SFT 2024 - Lectures on Statistical Field Theories, GGI, Florence
Feb 05-16, 2024

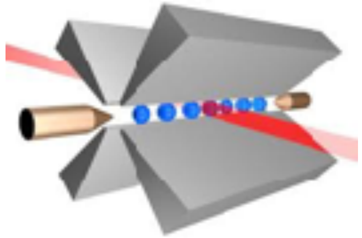
Lectures: Theoretical Quantum Optics

Motivation: Quantum Computing & Simulation with Atomic Platforms

Peter Zoller

350 universität innsbruck
IQI
PASQuanS
FWF Der Wissenschaftsfonds.
AFOSR MURI (JILA)
Q-SEnSE
Simons Collaboration on Ultra-Quantum Matter
AQT

Trapped ions



UIBK & IQOQI

Outline

Lectures 1-4: **Theoretical Quantum Optics**

Part I: Hamiltonian engineering & quantum optical toolbox

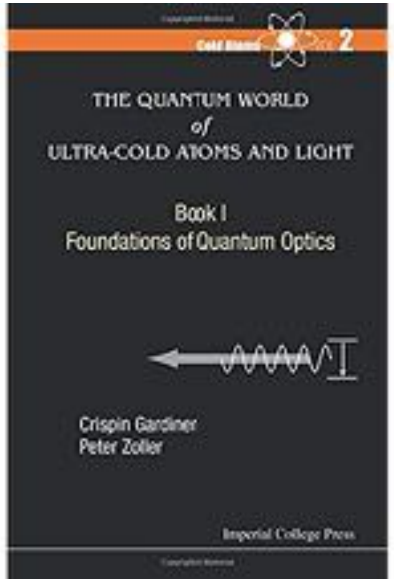
Part II: quantum noise & open quantum systems

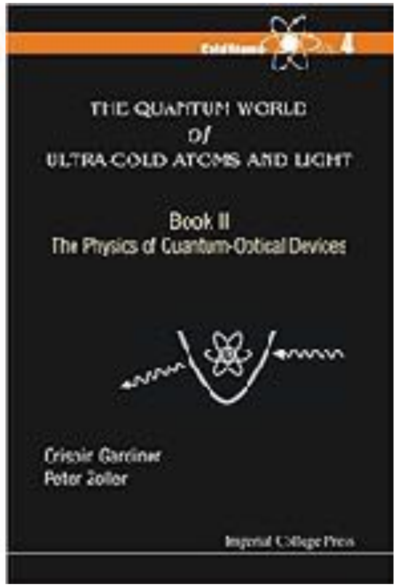
… basic concepts & minimal models

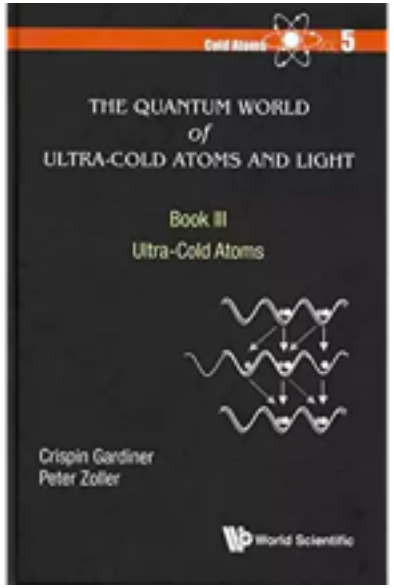
… how we "think" about quantum noise in quantum optics

Seminar: **Programmable Quantum Simulators with Atoms and Ions**

Literature



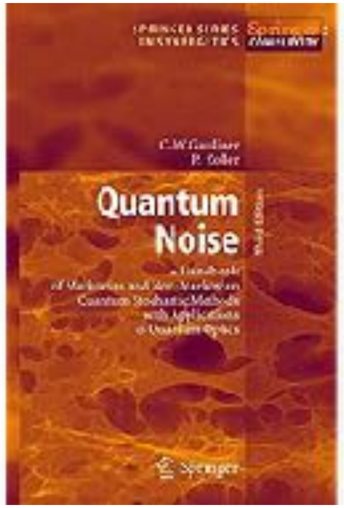




The Quantum World of Ultra-Cold Atoms and Light:

Book I: Foundations of Quantum Optics
Book II: The Physics of Quantum-Optical Devices
Book III: Ultra-cold Atoms

by Crispin W Gardiner and Peter Zoller



Quantum Noise

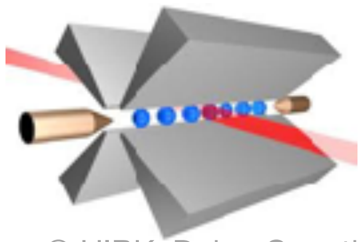
A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics

by Crispin W Gardiner and Peter Zoller

Motivation

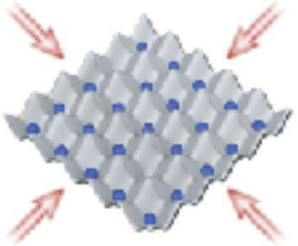
Engineered Quantum Many-Body Systems with Quantum Optical Systems

Trapped ions



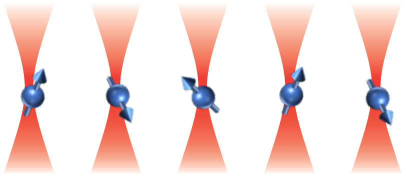
© UIBK, Duke, Quantinuum …

Optical Lattices



© MPQ, MIT, ETH,…

Rydberg Arrays



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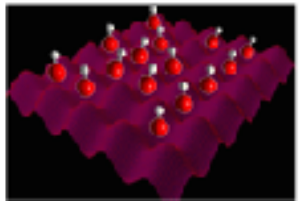
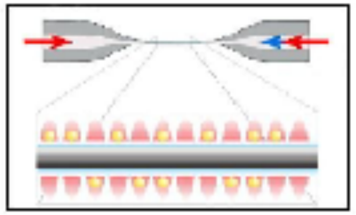
Polar Molecules



© JILA

CQED & Photonic



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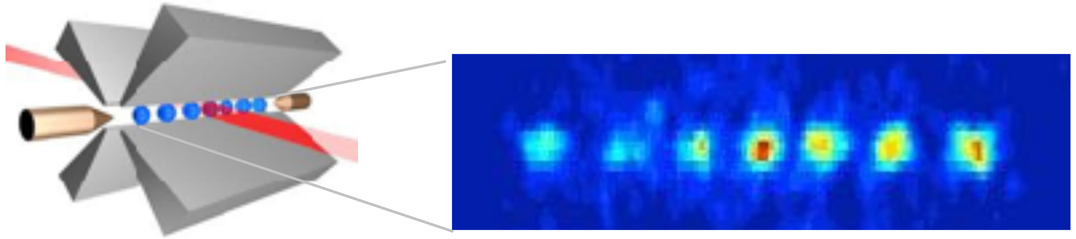
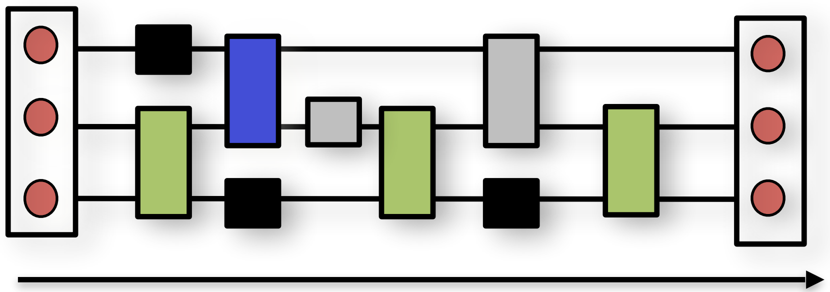
Quantum Computing [Digital]

trapped ions



theory: JI Cirac & PZ PRL 1995
exp.: UIBK, Duke, NIST, …;

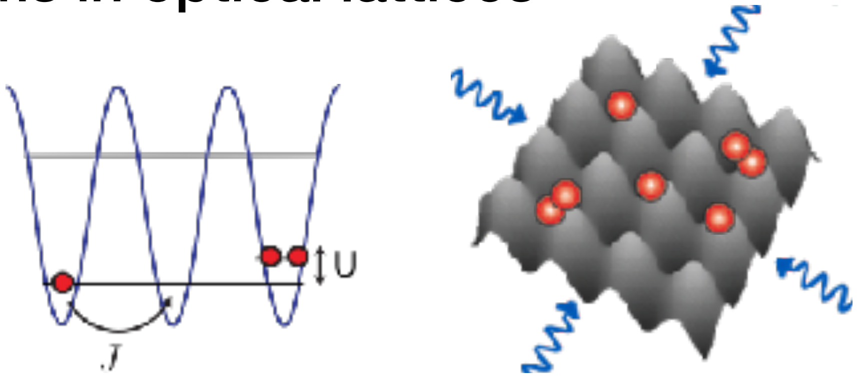
quantum logic network model



… demonstrating quantum algorithms

Quantum Simulation [Analog]

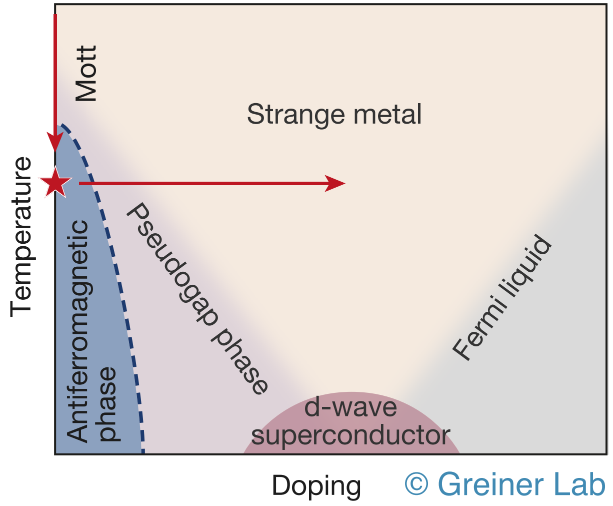
atoms in optical lattices



theory: Jaksch et al. PRL 1998

exp.: Munich, ETH,, Harvard, MIT, Hamburg, UIBK, Heidelberg …

(non-)equilibrium many-body physics



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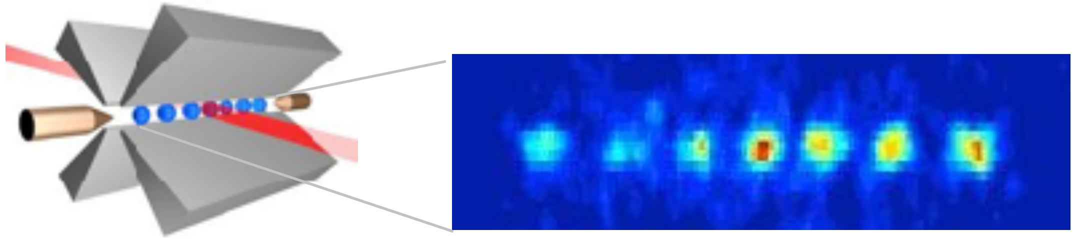
Fermi-Hubbard Model in 2D (high Tc)

… many-body quantum physics / cond mat

Quantum Computing [Digital]

trapped ions



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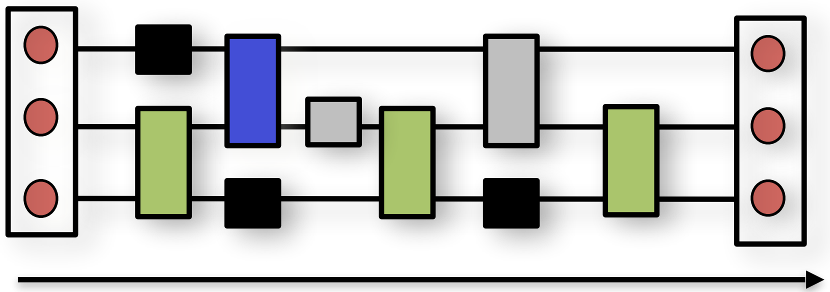


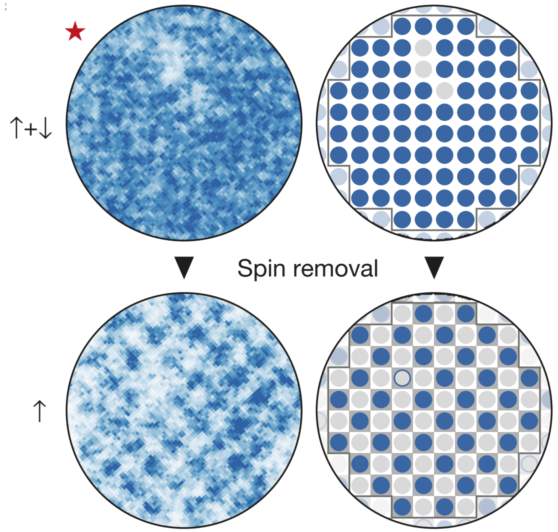
… demonstrating quantum algorithms

Quantum Simulation [Analog]

atoms in optical lattices

quantum gas
microscope

`seeing single atom in
a single shot'



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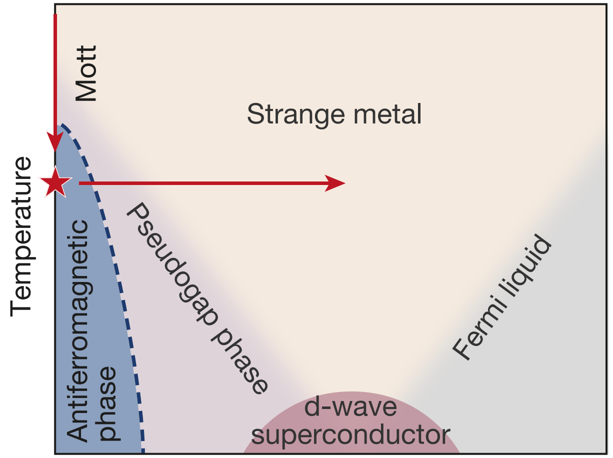

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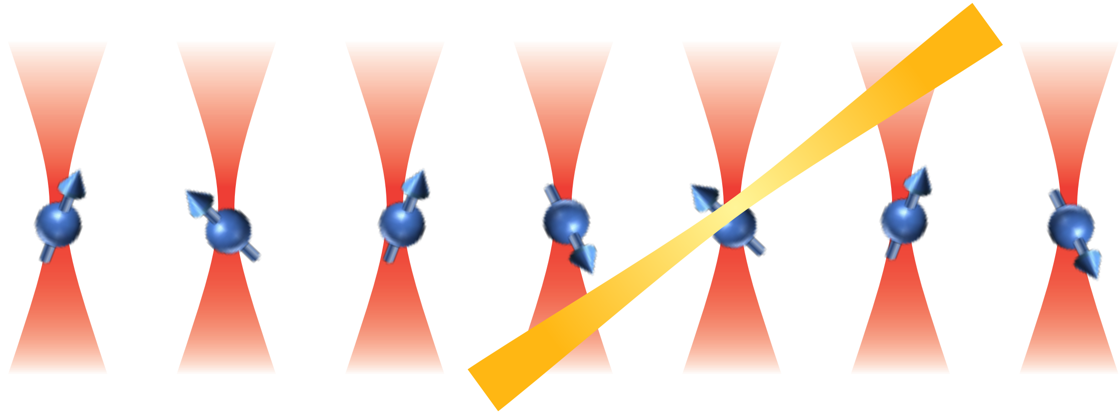
Fermi-Hubbard Model
in 2D (high Tc)

… many-body quantum physics / cond mat

Programmable Analog Quantum Simulators

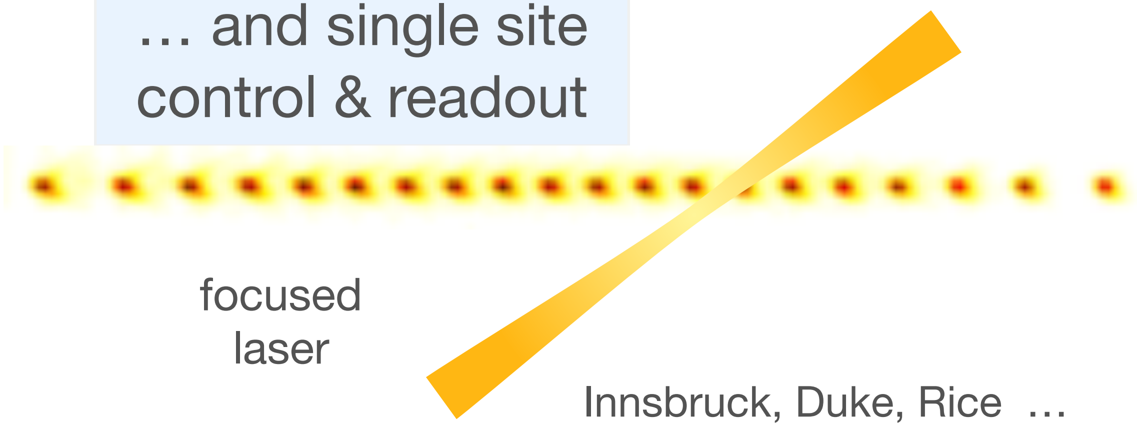
Rydberg Tweezer Arrays [1D,2D,3D]



Harvard - MIT, Palaiseau, JILA, Caltech, Wisconsin, Sandia, …

Trapped-Ions [1D, 2D]

… and single site control & readout



Innsbruck, Duke, Rice …

Engineered Spin Models & Hamiltonians

$$\hat{H} = \sum_i \frac{1}{2}\Omega_i \hat{\sigma}_x^i - \sum_i \Delta_i \hat{n}_i + \sum_{i<j} V_{ij}\hat{n}_i\hat{n}_j$$

$$V_{ij} = C_6 / r_{ij}^6$$

spin-spin interaction as Rydberg Van der Waals

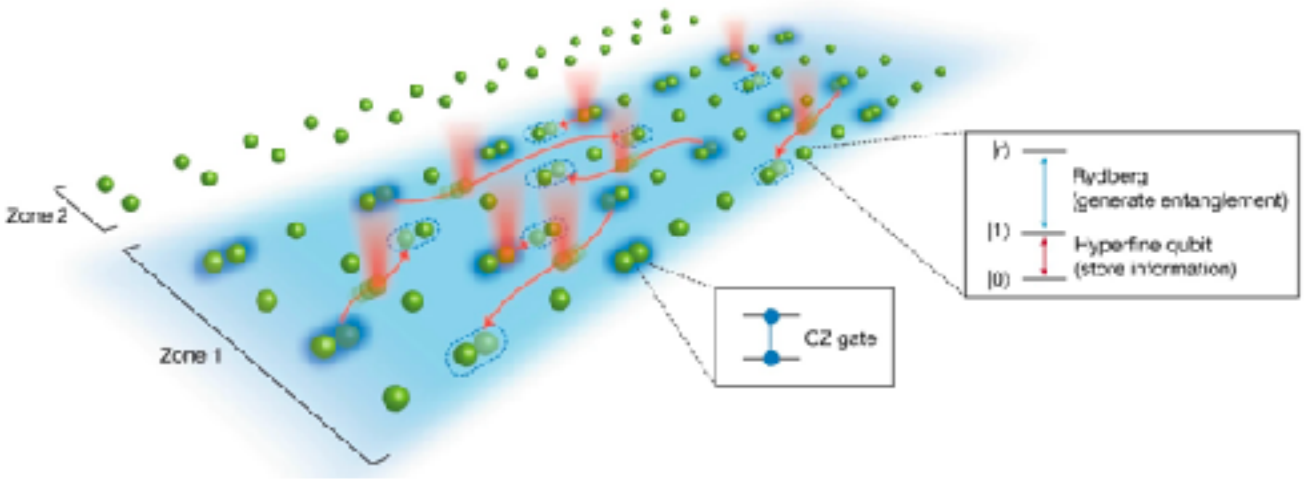
$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x + B\sum_i \hat{\sigma}_i^z$$

$$J_{ij} \sim \frac{1}{|i-j|^\alpha} \quad \alpha = 0\ldots3$$ long range

phonon-mediated spin-spin interaction

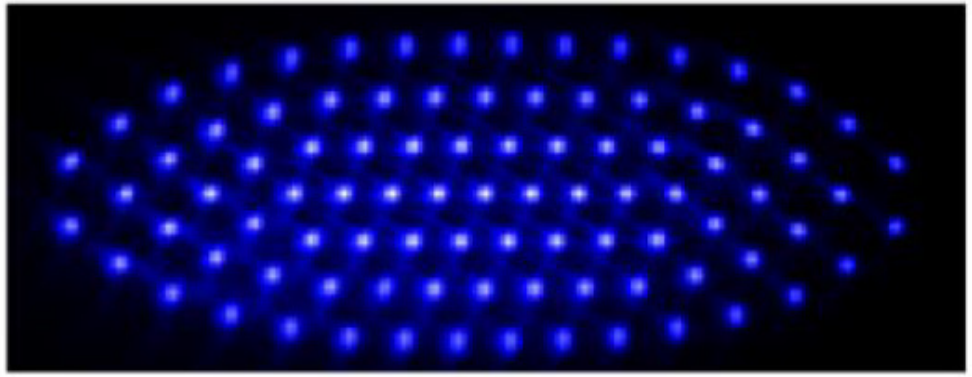
Programmable Analog Quantum Simulators

Rydberg Tweezer Arrays [1D,2D,3D]



D. Bluvstein et.al.,Nature 604, 451 (2022)
D. Bluvstein et.al., Nature Dec 6 (2023)

Trapped-Ions [2D]



Innsbruck ~200 ions, Tsinghua, ~1000 ions

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

AQT

AQT QUANTUM COMPUTER

INSIDE INDUSTRY-STANDARD 19” RACK

Performance:

- Single qubit gates
  - T_{1q} = 20µs
  - $Error_{1q} = 3.5 \cdot 10^{-4}$
- Two qubit gates
  - T_{2q} = 250µs
  - $Error_{2q} = 1.8(2) \cdot 10^{-2}$
- Memory
  - T1 = 1.14(6) s
  - T2 = 0.45(7) s
  - T2*=1.19(9) s (with spin echo)

All values measured in 8 qubit register

AQT DEMONSTRATED:

- 50+ ions
- 24-qubit entanglement
- Quantum volume 128
- Fault-tolerant performance
- Demo‘d Shor’s algorithm
- Demo’d finance applications
- Demo’d security applications
- Demo’d chemistry applications
- …

WITH OUR SYSTEM BEING:

- Rack-mounted
- Cloud-accessible
- Data-center compatible

QUANTINUUM

System Model H2: Accelerating your path to fault-tolerant quantum computing

A quantum revolution is on the horizon

Read the Announcement

99.997%

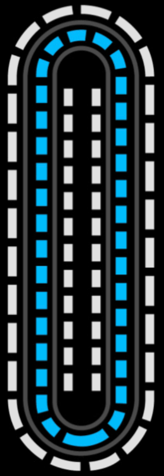
single-qubit gate fidelity

99.8%

two-qubit gate fidelity

- Highest commercially available two-qubit gate fidelity
- All-to-all connectivity
- Qubit reuse
- Mid-circuit measurement with conditional logic

H2
POWERED BY HONEYWELL



Entering a New Phase of Quantum Computing with our Second-generation System

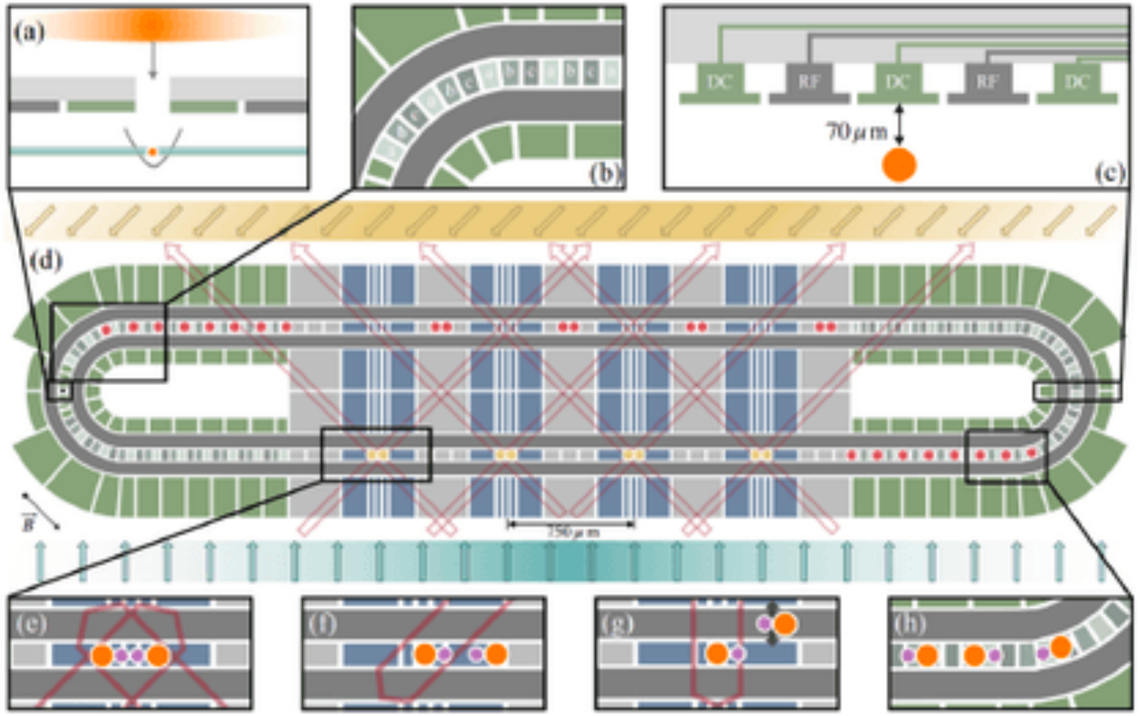
The System Model H2, Powered by Honeywell, is our latest generation of quantum computers with a new racetrack-shaped trap. Featuring 32 fully-connected qubits and an all-new architecture, Quantinuum's H2 provides a quantum volume of 65,536 (2^{16}) and the largest GHZ-state.

Quantinuum's System Model H2 includes numerous hallmark features that set it apart from other types of quantum computers, including:

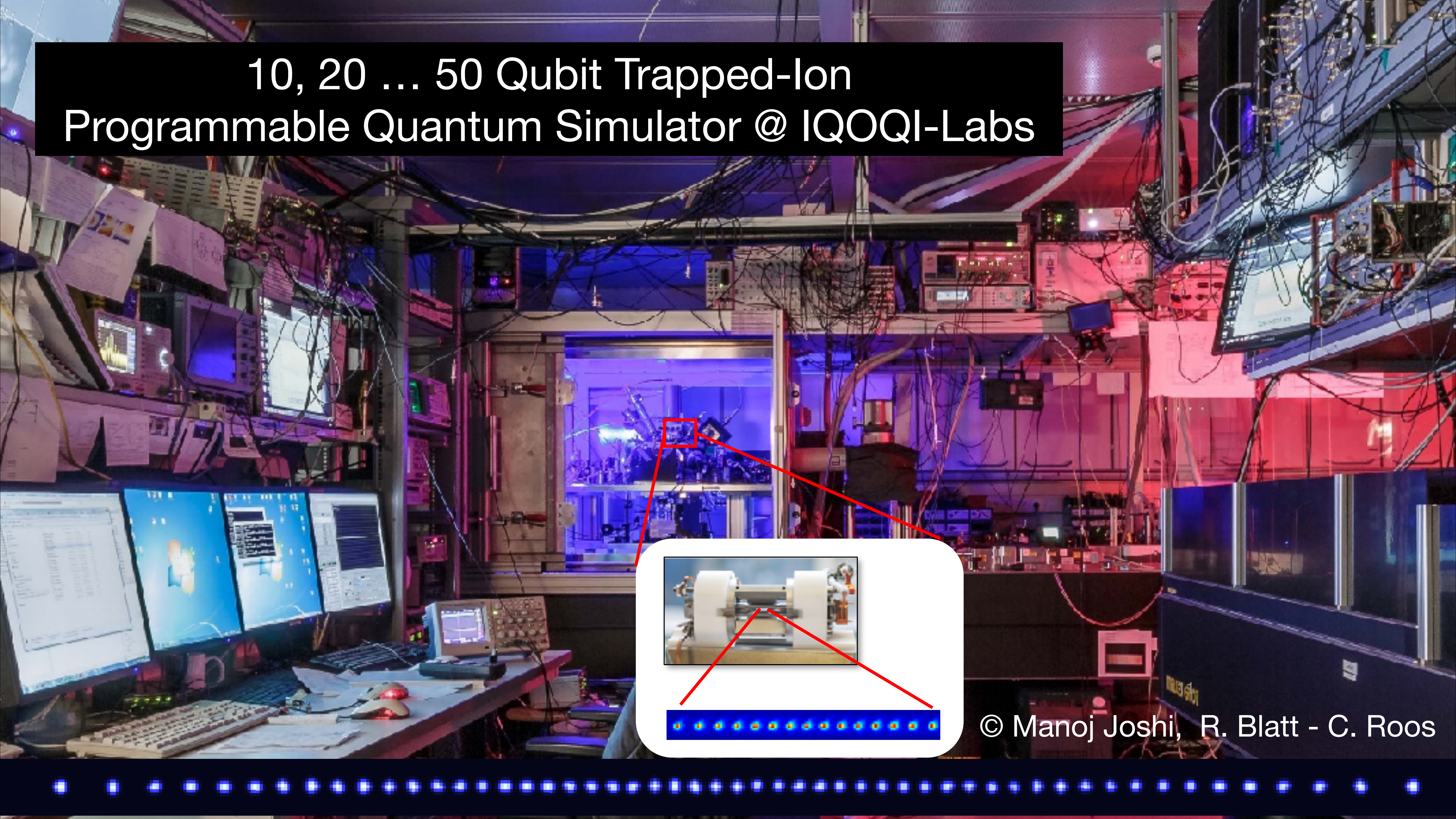
32

fully-connected qubits

65,536 (2^{16})



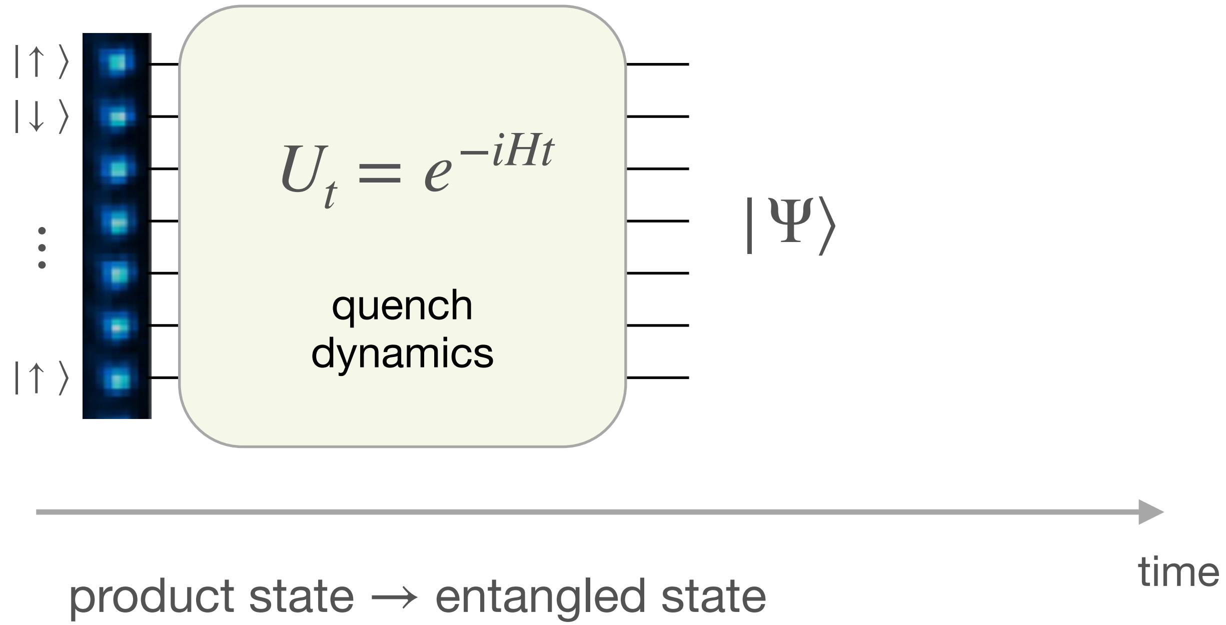
10, 20 … 50 Qubit Trapped-Ion
Programmable Quantum Simulator @ IQOQI-Labs



© Manoj Joshi, R. Blatt - C. Roos

Analog Quantum Simulators

What physics can we do … ?

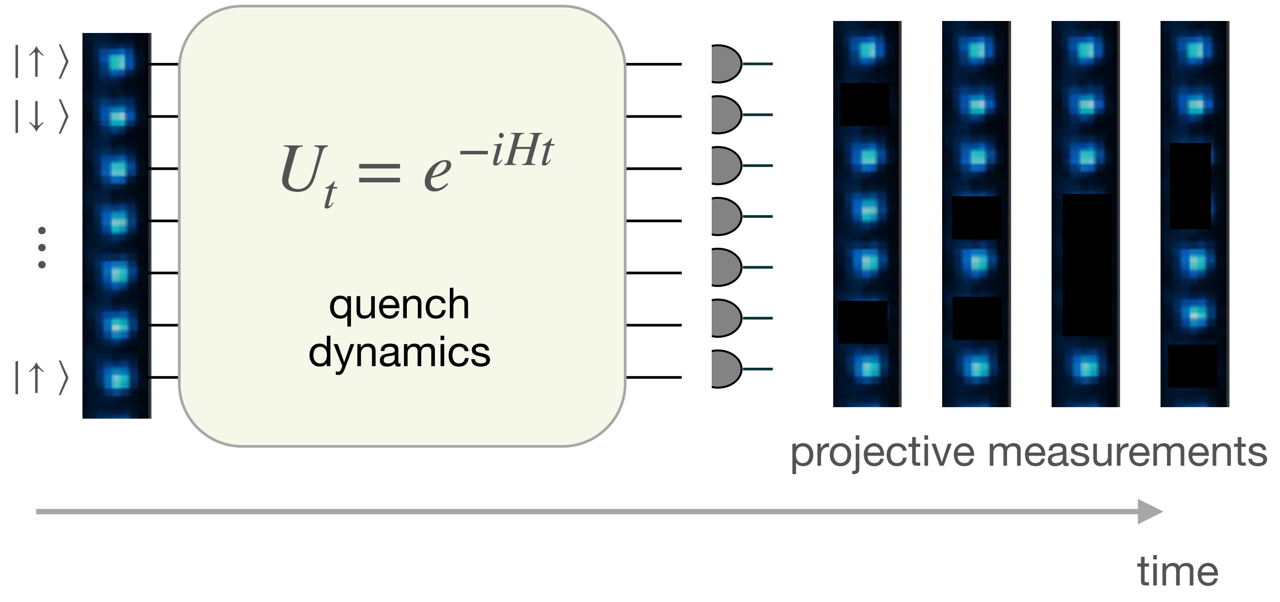


Native Hamiltonian

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

Analog Quantum Simulators

What physics can we do … ?



Repeat

Probability to see a specific configuration of bit string $\mathbf{s}$

$$P_\Psi(\mathbf{s})$$

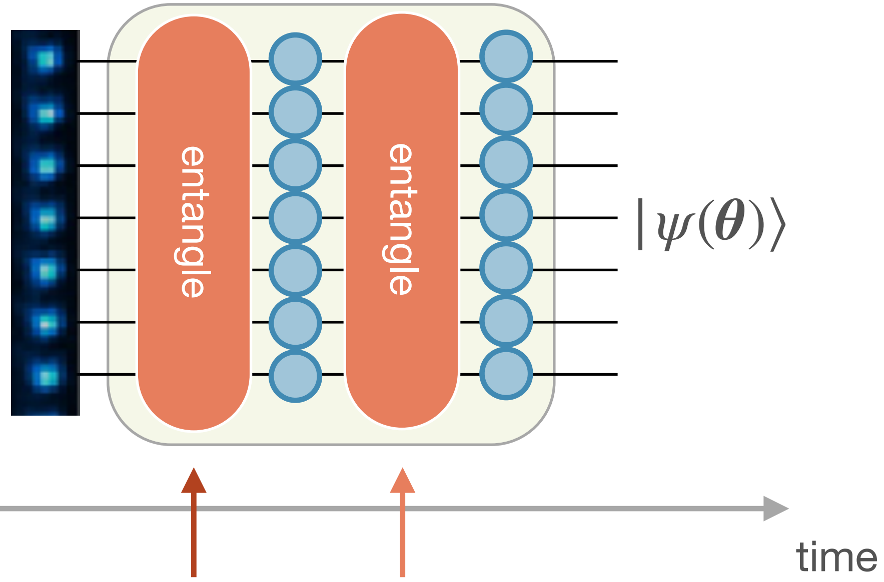
quantum
expectation values

Native Hamiltonian

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

`Programming' Quantum Simulators

programming quantum circuits



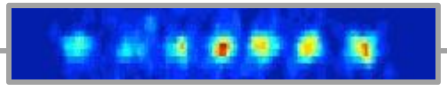
Native Hamiltonian

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

… as resource for high-fidelity N-body gate

family of entangled states

$$|\psi(\boldsymbol{\theta})\rangle = \hat{U}_N(\theta_N)\ldots\hat{U}_2(\theta_2)\hat{U}_1(\theta_1)|\psi_0\rangle$$



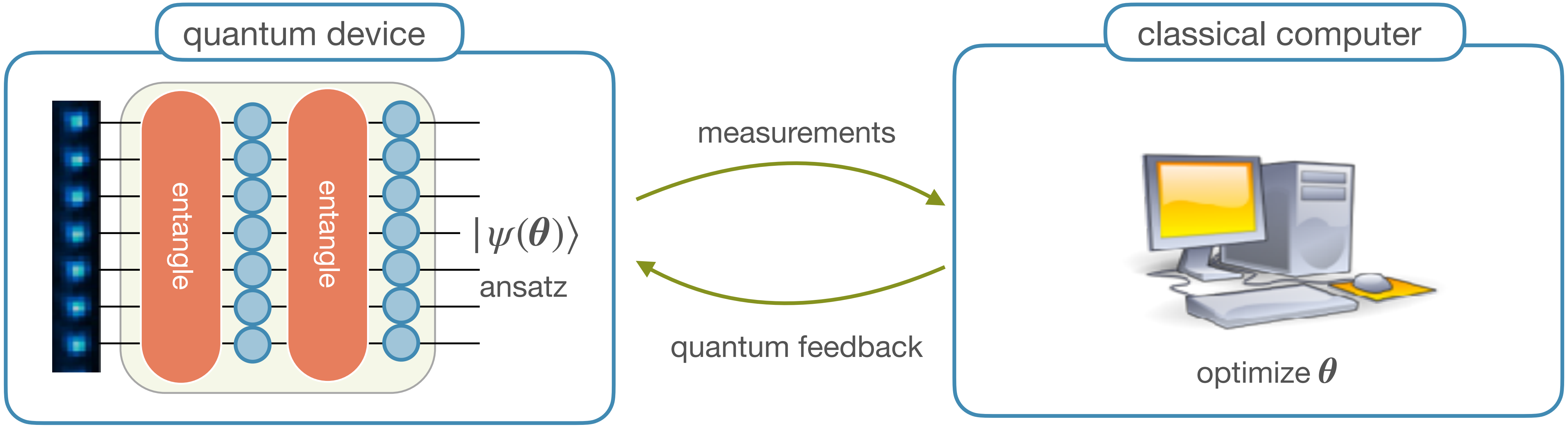
trapped ion quantum resources

$\hat{U}_1(\theta) = e^{-i\theta \sum_{ij} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x}$ entangle (Ising)

$\hat{U}_{2,i}(\theta) = e^{-i\theta \mathbf{n}\cdot\hat{\sigma}_i}$ local rotations

- in general not universal gate set
- scalable

`Programming’ Quantum Simulators



Variational Classical-Quantum Algorithms

target Hamiltonian (e.g. lattice model)

$$\hat{H}_T = \sum_{n\alpha} h_n^\alpha \hat{\sigma}_n^\alpha + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta + \dots$$

Variational Quantum Eigensolver (VQE)

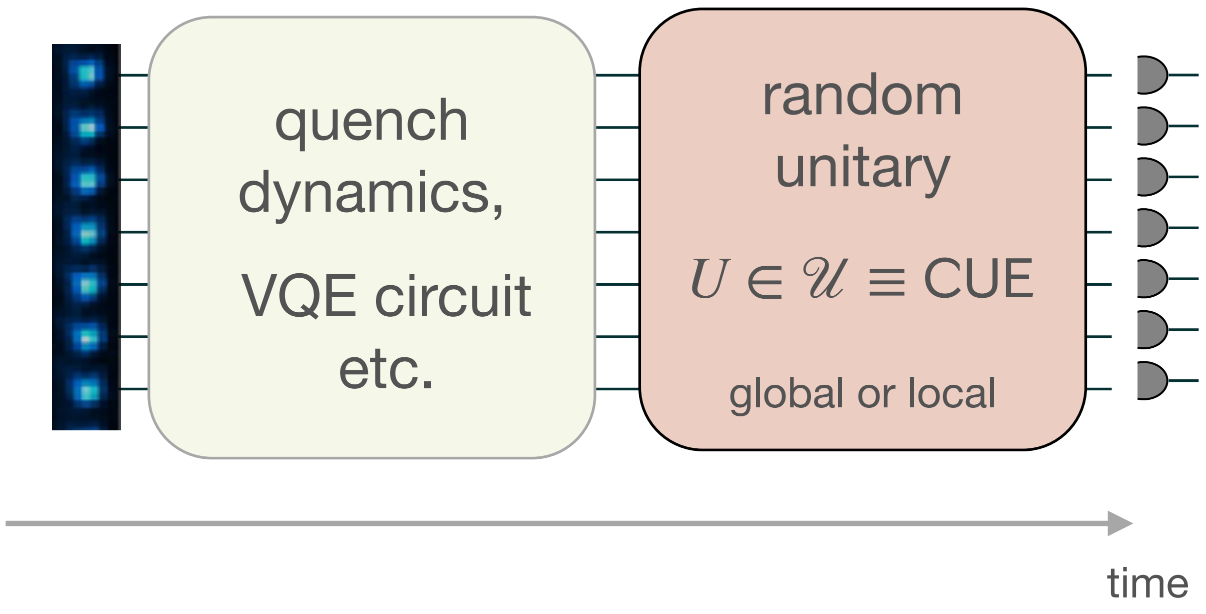
$$\text{Energy}(\theta) = \langle \psi(\boldsymbol{\theta}) | \hat{H}_T | \psi(\boldsymbol{\theta}) \rangle \to \text{min}$$

… computing ground states

QAOA, E Farhi, J Goldstone, S Gutman, arXiv:1411.4028,
Review: M Cerezo, A Arrasmith, R Babbush, SC Benjamin, S Endo, K Fujii, JR McClean, K Mitarai, X Yuan, L Cincio, PJ Coles, Nature Reviews Physics 3, 625 (2021)

Analog Quantum Simulation + Postprocessing

Measurement post-processing



Repeat

Probability for bitstring s

$$P_U(\mathbf{s})$$

is a random variable depending on U

(Cross-) Correlation of probabilities

$$\mathbb{E}_{U\sim\mathscr{U}}[P_U(\mathbf{s})P_U(\mathbf{s}')]$$

'Noise' or ensemble average (e.g. CUE)

experiment `day 1, lab 1'

experiment `day 2 lab 2'

2-design, …

… hybrid classical-quantum protocols

Part I: Engineered Many-Body Systems

Trapped-Ion Quantum Computing / [Simulation]

Optical Manipulation of Trapped Ions

Quantum Computing with Trapped Ions

1 Ions in a Linear Trap

2 Two-Qubit Quantum Gates

’95 gate, geometric gate, …

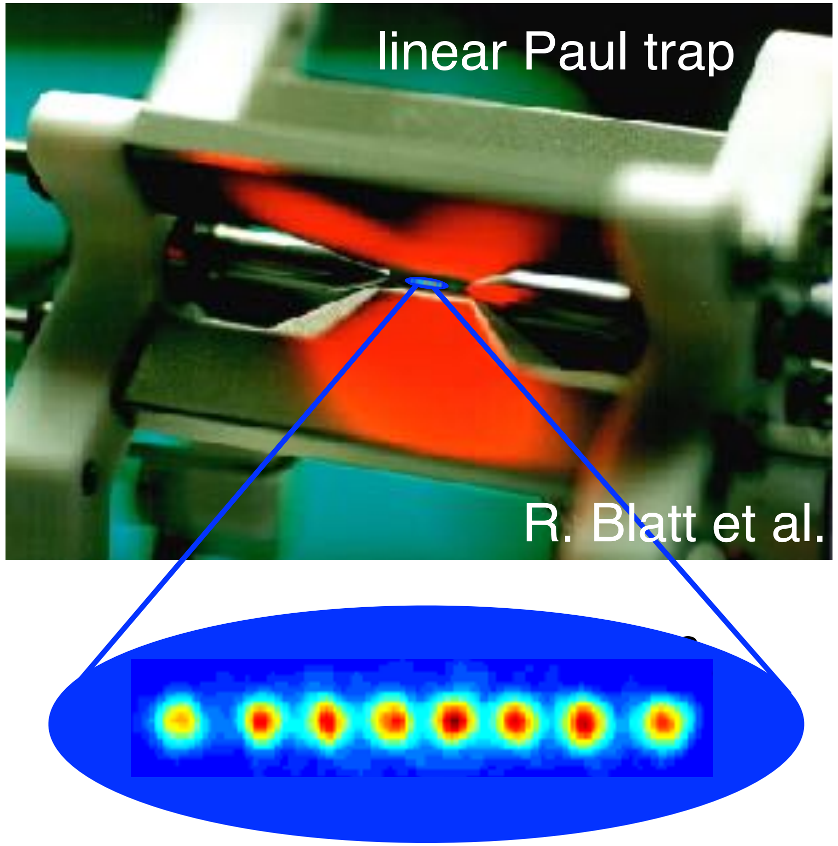
Quantum Simulation with Trapped Ions

1 Hamiltonian

Transverse Ising Hamiltonian

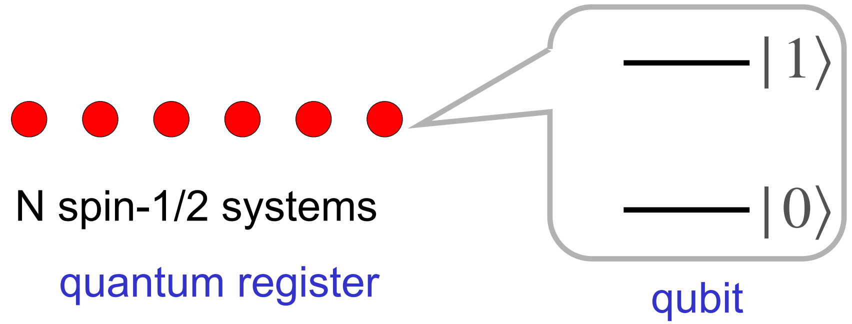
in class

extra reading material



Quantum Computing: what we want to implement …

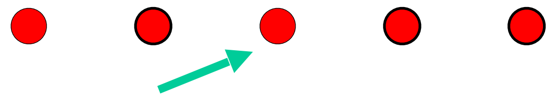
- quantum memory



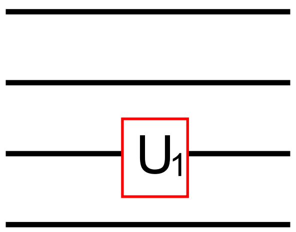
$$|\Psi\rangle = \sum_{x\in\{0,1\}^N} c_x |x_{N-1}x_{N-2}\ldots x_0\rangle$$

- quantum gates

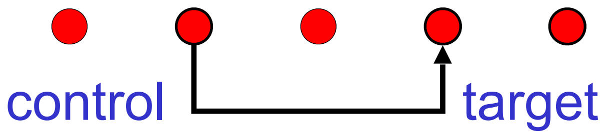
single qubit gate:



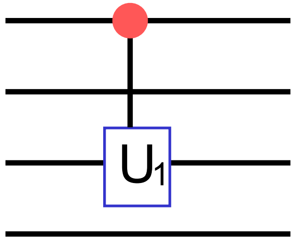
$\hat{U}_1$=rotation of a single qubit



two-qubit gate:



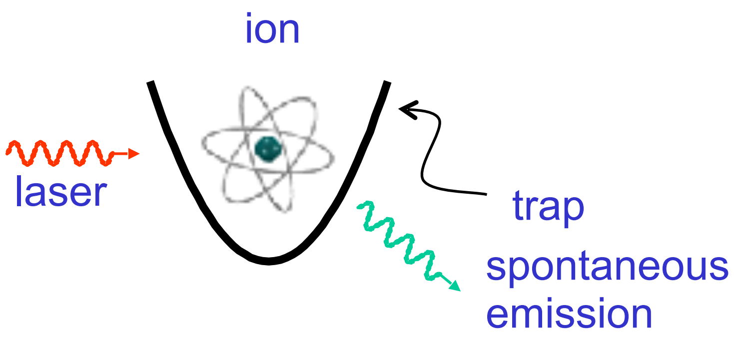
$$\hat{U} = |0\rangle_1\langle 0| \otimes \hat{1}_1 + |1\rangle_1\langle 1| \otimes \hat{U}_1$$



- read out
- [no decoherence]

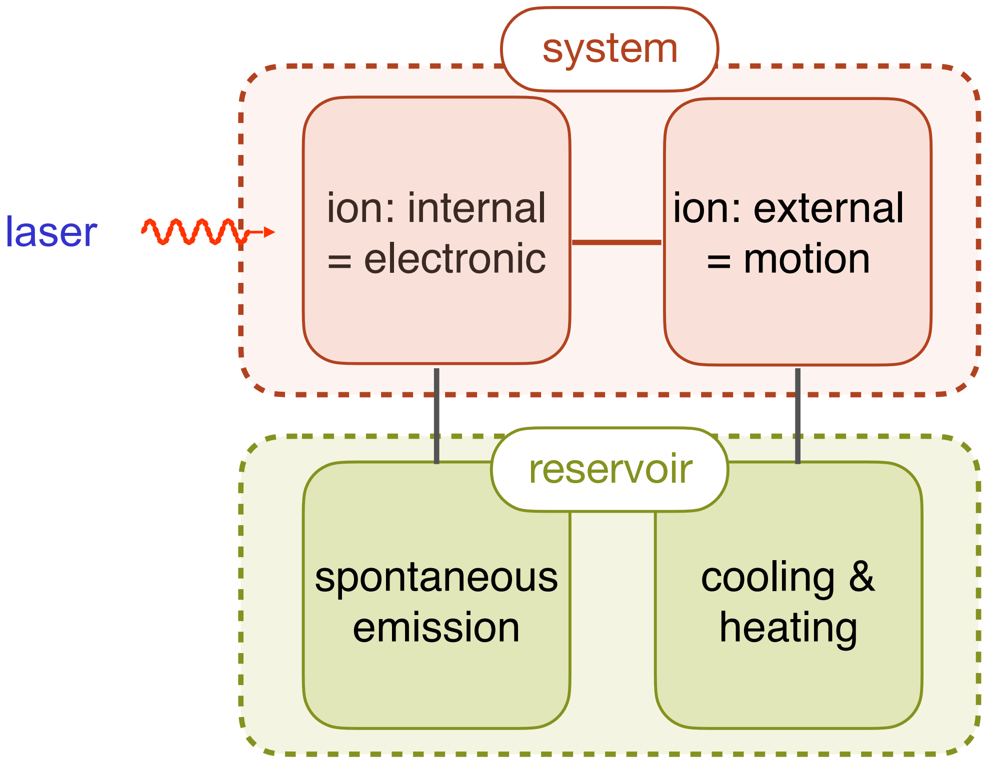
1. A single trapped ion

- a single laser driven ion in a trap



✓ system: atom + motion in trap:
goal: quantum engineering

✓ [open quantum system]



✓ control

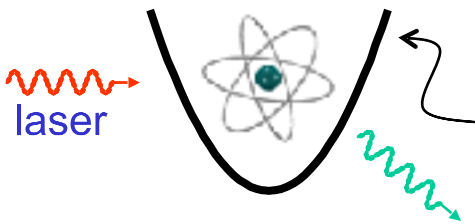
✓ decoherence
✓ preparation
✓ measurement

Development of the theory:

- system: Hamiltonian (control)
- reservoir: master equation & continuous measurement theory

1. A single trapped ion

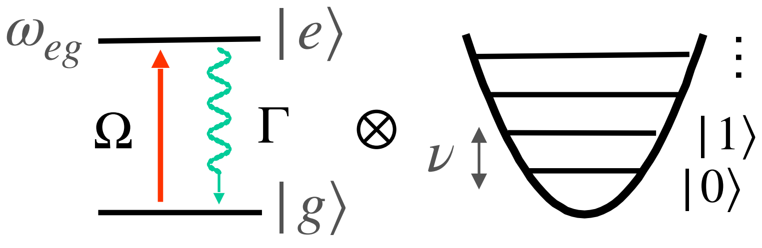
- a single laser driven ion in a trap



✓ system: atom + motion in trap:
goal: quantum engineering

✓ [open quantum system]

- model system: two-level atom + 1D harmonic oscillator



two-level system phonons

$$H = H_{0T} + H_{0A} + H_1$$

trap atom laser

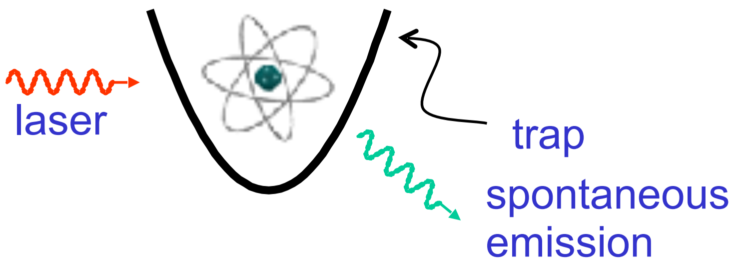
$$H_{0T} = \frac{\hat{P}^2}{2M} + \frac{1}{2}M\nu^2\hat{X}^2 \equiv \hbar\nu(a^\dagger a + \frac{1}{2})$$

$$H_{0A} = \hbar\omega_{eg}|e\rangle\langle e|$$

$$H_1 = -\boldsymbol{\mu}\cdot\boldsymbol{E}(\hat{X},t) \xrightarrow{\text{in RWA}} -\frac{1}{2}\hbar\Omega e^{ik_L\hat{X}-i\omega t}|e\rangle\langle g| + \text{h.c.}$$

1. A single trapped ion

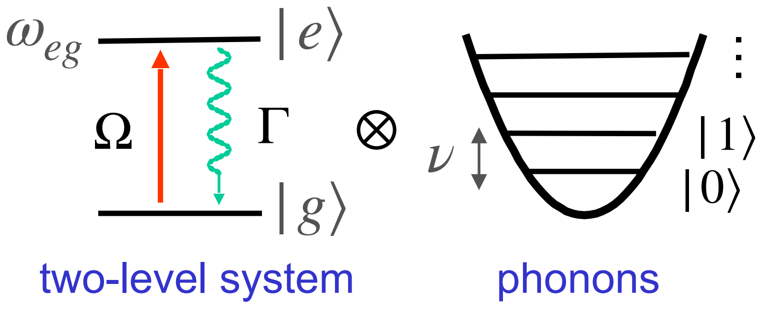
- a single laser driven ion in a trap



✓ system: atom + motion in trap:
goal: quantum engineering

✓ [open quantum system]

- model system: two-level atom + 1D harmonic oscillator



internal + external degrees-of-freedom

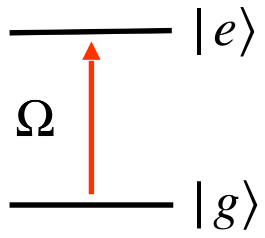
$$\tilde{H} = \frac{\hat{P}^2}{2M} + \frac{1}{2}M\nu^2\hat{X}^2 - \hbar\Delta|e\rangle\langle e| - \frac{1}{2}\hbar\Omega e^{ik_L\hat{X}}|e\rangle\langle g| + \text{h.c.}$$

harmonic trap | two-level atom | atom-laser interaction

in `rotating frame'

coherent control parameters: $\{\Delta \equiv \omega - \omega_{eg}, \Omega, \nu\} \overset{\text{RWA}}{\ll} \omega_{eg}, \omega$

- **laser absorption and recoil**



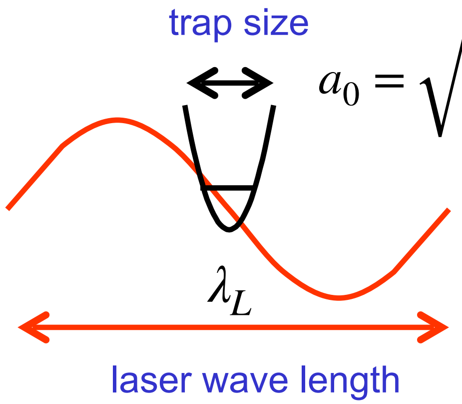
$$|g\rangle| \otimes |\text{motion}\rangle \xrightarrow{\tilde{H}_1} |e\rangle \otimes e^{ik_L\hat{X}}|\text{motion}\rangle$$

photon recoil kick

interaction $$\tilde{H}_1 = -\frac{1}{2}\hbar\Omega e^{ik_L\hat{X}}|e\rangle\langle g| + \text{h.c.}$$

laser photon recoil:
couples internal dynamics and center-of-mass

- **Lamb-Dicke limit**



$$a_0 = \sqrt{\frac{\hbar}{2M\nu}}$$

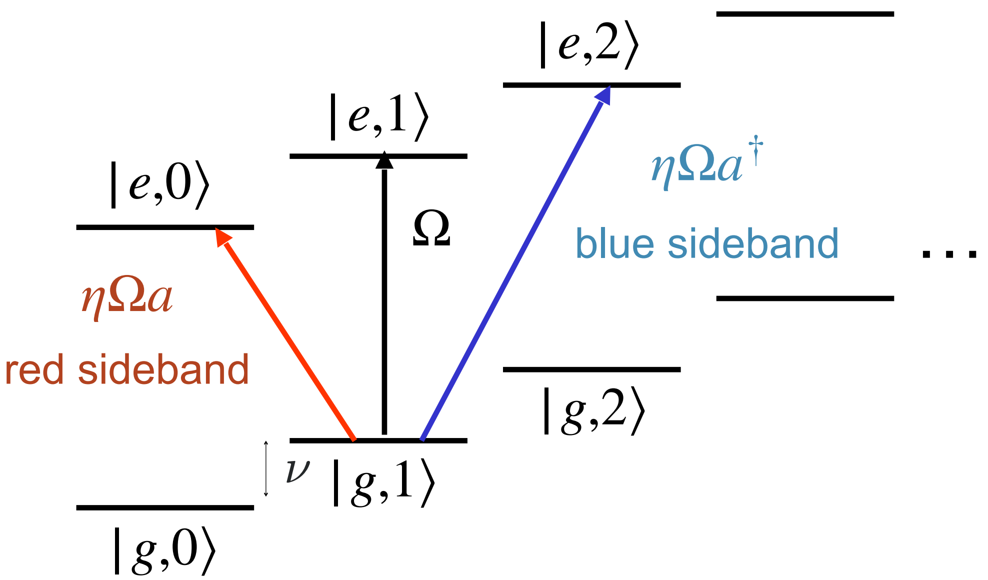
Lamb-Dicke expansion

$$e^{ik_L\hat{X}} = e^{i\eta(a^\dagger + a)}$$
$$= 1 + i\eta(a^\dagger + a) + \ldots$$

$$\eta = 2\pi\frac{a_0}{\lambda_L} \equiv \sqrt{\frac{\epsilon_R}{\hbar\nu}} \qquad \sim 0.1$$

Lamb-Dicke parameter

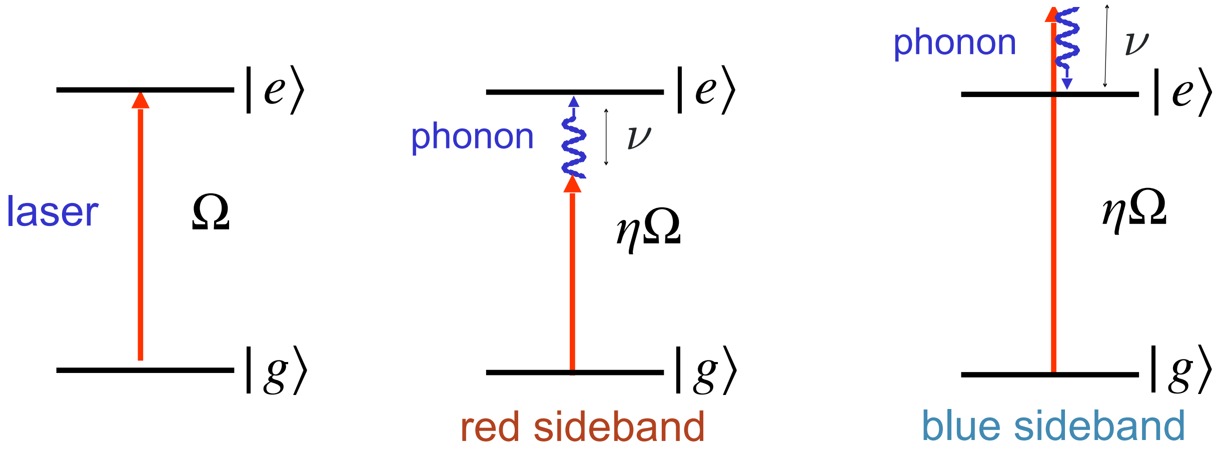
- spectroscopy: atom + trap



laser interaction

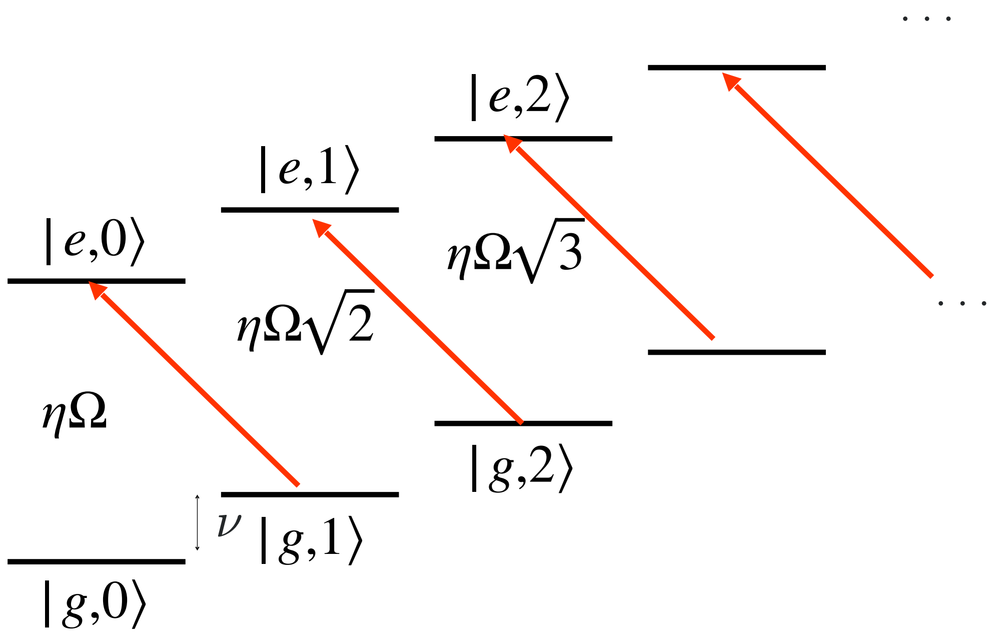
$$\frac{1}{2}\Omega e^{ik_L\hat{X}}|e\rangle\langle g| = \frac{1}{2}\Omega|e\rangle\langle g| + i\frac{1}{2}\Omega\eta a|e\rangle\langle g| + i\frac{1}{2}\Omega\eta a^\dagger|e\rangle\langle g| \ldots$$

- processes: “Hamiltonian toolbox for phonon state engineering”



laser assisted phonon absorption and emission

- example: "laser tuned to red sideband"



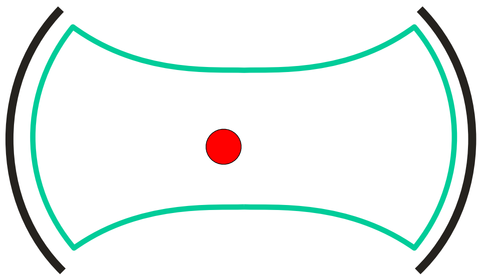
Jaynes-Cummings model

$$H = \hbar\nu a^\dagger a - \hbar\Delta |e\rangle\langle e| - \frac{1}{2}\hbar i \eta\Omega |e\rangle\langle g| a + \text{h.c.}$$

trap (↑ $\hbar\nu a^\dagger a$)

vacuum Rabi frequency ~ laser (switchable) (↑ $\eta\Omega$)

- Remark: Cavity QED



$$H_{\text{JC}} = \hbar\nu a^\dagger a + \hbar\omega_{eg} |e\rangle\langle e| - i\hbar g |e\rangle\langle g| a + \text{h.c.}$$

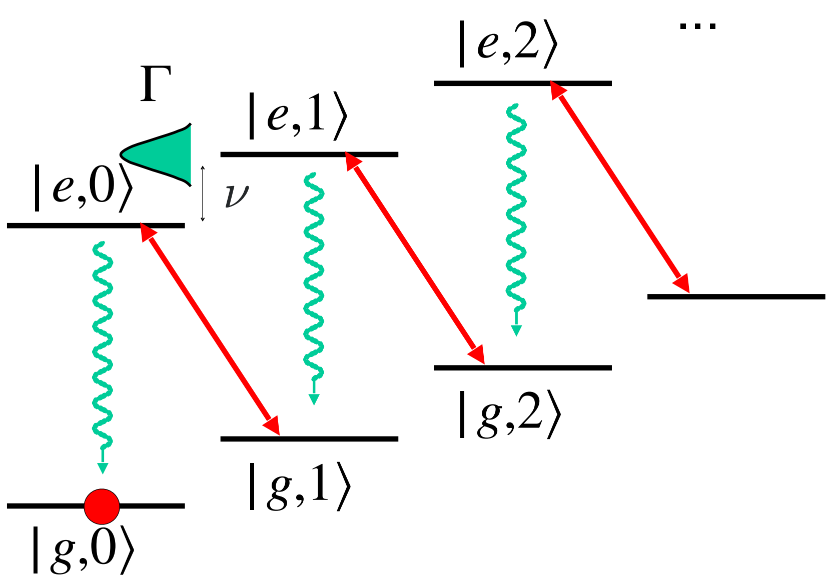
optical (↑ $\hbar\nu a^\dagger a$)

vacuum Rabi frequency (↑ g)

$\sim 1/\sqrt{\text{cavity volume}}$

[Dissipation: spontaneous emission]

- sideband cooling … as optical pumping to ground state



preparation of pure states

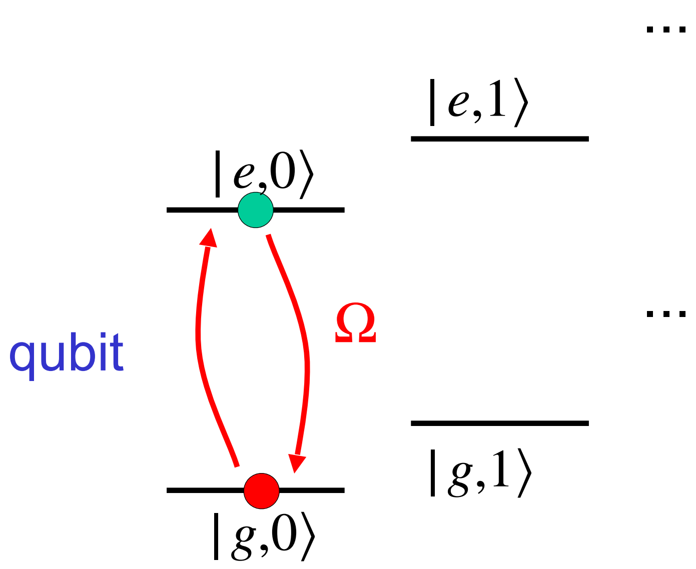
$$\rho_{\text{atom}} \otimes \rho_{\text{motion}} \to |g\rangle\langle g| \otimes |0\rangle\langle 0|$$

- measurement of internal states: quantum jumps …

qubit read out

Exercises in quantum state engineering

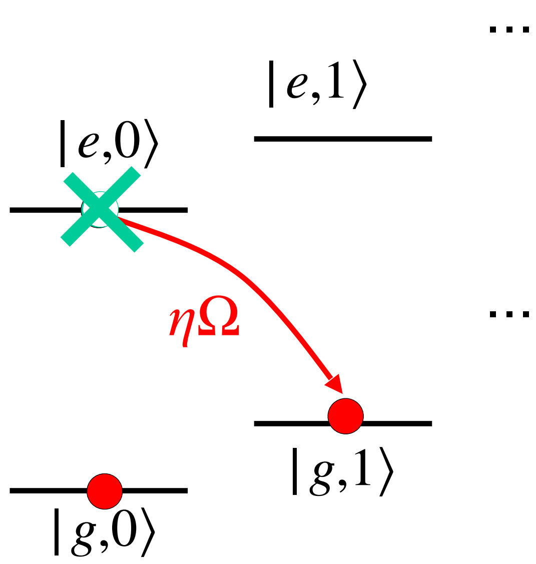
- Example 1: single qubit rotation



$$(\alpha|g\rangle + \beta|e\rangle \otimes |0\rangle \xrightarrow{\hat{U}_1} (\alpha'|g\rangle + \beta'|e\rangle \otimes |0\rangle$$

(1) we can rotate the qubit without touching the phonon state

- Example 2: swapping qubit to phonon mode



$$(\alpha|g\rangle + \beta|e\rangle \otimes |0\rangle \rightarrow |g\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$$

ion qubit phonon qubit

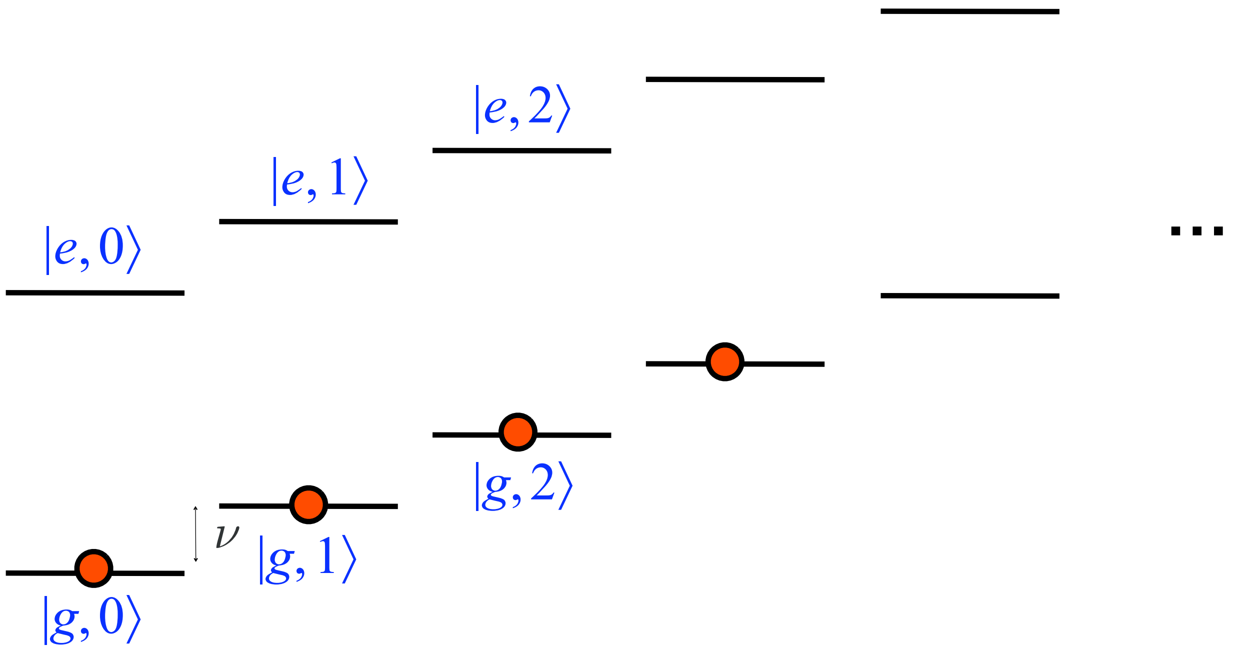
(2) Using a laser pulse we can swap qubits stored in ions to the phonon modes (and vice versa)

- **Example 3 [Exercise]**

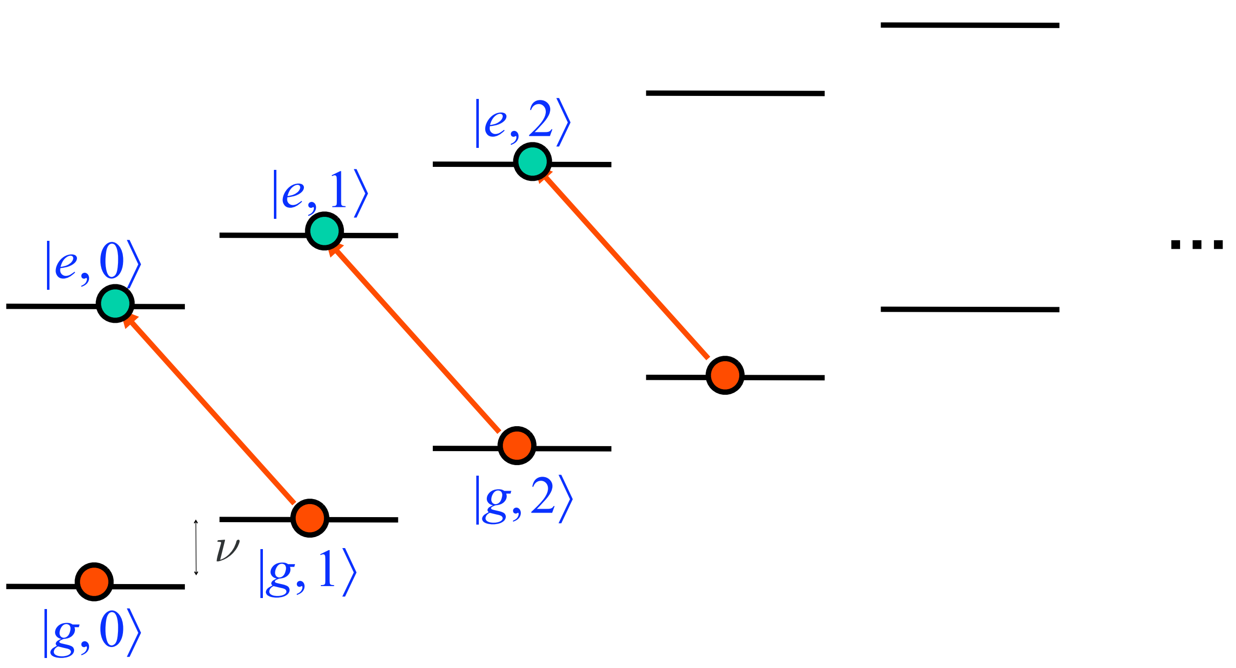
we can engineer an arbitrary superposition state of phonon states

$$|g\rangle \otimes |0\rangle \rightarrow |\Psi\rangle = |g\rangle \otimes \sum_{n=0}^{N} c_n |n\rangle$$

for given coefficients c_n.

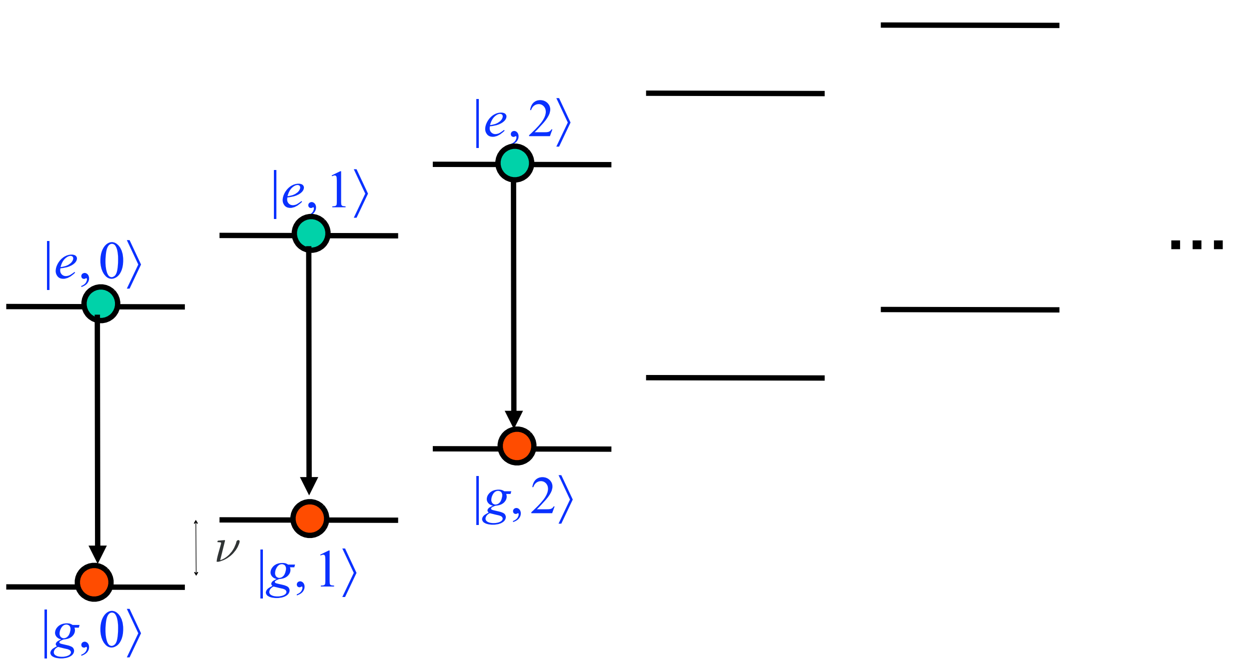


Idea: let us first consider the inverse of the problem - given the above superposition state we can want to find unitary transformations to obtain $|g\rangle \otimes |0\rangle$.

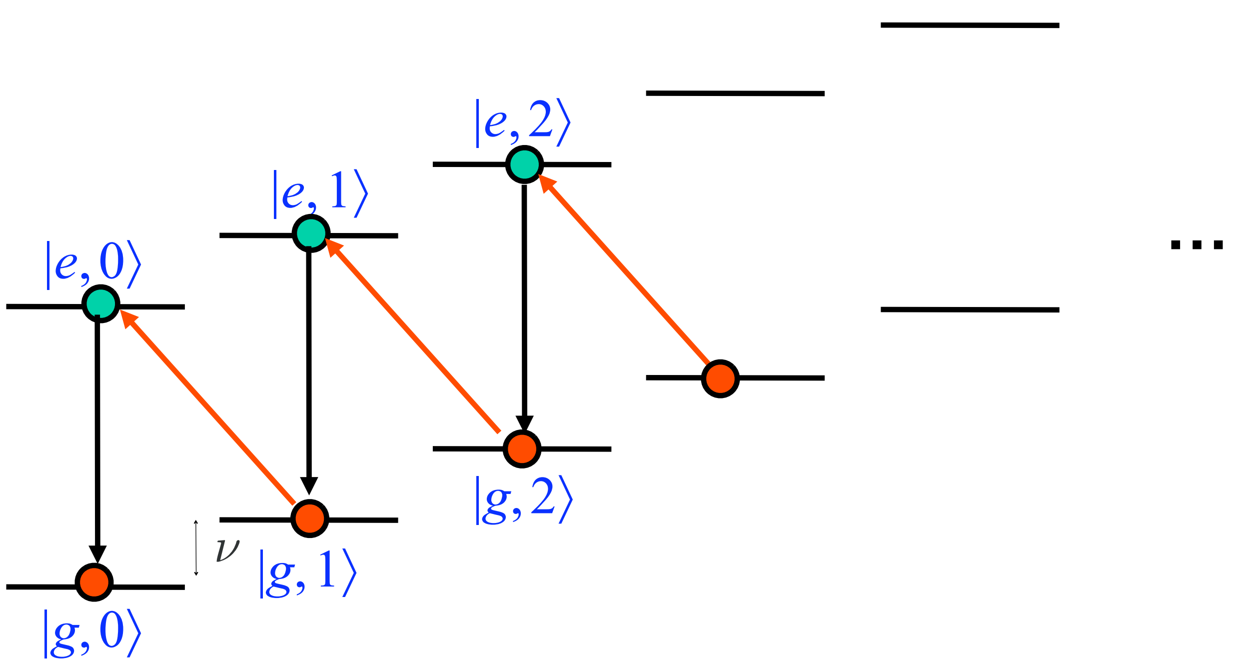


Procedure: Applying a laser on the red sideband we couple the states $|g\rangle|n\rangle \leftrightarrow |e\rangle|n-1\rangle$.

As a first step we apply a π-pulse so that we make the amplitude of $|g\rangle|N\rangle$ equal to zero by transferring the amplitude c_N to $|e\rangle|N-1\rangle$. But we now have a superposition of ground and excited state.



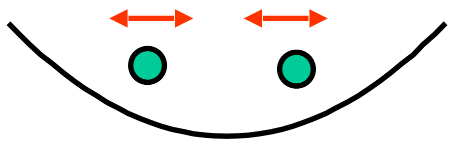
In the second step we apply a resonant laser so that we transform the known! superposition of $|g\rangle|N-1\rangle$, $|e\rangle|N-1\rangle$ to $|g\rangle|N-1\rangle$ with no amplitude left in $|e\rangle|N-1\rangle$. Now we repeat the argument until we have transformed the state to $|g\rangle|0\rangle$.



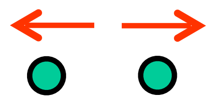
In the second step we apply a resonant laser so that we transform the known! superposition of $|g\rangle|N-1\rangle$, $|e\rangle|N-1\rangle$ to $|g\rangle|N-1\rangle$ with no amplitude left in $|e\rangle|N-1\rangle$. Now we repeat the argument until we have transformed the state to $|g\rangle|0\rangle$.

2. Many ions

- 2 ions & collective phonon modes

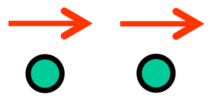


stretch mode

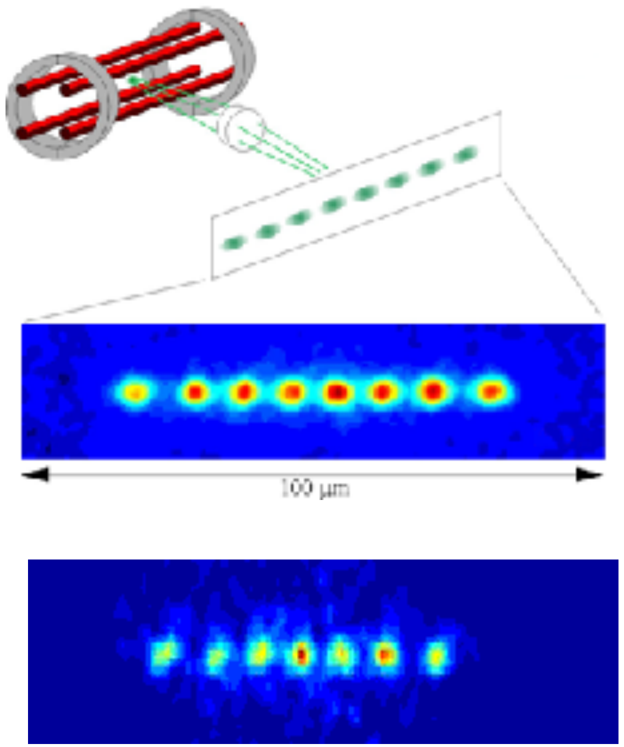


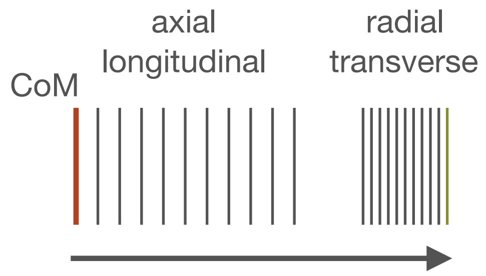
$\nu_r = \sqrt{3}\nu_c$

center-of-mass



$\nu_c = \nu$

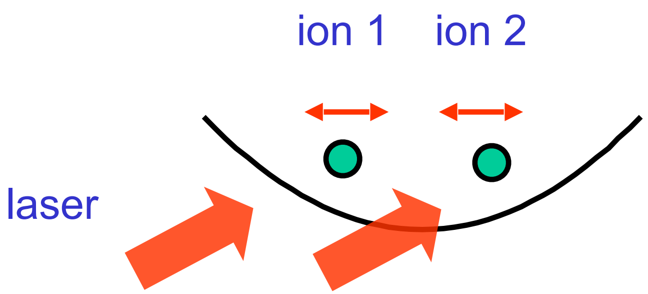


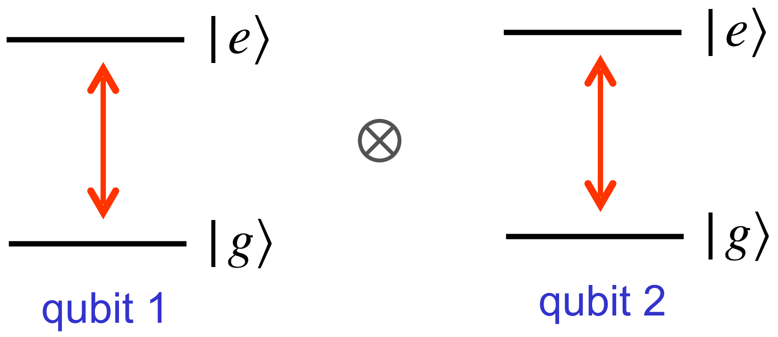


SL Zhu et al., PRL 2006

(3) We can swap a qubit to a *collective* mode via laser pulse

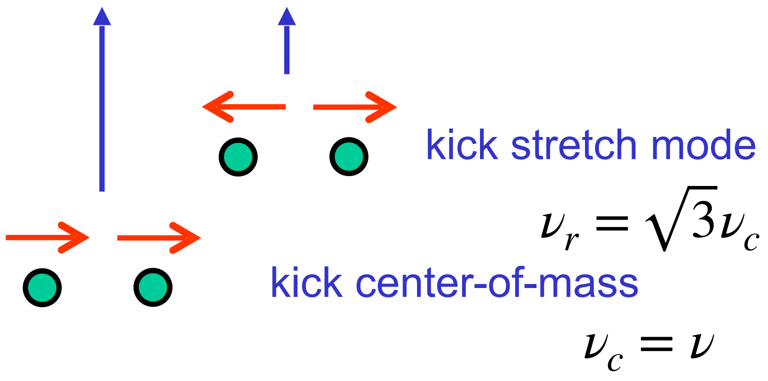
- Hamiltonian & control





$$H = \hbar\nu_c a^\dagger a + \hbar\nu_r b^\dagger b \; -\frac{1}{2}\hbar\Omega_1(t)e^{ik\hat{X}_1\otimes\hat{1}_2}|e\rangle_1\langle g| \otimes \hat{1}_2 - \frac{1}{2}\hbar\Omega_2(t)e^{ik\hat{1}_1\otimes\hat{X}_2}\hat{1}_1 \otimes |e\rangle_2\langle g| + \text{c.c.}$$

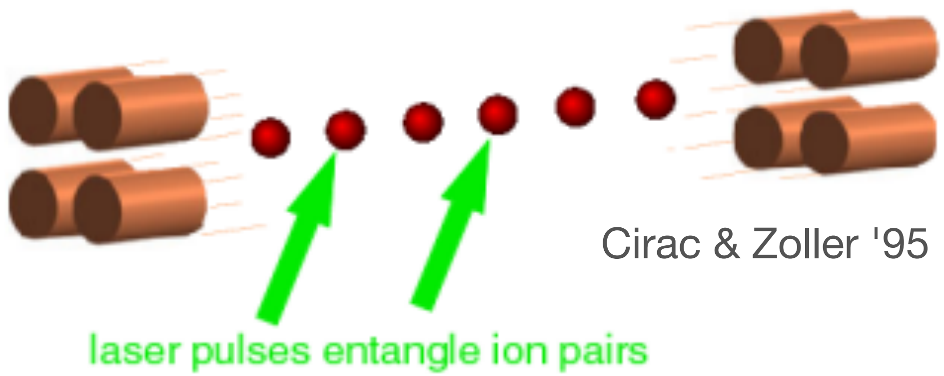
$$-\frac{1}{2}\hbar\Omega_1(t)e^{i\eta_c(a^\dagger+a)+i\nu_r(b^\dagger+b)}|e\rangle_1\langle g| \otimes \hat{1}_2 \; -\frac{1}{2}\hbar\Omega_2(t)e^{i\eta_c(a^\dagger+a)-i\nu_r(b^\dagger+b)}\hat{1}_1 \otimes |e\rangle_2\langle g| \; + \text{c.c.}$$



qubit 1 and 2 couple
to *both* collective modes
→ entangle qubits, make spin ½ interact

3. Trapped ion quantum computing (& quantum simulation)

- laser cooled ions in a linear trap



Cirac & Zoller '95

Qubits: internal atomic states

1-qubit gates: addressing ions with a laser

2-qubit gates: entanglement via exchange of phonons of quantized collective mode

- state vector

$$|\Psi\rangle = \sum_{x \in \{0,1\}^{\otimes N}} c_x |x_{N-1}\rangle \ldots x_0\rangle_{\text{atom}} \otimes |0\rangle_{\text{phonon}}$$

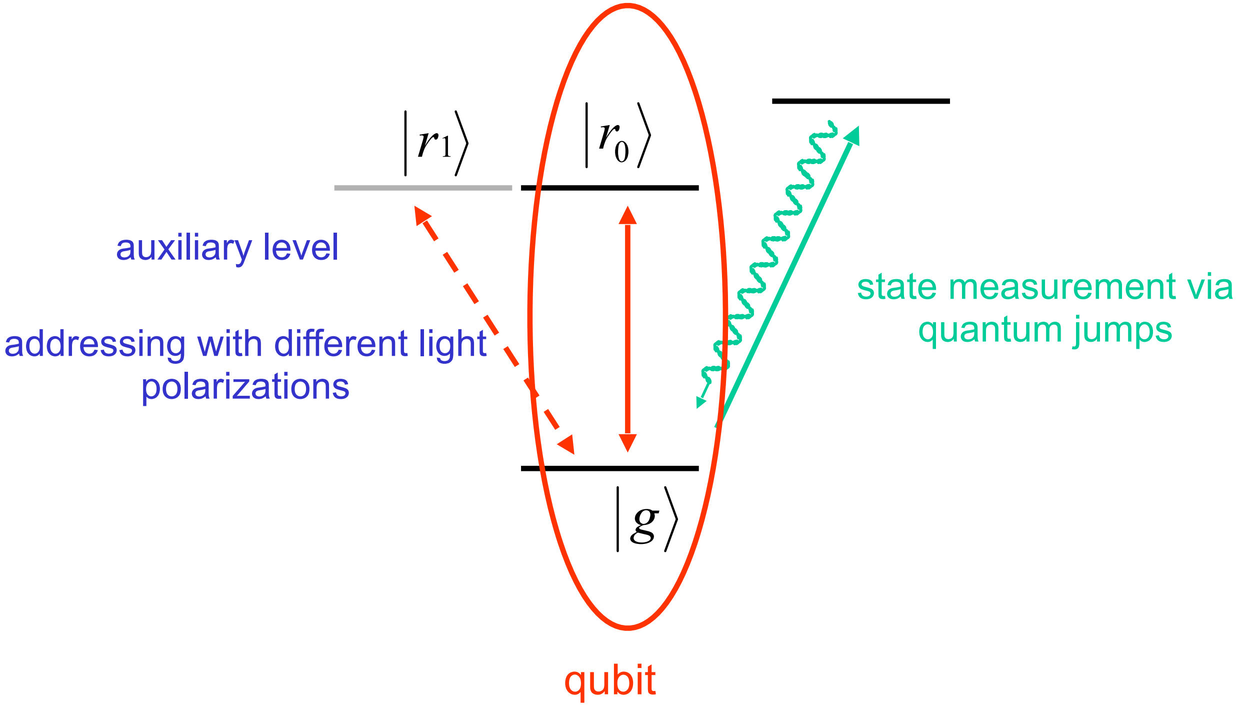
quantum register databus

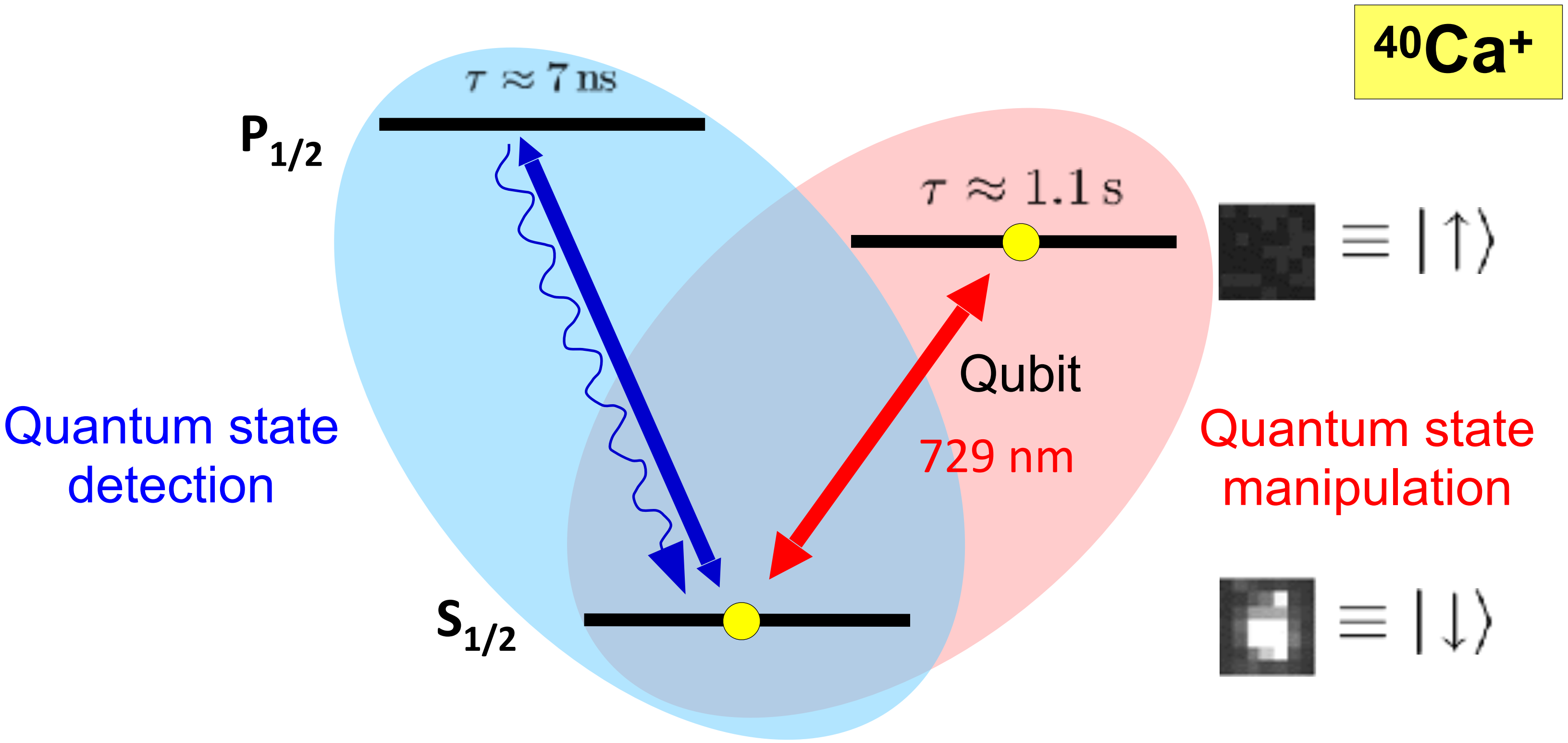
JI Cirac, P. Zoller, Quantum Computations with Cold Trapped Ions. *Phys. Rev. Lett.* **74**, 4091 (1995).

Review: P. Schindler et al. [Blatt-group], A quantum information processor with trapped ions. *New J. Phys.* **15**, 123012 (2013).

4. Entangling Gates: the ’95 gate

- level scheme





$^{40}Ca^+$
$\tau \approx 7\,\mathrm{ns}$
$P_{1/2}$
$\tau \approx 1.1\,\mathrm{s}$
$\equiv |\uparrow\rangle$
Qubit
729 nm
Quantum state detection
Quantum state manipulation
$S_{1/2}$
$\equiv |\downarrow\rangle$

A universal qudit quantum processor with trapped ions

Martin Ringbauer[1], Michael Meth[1], Lukas Postler[1], Roman Stricker[1], Rainer Blatt[1,2,3],
Philipp Schindler[1] and Thomas Monz[1,3]

NATURE PHYSICS | VOL 18 | SEPTEMBER 2022 | 1053–1057 | www.nature.com/naturephysics

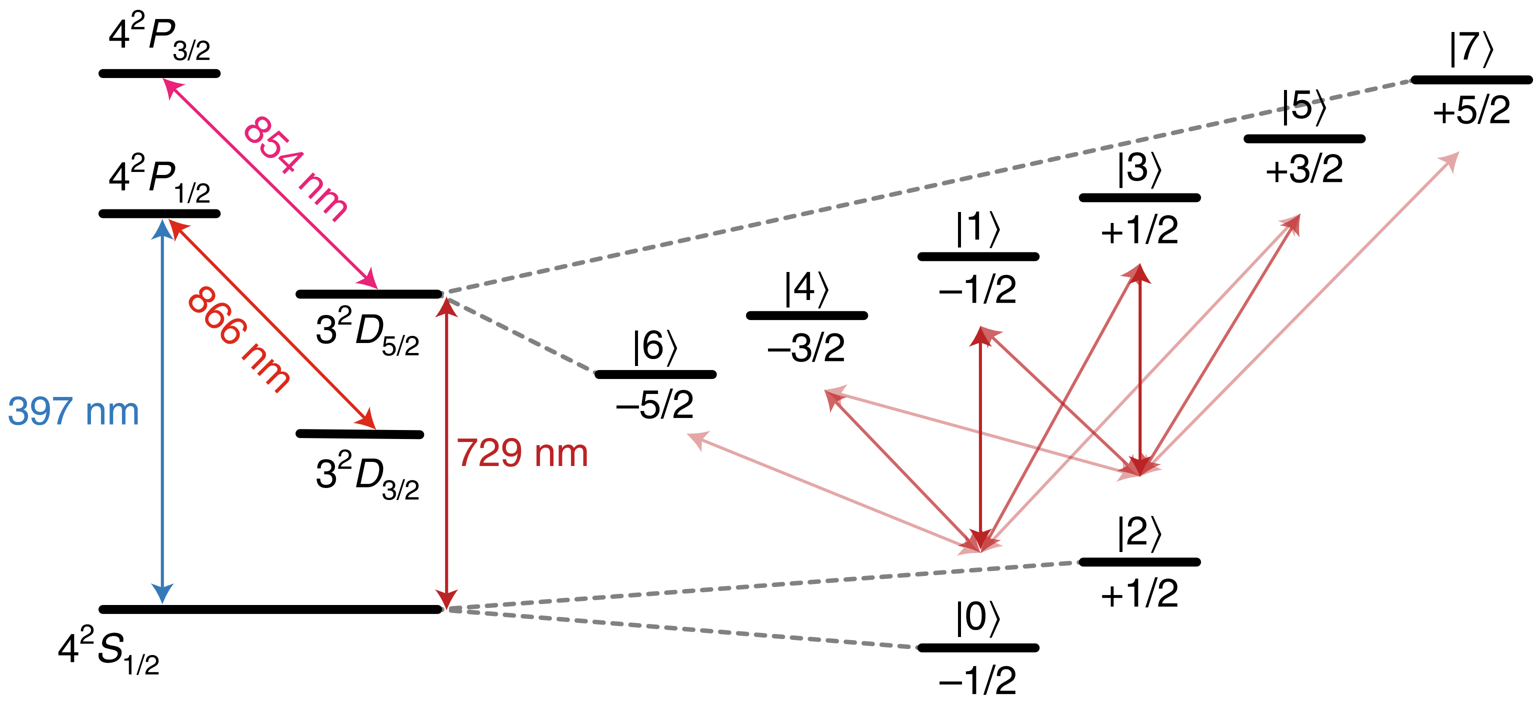
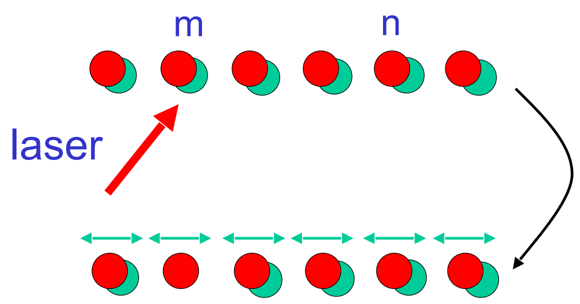


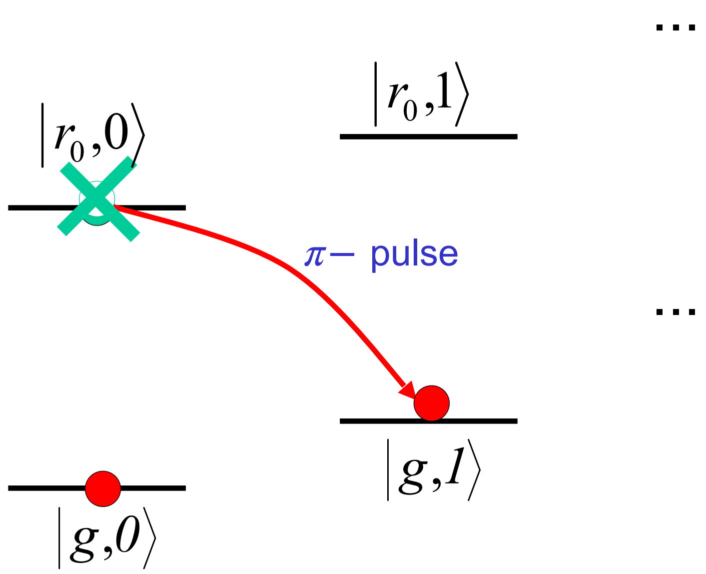
Fig. 1 | Level scheme of the $^{40}Ca^+$ ion. Quantum information is encoded in the $S_{1/2}$ and $D_{5/2}$ states, where each transition between S and D is accessible using a single narrowband laser at 729 nm.

’95 two-qubit phase gate

- step 1: swap first qubit to phonon bus



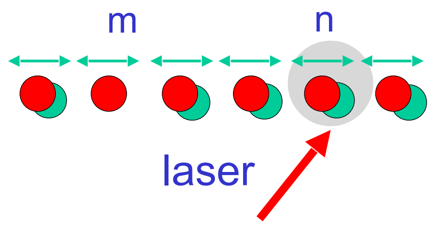
first atom: m

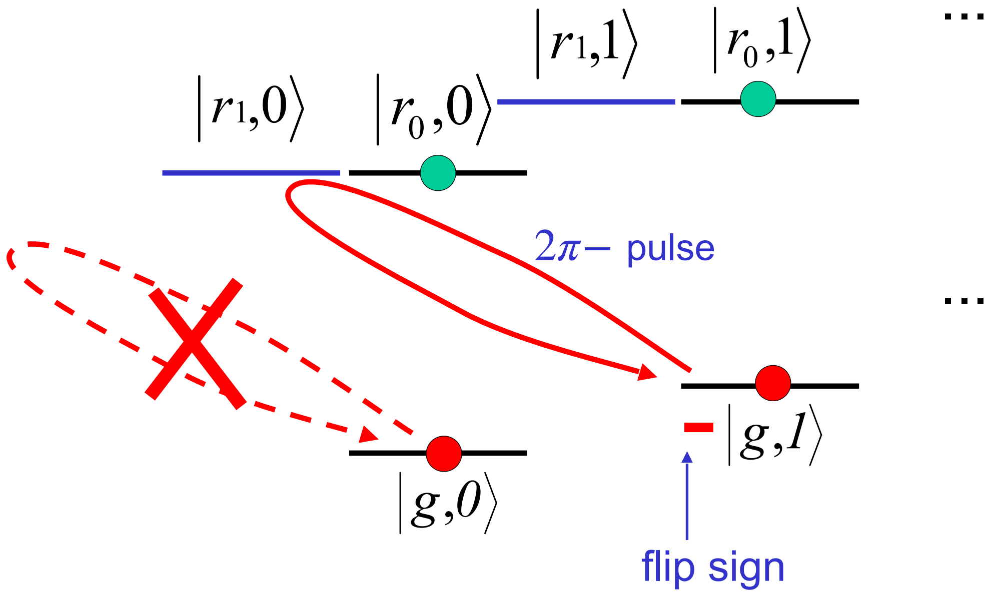


$$\begin{array}{rcr} & \hat{U}_m^{\pi,0} & \\ |g\rangle_m|0\rangle & \longrightarrow & |g\rangle_m|0\rangle \\ |r\rangle_m|0\rangle & \longrightarrow & -i|g\rangle_m|1\rangle \end{array}$$

'95 two-qubit phase gate

- step 2 conditional sign change

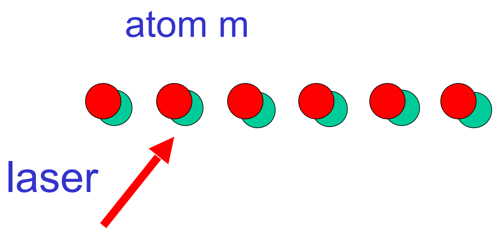




$$\begin{array}{rcr} & \hat{U}_n^{2\pi,1} & \\ |g\rangle_m|g\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|g\rangle_n|0\rangle \\ |g\rangle_m|r\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|r\rangle_n|0\rangle \\ -i|g\rangle_m|g\rangle_n|1\rangle & \longrightarrow & i|g\rangle_m|g\rangle_n|1\rangle \\ -i|g\rangle_m|r\rangle_n|1\rangle & \longrightarrow & -i|g\rangle_m|r\rangle_n|1\rangle \end{array}$$

’95 two-qubit phase gate

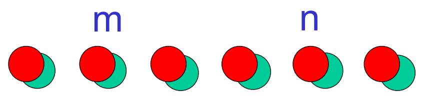
- step 3: swap phonon back to first qubit



$$|g\rangle_m \otimes \begin{array}{rcl} & \hat{U}_m^{\pi,0} & \\ |g\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|g\rangle_n \\ |r\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|r\rangle_n \\ i|g\rangle_n|1\rangle & \longrightarrow & |r\rangle_m|g\rangle_n \\ -i|r\rangle_n|1\rangle & \longrightarrow & -|r\rangle_m|r\rangle_n \end{array} \otimes |0\rangle$$

’95 two-qubit phase gate

- summary



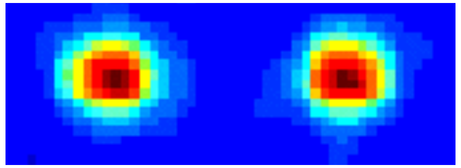
$$
\begin{aligned}
|g\rangle|g\rangle\;|0\rangle &\longrightarrow \quad |g\rangle|g\rangle\;|0\rangle, \\
|g\rangle|r_0\rangle\;|0\rangle &\longrightarrow \quad |g\rangle|r_0\rangle\;|0\rangle, \\
|r_0\rangle|g\rangle\;|0\rangle &\longrightarrow \quad |r_0\rangle|g\rangle\;|0\rangle, \\
|r_0\rangle|r_0\rangle|0\rangle &\longrightarrow \; - |r_0\rangle|r_0\rangle\;|0\rangle.
\end{aligned}
$$

phonon mode returned to initial state

$$
|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow (-1)^{\epsilon_1\epsilon_2}|\epsilon_1\rangle|\epsilon_2\rangle \quad (\epsilon_{1,2} = 0, 1)
$$

Excercise: write out all of these steps explicitly

- (addressable) 2 ion controlled-NOT + tomography



Realization of the Cirac–Zoller controlled-NOT quantum gate

Ferdinand Schmidt-Kaler, Hartmut Häffner, Mark Riebe, Stephan Gulde, Gavin P. T. Lancaster, Thomas Deuschle, Christoph Becher, Christian F. Roos, Jürgen Eschner & Rainer Blatt

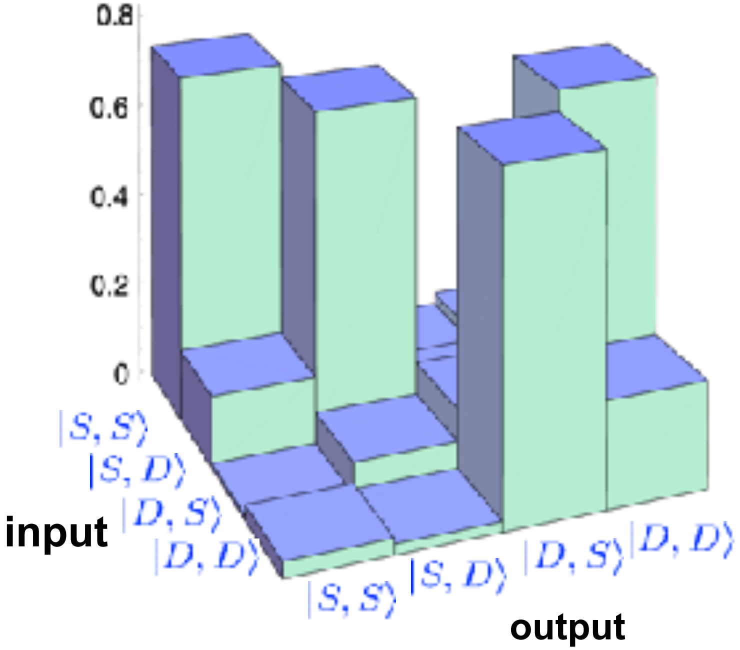
Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A 6020 Innsbruck, Austria

Experimental demonstration of a robust, high-fidelity geometric two ion-qubit phase gate

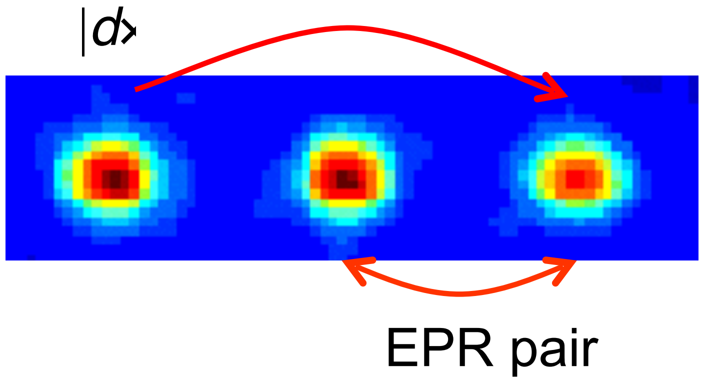
D. Leibfried*†, B. DeMarco*, V. Meyer*, D. Lucas*‡, M. Barrett*, J. Britton*, W. M. Itano*, B. Jelenković*§, C. Langer*, T. Rosenband* & D. J. Wineland*

truth table CNOT

Innsbruck



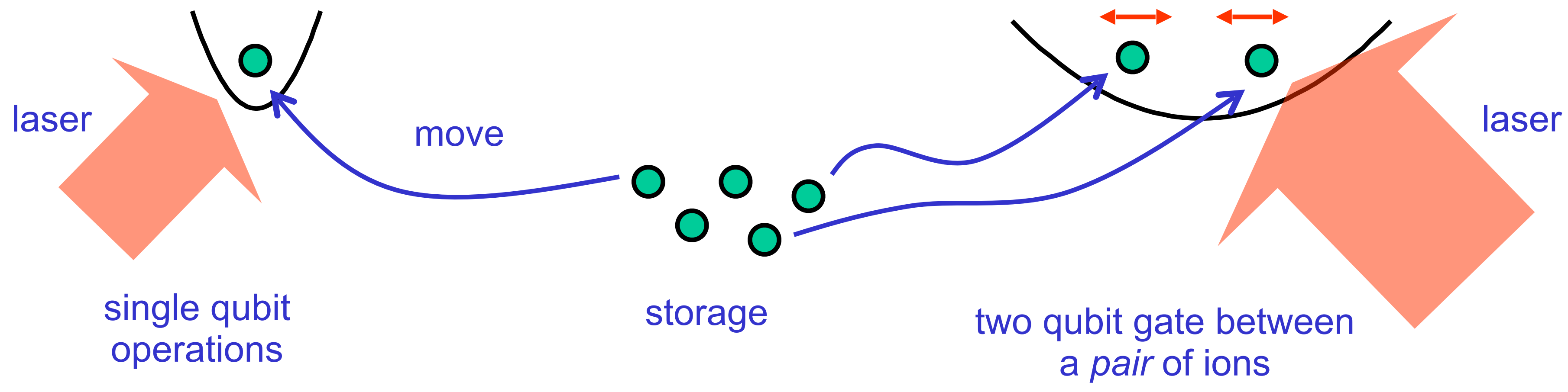
- teleportation Innsbruck / Boulder



- decoherence: quantum memory DFS 20 sec

Remarks: Scalability

- key idea: moving ions without destroying qubits



Remark: the wishlist

- fast: max # operations / decoherence [what are the limits?]

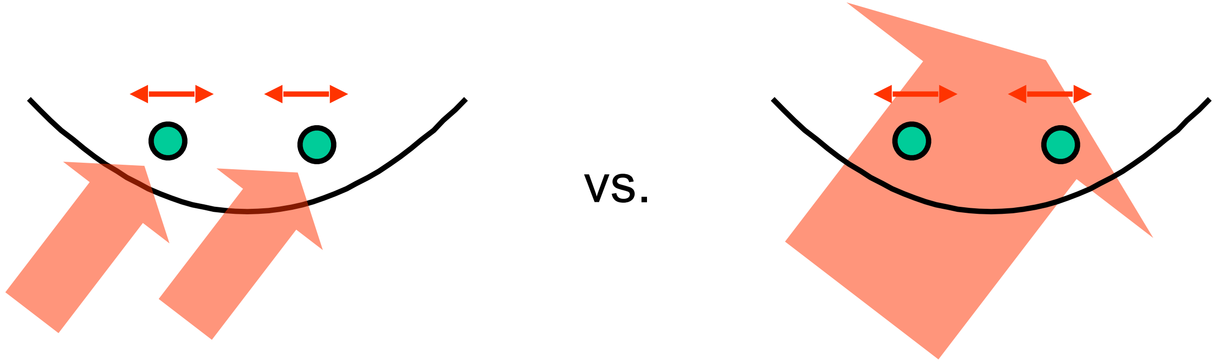
- NO temperature requirement: “hot” gate, i.e. NO ground state cooling

$$|\psi\rangle\langle\psi| \otimes \rho_{\text{motion}} \rightarrow \text{entangle via motion} \rightarrow |\psi'\rangle\langle\psi'| \otimes \rho'_{\text{motion}}$$

qubits motional state:
e.g. thermal

motional state factors out

- NO indivdual addressing

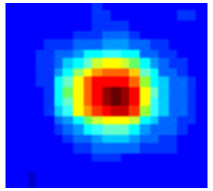


addressing:
large distance

vs.

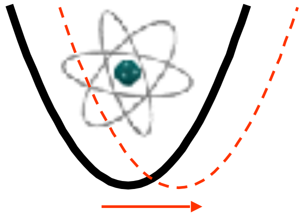
strong coupling:
small distance

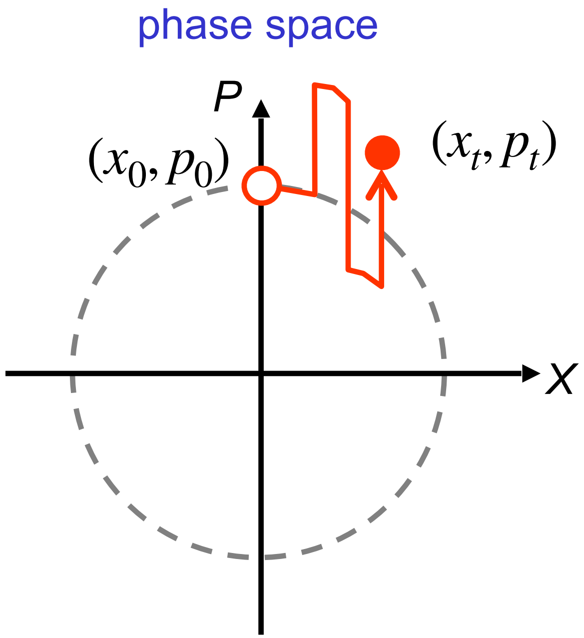
4.2 Geometric [Coherent Control] Gates: One Ion



- Goal: geometric phase by driving a harmonic oscillator
- Hamiltonian

$$H = \frac{1}{2}(\hat{p}^2 + \hat{x}^2) - f(t)\hat{x}$$





- Time evolution

$$|\psi_0\rangle = |z_0 = x_0 + ip_0\rangle \Longrightarrow |\psi_0\rangle = e^{i\phi_t}|z_t = x_t + ip_t\rangle$$

coherent state (initial); phase ($e^{i\phi_t}$); coherent state (final)

- Solution

$$\frac{d}{dt}z = -i\omega z + i\frac{1}{\sqrt{2}}f(t) \Longrightarrow z_t = e^{-i\omega t}\left[z_0 + \frac{i}{\sqrt{2}}\int_0^t d\tau f(\tau)\right]$$

$$\frac{d}{dt}\phi = \frac{1}{2\sqrt{2}}f(t)(z^\star + z)$$

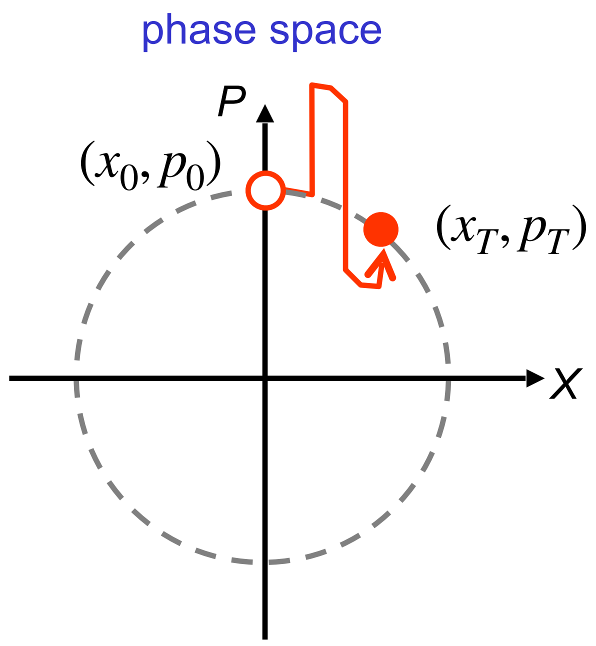
phase (ϕ); classical evolution; displacement

A. Sørensen, K. Mølmer, PRL 1999, PRA 2000; G Milburn arxiv 1999, JJ. Garcia-Ripoll, P Zoller, I Cirac, PRL 2003, PRA 2005

- Condition

After a given time T the coherent wavepacket is restored to the freely evolved state

$$\int_0^T d\tau\, e^{i\omega\tau} f(\tau) \stackrel{!}{=} 0$$

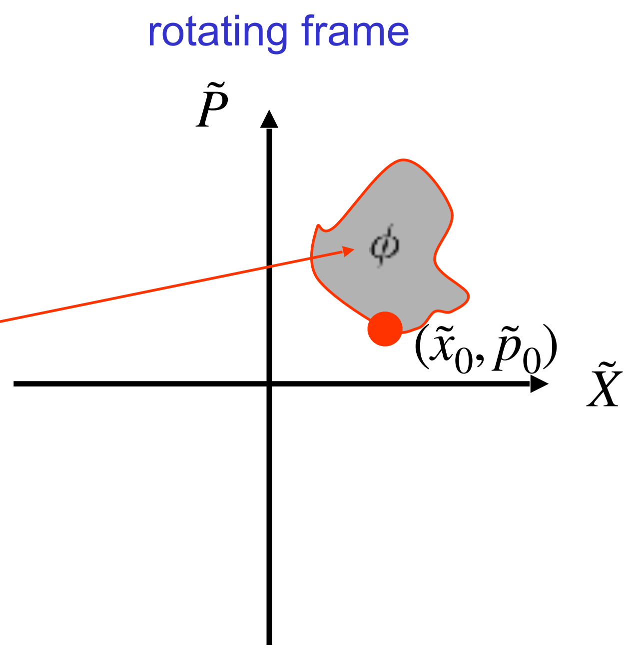


- Rotating frame

$$\tilde{z}_t \equiv \tilde{x}_t + i\tilde{p}_t = e^{i\omega t} z_t$$

$$\frac{d\tilde{z}}{dt} = ie^{i\omega t}\frac{1}{\sqrt{2}}f(t)$$

$$\frac{d\phi}{dt} = \frac{d\tilde{p}}{dt}\tilde{x} - \frac{d\tilde{x}}{dt}\tilde{p} = 2\frac{dA}{dt}$$



- Phase

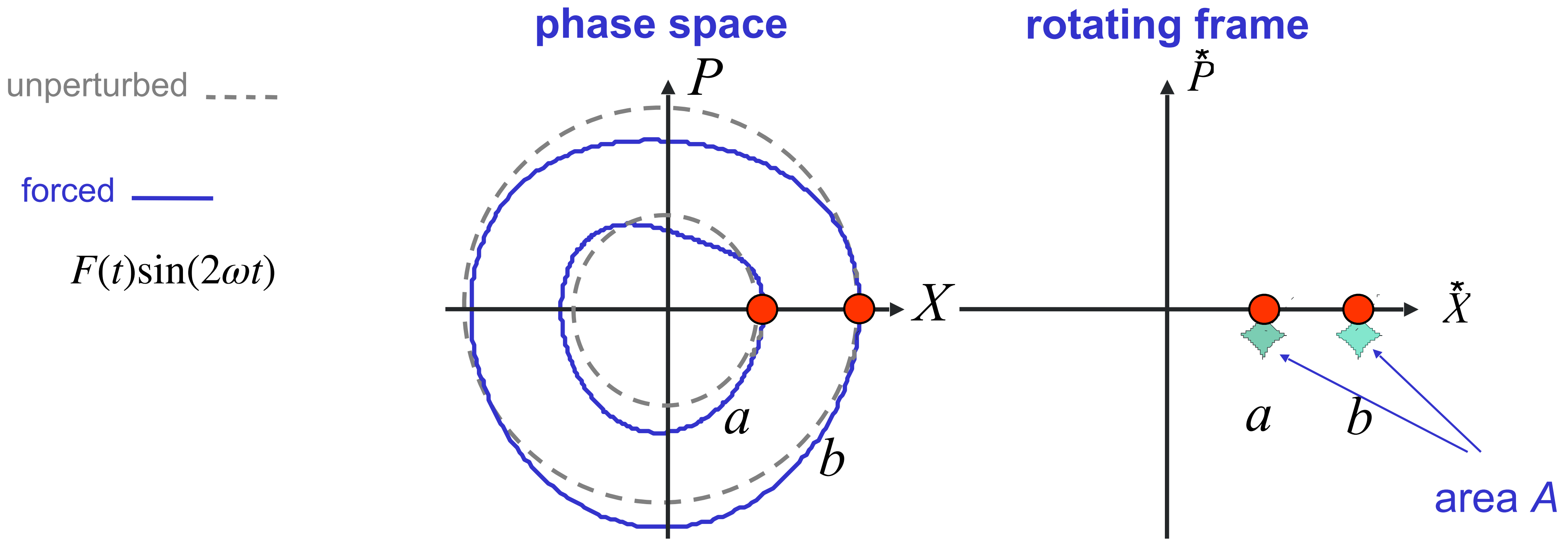
$$\phi(T) = \text{Im}\frac{i}{\sqrt{2}}\int_0^T d\tau e^{i\omega\tau} f(\tau)\tilde{z}_\tau^\star$$

$$= \text{Im}\frac{i}{\sqrt{2}}\left[\int_0^T d\tau e^{i\omega\tau} f(\tau)\right]\tilde{z}_0^\star + \frac{1}{2}\text{Im}\int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 e^{i\omega(\tau_1-\tau_2)} f(\tau_1) f(\tau_2)^\star$$

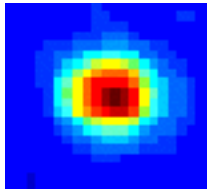
=0
return condition

The phase does *not* depend on the initial state, (x_0, p_0)

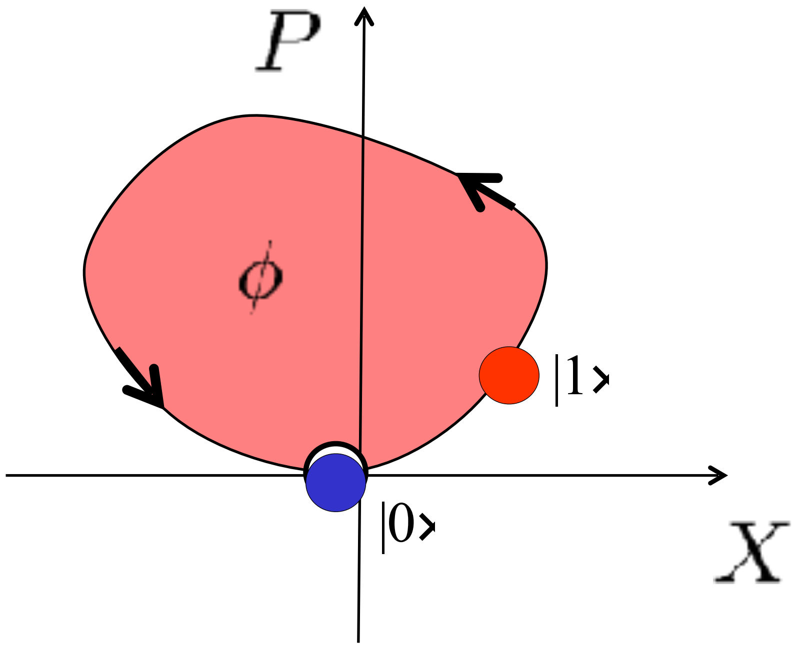
- Example



The phase does not depend on the initial state (x_0, p_0), i.e. temperature independent

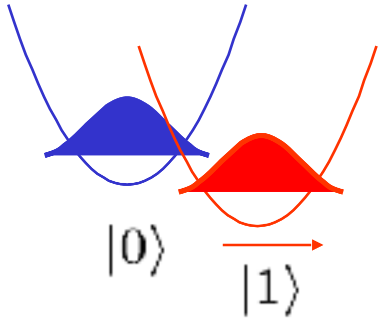


Geometric phase gate: single ion



- Hamiltonian

$$H = \frac{1}{2}(\hat{p}^2 + \hat{x}^2) - |1\rangle\langle 1| f(t)\hat{x}$$



- Time evolution operator

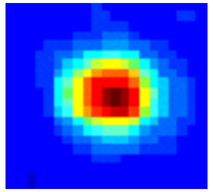
$$U(T) = e^{i\phi|1\rangle\langle 1|}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |z_0\rangle$$

$$\longrightarrow (\alpha|0\rangle + \beta e^{i\phi}|1\rangle) \otimes |z_0\rangle$$

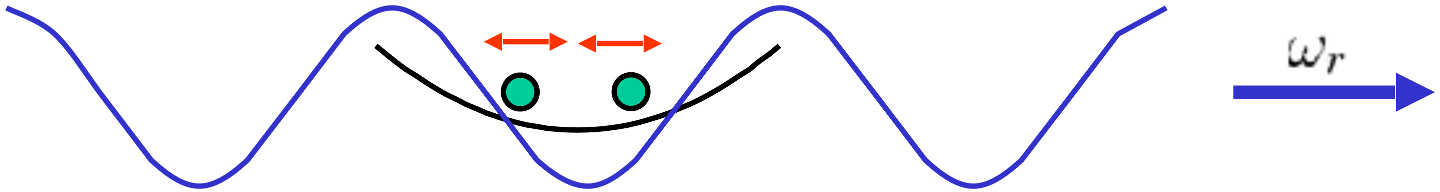
single ion phase gate

motion factors out



NIST Gate: Leibfried et al Nature 2003

- 2 ions in a running standing wave tuned to ω_r



$$H = \omega_r a^\dagger a - F(t)(\sigma_z^1 + \sigma_z^2)(a_r + a_r^\dagger)$$

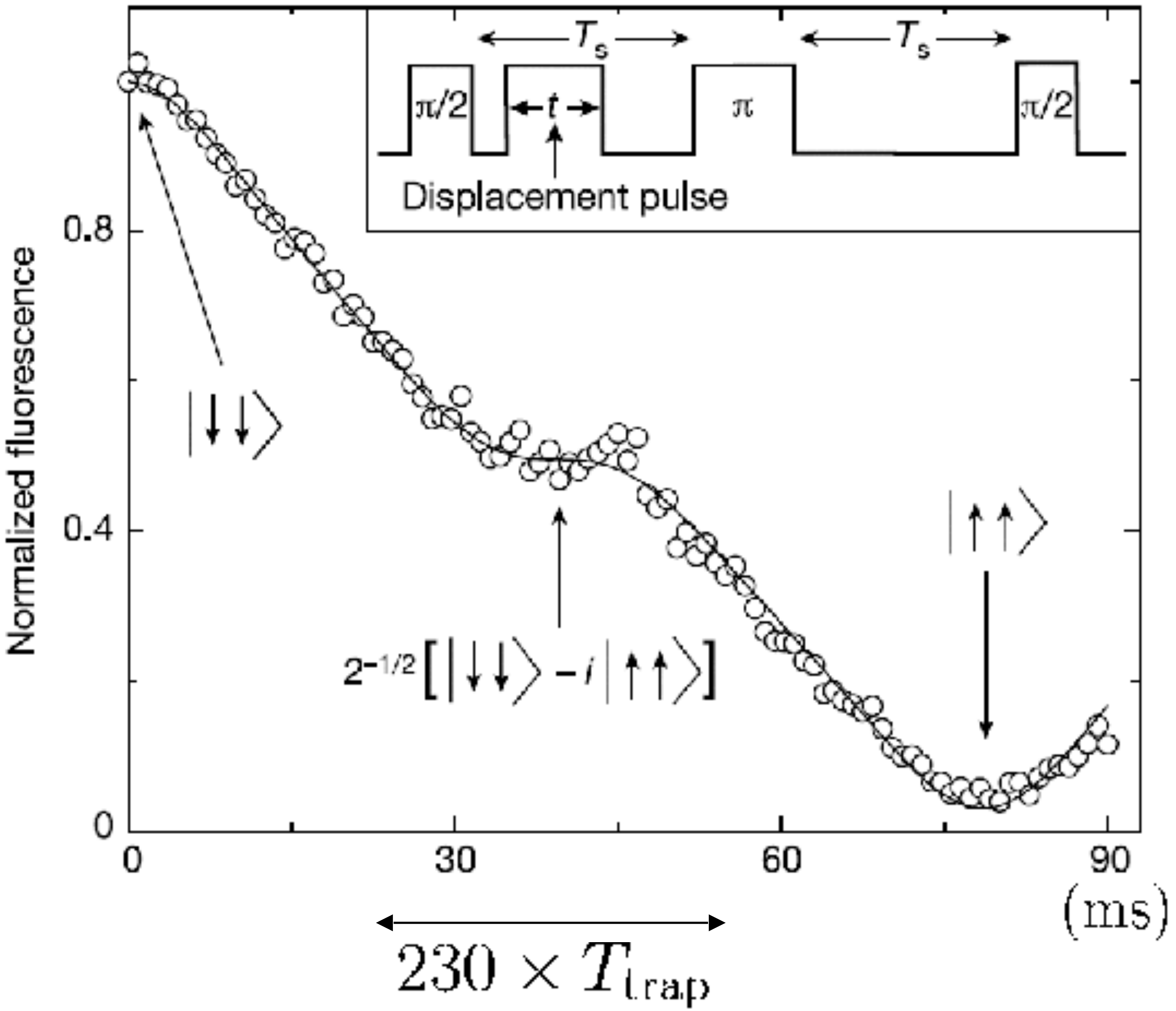
- If F(t) is periodic with a period multiple of ω_r, after some time the motional state is restored, but now the total phase is

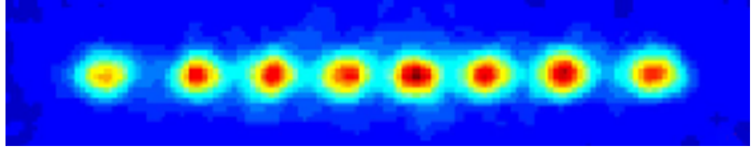
$$\phi = A\sigma_z^1\sigma_z^2 \qquad U(T) = \exp(i\phi\sigma_1^z\sigma_2^z)$$

- To address one mode, the gate must be slow ☹

$$T \gg 2\pi/\omega_r$$

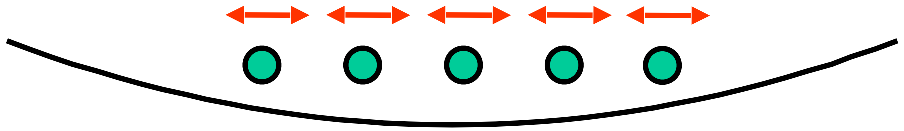
NIST Gate: Leibfried et al Nature 2003





N ions

- We will consider N trapped ions (linear traps, microtraps…), subject to state-dependent forces:



$$H = \sum_{i=1}^{N} \left[\frac{1}{2m} p_i^2 + V_{e,i}(x_i) - F_i(t)\sigma_z^i x_i\right] + \sum_{i<j} \frac{e^2}{4\pi\varepsilon_0} \frac{1}{|x_i - x_j|}$$

- normal modes

$$H = \sum_i \left[\frac{1}{2m} P_i^2 + \frac{1}{2} m\nu_k^2 Q_k^2\right] - \sum_k F_i(t)\sigma_z^i M_{ik} Q_k$$

integrable

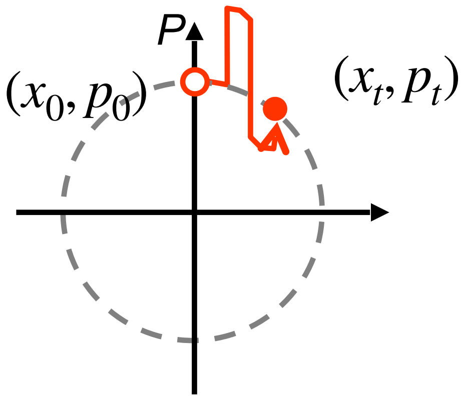
- unitary evolution operator

$$U(T) = \exp\left(i \sum_{ij} J_{ij}\sigma_z^i \sigma_z^j\right)$$

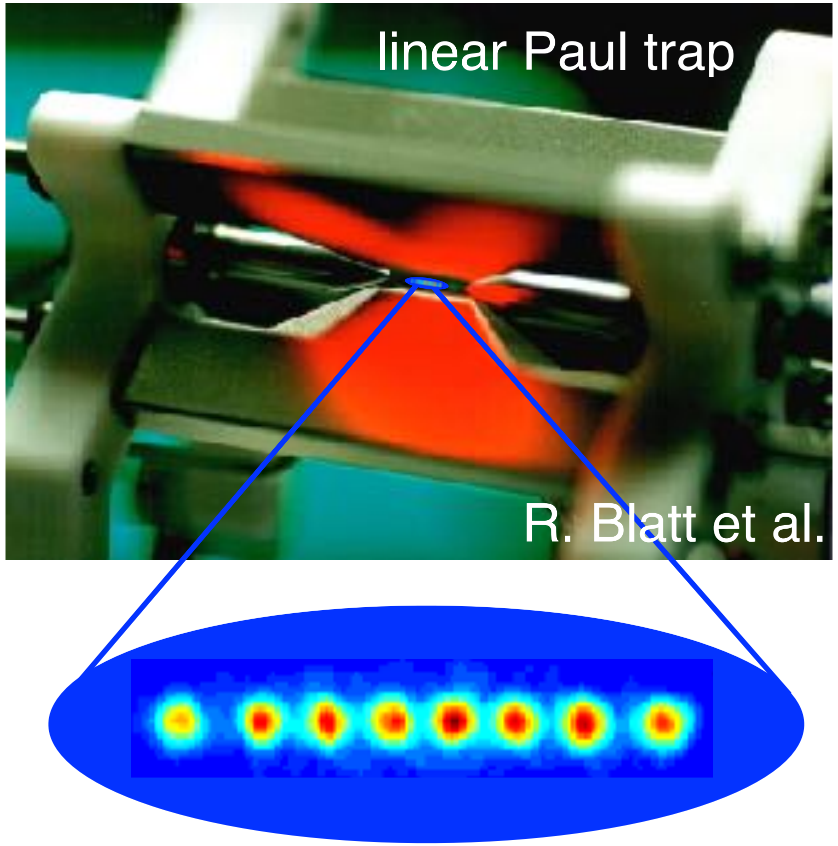
general Ising interaction

- constraints on forces

$$\int_0^T d\tau\, e^{i\omega_k \tau} F_i(\tau) = 0, \quad \forall i, k$$



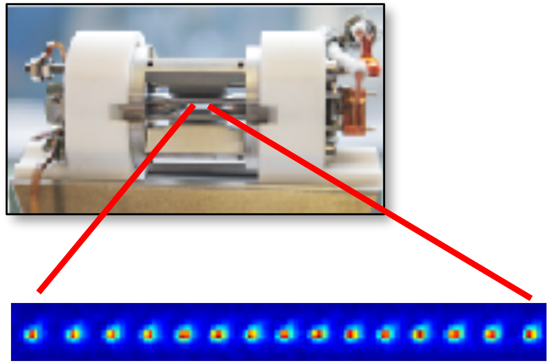
Trapped-Ion Quantum *Simulation*



String of Trapped Ions

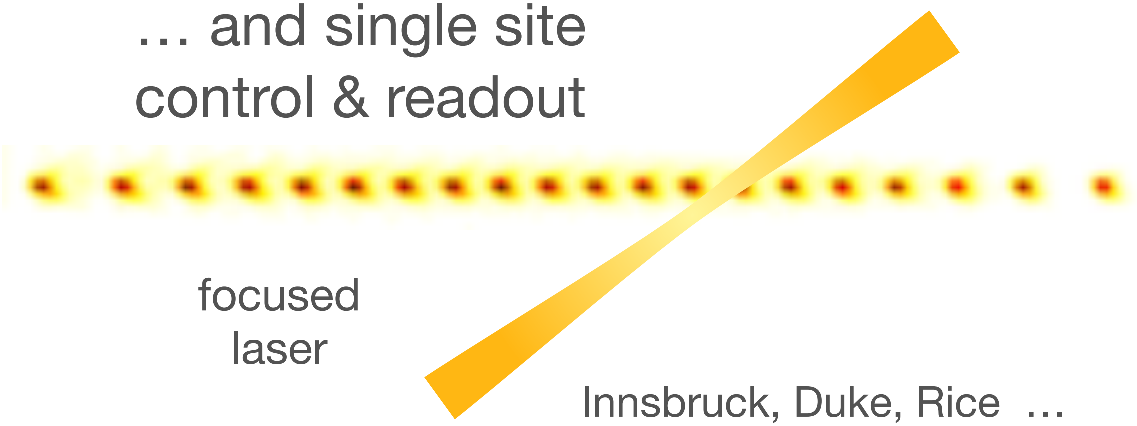
C. Monroe, W. C. Campbell, L.-M. Duan, Z.-X. Gong, A. V. Gorshkov, P. W. Hess, R. Islam, K. Kim, N. M. Linke, G. Pagano, P. Richerme, C. Senko, N. Y. Yao, *Programmable quantum simulations of spin systems with trapped ions. Rev. Mod. Phys.* **93**, 025001 (2021)

10, 20 … 50 Qubit Trapped-Ion Programmable Quantum Simulator @ IQOQI-Labs



Transverse long-range Ising model

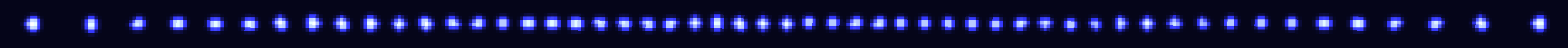
… and single site control & readout



Innsbruck, Duke, Rice …

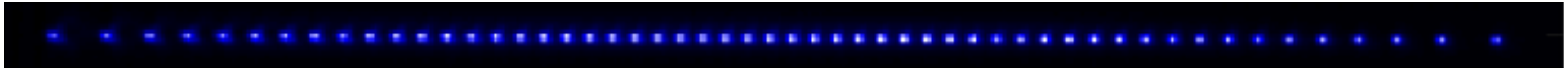
$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

C. Monroe et al., *Programmable quantum simulations of spin systems with trapped ions. RMP* (2021)



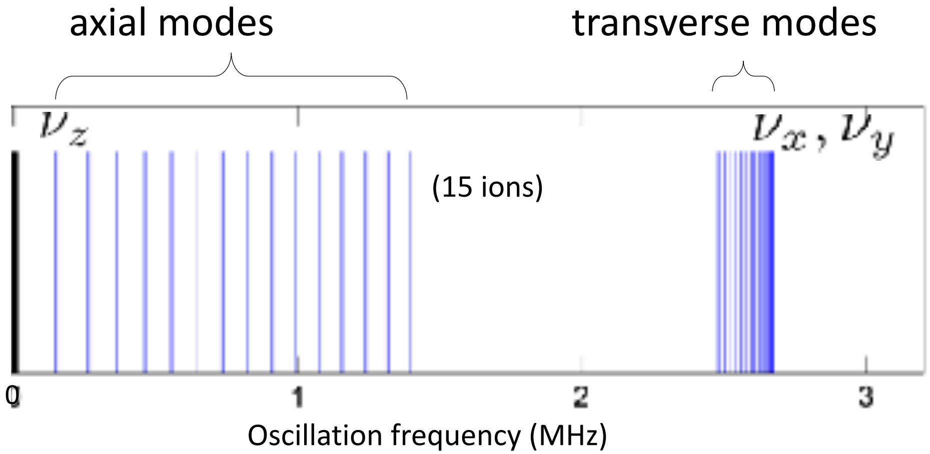
Crystal geometry and collective modes of motion

Linear strings of *N* ions : $\nu_x, \nu_y \gg \nu_z$

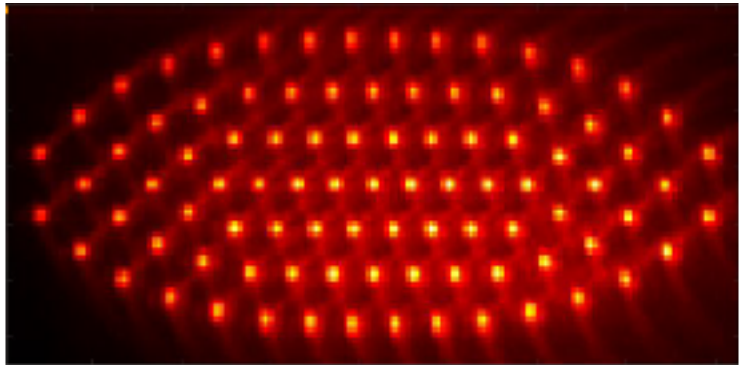


Collective motional modes:

- *N* axial modes
- 2 x *N* transverse modes
  that cluster in frequency space



Planar crystals: $\nu_x \gg \nu_y, \nu_z$

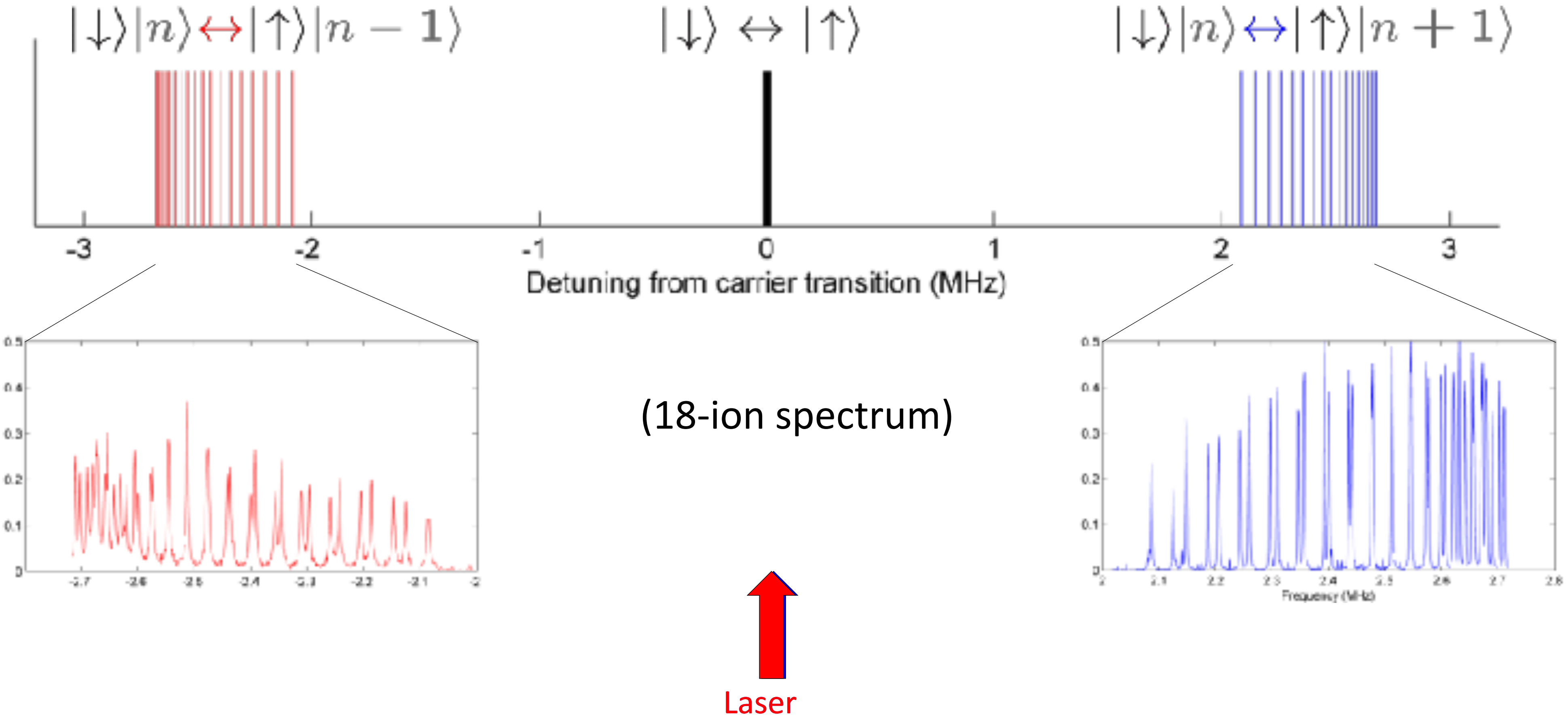


Collective motional modes:

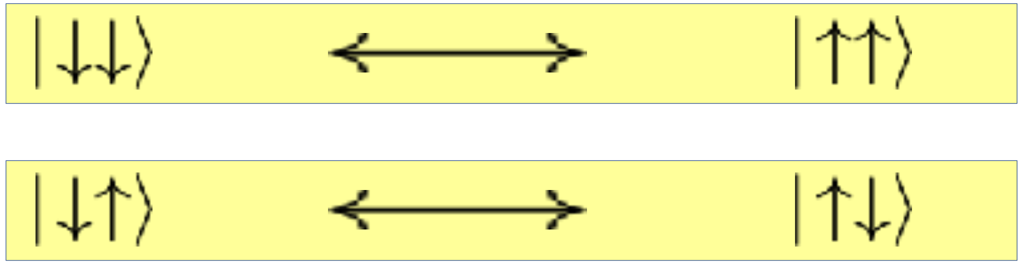
- *2N* in-plane modes
- *N* out-of-plane modes

slide credit: C. Roos

Entangling interactions mediated by transverse motional modes



Spin-spin interaction by off-resonant laser coupling to vibrational modes



$$H = \sum_{i<j} J_{ij}\sigma_i^x \sigma_j^x$$

slide credit: C. Roos

Variable-range entangling interactions (Ising)

$$H = \sum_{i<j} J_{ij}\sigma_i^x \sigma_j^x \qquad J_{ij} = \frac{J_0}{|i-j|^\alpha},\ 0 < \alpha < 3$$

Example: 11 ions

vibrational mode

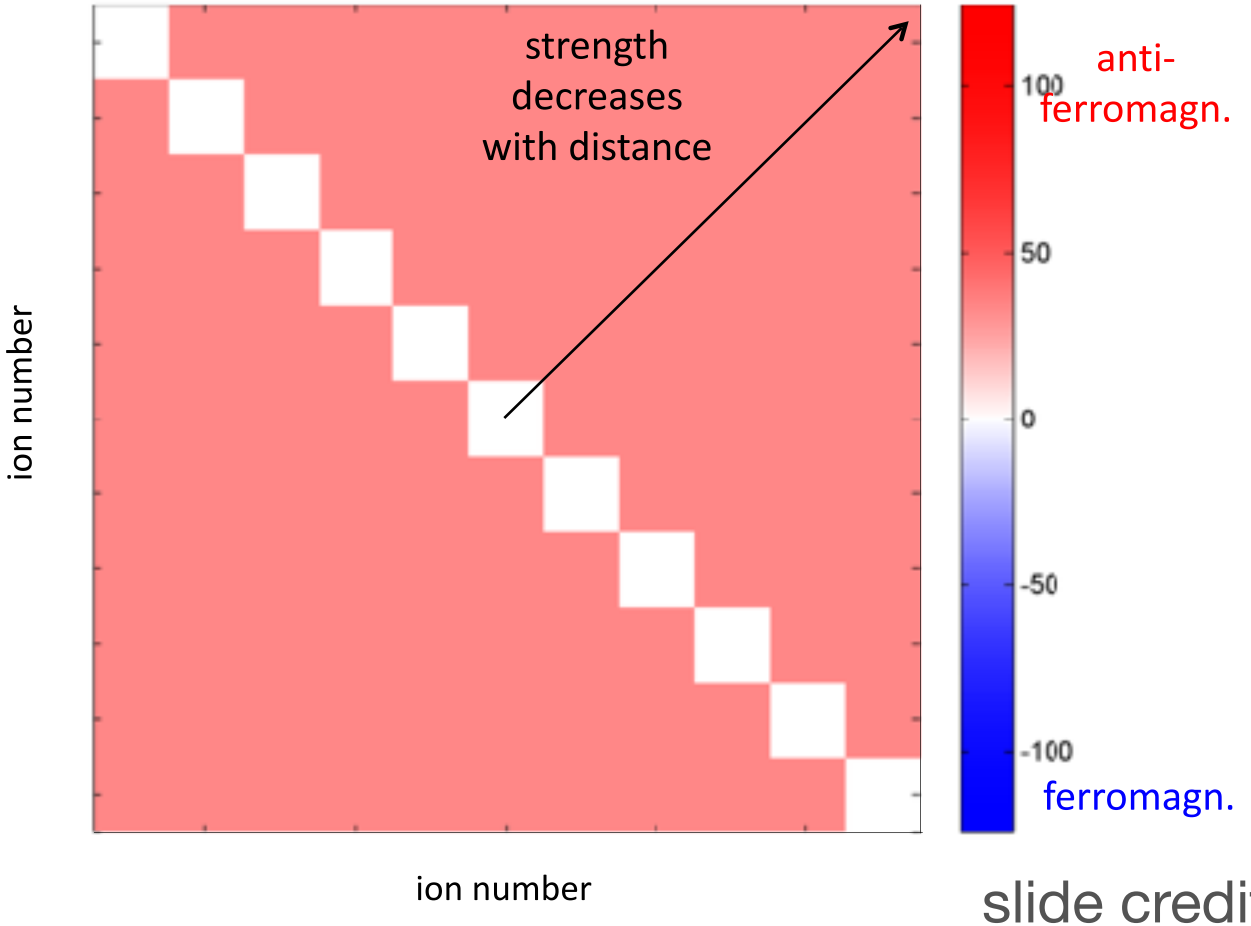
'COM'

'Tilt'

⋮

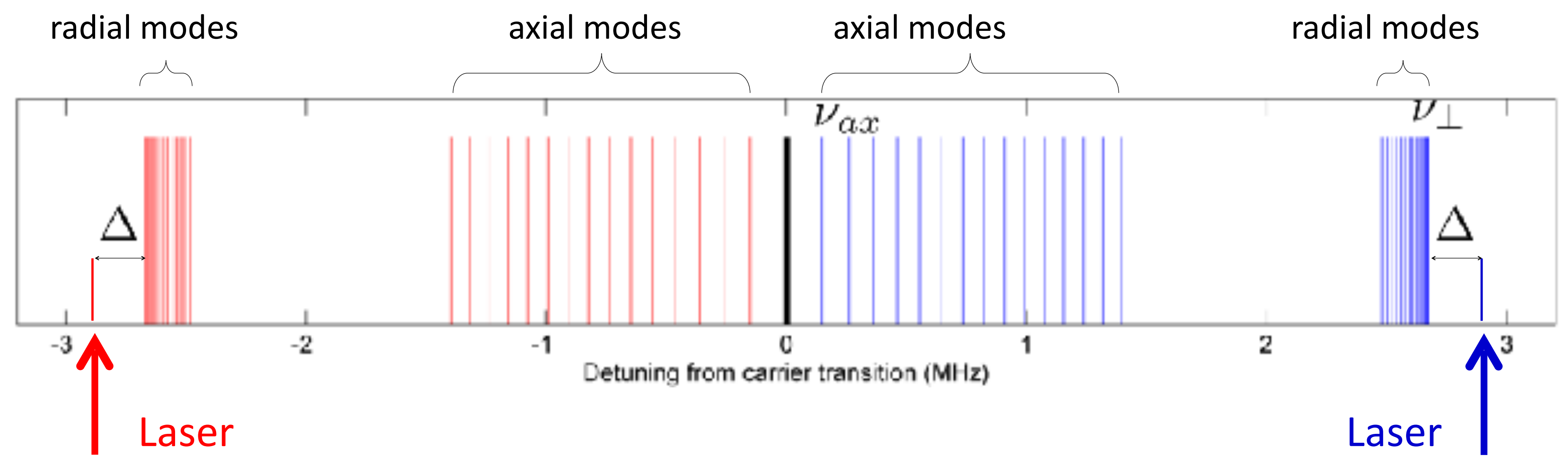
⋮

Spin-spin coupling J_{ij} (Hz)



slide credit: C. Roos

Variable-range entangling interactions (Ising)



$$H = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x \quad \text{with} \quad J_{ij} \approx \frac{J_0}{|i-j|^\alpha}$$

Interaction range: $0 < \alpha < 3$

couple only to center-of-mass (for $\alpha = 0$)

couple to all modes equally (for $\alpha = 3$)

Knobs to turn:

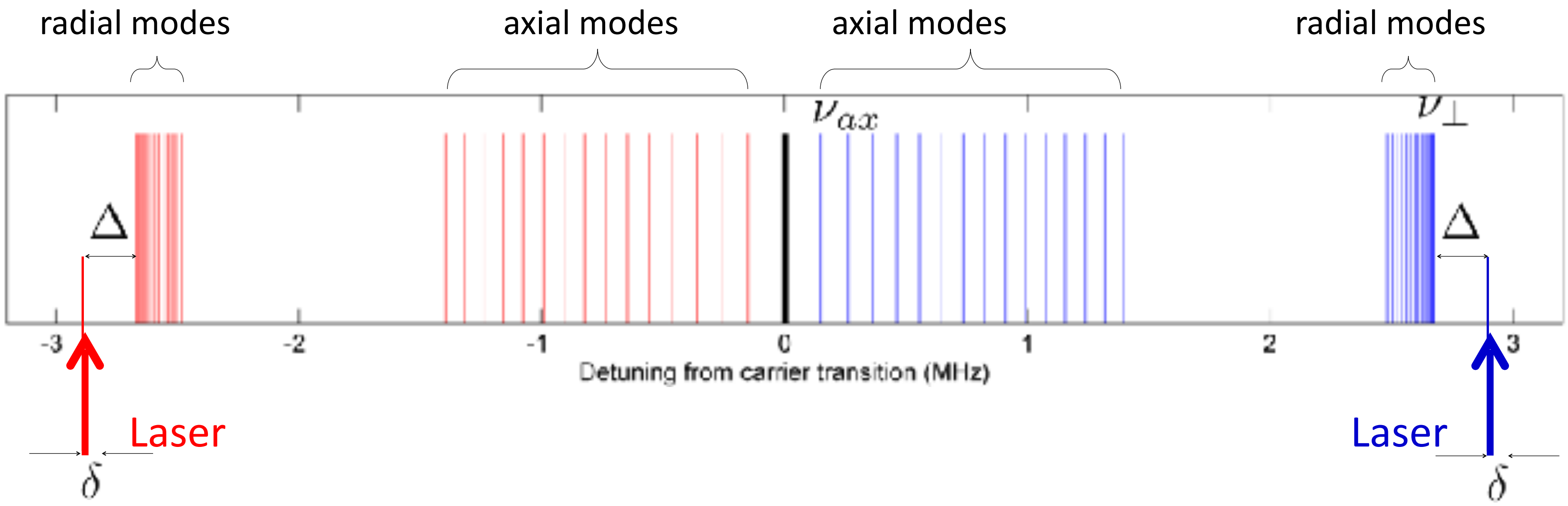
- laser detuning Δ
- spread of radial modes

slide credit: C. Roos

K. Kim et al, PRL 103, 120502 (2009)

J. Britton et al, Nature 484, 489 (2012)

Variable-range entangling interactions: XY model



$$H = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x + B\sum_i \sigma_i^z \qquad B = \delta/2$$

$$\approx \sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + h.c.) + B\sum_i \sigma_i^z \quad \text{for} \quad B \gg J$$

XY model: hopping of spin excitations

slide credit: C. Roos