Exercise set 2

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1. a) Derive the expression for the reduced density operator $\tilde{\rho}$ in terms of Kernels K.

   b) Assuming a factorizable initial condition, rewrite $\tilde{\rho}$.

2. Using a path integral representation for the Kernels K, derive the expression for the superpropagator and for the influence functional. What is their physicsl interpretation?

3. Show that the path integral of the k-th harmonic oscillator of the reservoir subject to a force $C_k x(t)$ is

$$K_{RI}^{(k))} = \left(\frac{m_k \omega_k}{2\pi i \hbar \sin(\omega_k t)} \right)^{1/2} \exp\left(\frac{i S_{cl}^{(k)}}{\hbar} \right)$$

   with $S_{cl}^{(k)}$ given by equation (71) of the script.

4. By assuming that at $t = 0$ the bath of harmonic oscillators is in thermal equilibrium at temperature T, show that

$$\rho_R(\boldsymbol{R}', \boldsymbol{Q}', 0) = \prod_k \rho_R^{(k)}(R'_k, Q'_k, 0) = \prod_k \left(\frac{m_k \omega_k}{2\pi\hbar \sinh(\hbar\omega_k/k_B T)} \right)^{1/2}$$
$$\times \exp\left\{ \frac{m_k \omega_k}{2\hbar \sinh(\hbar\omega_k/k_B T)} \left[(R_k'^2 + Q_k'^2)\cosh(\hbar\omega_k/k_B T) - 2R'_k Q'_k \right] \right\}$$

5. Perform the Gaussian path integrals in the influence functional to derive Eqs. 73-75 in the script.

6. a) Using the spectral function, you can transform the sum into an integral in $\alpha_R(\tau-\sigma)$ and $\alpha_I(\tau - \sigma)$. Discuss the meaning of the harmonic correction in the imaginary part of the exponent.

   b) Simplify the other integrals using the approximation

$$\frac{\sin \Omega(\tau - \sigma)}{\pi(\tau - \sigma)} \approx \delta(\tau - \sigma).$$

7. Discuss the problems concerning the inferior limit of integration $(t{-}0)$ and derive equation (78).

8. a) Perform a Wick rotation to derive the expression for the reduced density operator when the composite system is in thermal equilibrium at temperature T.

   b) Perform the path integrals and integrals to show that the reduced density operator finally reduces to Eqs. (87-90) in the script. Interpret the last term.

9. Discuss which would be the differences, if instead of considering a factorizable initial condition, you would have considered a more realistic case, i.e., the system is initially in thermal equilibrium with the bath, when you then perform a measure and disturb the system.