The SYK Hamiltonian (complex ed.)

$$\boxed{\hat{H} = \sum_{ijkl=1}^{N} J_{ijkl}\, c_i^\dagger c_j^\dagger c_k c_l}$$

- c_i Fermion annihilators
- $N \gg 1$ 'nuclear physics type' large
- J_{ijkl} all-to-all interaction. Often random

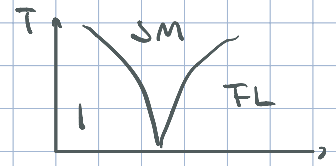
$$\langle J_{ijkl} \rangle = 0 \qquad \langle |J_{ijkl}|^2 \rangle = \text{const.} \times \frac{J^2}{N^3}$$

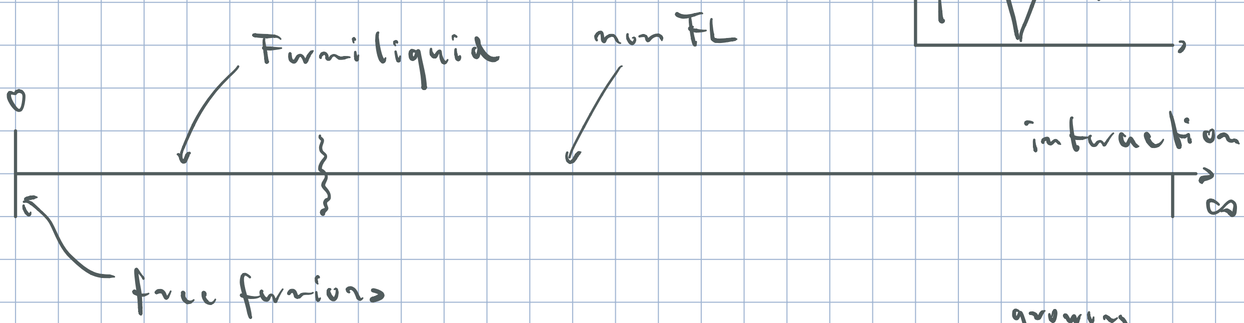
J: high energy scale (many body band width: $\sim J N^{1/2}$)

Why interesting: three perspectives

- Condensed matter
- Quantum Chaos
- Holography

Condensed Matter perspective





growing

$H = H_0 + H_{int}$

complexity

standard approach

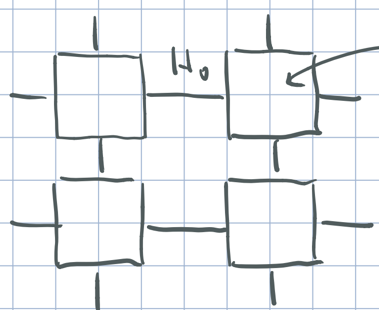
growing

$H = H_0 + H_{int}$

complexity

SYK approach.

SYK approach to correlated quantum matter



mesoscopically large cells with:

- N d.o.f. (fermions)
- maximum entropy all-to-all interaction
- ⇝ no Fermi liquid
- described by $H_{int} = H_{SYK}$

Quantum chaos perspective

· Early 70's nuclear physics

- strongly interacting,
- classically chaotic
- quantum liquid

N nucleons

< 70's : describe $D \sim e^N$-dim Hamiltonian as gaussian distributed random matrix ~ RMT ~ effective theory of quantum chaos

$$H = \begin{pmatrix} \ddots & & \\ & & \cdot \\ & & \end{pmatrix}$$

H_{nm} i.i.d. distributed 'knobs'

exp. large.

70's : ambition to 'inject' chaos into fermionic Fock space in more subtle ways

$$H = H_{SYK} \sim \text{only } O(N^4) \text{ 'knobs' } J_{ijkl}$$

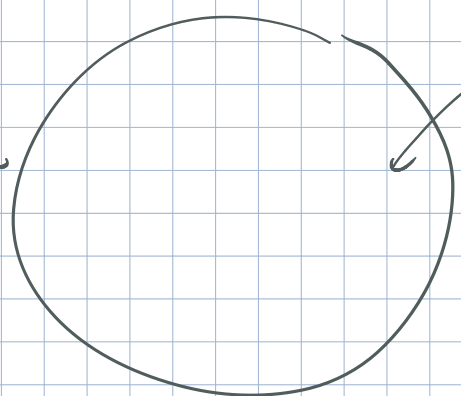
Model considered unsolvable (error!). Connections to chaos/RMT now understood

Holography perspective

Holography (vulgar cartoon)

large N quantum theory in d-1

- near conformal limit
- chaotic



gravitational theory in d-dimensions

- near AdS
- black hole

- No fully worked examples (except $N=4$ super Yang-Mills / AdS_5)

$\rightsquigarrow$ SYK : · chaotic ✓

· near conformal ✓

· $d=1$ (time = QM)

gravitational partner: Jackiw-Teitelboim gravity

Genesis of SYK models

· Early 70's: $\hat{H} = \sum_{ijkl=1}^{N} J_{ijkl}\, c_i^\dagger c_j^\dagger c_k c_l$

(Bohigas, French, Mello, Weidenmüller)

· Numerical results for many body DoS, spectral correlations

· Early 90's: rediscovered by Sachdev-Ye. Discover conformal symmetry

· 2015 Majorana version. Kitaev

$$\hat{H} = \sum_{ijkl} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

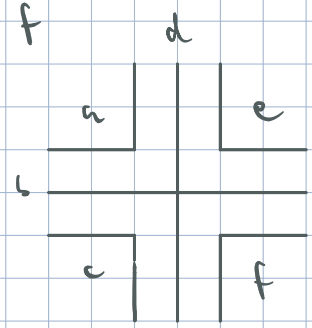
$$\{\chi_i, \chi_j\} = 2\delta_{ij} \qquad \langle |J_{ijkl}|^2 \rangle = \frac{6J^2}{2N^3}$$

· No particle number conservation.

· Model for a maximally random/interacting superconductor

• 2016 Model w/o randomness (Witten; Klebanov-Tarnopolski)

$$\hat{H} = g \sum_{a \ldots f} c^{abcf} c^{cdef} c^{ebf} c^{fda}$$



Physics: Energy-time scales

$G\Sigma$ theory →

exponential instabilities
scrambling
conformal symmetry (breaking)
collective quantum fl.

cond-mat

← nonlin σ model

quantum ergodicity/
collective spectral fl.
causal symmetry breaking
MBL (in extended models)

chaos

holography

Short time physics $t \sim \mathcal{O}(N)$

Goals:
- Explain non-Fermi liquid
- Conformal symmetry
- effective low energy theory

• applications

Mean field approach to the SYK non Fermi liquid

$$\hat{H} = \sum_{ijkl}^{\overset{\text{large}}{N}} \underbrace{J_{ijkl}}_{\text{all-to-all}} \chi_i \chi_j \chi_k \chi_l$$

$\rightsquigarrow$ mean field. But how?

• Pick good observable: $G(\tau) = \langle \chi_i(\tau)\, \chi_i(0) \rangle_{qu, J}$

building block in computation of various other observables

• Path integral approach.

$$Z = \int \mathcal{D}\chi \; e^{-\int d\tau \left(\sum_i \chi_i \partial_\tau \chi_i - H \right)}$$

• Compute G 'perturbatively' in H.

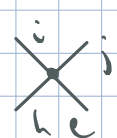
• 0th order

$G(\tau) =$

$$= \langle \tau | \frac{1}{-\partial_\tau} | 0 \rangle = \underbrace{\frac{1}{2}}_{\text{Majorana} = 1/2 \text{ Fermion}} \underbrace{\operatorname{sgn}(\tau)}_{\text{Fermi stat.}}$$

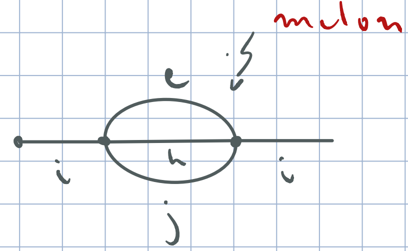
• 1st order

$H = \sum_{ijkl}$



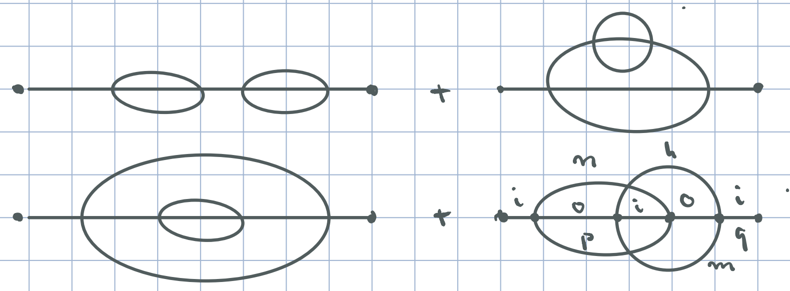
$\langle H \rangle_J = 0$

• 2nd order

$G(\tau) =$ 

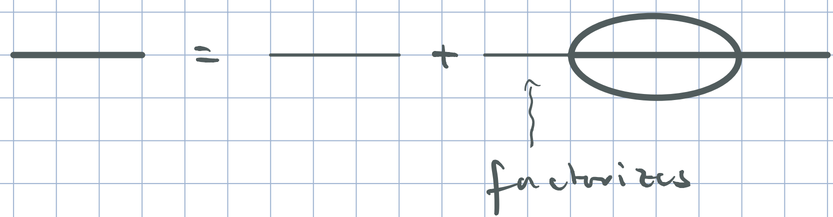
$$\sim \frac{J^2}{N^3} \times N^3$$

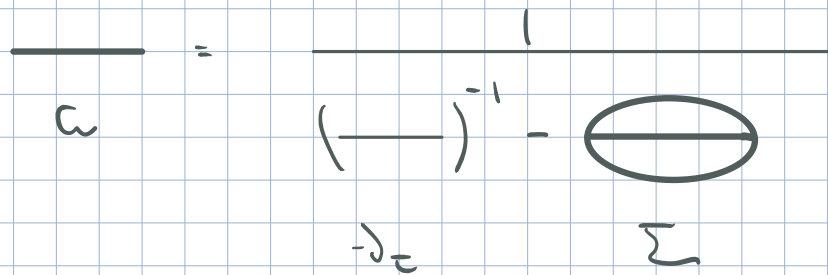
• 4th order



$$\rightsquigarrow \left(\frac{J^2}{N^3}\right)^2 N^5 \sim N^{-1}$$

$\rightsquigarrow$ self consistent melon approximation





algebraic representation:

$$\boxed{G = -\frac{1}{\partial_\tau + \Sigma} \qquad \Sigma = J^2 \; G^3}$$

$$\left(\Sigma(\tau,\tau') = J^2 \; (G(\tau,\tau'))^3 \right)$$

Solution: $G = \dfrac{1}{\cancel{\partial_\tau} - \Sigma}$

$\mathcal{O}$(energy scales of interest) $\ll$ $\mathcal{O}(J)$

algebraic equations: $G = \dfrac{1}{\Sigma} \qquad \Sigma = J^2 G^3$

Solution (e.g. by dimensional analysis $\rightsquigarrow$ ex.)

$$\boxed{\begin{aligned} G(\tau,\tau') &= -\frac{b}{J^{1/2}} \frac{\operatorname{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}} \\ \Sigma(\tau,\tau') &= -\underbrace{b^3}_{\text{numerical factor}} J^{1/2} \frac{\operatorname{sgn}(\tau-\tau')}{|\tau-\tau'|^{3/2}} \end{aligned}} \qquad (*)$$

Key feature: $[G] \sim [\chi\chi] \sim t^{-1/2} \Rightarrow [\chi] \sim t^{-1/4}$

Non-Fermi liquid

reminder: Fermi liquid $[\chi] = [c] = -1/2$

info e.g. from action: FL $S = \int d\tau \left(\chi \partial_\tau \chi - \chi h_0 \chi \right)$

$[S] = 1 \rightsquigarrow [\chi] = \tau^{-1/2}$

SYK $S = \int d\tau \left(\chi \cancel{\partial_\tau} \chi - J \chi^4 \right)$

$[\zeta] = \tau^{1/4}$

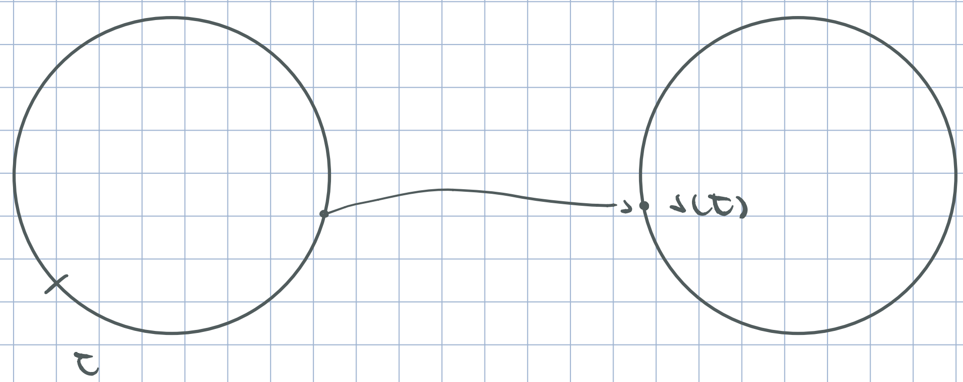
Q: Is (*) the only solution?

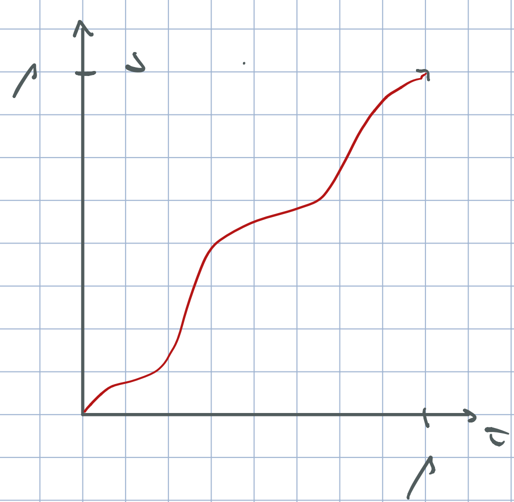
A: No, there are lots more

Conformal symmetry

Consider action $S = \int d\tau \left(\bar{\zeta} \cancel{\partial_\tau} \zeta - \underbrace{J \zeta^4}_{\text{local in time}} \right)$

It is invariant under reparametrizations of time





- 1-1
- smooth

} diffeomorphic

$s: [0,1] \longrightarrow [0,1] \in \mathrm{Diff}(S^1)$ Diffeomorphism group of circle

$\tau \longrightarrow s(\tau)$

- $\zeta(\tau) \longrightarrow \tilde{\zeta}(\tau) = \zeta(s(\tau)) \left(ds \right)^{1/4}$

$$S[\tilde{\xi}]_{\partial_\tau \to 0} = S[\xi] \qquad (\text{NCFT}_1)$$

nearly 1 dim

- Note: ∞ many functions in $\text{Diff}(S^1)$: an ∞ dim symmetry
  - By itself: useless
  - However, not in SYK
- **Q**: How does mean field respond to symmetry?

  Cf: $S_{\text{magnet}}[\uparrow] = S_{\text{magnet}}[R\uparrow]$

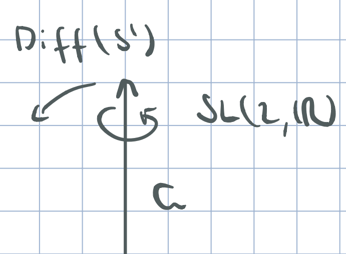
  mean field $\to$ Paramagnet $\nwarrow\uparrow \to \nwarrow\uparrow$ invariant
  mean field $\to$ Ferromagnet $\uparrow\uparrow\uparrow\uparrow\uparrow$ changes

A: $G(\tau,\tau') \sim \dfrac{\text{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}} \longrightarrow \tilde{G}(\tau,\tau') = \left(\dfrac{ds}{d\tau}\dfrac{ds}{d\tau'}\right)^{1/4} G(s(\tau), s(\tau'))$

$\tilde{G}(\tau,\tau') \neq G(\tau,\tau')$ except for $s(\tau) = \dfrac{a\tau + b}{d\tau + c}$, $ad-bc=1$

special conformal trafo. Parametrized by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $ad-bc=1$ $\sim SL(2,\mathbb{R})$

$\sim$ magnetic analogy:

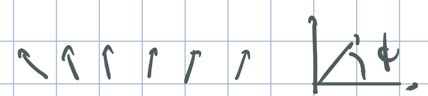


Goldstone mode MF : $\mathrm{Diff}(S^1)/SL(2,\mathbb{R})$ (vs. $O(3)/O(2)$)

Note: We are in $d=1$ (time). Mermin-Wagner theorem: Goldstone mode fluctuations become large at long time scales.

Q: What effective action governs these fluctuations

analogy: $\uparrow B$ - explicit sym breaking



$$S[\phi] = \int B \cos\phi$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$s(\tau) \qquad\qquad {}^{\backprime}\partial_\tau{}'$$

expect $S[s] = \int d\tau\, f(\partial_\tau s)$

Derivation of effective action

- Starting point $G\Sigma$- functional (Sachdev-Ye, 91)

$$\mathrm{tr}\left(e^{-\beta H}\right) \underset{\text{exact}}{=} \int DG\, D\Sigma\; e^{-S[G,\Sigma]}$$

replica index

$$G = \{G^{rr'}(\tau,\tau')\}$$

$$\Sigma = \{\Sigma^{rr'}(\tau,\tau')\}$$

$$\boxed{S[G,\Sigma] = -\frac{N}{2}\Big\{\mathrm{tr}\ln\left(\partial_\tau + \Sigma\right) + }$$

sym. breaking

$$+\int_0^\beta d\tau d\tau' \left(\frac{J}{4} G^4_{\tau,\tau'} + \Sigma_{\tau,\tau'} G_{\tau'\tau} \right) \Big\}$$

- Note: · Prefactor N stabilizes mean field

  · Variation $\frac{\delta S}{\delta G} = \frac{\delta S}{\delta \Sigma} = 0$ produces m.f. eq.

  · $\rightsquigarrow$ Fields G and Σ represent phys. G and Σ, resp.

- Strategy: Substitute $\tilde{G}(\tau,\tau') = \hat{G}_s(\tau,\tau')$ into
  $\tilde{\Sigma}(\tau,\tau') = \hat{\Sigma}_s(\tau,\tau')$

  functional integral and expand in $\{s, \partial_\tau\}$. $\rightsquigarrow$
  $\rightsquigarrow$ a lengthy calculation plagued by UV issues
  (A, Bagrets, Kamenev 17)

$$\boxed{S[s] = \frac{M}{2} \int d\tau \left(\frac{s''}{s'}\right)^2 = -M \int d\tau \, \{s,\tau\}}$$

Schwarzian action

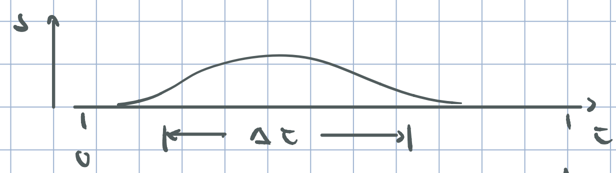
- $M = \frac{b^2}{32 J} N \ln N$

Schwarzian derivative:

$$\{s,\tau\} = -\frac{1}{2}\left(\frac{s''}{s'}\right)^2 + \underbrace{\left(\frac{s''}{s'}\right)'}_{\text{surface term}}$$

- Comments:

- Consider fluctuation



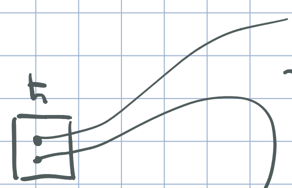
action: $\sim M\,\Delta\tau\,\Delta\tau^{-2} \sim M\,\Delta\tau^{-1}$

$\int \{s,\tau\}$

fluctuations on large time scales $\Delta\tau > M$ cost little action.

M defines threshold between semi-classical short time regime and (quantum) fluctuation dominated long time regime.

- Note: semiclassical time scales in quantum chaos:

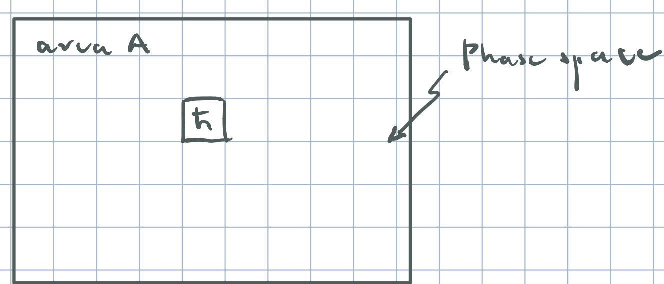


$\sim \hbar\, e^{\lambda t} \sim \Delta S_{cl}$

$\Rightarrow t \sim \frac{1}{\lambda} \ln\left(\frac{\Delta S_{cl}}{\hbar}\right)$

Ehrenfest time

What is $\hbar$?



Dimension Hilbert space $D \sim \frac{A}{\hbar} \Rightarrow D \sim \hbar^{-1}$

Here: $D \sim 2^N \Rightarrow \hbar \sim N$

Conclusion: I: Ehrenfest time of quantum chaos compatible with semiclassical time scale M. II: Ehrenfest time often defined differently, see below.

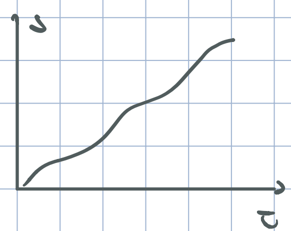
- Schwarzian action not friendly. +

+ complicated measure

$$\int_{\mathrm{Diff}(S^1)/SL(2,\mathbb{R})} \mathcal{D}s \; e^{-S[s]}$$

Liouville action

- Idea: use freedom of reparametrization to obtain more manageable description.



$$\rightsquigarrow \; s'(\tau) = e^{\phi(\tau)}$$

$$\rightsquigarrow \; Z = \int \underbrace{\mathcal{D}\varphi}_{\text{flat}} \; e^{-S[\varphi]}$$

$$\boxed{S[\varphi] = M \int d\tau \left(\tfrac{1}{2}\dot{\varphi}^2 + 2e^{-\varphi} \right)}$$

Liouville action (A-Bagrets-Kamenev, 17)

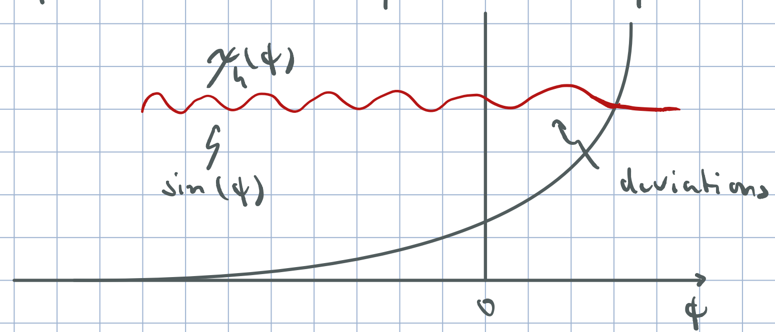
SYK model and Liouville QM

- Low energy ($\ll J$) physics of SYK model described in terms of Liouville action. D.o.f. reparametrization coordinate φ.
- At time scales $\tau \gtrsim M$: Quantum fluctuations strong

Q: What is Liouville QM?

A : • simple

• $S[\varphi] = M \int d\tau \left(\frac{1}{2}\dot{\varphi}^2 + 2e^{-\varphi} \right)$ is action of point particle in exponential potential (almost half a space)



Eigenfunctions/-values of $\hat{H} = -\frac{\partial_\phi^2}{2M} + \gamma e^{\phi}$
known: $E_k = \frac{k^2}{2M}$

$$\psi_k(\phi) \sim K_{2ik}\left(c \cdot e^{\phi/2}\right) \sim e^{ik\phi} \qquad \phi \ll 0$$

Most crucial feature: Liouville universality (Tsvelik-Shelton, 98)

For all local operators $\mathcal{O} = \varphi, \partial\varphi, \varphi^n, \ldots$

$$\langle \mathcal{O}(\tau)\, \mathcal{O}(\tau') \rangle \sim \frac{1}{|\tau - \tau'|^{3/2}}$$

This power law determines the temporal behavior of pretty much all late time SYK correlation functions!

Greens function and Liouville QM

• Recall $S(\tau)$ $SL(2,\mathbb{R})$ $\qquad G(\tau,\tau') \sim \frac{1}{|\tau - \tau'|^{1/2}}$

$$G_s(\tau,\tau') = \left(\frac{ds}{d\tau}\,\frac{ds}{d\tau'}\right)^{1/4} G(s(\tau), s(\tau'))$$

Q: What is $\langle G_s(\tau,\tau') \rangle_s$

A: Expect $\langle G_s(\tau,\tau') \rangle_s \sim G(\tau,\tau') \qquad |\tau-\tau'| \ll M$

$\qquad\qquad\qquad\qquad \to 0 \qquad\qquad$ " $\gg$ "

$$\langle G_s(\tau,\tau') \rangle_s \underset{s'=e^{\phi}}{=} \int \mathcal{D}\phi \; e^{-S[\phi]} \; G_{s(\phi)}(\tau,\tau')$$

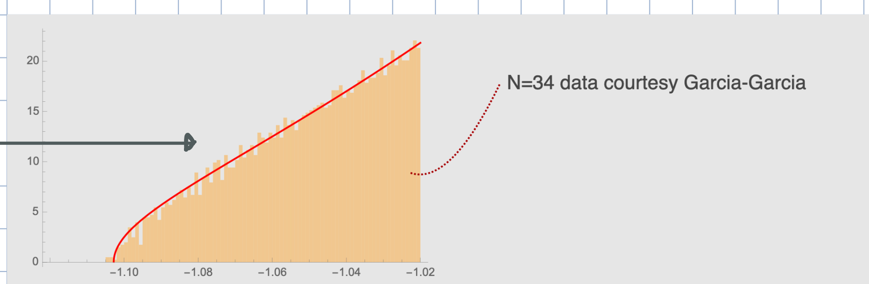
spectral decomposition

$\longleftarrow$ Eq. (16) - (25) DAK

- Another 'fundamental' observable: many body DoS:

$$Z = \mathrm{tr}\left(e^{-\beta \hat{H}}\right) \sim \int \mathcal{D}\varphi \; e^{-S[\varphi]} \sim \dots \sim \int_0^\infty d\epsilon \; \underbrace{\rho(\epsilon)}_{\text{DoS}} \; e^{-\beta\epsilon}$$

$$\rho(\epsilon) \sim \sinh\left(2\pi\sqrt{M\epsilon}\right)$$



Applications I: Early time ($t < e^N$) quantum chaos

- Principal diagnostics: Out of time ordered correlation functions (OTOC, Larkin-Ovchinnikov, 68)

Q: What are OTOCs?

A: The observables testing for quantum signatures of exp. chaotic instabilities

$$F(t) = \mathrm{tr}\left(e^{-\beta \hat H} \hat X \hat Y(t) \hat X \hat Y(t)\right)$$

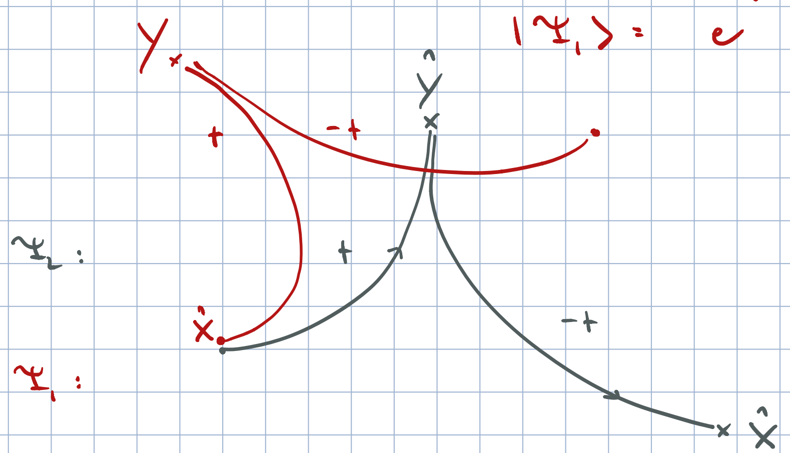
$\hat X, \hat Y$ some (non-commuting operators)

Interpretation as quantum butterfly observable

Consider $T=0$: $e^{\beta H}$ projects on GS. Draw

$F(t) = \langle \Psi_1 | \Psi_2 \rangle \qquad |\Psi_2\rangle = \hat X e^{i\hat H t} \hat y e^{-i\hat H t} |GS\rangle$

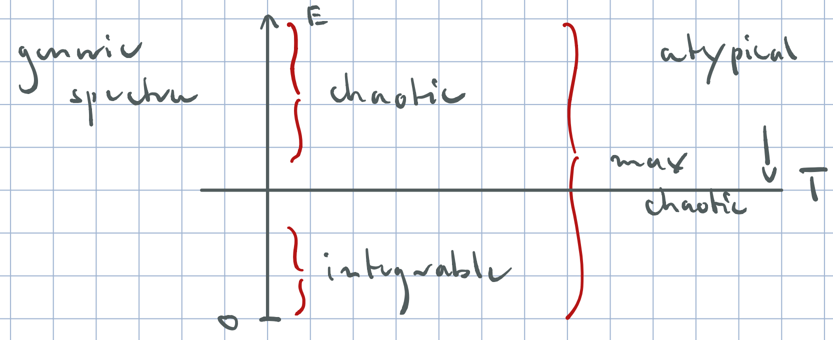
$|\Psi_1\rangle = e^{i\hat H t} \hat y e^{-i\hat H t} \hat X |GS\rangle$



$F(t)$ of classically chaotic quantum system expected to show $\sim \exp(\lambda t)$ instability.

Q: What is the maximal value of λ a chaotic system can show (in OTOC)?

Consider 'modified' OTOC: $F(t) = \mathrm{tr}\big(e^{-\beta \frac{\hat H}{4}} \hat X e^{-\beta \frac{\hat H}{4}} \hat Y(t) e^{-\beta \frac{\hat H}{4}} \hat X e^{-\beta \frac{\hat H}{4}} \hat Y(t)$



blocks access to spectrum higher than T.

$\lambda_{OTOC} \xrightarrow[T\to 0]{} 0$ $\qquad\qquad$ $\lambda_{OTOC} \longrightarrow T$

Q: But what happens after 'semiclassical' time scales? (A seldom-ly asked question.)

OTOCs of SYK model

- $\hat{X} = \gamma_i \quad \hat{Y} = \gamma_j$ holographic interpretation suggests maximal chaos ('black hole') $\sim$ $\lambda = T$

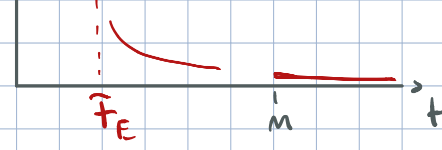
- Calculation: Trotterize $F(t) \sim$ integral over reparametrizations.

- Results:

  - Short times - mean field analysis of $G\Sigma$.

$$F(t) = 1 - \frac{1}{64\pi MT} e^{-2\pi t T} \qquad \text{(Maldacena et al. 16)}$$

Chaos bound!



Something must happen at $\tilde{t}_E \sim T^{-1} \ln(MT) \sim T^{-1} \ln(N)$

t_E defines (Ehrenfest time)$_{\text{OTOC}}$. For single particle chaotic systems this agrees parametrically with $\tilde{t}_E \sim \ln \hbar^{-1}$. But not here. $\tilde{t}_E + t_E \sim M$

- times larger than t_E: Stationary phase 'Newton eq.' for Liouville action

$$F(t) - \ln(MT)\, e^{-nT(t-t_E)}$$

ABK 17

- Semiclassics breaks down at $t \sim M$. Integration over repara's yields

$$F(t) \sim \left(\frac{M}{t}\right)^6 \quad \text{power law !}$$

ABK 17

Interpretation of power law OTOCs

- Liouville QM is gapless ($E_k = \frac{k^2}{2}$). Exp. time dependence needs gaps. Quantum chaos 'softer' than classical chaos

- Liouville universality

$$F(t) \sim \operatorname{tr}\left(z_1 e^{-i\hat{H}t} z_2 e^{i\hat{H}t} z_1 e^{-i\hat{H}t} z_2 e^{i\hat{H}t} \right)$$

$\rightsquigarrow$ 4 Liouville correlation functions in time

$$\sim (t^{-3/2})^4 = t^{-6}$$

- Hard quantum chaos interpretation:

Lehmann repr.

$$F(t) \sim \sum_{m_1 \dots m_4} \underbrace{\langle m_1 | \chi_i \rangle}_{\text{exact many body ef}} \langle m_2 | \chi_j \dots \; m_4 \rangle \times$$

$$\times\, e^{\epsilon_{m_1}(it - \beta/4)} \cdots e^{\epsilon_{m_4}(it - \beta/4)}$$

assume statistical ind. of wave functions and eigenvalues.

$$\sim \left| \sum_m e^{-(\beta/4 + it)\epsilon_m} \right|^4 \sim \int_0^\infty d\epsilon\, \underbrace{\rho(\epsilon)}_{\mathrm{sh}(\sqrt{\epsilon M}) \sim \sqrt{\epsilon}} e^{-(\beta/4 + it)\epsilon} \sim$$

$$\sim t^{-6}$$

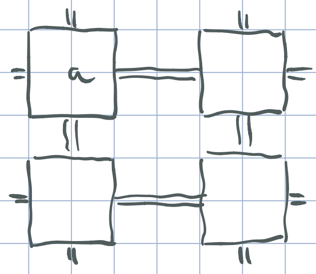
Summary

- SYK maximally chaotic non FL
- Strong symmetry breaking principle
- Goldstone modes $\rightsquigarrow$ Liouville QM

• System under analytic control (for times $t < \exp N$)

Applications II: Phenomenology of strong correlation physics

• Idea: Model strongly correlated fermion matter as SYK arrays (Balents et al 17, Berg et al 17, ..)



$$\hat{H} = \sum_a \hat{H}_{SYK}(\chi^a) + \hat{H}_T$$

$$\hat{H}_T = \frac{i}{2}\sum_{\langle ab\rangle} V^{ij}_{ab}\,\chi^a_i\,\chi^b_j$$

$$\langle (V^{ij}_{ab})^2\rangle = \frac{v^2}{N}$$

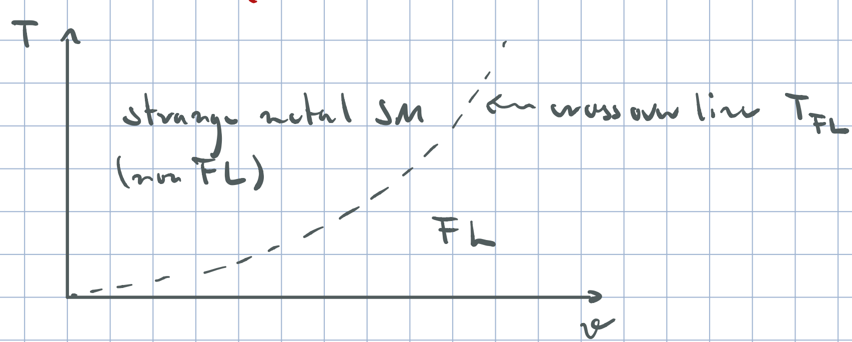
• mean field approach (Balents et al. 17)

Dimensional analysis $N \to \infty$ (reparas irrelevant)

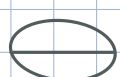
$$[\chi] = t^{-1/4} \quad \sim \quad S_T = \int d\tau\, \chi V \chi \sim t^{1-2\times(\frac{1}{4})} = t^{1/2}$$

SYK$_2$ operator $\chi_i V^{ij} \chi_j$ is RG relevant.

Consequence:



Q: When does SM/FL crossover take place?

Compare (ex.)  SYK self energy $\left(\sim \int_0^\beta d\tau\, G^3\right)$

$T_{FL} \sim \frac{v^2}{J}$

 tunneling self energy

Q: What happens for finite N?

Warfare between Goldstone modes and tunneling

- Consider lowering T at infinitesimally weak v but finite N.

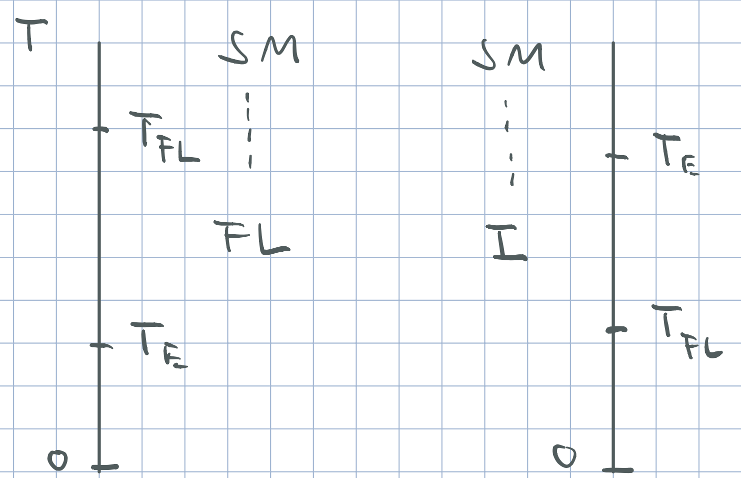
  Below $T \sim M^{-1}$: $\quad G(\tau,\tau') \sim |\tau-\tau'|^{-3/2} \Rightarrow [\zeta] = t^{-3/4}$

  $$S_T \sim \int d\tau \, \zeta V \zeta \sim t^{1-2\times\frac{3}{4}} = t^{-1/2}$$

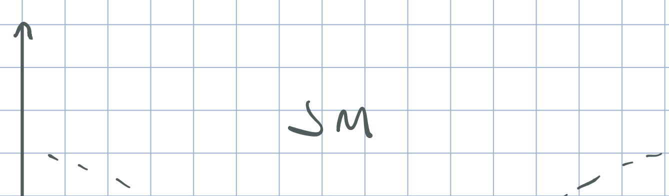
  RG irrelevant

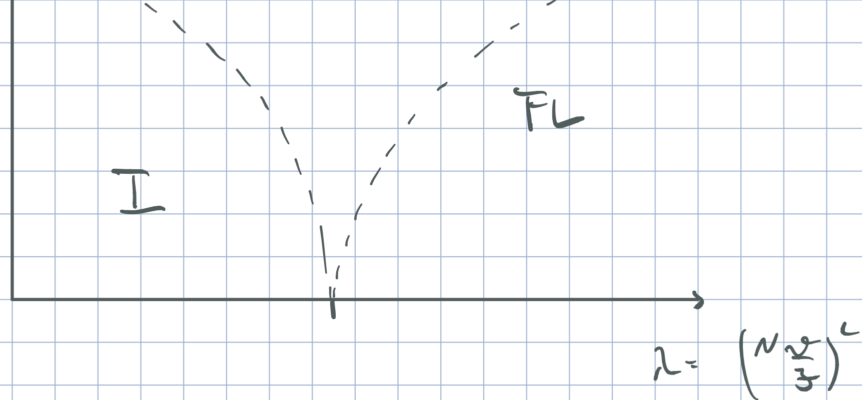
  Goldstone modes kill tunneling by quantum fluctuations (Lunkin et al. 18) $\Rightarrow$ system becomes insulator

- Two scenarios:



Phase diagram





← ABK 19

Applications

- Phenomenological modeling of correlated phases. Above phase diagram 'has the right features' 1) Coexistence of different phases in close parametric proximity 2) separated by QPT and showing insulating, FL, SM signatures. Problem: difficult to teach the approach superconductivity (Wang et al 2002.11757)

- Physics of nanoscopic quantum transport → single large molecule → C (1908.11351)

Summary (Physics of correlations)

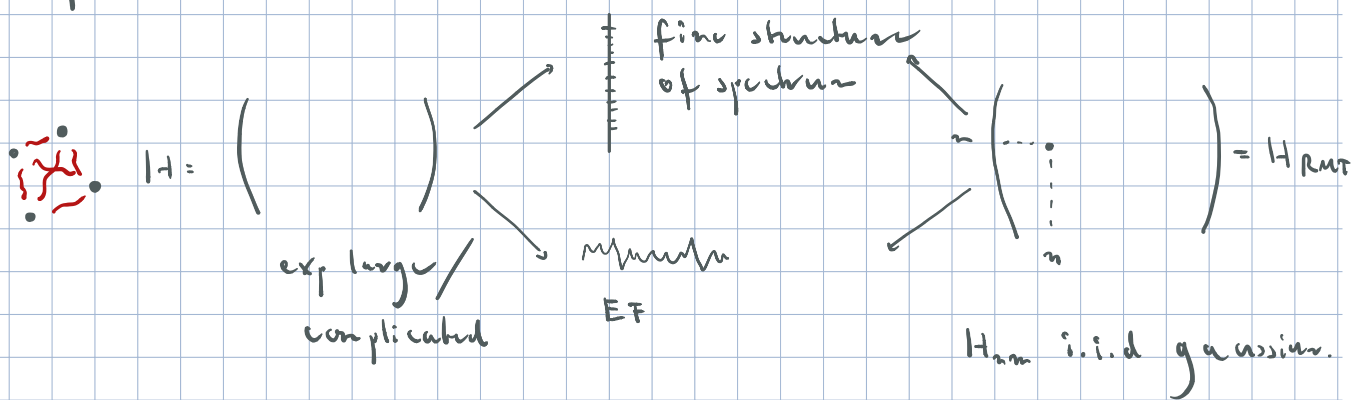
- SYK approach: Model correlations starting from hard core interacting cells ⇝ Strong phenomenology with little effort!

- Need more activity.

Long time physics ($t \sim \mathcal{O}(2^N)$)

- Probe structures at level spacing scales of a hard chaotic system

- Bohigas-Giannoni-Schmit (BGS) conjecture
physics at these time scales coincides with that of random matrix ensembles

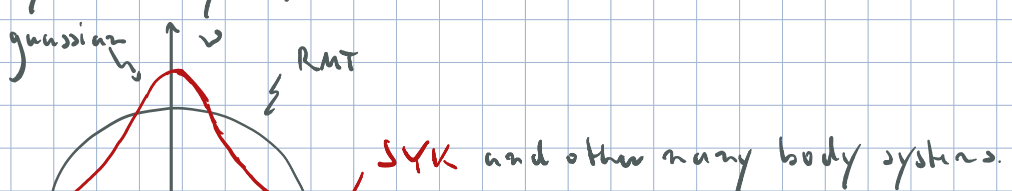


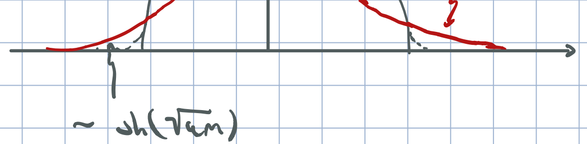
Q: How do we characterize fine structure of quantum spectra

A: • Distinguish symmetry classes - symmetry of H under Time reversal, Charge conjugation, Chiral symmetries. ∃ 10 classes in total. Depending on value of N mod 8 SYK realizes 8 of them! (1508.00995)

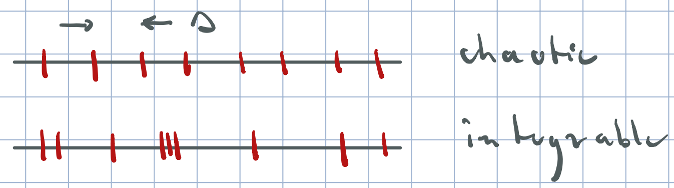
Here: class A / GUE (just hermiticity)

- (Many body) density of states (dos)

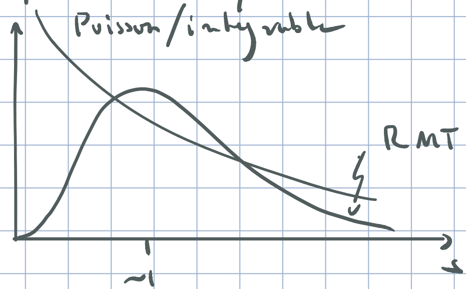




• correlations of spectra

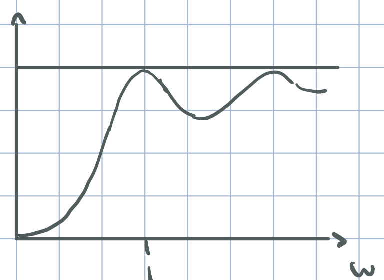


• $P(s)$ probability to find levels at separation $s = \frac{R}{2}\frac{\varepsilon}{\Delta}$



• $R_2(s) = \frac{1}{\langle\nu\rangle^2}\left(\langle \nu(E+\omega)\,\nu(E)\rangle - \langle\nu\rangle\langle\nu\rangle\right)$

only weakly ε-dependent



$\sim \frac{\sin^2(s)}{s^2}$

• $\Sigma_2(\omega)$ = var # of levels in interval ω