GGI - Galileo Galilei Institute
SFT 2022 - 7-18 February 2022
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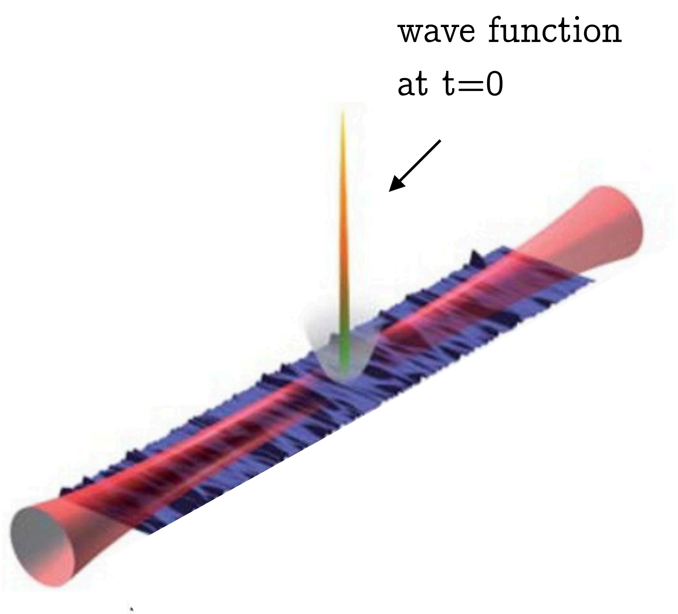
An introduction to Many-Body Localization

slides of support

Laboratoire de Physique Théorique et Modèles Statistiques
Laboratoire d'Excellence Physique : Atomes Lumière Matière



Put a condensate in a trap.

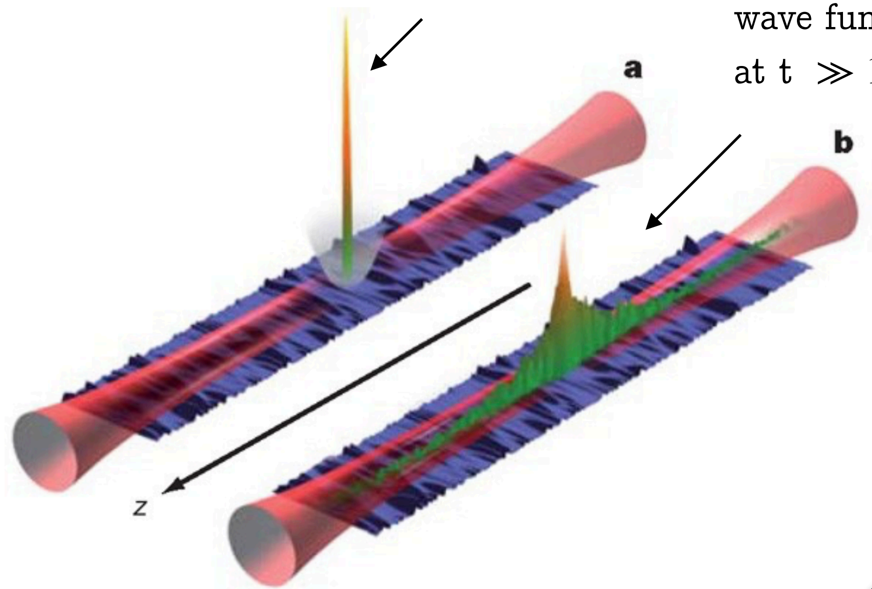
Release the trap,
come back at much later time.

What do you expect?

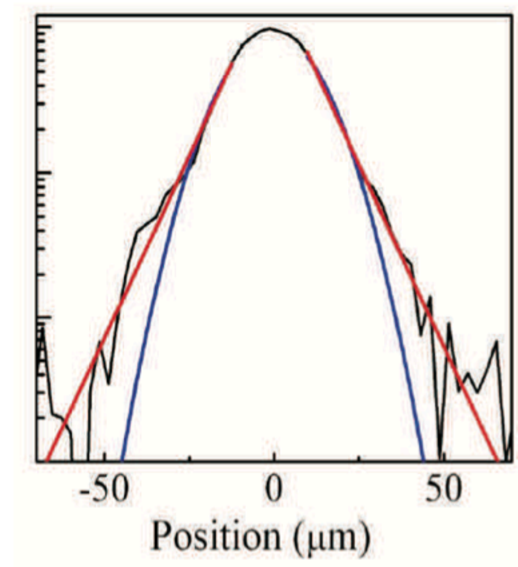
If the potential is random:

wave function
at t=0

wave function
at t $\gg$ 1



atomic density



data collected in Florence (at LENS)!

Roati et al, Nature 453 (2008)

and in Paris, too:

Billy et al, Nature 453 (2008)

[in a nutshell: looking at $t \gg 1$, can guess where particle was at $t = 0$]

Part I: MBL in 5 questions

What:
1. What does out-of-equilibrium mean *in MBL*?
2. What would *equilibrium* even mean, actually?

How:
3. How disorder generates localization?
4. How can we understand everything in a unified framework?

Why:
5. Why challenging & why worth the effort?

Part II: analytical arguments for localization

- Decay rates of local excitations: a functional order parameter
- Anderson's reasoning: resummed pert. theory & its convergence
- If time: Many-body case, diagramamtics & integrals of motion

References & material: https://mycore.core-cloud.net/index.php/s/h1N9JQoE4oKqeNq

[what]

What does "out-of-equilibrium" mean *in MBL*?

A tentative definition:

Many-Body Localized (MBL) systems are
thermodynamically-large systems of locally-interacting d.o.f.
(spins, cold atoms, qbits, electrons...)
in quenched disorder & evolving with unitary dynamics,

that remain permanently and robustly Out-Of-Equilibrium (OOE)
because transport at large lengthscale is suppressed:
they "dephase without dissipating".

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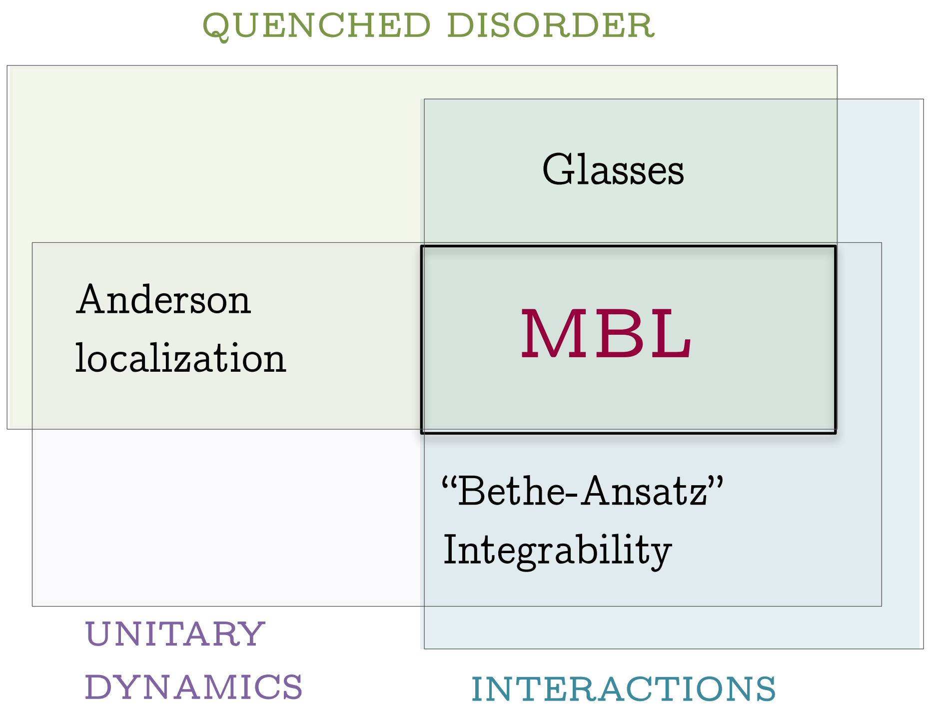
QUENCHED DISORDER
Glasses
Anderson
localization
MBL
“Bethe-Ansatz”
Integrability
UNITARY
DYNAMICS
INTERACTIONS

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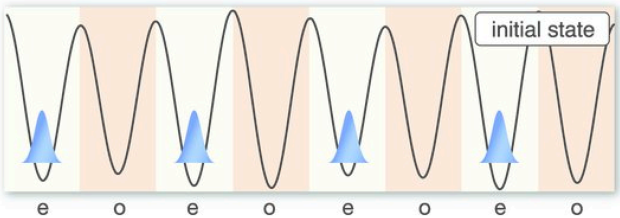
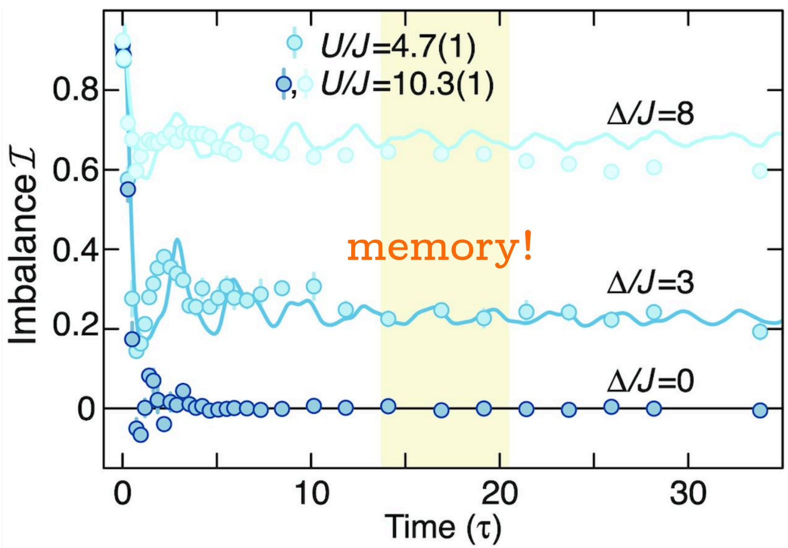
$$H_{\mathrm{MB}} = \sum_i \Delta\epsilon_i n_i + \sum_{\langle i,j\rangle} J\left(c_i^\dagger c_j + c_j^\dagger c_i\right) + U\, n_i n_j$$

(i) initialize system in a state $|\psi_0\rangle$ with well-defined local structure



(ii) Let it evolve with $U(t) = e^{-itH_{\mathrm{MB}}}$. Measure Imbalance $\mathrm{I} = \frac{2}{L}\sum_{i=1}^{L}(-1)^i \langle\psi_0|\, n_i(t)\, \psi_0\rangle$



Schreiber et al, Science 349 (2015)

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PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

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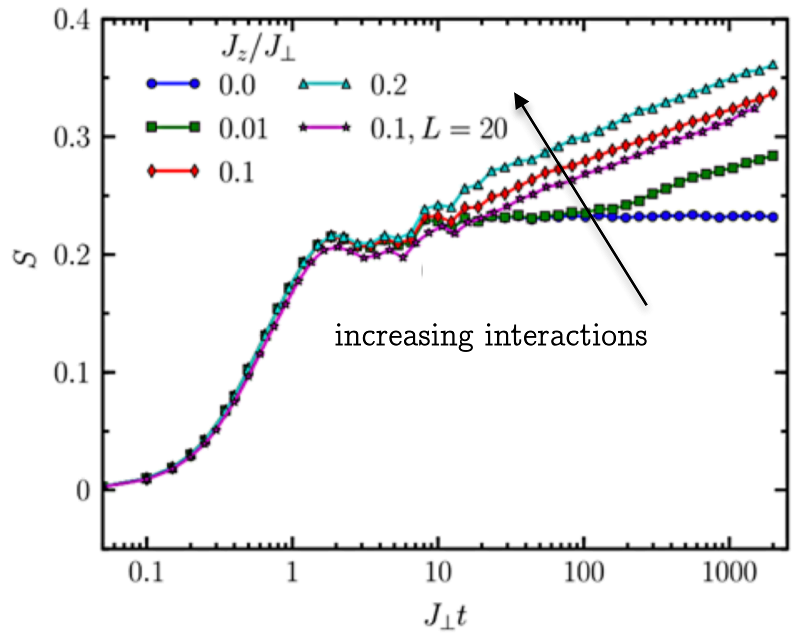
$$|\psi_0\rangle = |\sigma_1\rangle \otimes \cdots \otimes |\sigma_L\rangle \text{ product state}$$

Reduced Density Matrix of half system $\rho_1(t) = \mathrm{Tr}_2 \left\{ |\psi(t)\rangle\langle\psi(t)| \right\}$

Half-system entanglement entropy $S_{1/2}(t) = -\mathrm{Tr}\left\{\rho_1(t) \log \rho_1(t)\right\} \sim \log t$



Bardarson et al, PRL 109 (2012)

De Chiara et al, J.Stat Mech (2005)

Results obtained with tDMRG, based on Matrix Product State — M. C. Banuls lectures!

	In general:	In MBL:
TRANSPORT (cond.mat)	Conserved quantities (energy, particles, spin) transported ballistically or diffusively	Vanishing d.c dissipative part of conductivity
MEMORY (quantum info)	Local observables relax to stationary values that are independent of details of initial state	Local observables DO relax, but stationary values initial-state dependent
THERMALIZATION (stat.phys)	Thermal equilibrium emerges from unitary dynamics	Failure of conventional thermodynamic ensembles

[what]

Actually, what would “equilibrium” even mean?

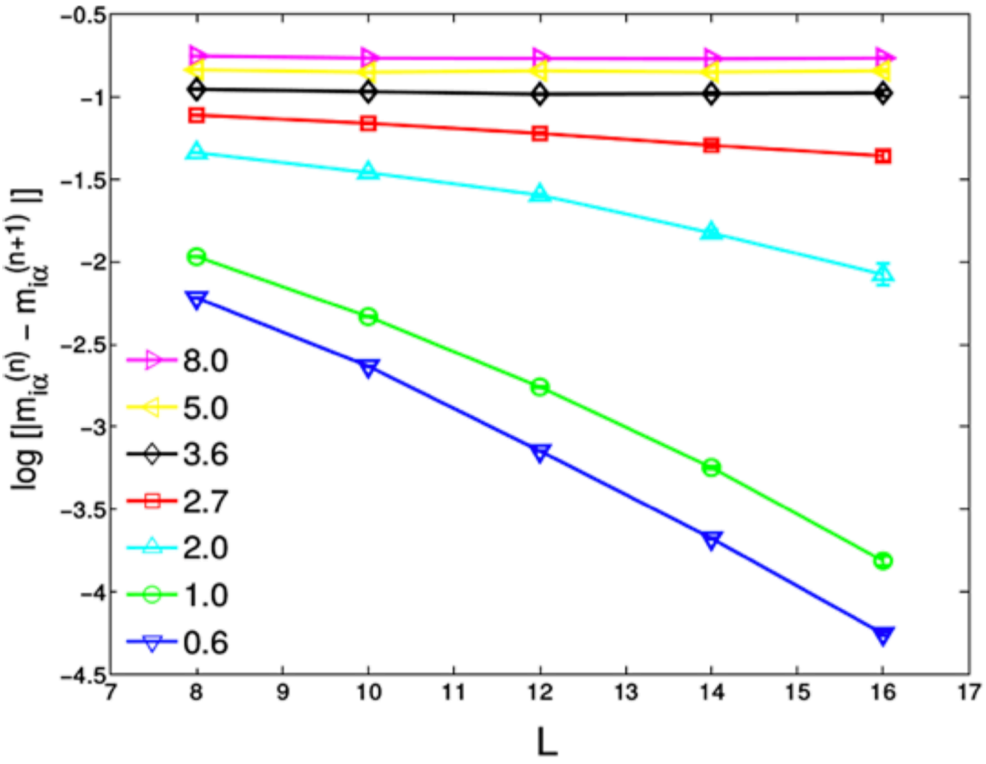
Breakdown of thermalization: local operators

Pal & Huse, PRB 82 (2010): 174411

$$H_{xxz} = \sum_{i=1}^{L} h_i \sigma_i^z + \sum_{i=1}^{L}\left[J_\perp\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \sigma_i^z \sigma_{i+1}^z\right]$$

$h_i \in \left[-h/2, h/2\right]$

$J_\perp = 1$



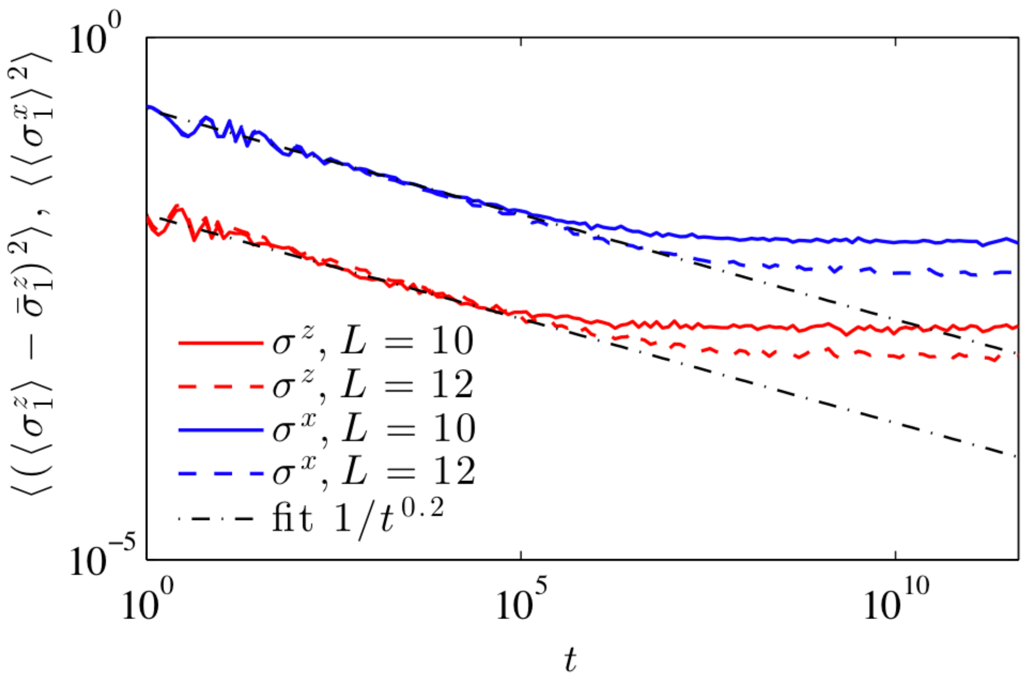
$$m_{i\alpha}(n) = \langle E_n | \sigma_i^z | E_n \rangle_\alpha$$

α disorder realization

Eigenstates in the same energy shell are locally distinguishable (no ETH)

Relaxation, though: local operators

Serbyn et al, PRB 90 (2014): 174302



$|\psi_0\rangle$ = random product state - σ_i^z basis

$$[\delta A(t)]^2 = \left(\langle A(t)\rangle - \overline{\langle A\rangle}\right)^2$$

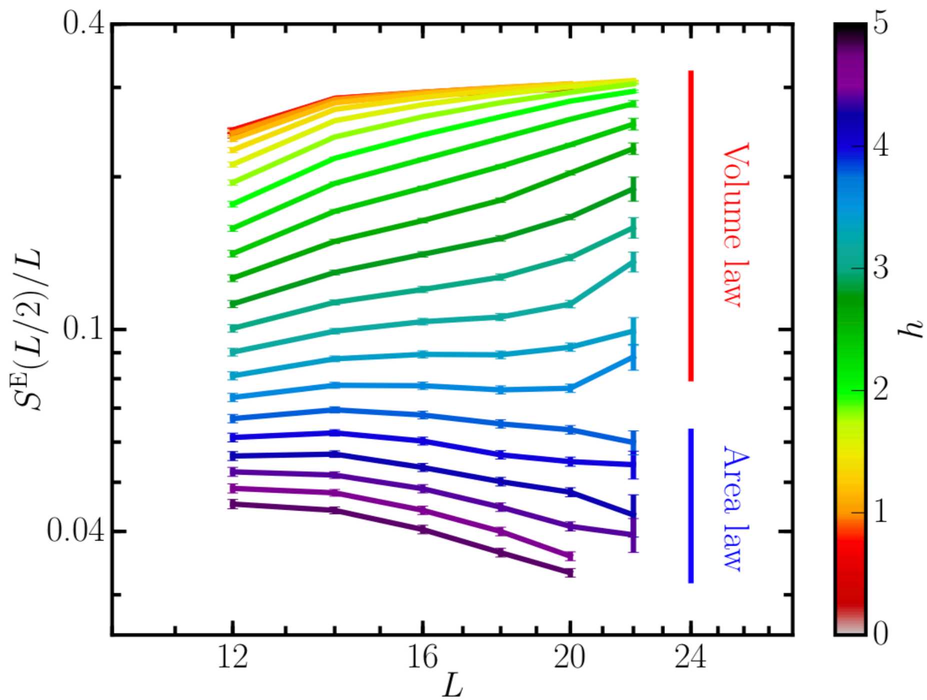
$$\sim 1/t^a$$

$$A = \sigma_1^z, \sigma_1^x$$

Local observables do equilibrate, slowly (power-law), to non-thermal values.

Breakdown of thermalization: entanglement

Luitz et al, PRB 91 (2015): 081103



Bipartite EE:

$$S_{1/2}(n) = -\,\mathrm{tr}\{\rho_1(n)\log\rho_1(n)\}$$

$$\rho_1(n) = \mathrm{tr}_2\{|E_n\rangle\langle E_n|\}$$

Volume-law: $S_{1/2} \propto L$

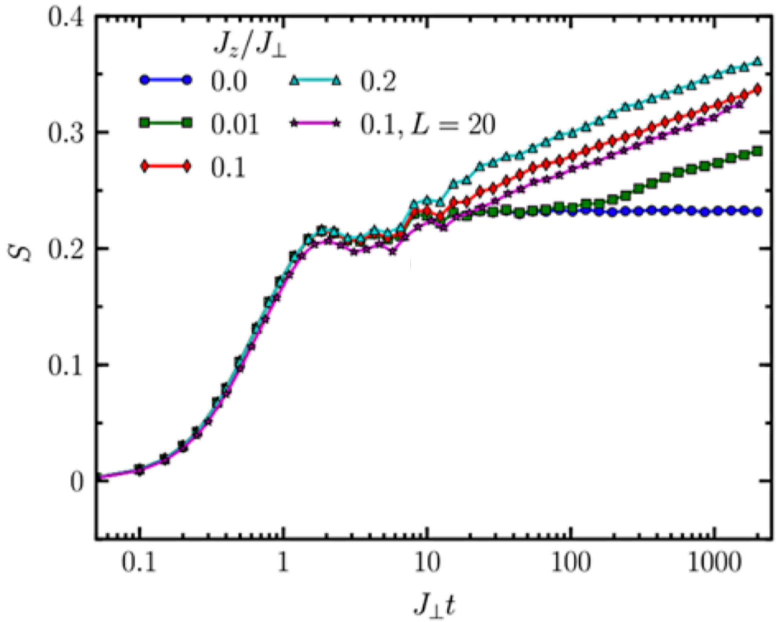
Area-law: $S_{1/2} \sim \mathcal{O}(1)$

Eigenstates have low, non-thermal entanglement:

<u>like ground states</u> of gapped Hamiltonians

Dephasing, though: entanglement

Bardarson et al, PRL 109 (2012): 017202



$|\psi_0\rangle =$ random product state

$$S_{1/2}(t) = -\mathrm{tr}\{\rho_1(t)\log\rho_1(t)\} \sim \log(t)$$

$$\rho_1(t) = \mathrm{tr}_2\{|\psi(t)\rangle\langle\psi(t)|\}$$

Entanglement Entropy growth is unbounded, but slow (no ballistic). Saturates to extensive sub-thermal value.

[How]

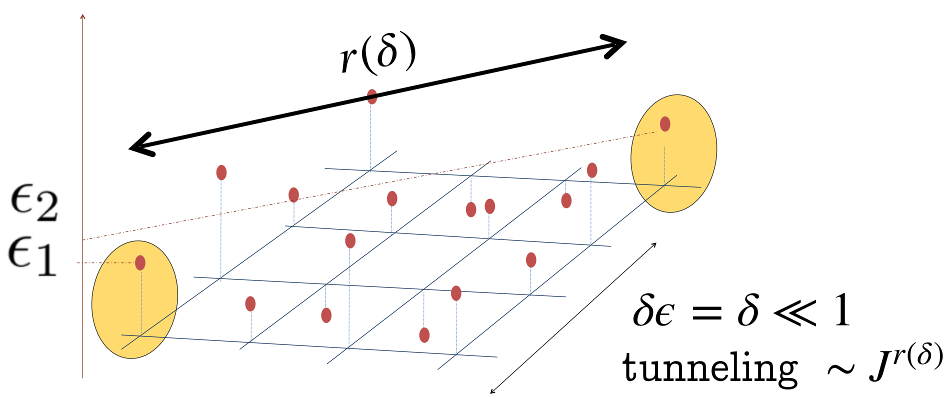
How disorder generates localization?

Can this survive in infinite lattice?

$$H = \sum_{i \in \Lambda} \epsilon_i n_i + J \sum_{\langle i,j \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right)$$

There will be resonances. But between nearby sites.

At large scales tunneling typically weaker than energy splitting.



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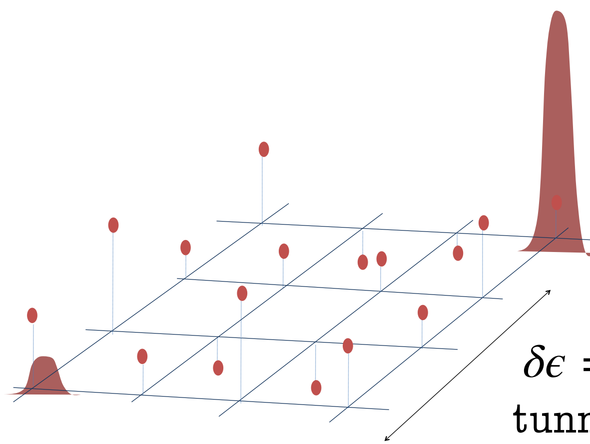

$\delta\epsilon = \delta \ll 1$

tunneling $\sim J^{r(\delta)}$

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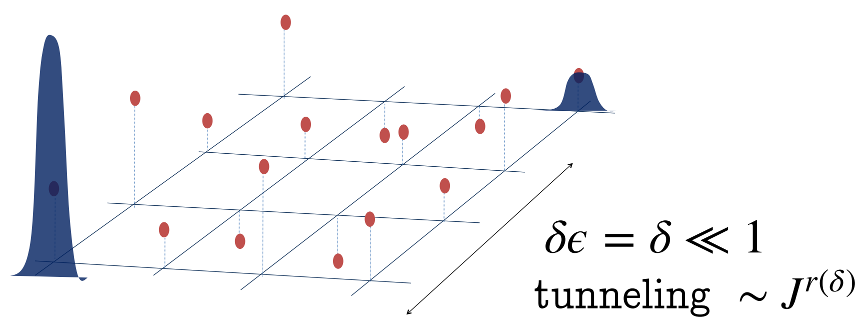

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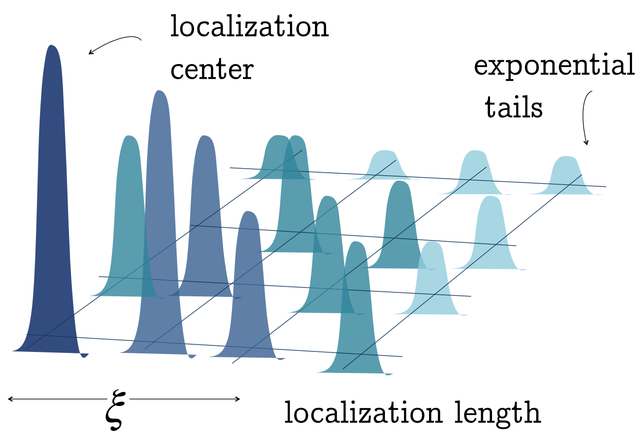


$\rightarrow$ exponentially localized eigenstates

[why]

Why challenging & why worth the effort?

Why challenging?

- **Highly-excited states**, "middle of band": no GS, no low-E effective (field) theories,...
- **No structure of clean integrability**: Yang-Baxter, Bethe Ansatz, GGE,...
- **No equilibrium tools**: replica formalism for $\mathscr{Z}_\beta$, free-energy landscapes...
- **Numerics at small sizes:** finite size effects, statistics over disorder needed....

Why worth? — a school-oriented selection

Quantum computing, quantum devices [→ Schoutens]

- Local quantum memory: density patterns (imbalance), *but also* relative phases
- Can we use the I_α as qubits?

A dynamical transition [→Banuls, Sachdev]

Restore:
diffusion, ETH in eigenstates, random matrix spectra, area law entanglement

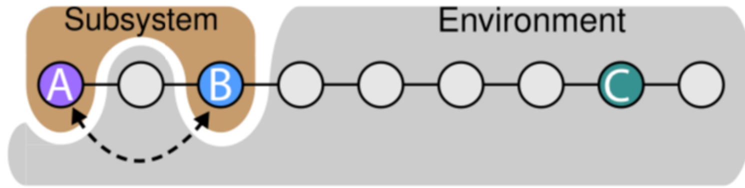
- what drives transition? Resonances, avalanches,...
- are there critical exponents?
- dipendence on dimension, energy? ...

Stabilize ordered phases [→Cooper, Sachdev]

- Circumvent equilibrium no-go theorems forbidding, e.g., long-range order in $1d$ (Peierls "energy *vs* entropy" arguments): localize the excitations that would destroy order.
- Example: Random Ising chain with interactions: "spin-glass" phase in eigenstates at E=0; (Majorana: edge modes)
- Which symmetries can be protected?

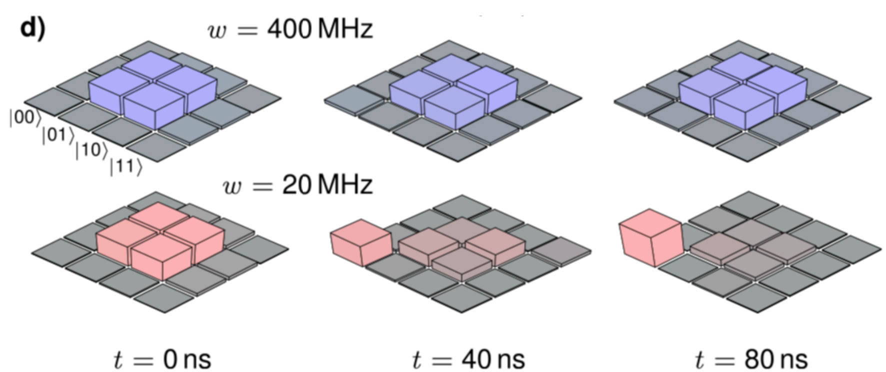
Measuring quantum coherence: an example

Chiaro et al, arXiv:1910.06024



$$|\psi_0\rangle = \left(\frac{|01\rangle+|10\rangle}{\sqrt{2}}\right)_{AB} \otimes |1\rangle_C \otimes |0, \ldots, 0\rangle_{\text{Other}}$$

Reduced density matrix: localized vs delocalized



[Coupled superconducting qbits]

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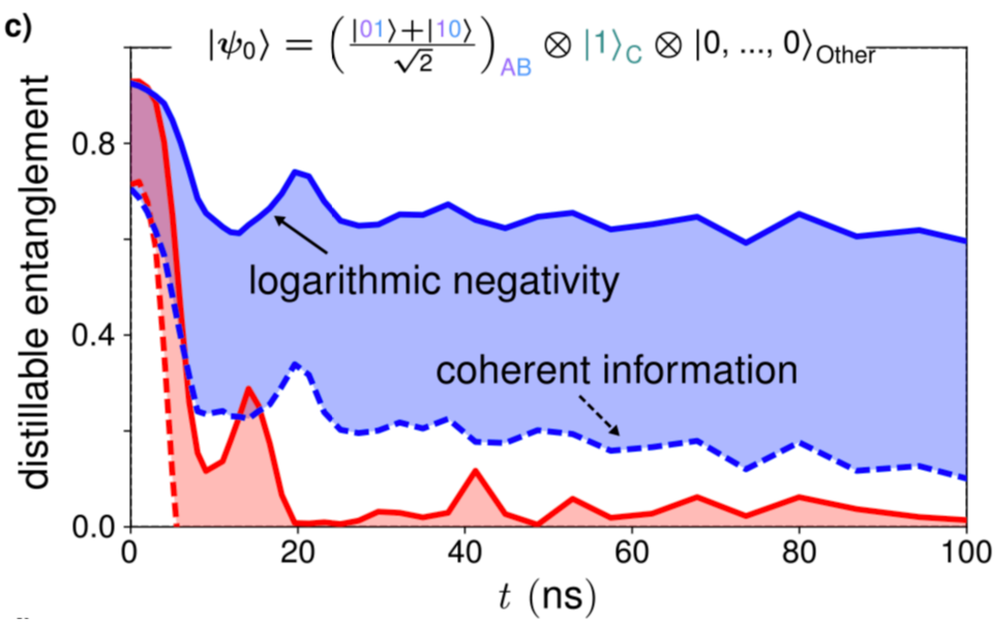

Monogamy of Entanglement: Two maximally entangled particles cannot be entangled with any others. But the exact meaning of entanglement, like matrimony, is a matter of definition.

$$\underbrace{E_N(\rho_2) = \log_2 \|\rho_{29}^{T_A}\|_1}_{\text{“ENTANGLEMENT NEGATIVITY”}} \geq \underbrace{E_D(\rho_{29})}_{\text{DISTILLABLE ENTANGLEMENT}} \geq \underbrace{S(\rho_{19}) - S(\rho_{29})}_{\text{“COHERENT INFORMATION ENTROPY”}}$$

[MBL in 5 questions]

Summary of Part I

What does out-of-equilibrium mean in MBL?

Suppression of transport at large scales, which implies persistent memory of initial conditions & failure of thermalization

What would equilibrium even mean, actually?

Observables in finite subsystems relax, towards thermal Gibbs-like value. ETH.
In MBL, relaxation towards non-thermal values

How disorder generates localization?

Resonances are local in space, do not proliferate at asymptotically large distances.
Battle of exponentials: energy denominators win.

How can we understand everything in a unified framework?

MBL systems are a peculiar type of integrable system: disorder-dependent, interacting conserved quantities with exponentially localized norm.

Why challenging & why worth the effort?

“More robust” and “more controllable” out-of-equilibrium phase: quantum computing, protecting order, dynamical phase transition.