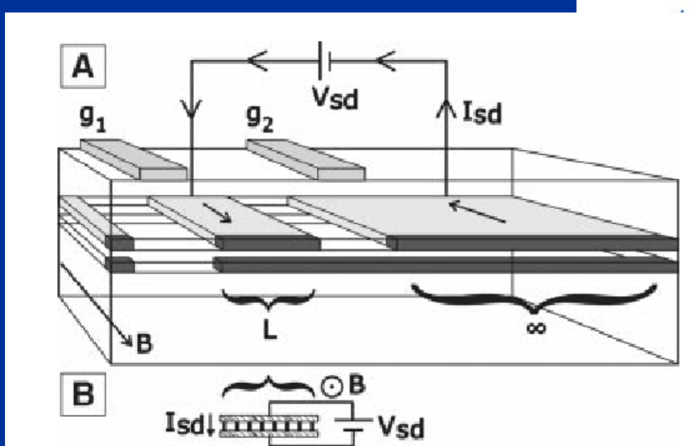
$$\psi = |\psi| e^{i\theta}$$

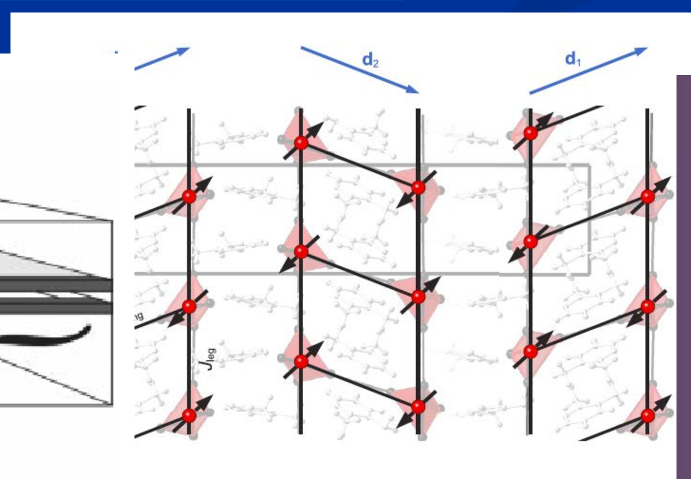
Difficult to order

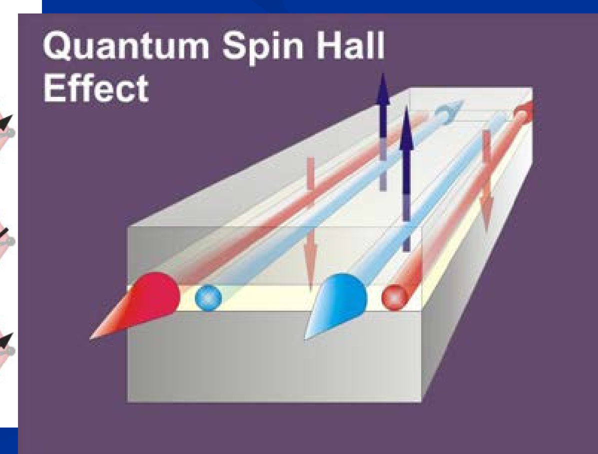
Many CM or cold atoms Systems











Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

Drastic evolution of the 1d world

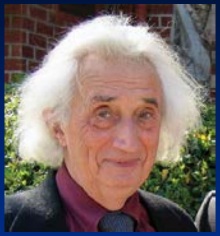
- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

One dimension

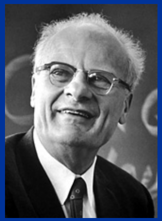
How to treat ?

- ``Standard” many body theory

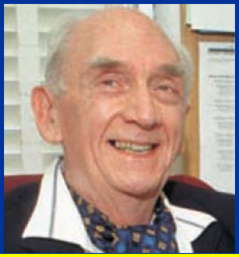
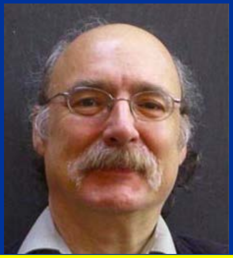



- Exact Solutions (Bethe ansatz)

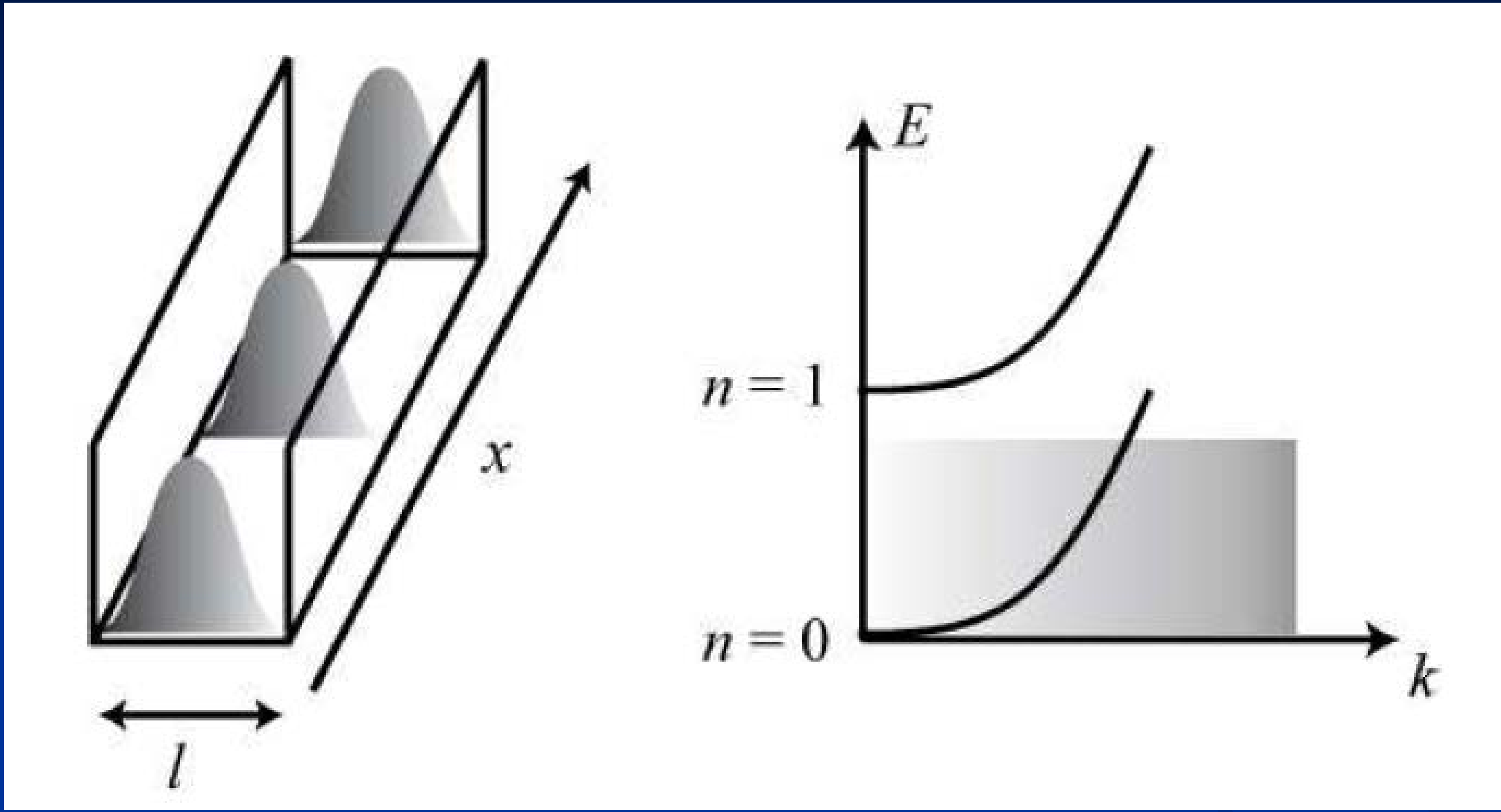



- Field theories (bosonization, CFT)

- Numerics (DMRG, MC, etc.)


And now we start….

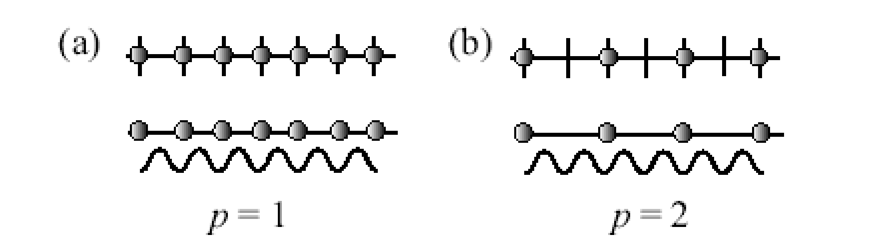

x
l
E
n = 1
n = 0
k

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu\int dx\, \rho(x)$$

- Lattice:



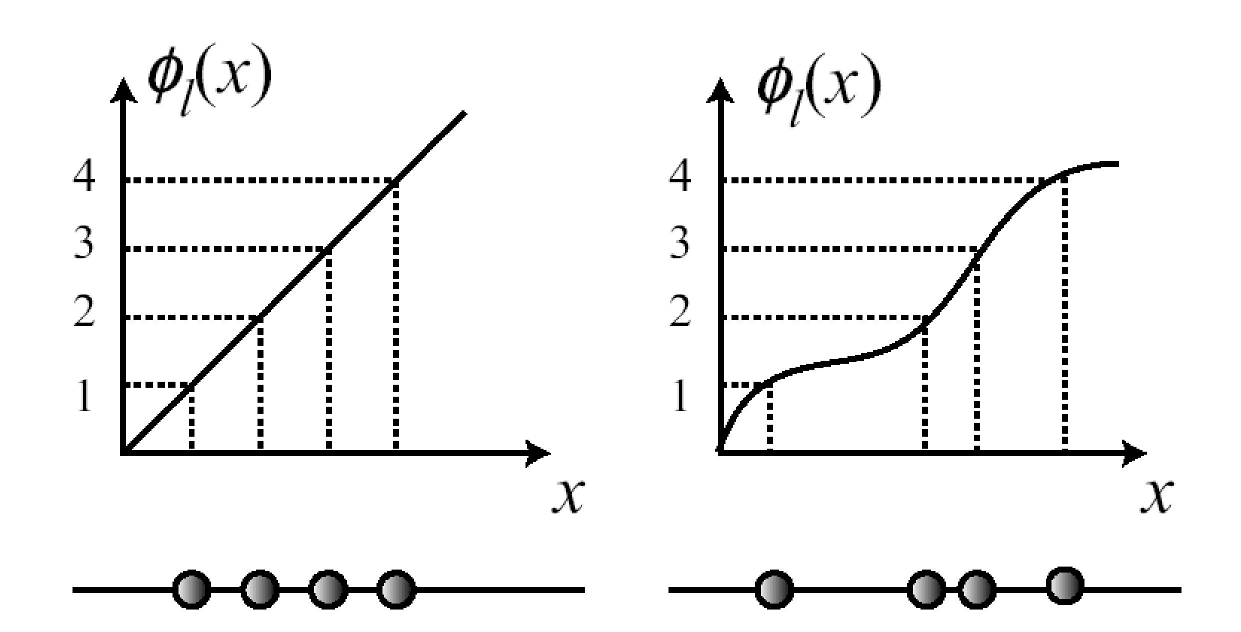
$$H = -J\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

Luttinger liquid physics

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

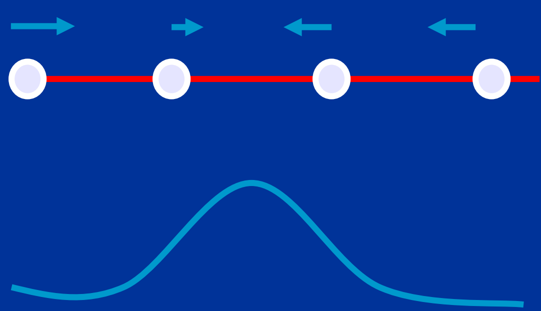
1D: unique way of labelling



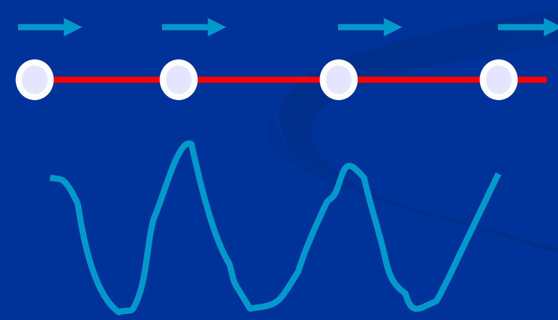
$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right] \sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly



$q \sim 0$



CDW

$q \sim 2\pi\rho_0$

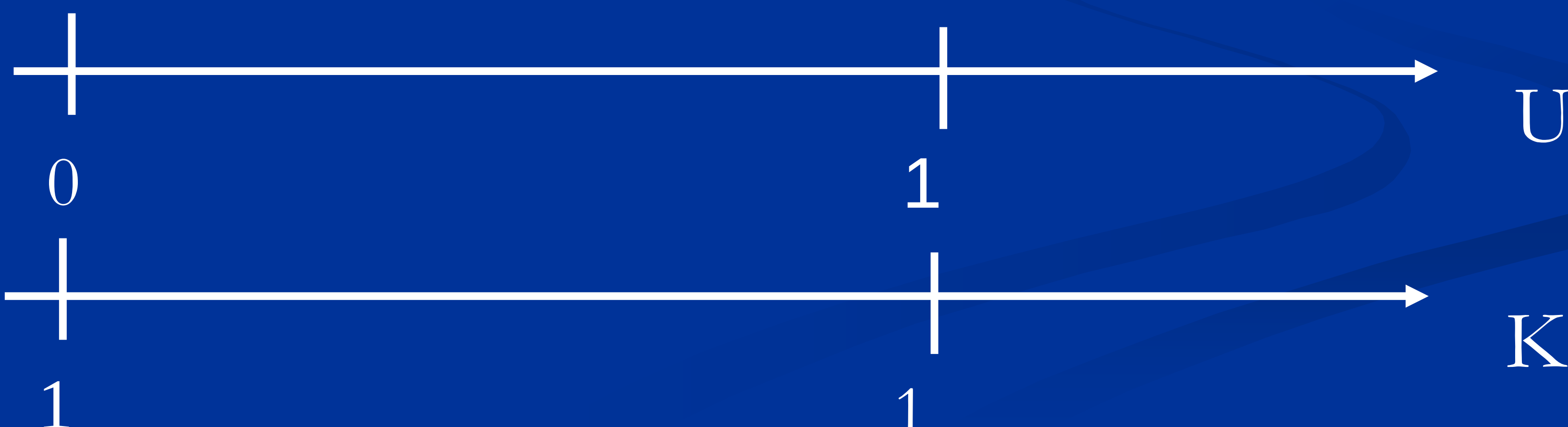
$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

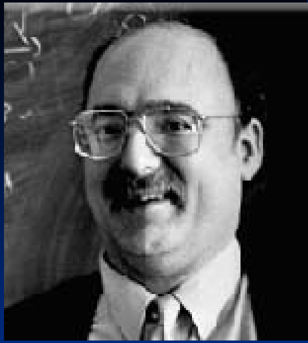
$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$H = \frac{\hbar}{2\pi}\int dx [\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$



Luttinger liquid concept

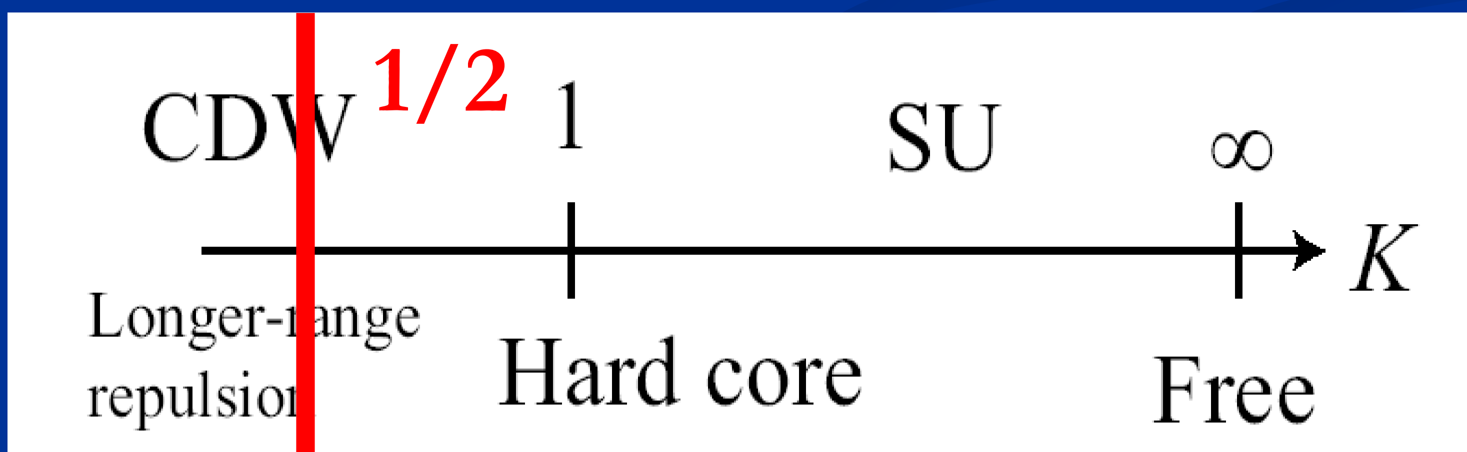


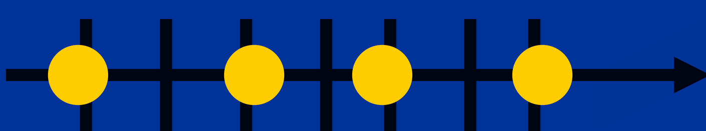
- How much is perturbative ?
- Nothing (Haldane):
  provided the correct u,K are used
- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

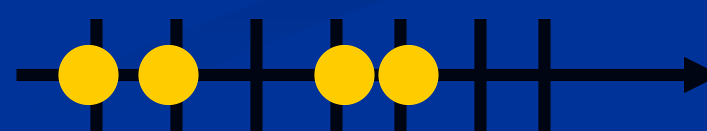
Correlations

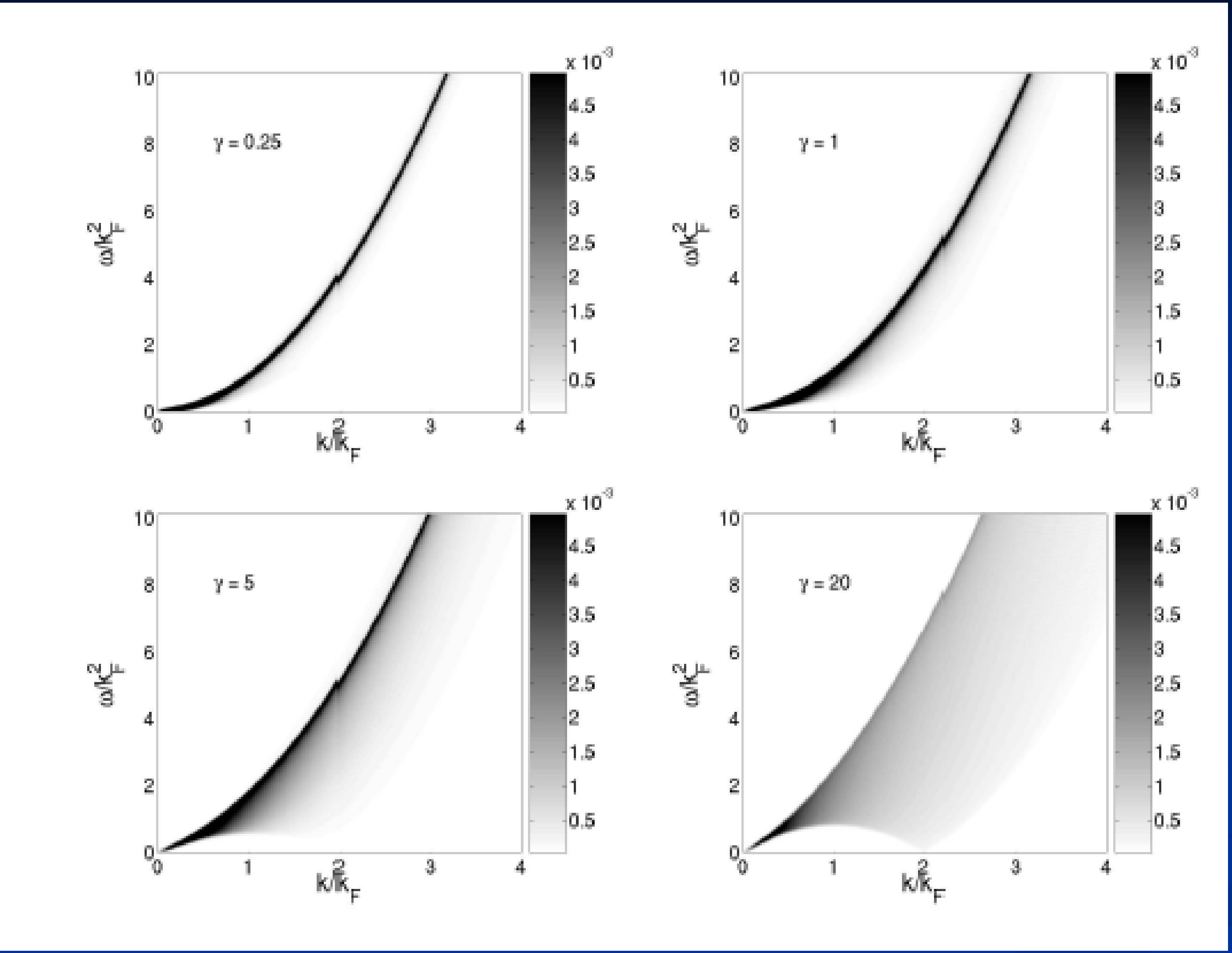
$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$





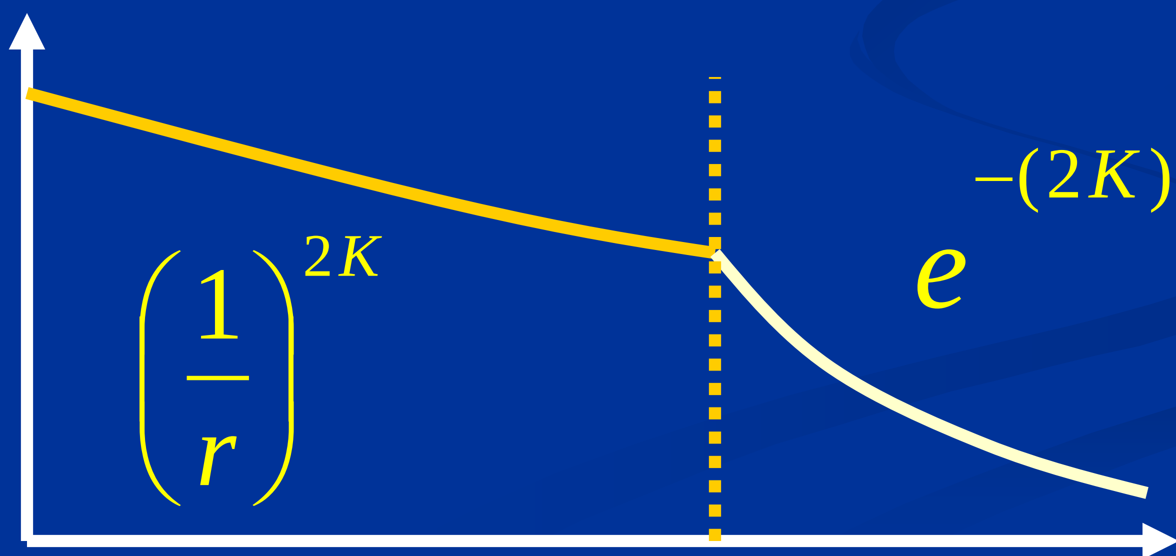




S(q,!) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory



$$\left(\frac{1}{r}\right)^{2K}$$

$$e^{-(2K)\frac{\pi x}{\beta}} = e^{-x/\xi_\beta}$$

Other 1D systems

Spins

Use boson or fermions mapping

$$S^+ = (-1)^i e^{i\theta} + e^{i\theta}\cos(2\phi)$$

$$S^z = \frac{-1}{\pi}\nabla\phi + (\text{-}1)^i \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x,0)S^z(0,0)\rangle = C_1\frac{1}{x^2} + C_2(-1)^x\left(\frac{1}{x}\right)^{2K}$$

$$\langle S^+(x,0)S^-(0,0)\rangle = C_3\left(\frac{1}{x}\right)^{2K+\frac{1}{2K}} + C_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

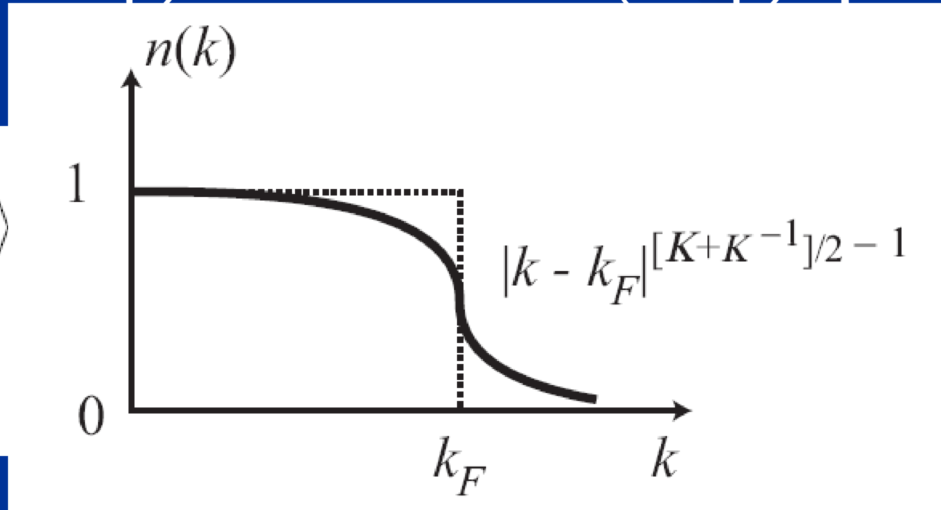
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles

$\langle\rho(x,\tau)\rho(0)\rangle$



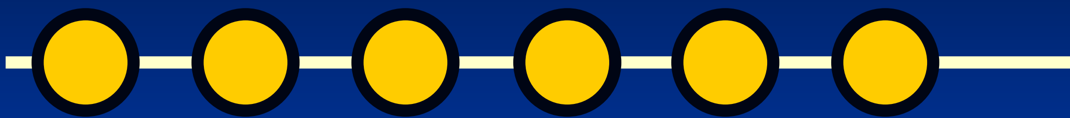
$\rho_0 x) \left(\frac{\alpha}{r}\right)^{2K}$

$\pi\rho_0 x) \left(\frac{\alpha}{r}\right)^{8K} + \cdots$

Calculation of Luttinger parameters

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = 1 : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

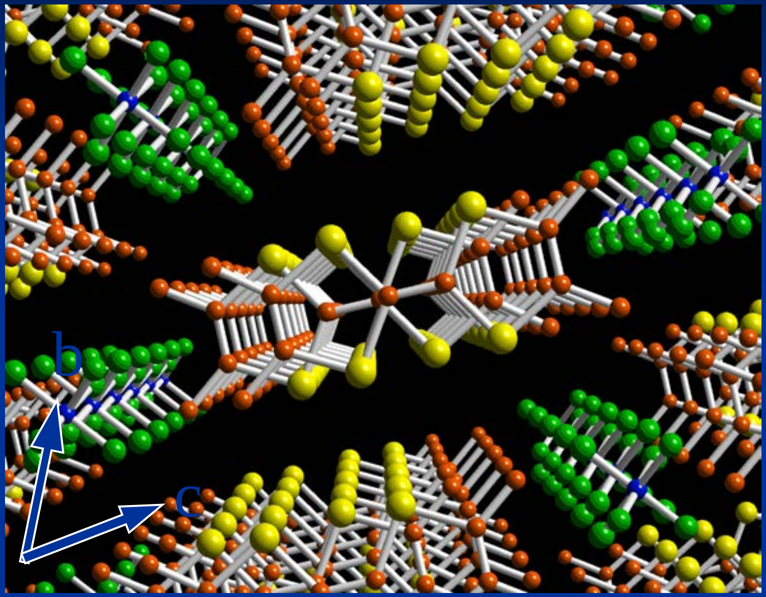
Free fermions: $$\langle \rho(x)\rho(0)\rangle \propto \cos(2k_F x)\left(\frac{1}{x}\right)^2$$

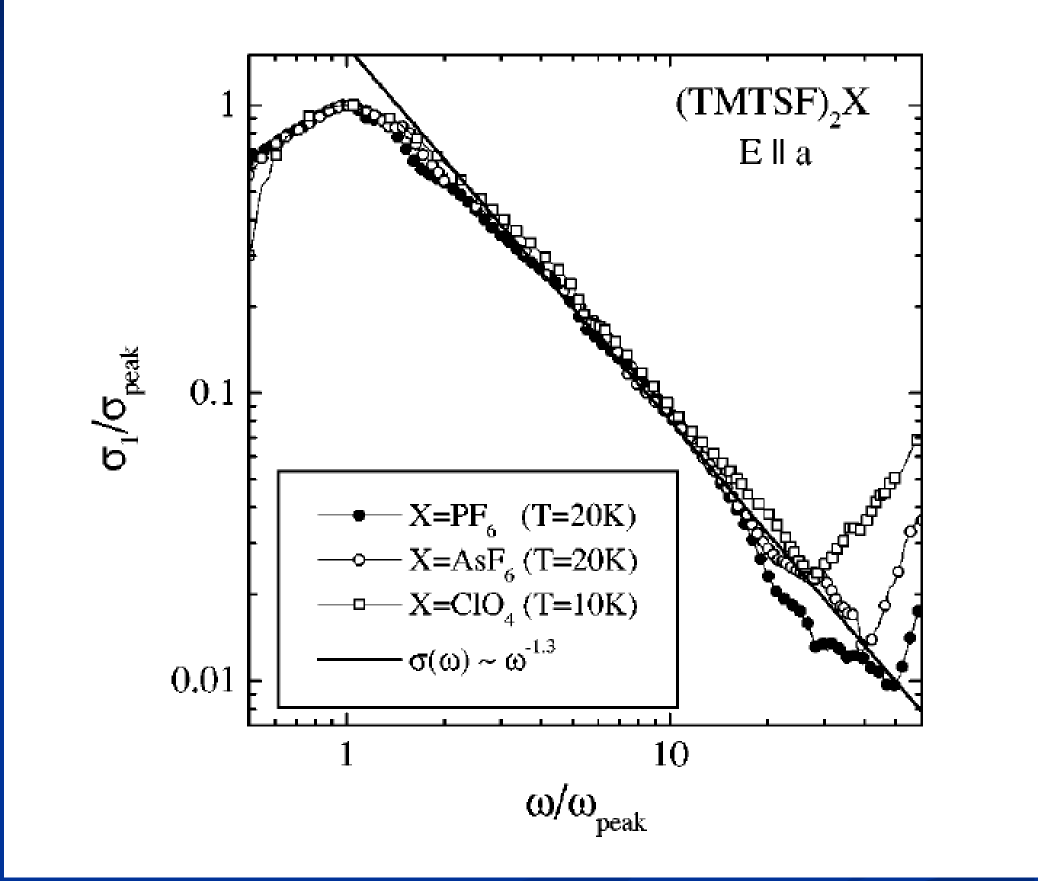
K=1

Note: $$\langle \psi_B(x)\psi_B(0)^\dagger\rangle \propto \left(\frac{1}{x}\right)^{1/2}$$

Tests of Luttinger liquids

Organic conductors



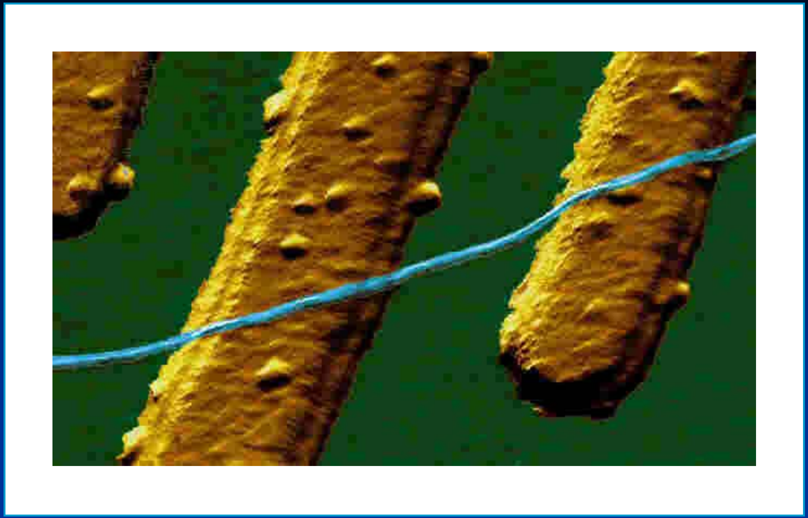


$$\sigma(\omega) \sim \omega^{-\nu}$$

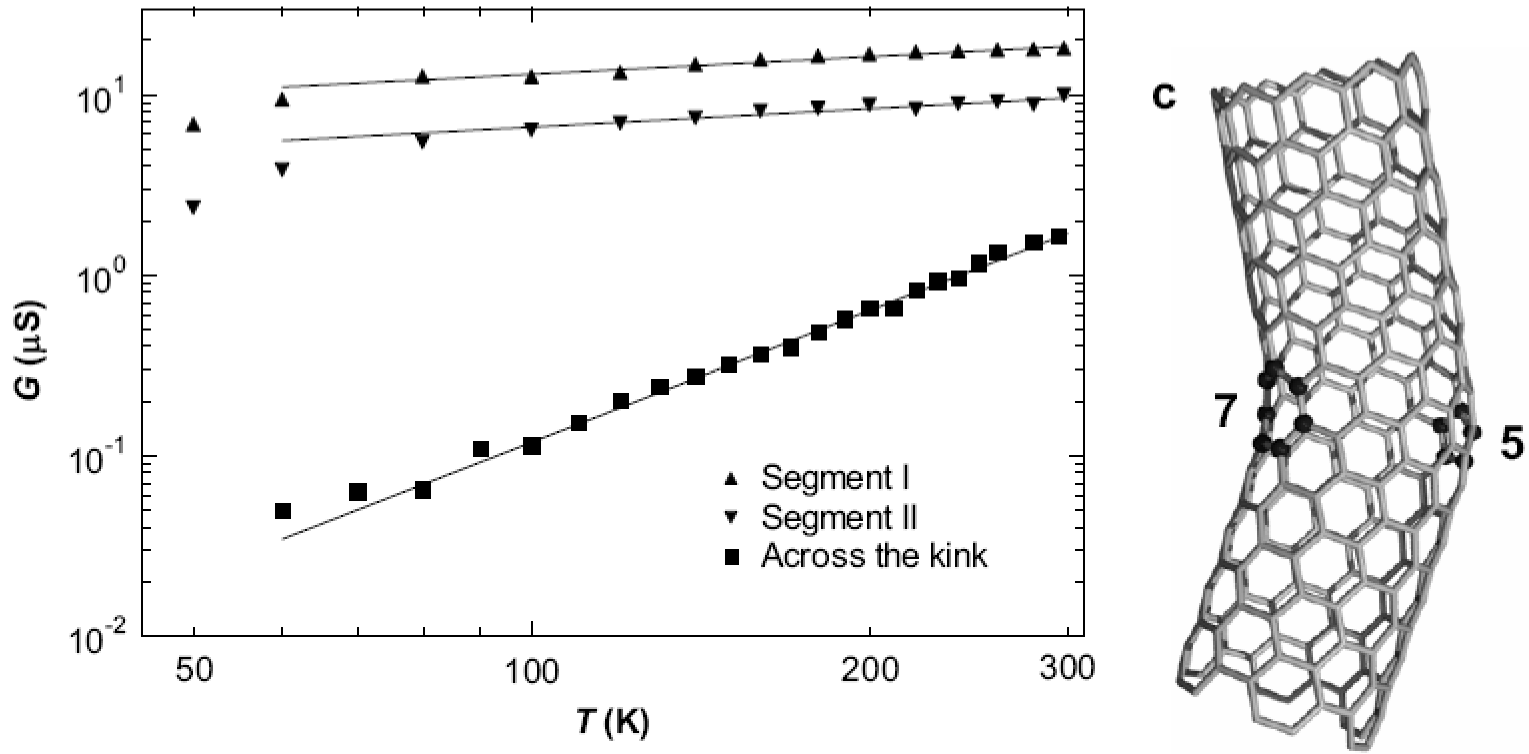
TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!



Z. Yao et al. Nature 402 273 (1999)



And many others…..

Edge states in quantum hall effect

Josephson junction arrays

Helium in capillaries

Photons in waveguides

Adatoms on vicinal surfaces

………..

Cold atoms

General references

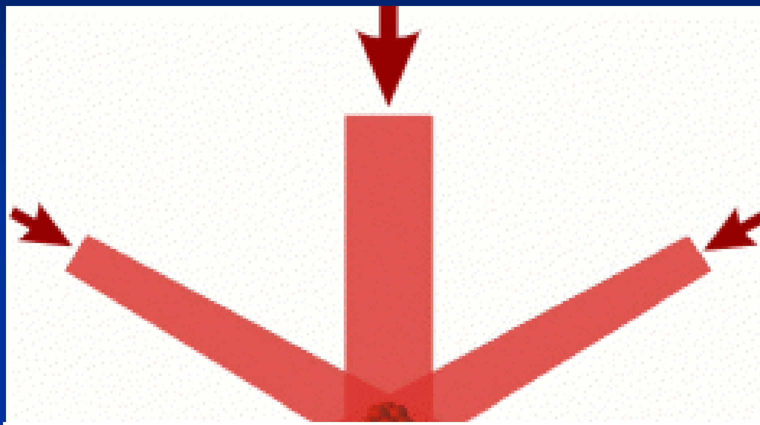
References:

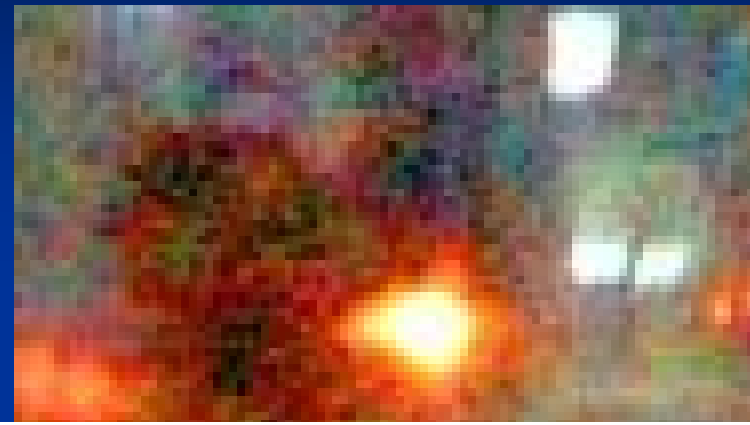
Immanuel Bloch, Jean Dalibard, and Wilhelm Zwerger
Rev. Mod. Phys. **80**, 885 (2008)

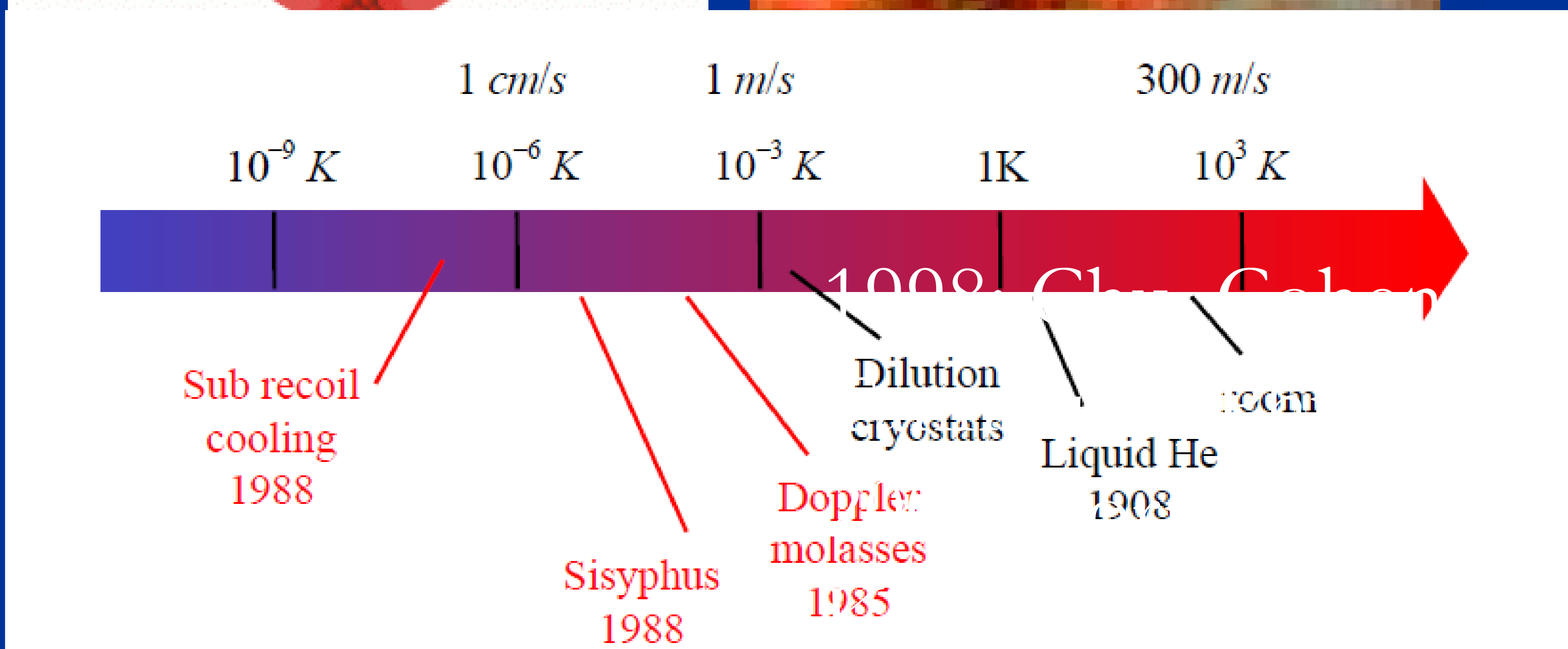
T. Esslinger:
Condensed Matter Physics 1 (2010).

A. Georges, T. Giamarchi
Les houches 2012, arXiv:1308.2684

Atom trapping and cooling

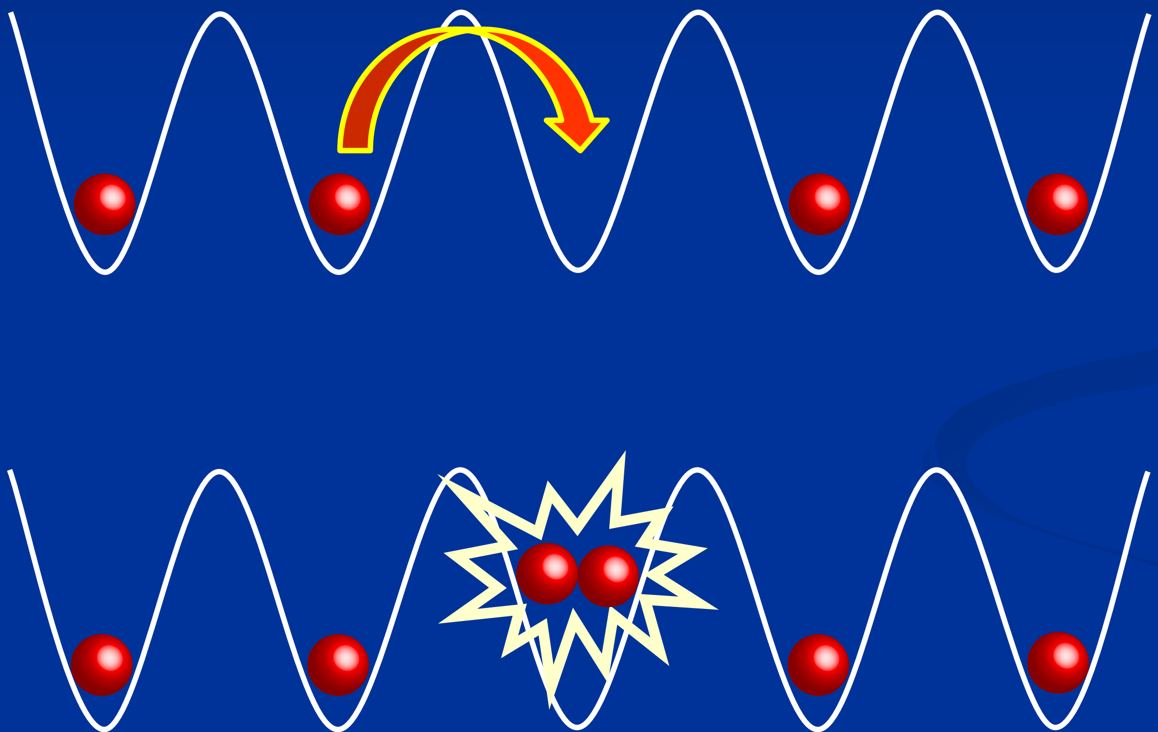






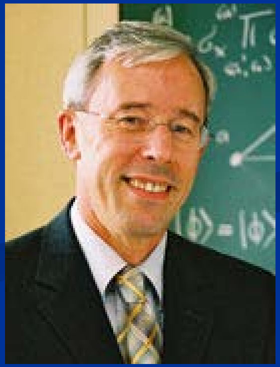
Groupe: T. Esslinger (ETH, Zurich)

Virtual solids



Tunnelling

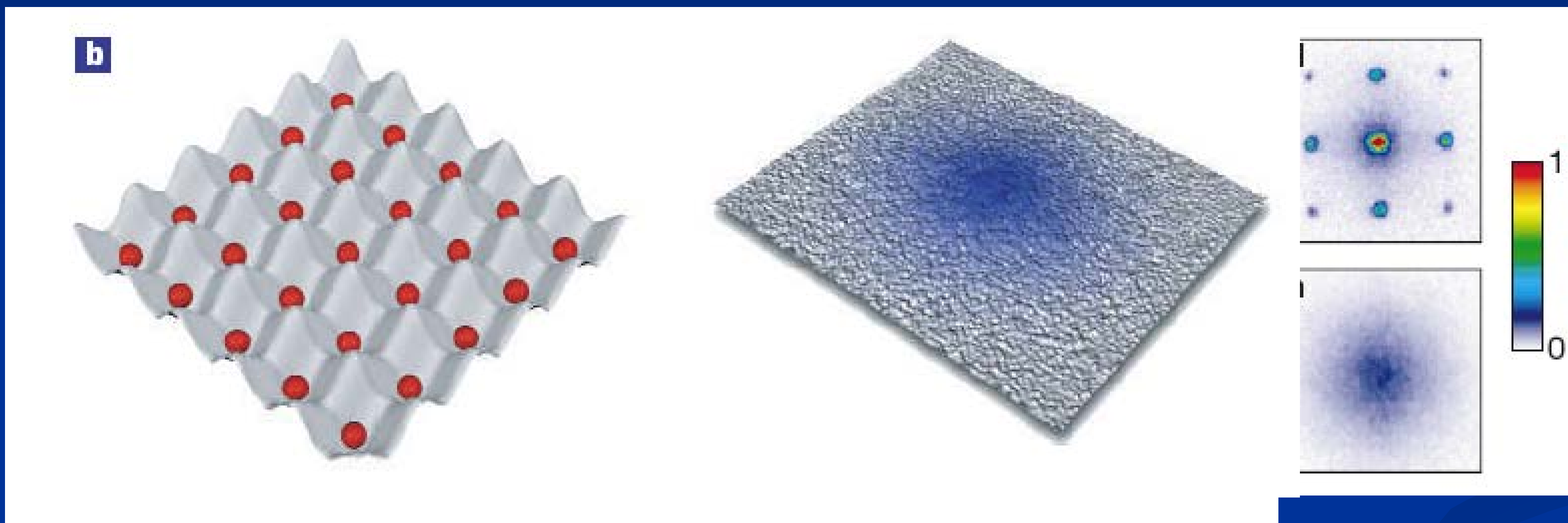
Short range interaction



P. Zoller

Proposal: D. Jaksch et al PRL81 3108 (98)

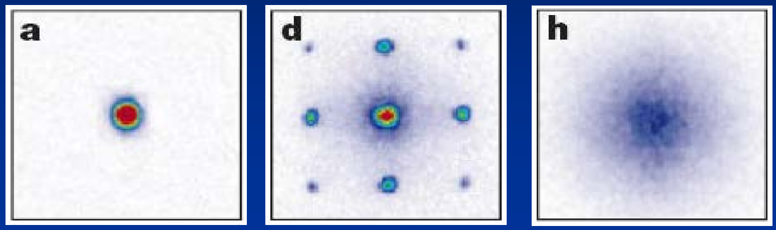
Simulators for condensed matter



M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, I. Bloch, Nature 415 39 (2002)

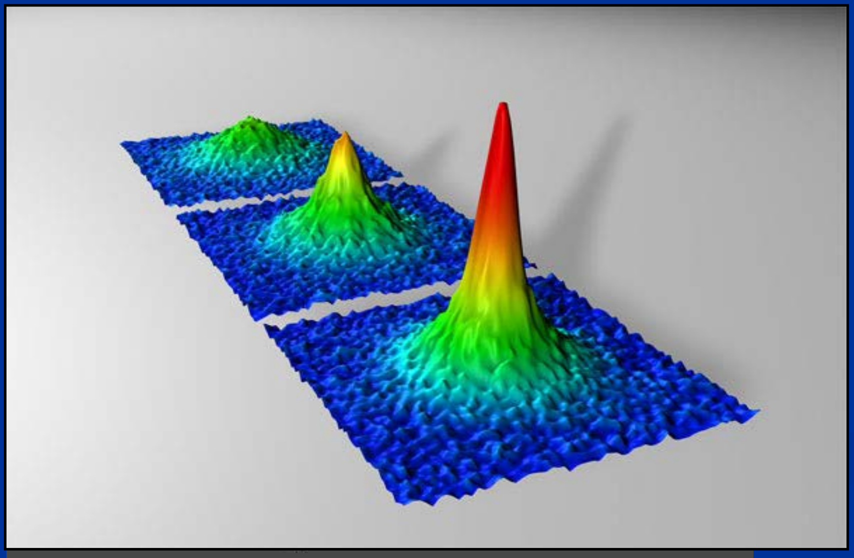
Good control on the system

Interactions
(Lattice, Feshbach resonnance)

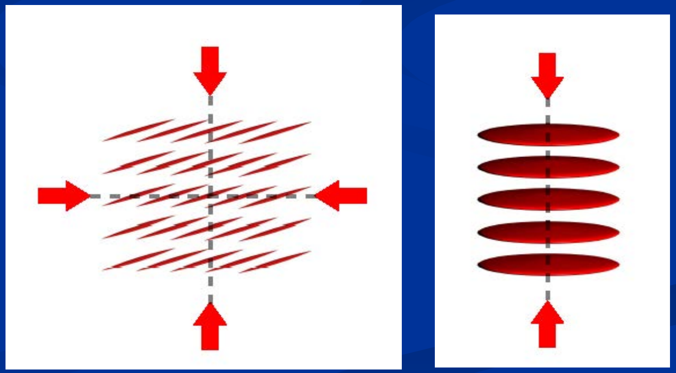


Statistics

Bosons Fermions



Dimensionality

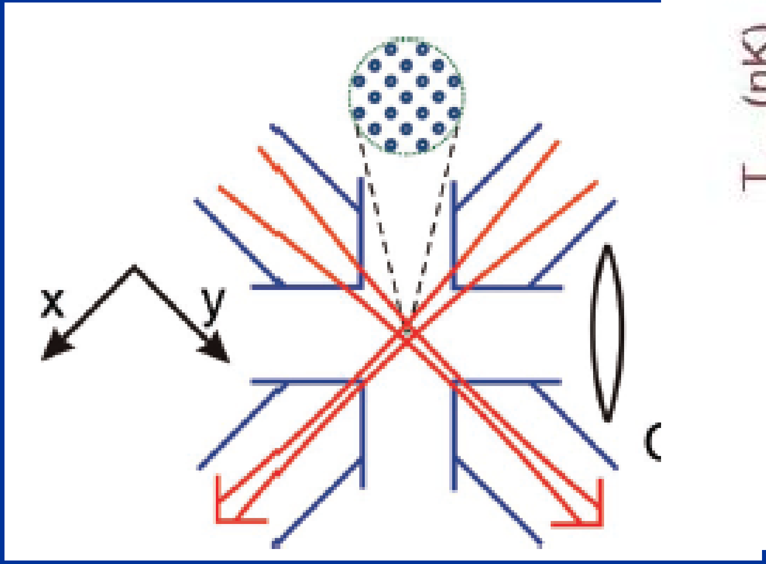


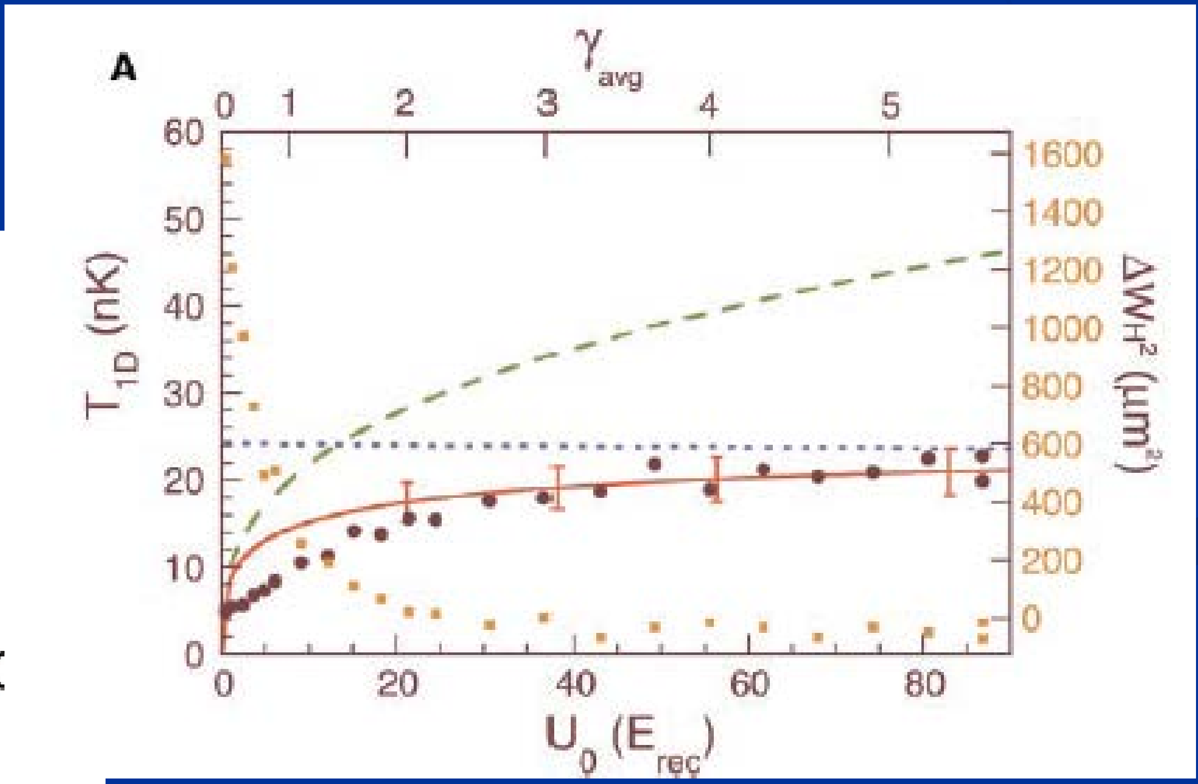
Bosons (continuum)

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

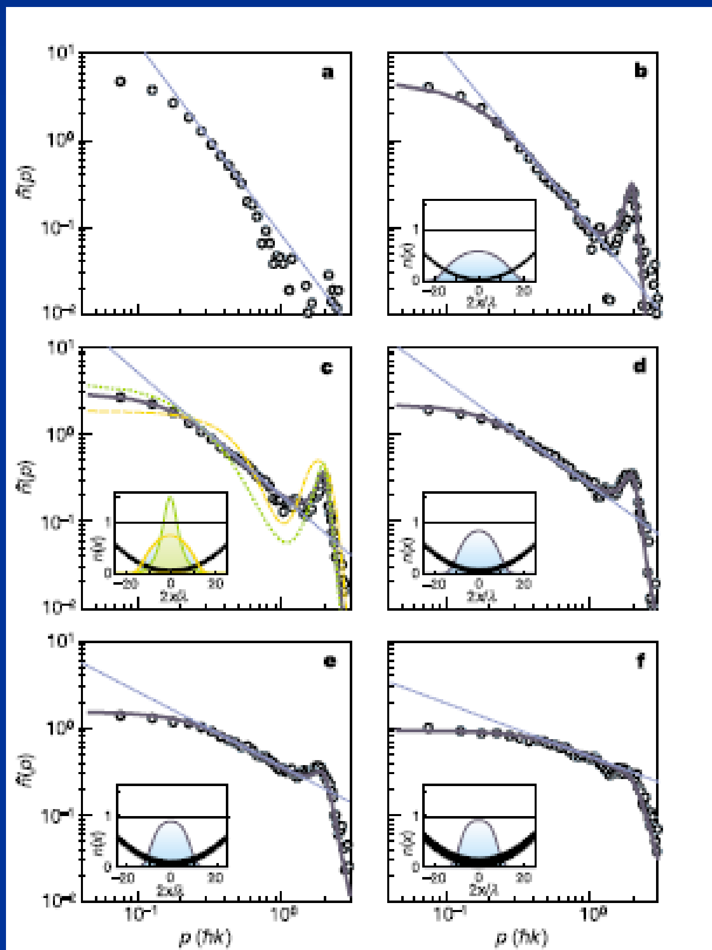
SCIENCE VOL 305 20 AUGUST 2004 1125

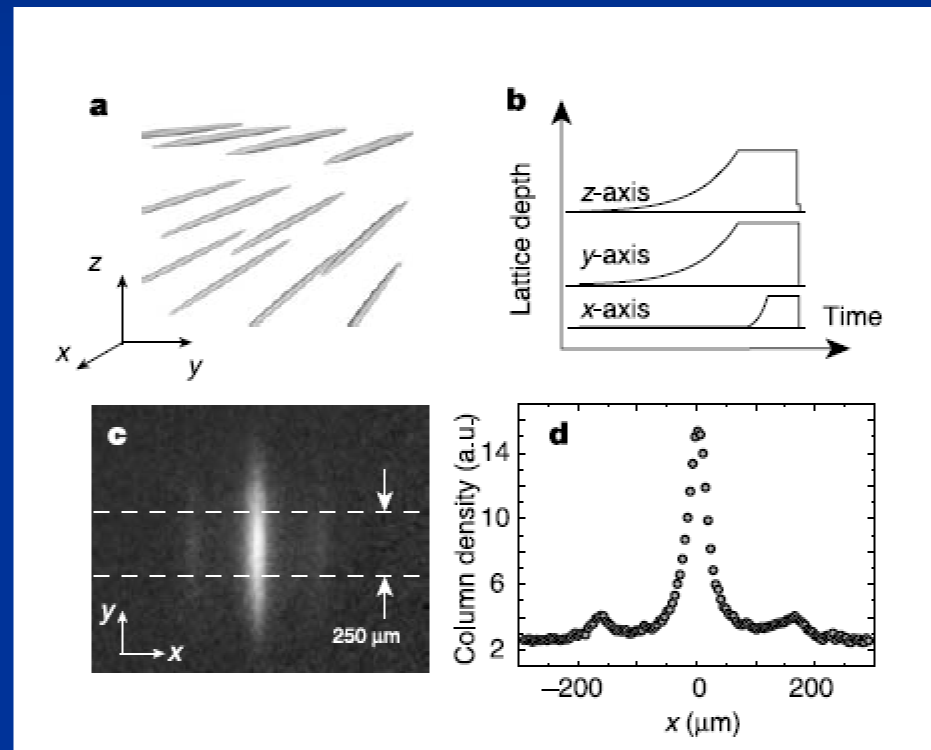




Optical lattices (dilute)

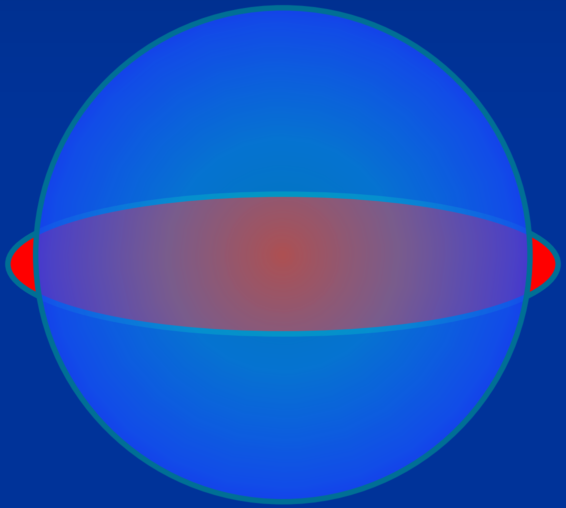
B. Paredes et al., Nature 429 277 (2004)





$$n(k)=\int dx e^{ikx}\langle\psi^{\dagger}(x)\psi(0)\rangle$$

Confining potential / Trap

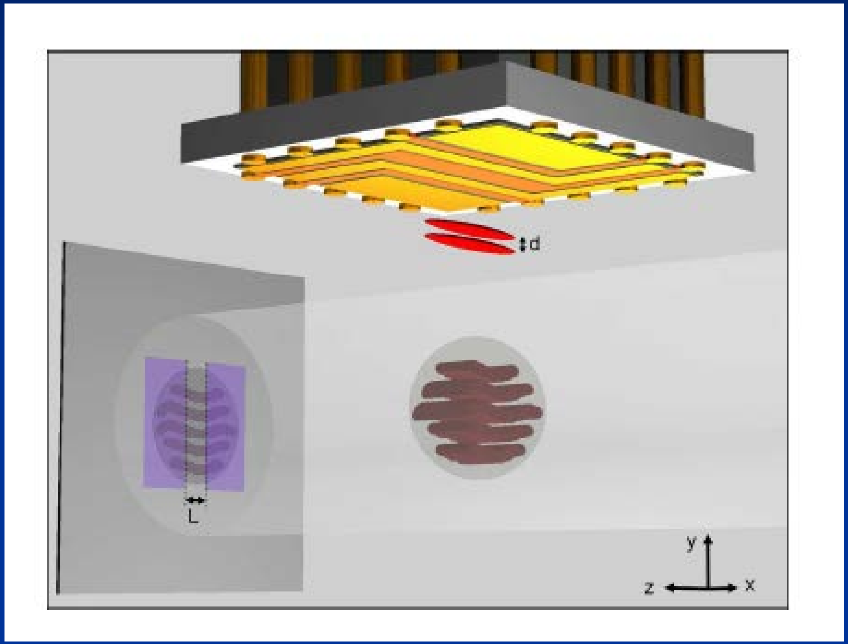


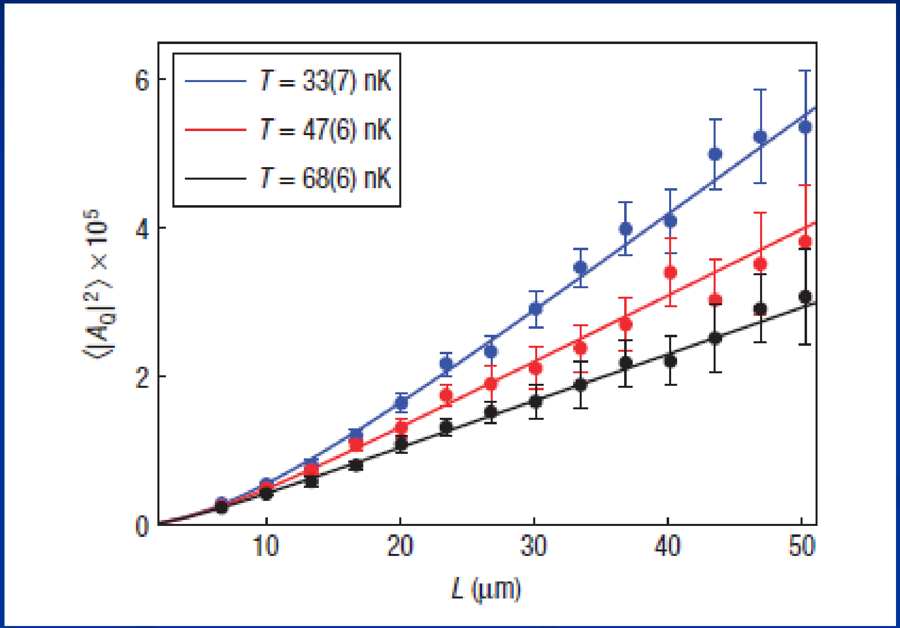
- No homogeneous phase !

$$H = \int d^3 r \frac{1}{2} \omega_0^2 r^2 \rho(r)$$

I am your worst nightmare

Atom chips





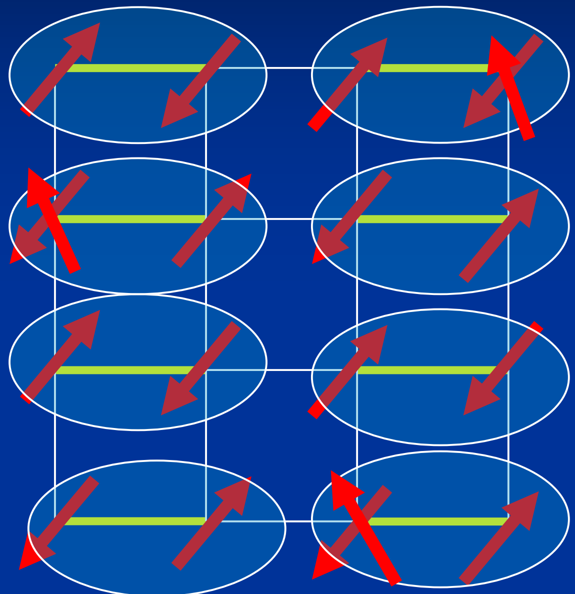
$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

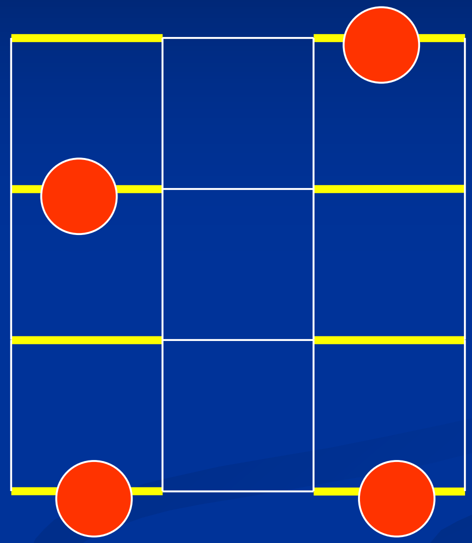
K large (42)

S. Hofferberth et al. Nat. Phys 4 489 (2008)

Magnetic insulators

triplon = hard core boson





h ~ h_c dilute limit: « free » bosons

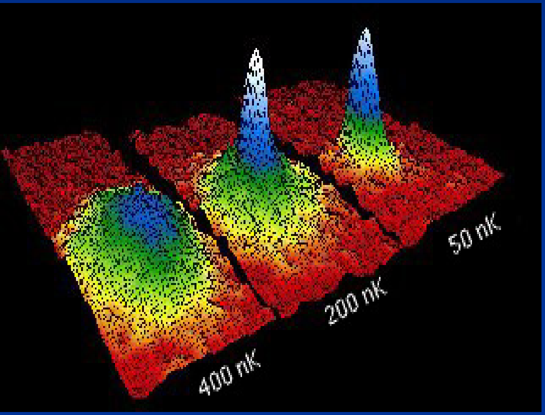
Bose Einstein Condensation

(TG and A. M. Tsvelik PRB 59 11398 (1999))

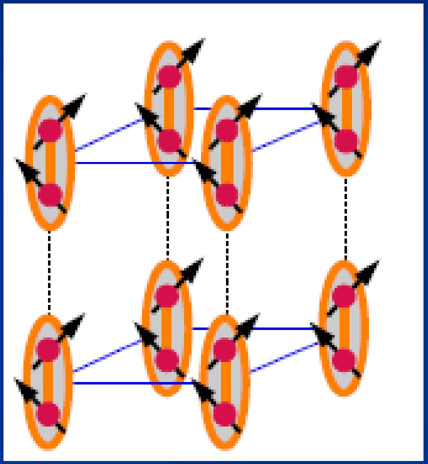
BEC vs BEC

Cold atoms

Dimers/Spins



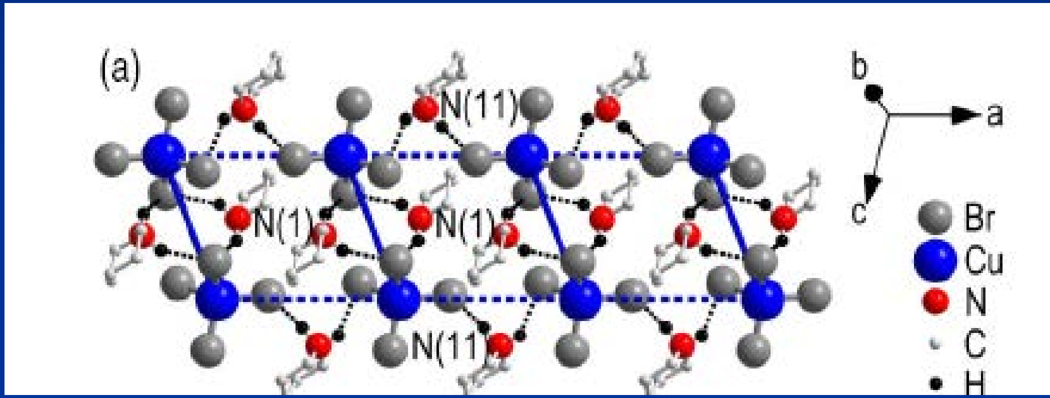
TG, Ch. Rüegg, O. Tchernyshyov, Nat. Phys. 4 198 (08)



Bose gas	Antiferromagnet
Particles	Spin excitations ($\Delta S^z = \pm 1$)
Boson number N	Spin component S^z
Charge conservation U(1)	Rotational invariance O(2)
Condensate wavefunction $\langle \psi(\mathbf{r}) \rangle$	Transverse magnetic order $\langle s_i^x + i s_i^y \rangle$
Chemical potential μ	Magnetic field B
Superfluid density ρ_s	Transverse spin stiffness
Mott insulating state	Integer magnetization plateau

Spin dimer systems

B. C. Watson et al., PRL 86 5168 (2001)

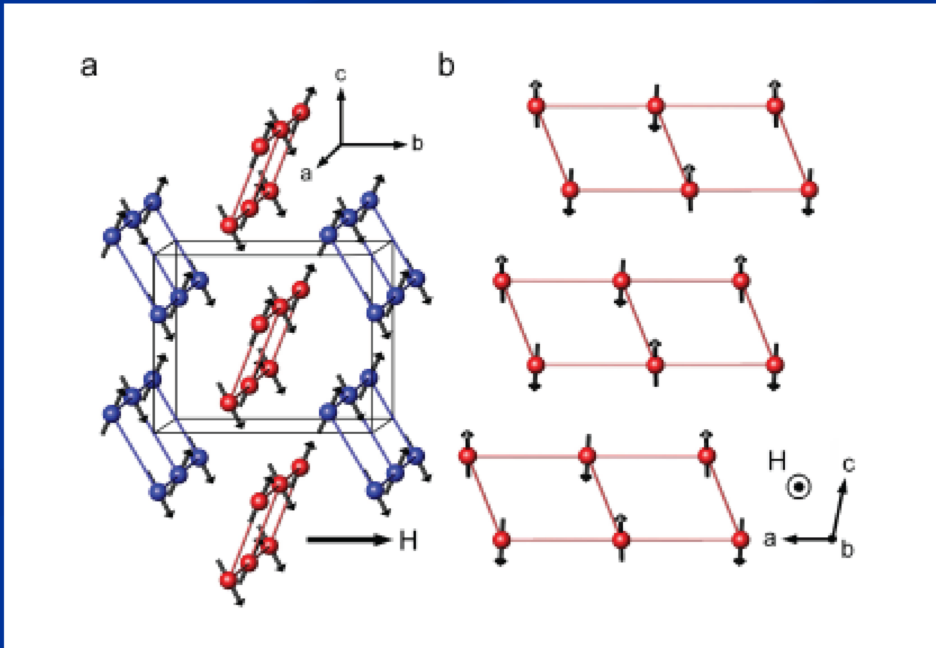


M. Klanjsek et al.,

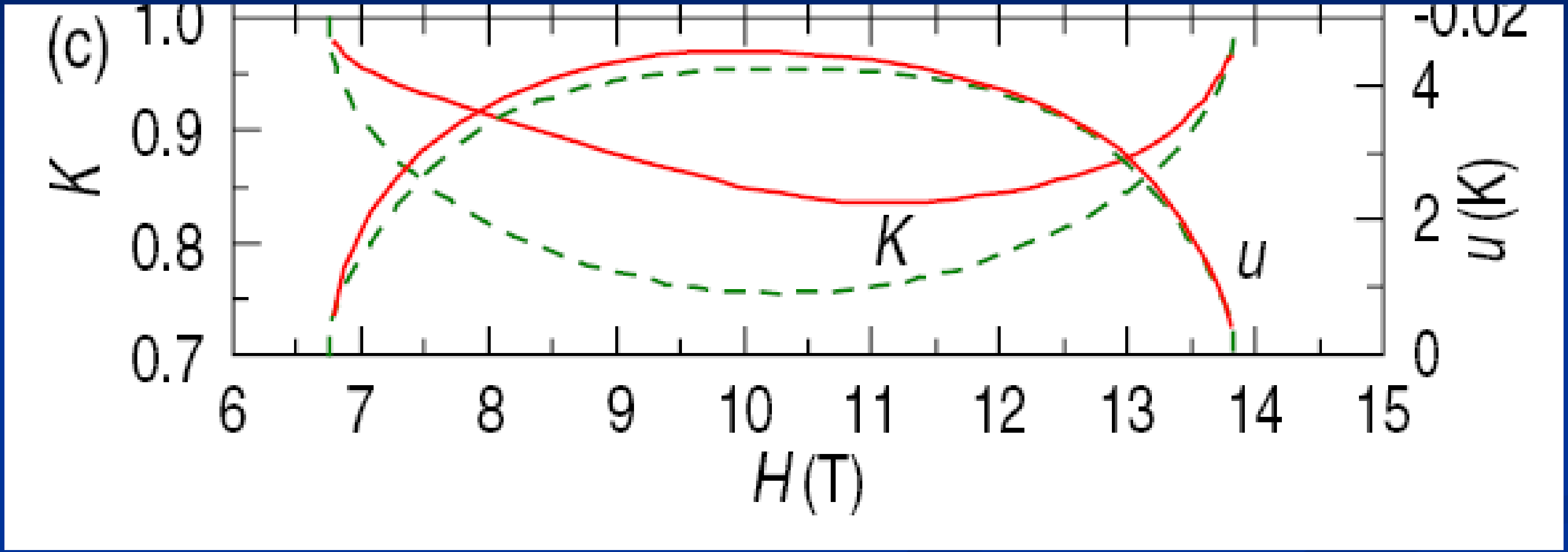
PRL 101 137207 (2008)

B. Thielemann et al.,

PRB 79 020408 ® (2009)



Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

Red : Ladder (DMRG)

Green: Strong coupling ($J_r \rightarrow 1$) (BA)

Correlation functions

M. Klanjsek et al., PRL 101 137207 (2008)

R. Chitra, TG PRB 55 5816 (97); TG, AM Tsvelik PRB 59 11398 (99)

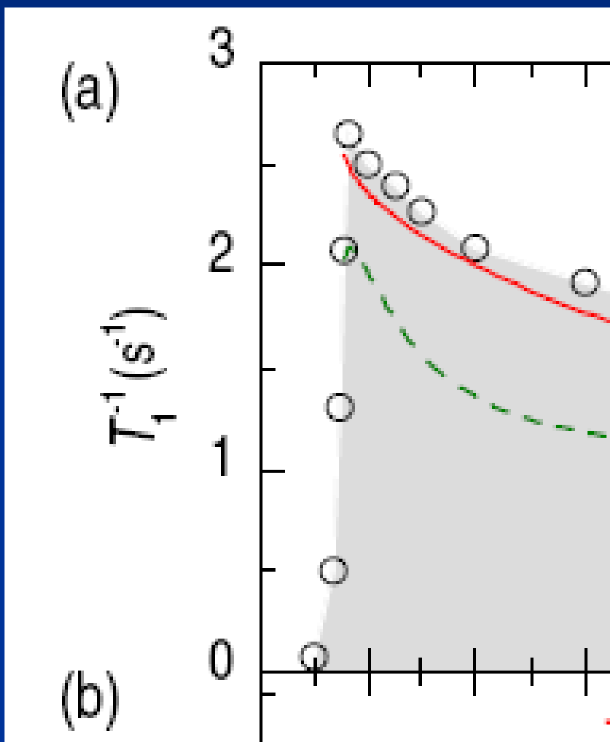
- NMR relaxation rate:

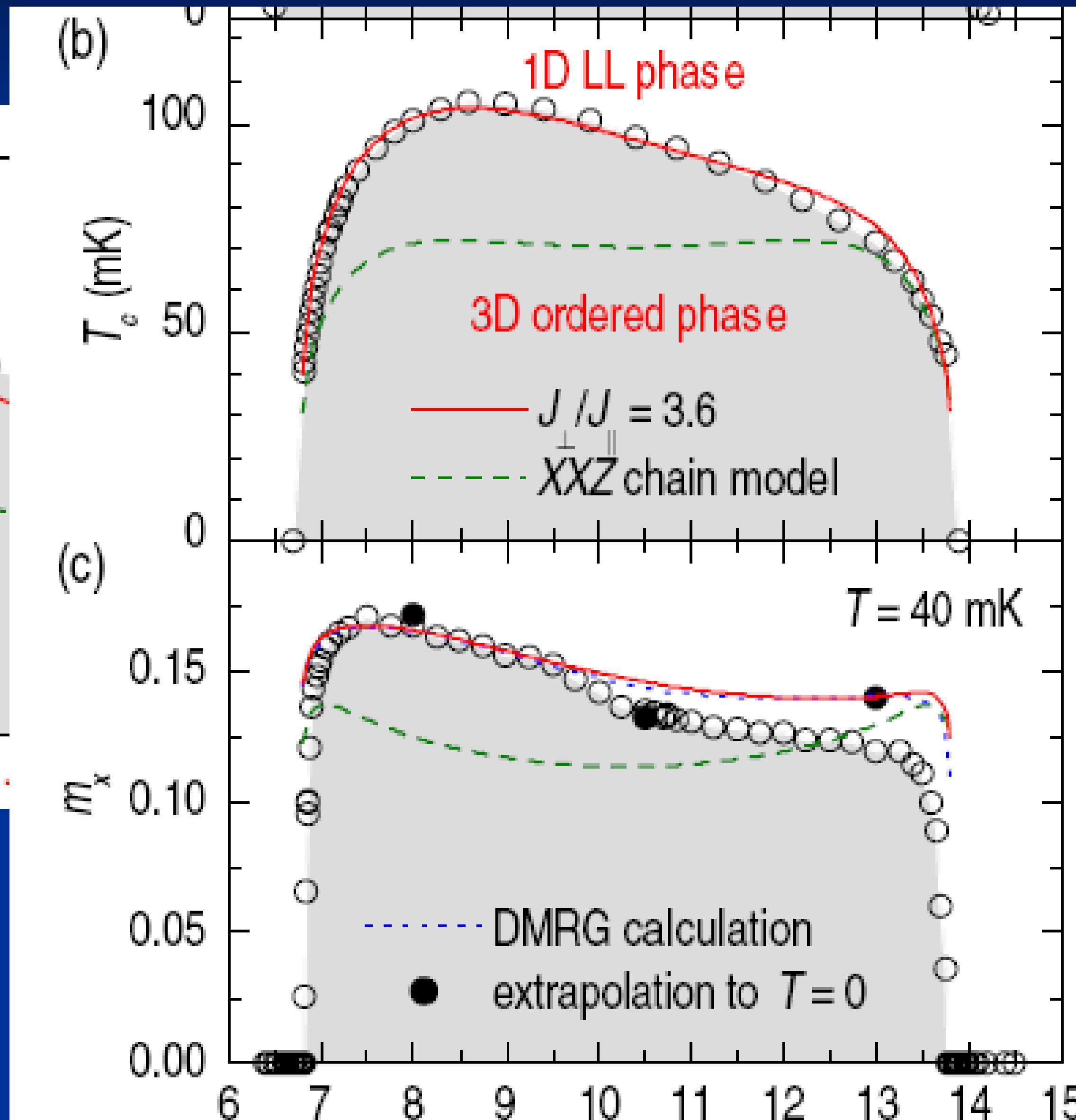
$$T_1^{-1} = \frac{\hbar \gamma^2 A_\perp^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right) \left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$

- Tc to ordered phase: $1/J' = \chi_{1D}(T_c)$

$$T_c = \frac{u}{2\pi}\left[\sin\left(\frac{\pi}{4K}\right) B^2\left(\frac{1}{8K}, 1 - \frac{1}{4K}\right) \frac{zJ' A_0^x}{2u}\right]^{2K/(4K-1)}.$$

NMR





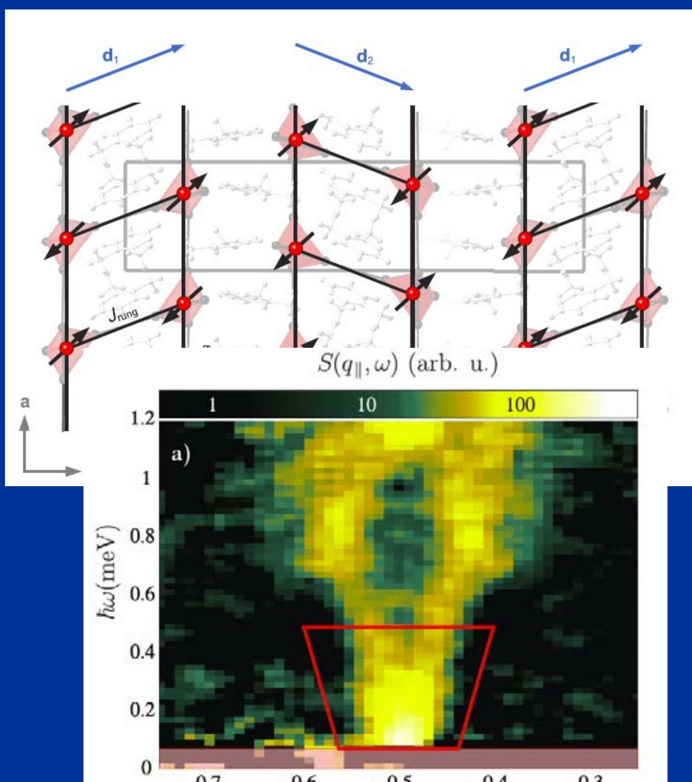
M. Klanjsek et al.,

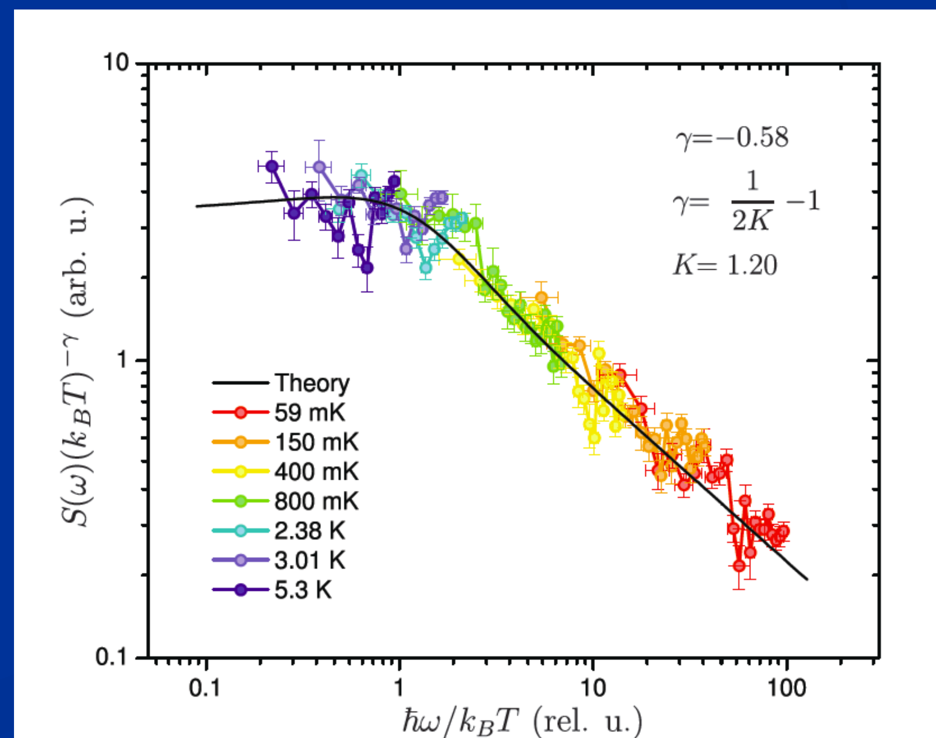
PRL 101 137207 (2008)

Spin ladders

D. Schmidiger et al. PRL 108 167201 (2012):
K. Yu et al. PRB 91 020406(R) (2015)

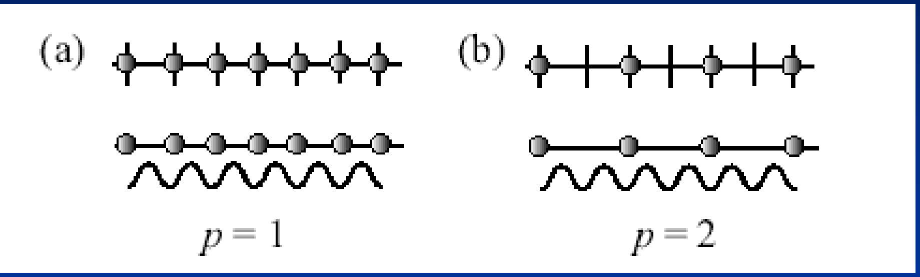
$$\langle S^- S^+ \rangle_{q,\omega} = \langle \psi \psi^\dagger \rangle_{q,\omega}$$





Sine-Gordon

Periodic lattice



$$H = \int dx V_0 \cos(Qx)\rho(x)$$

$$H = \int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

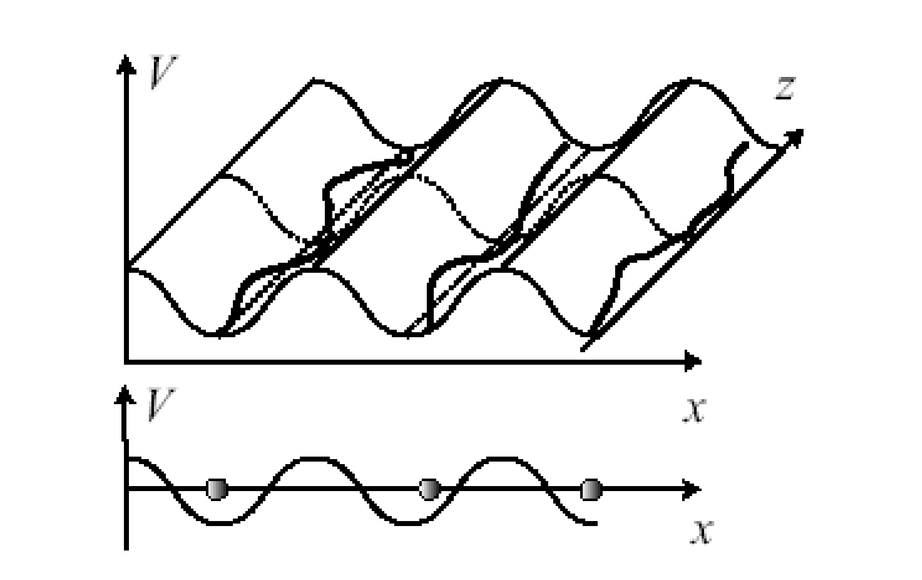
- Commensutate: $Q = 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

$$S_0 = \int \frac{dxd\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

$$S_L = -V_0 \rho_0 \int dxd\tau \cos(2\phi(x))$$



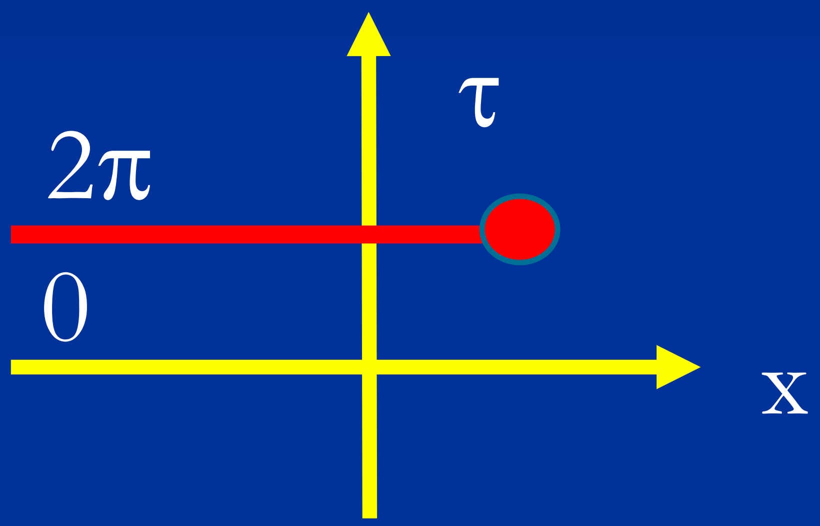
Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter

Vortex operator

$$e^{iaP}\,|\,x\rangle = |\mathrm{x+a}\rangle$$

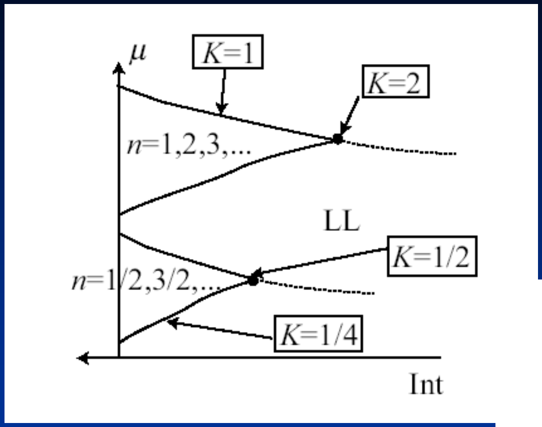
$$\phi(x,\tau)=\pi\int_{-\infty}^{x}dx'\Pi_{\theta}(x',\tau)$$

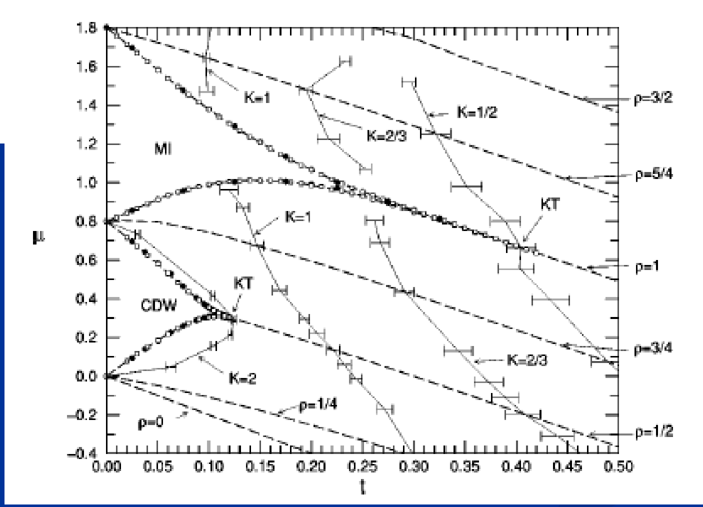


$$\cos(2\phi(x_1,\tau_1))$$

- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity

$$S=\frac{K}{2\pi}\int dx d\tau\left[\frac{1}{u}(\partial_{\tau}\theta)+u(\partial_{x}\theta)^{2}\right]-g\int dx\cos(2\phi)$$





T. Kuhner et al. PRB 61 12474 (2000)

Mott insulator:

ϕ is locked

Density is fixed

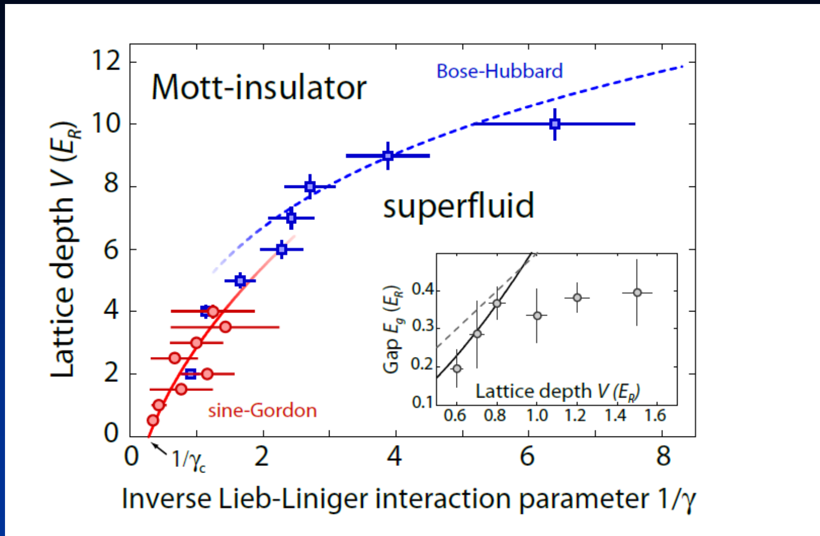
Gap in the excitation spectrum

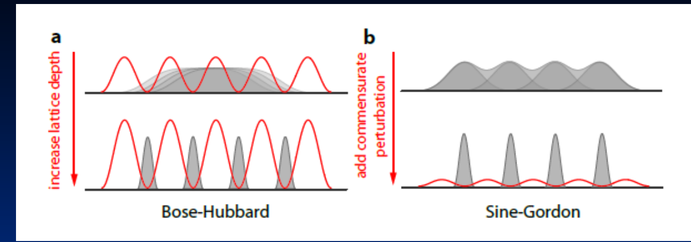
$$G(r) \propto e^{-r/\xi}$$

TG, Physica B 230 975(97): arXiv/0605472 (Salerno lectures);

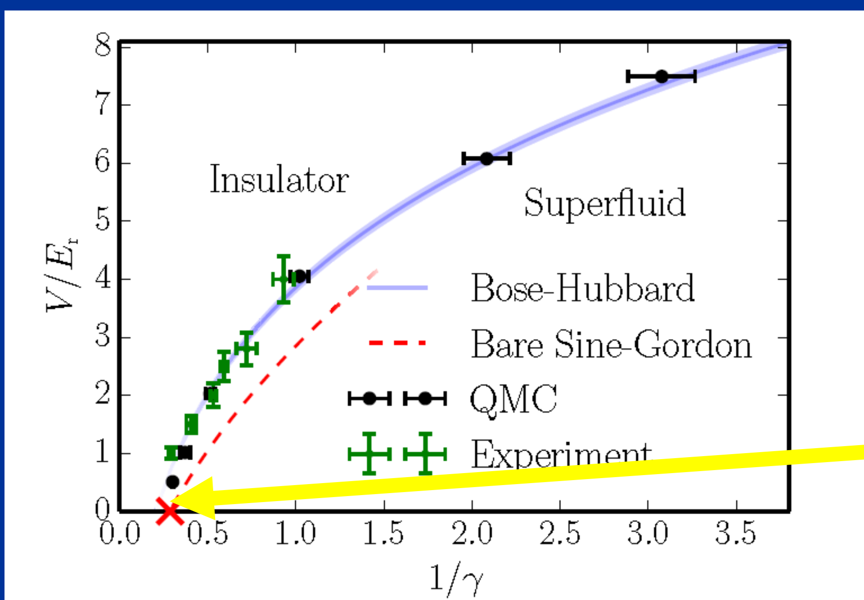
Oxford (2004);

M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)





E. Haller et al. Nature 466 597 (2010)



Renormalized Sine-Gordon

Shows:

K* = 2

G. Boeris et al. PRA 93 93, 011601(R) (2016)

Non local (topological) order

$$\rho(x) \sim \nabla \phi(x)$$

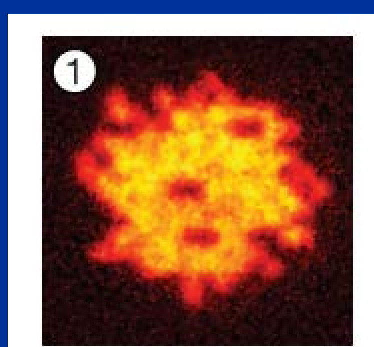
$$\mathcal{O}_{\mathrm{P}}^{2} = \lim_{l \to \infty} \mathcal{O}_{\mathrm{P}}^{2}(l) = \lim_{l \to \infty} \left\langle \prod_{k \leq j \leq k+l} e^{i\pi\delta\hat{n}_j} \right\rangle$$

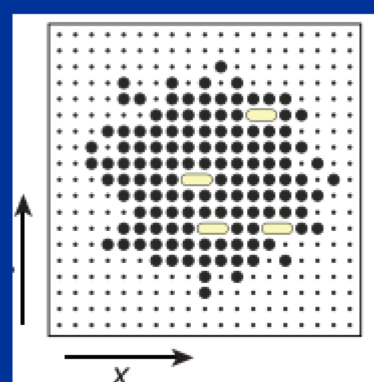
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, *Phys. Rev. B* **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

M. Endres,[1*] M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]

Science (2011)

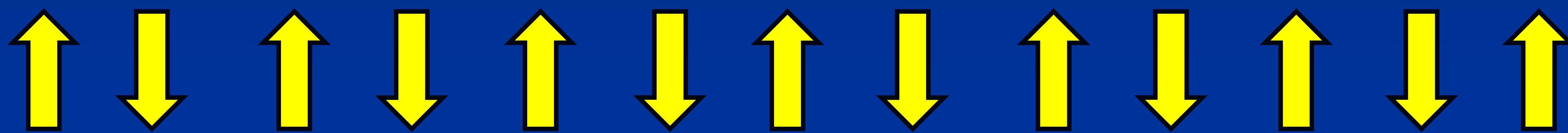




Other important 1D properties Fractionalization of excitations

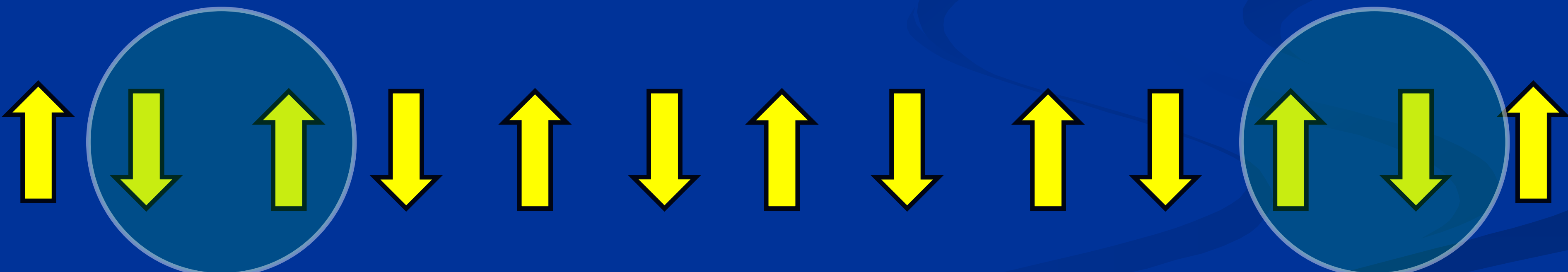
Fractionalization of excitations

$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$



$\Delta S = -1 \quad E = \epsilon(q)$

Magnon



$\Delta S = -1/2$

Spinons

$\Delta S = -1/2$

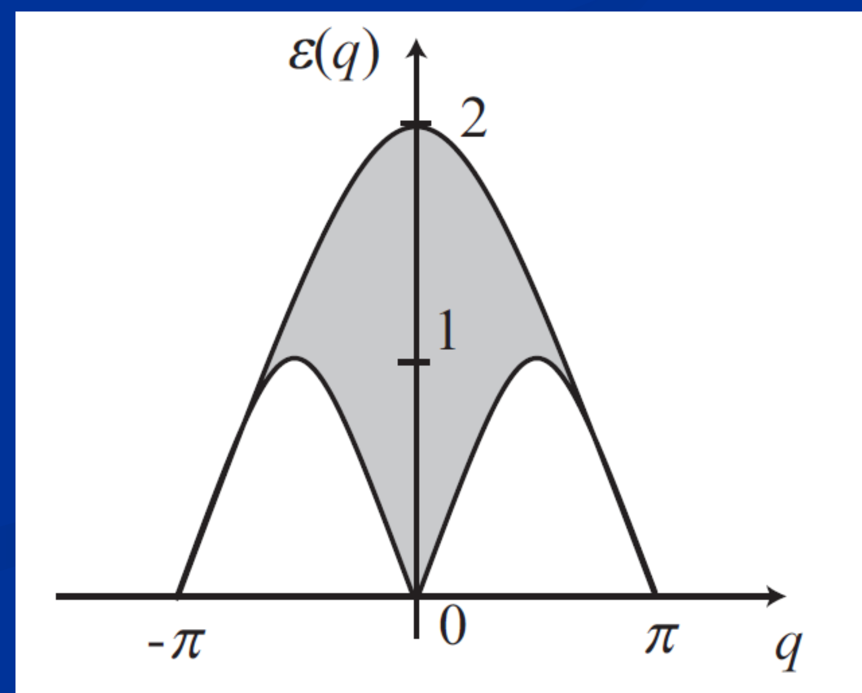
Magnons and spinons: 1 = ½ + ½

- Hidden (topological) order parameters

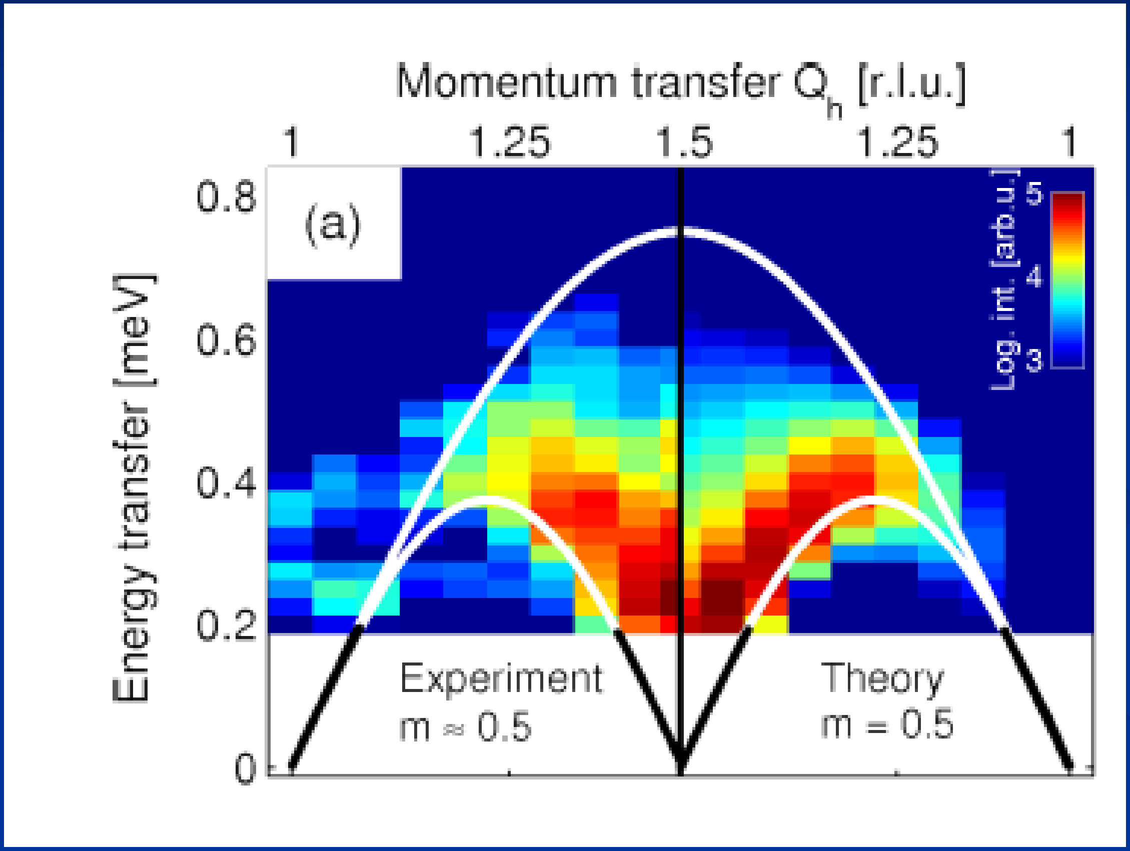
- Continuum of excitations

$$E(k) = \cos(k_1) + \cos(k_2)$$

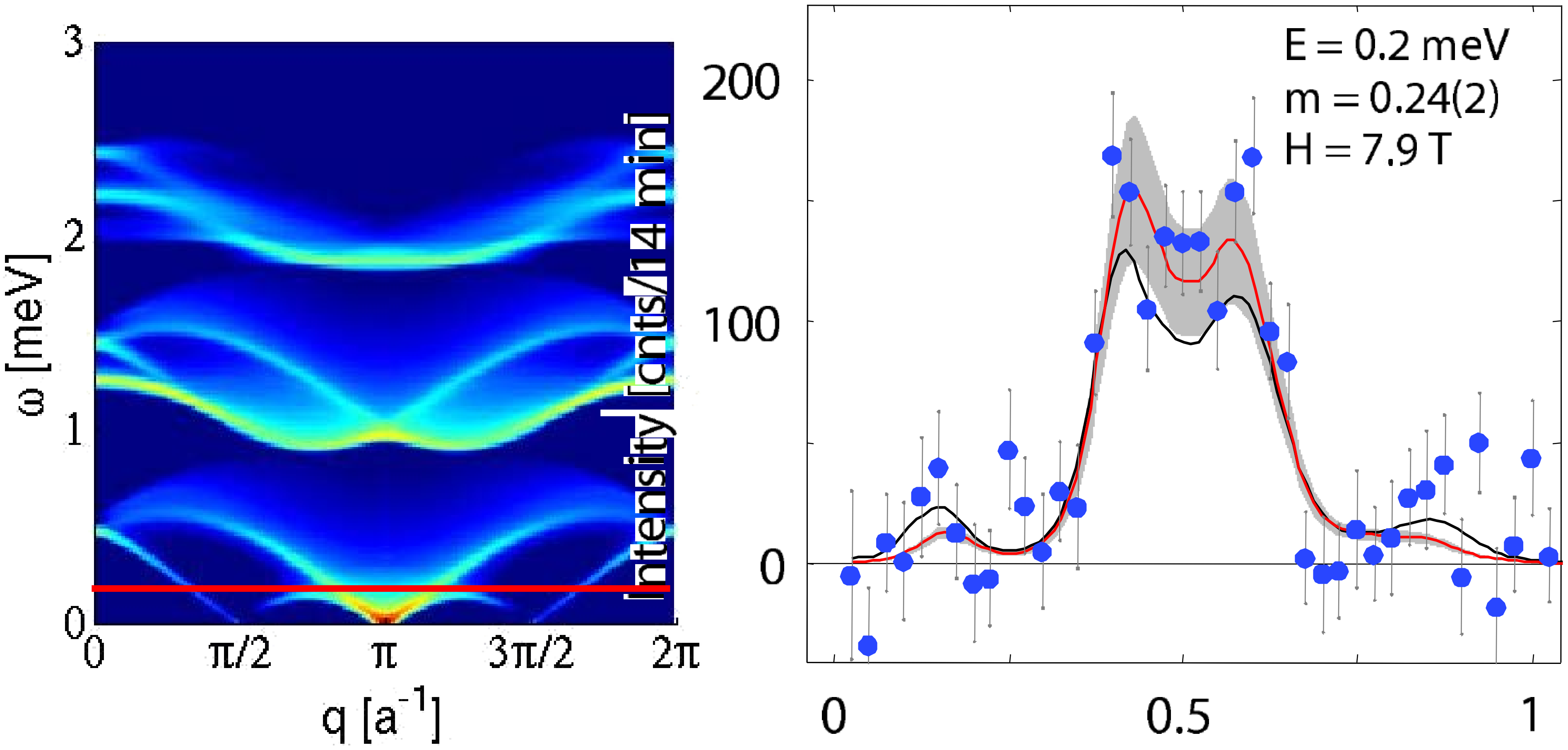
$$k = k_1 + k_2$$



Neutron scattering



B. Thieleman et al. PRL 102, 107204 (2009)



P. Bouillot et al. PRB 83, 054407 (2011)

Fermions

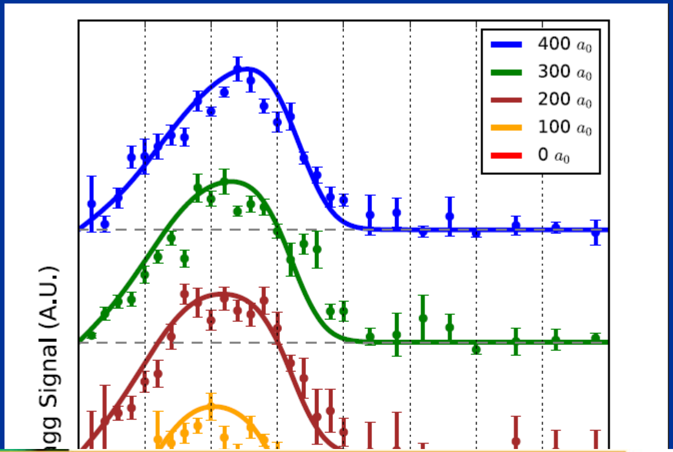
Charge velocity

PHYSICAL REVIEW LETTERS **121,** 103001 (2018)

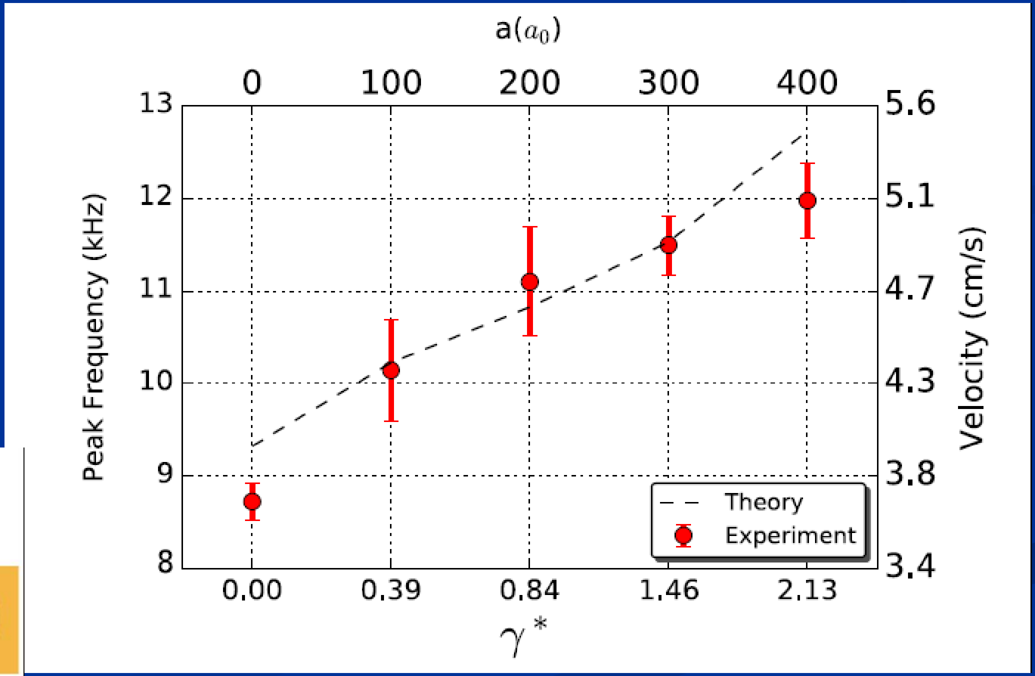
Editors' Suggestion

Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas

T. L. Yang,[1] P. Grišins,[2] Y. T. Chang,[1] Z. H. Zhao,[1] C. Y. Shih,[1] T. Giamarchi,[2] and R. G. Hulet[1]

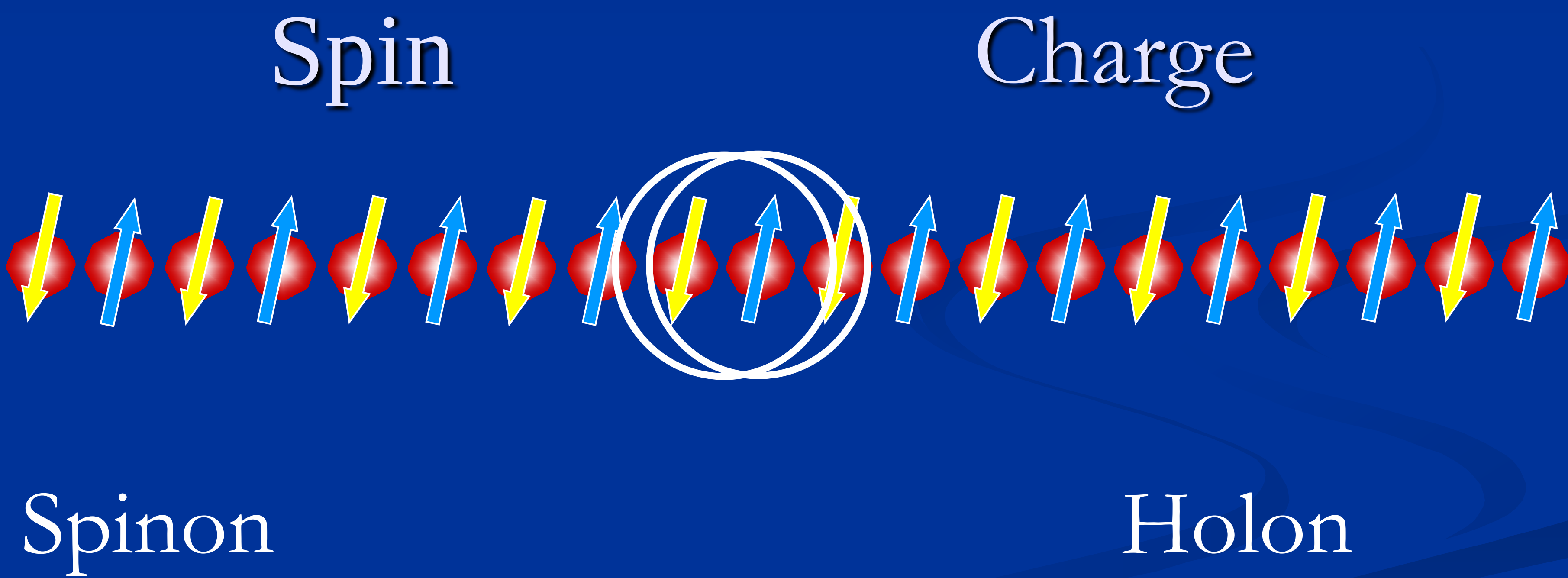


ULTRACOLD ATOMS CONFIRM 55-YEAR-OLD PHYSICS THEORY



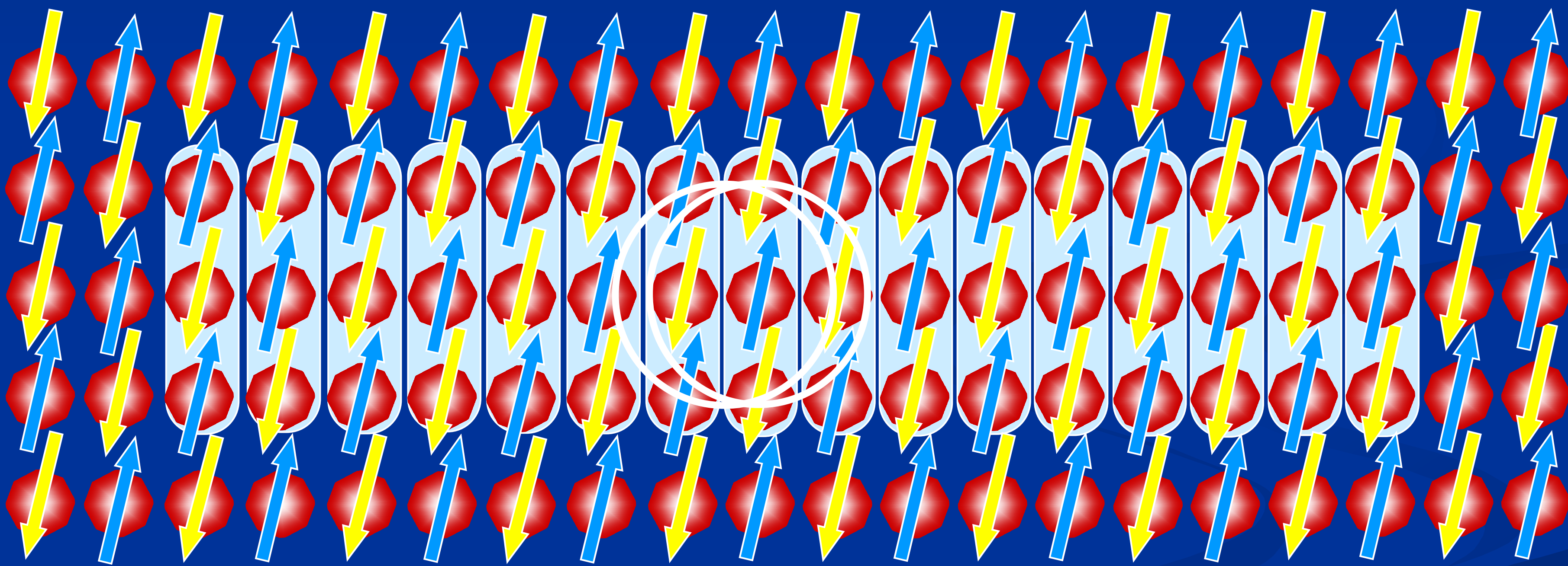
https://www.futurity.org/one-dimensional-electrons-physics-1858622/

Deconstruction of the electron: spin-charge separation



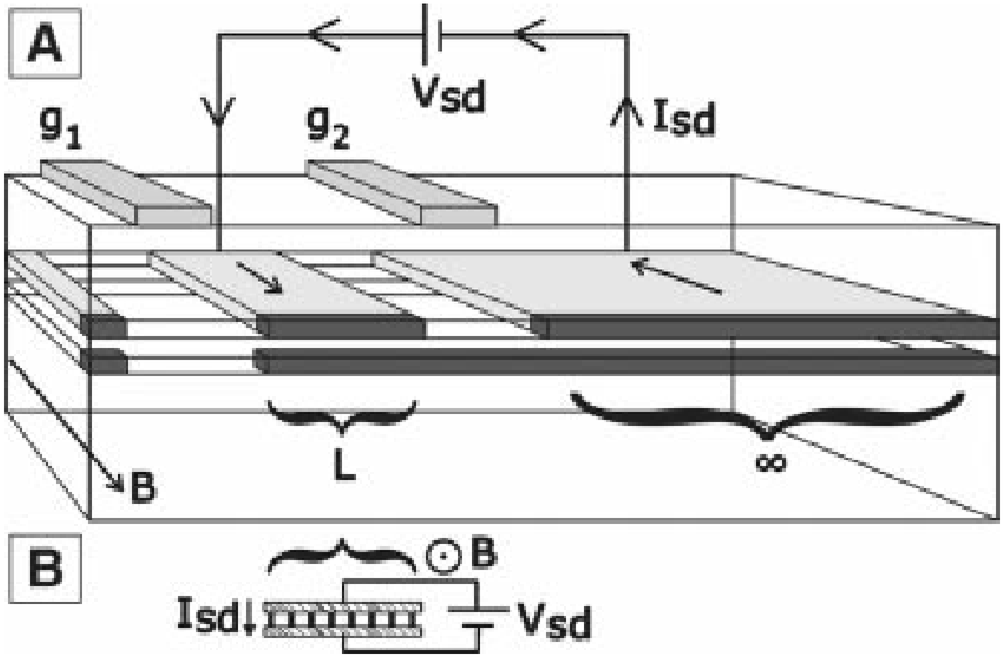
Spin-Charge Separation higher D ?

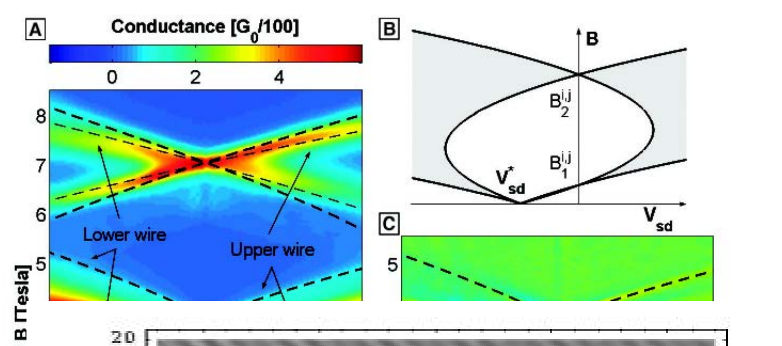
Spin Charge

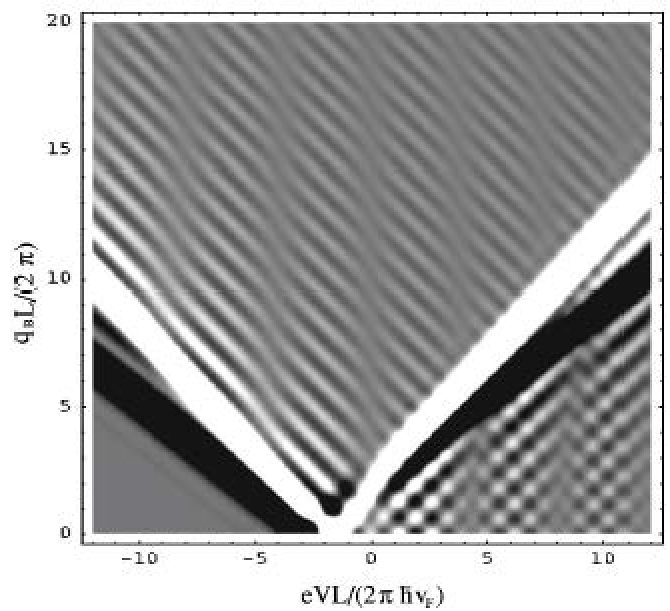


Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »







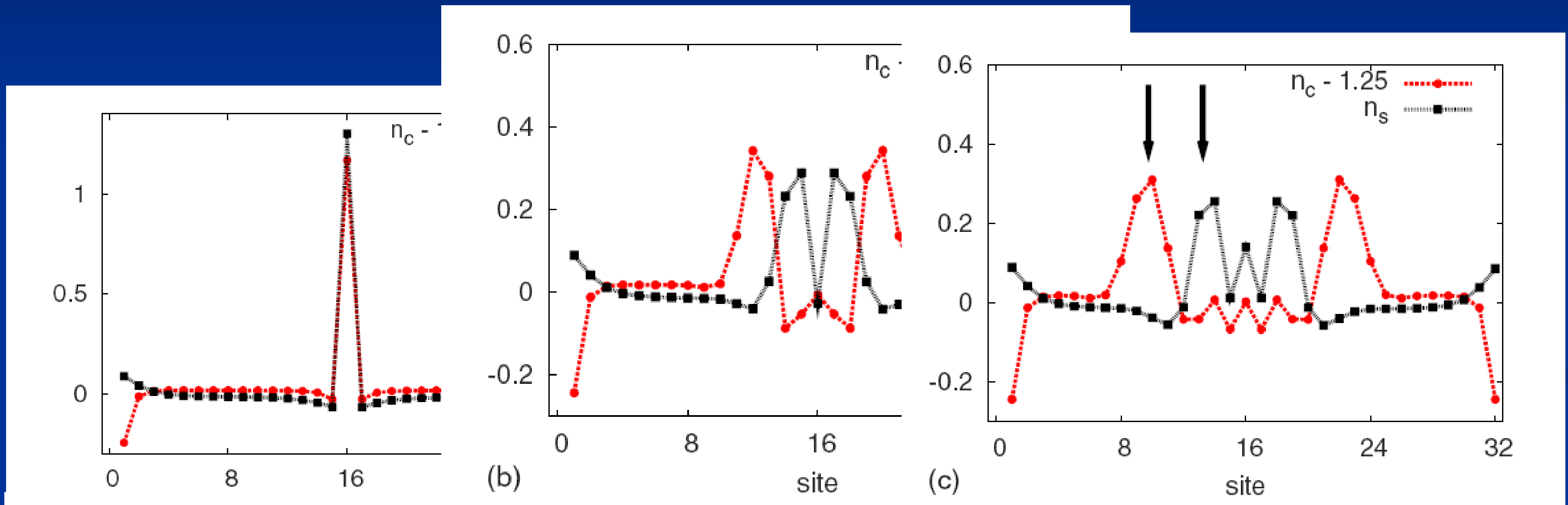
O.M Ausslander et al., Science **298** 1354 (2001)

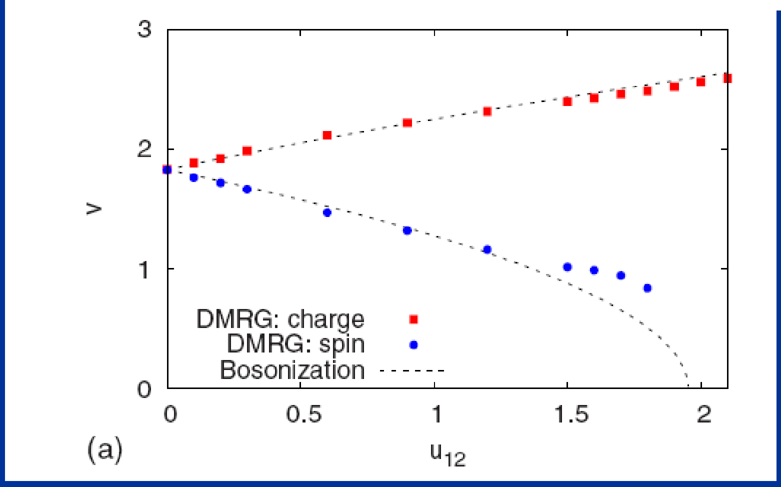
Y. Tserkovnyak et al., PRL **89** 136805 (2002)

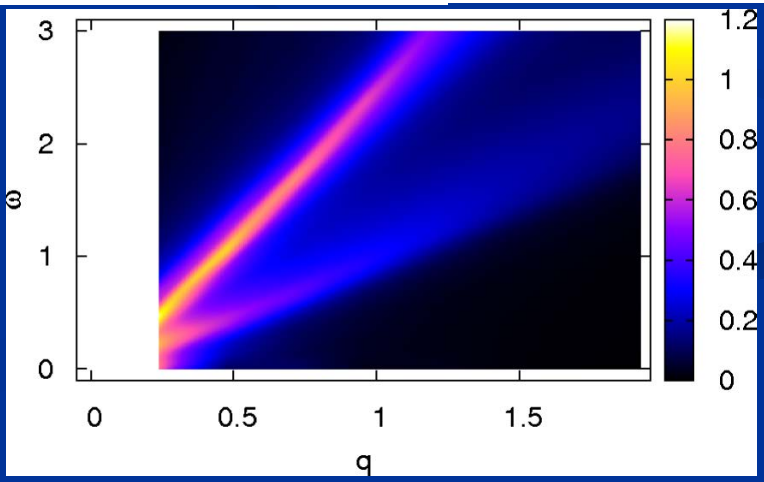
Y. Tserkovnyak et al., PRB **68** 125312 (2003)

Proposal for cold atoms (Rb)

A. Kleine, C. Kollath et al. PRA 77 013607 (2008); NJP 10 045025 (2008)







To boldly go where no theorist has gone before….

Beyond Luttinger liquids

- 1D additional perturbation:

Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures, ….

- New type of quantum critical points (e.g. topological)…

Topological phase transitions

Dual field double sine-Gordon

$$S = \frac{K}{2\pi}\int dx d\tau \left[\frac{1}{u}(\partial_\tau \theta) + u(\partial_x \theta)^2\right] - g_\phi \int dx \cos(p\phi) - g_\theta \int dx \cos(q\theta)$$

- m=4, n=1 : XXZ + staggered magnetic field

$$-J_z \cos(4\phi) - h_x \cos(\theta)$$

K = ½ ; K = 1/8

K=1/4

$$-J_z \psi_R^\dagger \psi_L - h_x \psi_R^\dagger \psi_L^\dagger + \text{h.c.}$$

Ising-like transition (c=1/2)

- m=4, n=2 : XXZ + XY anisotropy

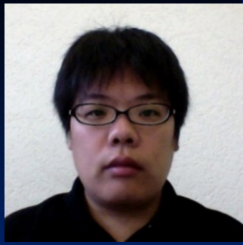
TG + H.J Schulz J. Physique 49 819 (88)

$$-J_z \cos(4\phi) - h_x \cos(2\theta)$$

K = ½ ; K = ½

BKT-like transition (c=1)

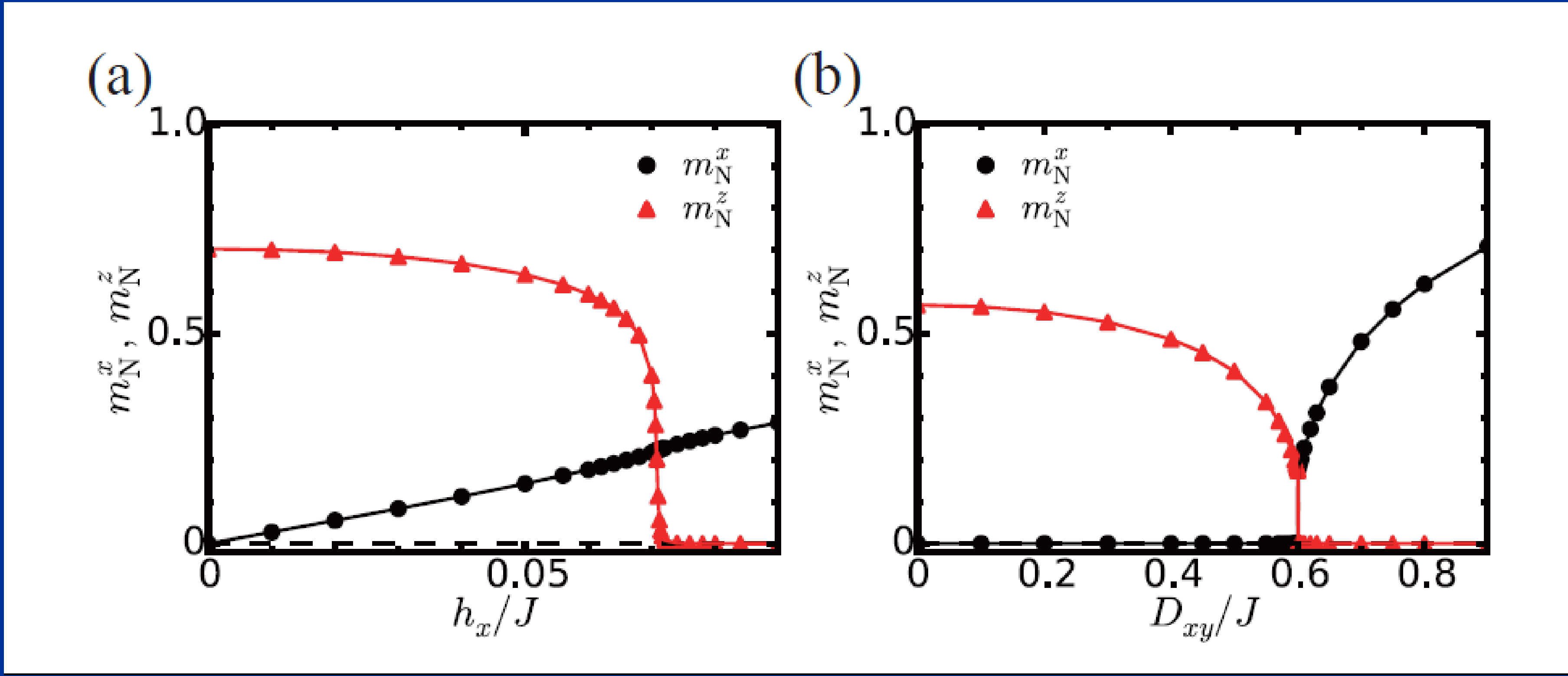
Two types of transition



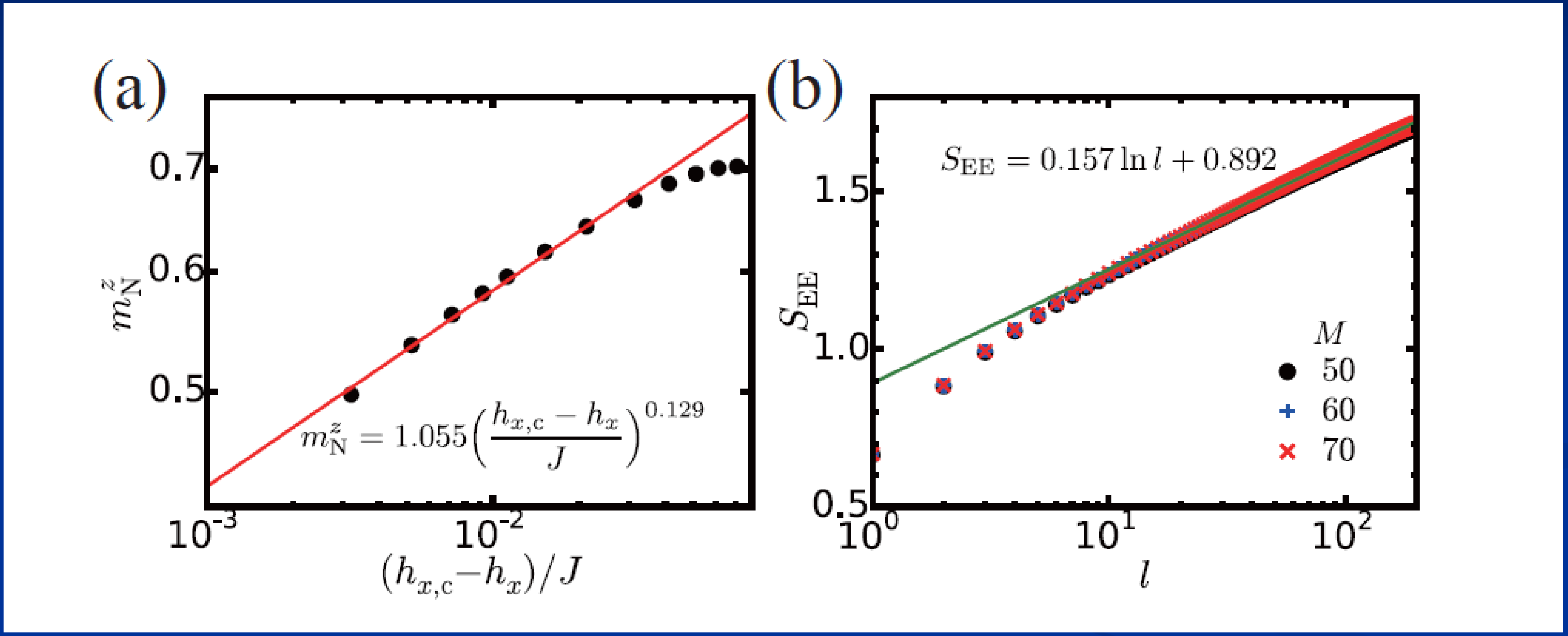
Topological transition between competing orders in quantum spin chains

Shintaro Takayoshi, Shunsuke C. Furuya, and Thierry Giamarchi

Phys. Rev. B **98**, 184429 (2018) - Published 27 November 2018



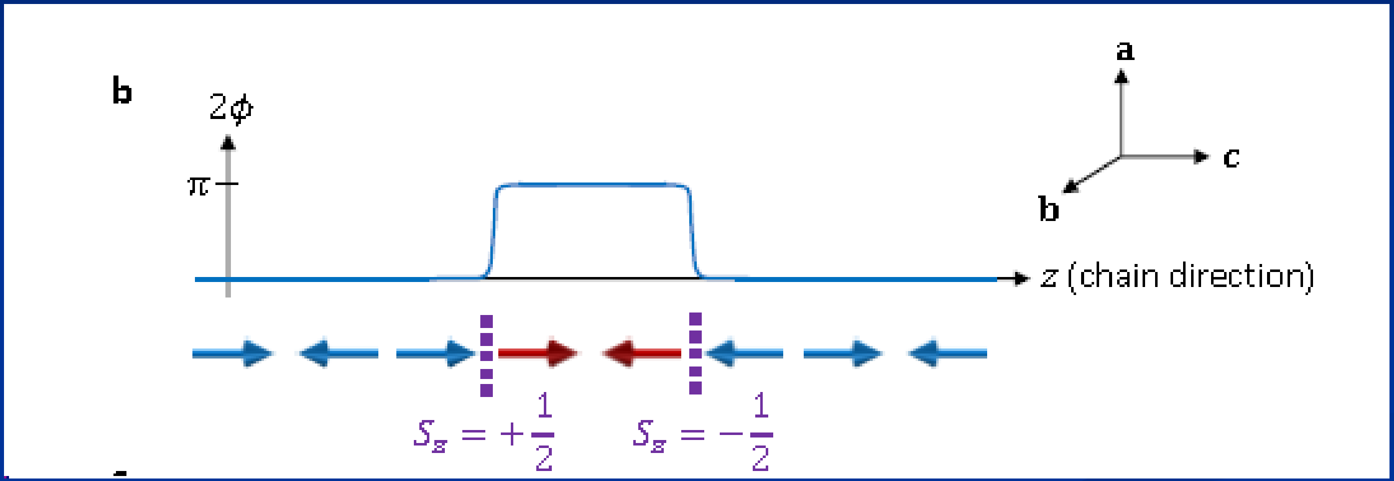
M=4, N=1

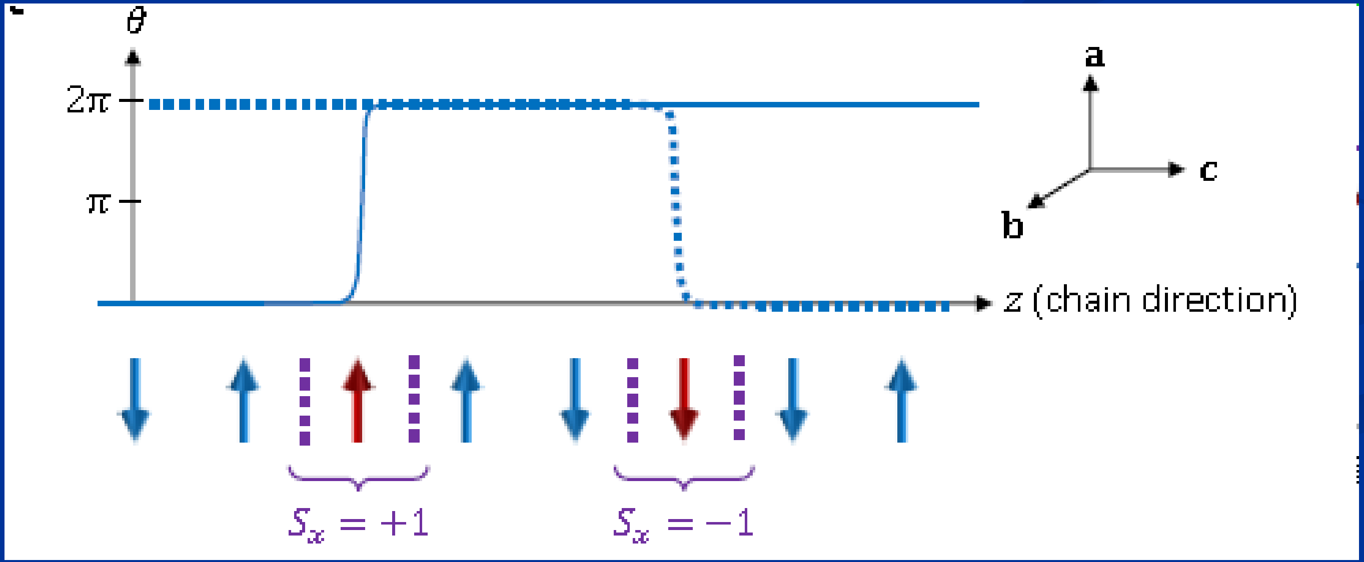


$$S_{EE} = \frac{c}{3}\log(l) + Cste$$

$$\beta = 0.129 \quad c = 0.471$$

Dual topological excitations





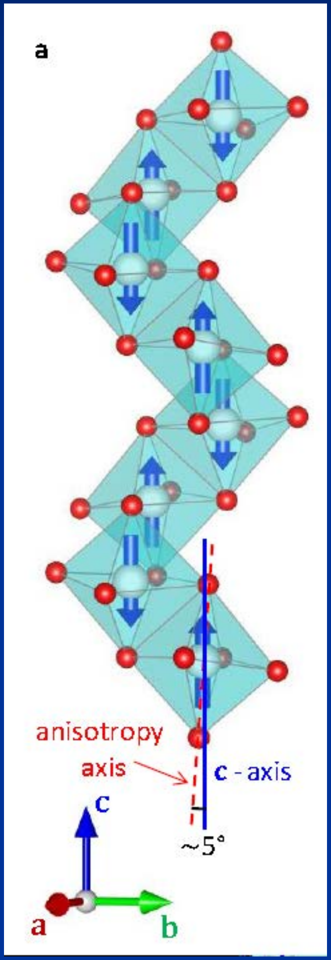
Realization in a quantum spin chain

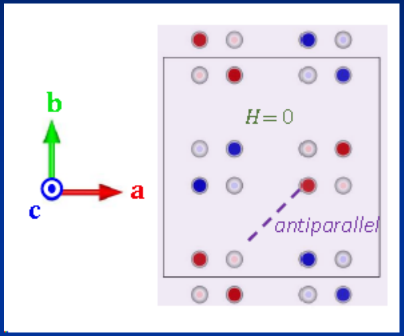
Topological quantum phase transition in the Ising-like antiferromagnetic spin chain BaCo2V2O8

Q. Faure, S. Takayoshi, et al

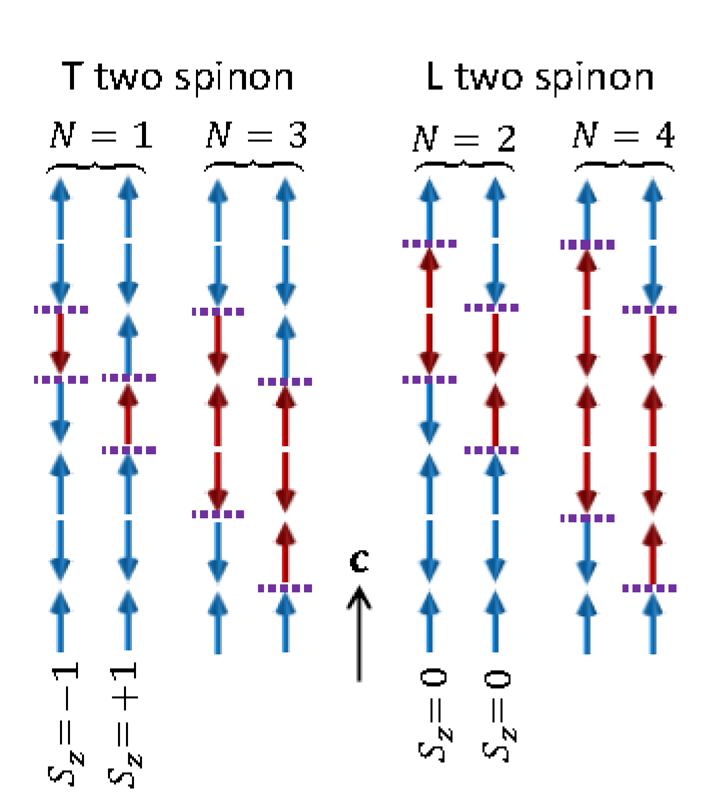
Nature Physics 14, 867 (2018)

Ising-like chains

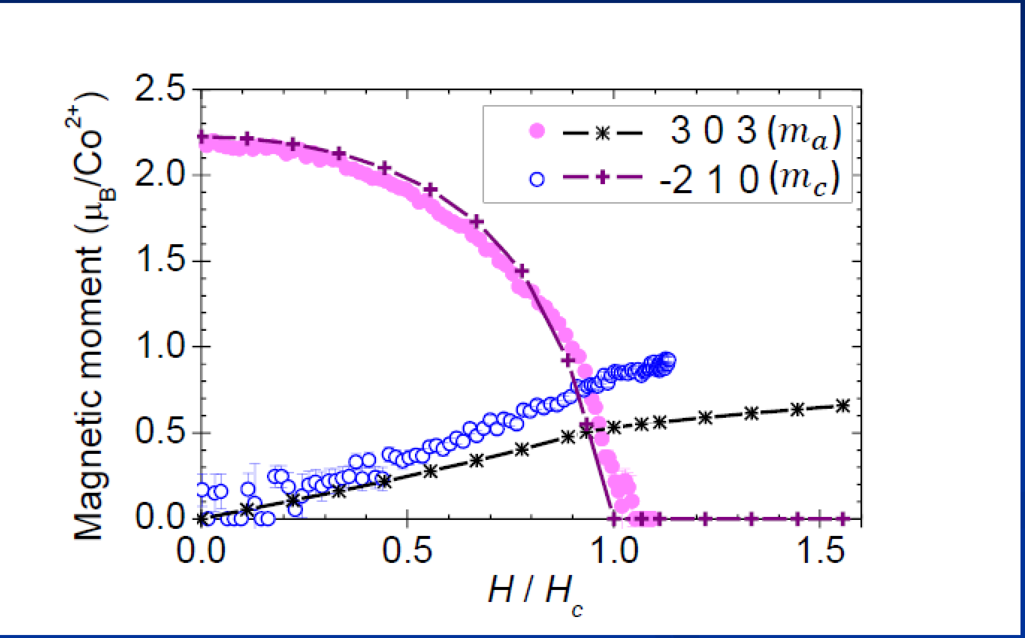




$$\mathcal{H}_{XXZ} = J\sum_{n,\mu}[\epsilon\left(S^x_{n,\mu}S^x_{n+1,\mu} + S^y_{n,\mu}S^y_{n+1,\mu}\right) + S^z_{n,\mu}S^z_{n+1,\mu}]$$
$$-\sum_{n,\mu}\tilde{g}\mu_B\mathbf{H}\cdot\mathbf{S}_{n,\mu} + J'\sum_{n}\sum_{\mu,\nu(\mu\neq\nu)} S^z_{n,\mu}S^z_{n,\nu} \quad (1)$$

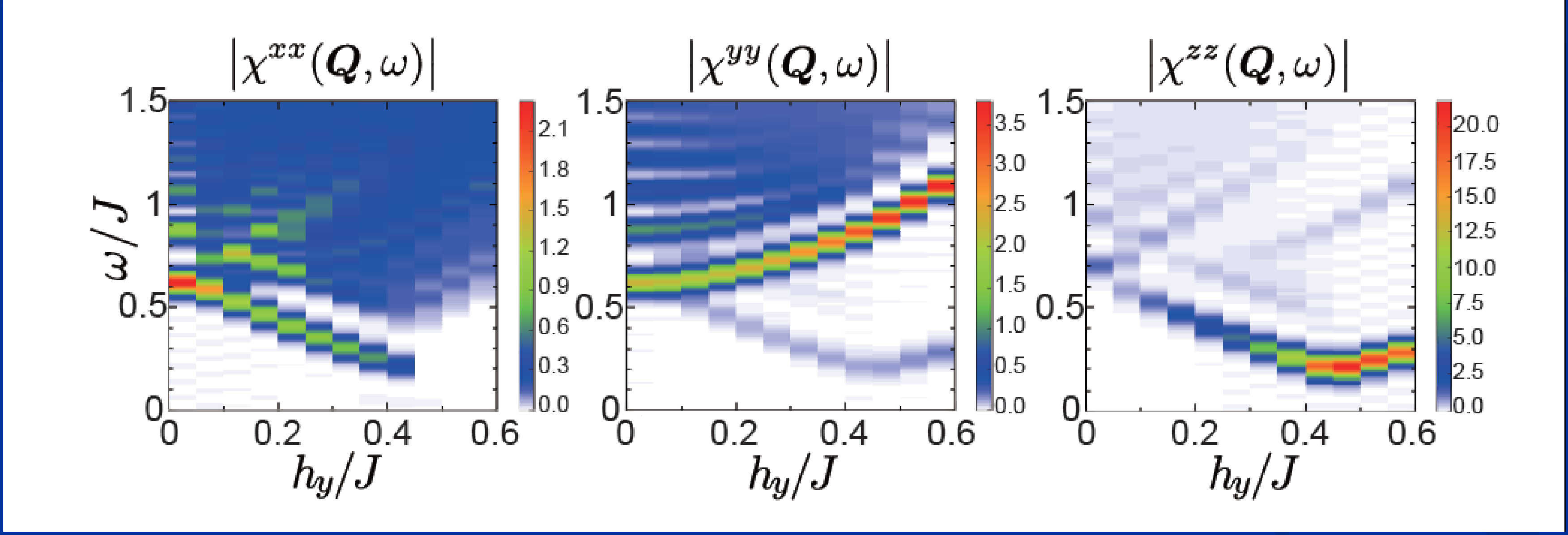


Transition in magnetic field

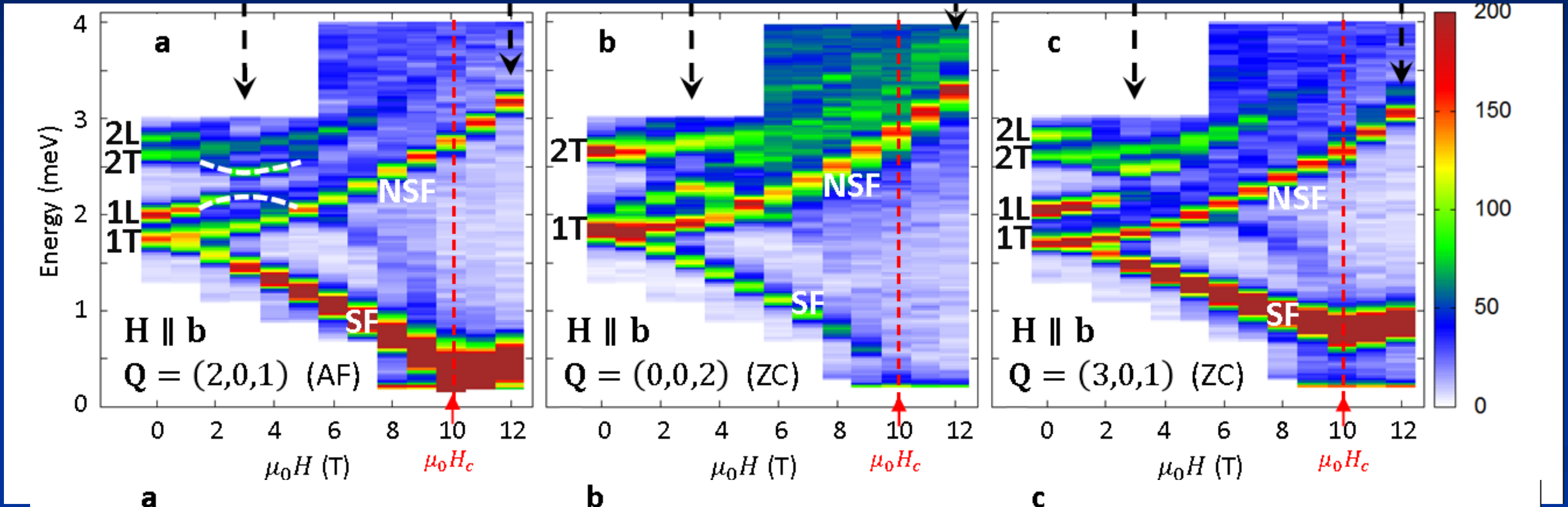


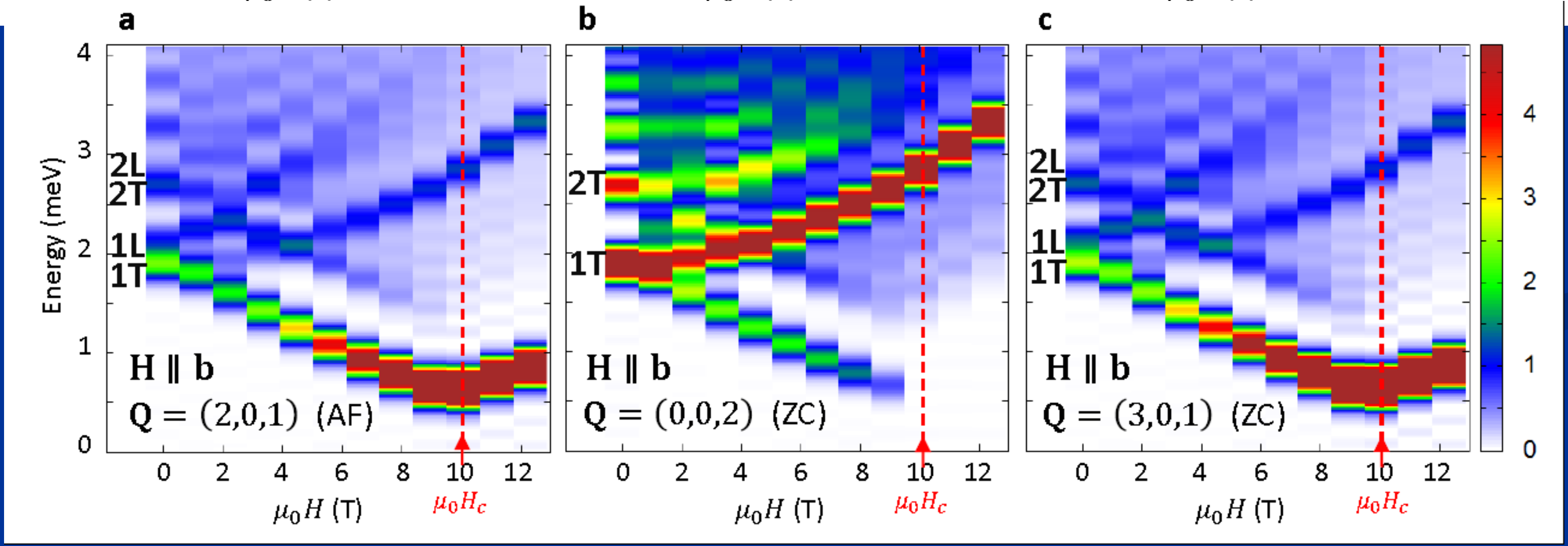
- Nature of the transition ?
- DMRG: not the transition in unif. field
- Effective staggered field hx (g- tensor)

Spin-Spin correlations



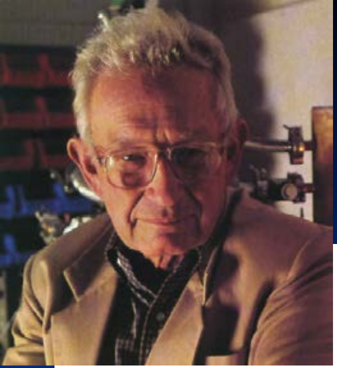
Neutrons (un-pol. and polarized)

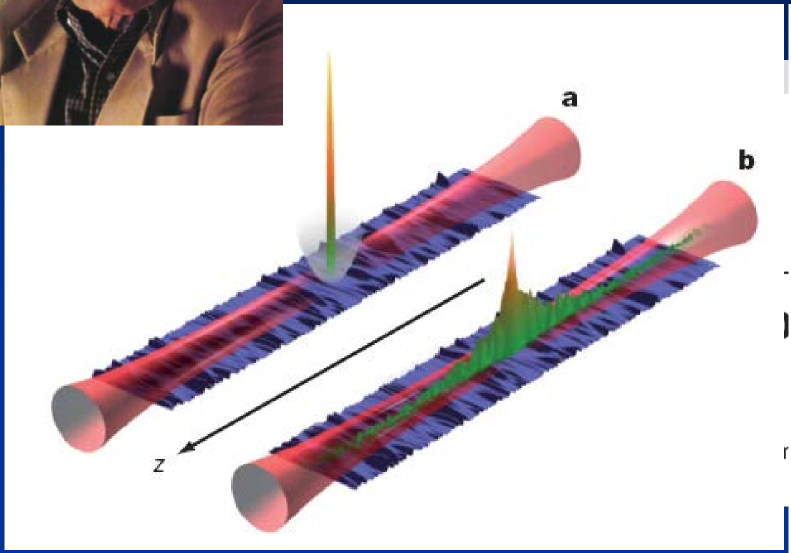


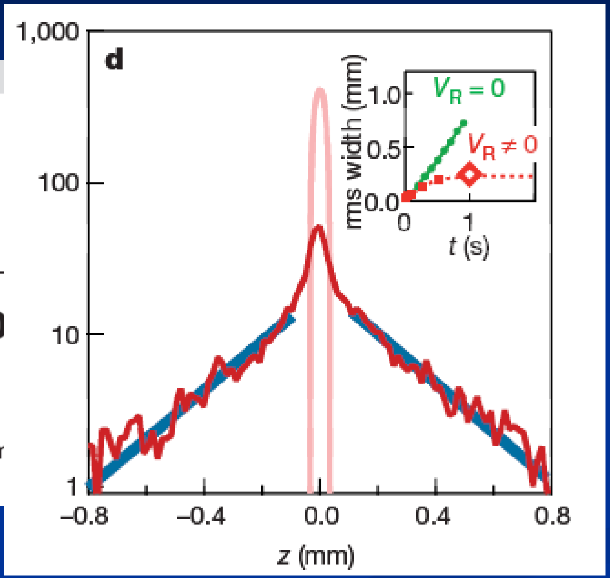


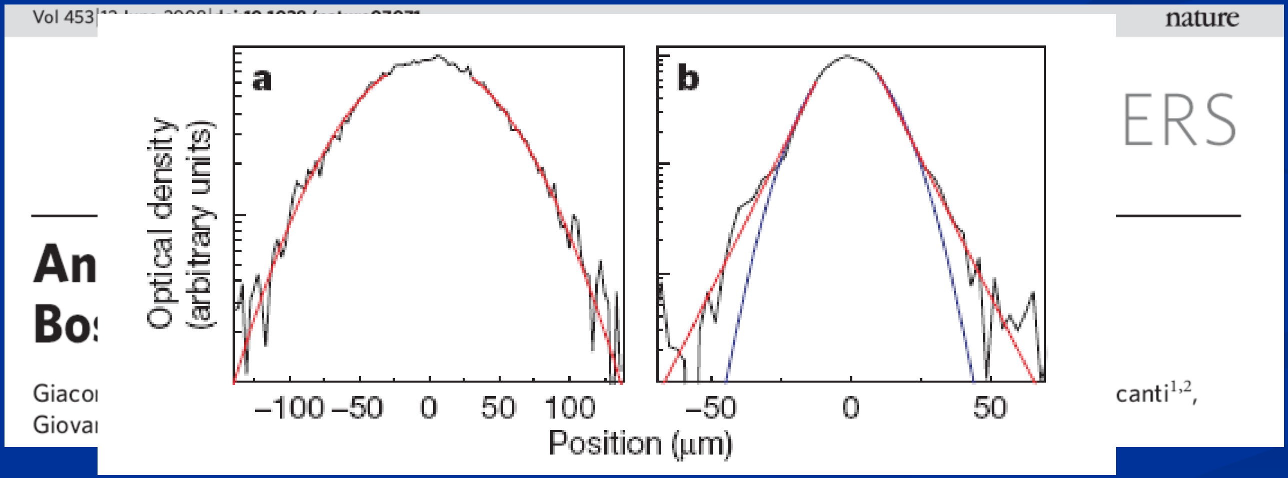
Disorder

Anderson localization





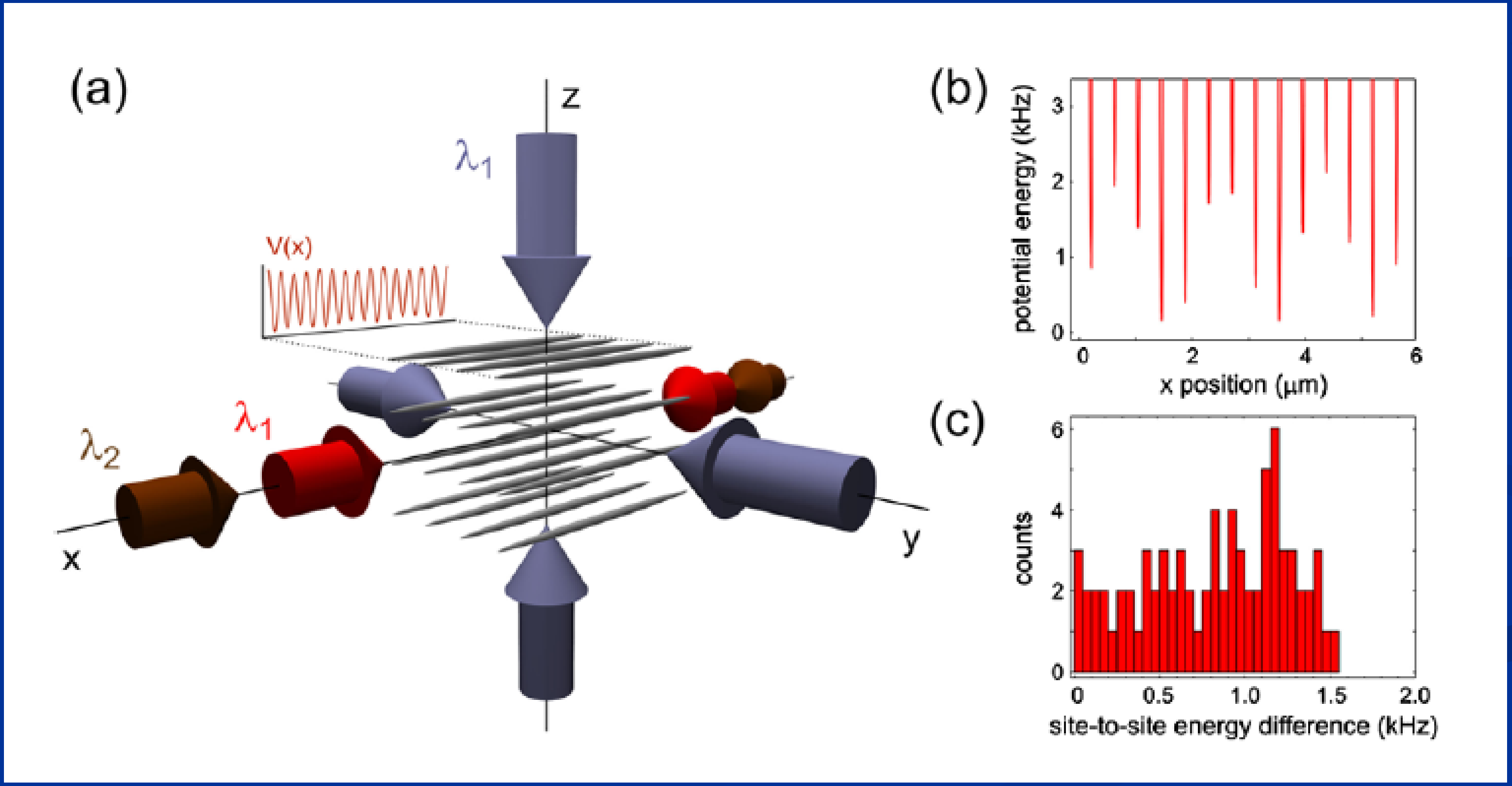




Aubry-Andre Model (Ann. Isr. Phys. Soc. **3, 133 1980**)

Experiments with interactions !

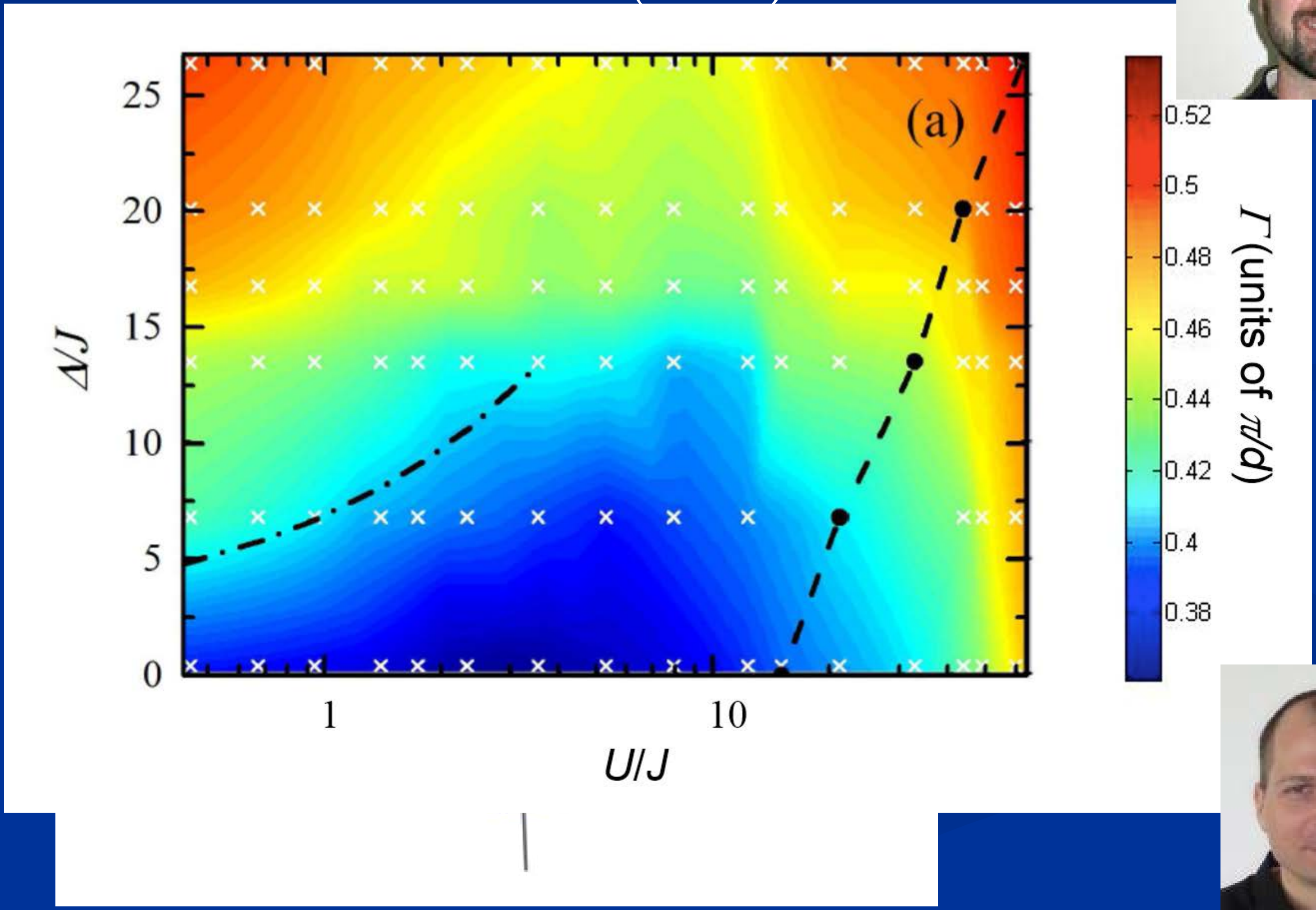
L. Fallani et al. PRL 98, 130404 (2007)



Quasi-periodics and interactions

C. D'Errico, E. Lucioni et al. PRL (2014);

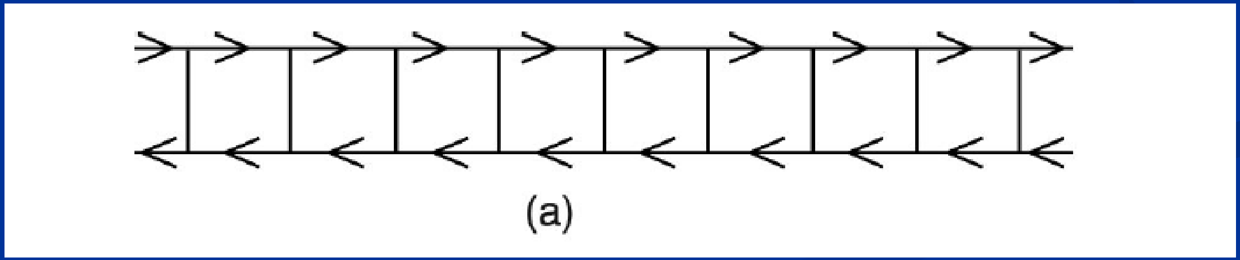
L. Gori et al PRA 93 033650 (2016)



Gauge fields

Meissner effect in bosonic ladders

E. Orignac, TG, PRB 64 144515 (2001)



$$H = -t_{\parallel} \sum_{i,p=1,2} \left(b^{\dagger}_{i+1,p} e^{ie^* a A_{\parallel,p}(i)} b_{i,p} + b^{\dagger}_{i,p} e^{-ie^* a A_{\parallel,p}(i)} b_{i+1,p} \right)$$

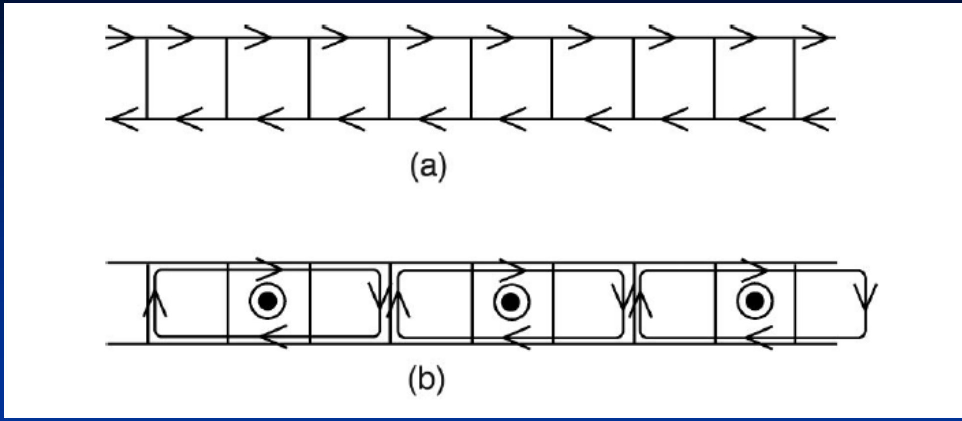
$$- t_{\perp} \sum_{i} \left(b^{\dagger}_{i,2} e^{ie^* A_{\perp}(i)} b_{i,1} + b^{\dagger}_{i,1} e^{-ie^* A_{\perp}(i)} b_{i,2} \right)$$

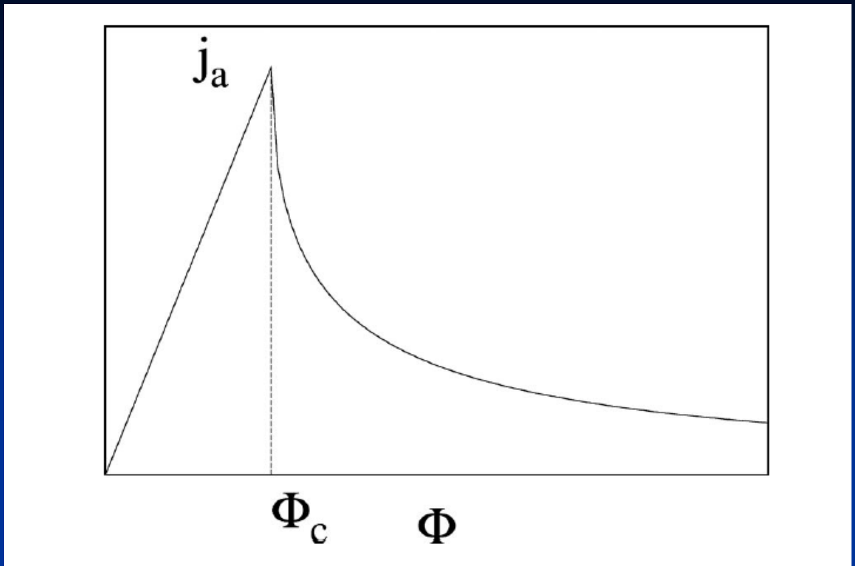
$$+ U \sum_{i,p} n_{i,p}(n_{i,p} - 1) + V n_{i,1} n_{i,2}, \qquad (1)$$

$$\int \vec{A} \cdot \overrightarrow{dl} = \Phi$$

$$H = H^0_s + H^0_a - \frac{t_{\perp}}{\pi a} \int dx \cos[\sqrt{2}\,\theta_a + e^* A_{\perp}(x)]$$

$$+ \frac{2Va}{(2\pi a)^2} \int dx \cos\sqrt{8}\,\phi_a,$$



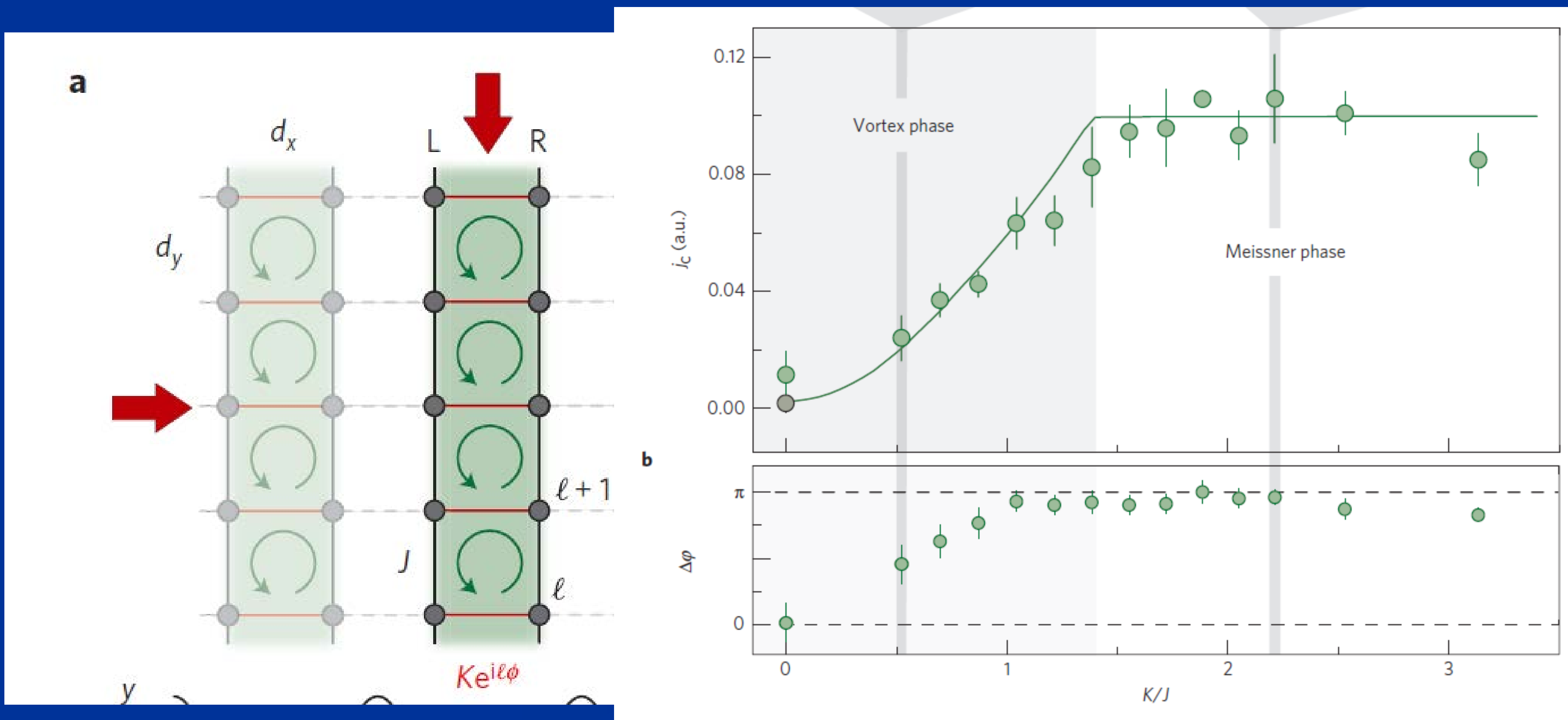


Orbital currents (``Meissner" effect)

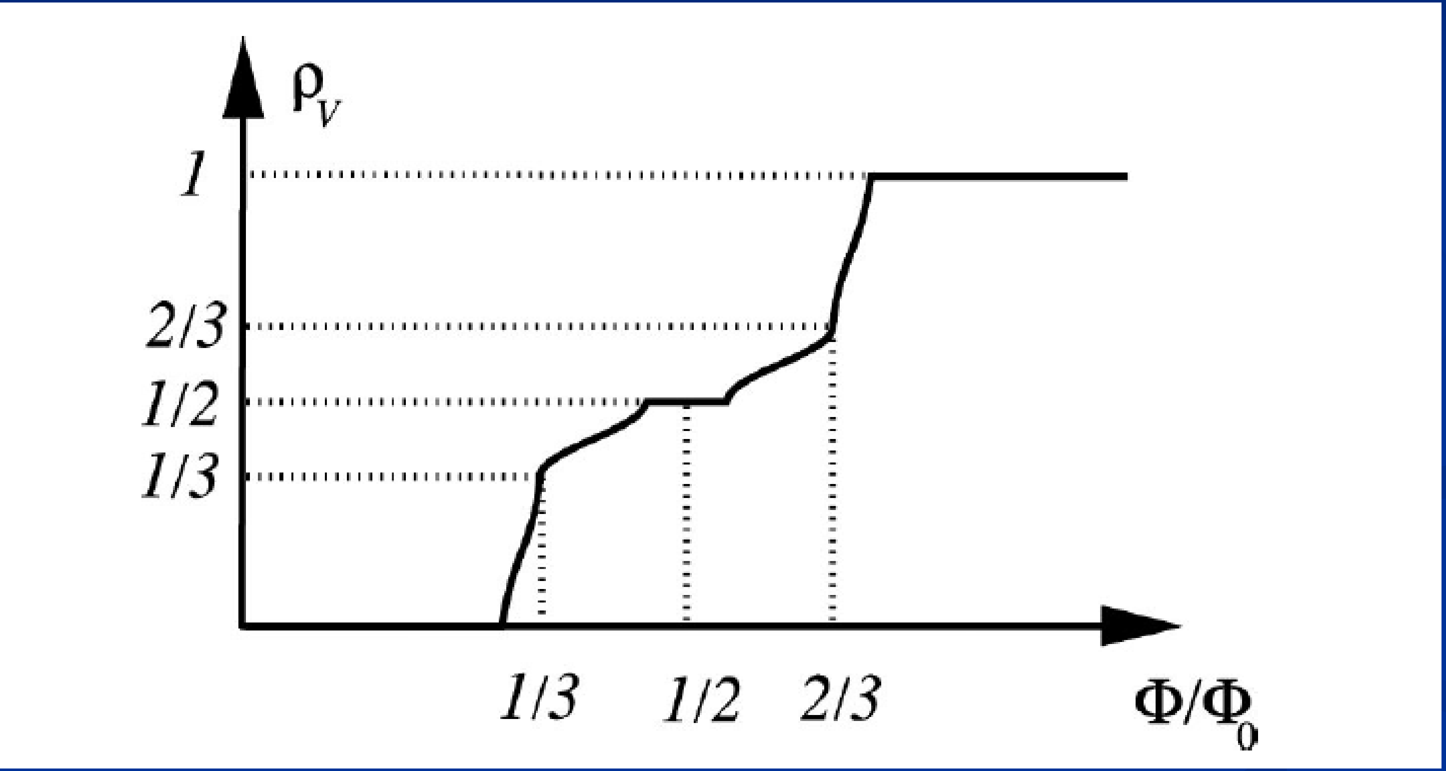
Field ``Hc1": appearance of vortices

Artificial gauge field (cold atoms)

M Atala et al. Nat Phys, 10 588 (2014)

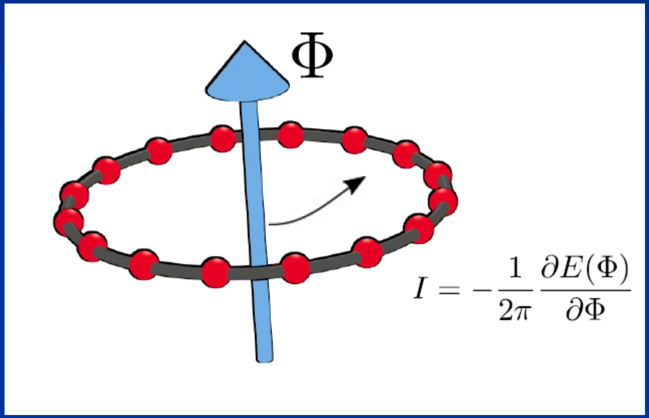


Interactions: devil's staircase

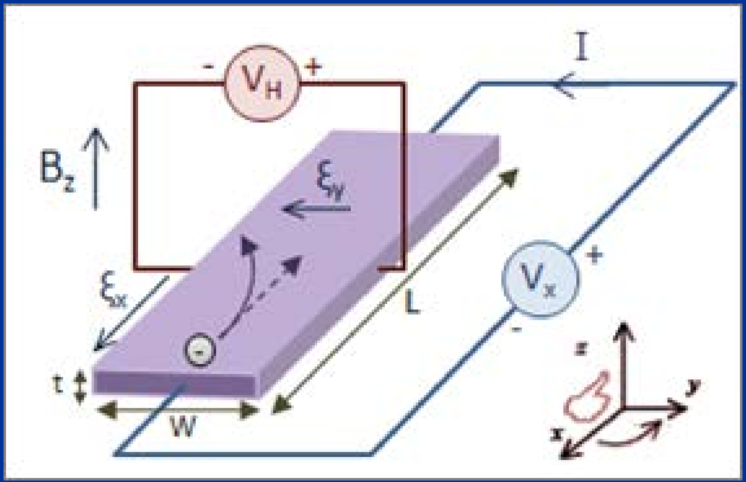


Plateau at ½ should be visible

Magnetic field effects (artificial gauge fields)



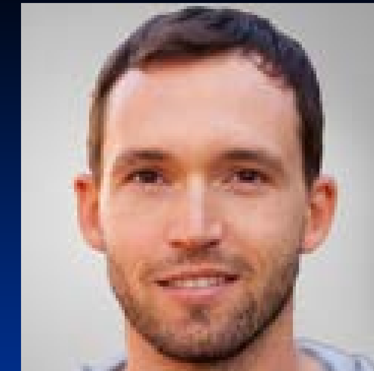
Flux: can induce a current



Hall effect

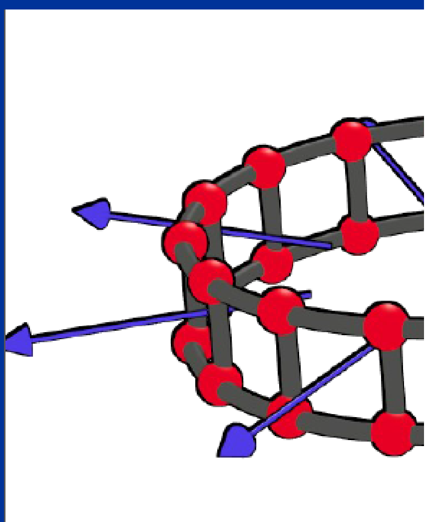
Public Domain, https://en.wikipedia.org/w/index.php?curid=22918777

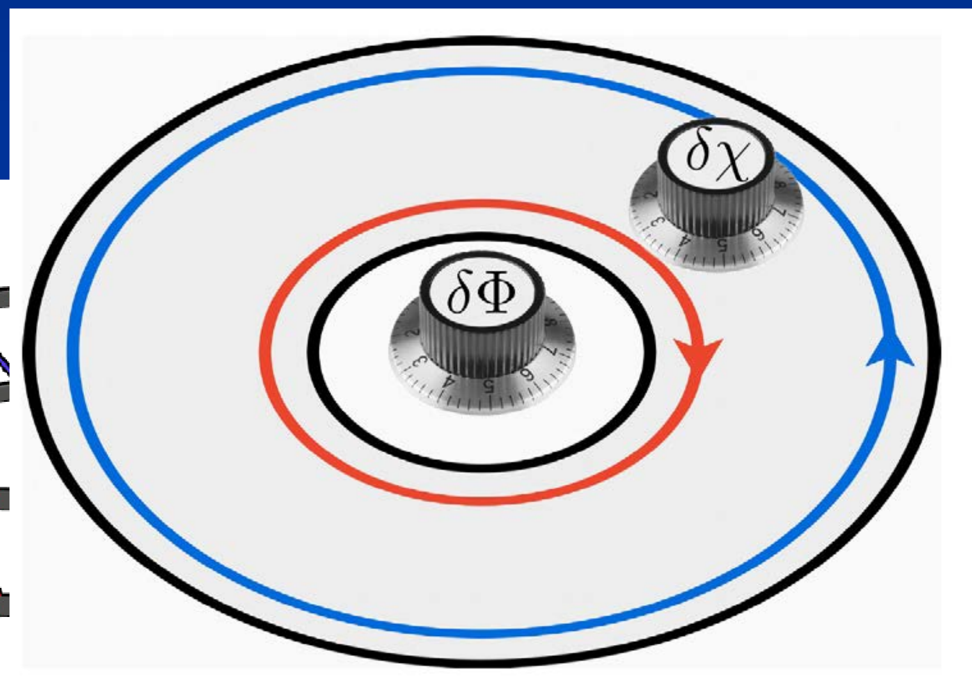
Pumping



M. Filippone

C. Bardyn





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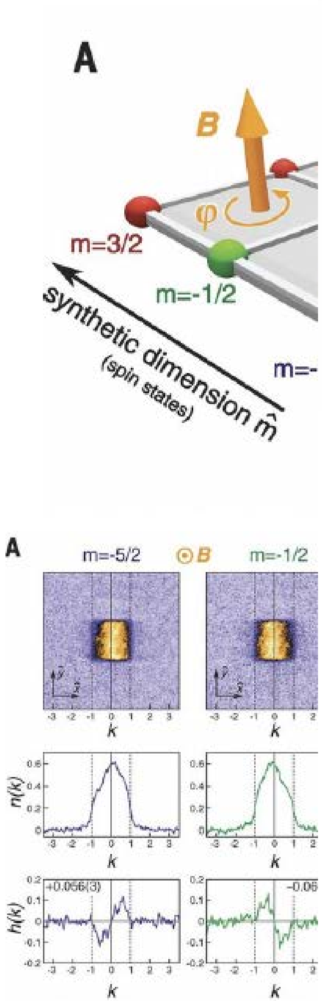
etic field

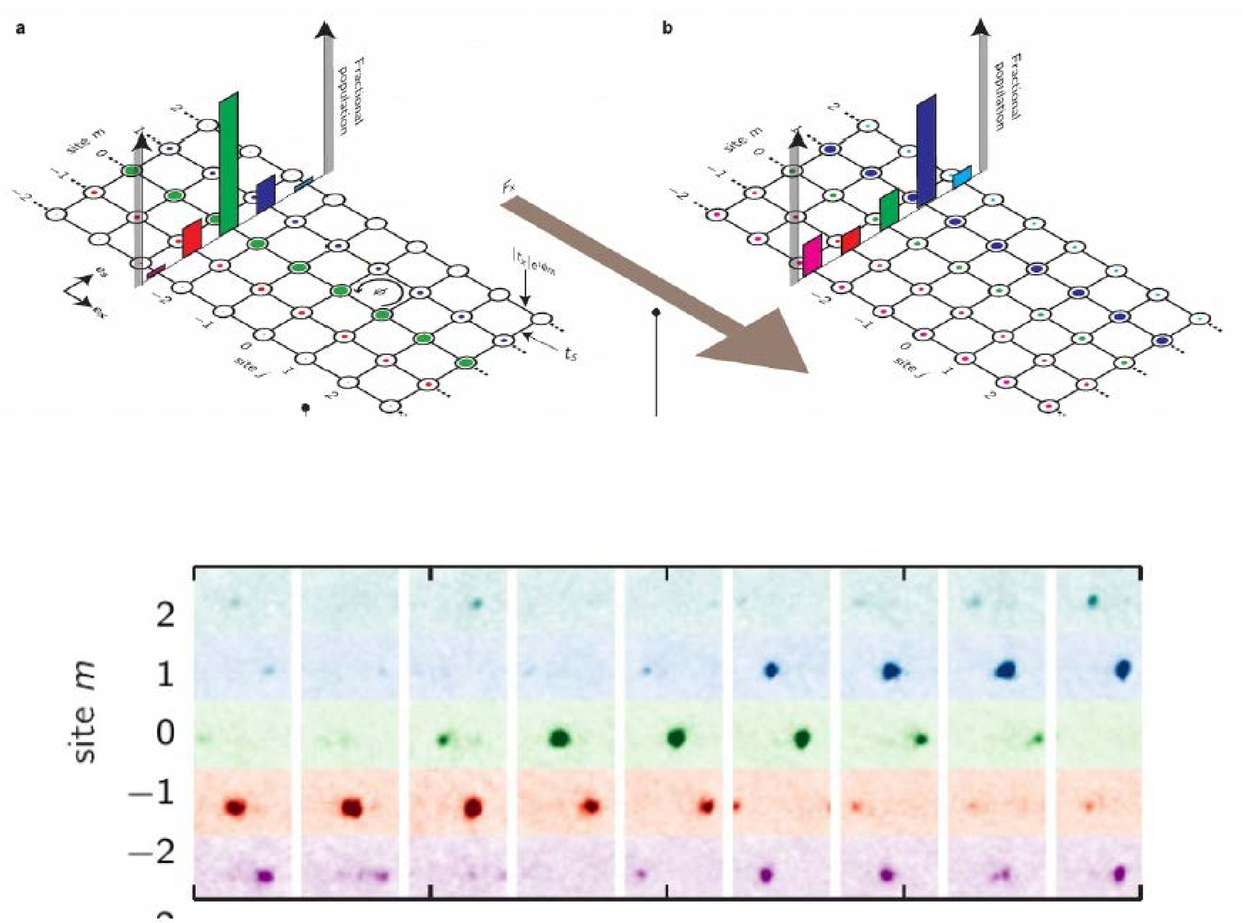
M. Filippone, C.E. Bardyn, TG PRB **97**, 201408(R) (2018)

C.E. Bardyn, M. Filippone, TG arXiv:1807.01710

Hall effect



Fallani's group (Florence) **Scien**



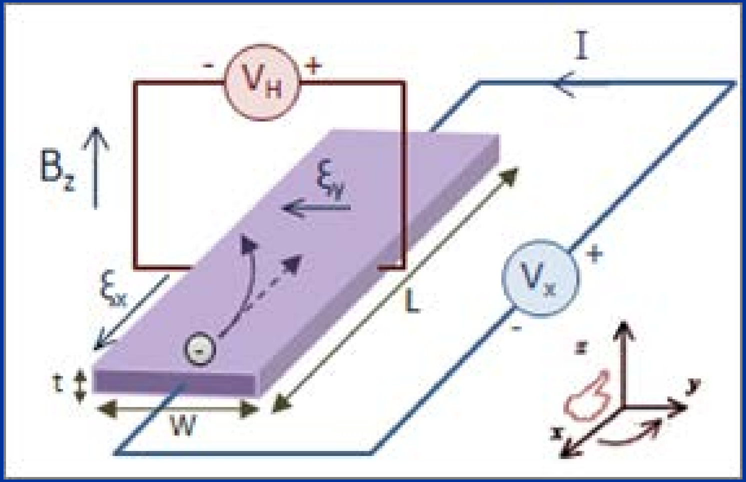
Spielman's group (Maryland) **arXiv:1804.06345**

Hall effect

S. Greshner, M. Filippone, TG, arXiv:1809.10927

S. Greshner

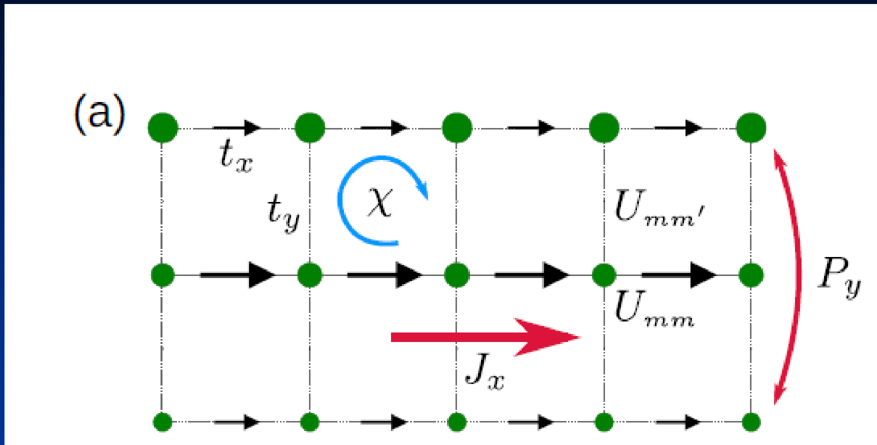
M. Filippone



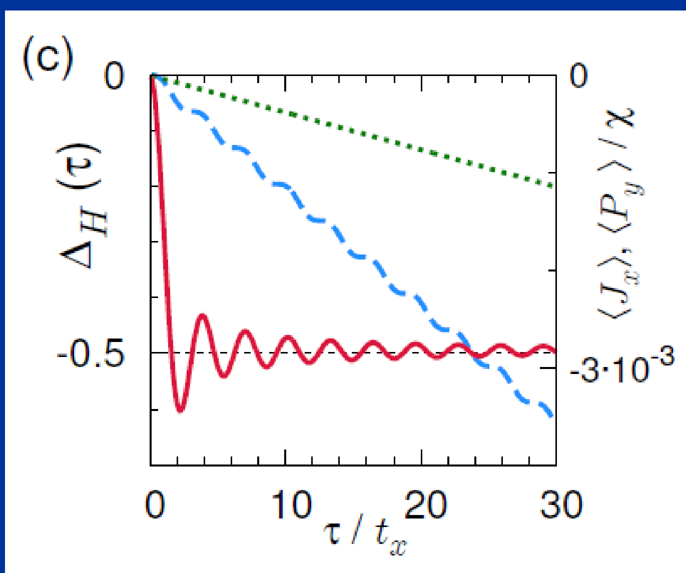
Non interacting and ``simple”

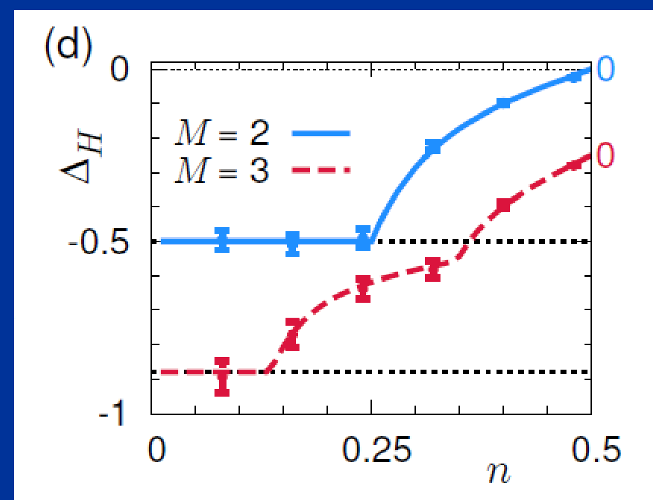
$$R_h = \frac{V_\perp}{I_\| B} \propto \frac{1}{n}$$

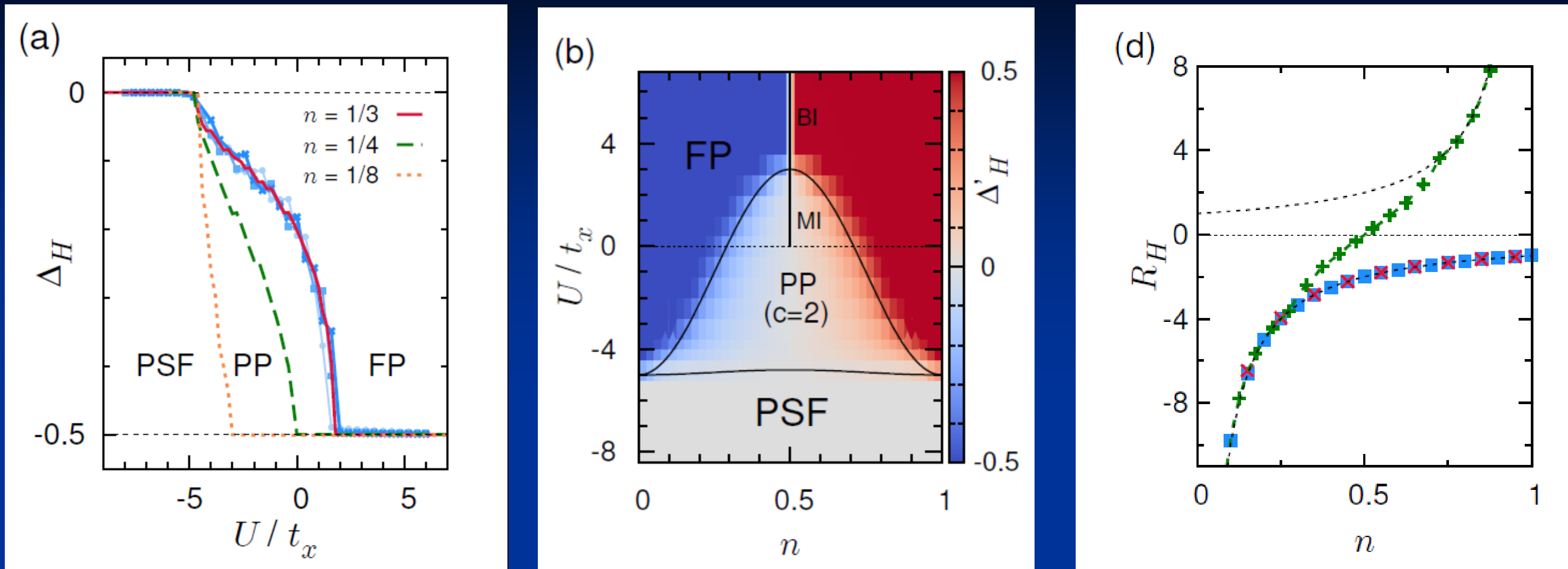
With interactions: open question



$$\Delta_{\rm H} = \left. \frac{\langle P_y \rangle}{\chi \cdot \langle J_x \rangle} \right|_{\chi \to 0}$$







Dependence of Rh in interactions and filling

Some universality of Rh if one band is occupied

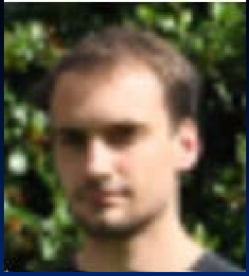
Universality carries between fermions and bosons

Beyond Luttinger liquids

- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures, ….

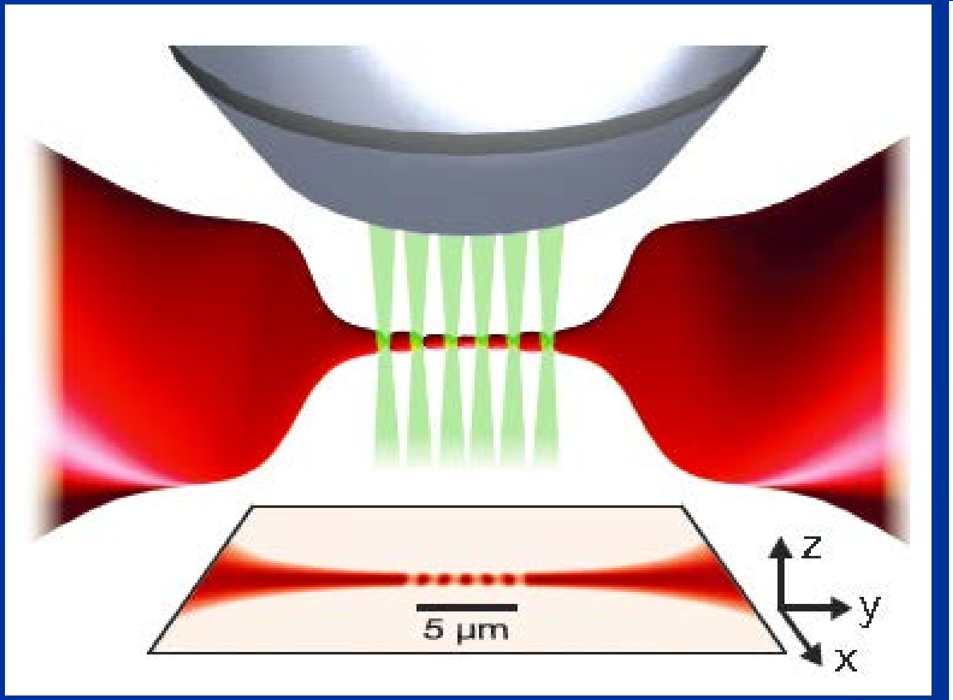
- New type of quantum critical points (e.g. topological)…

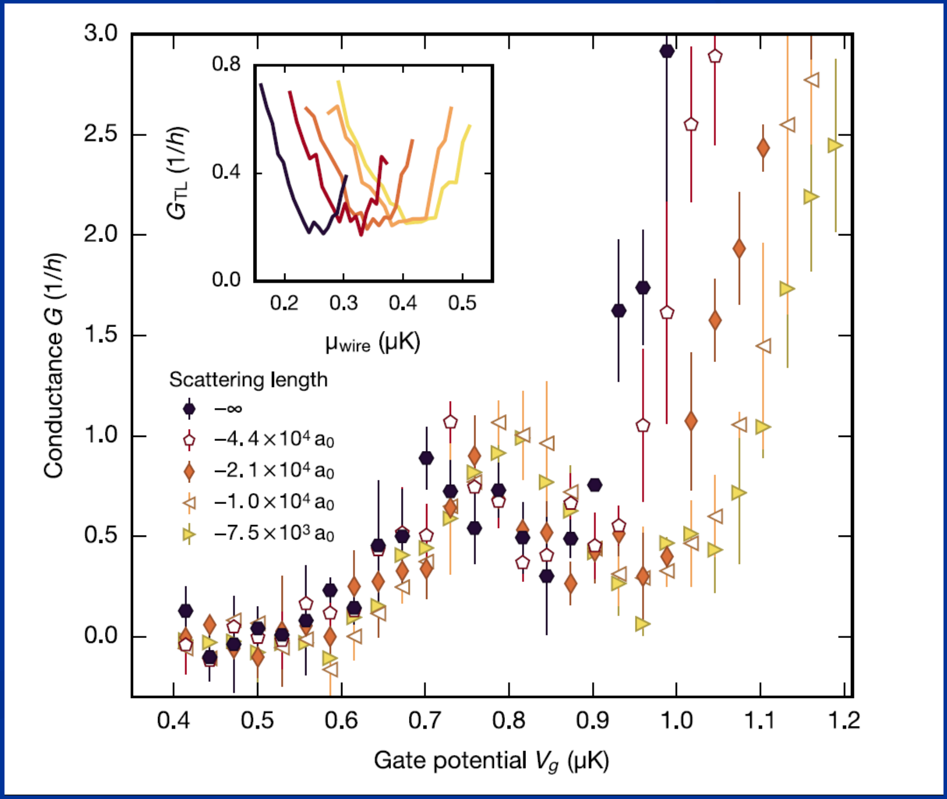
- Transport; Out of equilibrium situations.



Quantum transport in 1D

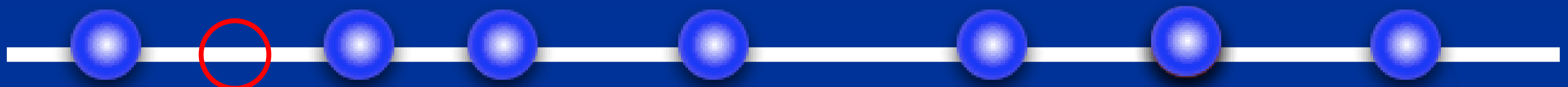
M. Lebrat, P. Grisins et al., PRX 8 011053 (2018)





Out of equilibrium situations

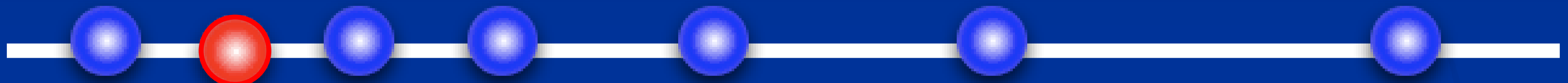
- Local quench (impurity)



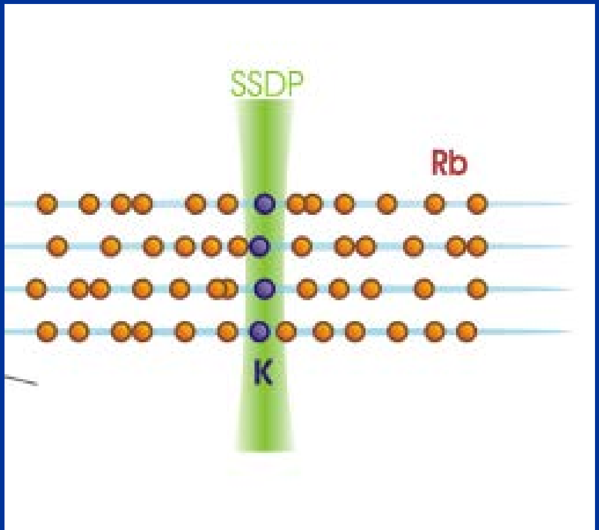
M. Zvonarev, V. Cheianov, TG PRL (2007);

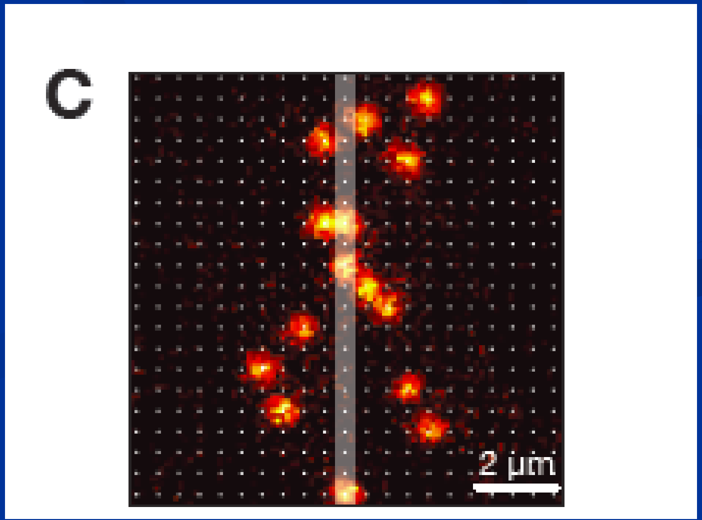
Out of equilibrium situations

- Local quench (impurity)



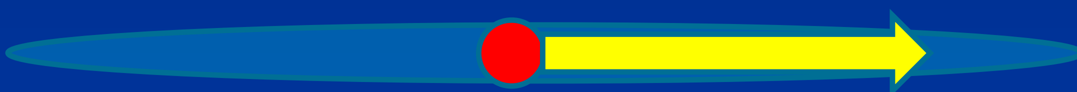
M. Zvonarev, V. Cheianov, TG PRL (2007);
J. Catani et al, PRA 85 023623 (2012) ; T. Fukuhara, A. Kantian et al. Nat Phys. (2013)
A. Kantian, U. Schollwoeck, TG PRL 113 070601 (2015)





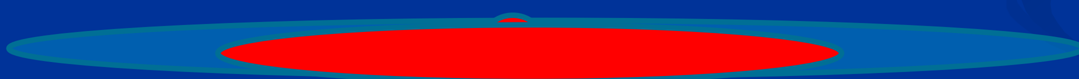
Driven impurity vs diffusion

- Normal transport



$$v = \mu F$$

$$v = f(F)$$



$$\langle x^2 \rangle \sim Dt$$

$$\langle x^2 \rangle \sim \log(t)$$

- Einstein relation: μ = D

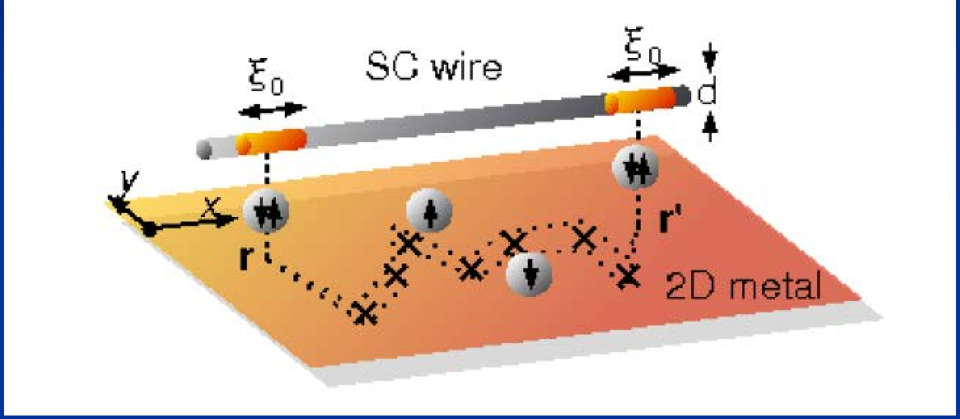
Beyond Luttinger liquids

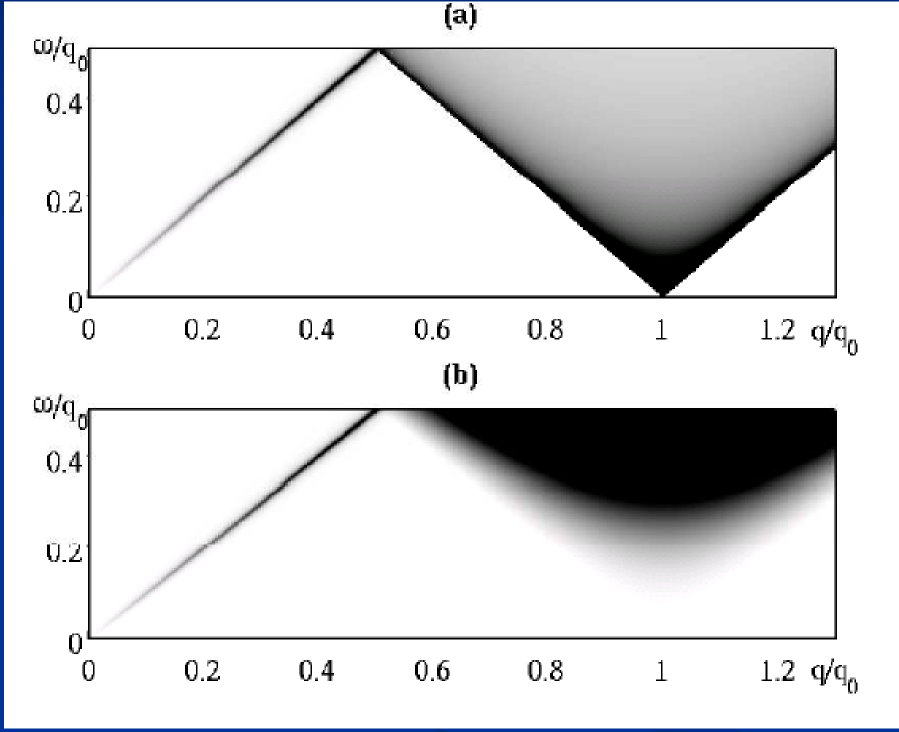
- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures, ….

- New type of quantum critical points (e.g. topological)…

- Transport; Out of equilibrium situations.

- Dimensional crossover 1d – 2d/3d

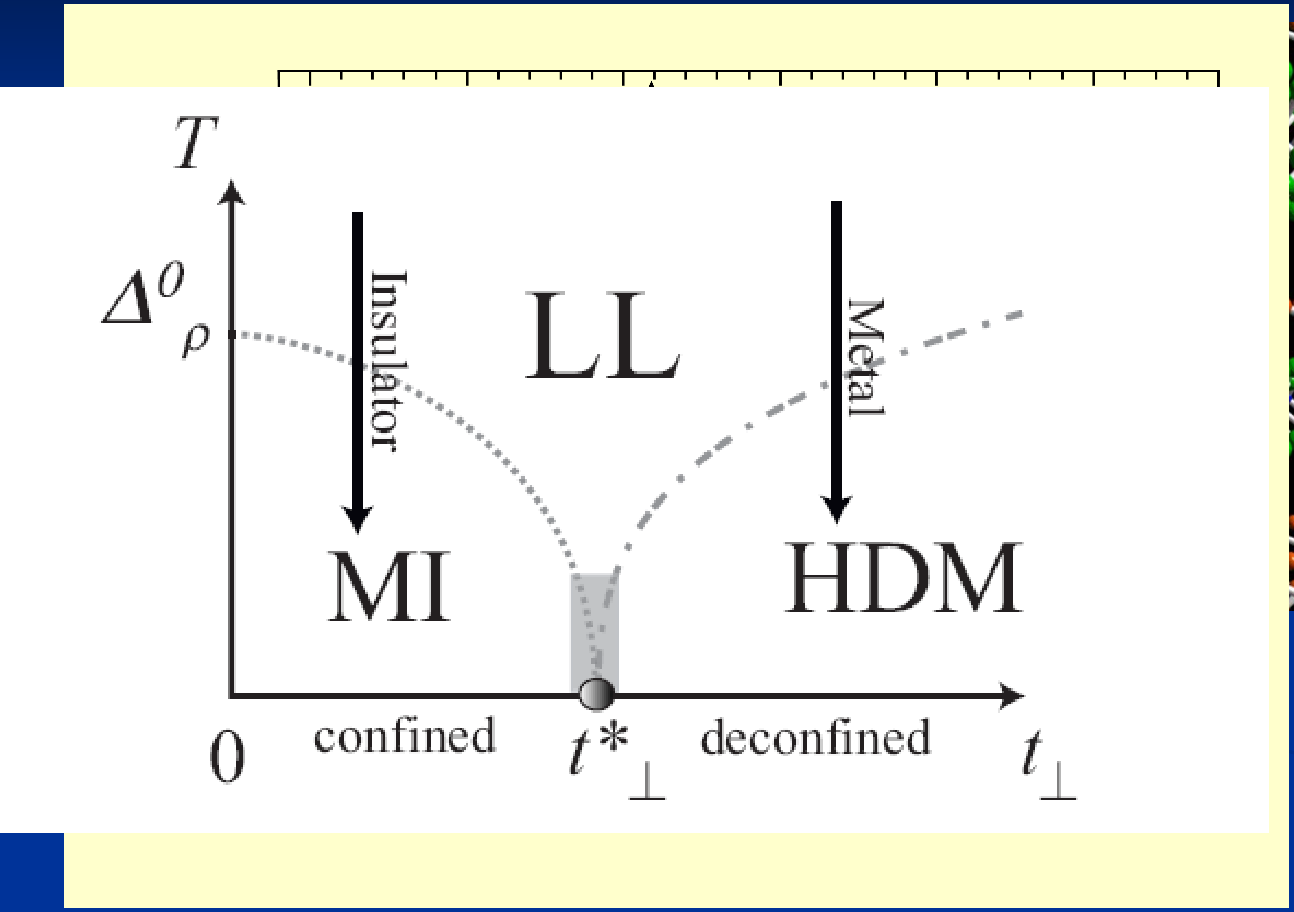


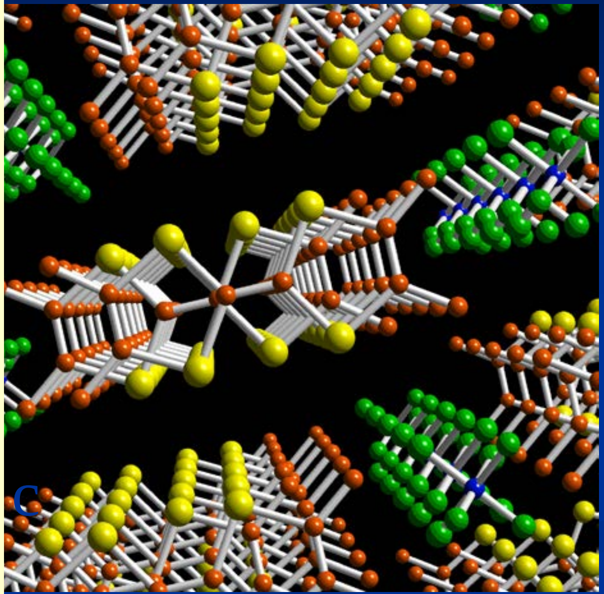


A. Lobos, A. Iucci, M. Muler, TG PRB 80 214515 (09)

E. Dalla Torre, E. Demler,TG, E. Altman, Nat. Phys. 6 806 (2010)

Deconfinement



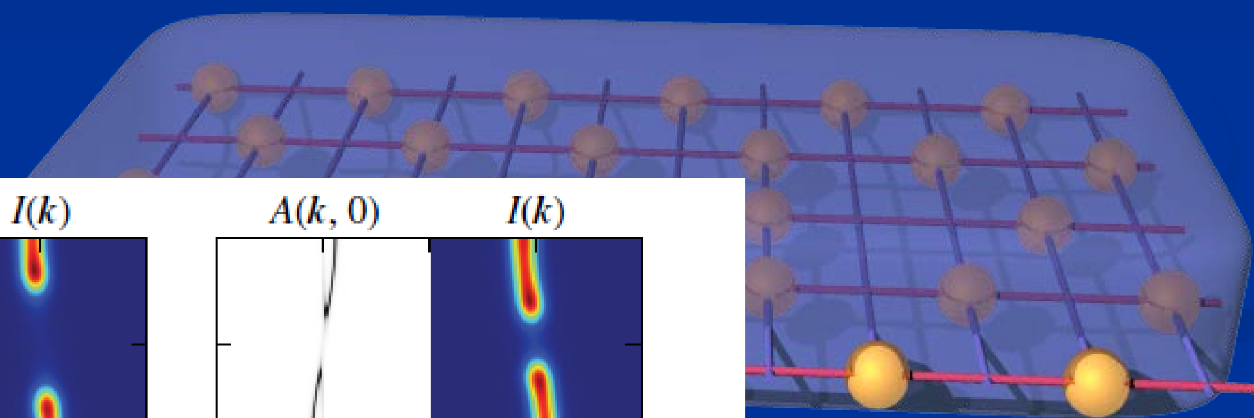


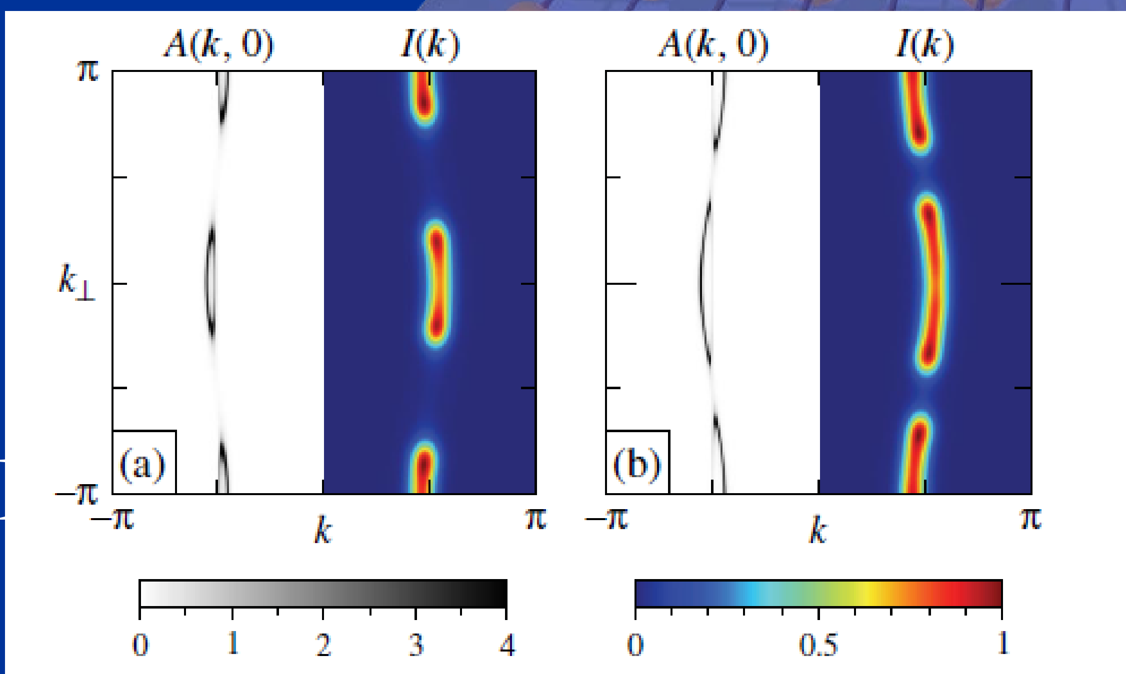
TG Chemical Review 104 5037 (2004)

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, C.A. Kuntscher, PRB 81 125109 (2010)
D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Back to the (self-consistent) bath

S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)





determined

Conclusions

- Tour of one dimensional physics
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Beautiful and challenging questions going beyond the Luttinger liquids
- Requires interplay of analytical and numerical techniques (and new ideas!) to make progress
- Many experimental realizations both in condensed matter and in cold atoms